**Chapter 1: Introduction**

**1.1 Problem Statement**

**The National Health Service (NHS) in the United Kingdom is confronted with persistent challenges in managing elective surgical procedures, where waiting lists have escalated to unprecedented levels in the wake of the COVID-19 pandemic. This issue is particularly evident within the Cambridgeshire and Peterborough Integrated Care Board (ICB), where cataract surgery—a high-volume elective procedure—illustrates systemic inefficiencies stemming from fragmented Patient Treatment Lists (PTLs). Under the current model, NHS hospitals and independent sector providers each maintain independent waiting lists, leading to disparities in resource utilisation and patient access. For instance, NHS providers, often teaching hospitals, frequently experience waiting times surpassing the NHS constitutional standard of 18 weeks for routine cases, while independent providers may have waits as low as two weeks, reflecting underutilised capacity.**

**The patient pathway for cataract surgery begins with an initial assessment by an optometrist, who diagnoses the need for surgery and refers the patient to a specific provider. This referral process can introduce biases, such as preferences influenced by commissions or incentives, although patients retain the ultimate right to select their provider. Once assigned, patients are added to that provider's PTL, with no mechanism for dynamic reallocation based on system-wide availability. Waiting times are calculated from the date of PTL addition to the treatment date, and cases are prioritised into three categories: Urgent (requiring immediate attention), Two-Week Wait (TWW, for potentially time-sensitive conditions), and Routine (standard elective cases).**

**Cataract procedures are categorised using Healthcare Resource Groups (HRGs), which account for procedural complexity and patient comorbidity scores (CC scores, a measure of additional health conditions impacting risk and duration). These HRGs range from least to most complex: (1) Minor Cataract or Lens Procedures (5–10 minutes operating time); (2) Intermediate Cataract or Lens Procedures with CC Score 0–1 (10–15 minutes); (3) Intermediate with CC Score 2+ (15–20 minutes); (4) Phacoemulsification Cataract Extraction and Lens Implant with CC Score 0–1 (≤10 minutes); (5) Phacoemulsification with CC Score 2–3 (10–15 minutes); (6) Phacoemulsification with CC Score 4+ (15–20+ minutes); (7) Complex Cataract or Lens Procedures with CC Score 2+ (20–30 minutes); (8) Very Major Cataract or Lens Procedures with CC Score 0–1 (30–40 minutes); and (9) Very Major with CC Score 2+ (40–60+ minutes). The ranking integrates technical demands (e.g., phacoemulsification involves ultrasonic lens fragmentation) and patient risk, with distributions showing tight clustering for low-complexity cases (low standard deviation) and broader tails for high-complexity ones (e.g., exceeding 60 minutes in 10–15% of cases).**

**Operating times vary significantly by provider type and surgeon experience. Analysis of 11,067 cases indicates mean durations of 19±10 minutes for consultants, 30±11 minutes for juniors, 27±12 minutes for intermediates, 24±10 minutes for senior trainees, and 31±11 minutes for fellows [1], [2]. NHS providers, as teaching institutions, handle a disproportionate share of complex cases (e.g., Phacoemulsification with CC Score 4+), resulting in longer times and lower throughput (approximately 16 procedures per day, structured as two 4-hour programmed activities with 8 procedures each). Independent providers, conversely, focus on routine cases, achieving up to 30 procedures per day due to streamlined operations and lower complexity.**

**Data from the ICB comprises two datasets: one spanning three financial years with treatment details (e.g., procedure type, provider, HRG, but lacking waiting times) and another for the latest year including actual waiting times, urgency levels, and patient demographics. Additional variables include Lower Layer Super Output Areas (LSOAs) for geographical recommendations and estimated capacities (surgeons, theatres, utilisation) derived from historical patterns and NHS-provided information on programmed activities. The proposed intervention consolidates PTLs into a single list, allocating patients to the provider with the shortest wait while evaluating a shift in triage from optometrists to a central NHS level to mitigate biases and enhance equity.**

**1.2 Significance and Context**

**Resolving PTL fragmentation has profound implications for NHS sustainability, patient outcomes, and resource efficiency. Surgical services constitute 10–30% of hospital budgets, and targeted optimisation could generate substantial savings while advancing equitable care [3]. In cataract surgery, untreated delays exacerbate vision impairment, affecting mobility, independence, and quality of life for over 2.5 million UK adults, with associated economic burdens exceeding £22 billion annually, including direct healthcare costs, social care, and lost productivity [4].**

**The NHS Long Term Plan (2019) and the 2025/26 Operational Planning Guidance prioritise elective recovery and integrated care systems (ICSs) to minimise variation and improve access [5], [6]. The Health and Care Act 2022 facilitates cross-provider collaboration, yet practical adoption of consolidated lists lags, despite evidence from OECD countries showing centralised systems reduce waits and costs [7]. Post-COVID backlogs have intensified the need, as non-critical surgeries were deferred, overloading existing capacity.**

**Contextually, NHS providers manage diverse complexities for training purposes, while independents prioritise efficiency in routine cases, leading to imbalances: NHS handles ~80% of high-complexity procedures, with independent providers executing simpler ones more rapidly. Geographical factors (via LSOAs) add layers, as allocations must balance wait times with travel feasibility.**

**Insights from analogous domains reinforce consolidation's viability. Airport systems employ centralised queueing to manage passenger flows, improving utilisation by 20–30% [8]. Traffic engineering uses network models to distribute loads, preventing congestion [9]. In healthcare, emergency department queueing has yielded 15–78% efficiency gains [10], [11]. For cataract services, simulation studies predict demand surges and identify triage as a key inefficiency [12]. This research adapts these principles to NHS realities, quantifying benefits like 15–40% utilisation increases and 25–40% wait reductions.**

**1.3 Motivation and Objectives**

**This study is driven by the imperative to bridge theoretical operational research with NHS practice, using ICB-specific data to evaluate PTL consolidation. Existing literature often emphasises single-site optimisation or abstract models, neglecting NHS nuances like provider heterogeneity, patient choice, and triage biases [3], [13]. The availability of detailed datasets, combined with methodologies from diverse fields, enables a comprehensive framework to inform policy.**

**Objectives include:**

1. **Modelling fragmented PTLs via queueing theory (M/M/c or M/G/c) to derive baseline metrics, incorporating Poisson arrivals, general service distributions, and multi-server capacities.**
2. **Simulating scenarios with DES (using SimPy) to capture stochastic elements like priorities, cancellations (95% utilisation cap), and triage shifts, validating against historical data.**
3. **Optimising allocation with RL, starting with CEM for baseline policy refinement and advancing to DQN for handling discrete actions (provider selection) and high-dimensional states (27 features, e.g., queues, capacities, HRGs).**
4. **Forecasting demand via time series (ARIMA, Prophet) to project patient loads, integrating with models for proactive planning [14], [15].**

**These aim to demonstrate tangible improvements, supporting NHS elective recovery goals.**

**1.4 Thesis Overview**

**Chapter 2 surveys literature and foundations. Chapter 3 outlines methodology. Chapter 4 details execution. Chapter 5 evaluates results. Chapter 6 summarises contributions and future work.**

**Chapter 2: Background**

**2.1 Problem-Specific Literature Review**

**Healthcare resource allocation research has increasingly focused on waiting list inefficiencies, particularly in elective care where fragmentation exacerbates disparities. Cardoen et al. (2010) synthesised operating room scheduling literature, identifying that siloed systems cause 10–30% underutilisation, with pooling and dynamic allocation offering remedies [3]. In emergency contexts, Green et al. (2006) used queueing to optimise staffing, reducing overcrowding through reallocation [10]. Cochran and Roche (2009) extended multi-class models to departments, improving flow by integrating acuity levels [16].**

**Surgical waiting lists have been addressed through simulation and optimisation. Beliën and Demeulemeester (2007) proposed cyclic scheduling, levelling bed occupancy by 15–20% via balanced allocation [13]. Santibañez et al. (2009) simulated cancer pathways, achieving 23% wait reductions and 16% utilisation gains via centralisation [11]. UK-focused studies like Harrison and Appleby (2009) analysed NHS data, linking 40% of wait variance to provider differences [17]. Monks et al. (2016) evaluated pooled orthopaedic lists, cutting maximum waits by 31% without added resources [18].**

**Cataract surgery literature emphasises standardisation and variability. Day et al. (2019) studied 72 trusts, attributing 28% productivity gaps to scheduling, with flexible systems boosting utilisation by 18% [19]. Hussain et al. (2019) applied DES to forecast demand, pinpointing triage bottlenecks [12]. Siciliani et al. (2013) compared OECD policies, showing centralised systems enhance efficiency and equity [7].**

**Machine learning integrations advance beyond traditional methods. Moosavi et al. (2023) fused forecasting and optimisation, yielding 10.81% performance boosts and 8.17% wait cuts with low error [20]. Zhang et al. (2018) used deep learning for flow prediction, outperforming classics by 15–20% [15]. RL applications include Yu and Liu (2020)'s survey of 127 studies, noting 15–25% gains in scheduling [21]. Liu et al. (2021) reduced surgical waits by 18% via RL [22].**

**Gaps include limited NHS-specific integrations of queueing, DES, and RL, often ignoring mixed providers, choice, and complexities. This study addresses these using ICB data.**

**2.2 Queueing Systems and Discrete Event Simulation Foundations**

**Queueing theory models waiting via Kendall's notation (A/B/c/k/m/z), focusing on arrival (A), service (B), servers (c), capacity (k), population (m), and discipline (z) [23], [24]. Healthcare arrivals are Poisson (M), with service variability necessitating general distributions (G). M/M/c assumes exponential times, but cataract data shows log-normal fits due to HRG complexities and surgeon levels [1], [2], favouring M/G/c.**

**Little's Law (L = λW) connects length (L), arrivals (λ), and wait (W) [25]. For M/G/1, Pollaczek-Khinchine gives W\_q = (λσ² + ρ²)/(2(1-ρ)μ), with variance σ² capturing 0.6–0.8 coefficients in providers. M/G/c uses approximations like Allen-Cunneen for multi-server waits. Stability requires ρ <1, challenging in NHS with backlogs.**

**DES simulates entities through events, handling stochasticity absent in analytics [26]. Brailsford et al. (2007) reviewed healthcare DES, noting superiority for constraints [27]. Jun et al. (1999) surveyed clinics, identifying bottlenecks [28]. Harper and Shahani (2002) modelled beds, improving capacities [29]. Günal and Pidd (2010) reviewed 182 studies for policy testing [30]. Landa et al. (2021) confirmed 15–40% gains with optimisation [31].**

**SimPy enables Python-based DES for cataract flows: stochastic arrivals (Poisson), services (log-normal per HRG), priorities, cancellations, and scenarios (fragmented/consolidated, triage shifts). Validation uses runs (10+), warm-ups, and queueing baselines. Hybrids like Cochran and Roche (2009) validate staffing [16].**

**2.3 Reinforcement Learning Foundations**

**RL solves MDPs with states, actions, transitions, and rewards [32]. Q-learning updates Q(s,a) = Q(s,a) + α[r + γ max\_{a'} Q(s',a') - Q(s,a)] [32]. For PTLs, states (27D: queues, capacities, HRGs, priorities), actions (6 providers), rewards (wait minimisation).**

**CEM refines policies stochastically. DQN approximates Q via networks, with replay and targets for stability [33]. It fits discrete actions and states, outperforming Q-learning (state explosion), SARSA (slow), PPO (expensive) [21].**

**Healthcare RL reduces waits by 18% [22], optimises sepsis [34]. Yu and Liu (2020) surveyed gains [21]. DES training enables safe learning, with constraints for ethics [35].**

**2.4 Synthesis and Research Gaps**

**Queueing provides baselines, DES realism, RL adaptivity, but NHS integrations are rare [20]. Gaps: empirical NHS validation, heterogeneity handling, multi-timescale planning (forecasting with ARIMA/Prophet [36], [37]). This study synthesises these, quantifying consolidation using data.**

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