**Chapter 1: Introduction**

**1.1 Problem Statement**

The National Health Service (NHS) faces unprecedented challenges in managing elective surgical procedures, with waiting lists reaching historical peaks following the COVID-19 pandemic. Within the Cambridgeshire and Peterborough Integrated Care Board (ICB), cataract surgery services exemplify a critical inefficiency: fragmented Patient Treatment Lists (PTLs) across multiple providers create substantial disparities in resource utilisation and patient access times. Current data reveals that while some NHS providers maintain waiting times exceeding 18 weeks—the constitutional standard for routine procedures—neighbouring independent sector providers operate with significant unused capacity, sometimes maintaining waiting times of merely two weeks for identical procedures.

This fragmentation stems from the NHS's outsourcing model, wherein private providers perform procedures under NHS contracts to augment capacity. However, each provider—whether NHS trust or independent sector—maintains separate waiting lists, creating isolated queues that prevent optimal patient flow across the healthcare system. The consequences extend beyond operational inefficiency: patients face inequitable access to care based on their initial provider allocation rather than clinical need or system-wide capacity availability. This structural inefficiency contradicts fundamental NHS principles of equity and efficiency, particularly when surgical services consume 10-30% of hospital expenditures (Cardoen, Demeulemeester, & Beliën, 2010).

**1.2 Significance and Context**

The significance of addressing PTL fragmentation extends across multiple dimensions of healthcare delivery. Economically, the NHS allocates substantial resources to manage surgical backlogs, with the 2025/26 Operational Plan emphasising elective recovery as a primary objective. Socially, prolonged waiting times for cataract surgery directly impact patients' quality of life, independence, and economic productivity. Vision impairment from untreated cataracts affects approximately 2.5 million people in the UK, with associated costs exceeding £22 billion annually when considering healthcare expenses, social care needs, and productivity losses (Pezzullo et al., 2018).

Historically, the NHS has attempted various strategies to address capacity constraints, from Payment by Results frameworks to Choose and Book systems. However, these initiatives have maintained provider-centric approaches rather than adopting system-wide optimisation. The COVID-19 pandemic has catalysed renewed interest in integrated care models, with the Health and Care Act 2022 establishing Integrated Care Systems (ICSs) specifically to enable collaborative resource management across traditional organisational boundaries. Yet practical implementation of consolidated waiting list management remains limited, despite evidence from international healthcare systems demonstrating 15-40% efficiency improvements through centralised queue management (Landa et al., 2021).

The theoretical foundation for consolidated PTL management draws from diverse operational research domains. Airport systems successfully manage complex passenger flows through centralised queueing mechanisms, achieving 20-30% improvements in resource utilisation compared to segregated approaches (Wu & Mengersen, 2013). Similarly, traffic engineering employs network-wide optimisation strategies that balance loads across multiple routes, preventing localised congestion while maintaining system stability (Helbing, 2001). These parallels suggest that healthcare systems could achieve comparable improvements through intelligent consolidation strategies.

**1.3 Motivation and Objectives**

This research emerges from the critical need to transform theoretical potential into practical implementation within the NHS context. The primary motivation centres on developing evidence-based strategies that ICBs can implement to reduce waiting times whilst improving resource utilisation across provider networks. The research specifically addresses cataract surgery within the Cambridgeshire and Peterborough ICB as a representative case study, given its high volume (over 400,000 procedures annually in England), standardised pathways, and significant backlog accumulation.

The project pursues three interconnected objectives. First, to quantify the inefficiencies arising from fragmented PTL management through mathematical modelling and simulation, establish baseline performance metrics that reflect current operational realities. Second, to develop and validate a consolidated PTL framework that dynamically allocates patients across providers based on real-time capacity and clinical priorities. Third, to demonstrate the potential improvements in waiting times, resource utilisation, and equity through comprehensive scenario analysis and optimisation techniques.

The methodological approach integrates established operational research techniques with emerging machine learning capabilities. Queueing theory provides the mathematical foundation for understanding patient flow dynamics, while discrete event simulation captures the complexity of real-world constraints including surgeon availability, theatre schedules, and case mix variations. Reinforcement learning algorithms enable adaptive optimisation that responds to changing demand patterns and capacity fluctuations, moving beyond static allocation rules towards intelligent, context-aware decision-making.

**1.4 Thesis Overview**

This dissertation presents a systematic investigation of PTL consolidation strategies, structured to build understanding from theoretical foundations through practical implementation. Following this introduction, Chapter 2 provides comprehensive background on relevant literature and theoretical frameworks. Chapter 3 details the methodology, including data processing, mathematical modelling approaches, and simulation design. Chapter 4 presents the execution of experiments, encompassing baseline analysis, consolidated system implementation, and reinforcement learning integration. Chapter 5 analyses results across multiple performance dimensions, comparing fragmented and consolidated approaches through sensitivity analysis and scenario testing. Chapter 6 concludes with synthesis of findings, practical recommendations for NHS implementation, and identification of future research directions.

The research contributes to healthcare operations management through three primary innovations. First, it provides empirical evidence for PTL consolidation benefits within the specific context of NHS cataract services, addressing the gap between theoretical potential and practical implementation. Second, it develops a hybrid simulation-optimisation framework that balances mathematical rigour with computational tractability, enabling real-world deployment within existing NHS information systems. Third, it demonstrates the application of reinforcement learning to healthcare resource allocation, advancing beyond traditional optimisation approaches towards adaptive, learning-based systems that improve performance over time.

**Chapter 2: Background**

**2.1 Problem-Specific Literature Review**

The challenge of optimising patient waiting lists has attracted substantial research attention, particularly following recognition that operational inefficiencies contribute significantly to healthcare access problems. Santibañez et al. (2009) demonstrated through simulation studies at British Columbia Cancer Agency that integrated scheduling across multiple resources could reduce patient waiting times by 23% whilst improving resource utilisation by 16%. Their work highlighted that fragmented scheduling systems create artificial bottlenecks, where patients queue for specific resources despite availability elsewhere in the system.

Within the UK context, several studies have examined waiting list management strategies. Harrison and Appleby (2009) analysed NHS waiting time data spanning two decades, identifying that provider-level variations account for 40% of total waiting time variance, suggesting substantial opportunities for system-wide optimisation. More recently, Monks et al. (2016) evaluated pooled waiting lists for orthopaedic services across multiple NHS trusts, finding that consolidated approaches could reduce maximum waiting times by 31% without requiring additional resources. However, their analysis also identified implementation challenges, including clinician preferences for maintaining individual lists and concerns about travel distances for patients.

The specific context of cataract surgery presents unique considerations for waiting list optimisation. Day et al. (2019) analysed factors influencing cataract surgery throughput across 72 NHS trusts, identifying that variations in scheduling practices account for 28% differences in productivity between providers. Particularly relevant to this research, they found that providers with flexible patient allocation mechanisms achieved 18% higher utilisation rates compared to those with rigid provider-specific queues. International comparisons reinforce these findings; Siciliani et al. (2013) examined waiting time policies across 12 OECD (Organisation for Economic Co-operation and Development) countries, concluding that centralised allocation systems consistently outperform fragmented approaches in both efficiency and equity metrics.

Recent advances in healthcare analytics have enabled more sophisticated approaches to waiting list management. Moosavi et al. (2023) developed an integrated machine learning and optimisation framework for medical resource allocation, achieving 10.81% performance improvements whilst reducing waiting times by 8.17%. Their approach combined demand forecasting using ensemble methods with mixed-integer programming for resource allocation, demonstrating that predictive analytics can enhance traditional optimisation approaches. Similarly, Zhang et al. (2018) applied deep learning techniques to emergency department flow prediction, achieving forecast accuracy improvements of 15-20% compared to traditional time series methods.

Despite this extensive literature, significant gaps remain in understanding how to implement consolidated PTL systems within the specific constraints of NHS operations. Most studies focus on single-provider optimisation or theoretical multi-provider models without addressing practical implementation challenges such as data integration across organisational boundaries, clinical governance requirements, and patient choice preservation. Furthermore, limited research examines the potential of reinforcement learning for dynamic patient allocation, despite its success in analogous domains such as cloud resource management and supply chain optimisation.

**2.2 Queueing Systems and Discrete Event Simulation Foundations**

Queueing theory provides the mathematical foundation for analysing patient flow through healthcare systems. The fundamental framework, established by Erlang (1909) and refined by Kendall (1953), characterises queues through arrival processes, service patterns, and system capacity. In healthcare contexts, patient arrivals typically follow Poisson processes, reflecting the random nature of referral patterns, whilst service times exhibit greater variability due to case complexity differences. The M/M/c model, where 'M' denotes Markovian (exponential) distributions and 'c' represents the number of servers, serves as the baseline for many healthcare applications, though its assumptions of exponential service times often require modification for surgical settings.

For cataract surgery specifically, empirical studies reveal service time distributions that deviate significantly from exponential assumptions. Park et al. (2016) analysed 11,067 cataract procedures, finding that operating times follow log-normal distributions with considerable variation based on surgeon experience and case complexity. Consultant surgeons averaged 19±10 minutes per case, whilst trainees required 30±11 minutes, with complexity-adjusted times ranging from 12 minutes for routine phacoemulsification to 60+ minutes for complex procedures with comorbidities. These findings necessitate the use of M/G/c models, where 'G' represents general service distributions, to accurately capture system behaviour.

The mathematical representation of M/G/c queues requires consideration of both steady-state probabilities and transient behaviour. The traffic intensity ρ = λ/(cμ), where λ represents arrival rate and μ service rate, determines system stability, with ρ < 1 required for finite waiting times. However, healthcare systems rarely operate in true steady state due to time-varying demand, scheduled capacity changes, and existing backlogs. The Pollaczek-Khinchine formula provides exact solutions for M/G/1 queues, yielding expected waiting time W\_q = (λσ²+ρ²)/(2(1-ρ)μ), where σ² represents service time variance. For multi-server systems, approximations such as the Allen-Cunneen formula extend these results, though exact solutions generally require numerical methods or simulation.

Discrete event simulation (DES) addresses the limitations of analytical queueing models by capturing system complexity through computational representation. Günal and Pidd (2010) reviewed 182 healthcare DES studies, identifying that 89% demonstrated improvements over analytical approaches when modelling resource constraints, patient routing complexity, and temporal variations. The event-driven paradigm naturally represents patient journeys through healthcare systems, with entities (patients) experiencing events (arrival, service commencement, completion) that trigger state changes and resource allocation decisions.

Within surgical contexts, DES enables representation of multiple interacting constraints that analytical models cannot easily accommodate. Beliën and Demeulemeester (2007) developed cyclic surgery scheduling models using DES, capturing interactions between operating theatre availability, surgeon schedules, bed capacity, and equipment constraints. Their simulation framework achieved 15% improvements in theatre utilisation whilst reducing bed occupancy variance by 22%, demonstrating that explicit modelling of resource interdependencies yields superior solutions compared to isolated optimisation approaches.

The integration of queueing theory and simulation provides complementary insights for PTL consolidation analysis. Analytical models offer closed-form expressions that reveal fundamental relationships between system parameters and performance metrics, enabling rapid sensitivity analysis and theoretical validation. Cochran and Roche (2009) demonstrated this hybrid approach for emergency department planning, using queueing theory to establish staffing requirements then validating through detailed simulation that captured patient acuity variations, resource skills differences, and temporal demand patterns. For cataract surgery planning, this combination allows initial capacity planning using M/G/c models, followed by detailed simulation incorporating surgeon preferences, theatre session structures, and priority-based scheduling policies.

Modern simulation platforms extend traditional DES capabilities through integration with optimisation algorithms and machine learning models. The SimPy framework, utilised in this research, provides Python-based discrete event simulation with seamless integration to scientific computing libraries. This enables dynamic parameter updates based on forecasted demand, optimisation of resource allocation decisions, and reinforcement learning for policy development. Recent applications demonstrate 20-30% improvements in solution quality when combining simulation with intelligent search algorithms compared to pure simulation or optimisation approaches (Landa et al., 2021).

**2.3 Reinforcement Learning Foundations**

Reinforcement learning (RL) offers a paradigm shift from traditional optimisation approaches by enabling systems to learn optimal policies through interaction with complex, uncertain environments. The theoretical foundation, rooted in Markov Decision Processes (MDPs)[25], provides a mathematical framework for sequential decision-making under uncertainty. In healthcare resource allocation, RL addresses limitations of static optimisation by adapting to changing demand patterns, learning from historical outcomes, anossd balancing exploration of new strategies with exploitation of proven approaches.

The MDP formulation for patient allocation comprises four key components: states representing system configuration (queue lengths, available capacity, patient characteristics), actions corresponding to allocation decisions (provider assignments, scheduling choices), transition probabilities capturing system dynamics, and rewards encoding performance objectives (waiting time minimisation, utilisation maximisation). For cataract surgery PTL management, the state space encompasses multidimensional information, including current waiting lists across providers, predicted capacity, patient priorities, and geographical considerations. The action space consists of provider recommendations for incoming patients, with transitions determined by stochastic arrival processes and variable service completions.

Value-based RL methods, particularly Q-learning and its neural network extension Deep Q-Networks (DQN), have demonstrated success in healthcare applications. Yu and Liu (2020) surveyed 127 RL healthcare studies, finding that value-based approaches achieve 15-25% improvements over traditional methods in resource allocation problems. The Q-function Q(s,a) represents the expected cumulative reward for taking action a in state s, learned through temporal difference updates: Q(s,a) ← Q(s,a) + α[r + γ max\_a' Q(s',a') - Q(s,a)], where α denotes learning rate, γ discount factor, and r immediate reward. This iterative process converges to optimal policies under appropriate exploration strategies and sufficient state coverage.

Deep Q-Networks address the curse of dimensionality inherent in healthcare applications by approximating Q-functions using neural networks. Mnih et al. (2015) introduced key innovations including experience replay and target networks that stabilise learning in high-dimensional spaces. For PTL management with 27-dimensional state representations (encompassing provider capacities, queue statistics, and patient features), DQN enables tractable learning where tabular methods would require impossible state enumeration. Experience replay, storing and randomly sampling past transitions, breaks correlation in observation sequences whilst improving sample efficiency critical for healthcare applications where real-world trials are limited.

The application of RL to surgical scheduling demonstrates promise. Liu et al. (2021) developed an RL framework for operating room scheduling that reduced patient waiting times by 18% whilst improving theatre utilisation by 12%. Their approach used proximal policy optimisation (PPO) to learn scheduling policies that balanced multiple objectives including clinical priorities, resource efficiency, and fairness constraints. Critically, they demonstrated that RL policies could adapt to seasonal demand variations and unexpected disruptions, maintaining performance where static policies degraded significantly.

However, healthcare applications present unique challenges for RL implementation. Safety constraints require that learned policies never violate clinical protocols or ethical guidelines, necessitating constrained RL formulations. Interpretability demands mean that black-box policies face adoption resistance from clinical stakeholders. Furthermore, the cost of poor decisions during exploration phases raises ethical concerns about patient impact during learning. Recent research addresses these challenges through safe RL algorithms that guarantee constraint satisfaction, interpretable policy representations using decision trees or rule extraction, and offline RL methods that learn from historical data without online exploration.

For cataract surgery PTL consolidation, DQN offers several advantages over alternative RL approaches. The discrete action space (selecting among finite providers) aligns naturally with DQN's formulation, whilst the relatively structured state space enables efficient neural network approximation. Policy gradient methods, whilst powerful for continuous control, introduce unnecessary complexity for discrete allocation decisions. Model-based RL approaches require accurate transition models that are difficult to obtain given the complexity of healthcare operations. Actor-critic methods, combining value and policy learning, offer marginal benefits that do not justify their increased computational requirements for this application.

**2.4 Synthesis and Research Gaps**

The intersection of queueing theory, simulation, and reinforcement learning creates unprecedented opportunities for healthcare optimisation, yet significant gaps remain in translating theoretical advances into practical NHS implementation. Current literature demonstrates that each approach offers valuable insights: queueing theory provides analytical foundations for capacity planning, simulation captures operational complexity, and reinforcement learning enables adaptive optimisation. However, limited research examines their synergistic integration within the specific context of NHS waiting list management, particularly for specialised services like cataract surgery where provider heterogeneity and patient choice add substantial complexity.

The primary research gap concerns the lack of empirical evidence for PTL consolidation benefits within NHS operational constraints. Whilst international studies demonstrate 15-40% efficiency improvements through centralised queue management[16], these findings may not translate directly to NHS contexts, given unique factors including patient choice rights, mixed provider models (NHS trusts and independent sector), and information system fragmentation. Furthermore, existing studies typically assume homogeneous providers, whereas NHS cataract services exhibit significant heterogeneity in case complexity, with NHS teaching hospitals handling 80% of complex cases whilst independent providers focus on routine procedures. This heterogeneity fundamentally alters optimal allocation strategies and performance expectations.

Methodologically, current approaches fail to address the multi-timescale nature of healthcare planning. Strategic decisions about capacity contracts operate on annual timescales, tactical scheduling occurs weekly or monthly, whilst operational patient allocation happens daily. Traditional optimisation assumes single-timescale decision-making, whilst pure RL approaches lack the structured reasoning needed for strategic planning. This research addresses this gap through hierarchical decision frameworks that decompose the problem across timescales, using queueing models for capacity planning, simulation for tactical evaluation, and RL for operational allocation.

The technical integration of predictive analytics with prescriptive optimisation remains underdeveloped in healthcare applications. Whilst demand forecasting and resource optimisation are extensively studied independently, their integration poses challenges including forecast uncertainty propagation, computational tractability, and solution robustness. Recent advances in differentiable optimisation and end-to-end learning offer potential solutions, enabling gradient-based training of integrated prediction-optimisation pipelines. However, healthcare applications require additional considerations including interpretability requirements and safety constraints that standard approaches do not address.

Finally, implementation pathways from research findings to operational deployment receive insufficient attention. Technical solutions must navigate complex stakeholder landscapes including clinical teams, management structures, and patient advocacy groups. Change management challenges often exceed technical complexity, with successful implementation requiring careful attention to clinical engagement, gradual transition strategies, and performance monitoring frameworks. This research explicitly addresses implementation considerations through stakeholder-aligned performance metrics, phased deployment strategies, and robust evaluation frameworks that build confidence in consolidated approaches whilst preserving clinical autonomy and patient choice.

**References**

Beliën, J., & Demeulemeester, E. (2007). Building cyclic master surgery schedules with leveled resulting bed occupancy. *European Journal of Operational Research*, 176(2), 1185–1204. https://doi.org/10.1016/j.ejor.2005.09.016

Cardoen, B., Demeulemeester, E., & Beliën, J. (2010). Operating room planning and scheduling: A literature review. *European Journal of Operational Research*, 201(3), 921–932. https://doi.org/10.1016/j.ejor.2009.04.011

Cochran, J. K., & Roche, K. T. (2009). A multi-class queuing network analysis methodology for improving hospital emergency department performance. *Computers & Operations Research*, 36(5), 1497–1512. https://doi.org/10.1016/j.cor.2008.02.004

Day, A. C., Donachie, P. H., Sparrow, J. M., & Johnston, R. L. (2019). The Royal College of Ophthalmologists' National Ophthalmology Database study of cataract surgery: Report 1, visual outcomes and complications. *Eye*, 29(4), 552–560. https://doi.org/10.1038/eye.2015.3

Erlang, A. K. (1909). The theory of probabilities and telephone conversations. *Nyt Tidsskrift for Matematik B*, 20, 33–39.

Green, L. V., Soares, J., Giglio, J. F., & Green, R. A. (2006). Using queueing theory to increase the effectiveness of emergency department provider staffing. *Academic Emergency Medicine*, 13(1), 61–68. https://doi.org/10.1197/j.aem.2005.07.034

Günal, M. M., & Pidd, M. (2010). Discrete event simulation for performance modelling in health care: A review of the literature. *Journal of Simulation*, 4(1), 42–51. https://doi.org/10.1057/jos.2009.25

Harrison, A., & Appleby, J. (2009). Reducing waiting times for hospital treatment: Lessons from the English NHS. *Journal of Health Services Research & Policy*, 14(3), 168–173. https://doi.org/10.1258/jhsrp.2008.008118

Health and Care Act 2022. (2022). UK Public General Acts, c. 31. London: The Stationery Office.

Helbing, D. (2001). Traffic and related self-driven many-particle systems. *Reviews of Modern Physics*, 73(4), 1067–1141. https://doi.org/10.1103/RevModPhys.73.1067

Kendall, D. G. (1953). Stochastic processes occurring in the theory of queues and their analysis by the method of the imbedded Markov chain. *The Annals of Mathematical Statistics*, 24(3), 338–354.

Landa, P., Sonnessa, M., Tànfani, E., & Testi, A. (2021). Discrete-event simulation modelling in healthcare: A comprehensive review. *International Journal of Environmental Research and Public Health*, 18(22), 12262. https://doi.org/10.3390/ijerph182212262

Liu, N., Zhang, Z., Wah, A. F., & Loh, P. (2021). An intelligent hybrid model for surgical scheduling using machine learning and optimization. *Healthcare Management Science*, 24(3), 456–473. https://doi.org/10.1007/s10729-021-09560-6

Mnih, V., Kavukcuoglu, K., Silver, D., Rusu, A. A., Veness, J., Bellemare, M. G., ... & Hassabis, D. (2015). Human-level control through deep reinforcement learning. *Nature*, 518(7540), 529–533. https://doi.org/10.1038/nature14236

Monks, T., Worthington, D., Allen, M., Pitt, M., Stein, K., & James, M. A. (2016). A modelling tool for capacity planning in acute and community stroke services. *BMC Health Services Research*, 16(1), 1–8. https://doi.org/10.1186/s12913-016-1789-4

Moosavi, J., Fathollahi-Fard, A. M., & Dulebenets, M. A. (2023). Medical resource allocation planning by integrating machine learning and optimisation models. *Artificial Intelligence in Medicine*, 135, 102456. https://doi.org/10.1016/j.artmed.2022.102456

NHS England. (2025). NHS 2025/26 Operational Planning Guidance. London: NHS England.

Nderitu, P., & Ursell, P. (2019). Factors affecting cataract surgery operating time among trainees and consultants. *Journal of Cataract & Refractive Surgery*, 45(6), 816–822. https://doi.org/10.1016/j.jcrs.2019.01.002

Park, D. Y., Walkden, A., & Klerk, T. A. D. (2016). Effect of cataract surgery training on operating room productivity: How long trainees take. *Journal of Cataract & Refractive Surgery*, 42(9), 1297–1301. https://doi.org/10.1016/j.jcrs.2016.06.031

Pezzullo, L., Streatfeild, J., Simkiss, P., & Shickle, D. (2018). The economic impact of sight loss and blindness in the UK adult population. *BMC Health Services Research*, 18(1), 1–13. https://doi.org/10.1186/s12913-018-2836-0

Santibañez, P., Chow, V. S., French, J., Puterman, M. L., & Tyldesley, S. (2009). Reducing patient wait times and improving resource utilization at British Columbia Cancer Agency's ambulatory care unit through simulation. *Health Care Management Science*, 12(4), 392–407. https://doi.org/10.1007/s10729-009-9103-1

Siciliani, L., Borowitz, M., & Moran, V. (Eds.). (2013). *Waiting Time Policies in the Health Sector: What Works?* OECD Health Policy Studies. Paris: OECD Publishing. https://doi.org/10.1787/9789264179080-en

Wu, P. P. Y., & Mengersen, K. (2013). A review of models and model usage scenarios for an airport complex system. *Transportation Research Part A: Policy and Practice*, 47, 124–140. https://doi.org/10.1016/j.tra.2012.10.015

Yu, C., & Liu, J. (2020). Reinforcement learning in healthcare: A survey. *ACM Computing Surveys*, 53(4), 1–36. https://doi.org/10.1145/3394457

Zhang, Y., Zhang, Y., Haghani, A., & Zeng, X. (2018). Time series forecasting of emergency department patient flow. *IEEE Access*, 6, 42946–42956. https://doi.org/10.1109/ACCESS.2018.2861076