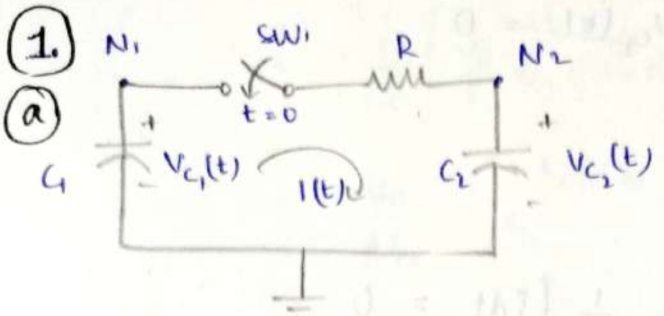
AEC Assignment - 2

1. (a)



Given, $V_{c_1}(0^-) = V_0$

$V_{c_2}(0^-) = 0.$

Let us come up with general conditions,

$i(t) \rightarrow \quad t = 0^{(-)} \rightarrow 0$

$t = 0^{(+)} \rightarrow \frac{V_0}{R}$

$t = \infty \rightarrow 0$

$V_{c_1}(t) \rightarrow \quad t = 0^{(-)} \longrightarrow V_0$

$t = 0^{(+)} \longrightarrow V_0$

$t = \infty \longrightarrow \frac{C_1}{C_1 + C_2} V_0$

$V_{c_2}(t) \rightarrow \quad t = 0^{(-)} \longrightarrow 0$

$t = 0^{(+)} \longrightarrow 0$

$t = \infty \longrightarrow \frac{C_1}{C_1 + C_2} V_0$

At t = infinity

charge must be conserved,

$$C_1 V_0 + C_2(0) = (C_1 + C_2) V_\infty$$

$$V_\infty = \frac{C_1}{C_1 + C_2} V_0$$

Now, apply KVL in the loop.

$$V_{c_1}(t) - IR - V_{c_2}(t) = 0$$

w.r.t $V_c = \frac{1}{c}\int i\,dt$

$$\Rightarrow -\frac{1}{C_1}\int I\,dt - IR - \frac{1}{C_2}\int i\,dt = 0$$

differentiate on both sides.

$$-I\left(\frac{1}{C_1}+\frac{1}{C_2}\right) = R\frac{dI}{dt}$$

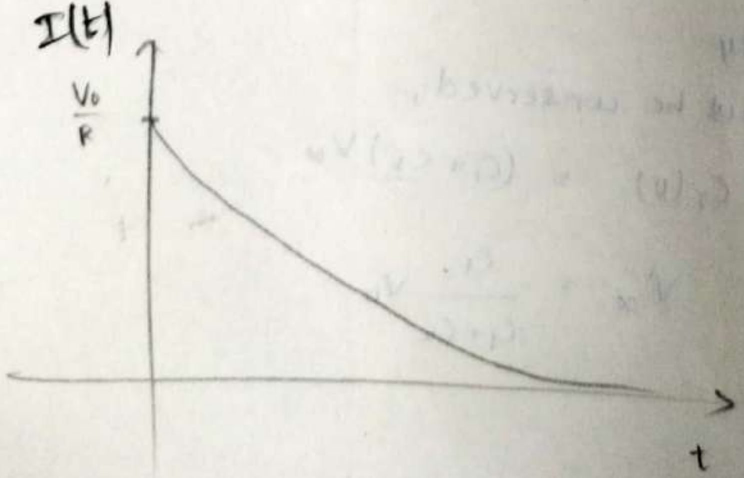
$$\frac{I}{C_{eq}} = -R\frac{dI}{dt}$$

$$\int_0^t \frac{dt}{-RC_{eq}} = \int_{I_0}^{I(t)} \frac{dI}{I}$$

$$\ln\frac{I(t)}{I_0} = -\frac{t}{RC_{eq}}$$

$$\therefore \boxed{I(t) = I_0 e^{-t/RC_{eq}}}$$

where, $I_0 = \frac{V_0}{R}$; $C_{eq} = \frac{C_1C_2}{C_1+C_2}$



$$V_{C_1}(t) = \frac{-1}{C_1} \int I(t)\, dt$$

$$= \frac{-1}{C_1} \int \frac{V_0}{R} e^{-t/RC_{eq}}\, dt$$

$$= +\frac{V_0}{RC_1} \left[\frac{e^{-t/RC_{eq}}}{-\frac{1}{RC_{eq}}} \right]_0^t + k$$

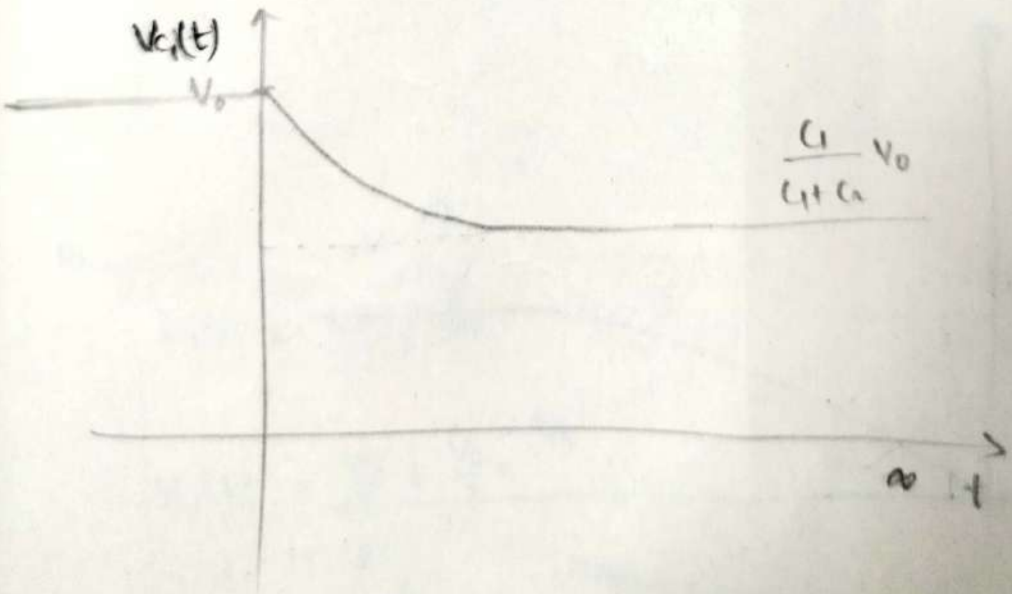
$$= \frac{V_0 C_2}{C_1 + C_2} \left[e^{-t/RC_{eq}} \right] + k$$

At $t = 0$

$$V_0 = \frac{V_0 C_2}{C_1 + C_2} + k$$

$$k = \frac{C_1 V_0}{C_1 + C_2}$$

$$\therefore \boxed{V_{C_1}(t) = \frac{C_1}{C_1 + C_2} V_0 + \frac{C_2}{C_1 + C_2} V_0 e^{-t/\tau}}$$



$$V_{C_2}(t) = \frac{1}{C_2}\int I(t)\,dt$$

$$= \frac{1}{C_2}\int \frac{V_0}{R}\, e^{-t/RC_{eq}}\, dt$$

$$= \frac{V_0}{C_2 R}\left[\frac{e^{-t/RC_{eq}}}{\frac{-1}{RC_{eq}}}\right] + k$$

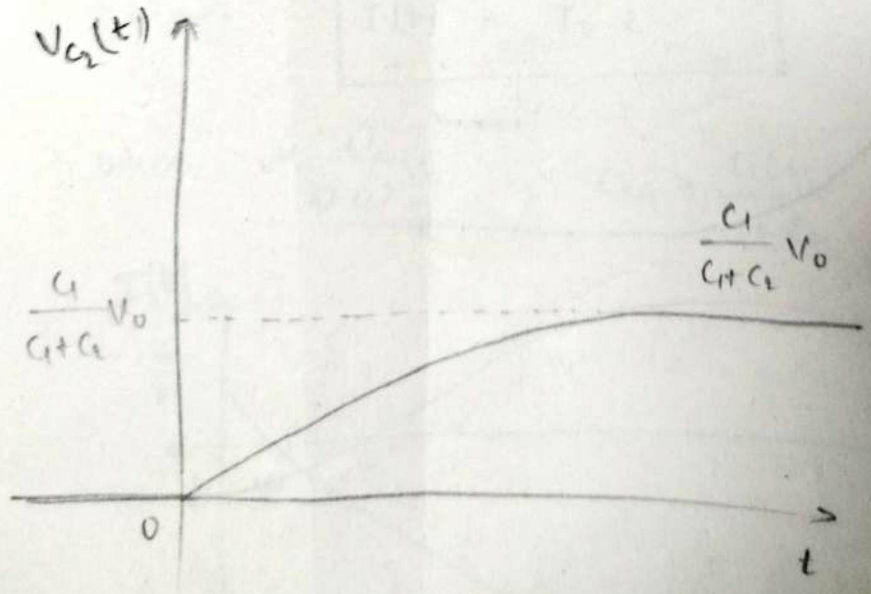
$$= \frac{-C_1 V_0}{C_1 + C_2}\left[e^{-t/RC_{eq}}\right] + k$$

at $t = 0$

$$0 = \frac{-C_1 V_0}{C_1 + C_2} + k$$

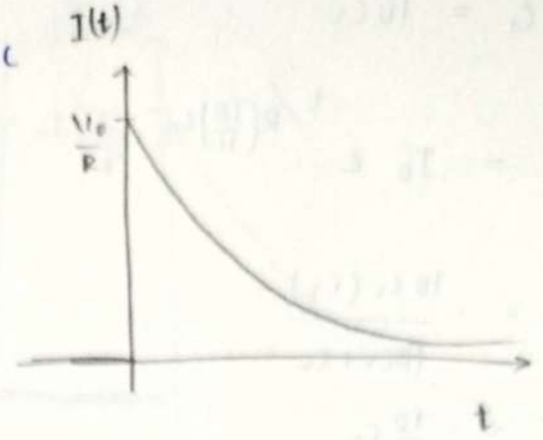
$$k = +\frac{C_1 V_0}{C_1 + C_2}$$

$$\Rightarrow \boxed{V_{C_2}(t) = \frac{C_1}{C_1 + C_2} V_0 \left[1 - e^{-t/RC_{eq}}\right]}$$



(ii) $C_1 = C_2$

$$I(t) = I_0 \, e^{-t/2RC}$$



$$C_{eq} = \frac{C \times C}{C + C} = C/2$$

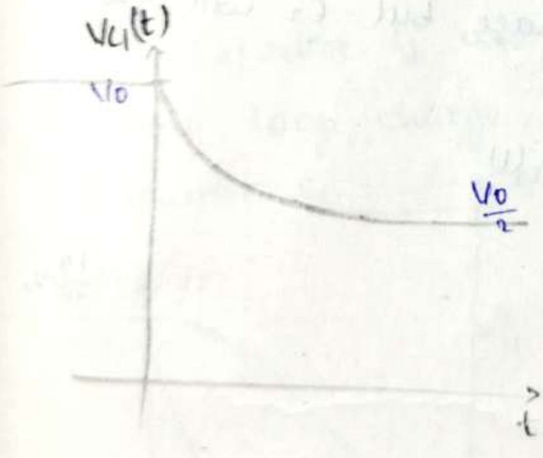
$V_{C_1}(t)$

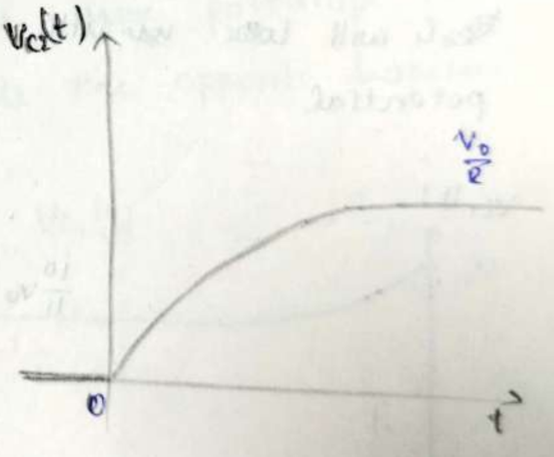
$$V_\infty = \frac{C_1}{C_1 + C_2} V_0 = \frac{V_0}{2}$$

$V_{C_2}(t)$

$$V_\infty = \frac{C_1}{C_1 + C_2} V_0 = \frac{V_0}{2}$$

Since, the value of C is equal, the ~~charge~~ voltage gets equally divided when capacitor is fully charged.



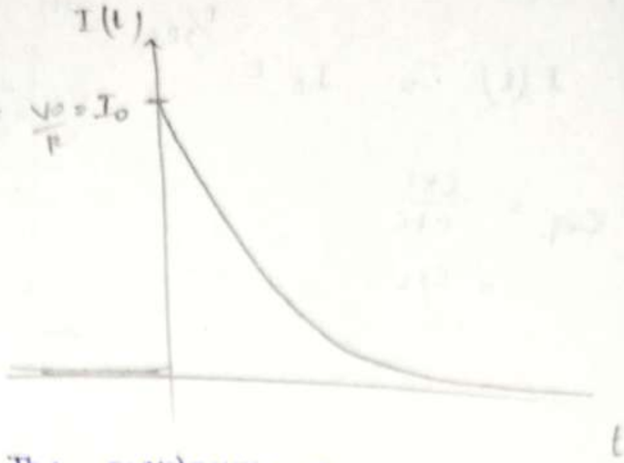


Here,

$$\boxed{\begin{aligned} I(t) &= \frac{V_0}{R} e^{-\frac{2t}{RC}} \\ V_{C_1}(t) &= \frac{V_0}{2} + \frac{V_0}{2} e^{-t/\tau} \\ V_{C_2}(t) &= \frac{V_0}{2}\left[1 - e^{-2t/RC}\right] \end{aligned}}$$

(ii) $C_1 = 10C_2$

$$I(t) = I_0 e^{-t/R\left(\frac{10}{11}\right)C_2}$$



$$C_{eq} = \frac{10C_2(C_2)}{10C_2 + C_2}$$

$$= \frac{10}{11} C_2$$

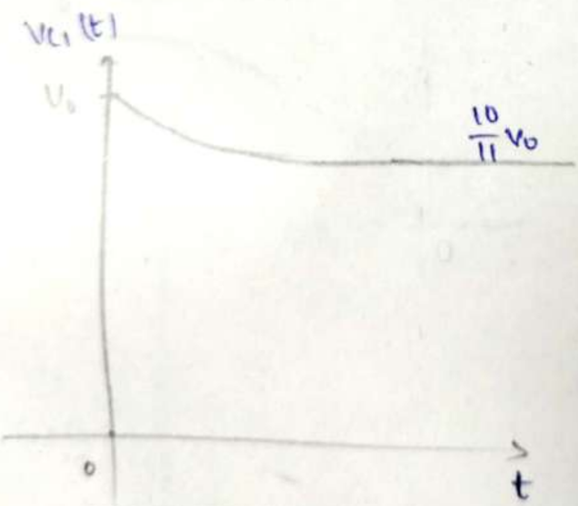
The maximum & min value of I(t) remain same, but the curve might be bend.

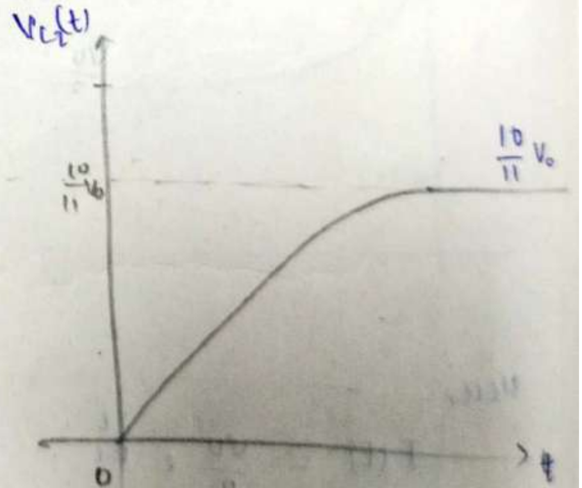
wk.t

$$V_\infty = \frac{C_1}{C_1 + C_2} V_0 \qquad V_\infty = \frac{10}{11} V_0$$

As $C_1 > C_2$, the V_∞ will be almost to V_0

~~so~~ C_1 will loose ~~very~~ little voltage but C_2 will gain potential





Here,

$$\boxed{\begin{aligned} I(t) &= \frac{V_0}{R} e^{-\frac{11t}{10RC_2}} \\ V_{C_1}(t) &= \frac{10}{11} V_0 + \frac{V_0}{11} e^{-\frac{11t}{10RC_2}} \\ V_{C_2}(t) &= \frac{10}{11} V_0 \left[1 - e^{-\frac{11t}{10RC_2}}\right. \end{aligned}}$$

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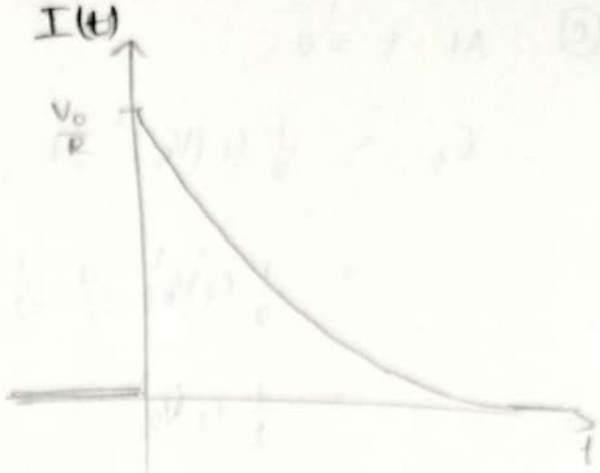
(iii) $C_1 = \frac{1}{10} C_2$

$$I(t) = I_0 e^{-t/R\left(\frac{10}{11}\right)C_1}$$

$$C_{eq} = \frac{(C_1)(10C_1)}{11C_1} = \frac{10}{11} C_1$$



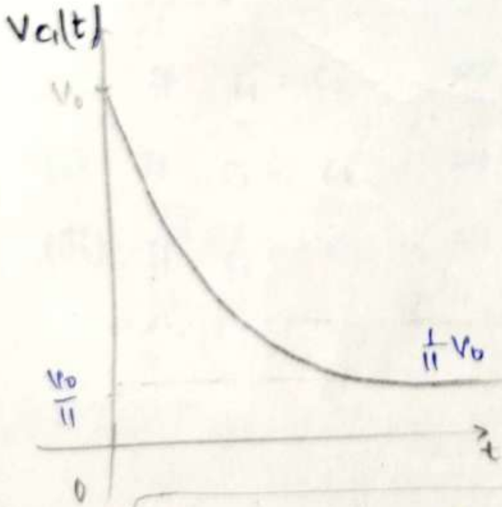
The max & min values of I(t) are same but the slope of curve changes acc to C values.

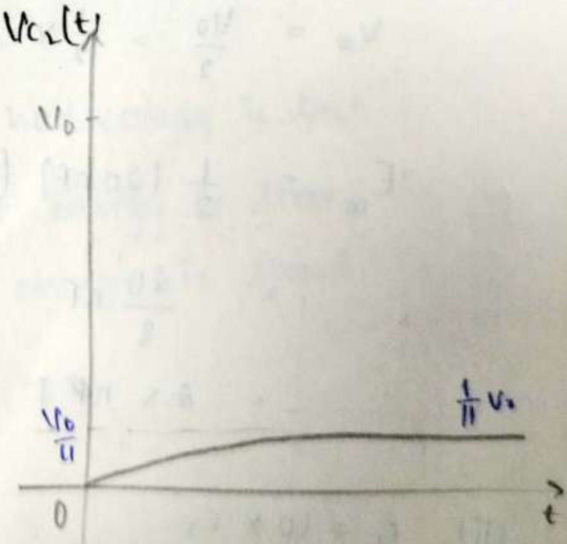
w.k.t

$$V_\infty = \frac{C_1}{C_1 + C_2} V_0 \qquad V_\infty = \frac{1}{11} V_0$$

As $C_1 < C_2$, the V_∞ will be closer to 0.

i.e Capacitor C_1 will lose more potential i.e gain large charge, while the opposite happens capacitor C_1





Here,

$$\boxed{\begin{aligned} I(t) &= \frac{V_0}{R} e^{-\frac{11t}{10RC_1}} \\ V_{C_1}(t) &= \frac{V_0}{11} + \frac{10}{11} V_0 e^{-\frac{11t}{10RC_1}} \\ V_{C_2}(t) &= \frac{V_0}{11}\left[1 - e^{-\frac{11t}{10RC_1}}\right] \end{aligned}}$$

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(b) Lt spice commands

$$.model\ SW1\ SW1\ (Ron = 1m\ \ Roff = 1Meg\ \ Vt = .5\ \ Vh = 0)$$

here SW — name of the switch.

Ron → this is the resistance when switch is closed, ~~its value~~,
- its value must be very low for current to completely flow.

R_{off} → resistance when switch is open,
- its value must be high so no current passes.

V_t → min threshold voltage for switch to change

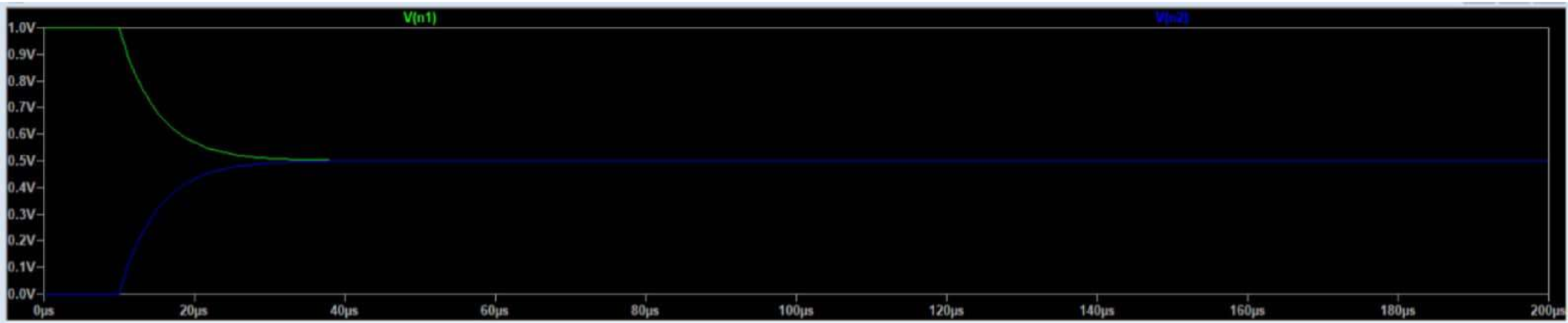
V_h → =0 show purely dependent circuit.

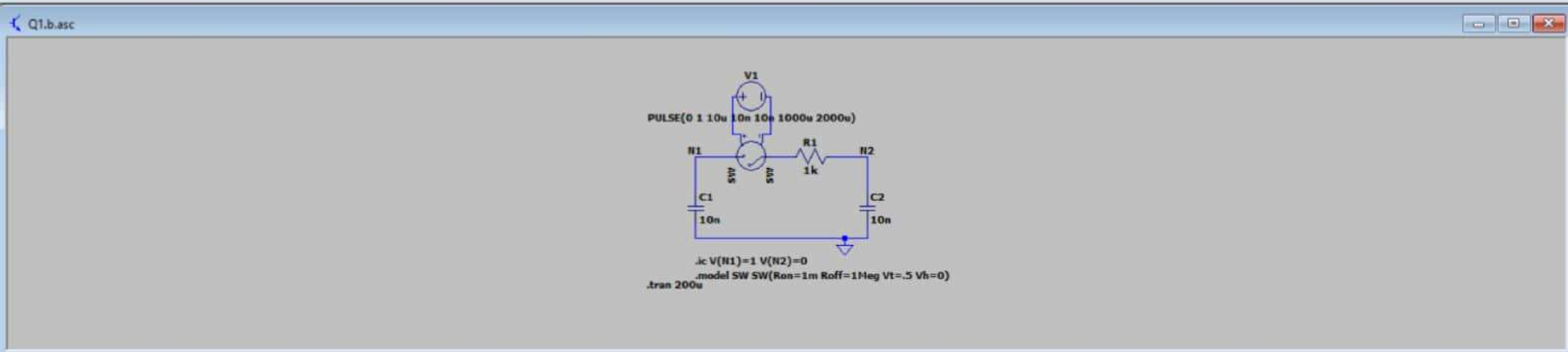
Hence, this explains the mechanism in which switch will be working, basing on the ~~i/p~~ voltage inflow

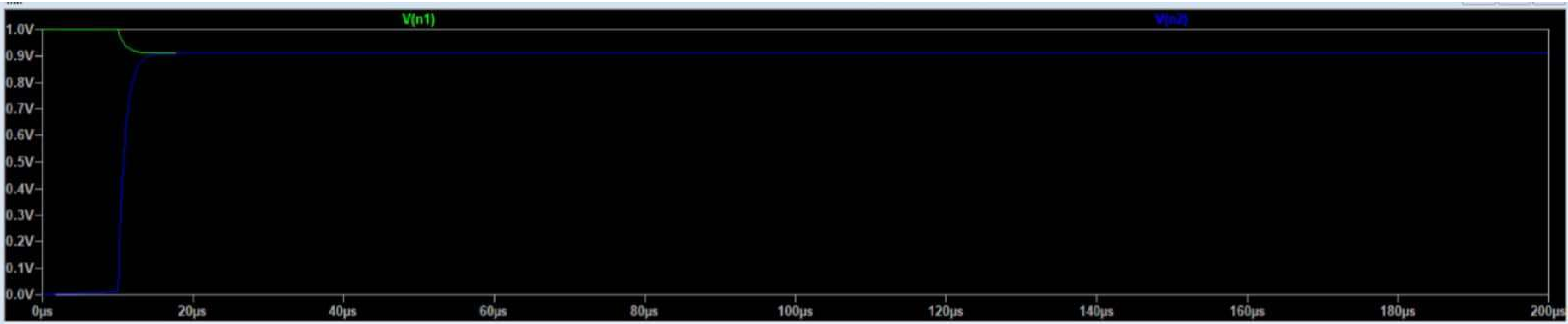
PULSE (0 1 10u 10n 10n 1000u 2000u)

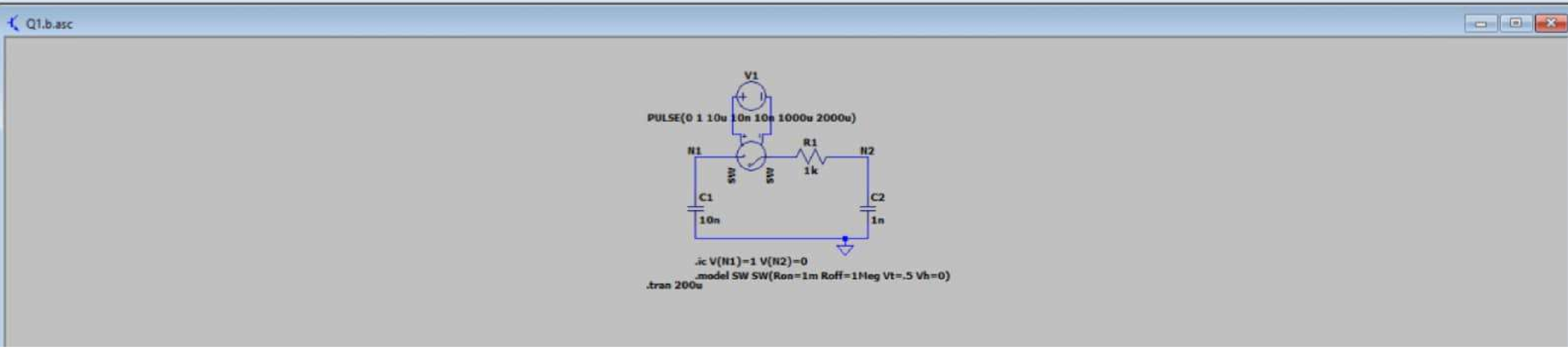
- 0 → $V_{initial}$
- 1 → V_{final}
- 10u → initial delay
- 10n → rise time
- 10n → fall time
- 1000u → width of pulse ⇓ during this span the pulse will be at V_{final}
- 2000u → Time period.

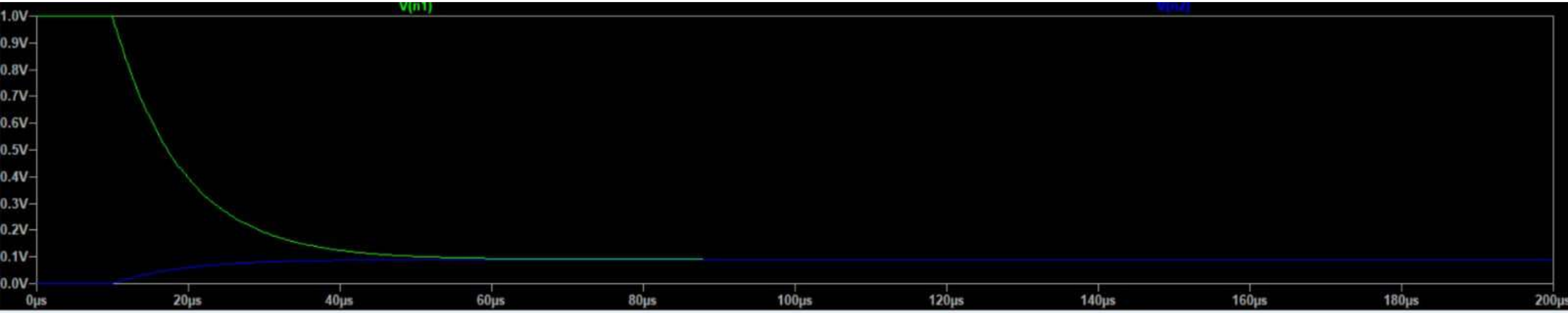
With these values, we can generate a pulse of voltage input that keeps changing periodically.

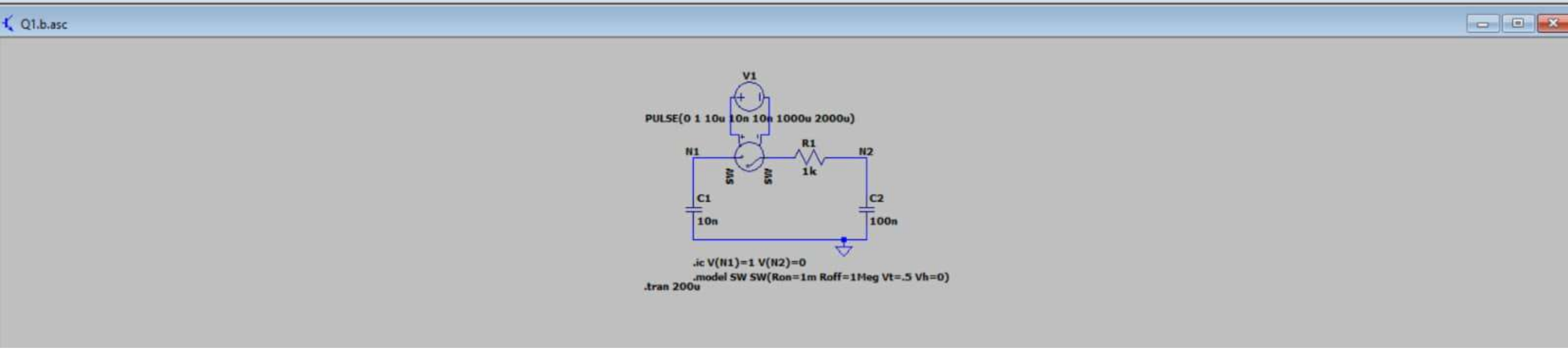

V(n1)
V(n2)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
0µs
20µs
40µs
60µs
80µs
100µs
120µs
140µs
160µs
180µs
200µs


Q1.b.asc
V1
PULSE(0 1 10u 10n 10n 1000u 2000u)
R1
N1
N2
1k
SW
SW
C1
C2
10n
10n
.ic V(N1)=1 V(N2)=0
.model SW SW(Ron=1m Roff=1Meg Vt=.5 Vh=0)
.tran 200u


V(n1)
V(n2)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
0µs
20µs
40µs
60µs
80µs
100µs
120µs
140µs
160µs
180µs
200µs


Q1.b.asc
V1
PULSE(0 1 10u 10n 10n 1000u 2000u)
N1
R1
1k
N2
SW
SW
C1
10n
C2
1n
.ic V(N1)=1 V(N2)=0
.model SW SW(Ron=1m Roff=1Meg Vt=.5 Vh=0)
.tran 200u


V(n1)
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
0µs
20µs
40µs
60µs
80µs
100µs
120µs
140µs
160µs
180µs
200µs


Q1.b.asc
V1
PULSE(0 1 10u 10n 10n 1000u 2000u)
R1
N1
N2
1k
C1
10n
C2
100n
.ic V(N1)=1 V(N2)=0
.model SW SW(Ron=1m Roff=1Meg Vt=.5 Vh=0)
.tran 200u

(c) At $t = 0$

$$E_0 = \frac{1}{2} C_1 (V_{C_1})^2 + \frac{1}{2} C_2 \left(V_{C_2}(t=0)\right)^2$$

$$= \frac{1}{2} C_1 V_0^2 + \frac{1}{2} C_2 (0)^2$$

$$= \frac{1}{2} C_1 V_0^2$$

$$E_\infty = \frac{1}{2} C_1 (V_\infty)^2 + \frac{1}{2} C_2 (V_\infty)^2$$

$$= \frac{1}{2} (C_1 + C_2) (V_\infty)^2 = \frac{C_1}{C_1 + C_2} (E_0)$$

From part (b)

$C_1 = 10nF$

$V_0 = 1V$

$$V_\infty = \frac{C_1}{C_1 + C_2} V_0$$

(i) $C_1 = C_2 = 10nF$

$$V_\infty = \frac{V_0}{2} = \frac{1}{2} V$$

$$E_\infty = \frac{1}{2} (20nF) \left(\frac{1}{2}\right)^2$$

$$= \frac{20}{8} nF$$

$$= 2.5 \text{ nJ}$$

(ii) $C_1 = 10 \times C_2$

$$C_2 = \frac{10n}{10} = 1nF$$

$V_\infty = 0.9090 \text{ V}$

$$E_\infty = \frac{1}{2} (10n + 1n) (0.909)^2 = \frac{1}{2} (11)(0.826)\, n$$

$$= 4.54 \text{ nJ}$$

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(iii) $C_1 = \frac{1}{10} C_2$

$C_2 = 100\,nF$

$V_\infty = 0.0909\ V$

$E_\infty = \frac{1}{2}(10+100)n \times (0.09)^2$

$= 0.45\ nJ$

Relation of C_1 & C_2	E_∞	E_∞ in terms of E_0
(i) $C_1 = C_2$	$2.5\ nJ$	$E_0/2$
(ii) $C_1 = 10 \times C_2$	$4.54\ nJ$	$\frac{10}{11} E_0$
(iii) $C_1 = \frac{C_2}{10}$	$0.45\ nJ$	$\frac{1}{11} E_0$

Conclusions :

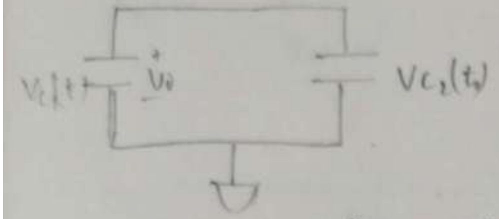
(i) if $C_1 = C_2$ $\Rightarrow$ only half energy is lost

(ii) if $C_1 >> C_2$ $\Rightarrow$ most energy is stored

(iii) if $C_1 << C_2$ $\Rightarrow$ less energy is stored

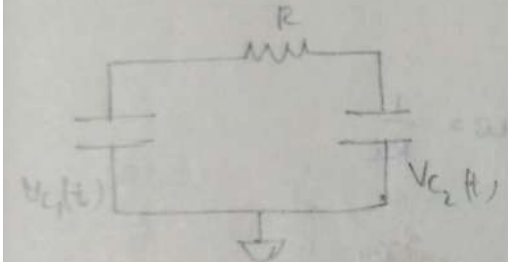
(c) Two capacitor Paradox

Generally, when two capacitors are connected as shown below and when they reach their steady state, the final energy isn't conserved



i.e $E_{initial} > E_{\infty}$

The best possible sol:
However, this ~~ea~~ lost energy can be shown to be present by placing a resistor between them,



i.e, the resistor dissipates that rest energy.

$$E_{initial} = \frac{1}{2} C_1 V_{C_1}^2 + \frac{1}{2} C_2 V_{C_2}^2$$

$$= \frac{1}{2} C_1 V_0^2$$

$$E_{\infty} = \frac{1}{2} C_1 V_{\infty}^2 + \frac{1}{2} C_2 V_{\infty}^2$$

$$= \frac{1}{2} (C_1 + C_2) V_{\infty}^2$$

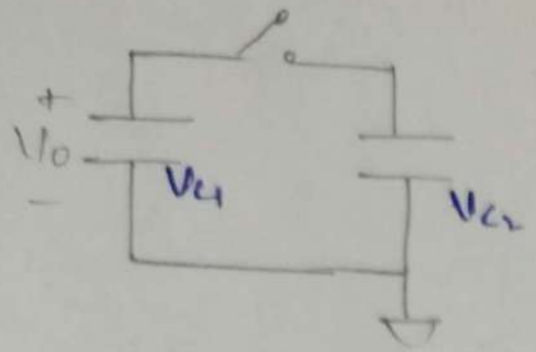
$$= \frac{1}{2} (C_1 + C_2) \left(\frac{C_1 V_0}{C_1 + C_2} \right)^2$$

$$= \frac{1}{2} C_1 V_0^2 \left(\frac{C_1}{C_1 + C_2} \right)$$

$$= E_0 \left(\frac{C_1}{C_1 + C_2} \right)$$

$$E_{dissipated\ by\ R} = \int i^2(t) R\, dt = \frac{C_2}{C_1 + C_2} E_0.$$

But for this case ~~when~~



The initial and final energy is lost similar to as where resistor is placed.

In that case the difference can be explained as the heat dissipated by the resistor.

However, in this case,
as the energy isn't conserved, the most probable case is that the resistance in the connecting wires can dissipate that ~~heat~~ energy in the form of heat / radiation.

(d) Effect of reducing R on settling time.

When there is a change in R, the time constant is differed and hence, settling time is altered.

$\tau = RC$

if $R\downarrow \Rightarrow \tau = \downarrow$

$\Rightarrow$ if time constant is $\downarrow$, then steady state is achieved faster.

$\therefore$ Reducing the value of R decreases the settling time and steady state is obtained faster.

Given, — $C_1 = C_2 = 10nF$

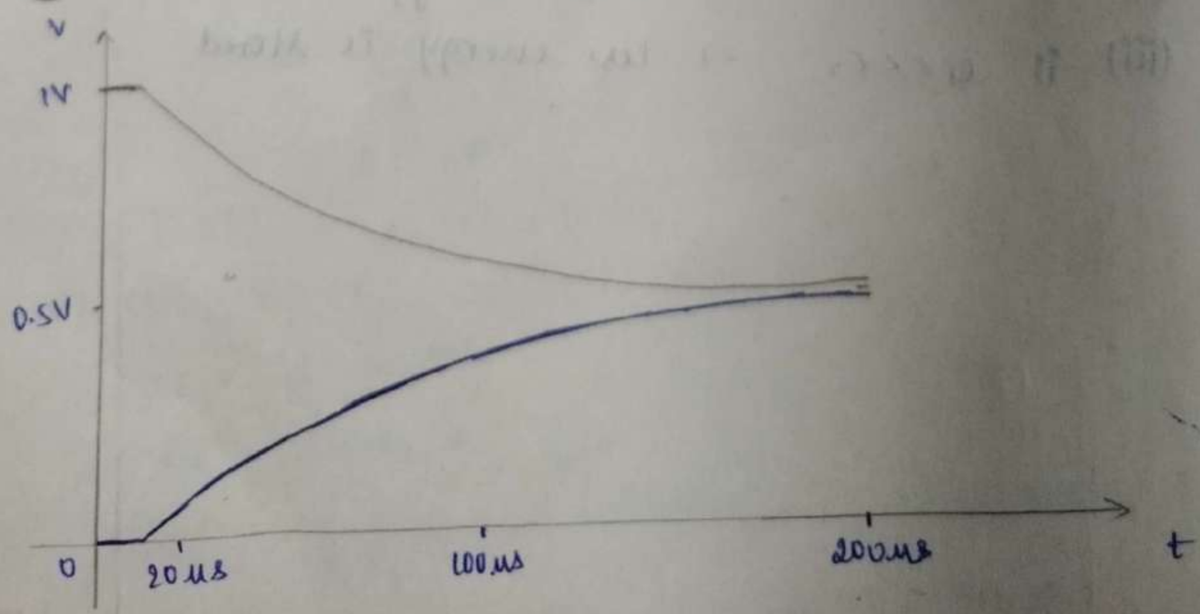
$V_{c_1}(0^-) = V_0 = 1V$

$V_{c_2}(0^-) = 0V$

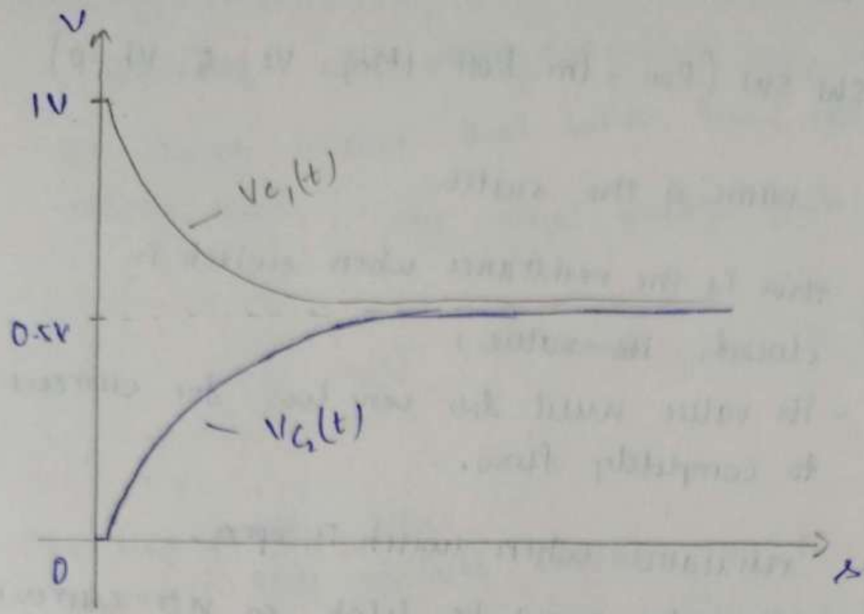
Equations $V_{c_1}(t) = \frac{C_1}{C_1+C_2} V_0 + \frac{C_2}{C_1+C_2} V_0 e^{-t/\tau}$

$V_{c_2}(t) = \frac{C_1}{C_1+C_2} V_0 \left[1 - e^{-t/RC_{eq}}\right]$

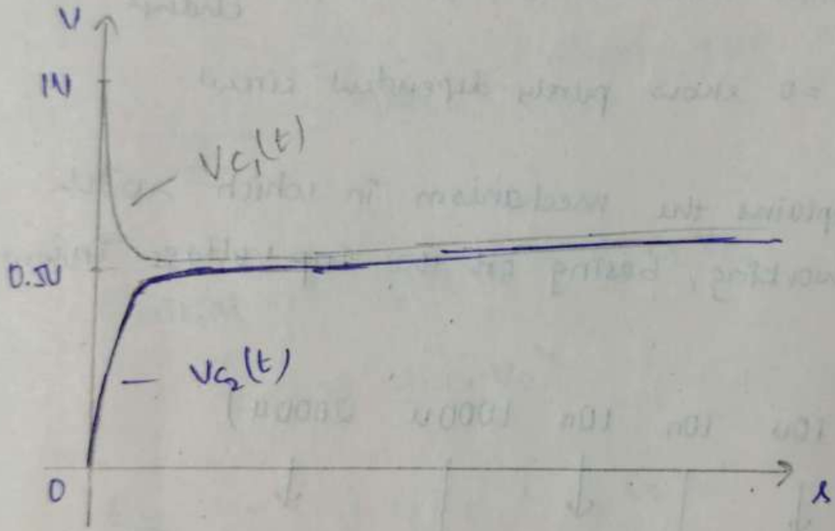
(a) $R = 10k\Omega$



(b) R = 10 Ω



(c) R = ~~k~~ 1 mΩ



For larger R values, say in (a) R = 10 kΩ, the settling time for $V_{C_1}(t)$ & $V_{C_2}(t)$ will be very long.

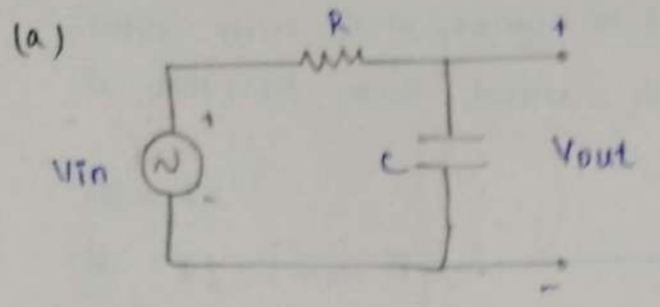
As we decrease the value of R i.e as in (b) R = 10Ω and (c) R = 1mΩ, the settling time is relatively shorter and faster.

∴ The effect of reducing R value is the reducing of settling time of $V_{C_1}(t)$ & $V_{C_2}(t)$.

Also, if R↓, then current flowing in circuit will ~~decrease~~ increase

2. RC circuits as filters

(a)



Given

$R = 20 M\Omega$

$C = 10 pF$

$V_c(0^-) = 0V$

(i)

also, $V_{in} = V_0 u(t)$

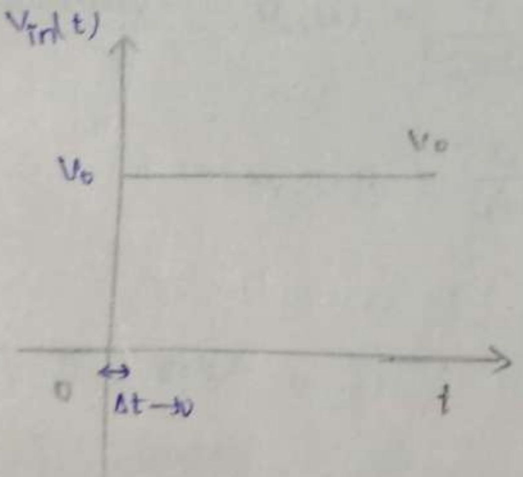
step func

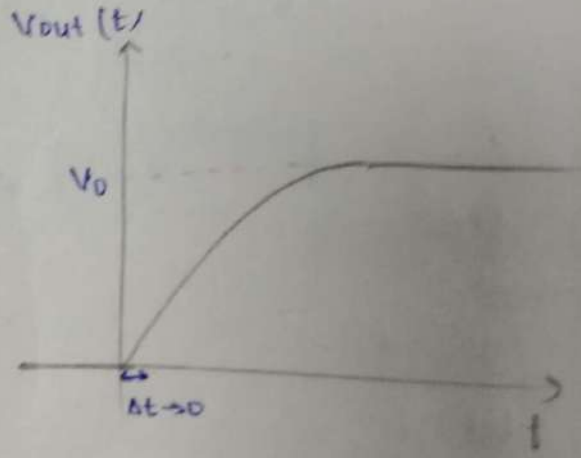
$V_{out}(t)$ — $t = 0^{(-)} \longrightarrow 0$

$t = 0^{(+)} \longrightarrow 0$ — since R is in series

$t = \infty \longrightarrow V_0$ — fully charged

* The curve of V_{out} is expected to rise exponentially (non-lin





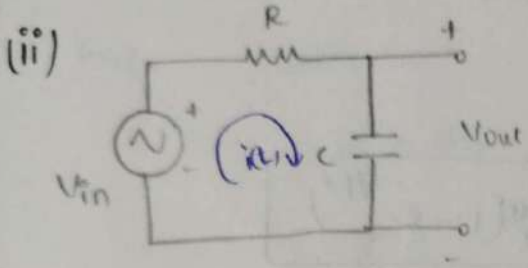
Here in Vin(t), just after a very small time Δt, the value of V_0 immediately went to V_0 without any delay.

- But this transition is not being seen in $V_{out}(t)$ graph. After small time Δt, $V_{out}(t)$ is still $<< V_0$. After some visible time, it changes to V_0.

This is because the RC combination in the circuit acts as a filter and filters out all of the fast transitions and also a delay occurs.

Hence, this RC circuit acts as a "low pass filter."

(ii)



$R = 20 M\Omega$

$C = 10 pF$

$V_c(0^-) = 0 \text{ V}$

$V_c(0^+) = 0 \text{ V}$ [$\because$ resistor is in series]

$V_c(\infty) = V_0$

Apply KVL

$$V_0 - iR - V_c(t) = 0.$$

$$V_0 - RC\frac{dV_c}{dt} - V_c(t) = 0.$$

$$V_0 - V_c = RC\frac{dV_c}{dt}$$

$$\int \frac{dt}{RC} = \int \frac{dV_c}{V_0 - V_c}$$

$$\frac{t}{RC} = -\left[\ln(V_0 - V_c)\right]_0^{V_c}$$

$$-\frac{t}{RC} = \ln(V_0 - V_c) - \ln V_0$$

$$-\frac{t}{RC} = \ln\left(\frac{V_0 - V_c}{V_0}\right)$$

$$e^{-t/RC} = 1 - \frac{V_c}{V_0}$$

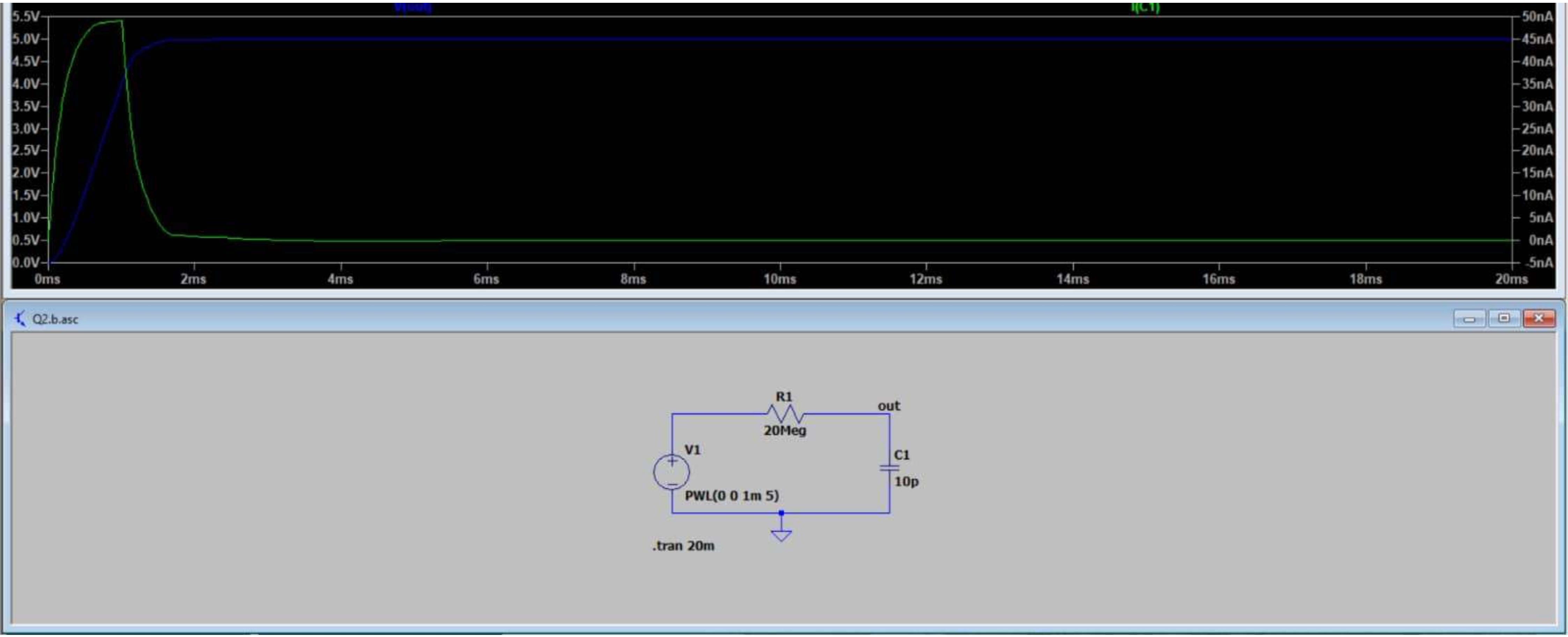
$$\boxed{V_c(t) = V_0 u(t)\left(1 - e^{-t/RC}\right)}$$

$$i(t) = C\frac{dV_c}{dt}$$

$$= C\left(-V_0 e^{-t/RC} \cdot \left(\frac{-1}{RC}\right)\right)$$

$$\boxed{i(t) = \frac{V_0 u(t)}{R} e^{-t/RC}}$$

$$V_c = V_{out} \Rightarrow \boxed{V_{out} = V_0 u(t)\left(1 - e^{-t/RC}\right)}$$


I(C1)
5.5V
5.0V
4.5V
4.0V
3.5V
3.0V
2.5V
2.0V
1.5V
1.0V
0.5V
0.0V
50nA
45nA
40nA
35nA
30nA
25nA
20nA
15nA
10nA
5nA
0nA
-5nA
0ms
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
20ms
Q2.b.asc
R1
20Meg
out
V1
C1
10p
PWL(0 0 1m 5)
.tran 20m

(iii) Transfer Function

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{Z_c}{Z_R + Z_c}$$

$$Z_c = \frac{1}{sC}$$

$$= \frac{1/sC}{1/sC + R}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{1}{1 + sCR}}$$

$$\left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = \left|\frac{1}{1 + j\omega CR}\right|$$

$$= \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\phi = -\tan^{-1}(\omega RC)$$

Take

$$20 \log \left| \frac{V_o}{V_i}(\omega) \right| = 20\left(\log \frac{1}{\sqrt{1+\omega^2R^2C^2}}\right)$$

$$= -20 \log \sqrt{1+\omega^2R^2C^2}$$

3dB frequency is the frequency at which

$$|H(j\omega)| = \frac{1}{\sqrt{2}}|H(0)|$$

$$\frac{1}{\sqrt{1+\omega^2R^2C^2}} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+0}}$$

$$\sqrt{1+\omega^2R^2C^2} = \sqrt{2}$$

$$\omega^2R^2C^2 = 1$$

$$\omega = \frac{1}{RC}$$

$$\boxed{f_c = \frac{1}{2\pi RC}}$$

$\therefore$ 3dB cutoff frequency is $\omega = \frac{1}{RC}$

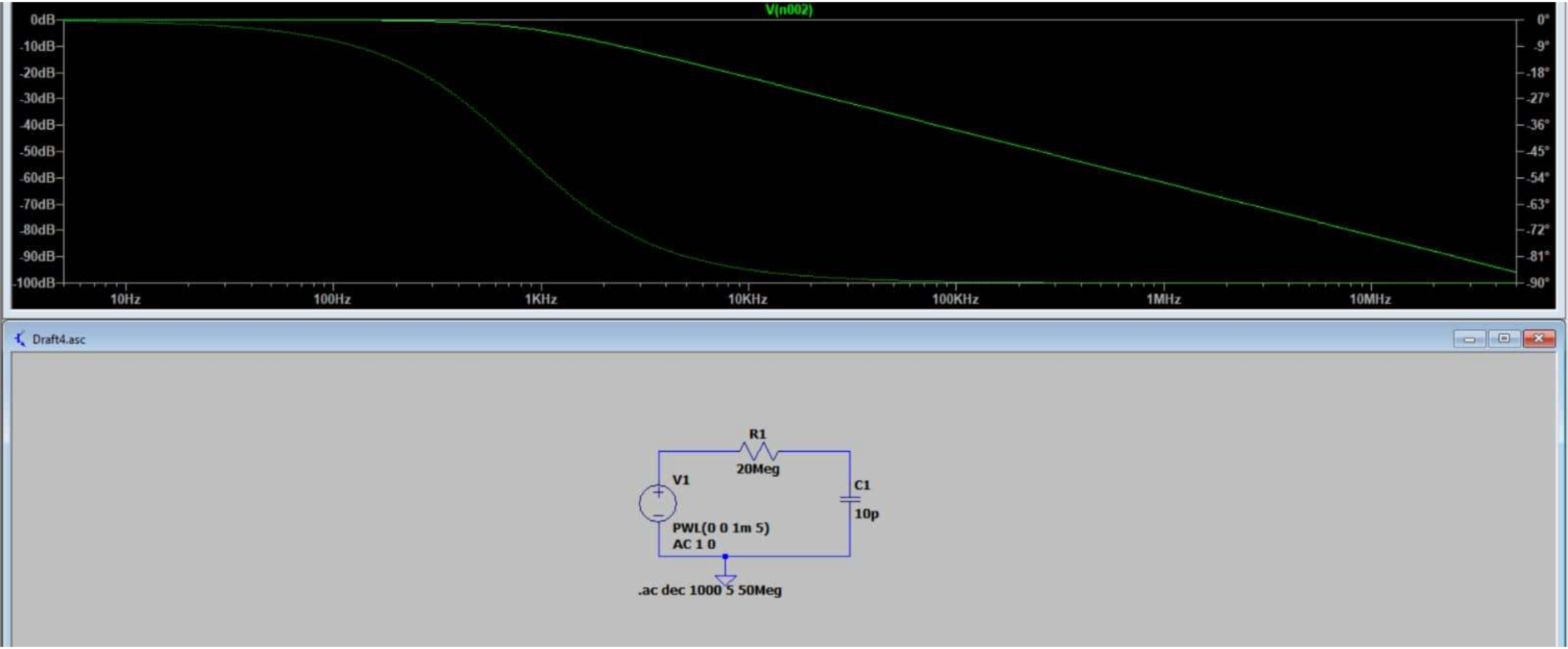
Gain $= 20\log|H(s)|$

$$= 20\log\left|\frac{1}{\sqrt{1+\omega^2R^2C^2}}\right|$$

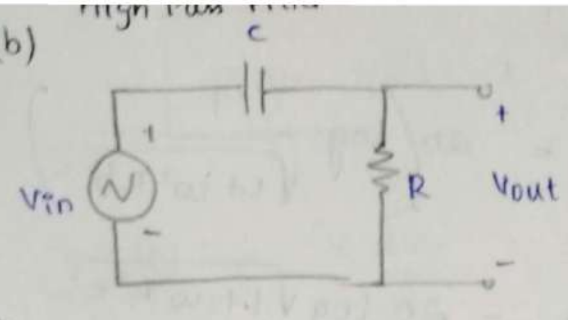
$$= -20\log\sqrt{1+\omega^2R^2C^2}$$

$$\boxed{\because \frac{V_{out}}{V_{in}} = \frac{1}{1+sRC}}$$

Phase $(\phi) = -\tan^{-1}(\omega RC)$


V(n002)
0dB
-10dB
-20dB
-30dB
-40dB
-50dB
-60dB
-70dB
-80dB
-90dB
-100dB
10Hz
100Hz
1KHz
10KHz
100KHz
1MHz
10MHz
0°
-9°
-18°
-27°
-36°
-45°
-54°
-63°
-72°
-81°
-90°
Draft4.asc
R1
20Meg
V1
C1
10p
PWL(0 0 1m 5)
AC 1 0
.ac dec 1000 5 50Meg

(b)



$R = 20k\Omega$
$C = 10pF$

$V_c(0^-) = 0$

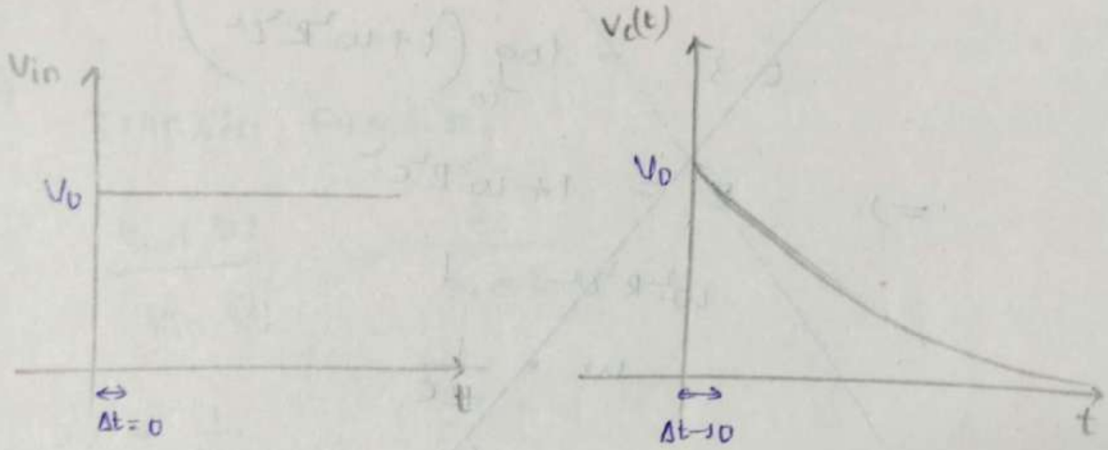
(i)

$V_{in} = V_0\, u(t)$

on analysing $V_c(t)$

$V_c(0^-) = 0$
$V_c(0^+) = V_0$

$V_c(\infty) = 0.$



* The curve of Vout is expected to fall exponentially (non-linear)

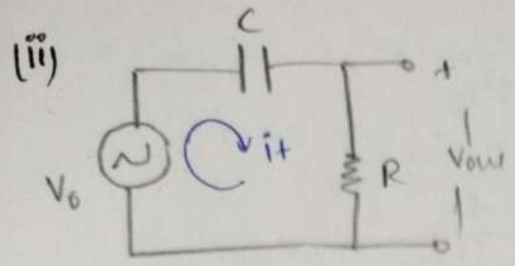
Here also, in $V_{in}(t)$, just after a very small time Δt, the value of V_0 immediately went to V_0 without any delay.

And in $V_{out}(t)$ for a same Δt, we are able to achieve a ~~sn~~ similar transition and jump to V_0 and slowly reduces.

∴ This is a High-Pass Filter.

→ A low pass filter attenuates ~~the~~ low f and passes high frequencies.

(iii)



Apply KVL,

$V_c(0^-) = 0$

$V_c(0^+) = V_0$

$V_c(\infty) = 0$

$V_0 - \frac{d}{dt} V_c(t) - iR = 0$

$V_0 - V_c - RC\frac{dV_c}{dt} = 0$

$V_0 - V_c = RC\frac{dV_c}{dt}$

$\int \frac{dt}{RC} = \int \frac{dV_c}{V_0 - V_c}$

$-\frac{t}{RC} = \ln(V_0 - V_c)\Big]_0^{V_c}$

$-\frac{t}{RC} = \ln\left(\frac{V_0 - V_c}{V_0}\right)$

$$\boxed{V_c = V_0 (1 - e^{-t/RC})}$$

$i_c(t) = C\frac{dV_c}{dt}$

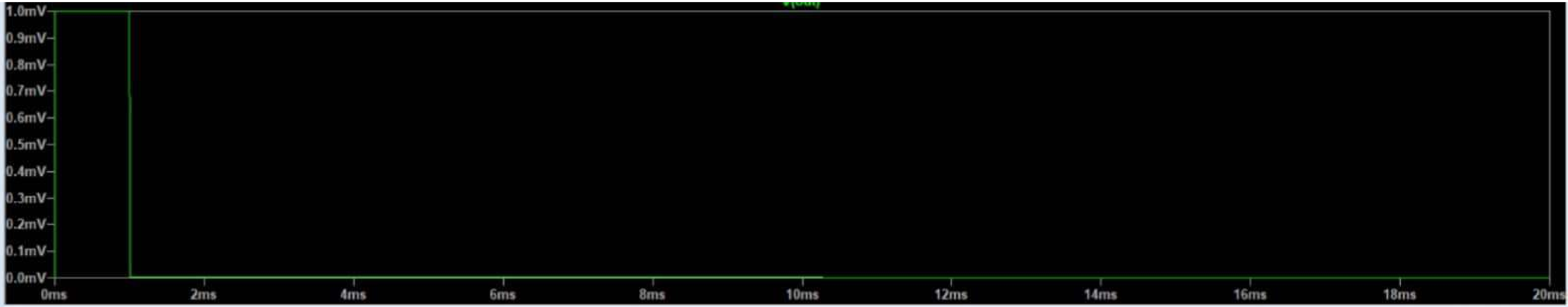
$= -C(V_0)\left(\frac{-1}{RC}\right) e^{-t/RC}$

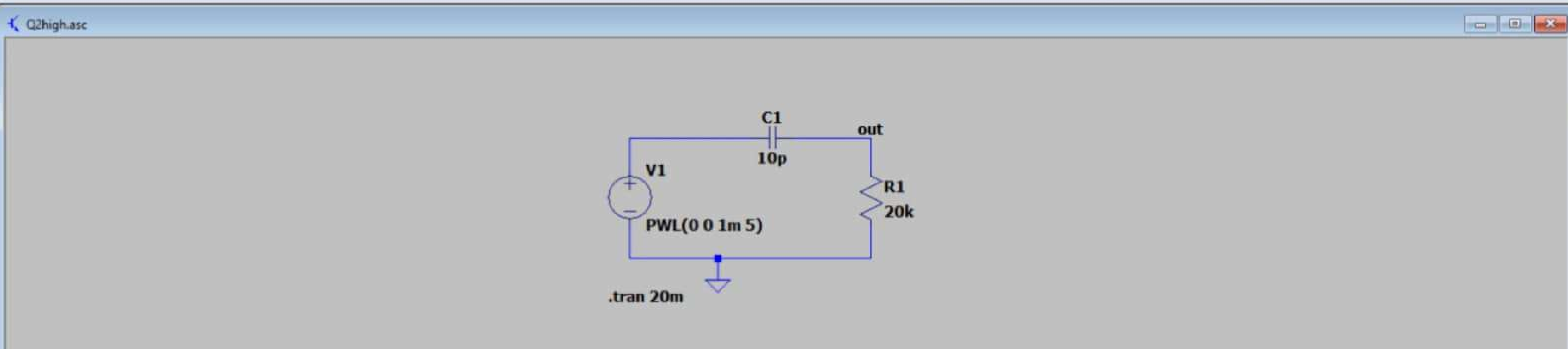
$\boxed{i_c(t) = \frac{V_0}{R} e^{-t/RC}}$ — current across capacit

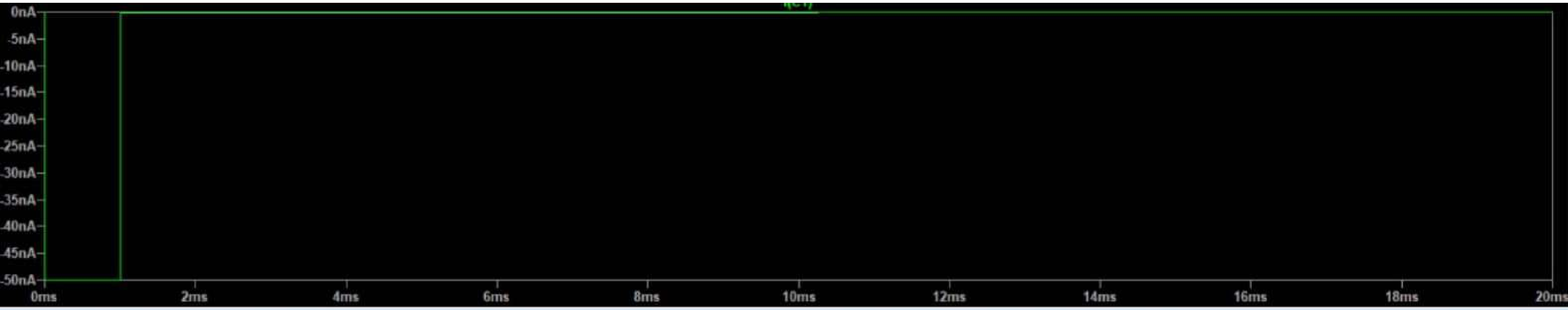
$V_{out} = iR$

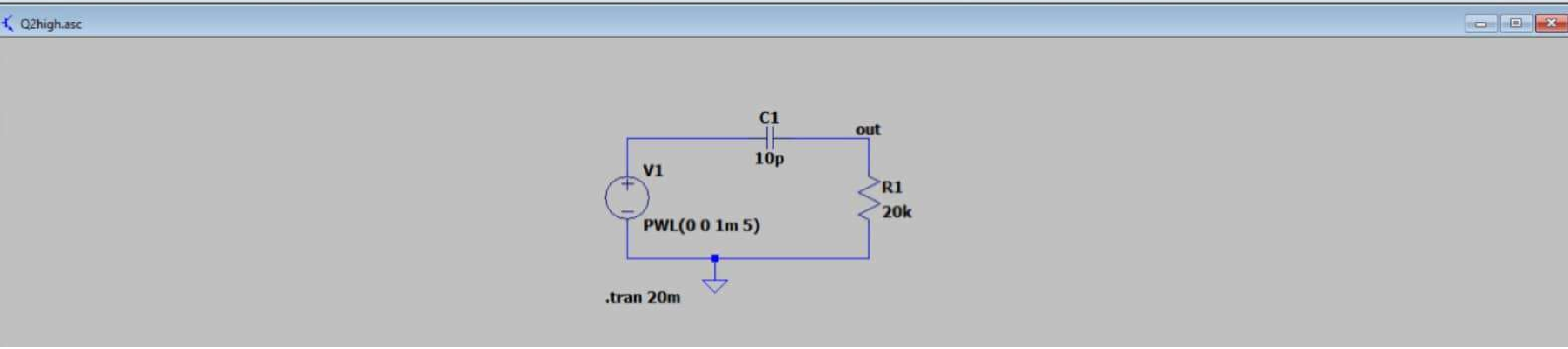
$= \frac{V_0}{R} e^{-t/RC} \times R$

$$\boxed{V_{out} = V_0 e^{-t/RC} u(t)}$$


V(out)
1.0mV
0.9mV
0.8mV
0.7mV
0.6mV
0.5mV
0.4mV
0.3mV
0.2mV
0.1mV
0.0mV
0ms
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
20ms


Q2high.asc
C1
out
10p
V1
R1
20k
PWL(0 0 1m 5)
.tran 20m


0nA
-5nA
-10nA
-15nA
-20nA
-25nA
-30nA
-35nA
-40nA
-45nA
-50nA
0ms
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
20ms


Q2high.asc
C1
10p
out
V1
R1
20k
PWL(0 0 1m 5)
.tran 20m

(iii)

Transfer Function

$$\frac{V_{out}(s)}{V_{in}} = \frac{R}{R + \frac{1}{sC}}$$

$$= \frac{sCR}{1 + sCR}$$

$$20 \log \left| \frac{V_{out}}{V_{in}} \omega \right| = 20 \log \left| \frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}} \right|$$

$$\left| \frac{V_{out}}{V_{in}} \omega \right| = \left| \frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}} \right|$$

$$H(j\omega) = 20 \log |\omega RC| - 20 \log \left| \sqrt{1+\omega^2R^2C^2} \right|$$

(i) $\omega << \frac{1}{RC}$ $\quad H(j\omega) = 20 \log \omega RC$

(ii) $\omega = \frac{1}{RC}$ $\quad H(j\omega) = -3\,dB$

(iii) $\omega >> \frac{1}{RC}$ $\quad H(j\omega) = 0.$

3dB cut-off frequency is the frequency at which

$$|H(s)| = \frac{1}{\sqrt{2}} |H(0)|$$

$$\frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}} = \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{1+0}}$$

$$\frac{\omega^2R^2C^2}{1+\omega^2R^2C^2} = \frac{1}{2}$$

$$\omega^2R^2C^2 = 1$$

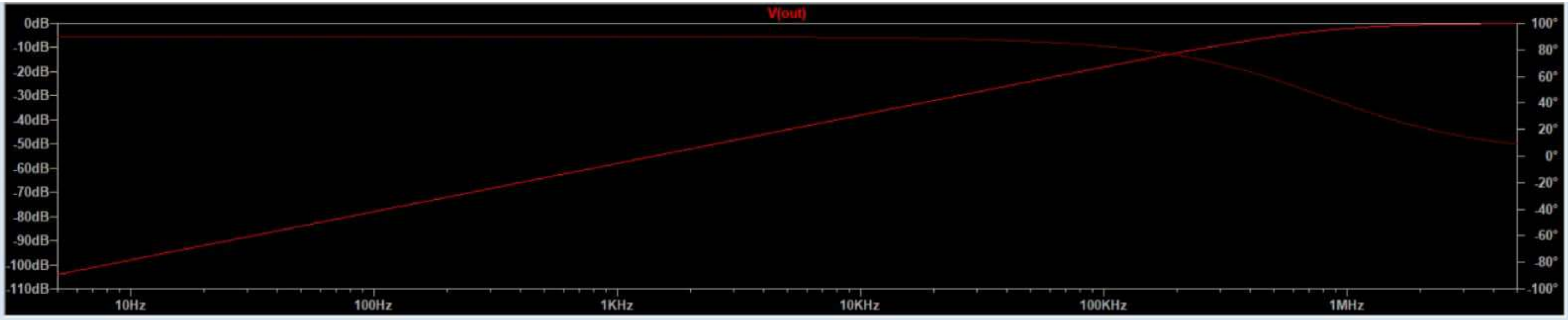
$$\boxed{\omega = \frac{1}{RC}}$$

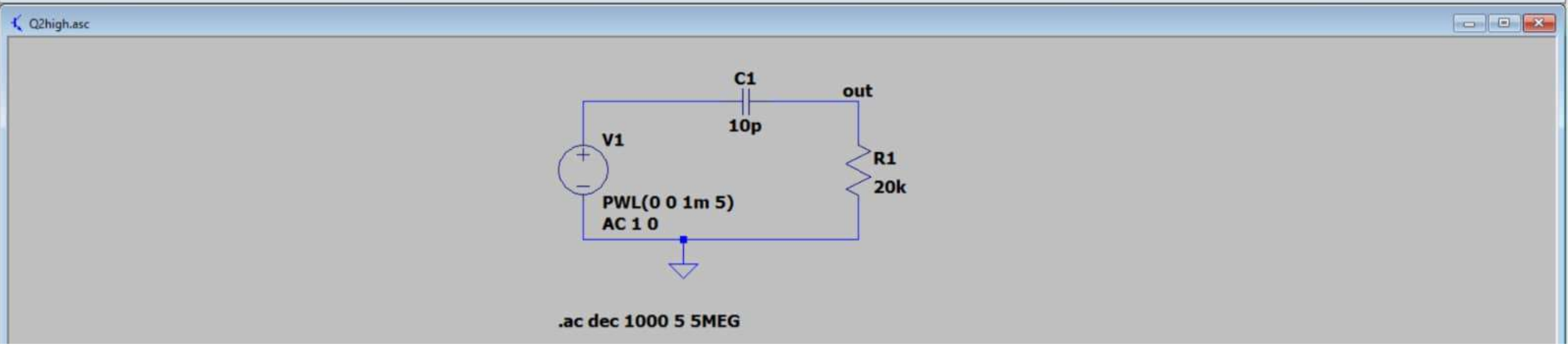
~~(iv)~~ Gain $= 20 \log |H(s)|$

$$= 20 \log \left| \frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}} \right|$$

$$= 20 \log |\omega RC| - 20 \log \sqrt{1+\omega^2R^2C^2}$$

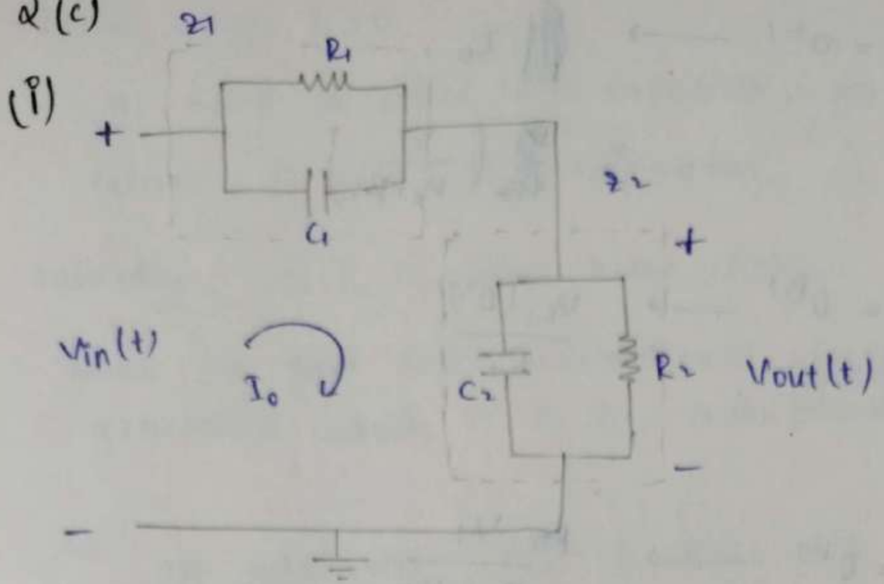
Phase ϕ = ~~tan~~ $\pi/2 - \tan^{-1}(\omega RC)$


V(out)
0dB
-10dB
-20dB
-30dB
-40dB
-50dB
-60dB
-70dB
-80dB
-90dB
-100dB
-110dB
100°
80°
60°
40°
20°
0°
-20°
-40°
-60°
-80°
-100°
10Hz
100Hz
1KHz
10KHz
100KHz
1MHz


Q2high.asc
C1
out
10p
V1
R1
20k
PWL(0 0 1m 5)
AC 1 0
.ac dec 1000 5 5MEG

2 (c)

(i)



Given $V_{in}(t) = \begin{cases} V_1 & t \le 0 \\ V_2 & 0 < t \le T_b \quad (V_2 \ge V_1) \\ V_1 & t \ge T_b \end{cases}$

and, $T_b \; (>> R_i C_i)$

Initially, $I_0(0^-) = \frac{V_0}{R_1 + R_2}$

Here $\boxed{\Delta V = V_2 - V_1}$

$V_{C_1}(t) \quad t = 0^{(-)} \longrightarrow \frac{R_1 V_1}{R_1 + R_2}$

$t = 0^{(+)} \longrightarrow V_{C_1}(0^-) + \Delta V_{C_1}$

[ΔV is shown in next page]

$= \frac{R_1 V_1}{R_1 + R_2} + \frac{C_2}{C_1 + C_2} \Delta V$

$V_{C_2}(t) \quad t = 0^{(-)} \longrightarrow \frac{R_2 V_1}{R_1 + R_2}$

$t = 0^{(+)} \longrightarrow V_{C_2}(0^-) + \Delta V_{C_2}$

$= \frac{R_2 V_1}{R_1 + R_2} + \frac{C_1}{C_1 + C_2} \Delta V$

$I_{R_1}(t)$ $\quad t = 0^{(-)} \longrightarrow I_0$

$$= \left(\frac{V_1}{R_1+R_2}\right)$$

$t = 0^{(+)} \longrightarrow \dfrac{V_{C_1}(0^+)}{R_1}$

$I_{R_2}(t)$ $\quad t = 0^{(-)} \longrightarrow \dfrac{V_1}{R_1+R_2}$

$t = 0^{(+)} \longrightarrow \dfrac{V_{C_2}(0^+)}{R_2}$

$I_{C_1}(t)$ $\quad t = 0^{(-)} \longrightarrow 0$

$t = 0^{(+)} \longrightarrow$ impulsive current for $\Delta t \to 0$ and again reaches steady state

$I_{C_2}(t)$ $\quad t = 0^{(-)} \longrightarrow 0$

$t = 0^{(+)} \longrightarrow$ impulsive current for small t and then steady state is obtained.

For $t < 0$,

the capacitor's are completely charged, hence no current flows into them. $I_{C_1}(0^-) = I_{C_1}(0^+) = 0$.

~~to~~ All the current flows into resistors and is equal in both of them

Now, when $t > 0$

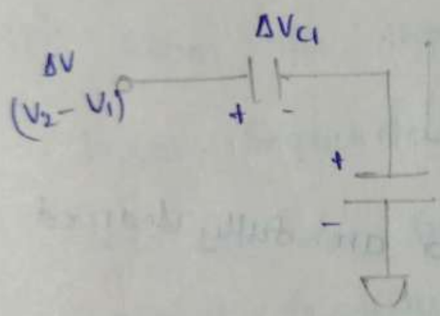
as no R in series with capacitor, an impulsive current I flows into capacitor.

Initially, C_1 & C_2 have some charge

Now, ~~we give~~ some additional charge is generated which is to be redistributed.

So, an additional ΔV_{C_1} & ΔV_{C_2} are generated at $t = 0^+$.

at $t=0$, equivalent circuit,



Q formed additionally

$$\Delta Q = \frac{C_1 C_2}{C_1 + C_2} \Delta V$$

$$= Q_1 + Q_2$$

$$= C_1 \Delta V_{C_1} + C_2 \Delta V_{C_2}$$

and $\Delta V = \Delta V_{C_1} + \Delta V_{C_2}$ —— (1)

Since charge must be conser—

$$C_1 \Delta V_{C_1} = C_2 \Delta V_2 \quad \text{—— (2)}$$

On solving (1), (2)

$$\Delta V_{C_1} = \frac{C_2}{C_1 + C_2} \Delta V$$

$$\Delta V_{C_2} = \frac{C_1}{C_1 + C_2} \Delta V$$

After $t>0$,

Impulsive current goes into capacitors.

But still small amount of I goes into the resistor

i.e

$$I_{R_1}(0^+) = \frac{V_{c_1}(0^+)}{R_1}$$

$$I_{R_2}(0^+) = \frac{V_{c_1}(0^+)}{R_2}$$

Because of this there are small transient change in capacitor and is not constant.

so, we need

$$\boxed{I_{R_1}(0^+) = I_{R_2}(0^+)}$$

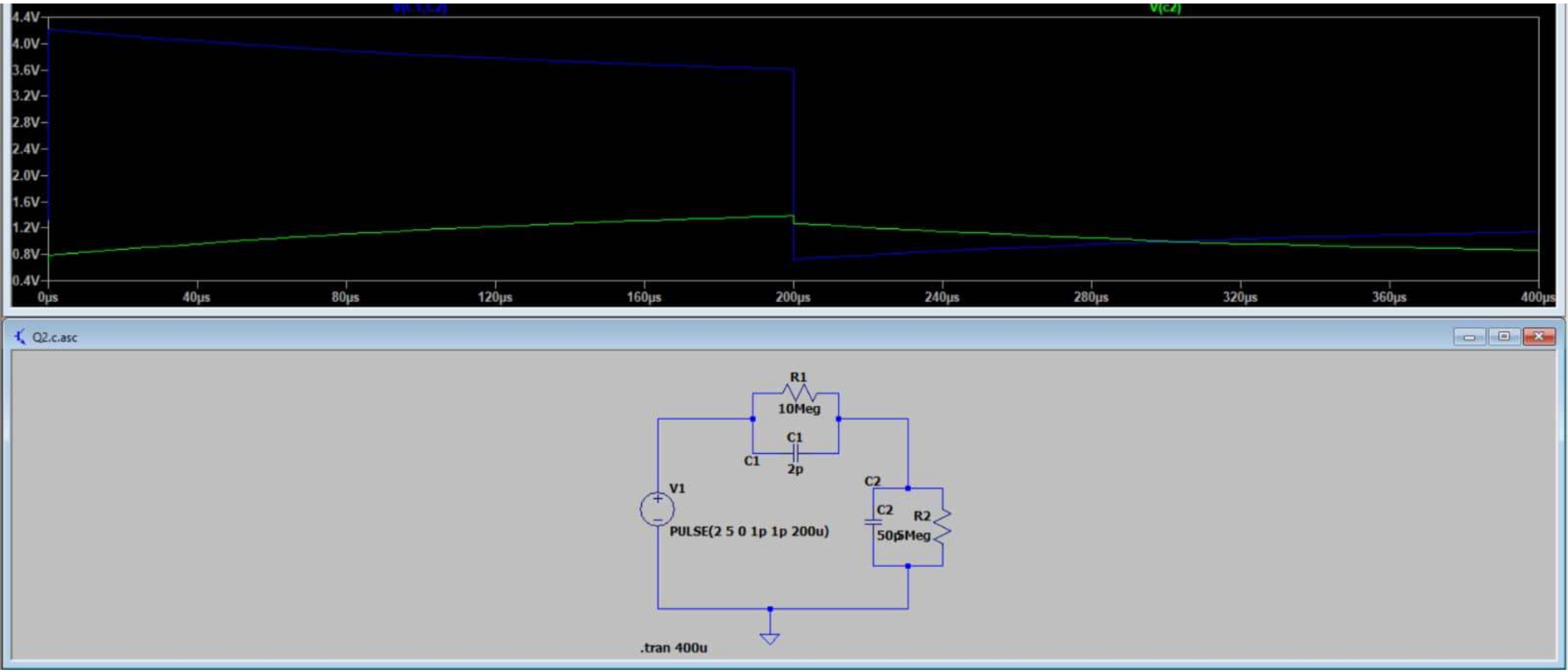
↓

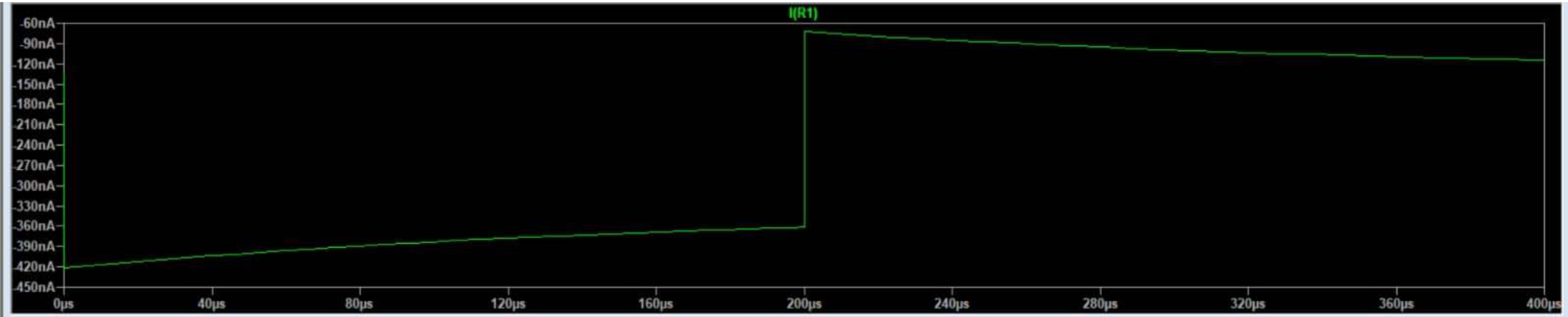
when this is achieved, capacitor are fully charged and reach steady state

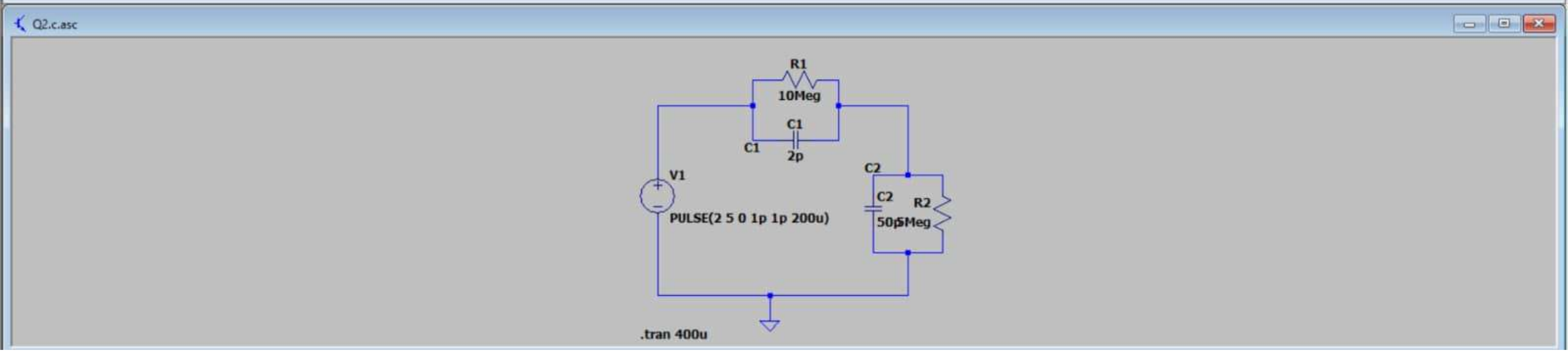
$$\Rightarrow \boxed{R_1C_1 = R_2C_2}$$

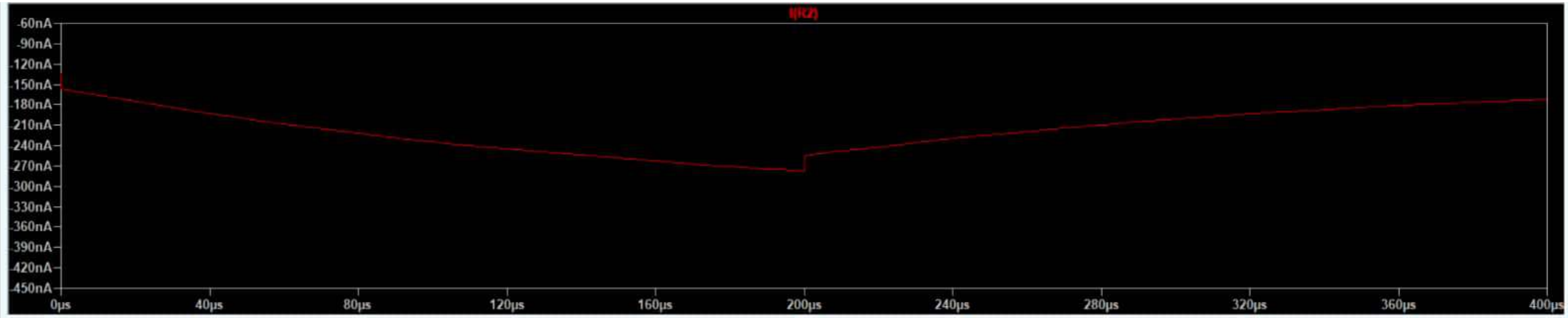
↓

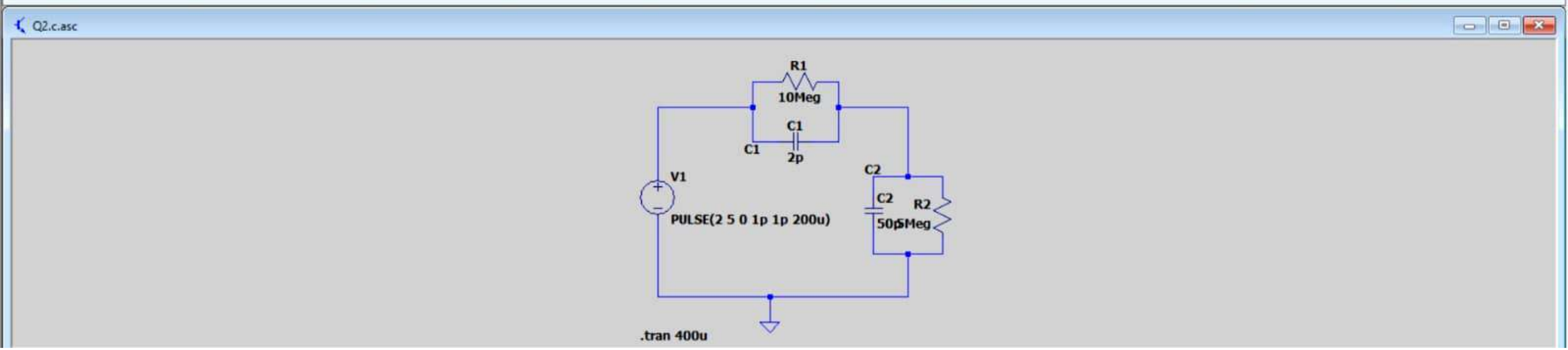
if this is possible, then $V_{c_1}(0^+)$ & $V_{c_2}(0^+)$ will remain fixed without transient changes.

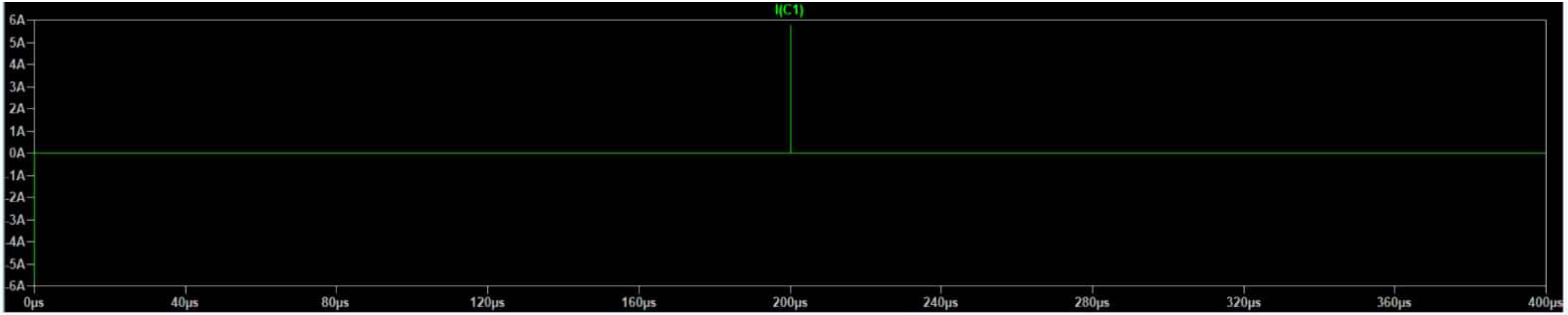

V(c2)
4.4V
4.0V
3.6V
3.2V
2.8V
2.4V
2.0V
1.6V
1.2V
0.8V
0.4V
0µs
40µs
80µs
120µs
160µs
200µs
240µs
280µs
320µs
360µs
400µs
Q2.c.asc
R1
10Meg
C1
C1
2p
V1
PULSE(2 5 0 1p 1p 200u)
C2
C2
R2
50p
5Meg
.tran 400u

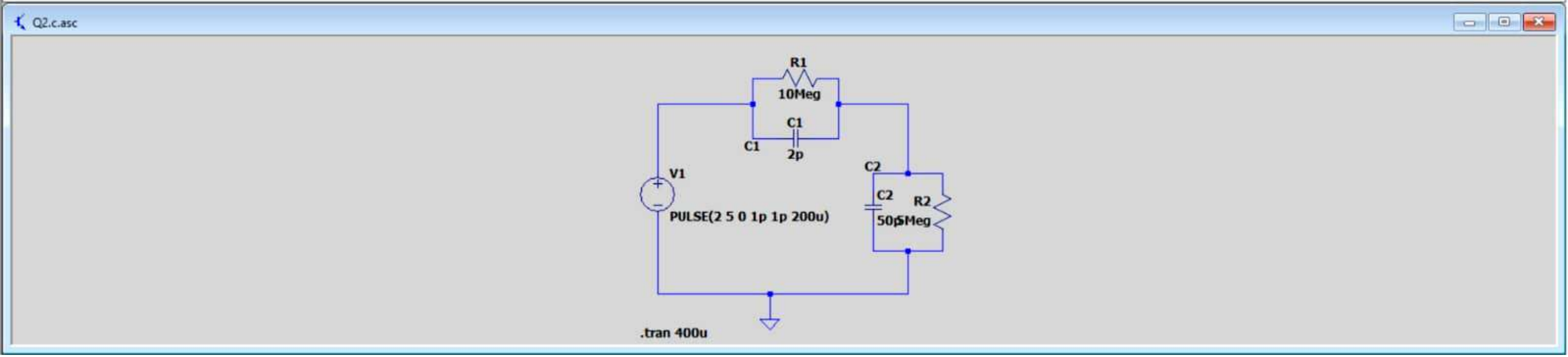

I(R1)
-60nA
-90nA
-120nA
-150nA
-180nA
-210nA
-240nA
-270nA
-300nA
-330nA
-360nA
-390nA
-420nA
-450nA
0µs
40µs
80µs
120µs
160µs
200µs
240µs
280µs
320µs
360µs
400µs

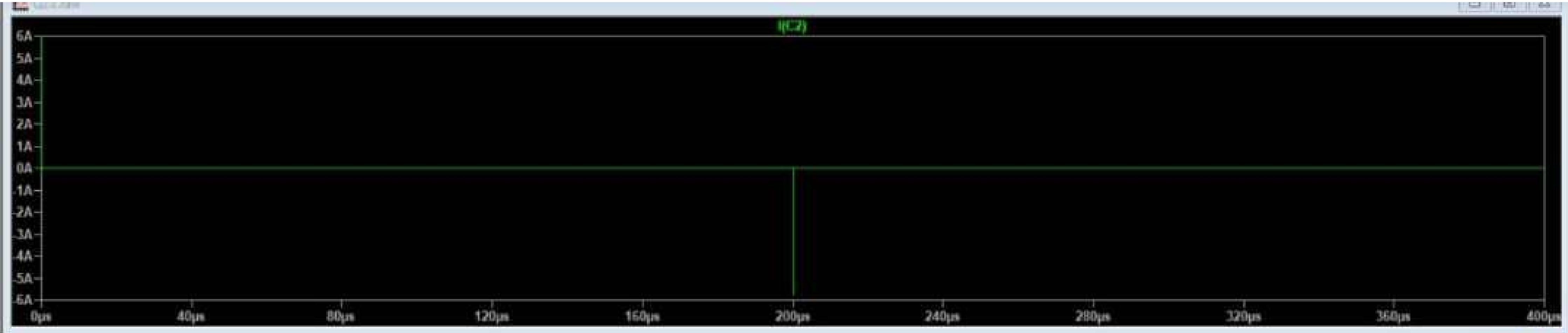

Q2.c.asc
R1
10Meg
C1
C1
2p
V1
PULSE(2 5 0 1p 1p 200u)
C2
C2
R2
50p
5Meg
.tran 400u

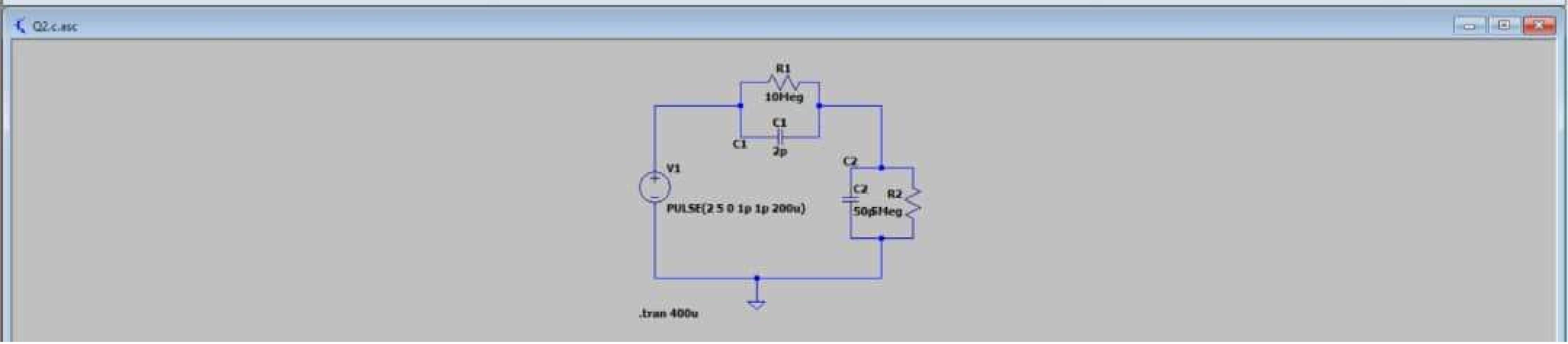

I(R2)
-60nA
-90nA
-120nA
-150nA
-180nA
-210nA
-240nA
-270nA
-300nA
-330nA
-360nA
-390nA
-420nA
-450nA
0µs
40µs
80µs
120µs
160µs
200µs
240µs
280µs
320µs
360µs
400µs

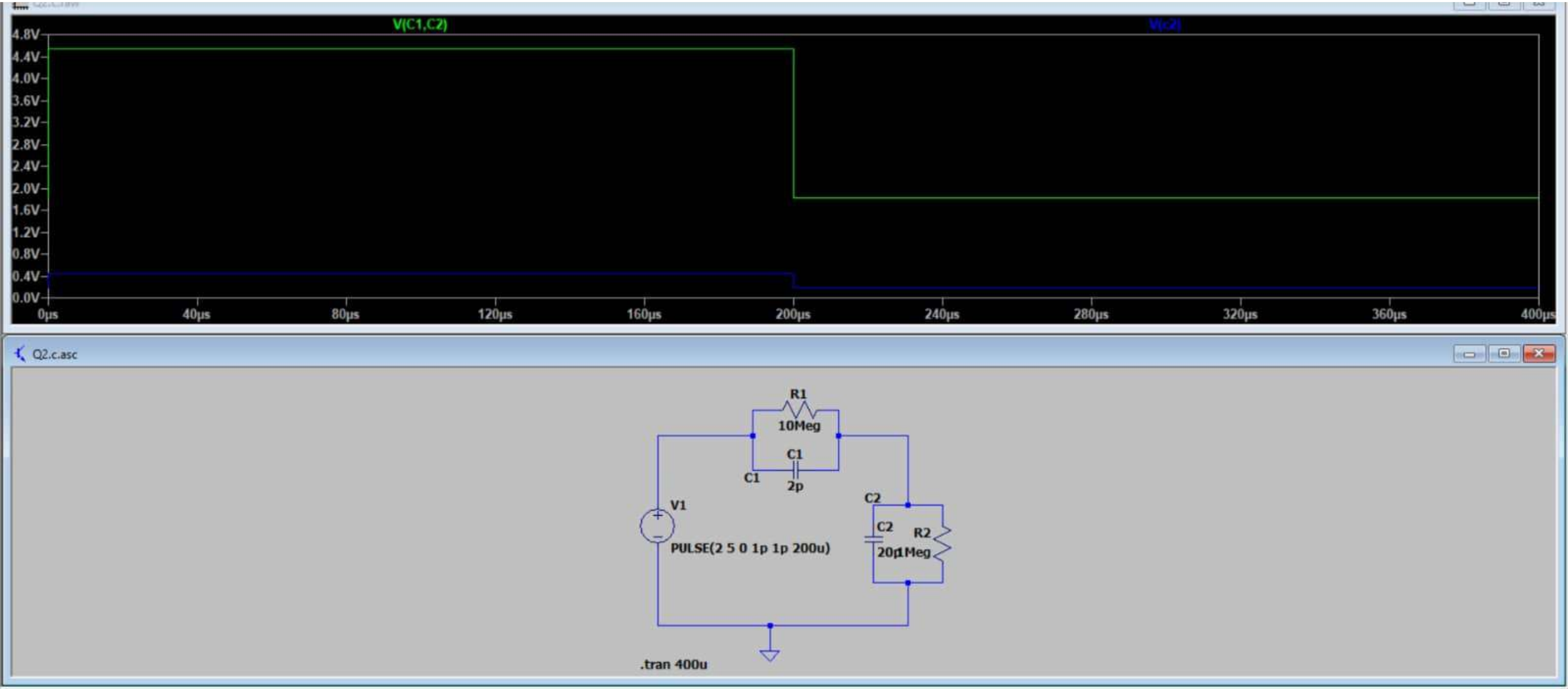

Q2.c.asc
R1
10Meg
C1
C1
2p
V1
PULSE(2 5 0 1p 1p 200u)
C2
C2
R2
50p
5Meg
.tran 400u


I(C1)
6A
5A
4A
3A
2A
1A
0A
-1A
-2A
-3A
-4A
-5A
-6A
0µs
40µs
80µs
120µs
160µs
200µs
240µs
280µs
320µs
360µs
400µs

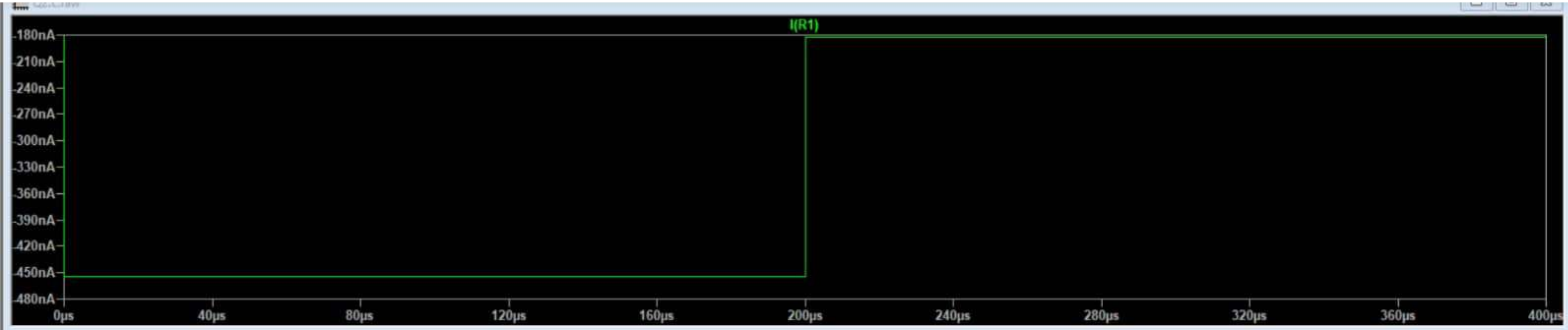

Q2.c.asc
R1
10Meg
C1
C1
2p
C2
V1
PULSE(2 5 0 1p 1p 200u)
C2
R2
50p
5Meg
.tran 400u

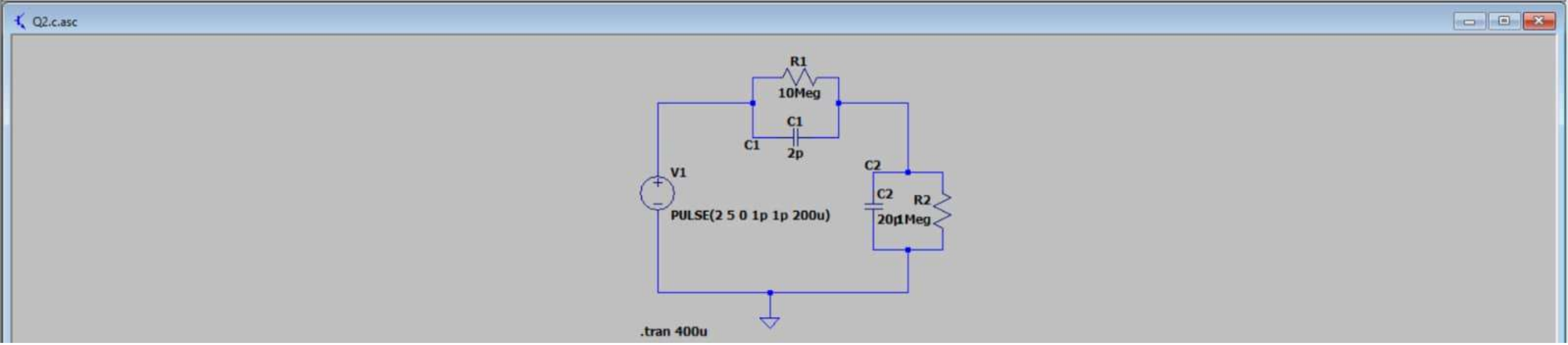

I(C2)
6A
5A
4A
3A
2A
1A
0A
-1A
-2A
-3A
-4A
-5A
-6A
0µs
40µs
80µs
120µs
160µs
200µs
240µs
280µs
320µs
360µs
400µs


Q2.c.asc
R1
10Meg
C1
C1
2p
V1
PULSE(2.5 0 1p 1p 200u)
C2
C2
R2
50p 5Meg
.tran 400u

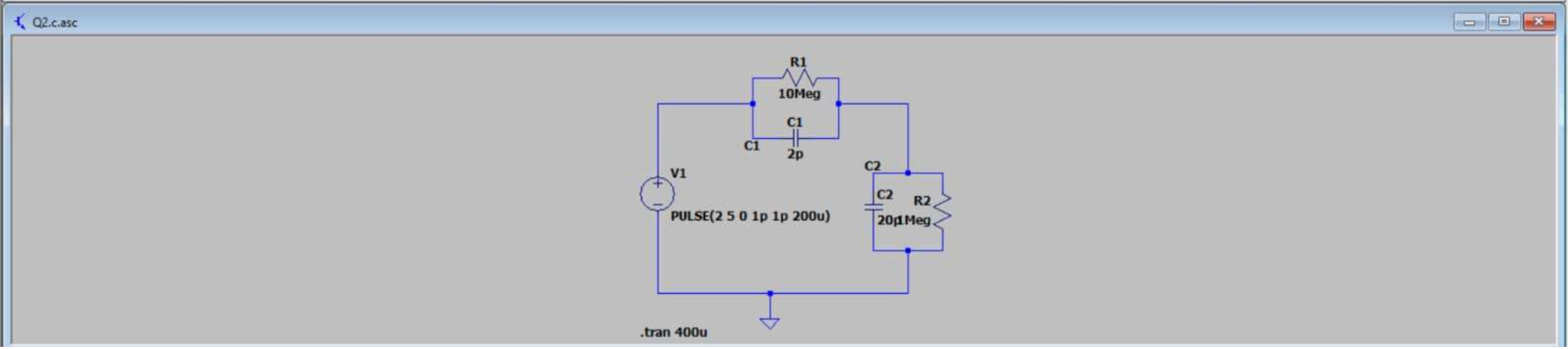


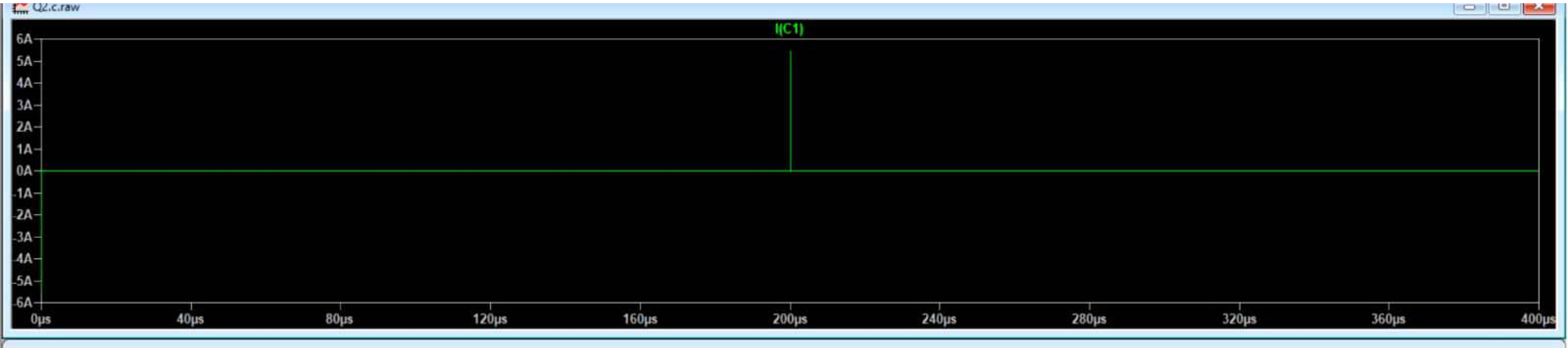
V(C1,C2)
V(c2)
4.8V
4.4V
4.0V
3.6V
3.2V
2.8V
2.4V
2.0V
1.6V
1.2V
0.8V
0.4V
0.0V
0µs
40µs
80µs
120µs
160µs
200µs
240µs
280µs
320µs
360µs
400µs
Q2.c.asc
R1
10Meg
C1
C1
2p
C2
V1
C2
R2
PULSE(2 5 0 1p 1p 200u)
20p
1Meg
.tran 400u

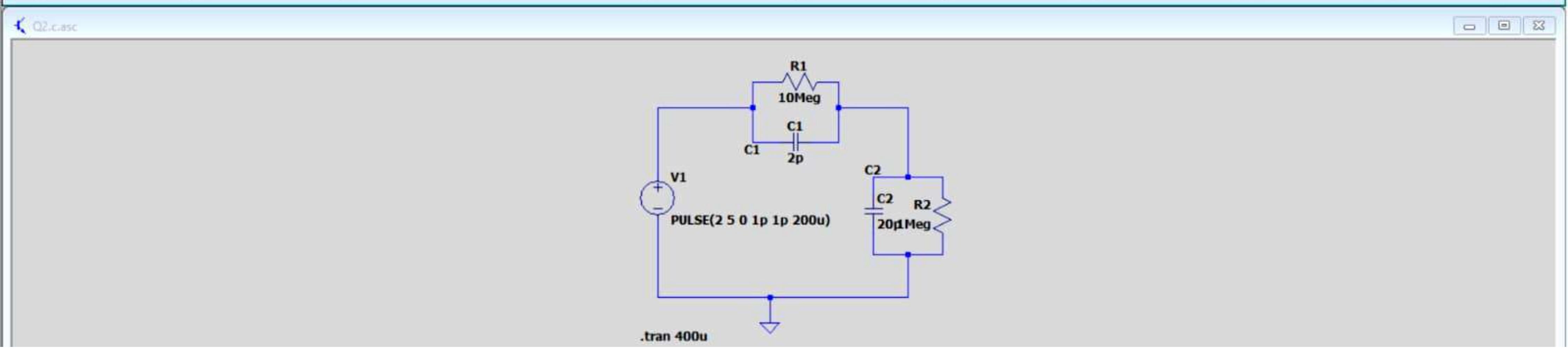

I(R1)
-180nA
-210nA
-240nA
-270nA
-300nA
-330nA
-360nA
-390nA
-420nA
-450nA
-480nA
0µs
40µs
80µs
120µs
160µs
200µs
240µs
280µs
320µs
360µs
400µs

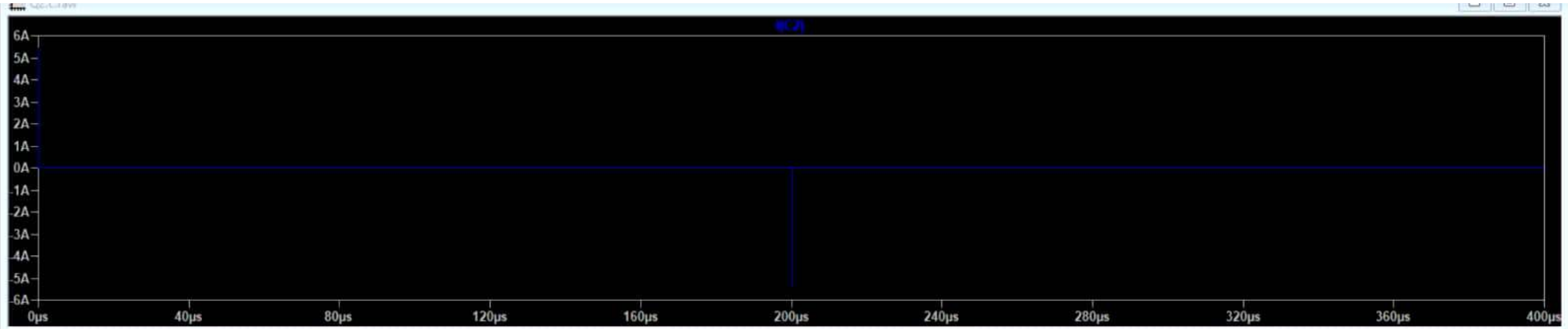

Q2.c.asc
R1
10Meg
C1
C1
2p
V1
PULSE(2 5 0 1p 1p 200u)
C2
C2
R2
1Meg
.tran 400u

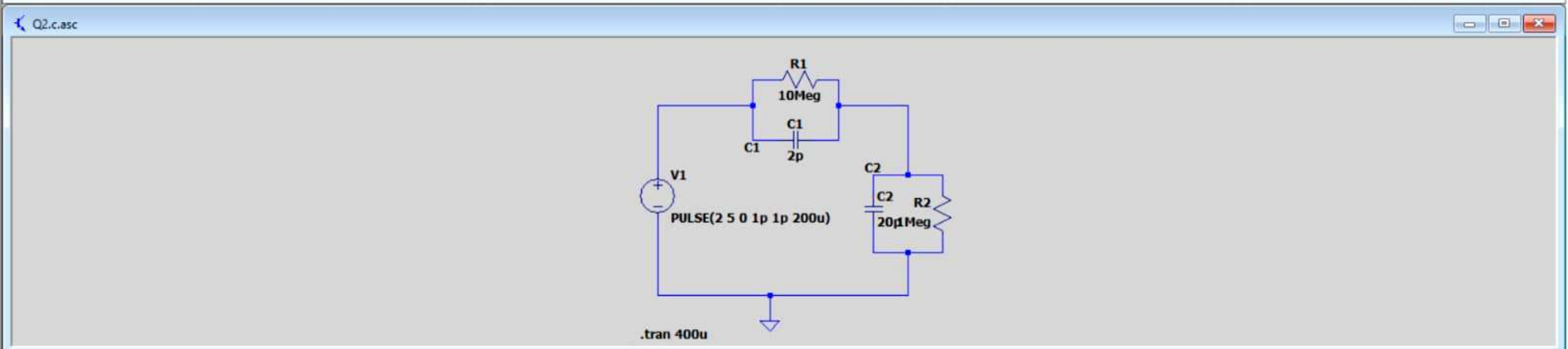

I(R2)
-180nA
-210nA
-240nA
-270nA
-300nA
-330nA
-360nA
-390nA
-420nA
-450nA
-480nA
0µs
40µs
80µs
120µs
160µs
200µs
240µs
280µs
320µs
360µs
400µs


Q2.c.asc
R1
10Meg
C1
2p
C1
V1
PULSE(2 5 0 1p 1p 200u)
C2
C2
R2
20µ 1Meg
.tran 400u


Q2.c.raw
I(C1)
6A
5A
4A
3A
2A
1A
0A
-1A
-2A
-3A
-4A
-5A
-6A
0µs
40µs
80µs
120µs
160µs
200µs
240µs
280µs
320µs
360µs
400µs


Q2.c.asc
R1
10Meg
C1
C1
2p
V1
PULSE(2 5 0 1p 1p 200u)
C2
C2
20p
R2
1Meg
.tran 400u


I(C2)
6A
5A
4A
3A
2A
1A
0A
-1A
-2A
-3A
-4A
-5A
-6A
0µs
40µs
80µs
120µs
160µs
200µs
240µs
280µs
320µs
360µs
400µs


Q2.c.asc
R1
10Meg
C1
C1
2p
V1
PULSE(2 5 0 1p 1p 200u)
C2
C2
R2
20µ
1Meg
.tran 400u

(iv)

Transfer Function :

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{Z_2}{Z_1 + Z_2}$$

$$= \frac{\dfrac{R_2}{1+sC_2R_2}}{\dfrac{R_2}{1+sC_2R_2} + \dfrac{R_1}{1+sC_1R_1}}$$

$$= \frac{R_2(1+sC_1R_1)}{R_1+R_2 + R_1R_2 s(C_1+C_2)}$$

$$= \frac{R_2}{(R_1+R_2)} \frac{(1+sC_1R_1)}{(1+sC_{eq}R_{eq})}$$

$$R_{eq} = \frac{R_1R_2}{R_1+R_2} \; ; \; C_{eq} = C_1+C_2$$

when,

$$R_1C_1 = R_2C_2$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2}{R_1+R_2}$$

Transfer function is constant.

Hence, it allows all the frequencies to pass through it.

∴ It is called an "ALL PASS FILTER".

Now,

if $R_1C_1 > R_2C_2$

$$\frac{R_1}{R_2} > \frac{C_2}{C_1}$$

in this case

as $\omega \uparrow \Rightarrow \frac{V_{out}}{V_{in}} \uparrow \Rightarrow$ High pass filter

if $R_1C_1 < R_2C_2$

here,

as $\omega \uparrow \Rightarrow \frac{V_{out}}{V_{in}} \downarrow \Rightarrow$ Low pass filter.

$\therefore$ When — Nature of circuit

$R_1C_1 = R_2C_2 \rightarrow$ All Pass circuit

$R_1C_1 > R_2C_2 \rightarrow$ High pass

$R_1C_1 < R_2C_2 \rightarrow$ low pass

(ii) Yes, the circuit allows to ~~qu~~pass quick transitions in input to the output in the cases where $R_1C_1 > R_2C_2$ [High pass]

And, It allows the slow transitions in input to output when $R_1C_1 < R_2C_2$ [Low Pass]

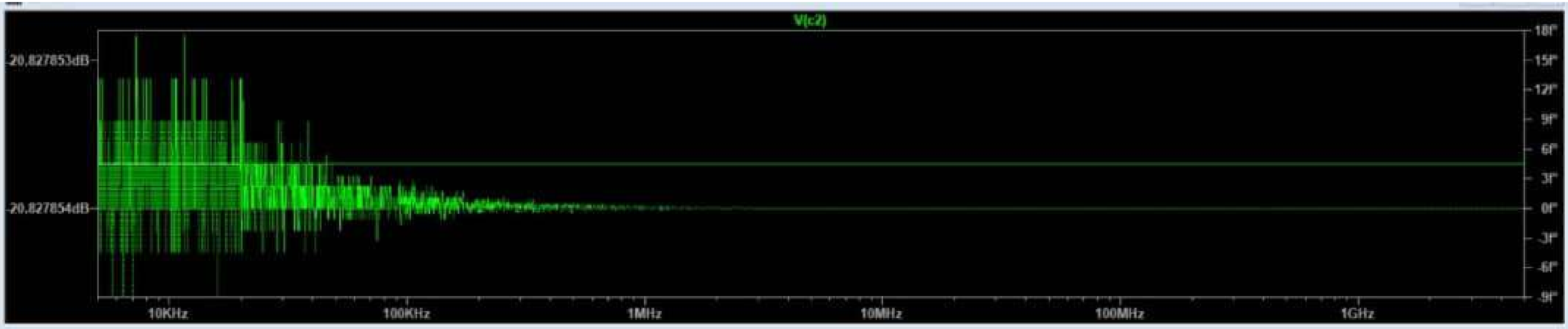
~~(iii)~~

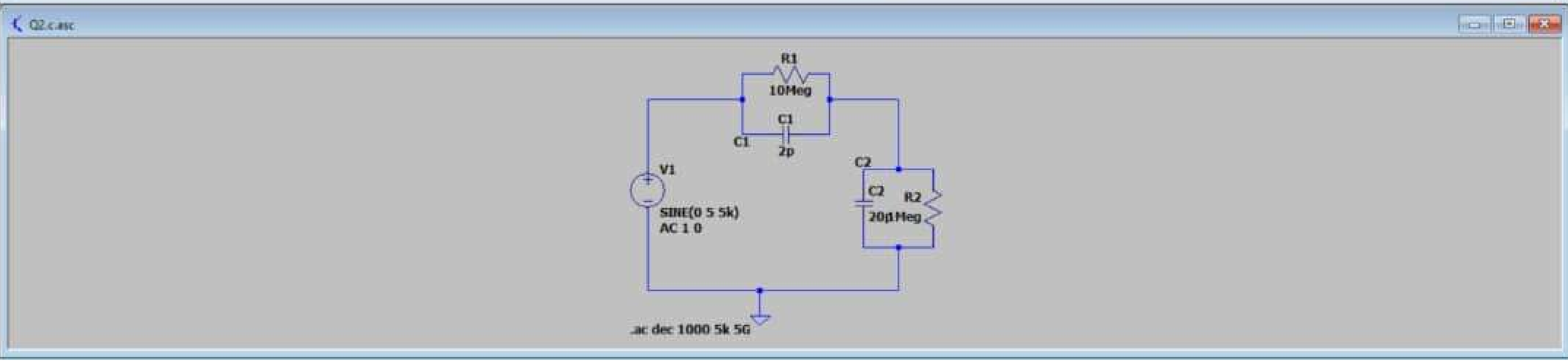
(v) By observing the ~~transfer function~~ Bode ~~that~~ plot below, we can clearly tell that transfer function of this circuit is constant when $R_1C_1 = R_2C_2$ at -20dB (approx).

Hence, there doesn't exist a -3dB bandwidth in the circuit.

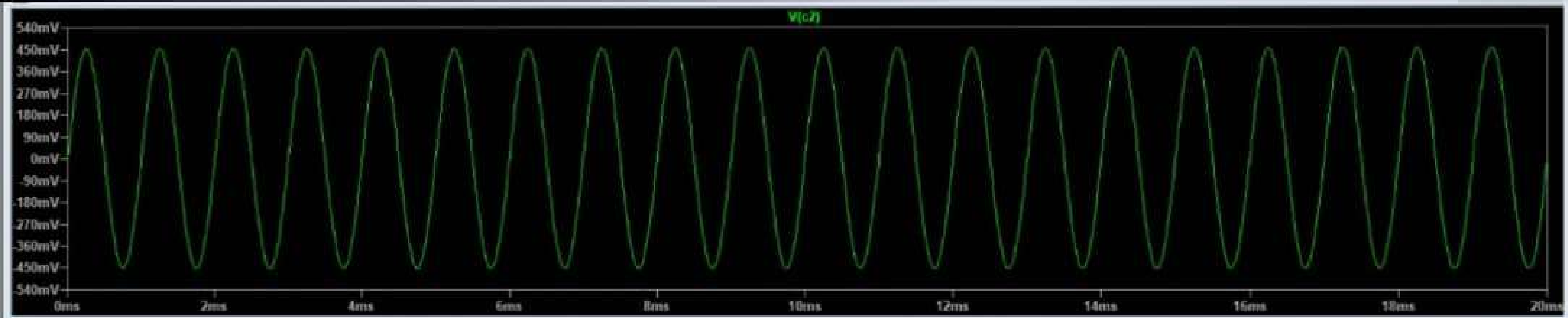
below at

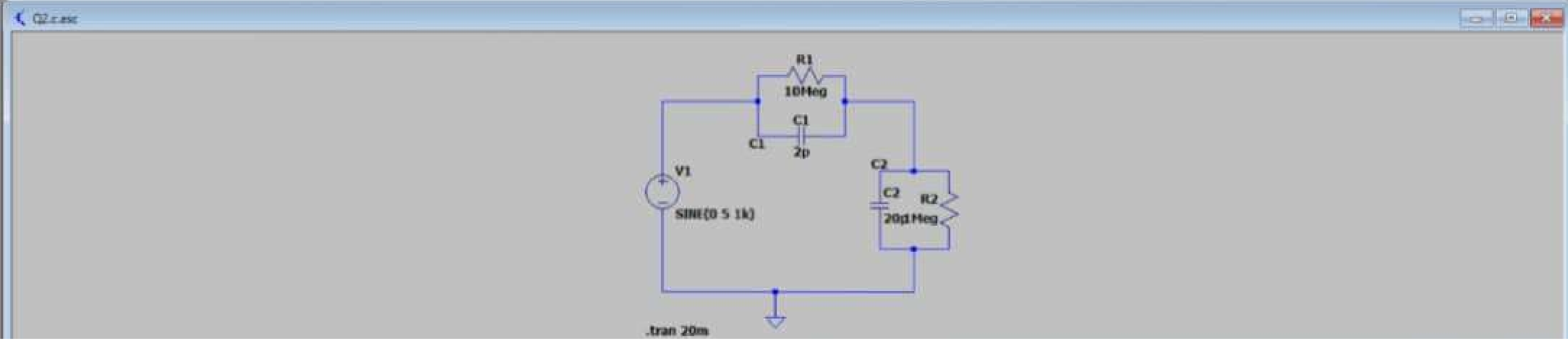
... plots shown

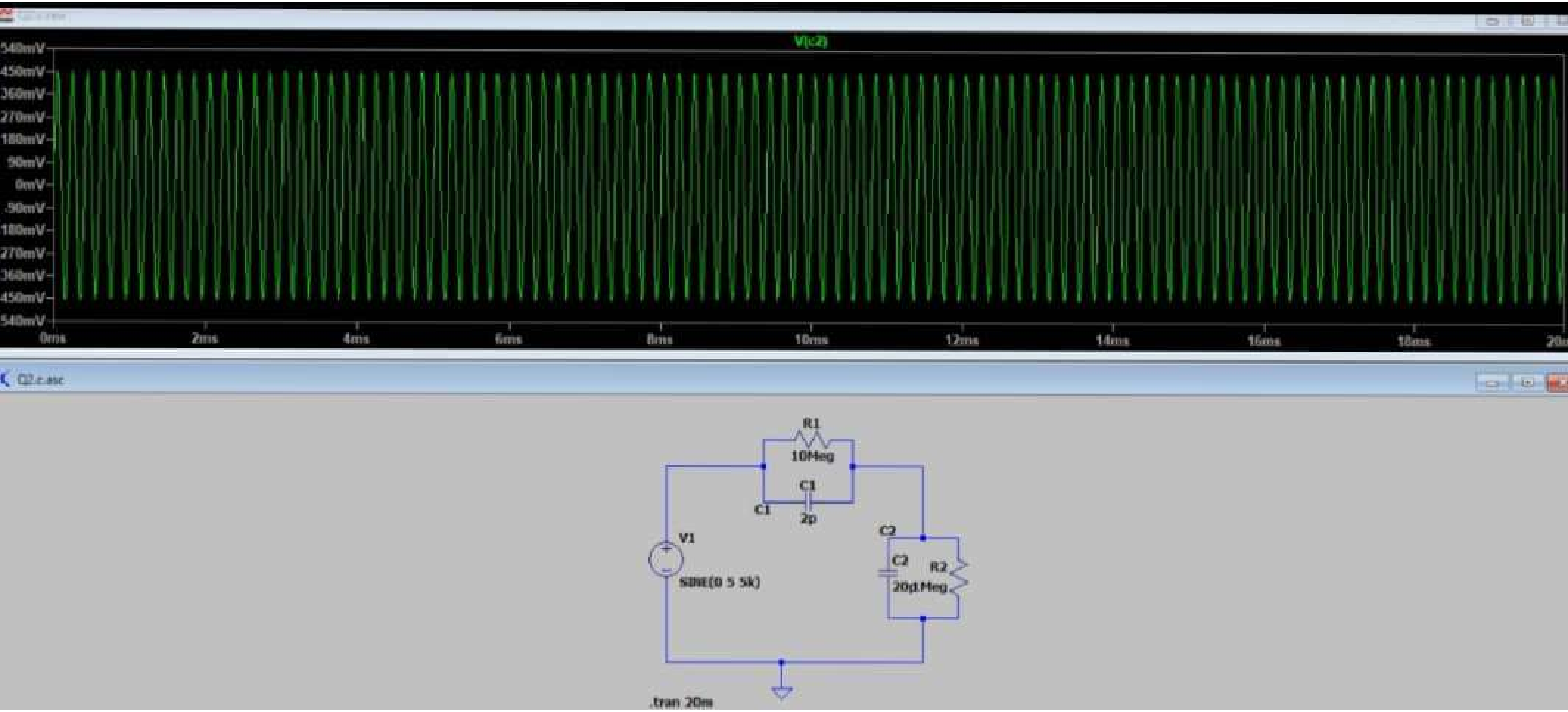

V(c2)
20.827853dB
20.827854dB
18f°
15f°
12f°
9f°
6f°
3f°
0f°
-3f°
-6f°
-9f°
10KHz
100KHz
1MHz
10MHz
100MHz
1GHz

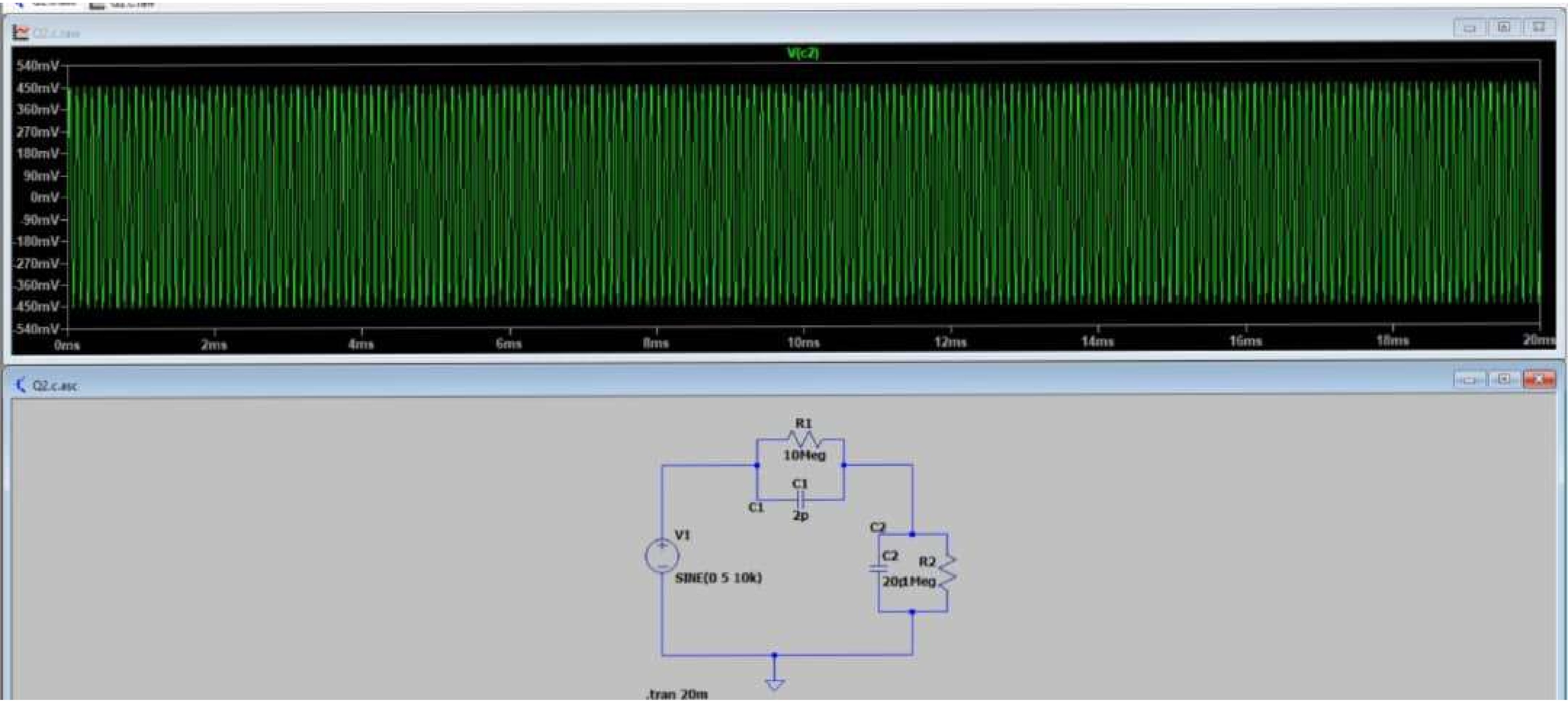

Q2.c.asc
R1
10Meg
C1
C1
2p
C2
V1
C2
R2
20µ
Meg
SINE(0 5 5k)
AC 1 0
.ac dec 1000 5k 5G

(vi) From the three plots shown below at input frequencies $f = 1kHz, 5kHz, 10kHz$, we can see that there is a change in the output frequency but not the amplitude.


V(c2)
540mV
450mV
360mV
270mV
180mV
90mV
0mV
-90mV
-180mV
-270mV
-360mV
-450mV
-540mV
0ms
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
20ms


Q2.c.asc
R1
10Meg
C1
C1
2p
V1
SINE(0 5 1k)
C2
C2
R2
20p
1Meg
.tran 20m


V(c2)
540mV
450mV
360mV
270mV
180mV
90mV
0mV
-90mV
-180mV
-270mV
-360mV
-450mV
-540mV
0ms
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
Q2.c.asc
R1
10Meg
C1
C1
2p
V1
SINE(0 5 5k)
C2
C2
R2
20p
1Meg
.tran 20m



V(c2)
540mV
450mV
360mV
270mV
180mV
90mV
0mV
-90mV
-180mV
-270mV
-360mV
-450mV
-540mV
0ms
2ms
4ms
6ms
8ms
10ms
12ms
14ms
16ms
18ms
20ms
Q2.c.asc
R1
10Meg
C1
C1
2p
V1
SINE(0 5 10k)
C2
C2
R2
20p
1Meg
.tran 20m

3) Bode Plots

(a) $H(j\omega) = 1 + j\omega T$

$$|H(j\omega)| = \sqrt{1+\omega^2 T^2}$$

$$\phi = \tan^{-1}(\omega T)$$

Take

$$20\log|H(j\omega)| = 20\log\sqrt{1+\omega^2T^2}$$

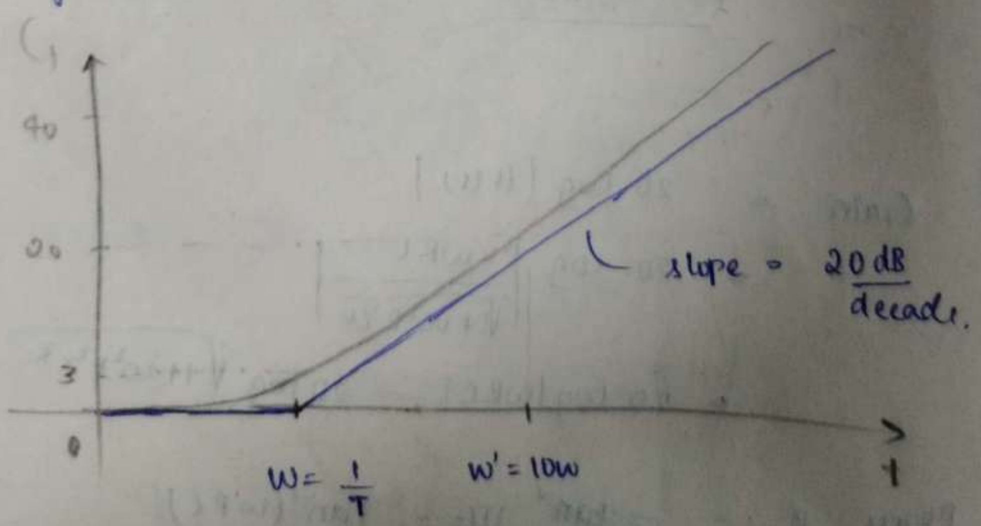
(i) $\omega << \frac{1}{T}$ $\quad H(j\omega) = 0\ dB$

(ii) $\omega = \frac{1}{T}$ $\quad H(j\omega) = 3\ dB$

(iii) $\omega >> \frac{1}{T}$ $\quad H(j\omega) = 20\log(\omega T)$

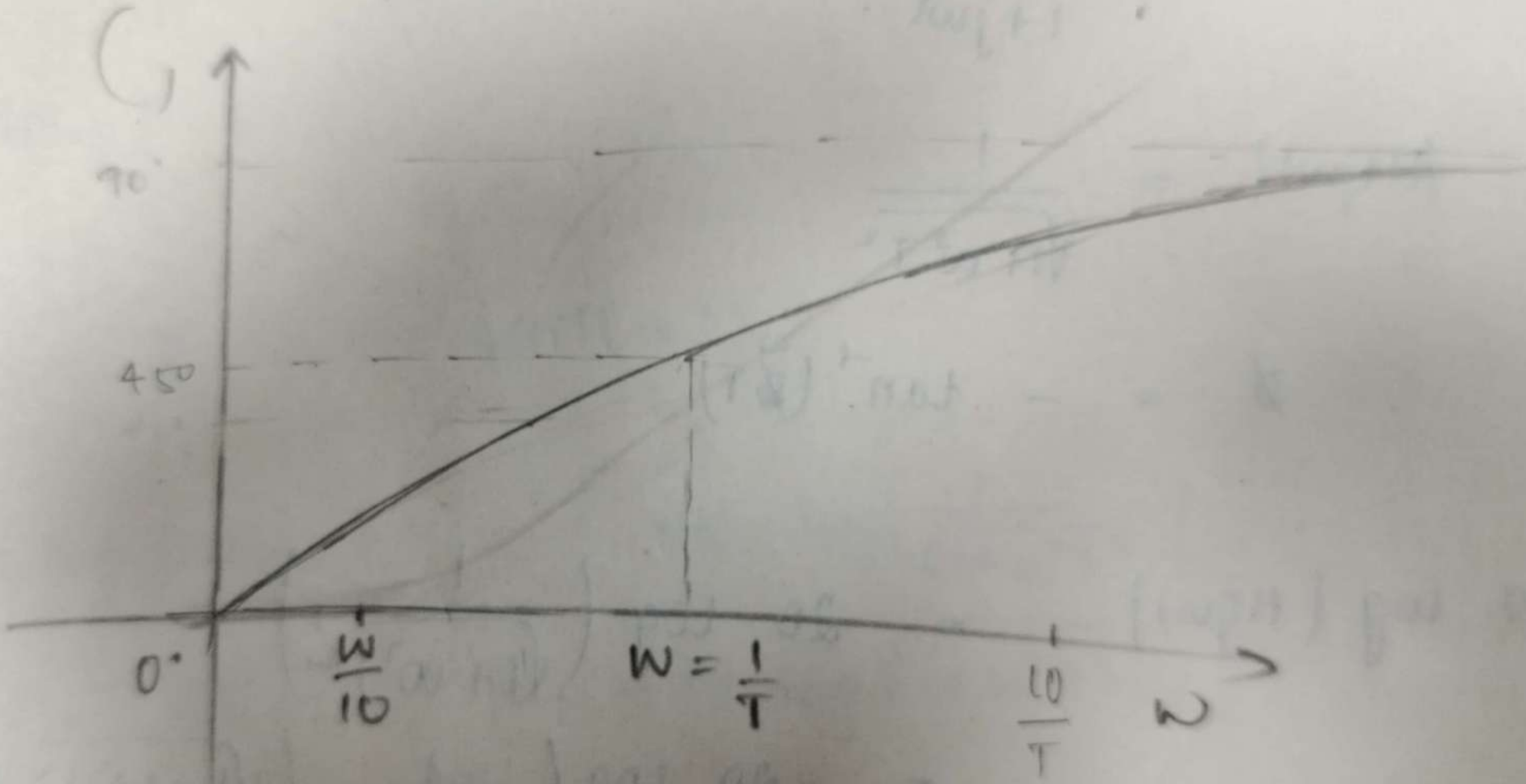
Magnitude Plot

$20\log|H(j\omega)|$



Phase Plot

$\phi = \tan^{-1}(\omega T)$



(b) $H(j\omega) = \dfrac{1}{1+j\omega T}$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2T^2}}$$

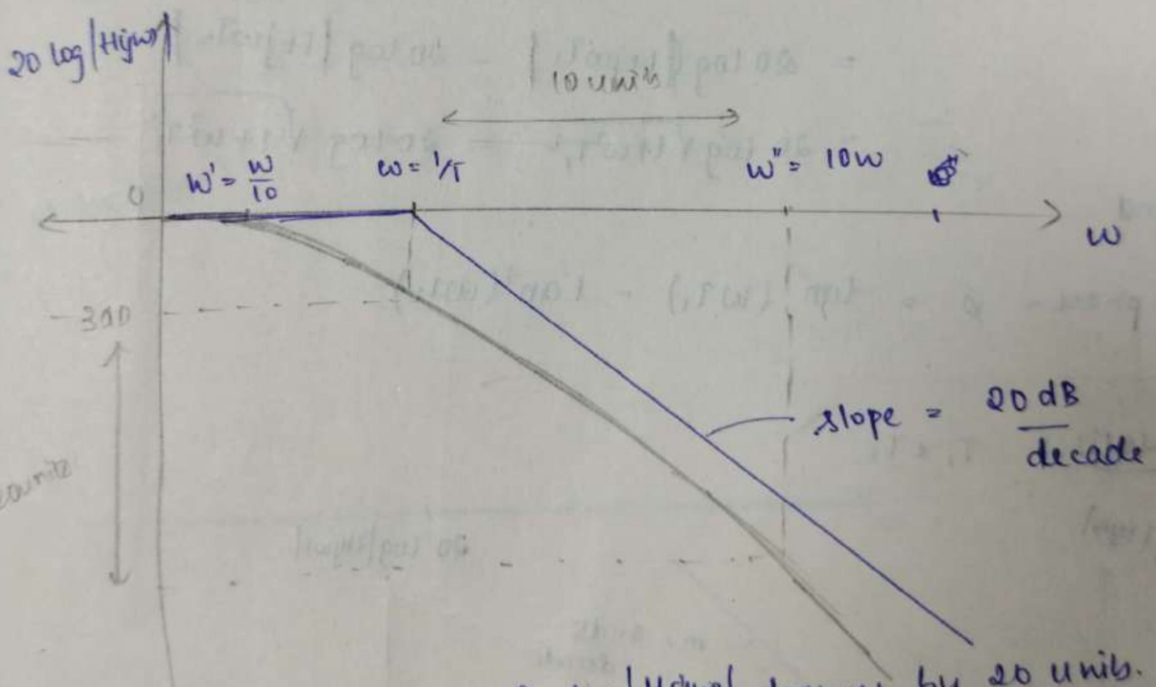
$$\phi = -\tan^{-1}(\omega T)$$

$$20\log|H(j\omega)| = 20\log\left(\frac{1}{\sqrt{1+\omega^2T^2}}\right)$$
$$= -20\log\sqrt{1+\omega^2T^2}$$

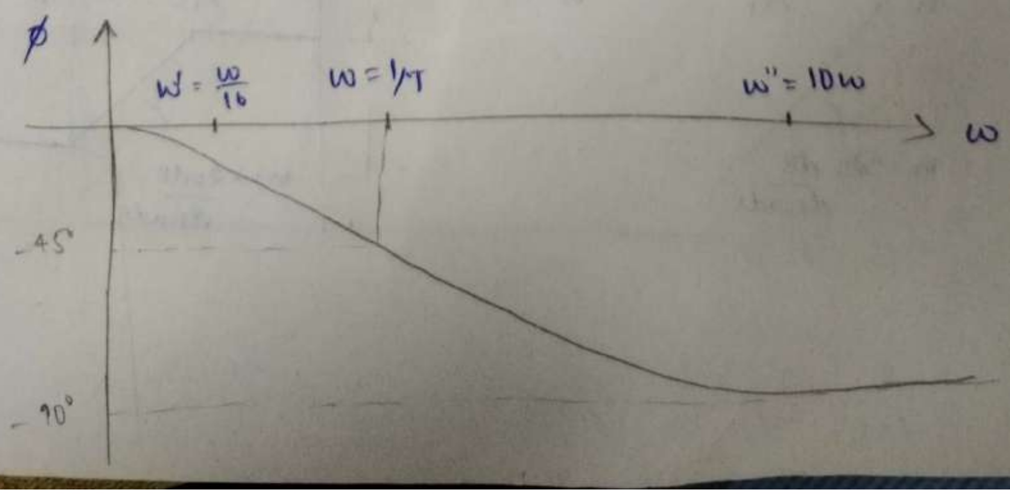
(i) $\omega T << 1 \Rightarrow 0$ dB

(ii) $\omega T = 1 \Rightarrow \cancel{20}\ -10\log 2 = -3$dB

(iii) $\omega T >> 1 \Rightarrow -20\log(\omega T)$



For every 10ω change, 20 log|H(jω)| changes by 20 units.



(c) $H(j\omega) = \dfrac{1+j\omega T_1}{1+j\omega T_2}$

$$20\log|H(j\omega)| = 20\log\left|\frac{1+j\omega T_1}{1+j\omega T_2}\right|$$

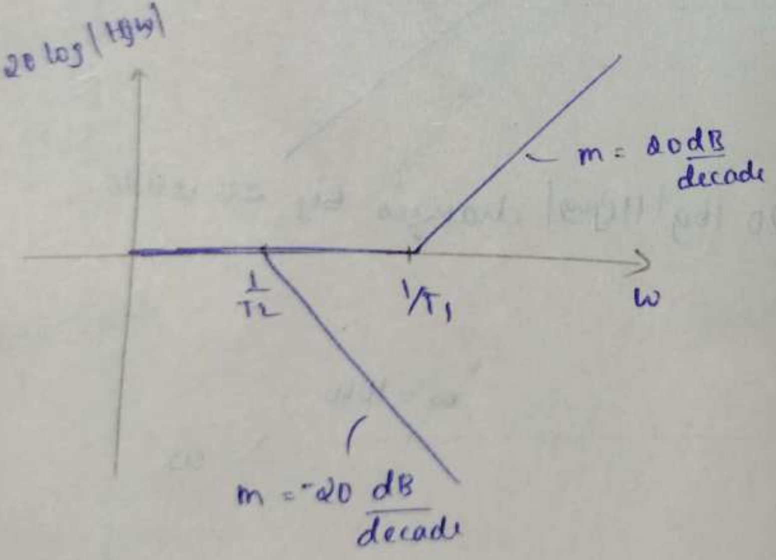
$$= 20\log|1+j\omega T_1| - 20\log|1+j\omega T_2|$$

$$= 20\log\sqrt{1+\omega^2 T_1^2} - 20\log\sqrt{1+\omega^2 T_2^2}$$

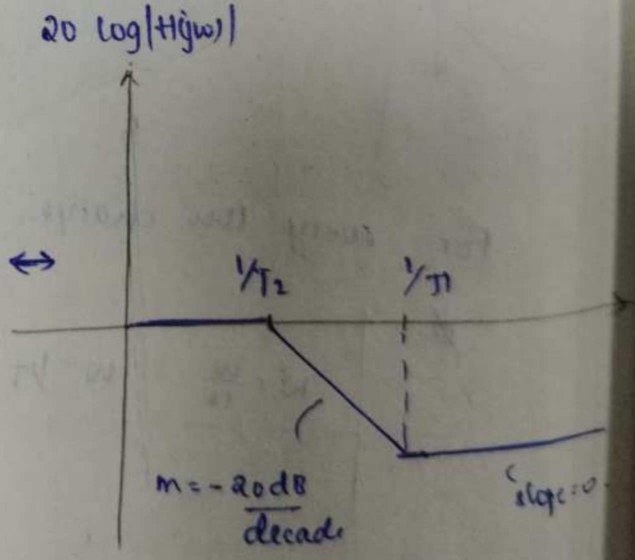
and

$$\text{phase} = \phi = \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

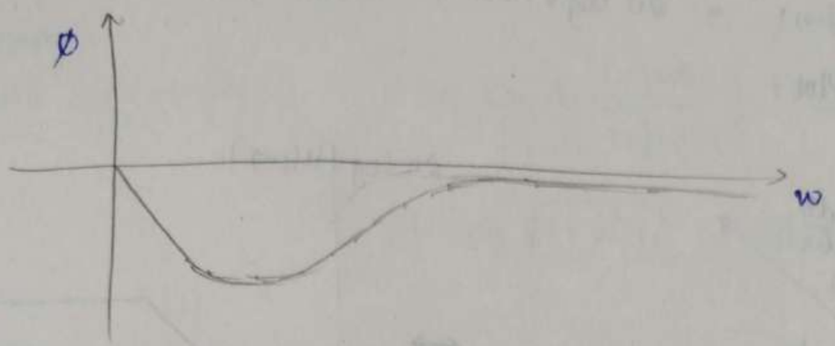
case (i) : $T_1 < T_2$



$\leftrightarrow$



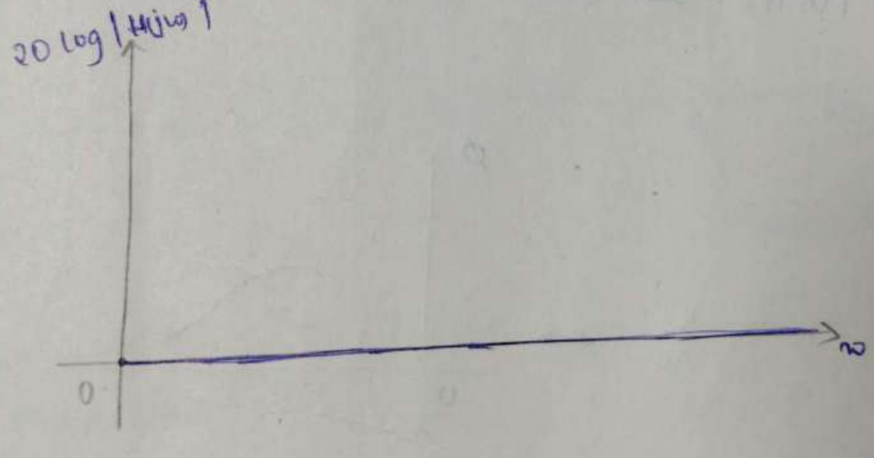
$\phi = \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$

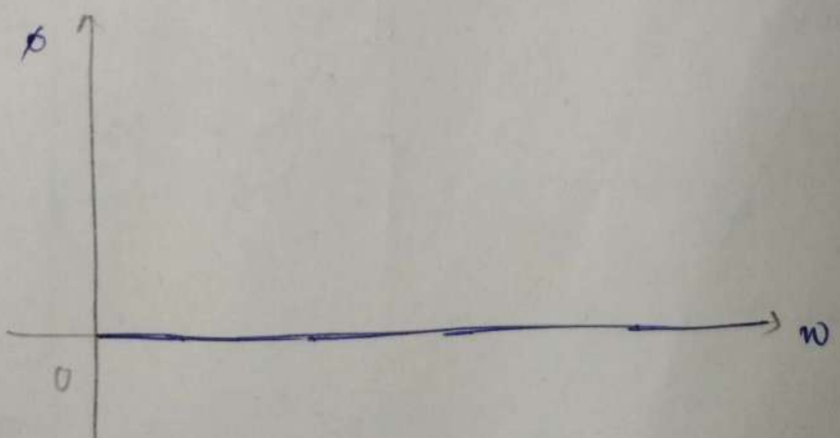


Case-(ii) - $T_1 = T_2$

$$20 \log |H(j\omega)| = 20 \log \sqrt{1+\omega^2 T_1^2} - 20 \log \sqrt{1+\omega^2 T_2^2}$$
$$= 0.$$

$$\phi = \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$
$$= 0.$$

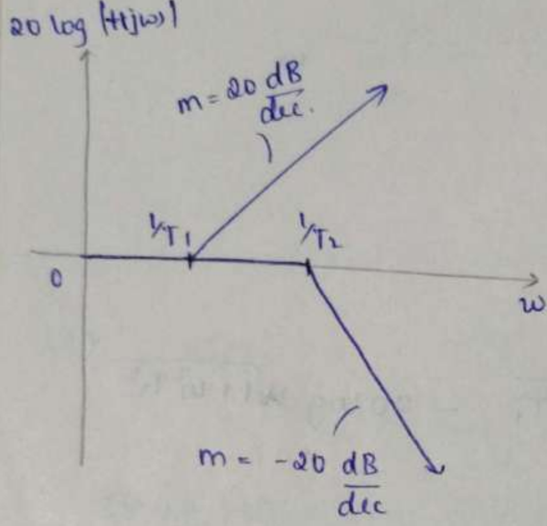




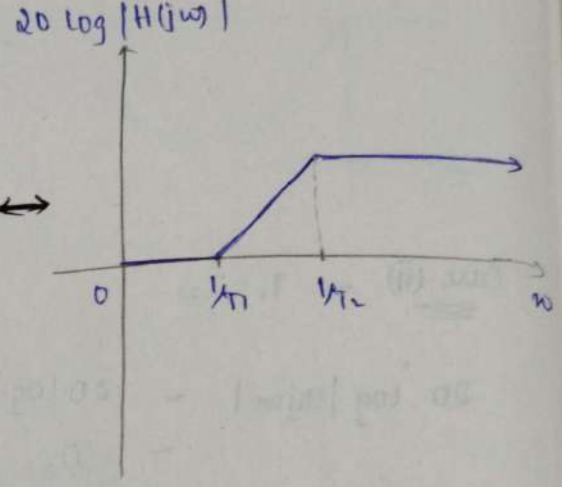
Case (iii) $T_1 > T_2$

$$20 \log |H(j\omega)| = 20 \log \sqrt{1+\omega^2 T_1^2} - 20 \log \sqrt{1+\omega^2 T_2^2}$$

Magnitude Plot :

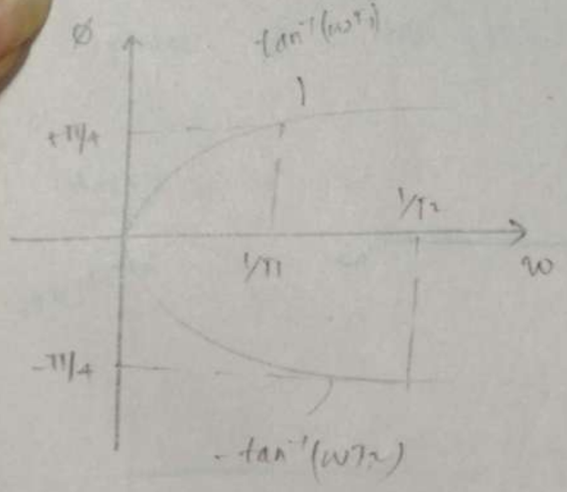


$\leftrightarrow$

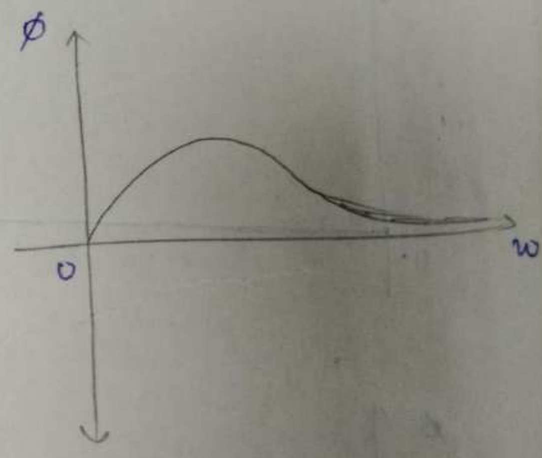


Phase Plot :

$$\phi = \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$



$\leftrightarrow$



(d) $H(j\omega) = \dfrac{1 - j\omega T_1}{1 + j\omega T_2}$

magnitude

$$20 \log |H(j\omega)| = 20 \log \left| \frac{1 - j\omega T_1}{1 + j\omega T_2} \right|$$

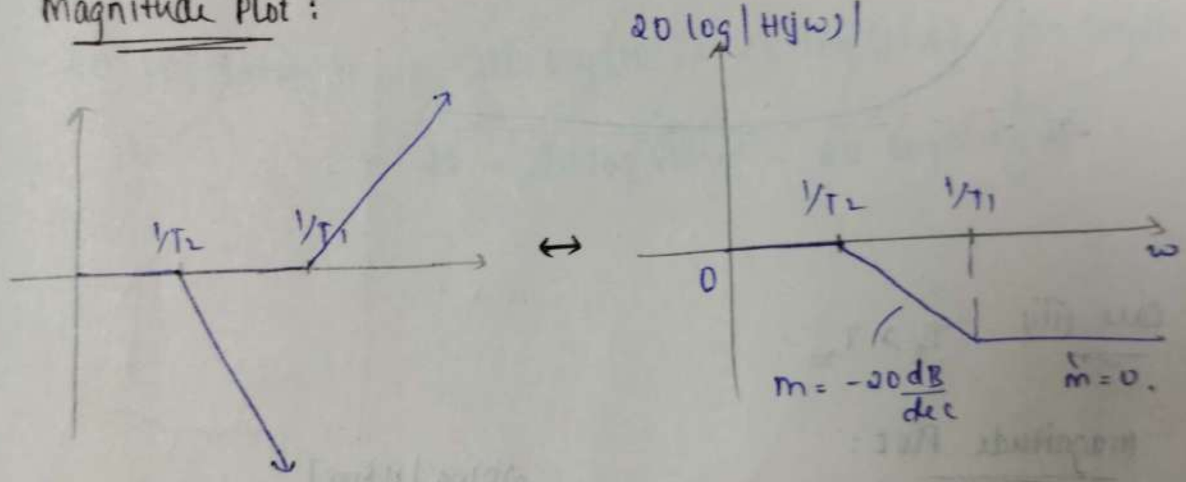
$$= 20 \log \sqrt{1 + \omega^2 T_1^2} - 20 \log \sqrt{1 + \omega^2 T_2^2}$$

Phase, $\phi = \tan^{-1}(-\omega T_1) - \tan^{-1}(\omega T_2)$

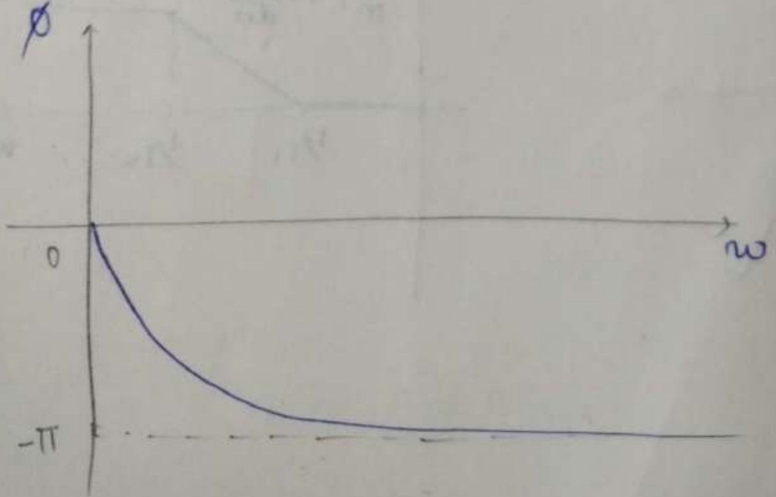
$$= -\left[\tan^{-1}(\omega T_1) + \tan^{-1}(\omega T_2)\right]$$

Case (i): $T_1 < T_2$

Magnitude Plot :



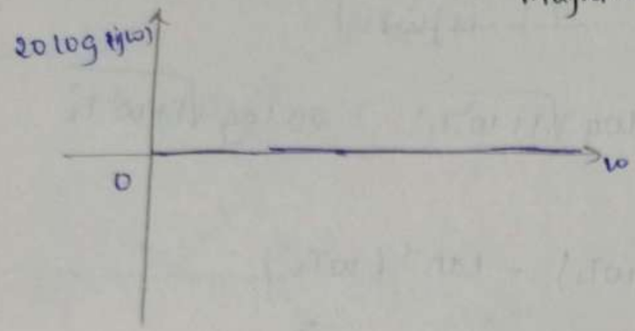
Phase Plot



Case (ii) – $T_1 = T_2$

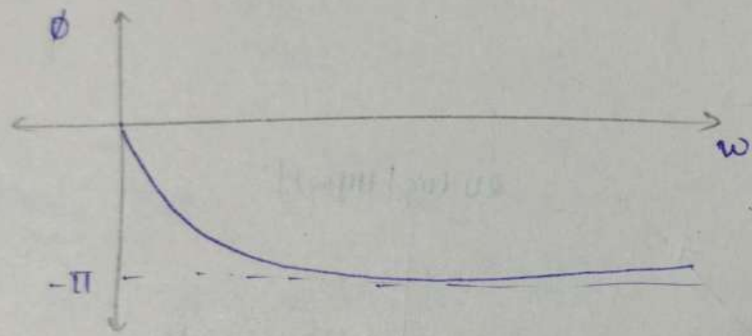
Magnitude = 0

Magnitude Plot



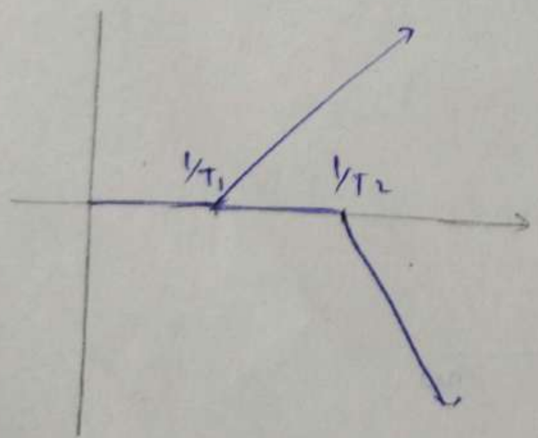
$\phi = -2\tan^{-1}(\omega T_1)$

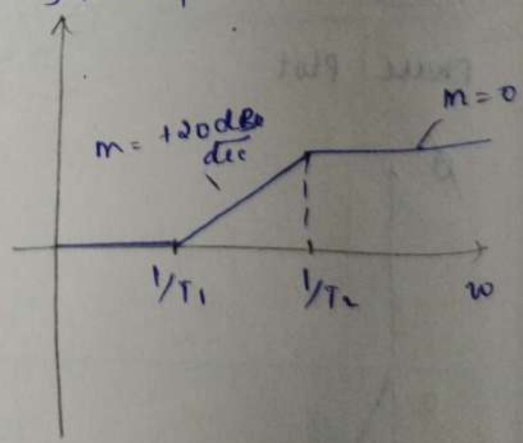
Phase Plot.



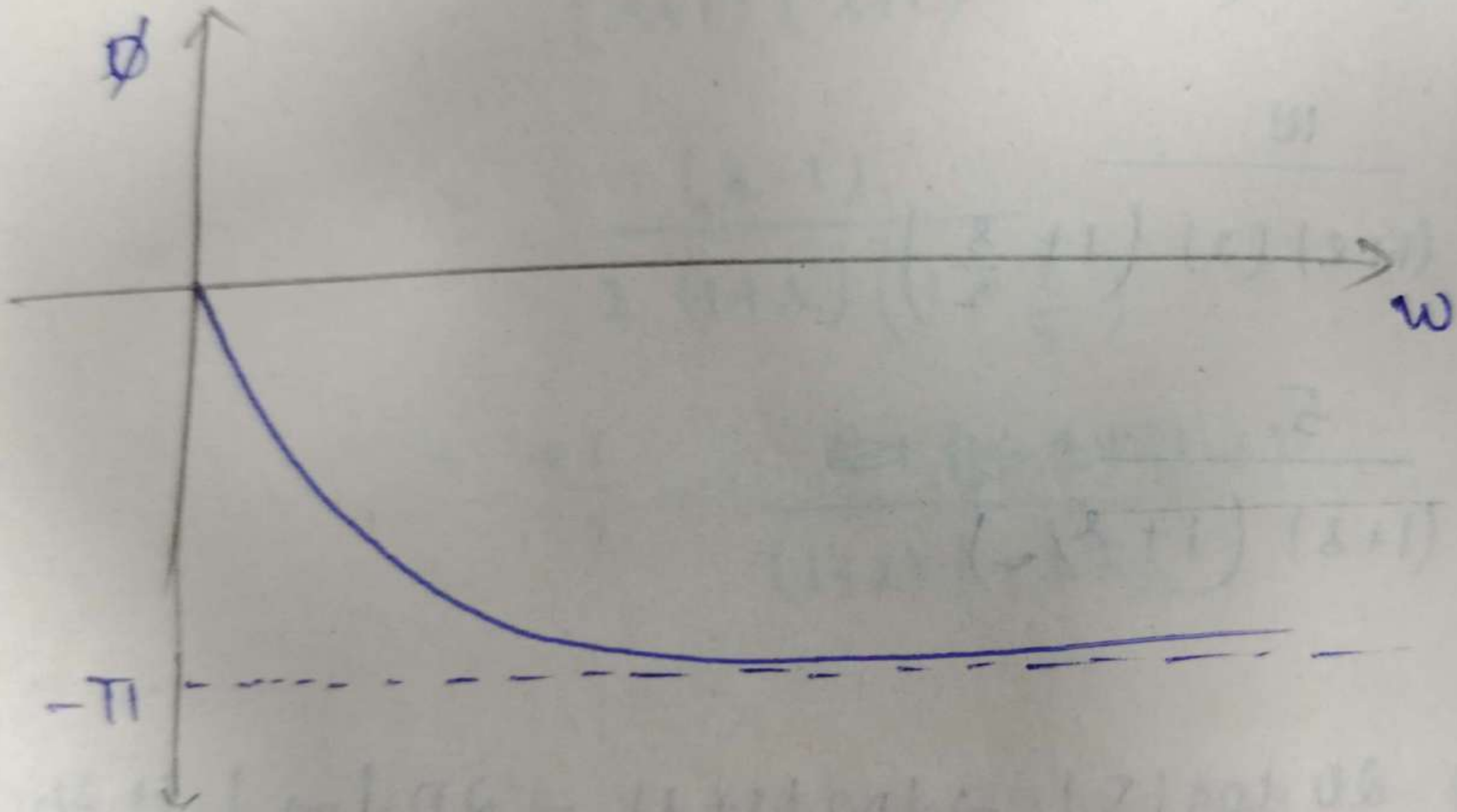
Case (iii) $T_1 > T_2$

Magnitude Plot:





Phase plot :



(e) $$H(s) = \frac{10}{(s+1)(s+2)}$$

$$= \frac{10}{(1+s)(2)\left(1+\frac{s}{2}\right)}$$

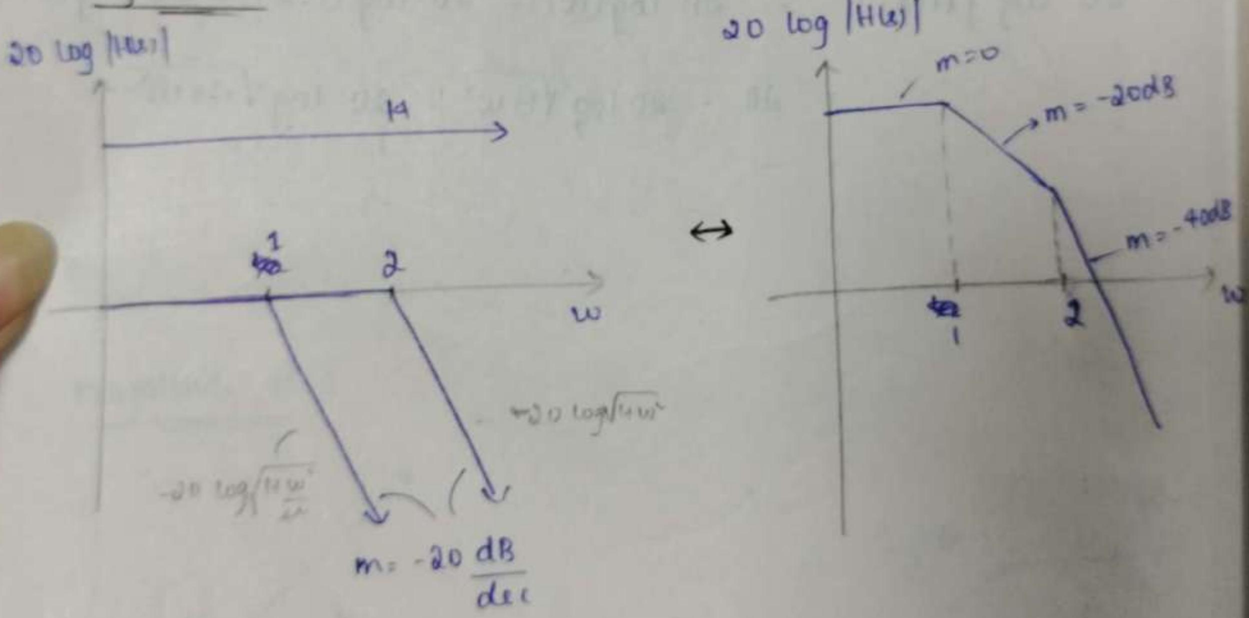
$$= \frac{5}{(1+s)\left(1+s/2\right)}$$

$$20 \log |H(s)| = 20\log(5) - 20\log|1+s| - 20\log\left|1+s/2\right|$$

$$= 14 - 20\log\sqrt{1+\omega^2} - 20\log\sqrt{1+\omega^2/4}$$

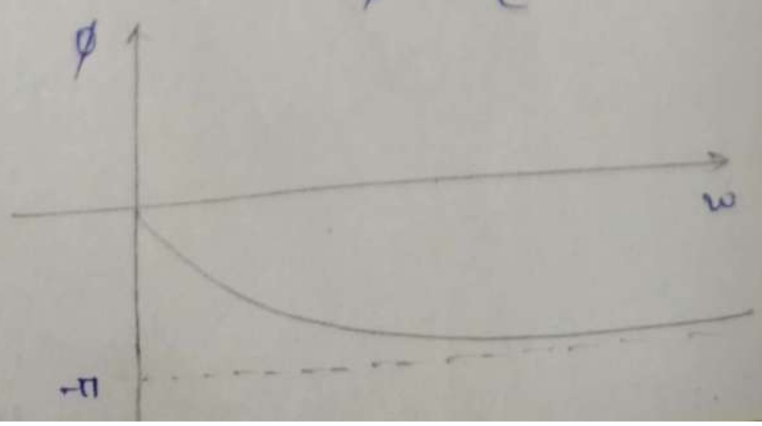
$$= 14 - 20\log\sqrt{1+\omega^2} - 20\log\sqrt{1+\omega^2\left(\tfrac{1}{2}\right)^2}$$

Magnitude Plot :



Phase Plot :

$$\phi = -\left[\tan^{-1}(\omega) + \tan^{-1}\left(\frac{\omega}{2}\right)\right]$$



(f) $H(s) = \dfrac{(s-1)}{(s+1)(s+2)}$

$= \dfrac{(s-1)}{2(1+s)\left(1+\frac{s}{2}\right)}$

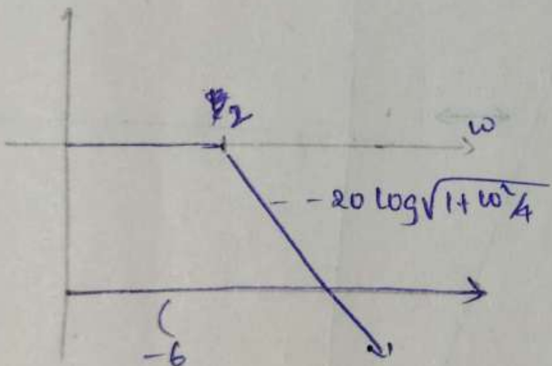
$= +\dfrac{1}{2} \dfrac{(s-1)}{(1+s)(1+s/2)}$

$20 \log |H(s)| = 20 \log |s-1| - 20 \log |1+s| - 20 \log \left|1+\frac{s}{2}\right| - 20 \log 2$

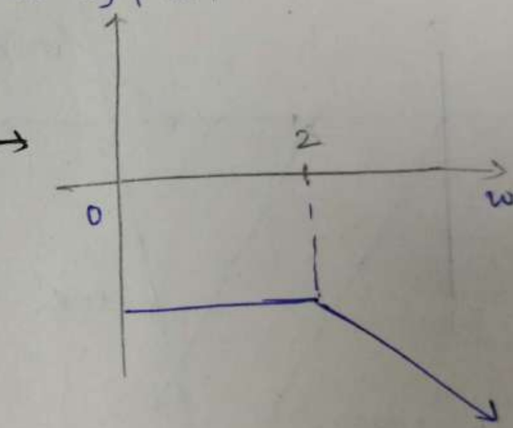
$= \cancel{20 \log \sqrt{1+\omega^2}} - \cancel{20 \log \sqrt{1+\omega^2}} - 20 \log \sqrt{1+\omega^2(1/2)^2} - 6$

$= -6 - 20 \log \sqrt{1+\omega^2(1/2)^2}$

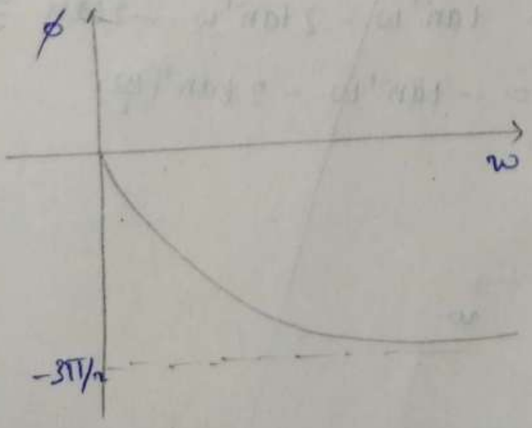
Magnitude Plot:



$\leftrightarrow$



Phase Plot:



$\phi = \tan^{-1}(-\omega) - \tan^{-1}(\omega) - \tan^{-1}(\omega/2)$

$= -2\tan^{-1}\omega - \tan^{-1}(\omega/2)$

(g) $H(s) = \dfrac{2(s+1)}{s^2(s+2)(s+0.5)}$

$= \dfrac{2(s+1)}{(s)(s)\cancel{(2)}\left(1+\frac{s}{2}\right)\cancel{(2)}\frac{1}{\cancel{2}}(1+2s)}$

$= \dfrac{2(s+1)}{(s)(s)\left(1+\frac{s}{2}\right)(1+2s)}$

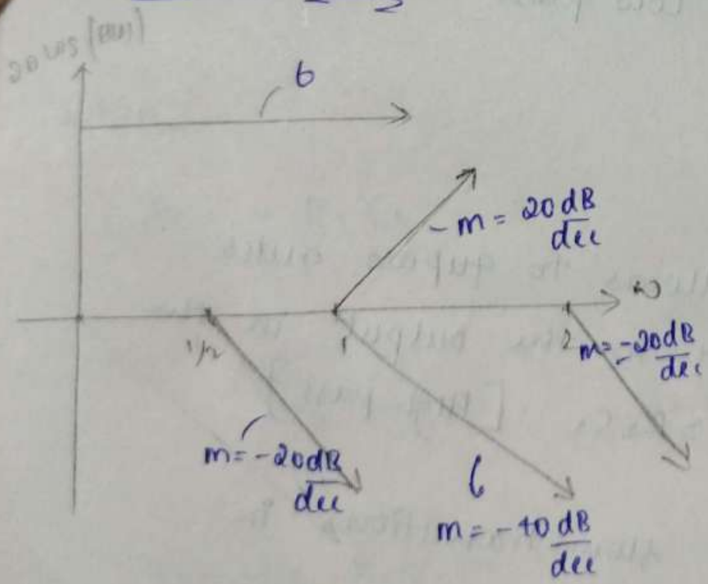
$20\log|H(s)| = 20\log 2 + 20\log|s+1| - 2(20)\log|s|$

$- 20\log|1+s/2| - 20\log|1+2s|$

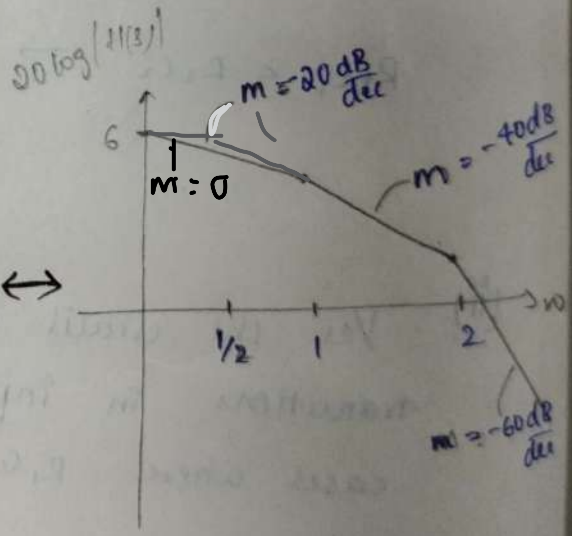
$= 6 + 20\log\sqrt{1+\omega^2} - 40\log\omega$

$- 20\log\sqrt{1+\frac{\omega^2}{4}} - 20\log\sqrt{1+4\omega^2}$

Magnitude Plot :

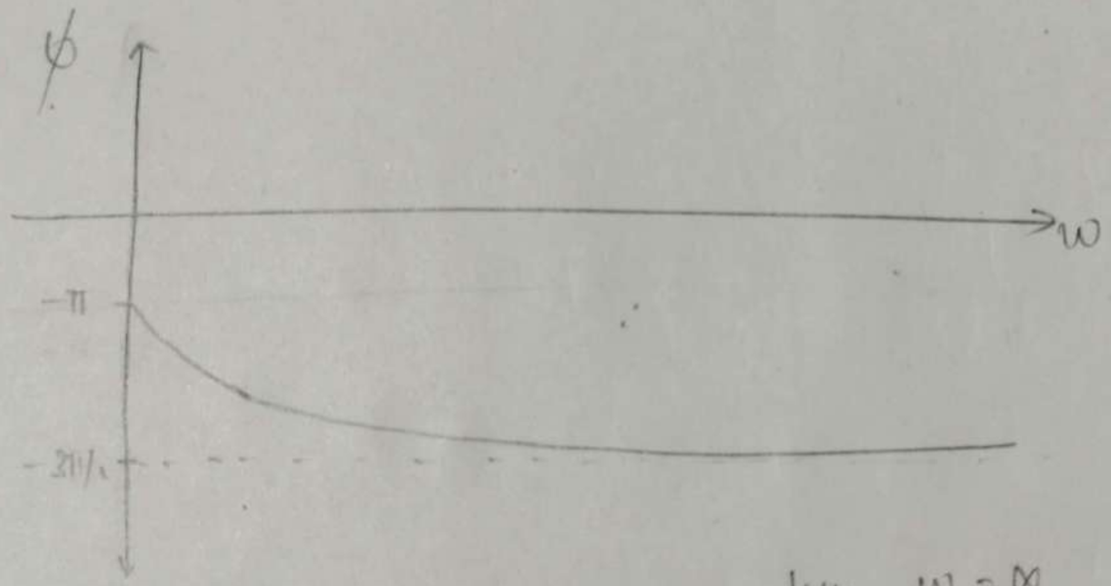


↔



Phase Plot:

$$\phi = \tan^{-1}\omega - \pi - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}(2\omega)$$



when $\omega = \infty$ $\quad \phi = -3\pi/2$

$\omega = 0$ $\quad \phi = -\pi$