1.48. Let z_0 be a complex number with polar coordinates (r_0, θ_0) and Cartesian coordinates (x_0, y_0). Determine expressions for the Cartesian coordinates of the following complex numbers in terms of x_0 and y_0. Plot the points z_0, z_1, z_2, z_3, z_4, and z_5 in the complex plane when $r_0 = 2$ and $\theta_0 = \pi/4$ and when $r_0 = 2$ and $\theta_0 = \pi/2$. Indicate on your plots the real and imaginary parts of each point.

(a) $z_1 = r_0 e^{-j\theta_0}$ **(b)** $z_2 = r_0$ **(c)** $z_3 = r_0 e^{j(\theta_0+\pi)}$

(d) $z_4 = r_0 e^{j(-\theta_0+\pi)}$ **(e)** $z_5 = r_0 e^{j(\theta_0+2\pi)}$

1.49. Express each of the following complex numbers in polar form, and plot them in the complex plane, indicating the magnitude and angle of each number:

(a) $1 + j\sqrt{3}$ **(b)** -5 **(c)** $-5 - 5j$

(d) $3 + 4j$ **(e)** $(1 - j\sqrt{3})^3$ **(f)** $(1 + j)^5$

(g) $(\sqrt{3} + j^3)(1 - j)$ **(h)** $\frac{2-j(6/\sqrt{3})}{2+j(6/\sqrt{3})}$ **(i)** $\frac{1+j\sqrt{3}}{\sqrt{3}+j}$

(j) $j(1 + j)e^{j\pi/6}$ **(k)** $(\sqrt{3} + j)2\sqrt{2}e^{-j\pi/4}$ **(l)** $\frac{e^{j\pi/3}-1}{1+j\sqrt{3}}$

1.50. **(a)** Using Euler's relationship or Figure P1.48, determine expressions for x and y in terms of r and θ.

(b) Determine expressions for r and θ in terms of x and y.

(c) If we are given only r and $\tan\theta$, can we uniquely determine x and y? Explain your answer.

as

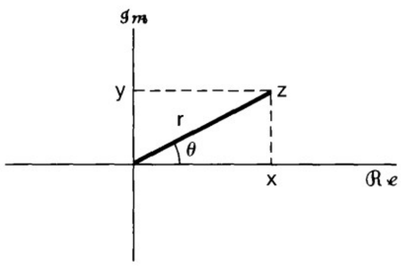

Figure P1.48

1.52. Let z denote a complex variable; that is,

$$z = x + jy = re^{j\theta}.$$

The *complex conjugate* of z is

$$z^* = x - jy = re^{-j\theta}.$$

Derive each of the following relations, where z, z_1, and z_2 are arbitrary complex numbers:

(a) $zz^* = r^2$

(b) $\frac{z}{z^*} = e^{j2\theta}$

(c) $z + z^* = 2\mathcal{Re}\{z\}$

(d) $z - z^* = 2j\mathcal{Im}\{z\}$

(e) $(z_1 + z_2)^* = z_1^* + z_2^*$

(f) $(az_1z_2)^* = az_1^*z_2^*$, where a is any real number

(g) $(\frac{z_1}{z_2})^* = \frac{z_1^*}{z_2^*}$

(h) $\mathcal{Re}\{\frac{z_1}{z_2}\} = \frac{1}{2}[\frac{z_1z_2^*+z_1^*z_2}{z_2z_2^*}]$

1.56. Evaluate each of the following integrals, and express your answer in Cartesian (rectangular) form:

(a) $\int_0^4 e^{j\pi t/2}dt$ **(b)** $\int_0^6 e^{j\pi t/2}dt$

(c) $\int_2^8 e^{j\pi t/2}dt$ **(d)** $\int_0^\infty e^{-(1+j)t}dt$

(e) $\int_0^\infty e^{-t}\cos(t)dt$ **(f)** $\int_0^\infty e^{-2t}\sin(3t)dt$

18.1 Trigonometric Form of the Fourier Series

1. Determine the fundamental frequency, fundamental radian frequency, and period of the following: (*a*) $5\sin 9t$; (*b*) $200\cos 70t$; (*c*) $4\sin(4t - 10°)$; (*d*) $4\sin(4t + 10°)$.
2. Plot multiple periods of the first, third, and fifth harmonics on the same graph of each of the following periodic waveforms (three separate graphs in total are desired): (*a*) $3\sin t$; (*b*) $40\cos 100t$; (*c*) $2\cos(10t - 90°)$.

3. Calculate a_0 for the following: (*a*) $4\sin 4t$; (*b*) $4\cos 4t$; (*c*) $4 + \cos 4t$; (*d*) $4\cos(4t + 40°)$.
4. Compute a_0, a_1, and b_1 for the following functions: (*a*) $2\cos 3t$; (*b*) $3 - \cos 3t$; (*c*) $4\sin(4t - 35°)$.

6. (*a*) Compute the Fourier coefficients a_0, a_1, a_2, a_3, a_4, b_1, b_2, b_3, and b_4 for the periodic function $g(t)$ partially sketched in Fig. 18.28. (*b*) Plot $g(t)$ along with the Fourier series representation truncated after $n = 4$.

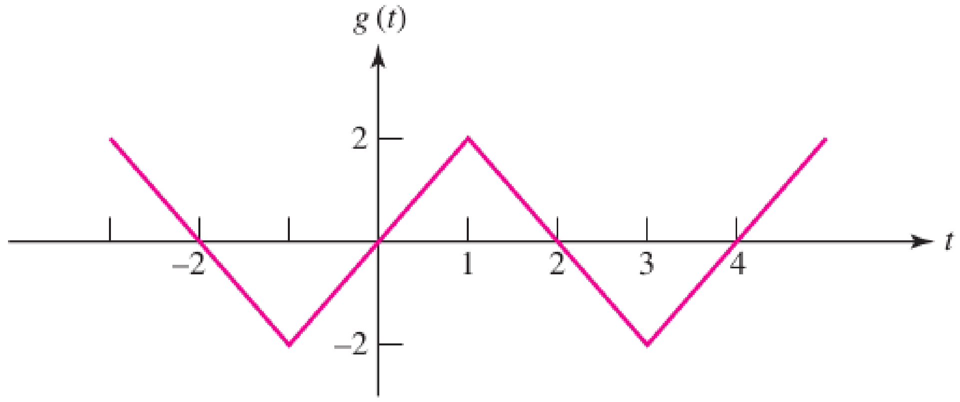

FIGURE 18.28