EC5.101 - Network, Signals and Systems
Assignment 1

Release date: 17 Aug 2023 Due date: 24 Aug 2023

Instructions:

1. The handwritten assignment must be submitted individually.
2. Students are free to refer to class notes and textbooks. Discussions are allowed but copying and plagiarism will attract strict penalty.
3. Late submission: 10 % penalty per day (up to at most 3 days after deadline).
4. Mention any additional assumptions you make that is not given in the question.
5. Clearly show the steps used to arrive at the solutions.

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1. [6] Identify whether the given signals are odd, even, or neither. For periodic signals, comment on their half-wave symmetry.
   (a) $x(t) = t^2\cos(3t)$
   (b) $x(t) = t\cos(70t) + t\sqrt{1+t^2}$
   (c) $x(t) = \sin^2(7t) + \sin(t)$
   (d) $x(t) = \log(\cos(t))$

2. [4] Let the trigonometric FS coefficients of a periodic signal $x(t)$ be written in the format $\{a_0, [a_1, a_2, \ldots], [b_1, b_2, \ldots]\}$. FS coefficients of two periodic signals $x_1(t)$ and $x_2(t)$ having the same period are given below:
   (a) $x_1(t) \longleftrightarrow \{1, [1, 0, 1], [2]\}$ and $x_2(t) \longleftrightarrow \{0, [-1, 0, -1], [-2]\}$
   (b) $x_1(t) \longleftrightarrow \{0, [1, 0, 1], [0, 2]\}$ and $x_2(t) \longleftrightarrow \{1, [0, 1, 0], [2, 0]\}$

   Assume that all other coefficients are zero. In each case, identify whether the pair of signals $x_1(t)$ and $x_2(t)$ are orthogonal or not. Justify your answer without performing explicit computations.

3. [4] Consider the periodic signal given below:

$$x(t) = \sin(2t) + \sin^2(t).$$

   (a) Find the trigonometric Fourier series (FS) coefficients of $x(t)$.
   (b) Find the complex FS coefficients of $x(t)$.

4. [8] The complex FS coefficients of a periodic signal $x(t)$ are $\{j, -1, 2, \underset{\uparrow}{0}, 2, -1, -j\}$ where $j = \sqrt{-1}$. Find the FS coefficients of the following signals:

   (a) $2x(2t)$
   (b) $x(2-t)$
   (c) $x(t) - x(-t)$
   (d) $2 - x(t)$

5. [6] For a periodic signal with period $T = 1$ it is given that,

$$\int_0^1 |x(t)|^2 dt = 4 = |a_{-1}|^2 + |a_0|^2 + |a_1|^2$$

   where a_k denote the complex FS coefficients.

   (a) If it is known that the signal is even and $a_{-1} = j$, then compute a_0.
   (b) If it is known that the signal is even and has half-wave symmetry, compute a_{-1}.

6. [12] Shiva is investigating alternate representations which can be used instead of trigonometric Fourier series for real periodic signals $x(t)$ of period $T = \frac{2\pi}{\omega_0}$. He proposes to replace the original basis signals $\sin(k\omega_0 t)$ and $\cos(k\omega_0 t), k \geq 1$ with their modified (quantized) versions given below:

$$q_k^{\sin}(t) = \begin{cases} 1, & \text{if } \sin(k\omega_0 t) \geq 0 \\ -1, & \text{if } \sin(k\omega_0 t) < 0 \end{cases}$$

$$q_k^{\cos}(t) = \begin{cases} 1, & \text{if } \cos(k\omega_0 t) \geq 0 \\ -1, & \text{if } \cos(k\omega_0 t) < 0 \end{cases}$$

   The modified series for a periodic signal is given by

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \, q_k^{\cos}(t) + \sum_{k=1}^{\infty} b_k \, q_k^{\sin}(t).$$

   (a) Sketch the modified basis signals $q_k^{\sin}(t)$ and $q_k^{\cos}(t)$ for $k = 1, 2$.
   (b) Of the four signals plotted in (a), identify all pairs of signals which are orthogonal over the period T. Prove your answers.
   (c) Assuming that a periodic signal $x(t)$ can be represented using the modified series, find the analysis equations, i.e., expressions for the coefficients a_k, b_k for $k \geq 1$ and a_0.
   (d) Find all the above coefficients for the following periodic signal with period T = 2,

$$x(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & 1 < t \leq 2. \end{cases}$$

   (e) (Optional) Show that every pair of signals in the set of modified basis signals is orthogonal over the period T.