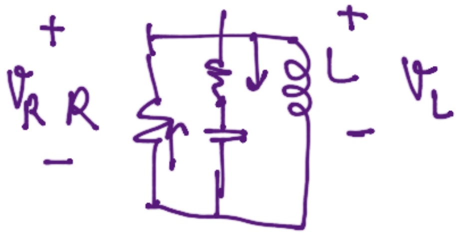
Chapter 8



$[i, v \rightarrow (t)] \rightarrow$ transient response

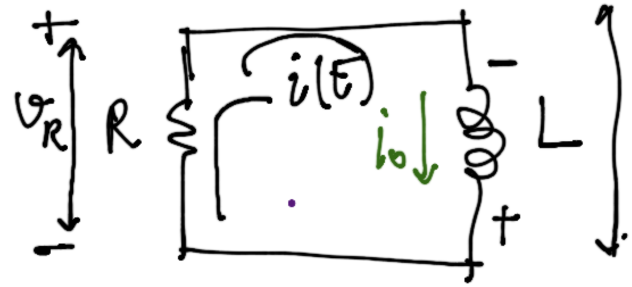
$[t \rightarrow \infty \quad i_\infty, v_\infty] \rightarrow$ Steady state value.

Natural response (R, L, C..)

Response : Natural (Source-free) + Forced response (external stimulation i, v)

$\underbrace{\text{Transient response}}_{\text{Complementary function}}$

(1) Source free R-L



i_0 dec.

v_L

KVL $\quad -iR + v_L = 0$

$$\Rightarrow -iR - L\frac{di}{dt} = 0$$

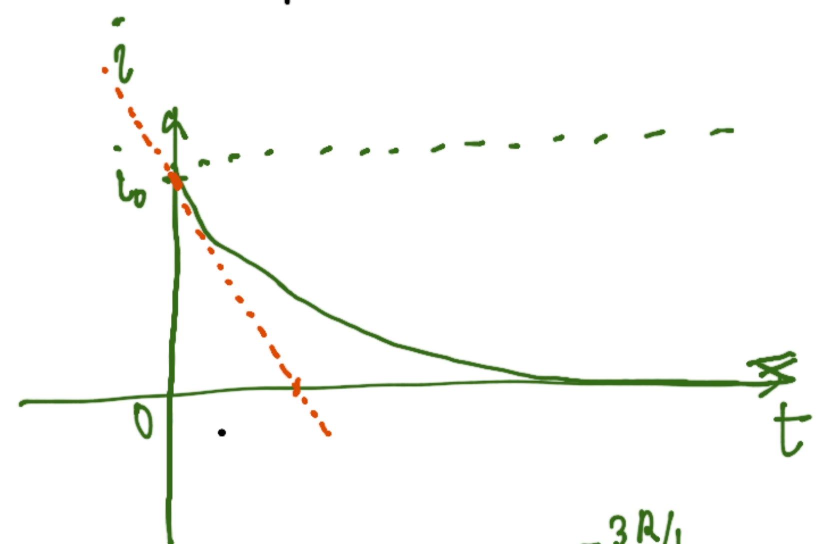
$i(t) \Rightarrow$ instantaneous current.

$$\Rightarrow \int_{i_0}^{i(t)} \frac{di}{i} = -\int_{t=0}^{t} \frac{R}{L}\, dt$$

$i : \quad i(t=0) \longrightarrow i(t)$

$\quad\quad \overset{\|}{i_0}$

$$[\ln i]_{i_0}^{i(t)} = -\frac{R}{L}(t-0)$$



$$\ln\frac{i(t)}{i_0} = -\frac{R}{L}t$$

$$\boxed{i(t) = i_0\, e^{-\frac{R}{L}t}}$$

$t=1 \quad e^{-R/L}$ $\quad t=2 \quad e^{-2R/L}$ $\quad t=3 \quad e^{-3R/L}$

$t=\infty \quad e^{-\infty} = \frac{1}{\infty} \to 0$

Another form of solution.

$$\boxed{\frac{di}{i} = \frac{-R}{L} dt}$$

$$\ln i = \frac{-R}{L} t + \text{Constant}$$

$t = 0 \quad i = i_0$

$$\ln i_0 = \text{constant}$$

$$\ln i = -\frac{R}{L} t + \ln i_0 \Rightarrow i = i_0 e^{-\frac{R}{L} t}$$

General Solution

$v, i \rightarrow$ exponential $\leftarrow e^{\omega t}$ $(\sin \omega t + i \cos \omega t)$

$\rightarrow i(t) = A e^{st}$: general solution

$\rightarrow \frac{di(t)}{dt} = A s e^{st}$

$\left\{ \begin{array}{l} \text{solve for } s = s_1, s_2, \ldots\ldots \\ A_1, A_2 \ldots A \end{array} \right.$

$$\frac{di}{dt} = -\frac{R}{L}\, i(t)$$

$$sAe^{st} = -\frac{R}{L} A e^{st}$$

$$sA = -\frac{R}{L} A$$

Solution : $s_1 = -\frac{R}{L}$

$i(t) = A e^{-\frac{R}{L}t}$: general solution.

$\uparrow\uparrow$

$i(t=0) \longrightarrow$ solve for A

General case.

$a \frac{df}{dt} + bf = 0$ where f is changing with time

$f = Ae^{st}$

$f' = Ase^{st}$

$\hookrightarrow a\,Ase^{st} + bAe^{st} = 0 \Rightarrow As + b = 0$

$s = -b/A$

Energy

$$i_L = i_R = i_0 e^{-\frac{R}{L}t}$$

$$P_R(t) = i^2 R = \left(i_0 e^{-\frac{Rt}{L}}\right)^2 \cdot R$$

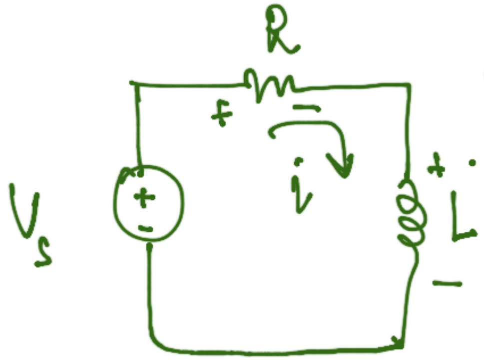
$$P_R(t) = i_0^2 R e^{-\frac{2Rt}{L}}$$

$$\text{Energy}_R = \int_0^t P_R(t) \cdot dt = i_0^2 R \int_0^t e^{-\frac{2Rt}{L}} dt$$

$$= i_0^2 \cancel{R} \left(\frac{1}{-2\cancel{R}/L}\right) \left[e^{-2Rt/L}\right]_0^t$$

$$= -\frac{1}{2} i_0^2 L \left(e^{-\frac{2Rt}{L}} - 1\right)$$

$t \to \infty$ $$\text{Energy}_R = -\frac{1}{2} i_0^2 L \left(\overset{0}{\cancel{\frac{1}{\infty}}} - 1\right) = \frac{1}{2} i_0^2 L$$



$\boxed{\text{L-R- circuit with } V_s}$

$\boxed{L \to S.C \quad t = \infty}$

$$V_s - iR - v_L = 0$$

$$v_L = L\frac{di}{dt}$$

$$V_s - iR - \underbrace{L\frac{di}{dt}} = 0$$

$\cancel{i\uparrow \Rightarrow \boxed{\frac{di}{dt}} > 0}$

$\cancel{v_L > 0}$

$$i = Ae^{st}$$

(I) $$V_s - iR - LAse^{st} = 0$$

(II) $$(V_s - iR) = +L\frac{di}{dt}$$

$$\int_{i_0}^{i(t)} \frac{di}{V_s - iR} = +\int_0^t \frac{dt}{L} \Rightarrow \left[-\frac{1}{R}\ln(V_s - iR)\right]_{i_0}^{i} = +\left[\frac{t}{L}\right]_0^t$$

$$\Rightarrow \left[\ln(V_s - iR)\right]_{i_0}^{i} = -\frac{R}{L}t$$

$$\ln\frac{V_s - iR}{V_s - i_0R} = -\frac{R}{L}t \Rightarrow \frac{V_s - iR}{V_s - i_0R} = e^{-\frac{R}{L}t}$$

$$V_s - iR = (V_s - i_0 R)\, e^{-\frac{R}{L}t}$$

$t = \infty$

∞

$t = \infty \;.\; i = \frac{V_s}{R}$

Incorrect response due to incorrect assumption of sign →

$$\cancel{i(t) = \frac{1}{R}\left(V_s - (V_s - i_0 R)\, e^{+\frac{R t}{L}}\right)}$$

→ Blows up. Does not satisfy

Checking

$$\boxed{t \to \infty \qquad L \text{ is S.C} \qquad i_\infty = \boxed{\frac{V_s}{R}}} = i_{max}$$

Rt/L

$$i(\infty) = \frac{1}{R} V_s \Rightarrow (V_s - i_0 R)\, e \longrightarrow 0$$

$\cancel{+V_L}$ $\quad i \to$ inc. → we go back to starting equation & change sign of V_L

$i \to$ decay

$$i(t) = \frac{1}{R}\left(V_s - (V_s - i_0 R)\, e^{-\frac{Rt}{L}}\right)$$

If $i_0 = 0$

$$\boxed{i(t) = \frac{V_s}{R}\left(1 - e^{-\frac{Rt}{L}}\right)}$$

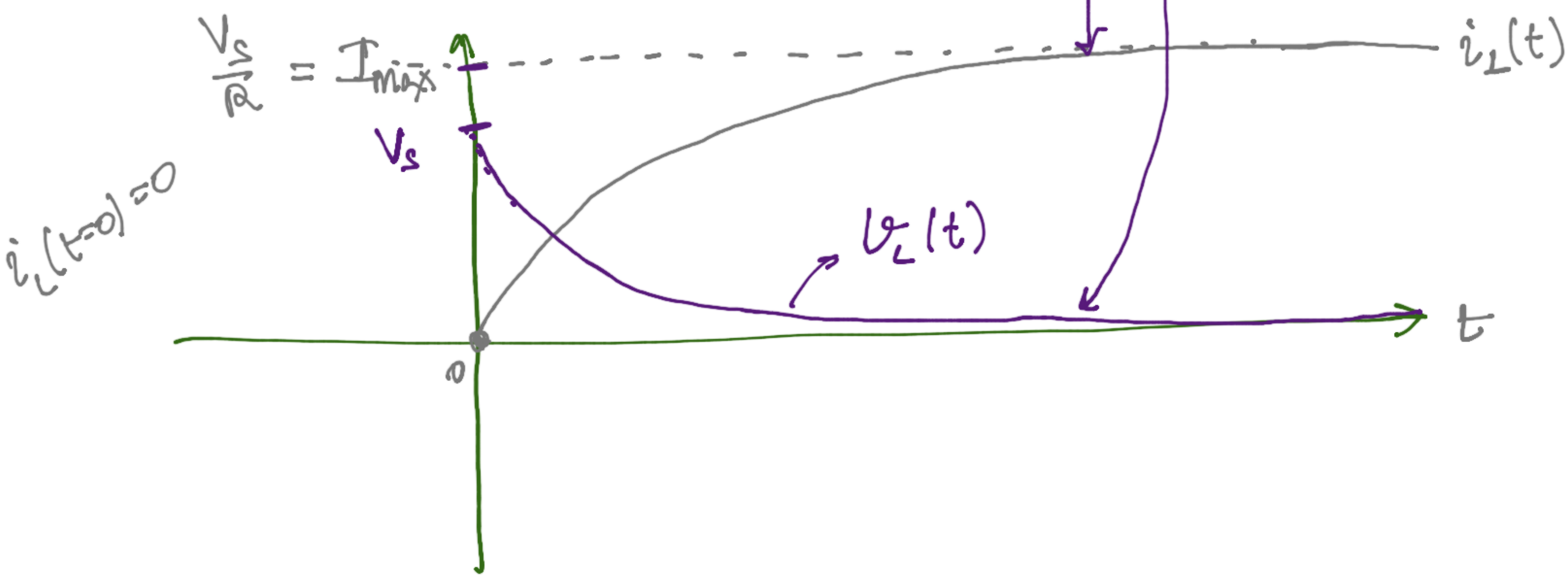
$$v_L = L \frac{di}{dt}$$

$$= \cancel{L} \frac{V_s}{\cancel{R}} \cdot \left(0 + \left(+\frac{\cancel{R}}{\cancel{L}}\right) e^{-\frac{Rt}{L}}\right)$$

$$\boxed{v_L = V_s e^{-\frac{Rt}{L}}}$$

$$i(t) = \frac{V_s}{R}\left(1 - e^{-\frac{Rt}{L}}\right).$$

$$v(t) = V_s e^{-Rt/L}$$



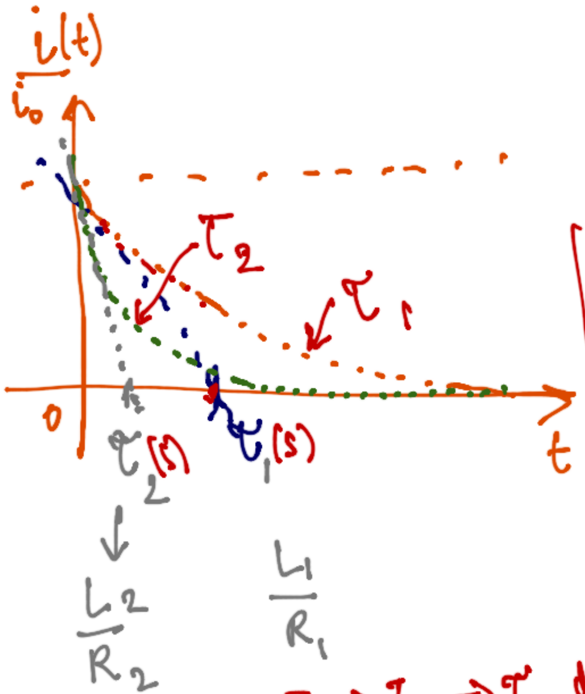
Slope at initial point $t=0$

R – L Source free $\quad i = i_0 e^{-\left(\frac{R}{L}\right)t}$

$$\frac{1}{i_0}\frac{di}{dt} = -\frac{R}{L}$$

slope

$$\frac{\Delta i}{i_0} \rightarrow \frac{\Delta t}{(L/R)} \quad \text{Decay}$$



Slope $\quad \frac{di}{dt} = -i_0 \frac{R}{L} e^{-\frac{Rt}{L}}$

Slope at $t=0$ $\quad \left.\frac{di}{dt}\right|_{t=0} = -i_0\left(\frac{R}{L}\right) = \frac{i_0}{\tau}$

τ = time constant : Units of time (Seconds or ms)

$= \frac{L}{R}$: how fast changing

$\tau_1 > \tau_2 \Rightarrow \tau_1$ decays slower

L is small or R is large $\tau \rightarrow$ small $\Rightarrow$ faster decay : $i = i_0 e^{-t/(L/R)}$

R is small or L is large $\tau \rightarrow$ large $\Rightarrow$ slower decay : $i = i_0 e^{-t/\tau_{small}}$

τ is large.	i_L large $\rightarrow$ decaying slower
τ is small	i_L small $\rightarrow$ decaying faster..

General $\boxed{\text{Gain} = \dfrac{V_{output}}{V_{input}}}$

Gain (mini- project)

$V_s \rightarrow$ input

$V_o(t) = [\text{ function }]\, V_s(t)$

$$\text{Gain} = \frac{V_o(t)}{V_s(t)}$$

<u>Time Constant</u>

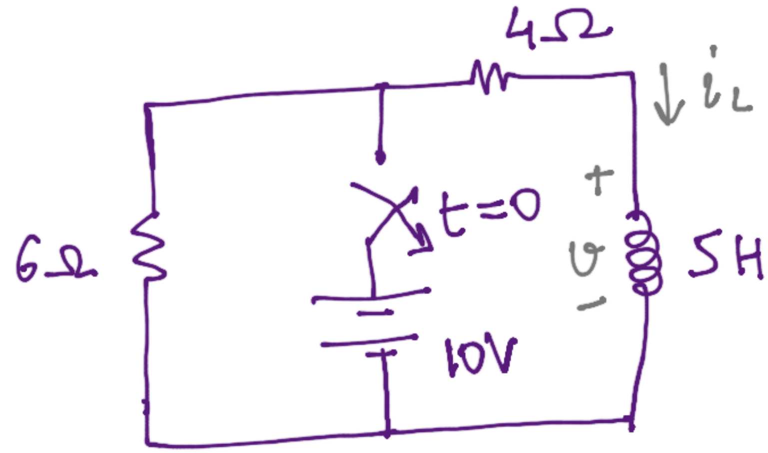
$\frac{L}{R} = \tau$ = unit is of time

$\tau \rightarrow$ determines speed of response. Larger $\tau \Rightarrow$ faster response.

Practice 8.2

Given circuit →

Find v for $t > 0$.

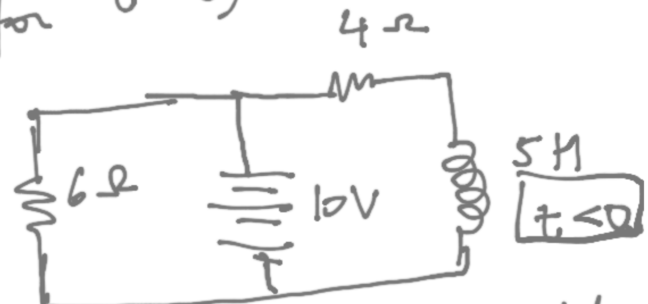


Facts: Switch is opened at $t=0$: Battery removed

For $t < 0$, { The circuit was operating with battery }
circuit was at steady state (unless mentioned)

Part 1 Find steady state v & i_L (for $t<0$)

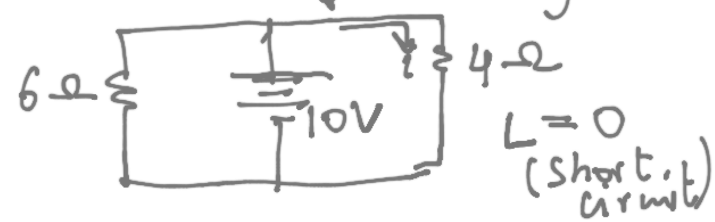
Part 2 Find solution to v & i_L (for $t>0$)



Part 1 ($t<0$) Circuit is given by

Inductor is short circuit at steady state.

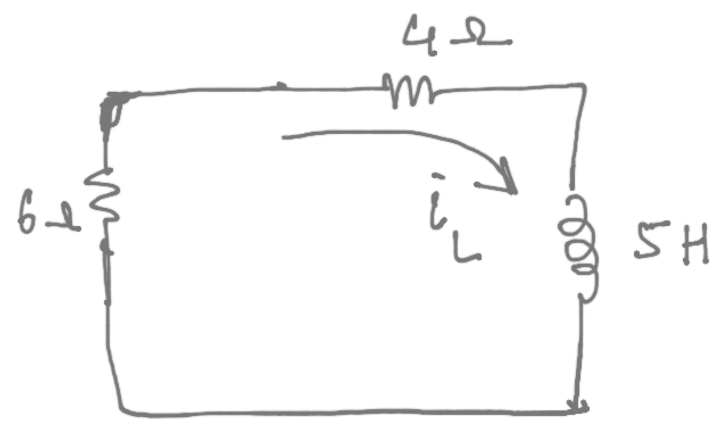
⇓ Steady state



$$\Rightarrow \quad i_L = \frac{10}{4} = 2.5A$$

$$v_L = 0$$

Part 2 $(t > 0)$



Voltage Source disconnected

Circuit will be driven

by inductor energy.

Now $i_L(0) = 2.5A$ (calculated from part I)

$v_L(0) = 0$

Write KVL $\underbrace{6i_L + 4i_L}_{V_R} - \underbrace{5\frac{di_L}{dt}}_{V_L} = 0$

$$v_L(t) = 5\frac{di_L}{dt}$$

$$v_R = 10\, i_L \qquad (v_R = v_L)$$

$$\Rightarrow 5\frac{di}{dt} - 10 i_L = 0$$

Let, $i = Ae^{st}$

$\Rightarrow \frac{di}{dt} = As\, e^{st}$

Using General solution

$$s i - 2 i = 0$$

$$s = 2$$

$$\Rightarrow i_L(t) = A e^{-2t} \Rightarrow t = 0 \quad i_L = 2.5 = A$$

from part 1 (initial condition)

$$v_L = 5 \frac{d}{dt}\left(Ae^{-2t}\right) = 5 * (-2) * (2.5)\, e^{-2t}$$

$$v_L = 25\, e^{-2t}$$