Linear Algebra(MA2.101), Spring 2024, IIIT Hyderabad

Quiz 1

Total Marks: 15

Answer any three questions out of five. Each question carries 5 marks.

1. Let

$$A = \begin{pmatrix} 3 & -6 & 2 & -1 \\ -2 & 4 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 1 & -2 & 1 & 0 \end{pmatrix}$$

For which values of $Y = (y_1, y_2, y_3, y_4)^T$ do the system of equation $AX = Y$ has a solution and under what conditions do the systems $AX = Y$ don't have any solution? Use row-reduced echelon form of A to justify your answer. [5 marks]

2. Prove the following:

   (a) Suppose $a \in \mathbb{F}$ and $\vec{v} \in \mathbf{V}$, where $\mathbf{V}$ is a vector space defined over $\mathbb{F}$. If $a\,\vec{v} = \vec{0}$, then prove that either $a = 0$ or $\vec{v} = \vec{0}$. [2 marks]

   (b) For every $\vec{v} \in \mathbf{V}$, $-(-\vec{v}) = \vec{v}$. [1 marks]

   (c) Every element in a vector space has a unique additive inverse. [1 marks]

   (d) A vector space has a unique additive identity. [1 marks]

3. If $b \in \mathbb{F}$, then the set $\{(x_1, x_2, x_3, x_4) \in \mathbb{F}^4 : x_3 = 5x_4 + b\}$ is a subspace of $\mathbb{F}^4$ if and only if $b = 0$. [5 marks]

4. Using elementary row operations, prove that A, where

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

is invertible if and only if $ad - bc \neq 0$. [5 marks]

$\bar{v} = 0$

$(a - a)\bar{v} = 0.$

$a\bar{v} + (-a)\bar{v} = 0$

$+ a\bar{v}$