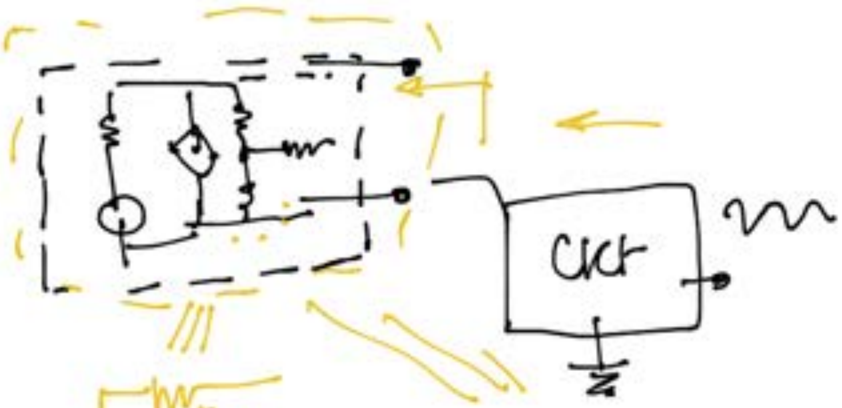
Summary of Basic Superposition Procedure

1. **Select one of the independent sources. Set all other independent sources to zero.** This means voltage sources are replaced with short circuits and current sources are replaced with open circuits. Leave dependent sources in the circuit.
2. **Relabel voltages and currents using suitable notation** (e.g., v', i_2''). Be sure to relabel controlling variables of dependent sources to avoid confusion.
3. **Analyze the simplified circuit to find the desired currents and/or voltages.**
4. **Repeat steps 1 through 3 until each independent source has been considered.**
5. **Add the partial currents and/or voltages obtained from the separate analyses.** Pay careful attention to voltage signs and current directions when summing.
6. **Do not add power quantities.** If power quantities are required, calculate only after partial voltages and/or currents have been summed. mA

$\left.\begin{matrix} V_1 \\ V_2 \end{matrix}\right\} \begin{matrix} P_1 \\ P_2 \end{matrix}$ $\quad \cancel{P_1 + P_2} \quad i^2R \quad i = i_1 + i_2$

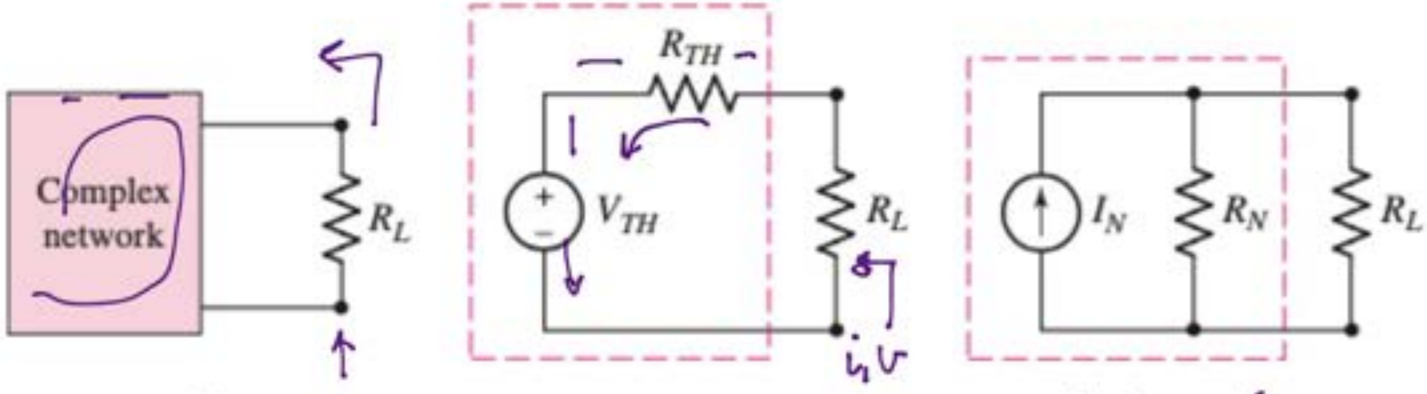
Norton & Thevenin Equivalent

Function generator



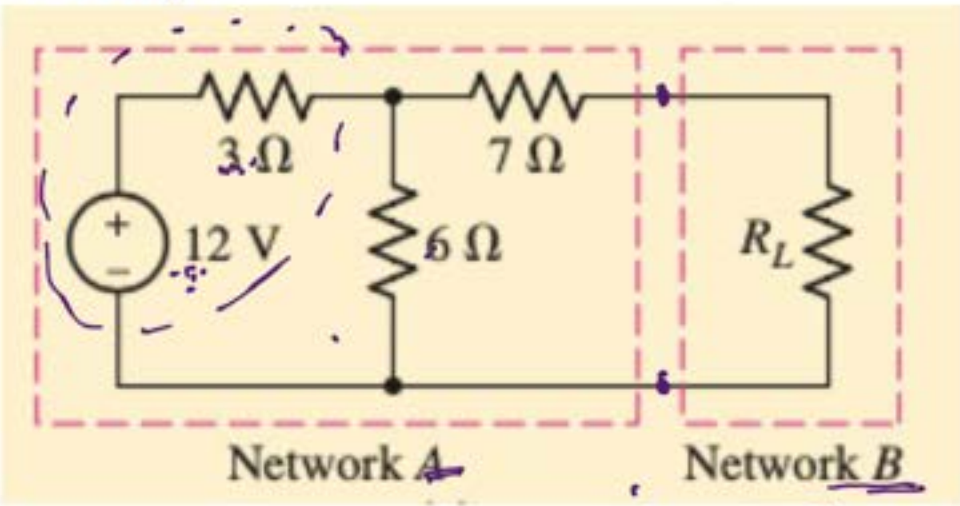
Thevenin Equivalent

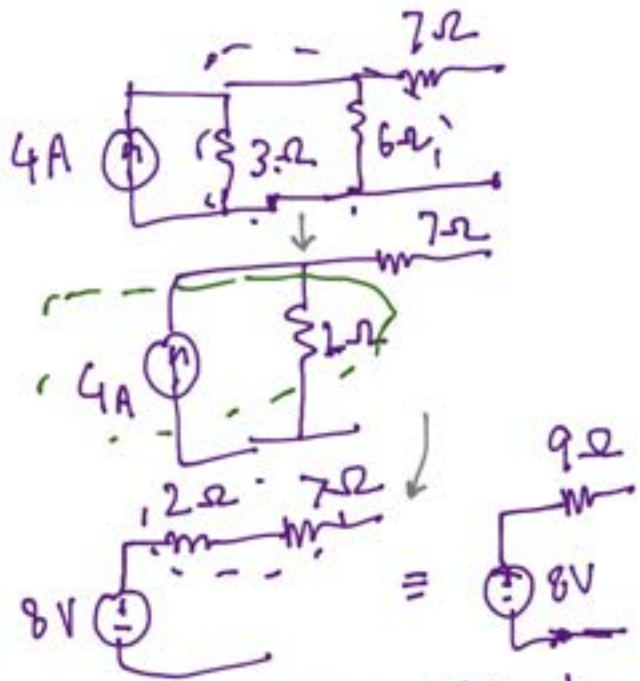
Norton Equivalent

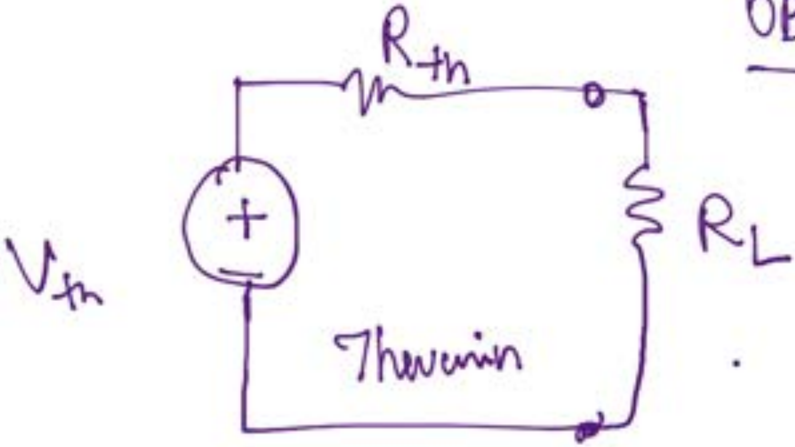


(a) (b) Thevenin Eq (c) Norton Eq.

Example 5.6





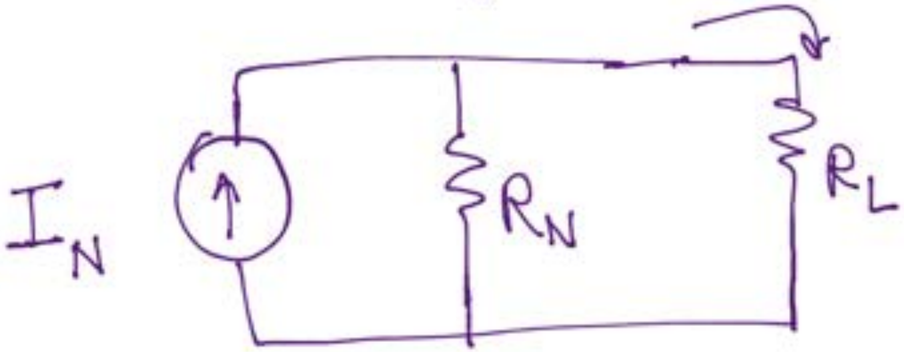


OBSERVATIONS

$R_L = \infty$
(open circuit)

$V_R = V_{th} = V_{max}$
$= V_{oc}$

$\Updownarrow$ Source Trans.



Norton.

$I_{max} = I_N = I_{sc}$

$R_L = 0$
(Short circuit)

Practice 5.5

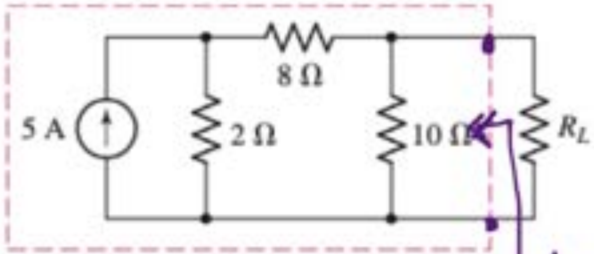
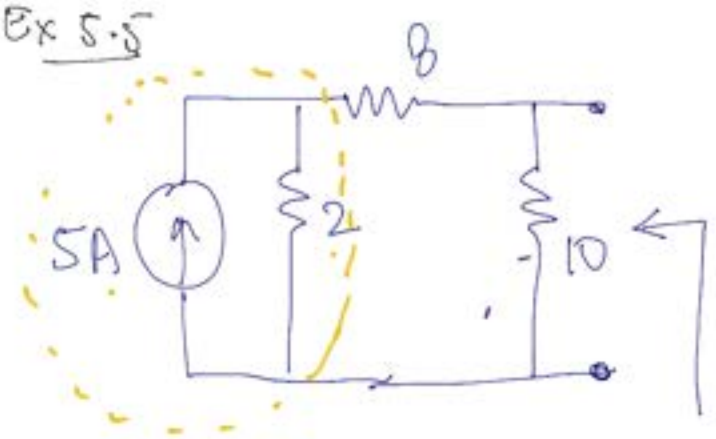


FIGURE 5.26

Norton Eq

Ex 5.5



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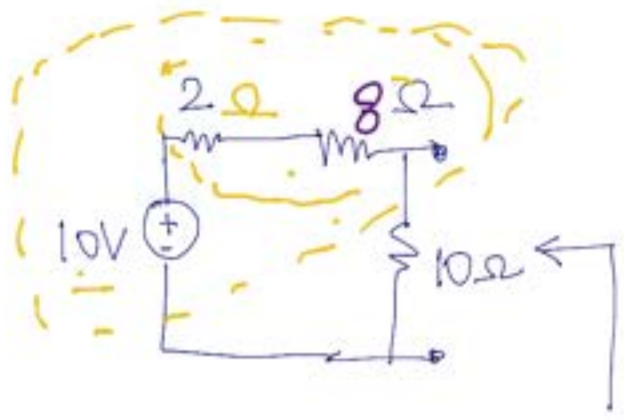

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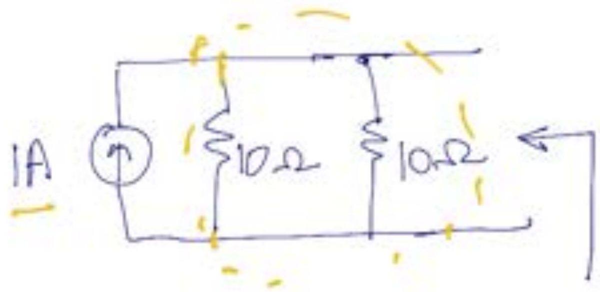

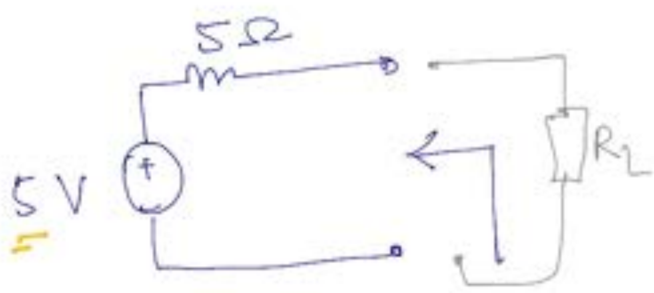

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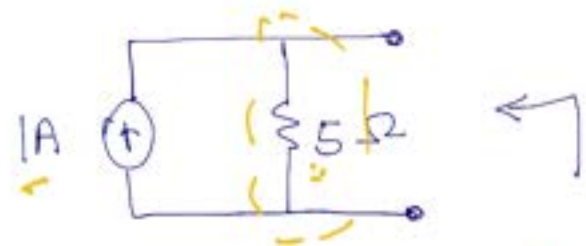
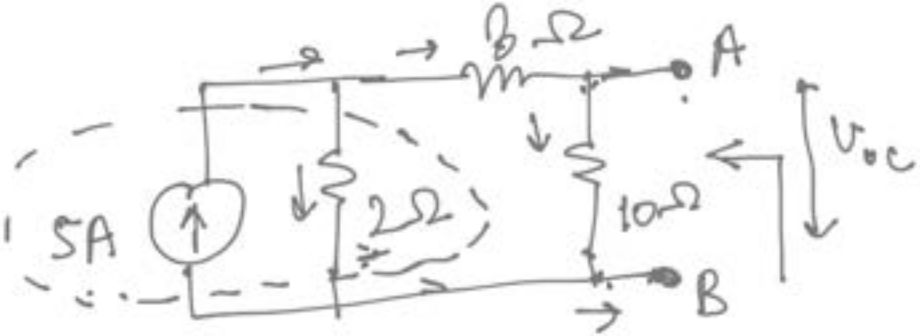
Thevenin Equivalent

≡



Norton Equivalent

Independent Sources

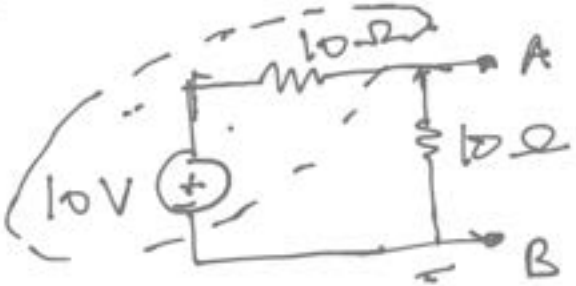


AB : Voltage & Resistor } Then

or Current & Resistor } Nort

$$V_{eq} = 5A * 2\Omega = 10V$$

$$R_{eq} = (R_s = R_s = R) = 2\Omega$$



$\equiv$

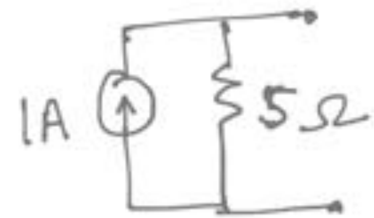
$$I_{eq} = \frac{V}{R} = \frac{10V}{10\Omega} = 1A$$

$$R_{eq} = R_s = R_p = 10\Omega$$

$\equiv$

$V_{eq} = 1A \times 5\Omega = 5V$

$\equiv$



Norton

A Statement of Thévenin's Theorem

1. **Given any linear circuit, rearrange it in the form of two networks, *A* and *B*, connected by two wires.** Network *A* is the network to be simplified; *B* will be left untouched.
2. **Disconnect network *B*.** Define a voltage v_{oc} as the voltage now appearing across the terminals of network *A*.
3. **Turn off or "zero out" every independent source in network *A* to form an inactive network.** Leave dependent sources unchanged. $\equiv R_{eq}$ R_{Th} R_N
4. **Connect an independent voltage source with value v_{oc} in series with the inactive network.** Do not complete the circuit; leave the two terminals disconnected.
5. **Connect network *B* to the terminals of the new network *A*.** All currents and voltages in *B* will remain unchanged.

I_{sc} = Short circuit current

s → Short, c → circuit

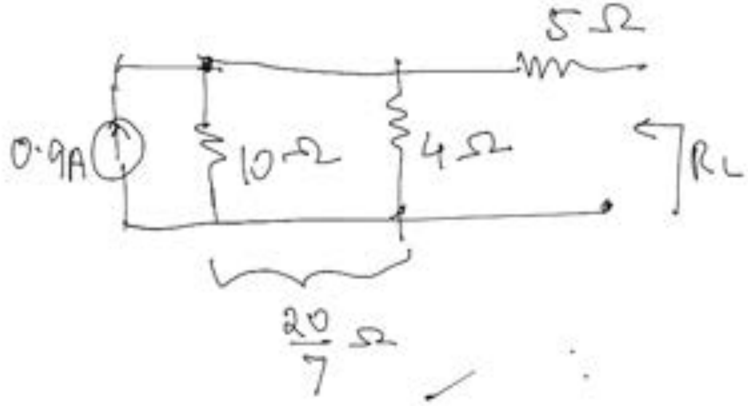
$V_{oc} = V_{AB}$

||

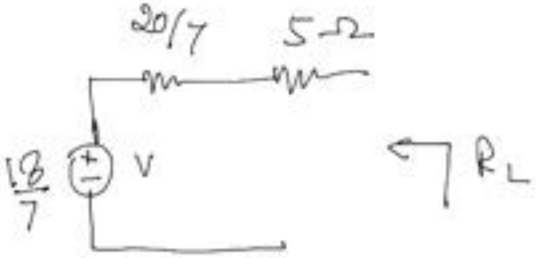
Open Circuit Voltage.

(o c)

P 5.6



⇒



⇓

$V_{th} = 18/7\ V$

$R_{th} = 7.857\ \Omega$

$R_L = 2\Omega$

$$I_L = \frac{18/7}{9.857} = 0.26\ A$$

Practice Prob 5.6

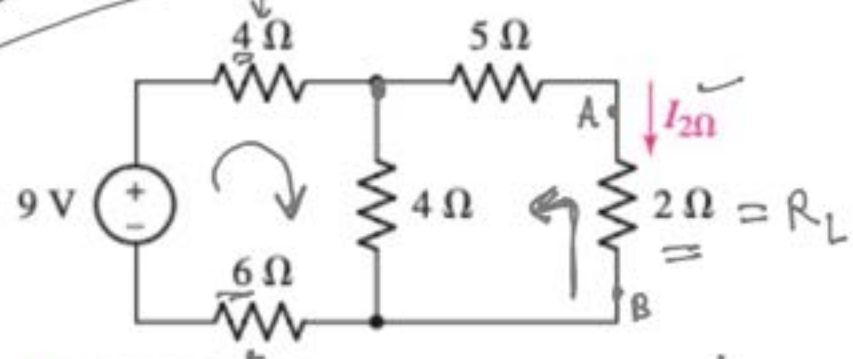


FIGURE 5.28

a) KCL

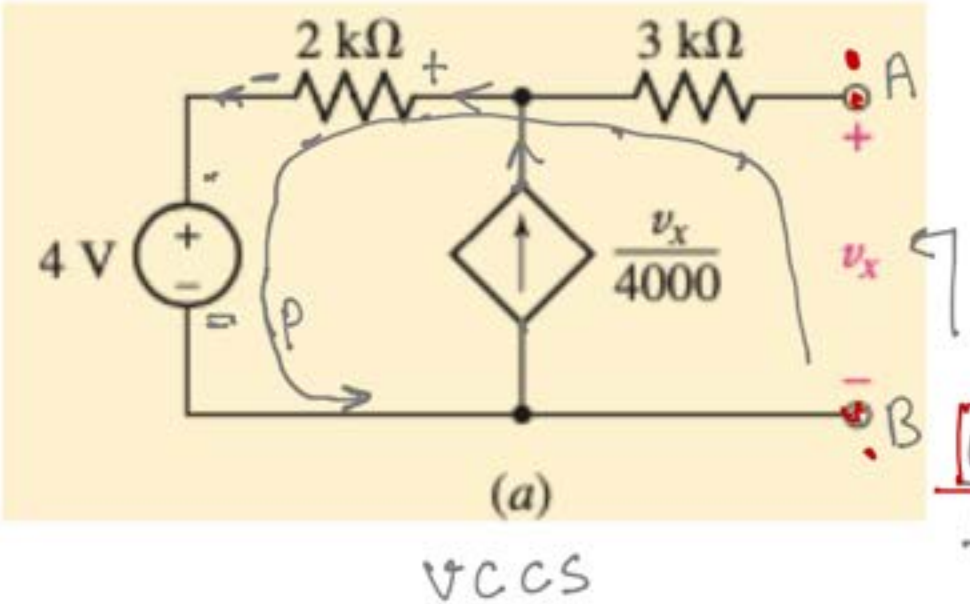
b) KVL

c) Thevenin Eq.

$I_{2\Omega} = ?$

Example 5.9

SPECIAL CASE



(a)

① $v_{oc} = v_{AB}$

$= v_x$

② R_{Th}

$\boxed{\text{Case } v_{oc}}$; No current flows through 3kΩ

Apply KVL (P)

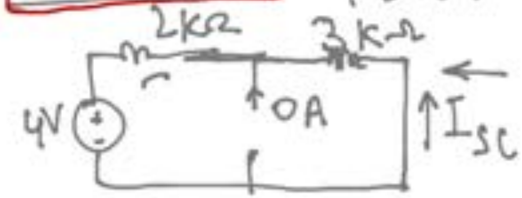
(open circuit v_x = open)

$$v_x - 2\times10^3 \times \frac{v_x}{4000} - 4 = 0$$

$$\frac{v_x}{2} = 4 \Rightarrow v_x = 8V$$

$$v_{oc} = v_x = 8V$$

$\boxed{\text{Case } I_{sc}}$ (Short A−B)



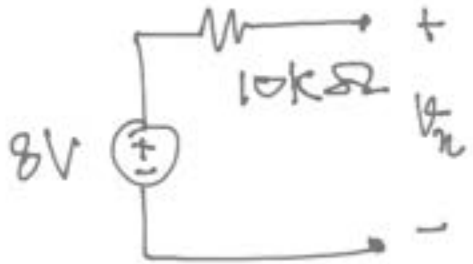
$$I_{sc} = \frac{4V}{5k\Omega} = 0.8mA$$

$$R_{TH} = \frac{v_{oc}}{I_{sc}} = \frac{8}{0.8m} = 10k$$

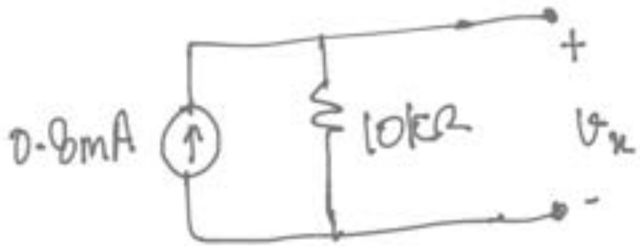
$$R_{TH} = \frac{v_{oc}}{I_{sc}}$$

v_{oc} → open circuit voltage ($R = \infty$)

I_{sc} → short circuit current ($R = 0$)

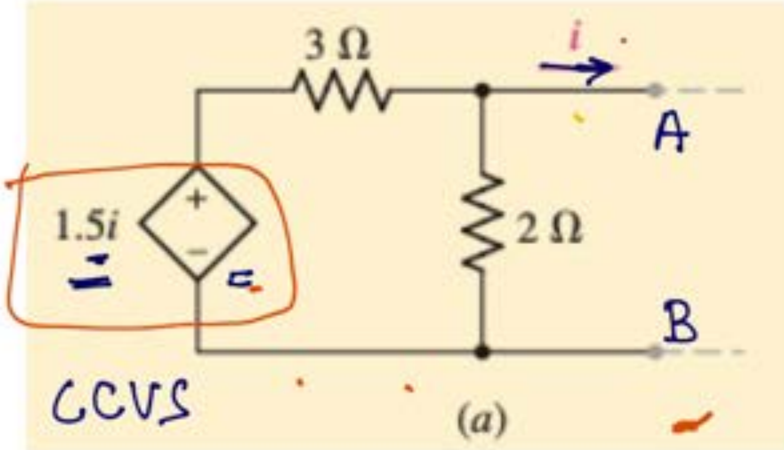


Thevenin Eq.



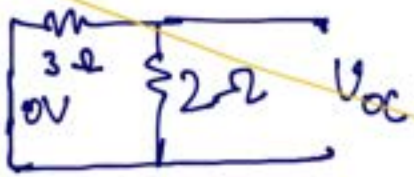
Norton Equivalent.

Example 5.10

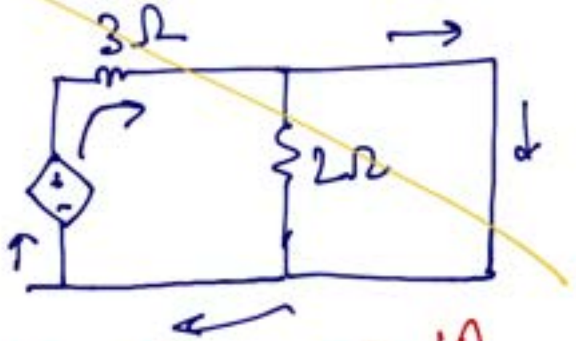


v_{oc} $\quad i = 0$

Dependent source is inactive →



$v_{oc} = ??$



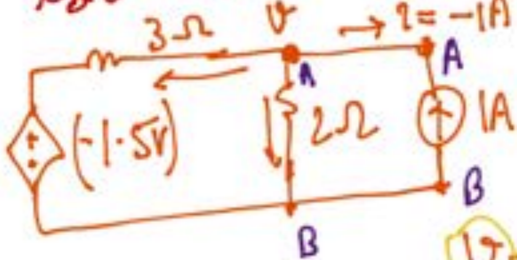
i_{sc} shorted terminal

i_x

2A

v_x ←

I apply a test current source $I_{test} = 1A$



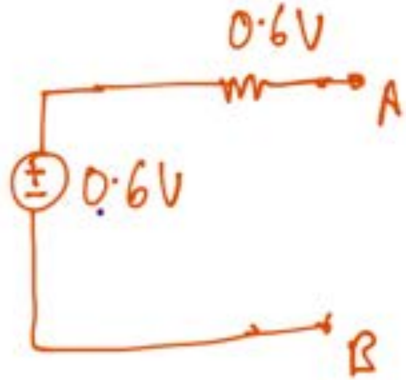
$$-\frac{v-(-1.5)}{3} - \frac{v}{2} + 1 = 0$$

$$-\frac{v}{3} - 0.5 - \frac{v}{2} + 1 = 0 \Rightarrow \frac{5}{6}v = 0.5 \Rightarrow \boxed{v = 0.6\ V} \Rightarrow$$

$$R = \frac{v}{i} = \frac{0.6V}{1A} = 0.6\,\Omega$$

$\underline{\underline{1A}}$ $\quad V_{AB}$, $R_{AB} = 0.6\,\Omega$

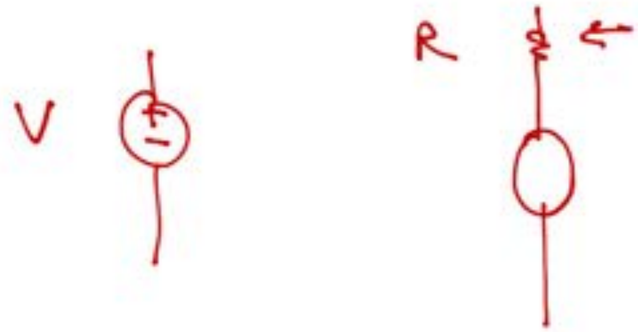
$$V_{AB} = 0.6V$$



Thevenin Eq

Norton.

Source Transformation



Source Resistance

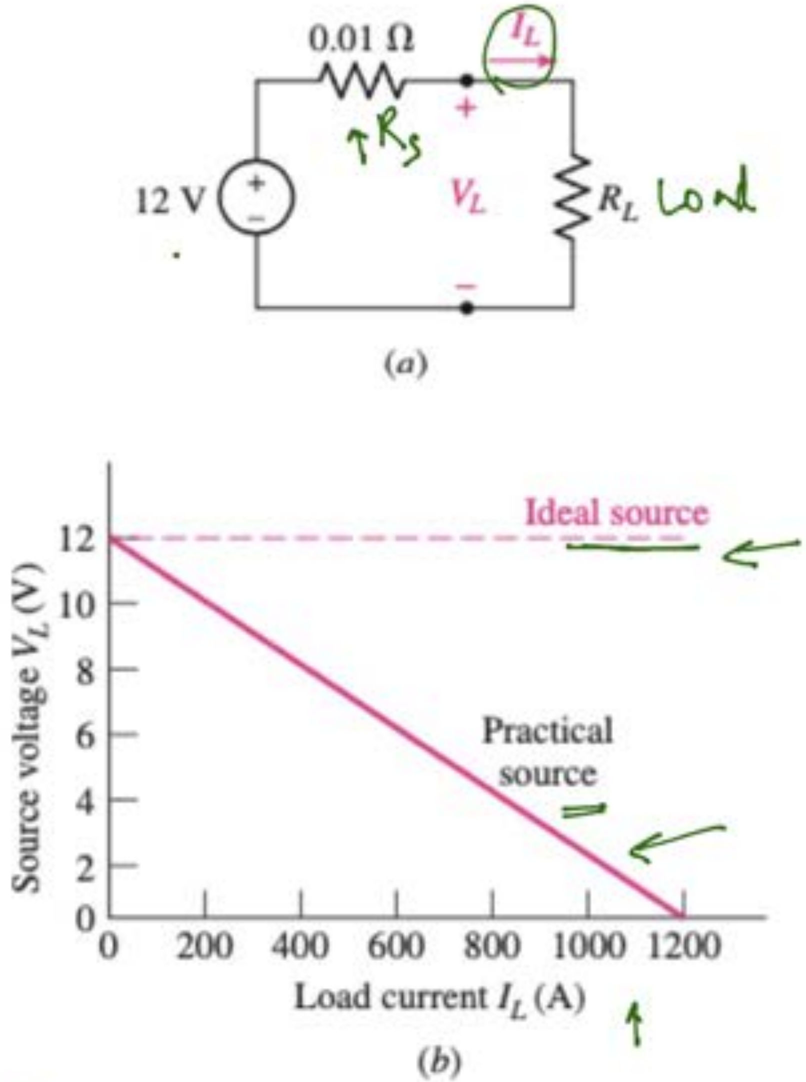


FIGURE 5.12 (*a*) A practical source, which approximates the behavior of a certain 12 V automobile battery, is shown connected to a load resistor R_L. (*b*) The relationship between I_L and V_L is linear.

Voltage Sources
(Non ideal)

R_s = Source Resistance

I_L = Current = Load Current

$$V_s = I_L R_L + \underbrace{I_L R_s}_{\text{Drop across source Resistance}}$$

$$\underbrace{'I_L R_L'}_{\text{Voltage across the Load}} = V_s - \underbrace{I_L R_s}_{\text{Drop } R_s}$$

Voltage Source

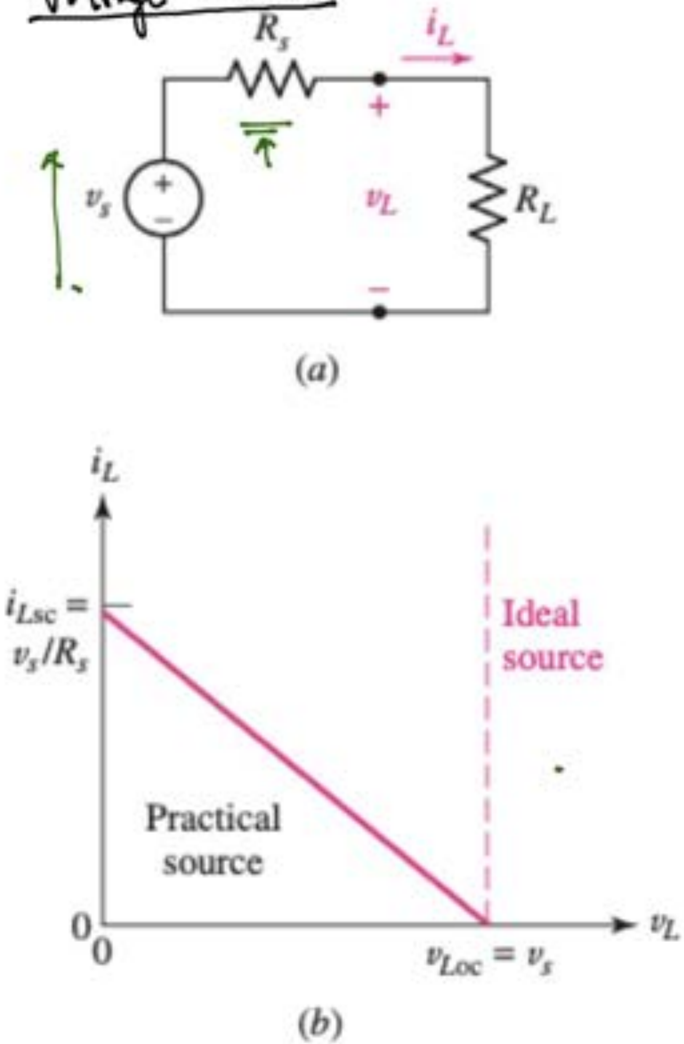
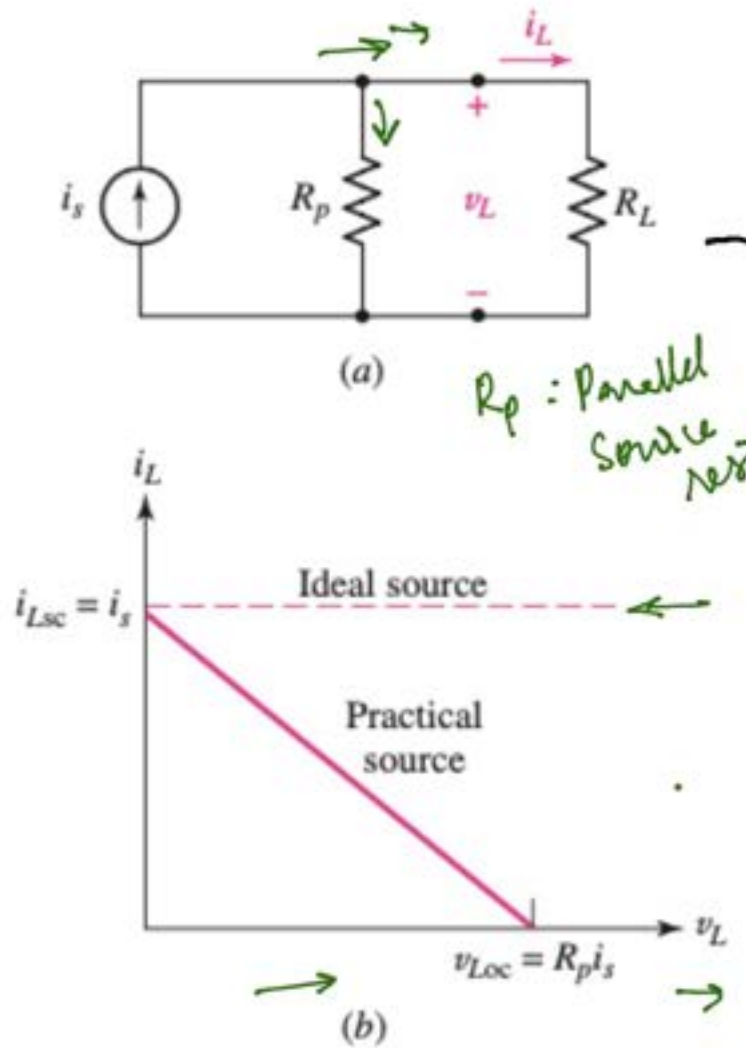


FIGURE 5.13 (*a*) A general practical voltage source connected to a load resistor R_L. (*b*) The terminal voltage of a practical voltage source decreases as i_L increases and $R_L = v_L/i_L$ decreases. The terminal voltage of an ideal voltage source (also plotted) remains the same for any current delivered to a load.

Current Source



R_p : Parallel Source resistance

FIGURE 5.14 (*a*) A general practical current source connected to a load resistor R_L. (*b*) The load current provided by the practical current source is shown as a function of the load voltage.

Voltage Source ⇔ Current Source

Voltage Source

Current Source

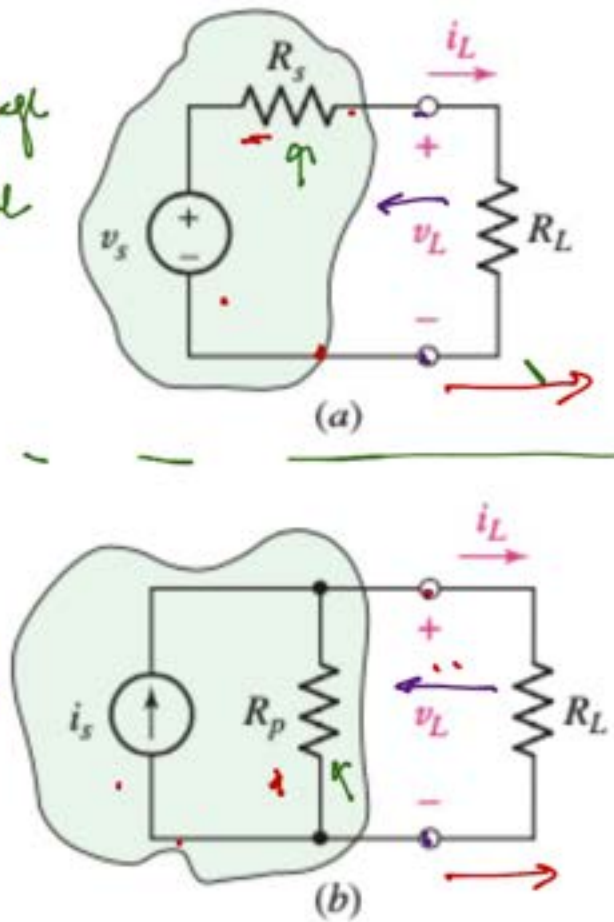


FIGURE 5.15 (*a*) A given practical voltage source connected to a load R_L. (*b*) The equivalent practical current source connected to the same load.

$$i_L = \frac{v_s}{R_s + R_L}$$

$$v_L = R_L \frac{v_s}{R_s + R_L}$$

$$i_L = \frac{R_p}{R_p + R_L} i_s$$

$$\Rightarrow v_L' = R_L \cdot \frac{R_p}{R_p + R_L} i_s$$

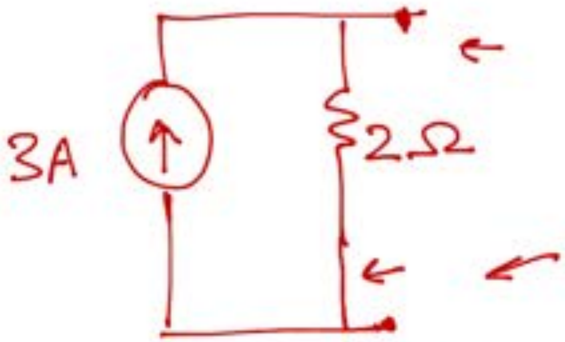
For R_L : v_L(voltage) = v_L(current source)

$$R_L \frac{v_s}{R_s + R_L} = R_L \frac{R_p}{R_p + R_L} i_s$$

$$\boxed{R_s = R_p = R}$$

$$\boxed{v_s = R i_s}$$

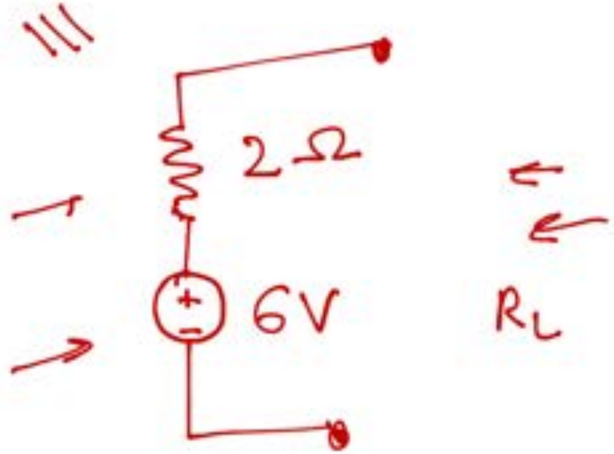
Example



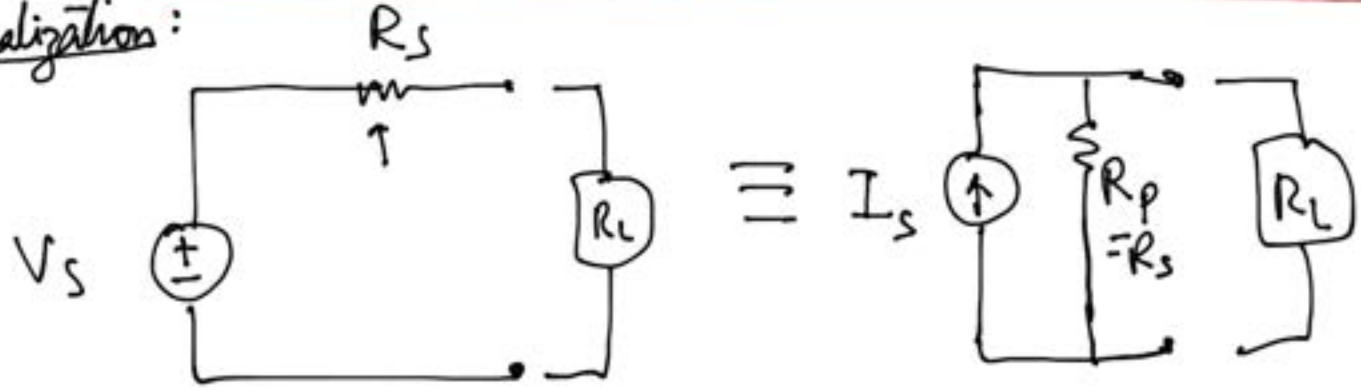
$i = 3A$

$R = 2\Omega$

$v = 3*2 = 6V$



Generalization:



Example 5.4

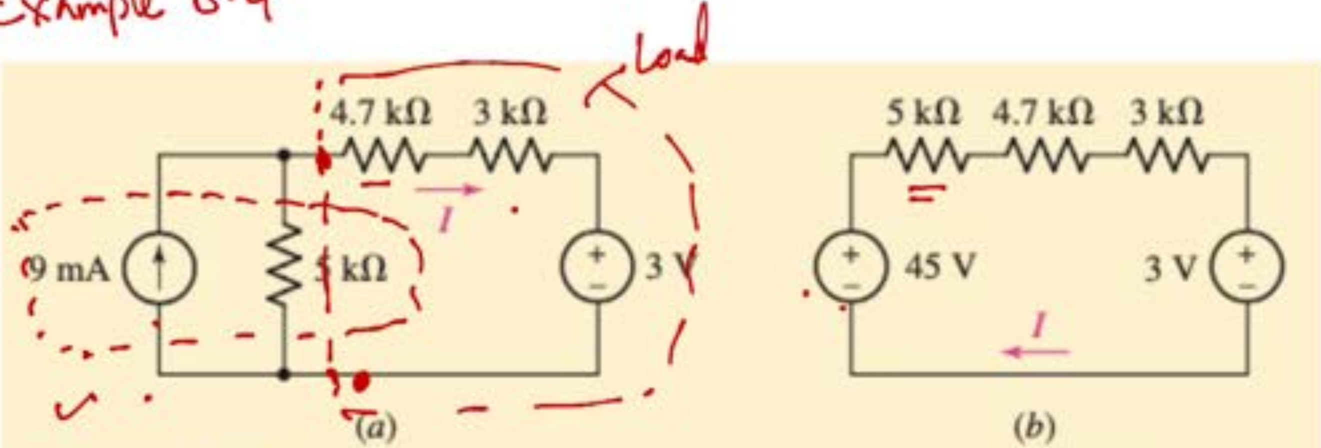
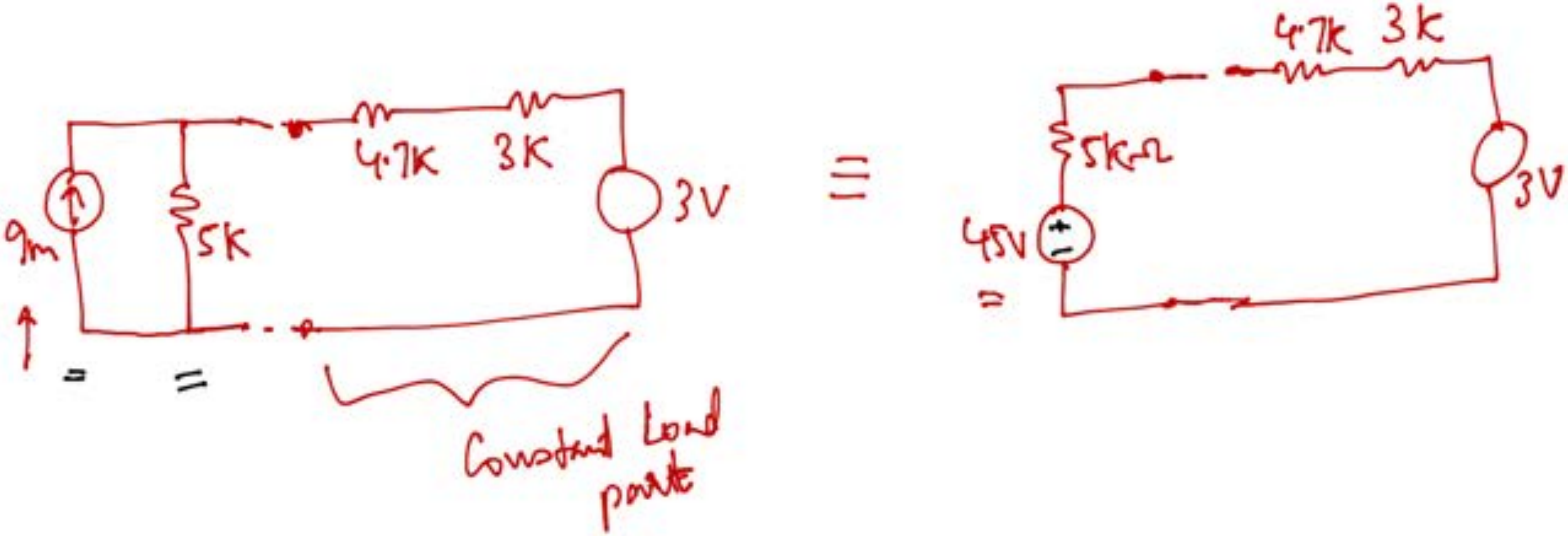


FIGURE 5.17 (*a*) A circuit with both a voltage source and a current source. (*b*) The circuit after the 9 mA source is transformed into an equivalent voltage source.

$$V_s = 9mA * 5K = 45V$$

$$R_s = 5K\Omega$$



Practice Prob 5.3

5.3 For the circuit of Fig. 5.18, compute the current I_X through the 47 kΩ resistor after performing a source transformation on the voltage source.

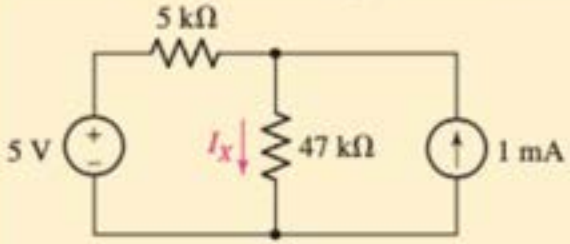


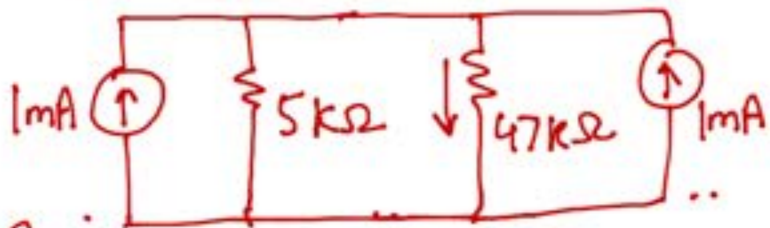
FIGURE 5.18

Ans: 192 μA.

$$V_s \rightarrow 5V \qquad R_s \rightarrow 5k\Omega$$

$$i_s = \frac{V_s}{R_s} = \frac{5}{5K} = 1mA$$

$$R_p = R_s = 5k\Omega$$



$$i_x = \frac{5}{52K} * 2mA$$

$$= 192\mu A$$

Example 5.5

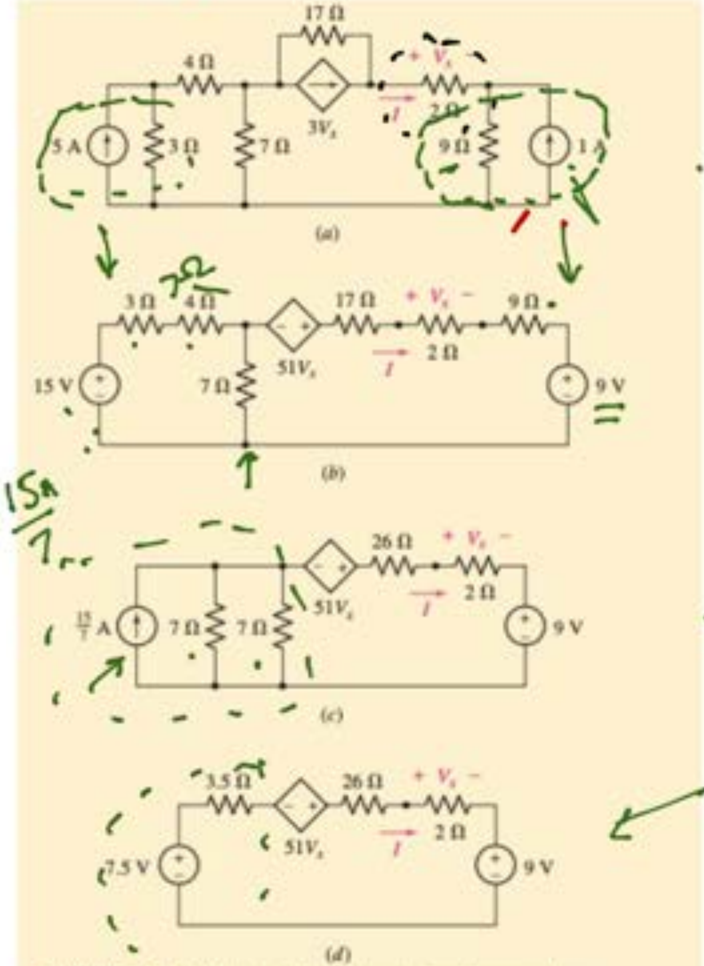
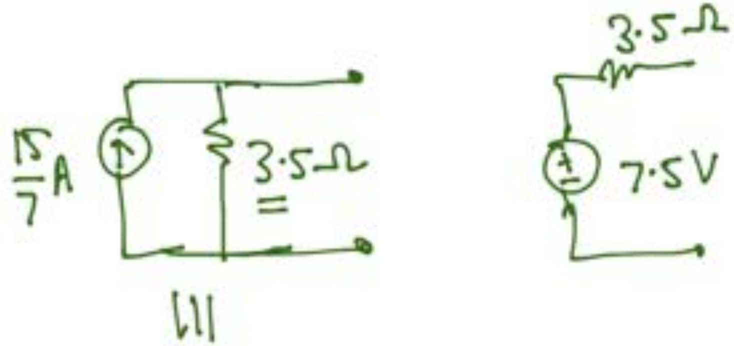


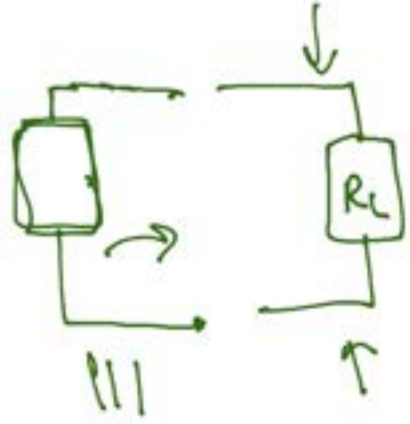
FIGURE 5.19 (*a*) A circuit with two independent current sources and one dependent source. (*b*) The circuit after each source is transformed into a voltage source. (*c*) The circuit after further combinations. (*d*) The final circuit.

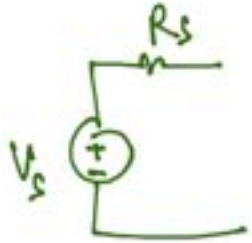


$$V = \frac{15}{7} * \frac{7}{2} = 7.5\,V$$

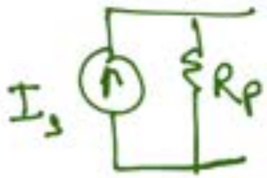
Summary of Source Transformation

1. **A common goal in source transformation is to end up with either all current sources or all voltage sources in the circuit.** This is especially true if it makes nodal or mesh analysis easier.
2. **Repeated source transformations can be used to simplify a circuit by allowing resistors and sources to eventually be combined.**
3. **The resistor value does not change during a source transformation, but it is not the same resistor.** This means that currents or voltages associated with the original resistor are irretrievably lost when we perform a source transformation.
4. **If the voltage or current associated with a particular resistor is used as a controlling variable for a dependent source, it should not be included in any source transformation.** The original resistor must be retained in the final circuit, untouched.
5. **If the voltage or current associated with a particular element is of interest, that element should not be included in any source transformation.** The original element must be retained in the final circuit, untouched.
6. **In a source transformation, the head of the current source arrow corresponds to the "+" terminal of the voltage source.**
7. **A source transformation on a current source and resistor requires that the two elements be in parallel.**
8. **A source transformation on a voltage source and resistor requires that the two elements be in series.**

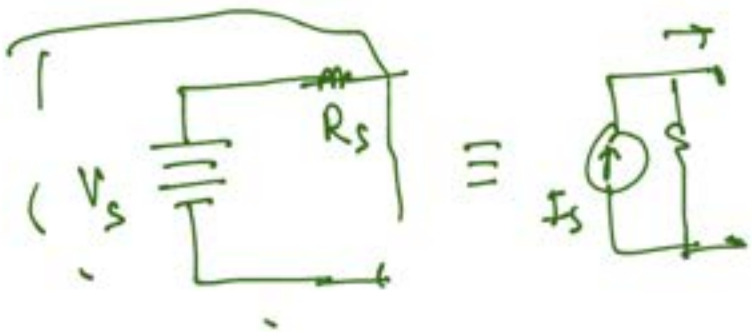




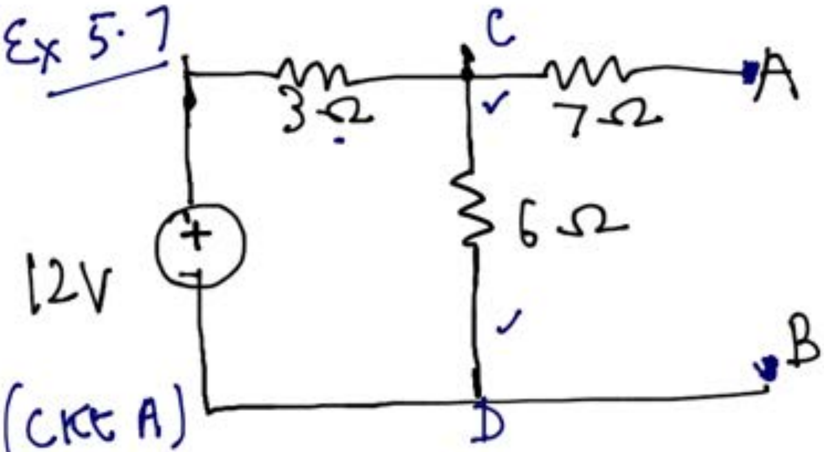
Thevenin eq

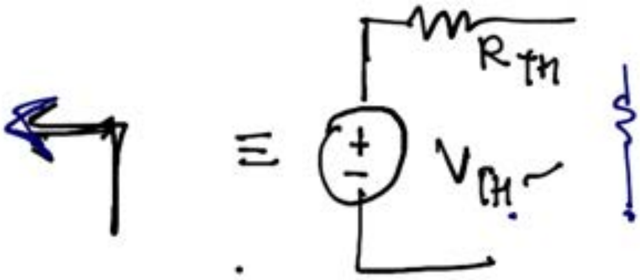


Norton Eq.



Ex 5.7



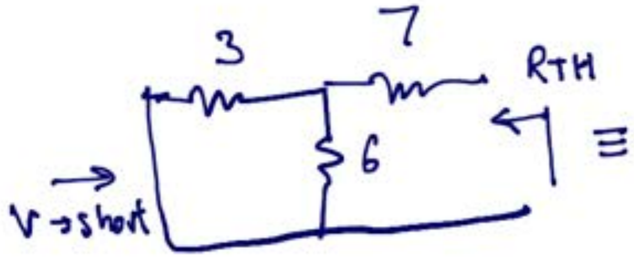


Find Thevenin equivalent between A & B

Step 1 R_{th} . Make current/voltage source $= 0$

$V_{source} \rightarrow$ Short Circuit

$I_{source} \rightarrow$ Open Circuit

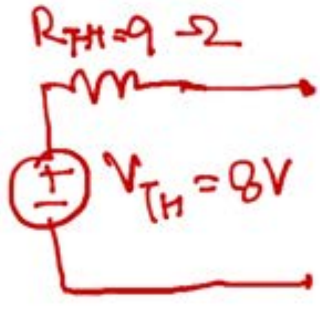


$\Rightarrow R_{TH} = 9\Omega$

Step 2 V_{TH} . Find V_{AB}

In Ckt A $V_{CD} = V_{AB}$; $V_{CD} = \frac{6\Omega}{6+3\Omega} * 12$

$V_{AB} = V_{TH} = V_{CD} = \frac{2}{3} \times 12 = 8\ V$



Thevenin Eq.

Steps : Thevenin Eq.

1(a) Know the terminals (AB) across which Equivalent has to be calculated.

1(b) Remove the load part

2 R_{TH} All independent sources → 0

i.e. Voltage Sources → Short Ckt. (S.C)

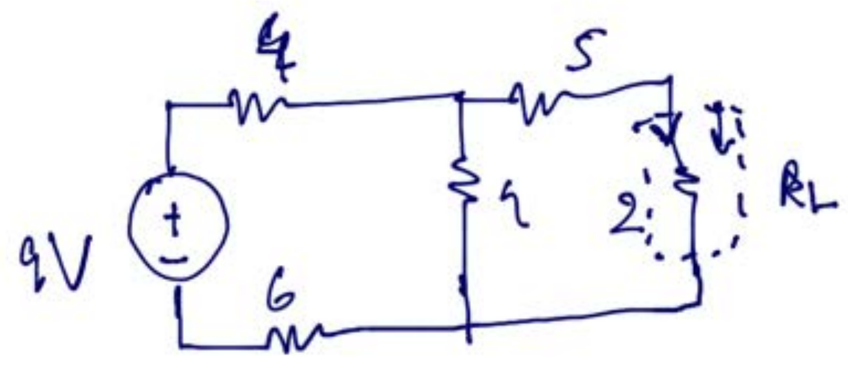
Current Sources → Open Ck (O.C)

Leave dependent sources as they are.

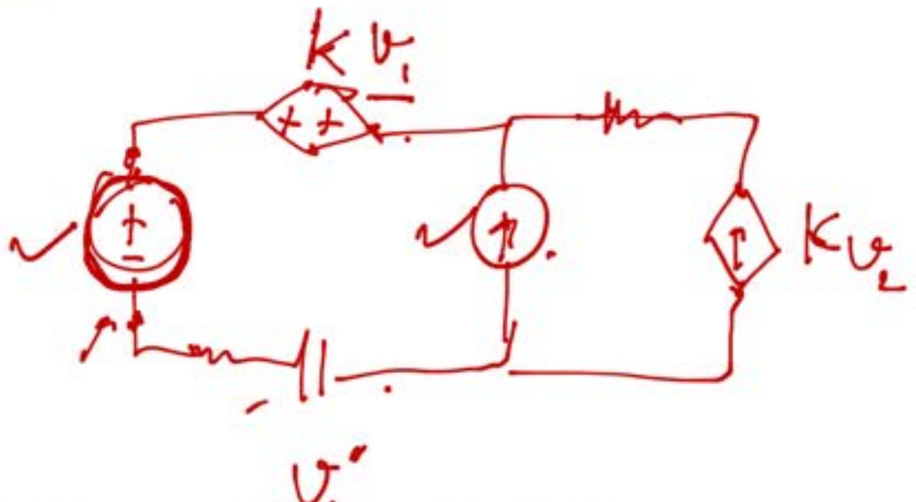
2(b) Find the Equivalent Resistance between A & B.

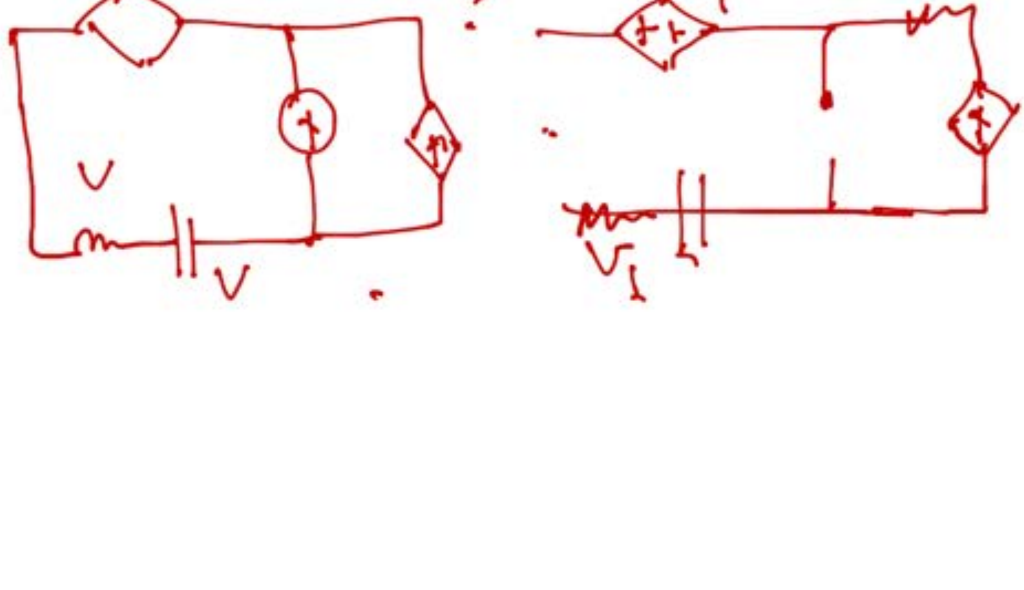
3. Find voltage V_{AB} across the two terminals.

5.6 Practice

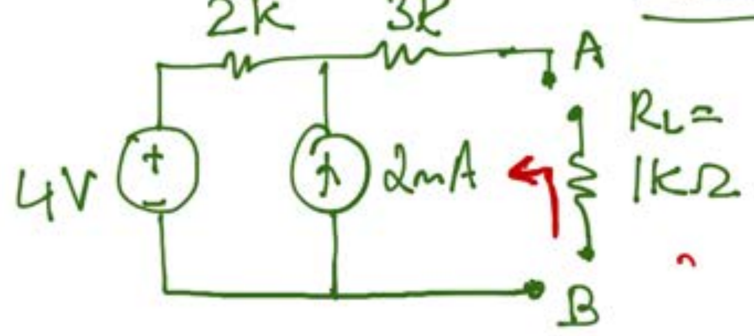


* In Superposition Theorem, we consider only independent voltage/current source.





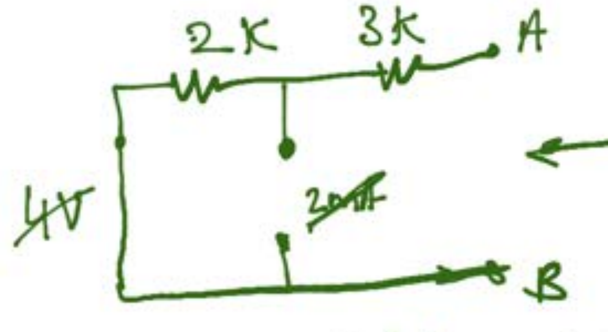
Norton Equivalent



Step 1 Find R_N : $V_{source} \rightarrow$ S.C

$I_{source} \rightarrow$ O.P

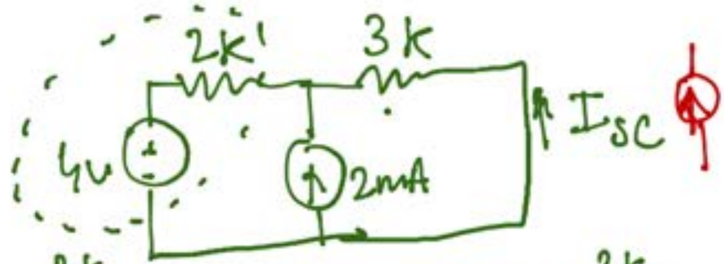
& Solve.



$R_N (= R_{Th})$
$= 5K$ ✓

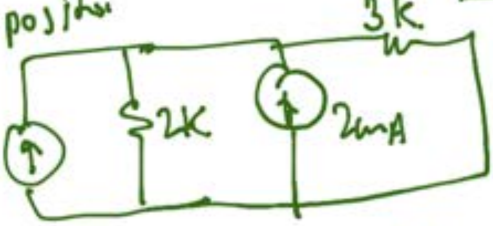
Step 2 I_N

S.C Terminals AB

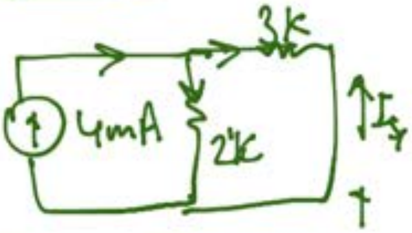


KVL, KCL, Superposition

Source Trans. :



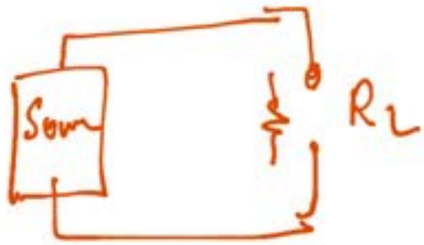
=

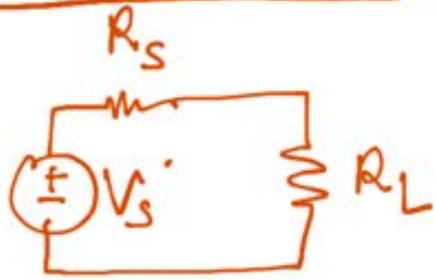


$\Rightarrow$ $I_{SC} = -\frac{2}{5} * 4mA = -\frac{8}{5} mA$

$= -1.6mA$

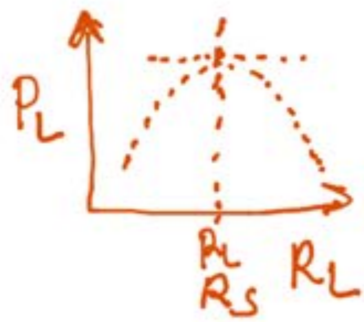
Maximum Power Transfer Theorem





Power transferred to the load = $\boxed{P_L}$

Plot the power vs R_L



$$\rightarrow \boxed{\frac{\partial P_L}{\partial R_L} = 0}$$

R_L in terms known value.

$R_S = R_L$:

$$\boxed{P_{L_{max}} = \ ?}$$

CHAPTER 7

Power $= Vi = V_c \, C \, \frac{dv_c}{dt}$

<u>Current-Voltage</u> Relationship $\quad i = C \frac{dv}{dt}$

$$\int_{t_0}^{t} dv = \frac{1}{C} \int_{t_0}^{t} i \, dt.$$

$$v(t) - v(t_0) = \frac{1}{C} \int_{t_0}^{t} i \, dt$$

$t_0 \rightarrow$ starting / initial time

<u>Q-V Relationship</u> $\quad Q = CV$ ($C \downarrow$ Capacitance)

Ex 7.1, 7.2.
Practice → 7.2

Energy & Power of a Cap

$$\text{Power} = C v_c \frac{dv_c}{dt}.$$

$$\text{Energy} = \int_{t_0}^{t} P \cdot dt$$

$$= C \int_{t_0}^{t} v_c \frac{dv_c}{dt} \cdot dt$$

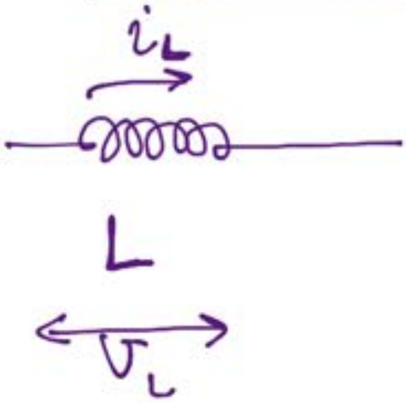
$$= C \int_{t_0}^{t} v_c \, dv_c = C \left[\frac{v_c^2}{2} \right]_{v_c(t_0)}^{v(t)}$$

$$\boxed{\text{Energy} = \frac{C}{2} \left[v_c^2(t) - v_c^2(t_0) \right]}$$

If initial v_c at t_0 $(v_c = 0)$

$$\text{Energy} = \frac{1}{2} C v_{(t)}^2$$

Inductor



$$v_L = L \frac{di_L}{dt}$$

L → inductor

'Inductance' . Units : Henry (H)

Current ~ Voltage Relation.

$$\int_{t_0}^{t} d i_L = \frac{1}{L} \int_{t_0}^{t} v_L \, dt$$

$$\boxed{i_L(t) - i_L(t_0) = \frac{1}{L} \int_{t_0}^{t} v_L \, dt}$$

Energy Storage

$$\text{Power} = p = v \cdot i$$

$$p_L = v_L \, i_L$$

$$= L \frac{di_L}{dt} \cdot i_L$$

$$\text{Energy} = \int_{t_0}^{t} p_L \cdot dt = L \int_{t_0}^{t} i_L \frac{di_L}{dt} dt$$

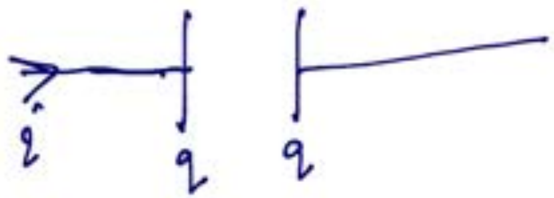
$$= L \left[\frac{i_L^2}{2} \right]_{i_L(t_0)}^{i_L(t)}$$

Energy stored in an inductor $= \frac{L}{2} \left(i_L^2(t) - i_L^2(t_0) \right)$

$w_L = \frac{1}{2} L i$ (if initial condition is zero)

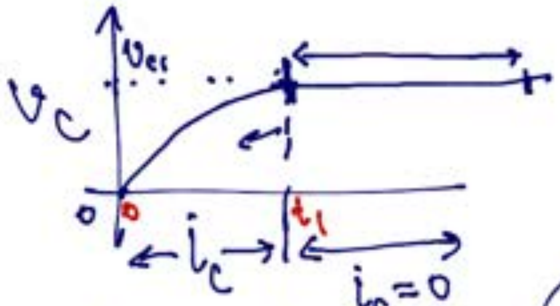
7.5, 7.6

C & L



$$i = C\frac{dv}{dt}$$

$$\left\{\begin{array}{l} v = \text{constant} : \text{D.C} \\ i = 0 \quad \Rightarrow \text{open circuit} \end{array}\right.$$

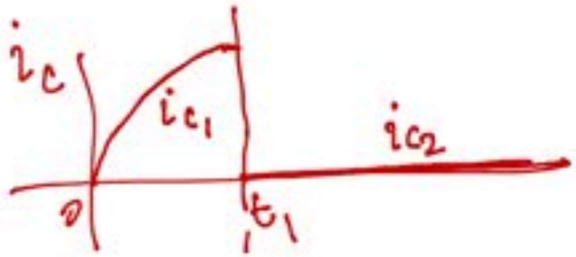


Energy building

C → O.C in D.C

v_{1c} $\quad$ v_{1c} $\quad$ $\underline{t > t_1}$

$$\text{Energy} = \frac{1}{2} C\left[V^2 - V^2(0)\right] = 0$$



$i_{c2} = 0 \quad \frac{dv_c}{dt} = 0 \quad (t > t_1)$

$t < t_1 \quad i = \frac{dv_{c1}}{dt} \quad E = \frac{1}{2} C\left(v_{c1}^2 - 0\right)$

Important Characteristics of an Ideal Capacitor

1. There is no current through a capacitor if the voltage across it is not changing with time. A capacitor is therefore an *open circuit to dc*.
2. A finite amount of energy can be stored in a capacitor even if the current through the capacitor is zero, such as when the voltage across it is constant.
3. It is impossible to change the voltage across a capacitor by a finite amount in zero time, as this requires an infinite current through the capacitor. (A capacitor resists an abrupt change in the voltage across it in a manner analogous to the way a spring resists an abrupt change in its displacement.)
4. A capacitor never dissipates energy, but only stores it. Although this is true for the *mathematical model*, it is not true for a *physical* capacitor due to finite resistances associated with the dielectric as well as the packaging.

ideal

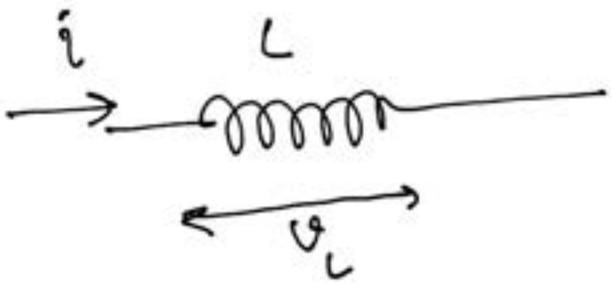
R=0

$i = C\frac{dv}{dt}$ $dt \to 0$ $i = \infty$ | finite value of 't'

Real capacitor

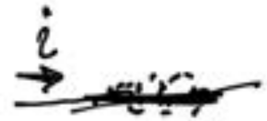
Important Characteristics of an Ideal Inductor

1. There is no voltage across an inductor if the current through it is not changing with time. An inductor is therefore a *short circuit to dc*.
2. A finite amount of energy can be stored in an inductor even if the voltage across the inductor is zero, such as when the current through it is constant.
3. It is impossible to change the current through an inductor by a finite amount in zero time, for this requires an infinite voltage across the inductor. (An inductor resists an abrupt change in the current through it in a manner analogous to the way a mass resists an abrupt change in its velocity.)
4. The inductor never dissipates energy, but only stores it. Although this is true for the *mathematical* model, it is not true for a *physical* inductor due to series resistances.



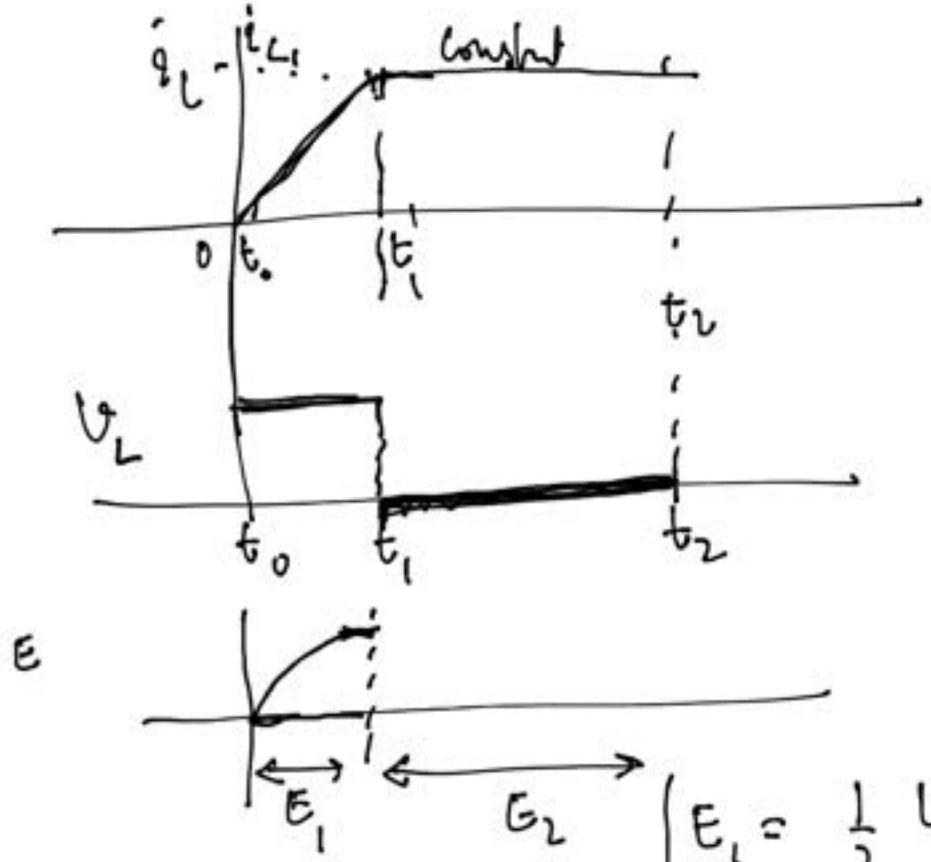
$$v_L = L \frac{di}{dt}$$

$i = \text{constant}$ $\quad v_L = 0$



Short Circuit

$$E = \frac{1}{2} L \left[i^2(t_1) - i^2(t_0) \right]$$



$$v_L = \frac{di}{dt} = \text{constant}$$

$$\times \left\{ dt \to 0 \Rightarrow v_L = \infty \right.$$

Always $dt \to$ finite.

stored

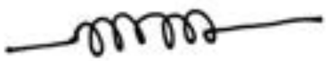
$\uparrow$

$$E_1 = \frac{1}{2} L \left[i(t_1)^2 - i(t_0)^2 \right] \to E_1$$

$$E_2 = \frac{1}{2} L \left[i(t_2)^2 - i(t_1)^2 \right] = 0$$

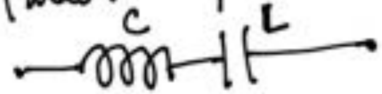
$$i(t_1) = i(t_2)$$

Ideal $(R = 0)$



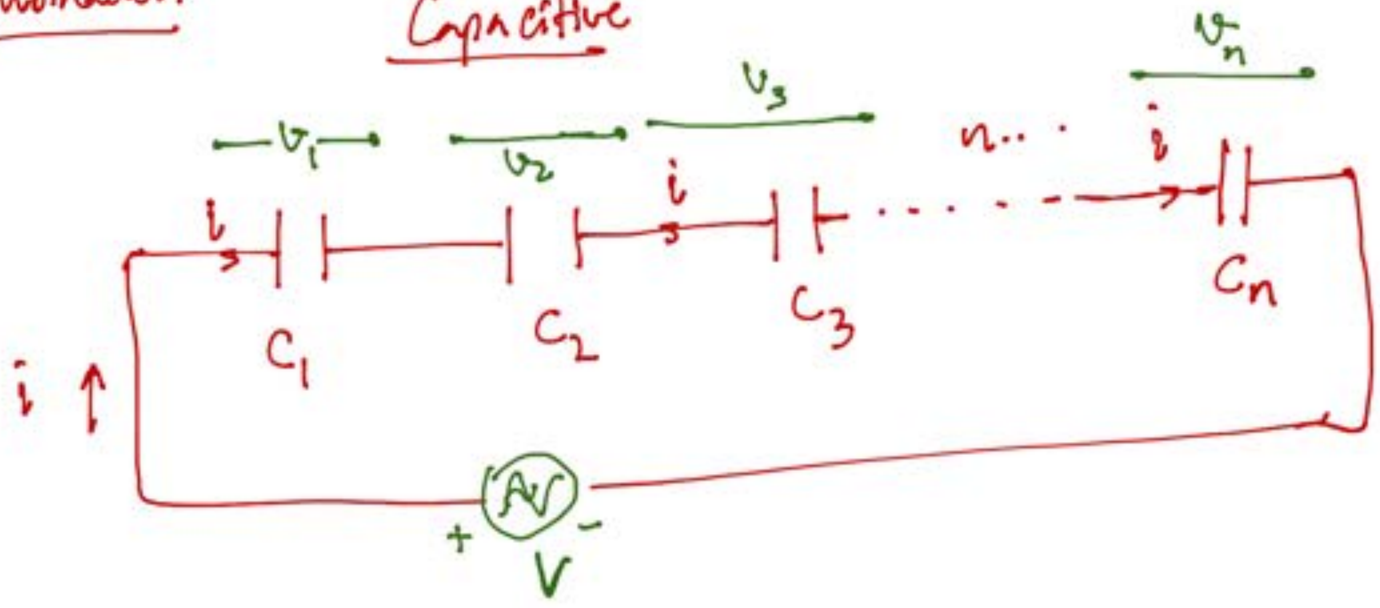
inductor : Induction cooking. Heater

Parasitic Cap & Inductor



Combination

Capacitive



$$V = v_1 + v_2 + \cdots + v_n$$

$$v = \frac{1}{C}\int_0^t i \cdot dt \qquad \left(i_c = C\frac{dv_c}{dt} \right)$$

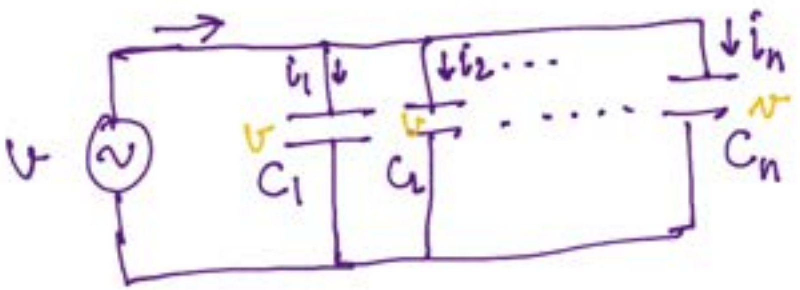
$$\int dv_c = \int \frac{1}{C} i_c \, dt$$

$$V = \frac{1}{C_1}\int_0^t i\,dt + \frac{1}{C_2}\int_0^t i\,dt + \cdots + \frac{1}{C_n}\int_0^t i\,dt$$

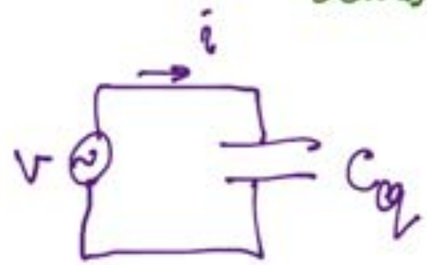
$$V = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}\right]\int_0^t i\,dt = \frac{1}{C_{eq}}\int_0^t i\,dt$$

$$\left(\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots\cdots + \frac{1}{C_n}\right)$$

C_{eqs} → Equivalent Capacitance. → Series Combination.



$\equiv$



$$i = i_1 + i_2 + \cdots\cdots + i_n$$

$$= C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + \cdots\cdots + C_n \frac{dv}{dt}$$

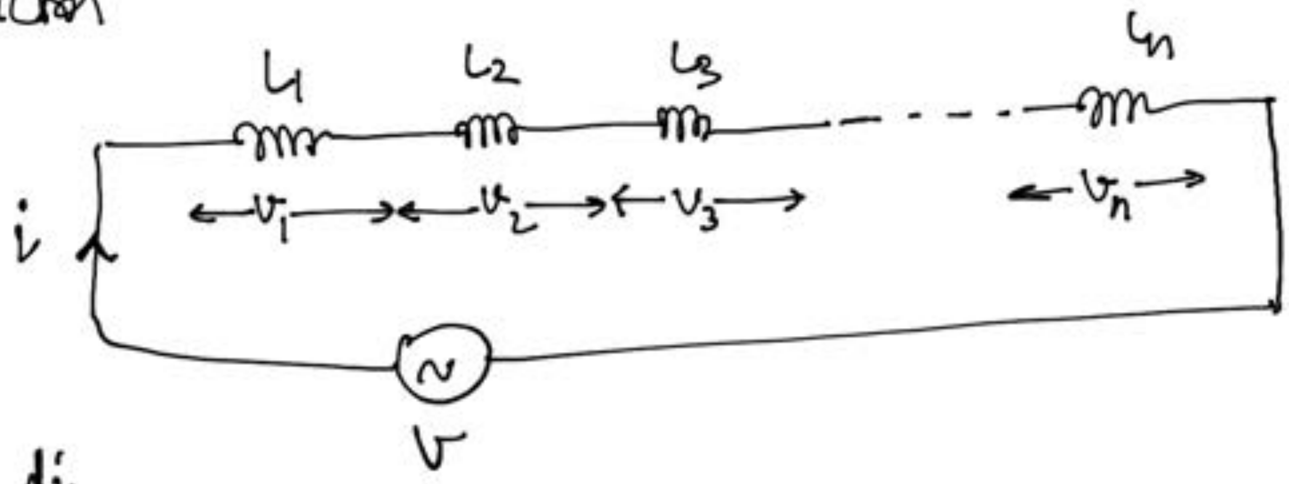
$$= (C_1 + C_2 + \cdots + C_n) \frac{dv}{dt}$$

$$= C_{eq\parallel} \frac{dv}{dt}$$

Parallel $C_{eq} = C_1 + C_2 + C_3 + \cdots C_n$

Inductor

Series



$$v_1 = L_1 \frac{di}{dt} \quad \text{or}$$

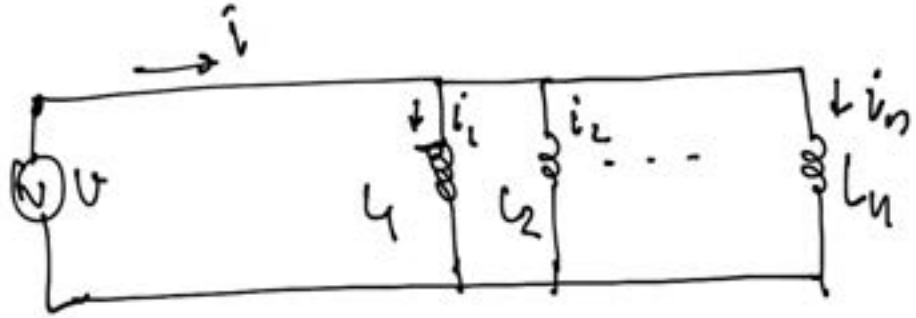
$$v = v_1 + \cdots \cdots + v_n$$

$$= L_1 \frac{di}{dt} + \cdots \cdots + L_n \frac{di}{dt}$$

$$= (L_1 + \cdots \cdots + L_n) \frac{di}{dt}$$

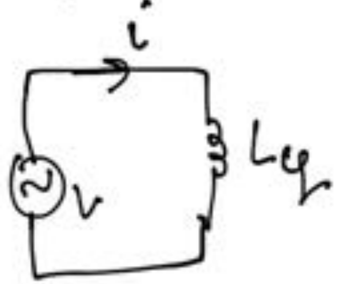
$$v = L_{eq} \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + \cdots + L_n$$: Series Equivalent Inductance



$$v = L\frac{di}{dt}$$

$$\Rightarrow i = \frac{1}{L}\int_0^t v\,dt$$



$$i = i_1 + \cdots\cdots + i_n$$

$$= \frac{1}{L_1}\int_0^t v\,dt + \cdots\cdots + \frac{1}{L_n}\int_0^t v\,dt$$

$$= \left(\frac{1}{L_1} + \cdots\cdots + \frac{1}{L_n}\right)\int_0^t v\,dt$$

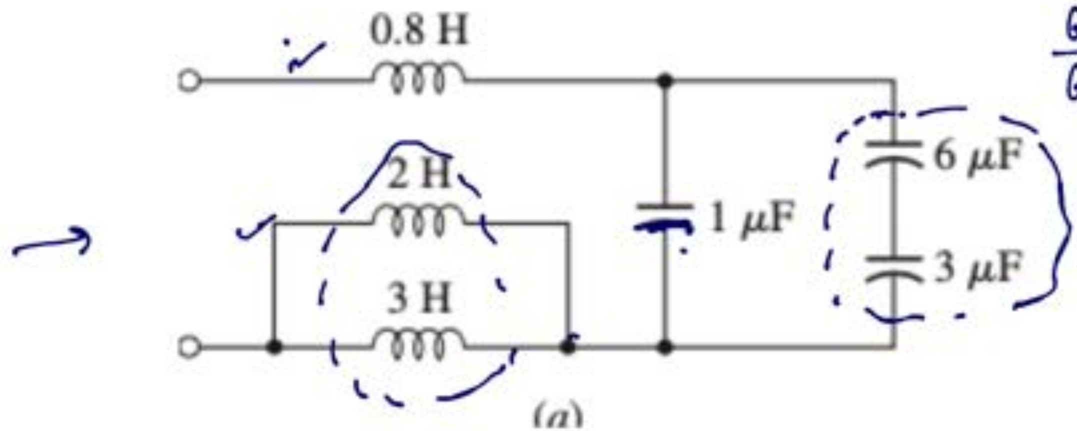
$$= \frac{1}{L_{eq}}\int_0^t v\,dt$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots\cdots + \frac{1}{L_n}$$

Equivalent Parallel Inductance

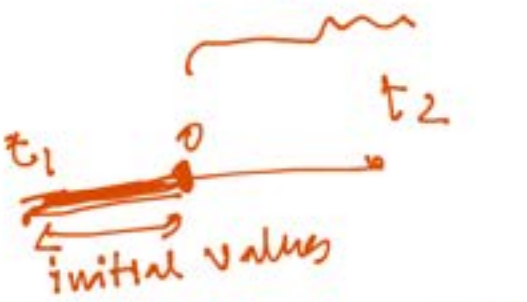
Example 7.8

Initial value of capacitor



$\frac{6\times3}{6+3} = 2\mu F$

observation



$\frac{1}{L} = \frac{1}{2} + \frac{1}{3} \Rightarrow L = \frac{6}{5} H = 1.2H$

$\boxed{L}$

$v = L\frac{di}{dt} \Rightarrow i = \frac{1}{L}\int_{t_1}^{t_2} v \cdot dt$

$i = \underbrace{\frac{1}{L}\int_{t_1}^{0} v \cdot dt}_{\text{Initial value of inductor}} + \frac{1}{L}\int_{0}^{t_2} v \cdot dt = i_{c_0} + \frac{1}{L}\int_{0}^{t_2} v dt$

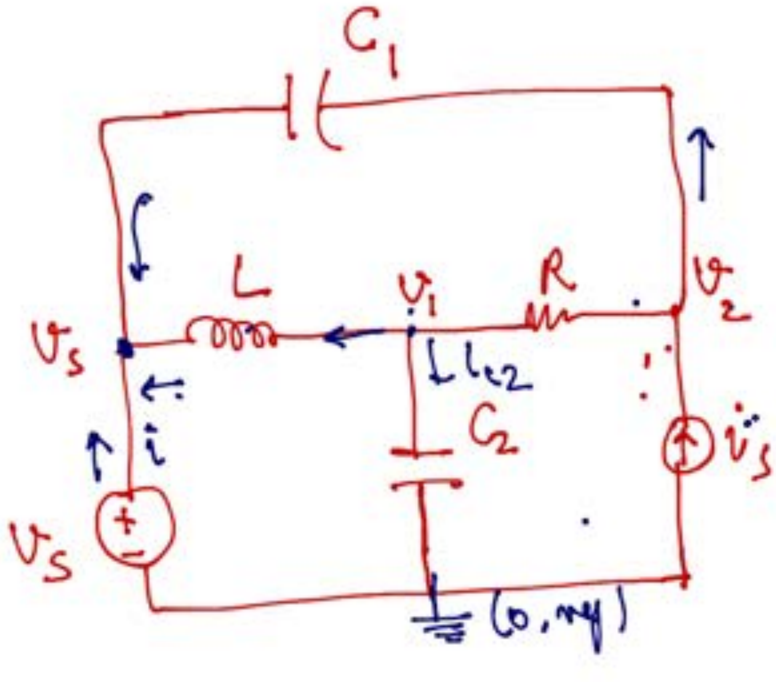
$\boxed{C}$

$i = C\frac{dv}{dt}$

$v = \frac{1}{C}\int_{t_1}^{t_2} i \cdot dt$

$= \underbrace{\frac{1}{C}\int_{t_1}^{0} i\, dt} + \frac{1}{C}\int_{0}^{t} i dt$

$v = v_{c_0} + \frac{1}{C}\int_{0} i\, dt$



KCL

Nodal

$$i_{c_1} = C_1 \frac{d(v_2 - v_s)}{dt}$$

$$i_L = \frac{1}{L}\int_0^t (v_1 - v_s)\,dt + i_{L0}$$

$\boxed{v_s}$ (in)

$$C_1 \frac{d}{dt}(v_2 - v_s) + \frac{1}{L}\int (v_1 - v_s)\,dt + i_{L0} + i = 0 \quad \text{--- (1)}$$

$$i_{c_2} = C_2 \frac{dv_1}{dt}$$

$\boxed{v_1}$

$$\frac{1}{L}\int (v_1 - v_s)\,dt + i_{L0} + C_2 \frac{dv}{dt} + \frac{v_1 - v_2}{R} = 0 \quad \text{--- (2)}$$

$$(v_2) \quad - C_1 \frac{d}{dt}(v_2 - v_s) - \frac{v_2 - v_1}{R} \cancel{+}\, i_s = 0 \quad \text{--- (3)}$$

Integro differential Equation

KCL, KVL are satisfied.

Linearity of L & C circuits

Equation 1

$$\text{LHS1} = \underline{R i} + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt + \underline{v_{C0}} = \underset{\text{RHS1}}{v_s}$$

Increase current 'K' time

$$\text{LHS2}: \underbrace{R i K + L K\frac{di}{dt} + \frac{K}{C}\int_0^t i\,dt}\ + \underline{K v_{C0}} =$$

$$i\,\textcircled{R} = K * \text{LHS1} = K * \underline{\text{RHS1}}$$

given that K is multiplied to constant V_{C0} also
i.e. v_{C0} as a input independent voltage source.
If all inputs are changed K $\Rightarrow$ output also change 'K'
$\Rightarrow$ Linearity ∴ Equation 1 is linear.