Tutorial : Discrete Random Variables

Go through the following questions for a better understanding of the topic. Some of the questions were discussed in the Tutorial on 21st February. Try solving them on your own.

Q1.)

In this problem, we would like to show that the geometric random variable is **memoryless**. Let $X \sim Geometric(p)$. Show that

$$P(X > m + l | X > m) = P(X > l), \text{ for } m, l \in \{1, 2, 3, \cdots\}.$$

We can interpret this in the following way: Remember that a geometric random variable can be obtained by tossing a coin repeatedly until observing the first heads. If we toss the coin several times, and do not observe a heads, from now on it is like we start all over again. In other words, the failed coin tosses do not impact the distribution of waiting time from this point forward. The reason for this is that the coin tosses are independent.

Q2.)

The **median** of a random variable X is defined as any number m that satisfies both of the following conditions:

$$P(X \geq m) \geq \frac{1}{2} \quad \text{and} \quad P(X \leq m) \geq \frac{1}{2}$$

Note that the median of X is not necessarily unique. Find the median of X if

a. The PMF of X is given by

$$P_X(k) = \begin{cases} 0.4 & \text{for } k = 1 \\ 0.3 & \text{for } k = 2 \\ 0.3 & \text{for } k = 3 \\ 0 & \text{otherwise} \end{cases}$$

b. X is the result of a rolling of a fair die.
c. $X \sim Geometric(p)$, where $0 < p < 1$.

Q3.) Prove the following.

Let X be a discrete random variable which has the binomial distribution with parameters n and p.

Then for $\lambda = np$, X can be approximated by a Poisson distribution with parameter λ:

$$\lim_{n \to \infty} \binom{n}{k} p^k (1-p)^{n-k} = \frac{\lambda^k}{k!} e^{-\lambda}$$

Solution :

Follow the video for a detailed solution, it will take only 5 minutes.

https://www.youtube.com/watch?v=ceOwlHnVCqo

Written version of the proof can be found here:-

https://medium.com/@andrew.chamberlain/deriving-the-poisson-distribution-from-the-binomial-distribution-840cc1668239

Following are some more questions, not discussed in Tutorial.

Q4.)

The number of customers arriving at a grocery store is a Poisson random variable. On average 10 customers arrive per hour. Let X be the number of customers arriving from $10am$ to $11:30am$. What is $P(10 < X \leq 15)$?

Q5.)

I roll a fair die repeatedly until a number larger than 4 is observed. If N is the total number of times that I roll the die, find $P(N = k)$, for $k = 1, 2, 3, \ldots$.

What is type of distribution does N follow?