CHAPTER 11

POWER

Instantaneous power.

inst voltage   inst. current

$$p(t) = v(t)\, i(t)$$

$$P_R(t) = v(t)\, i(t) = i^2(t) R = \frac{v^2(t)}{R}$$

$$P_L(t) = L\, i(t) \frac{di(t)}{dt} = \frac{1}{L} v(t) \int_{-\infty}^{t} v(t)\, dt$$

$$\left(v_L = L \frac{di(t)}{dt} \right)$$

$$P_C(t) = C\, v(t) \frac{dv(t)}{dt} = \frac{1}{C} i(t) \int i(t)\, dt$$

$$\left(i_C(t) = C \frac{dv(t)}{dt} \right)$$

$V_m(t)$ $I_m(t)$

Sinusoids: $p(t) = (V_m \cos \omega t)(I_m \cos(\omega t + \phi))$ $\left(\omega = \frac{2\pi}{T} \right)$

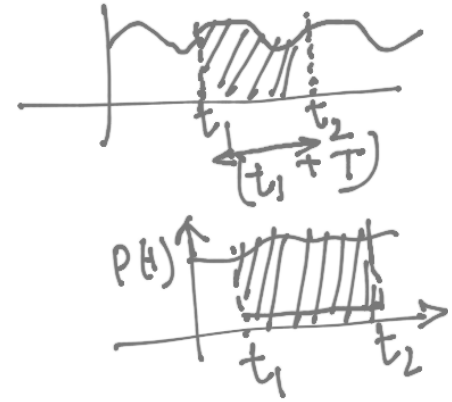
$$= \underbrace{\frac{V_m I_m}{2} \cos\phi}_{\text{Constant (non time varying)}} + \underbrace{\frac{V_m I_m}{2} (\cos 2\omega t + \phi)}_{\text{Time varying (period} = 2\omega) \text{ rad/s}}$$

$\frac{1}{2T}$

Average Power

$$P_{avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)\, dt$$

$\underbrace{\int_{t_1}^{t_2} p(t)\,dt}_{\text{energy}}$



For periodic (T) waveforms $t_2 = T + t_1$

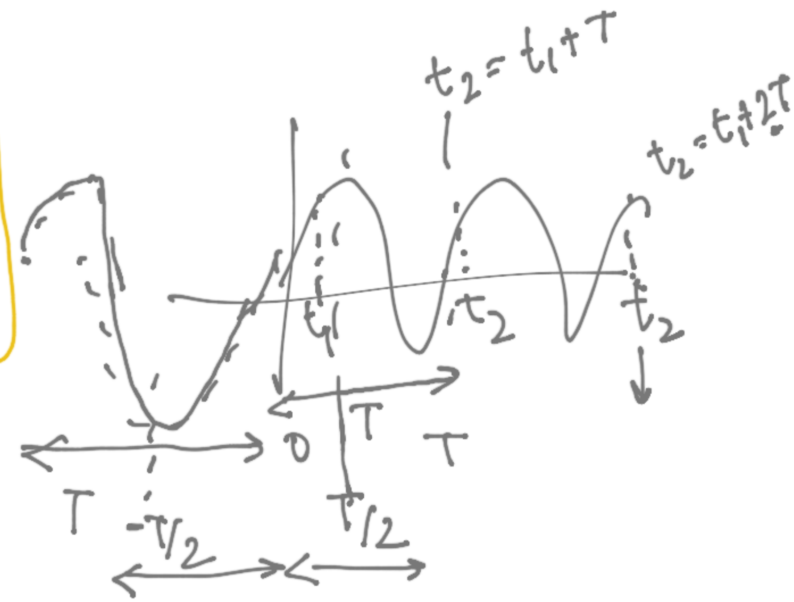
$$P_{avg} = \frac{1}{T} \int_{t_1}^{t_1+T} p(t)\, dt = \boxed{\frac{1}{T} \int_{-T/2}^{T/2} p(t)\,dt}$$

Time period of waveform

or $$P = \frac{1}{nT} \int_{t_1}^{t_1+nT} p(t)\, dt \qquad n = 1, 2, 3 \ldots$$

$(t_2 = t_1 + nT)$

$$\boxed{P_{avg}\ (\text{sin.}) = \frac{1}{nT} \int_{-nT/2}^{nT/2} p(t)\, dt}$$



In some cases $T \to \infty$ (specific case)

$$P_{avg} = \lim_{T \to \infty} \frac{1}{nT} \int_{-nT/2}^{nT/2} p(t)\,dt$$

Avg. of sin. power

$$\underset{(\text{sin forcing})}{P_{avg}} = \frac{1}{T}\int p(t)\,dt = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos\phi\, dt + \underbrace{\frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos(2\omega t+\phi)\, dt}_{=0}$$

On taking average over $T = \frac{2\pi}{\omega}$

$$\boxed{\underset{(\text{sin }f)}{P_{AVG}} = \frac{V_m I_m}{2}\cos\phi}$$

General case of forcing voltage $V = V_m \cos(\omega t + \theta)$, $\boxed{P_{avg} = \frac{V_m I_m}{2}\cos(\theta - \phi)}$

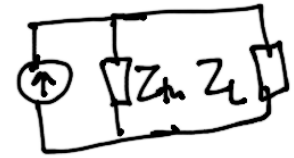
Resistor ($\phi = 0°$) I_m & V_m are in same phase (or in phase with each other)

$$\boxed{P_{avg\,R} = \frac{V_m I_m}{2}} \longrightarrow \text{Power absorbed (or dissipated) by resistor.}$$

Ideal L or Cap (no 'R') $\phi = 90°$ ($-90°$ for L & $+90°$ for C)

$\therefore$ $\boxed{P_{avg\,L} = 0}$ & $\boxed{P_{avg\,C} = 0}$

Max. Power Transfer



If Z_{th} is eq. (thevenin) impedance, max. power delivered to load is when $\boxed{Z_L = Z_{th}^*}$ $\underbrace{\phantom{Z_{th}}}_{\text{Conjugate}}$

Proof
HW 5
$P = ?$
$\angle\phi$

$\rightarrow Z_{th} = R + jX$
$Z_{th}^* = R - jX$

Average Power due to multiple currents :—

$$i(t) = I_{m1}\cos\omega_1 t + I_{m2}\cos\omega_2 t + \cdots + I_{mN}\cos(\omega_N t)$$

$$\boxed{P_{avg} = \frac{1}{2}\left(I_{m1}^2 + I_{m2}^2 + \cdots + I_{mN}^2\right)R}$$

$I_{m1}\cos\omega_1 t$
$I_{m2}\cos\omega_2 t$
$\vdots$
$I_{mN}\cos\omega_N t$

Effective Value

Eff. value of a periodic current is equal to D.C value of current which delivers same power to 'R' (resistor) when flowing through it.

(I_{eff} = D.C value)

Power absorbed $= I_{eff}^2 R = \frac{1}{T}\int_0^T i^2 R\,dt$ sinusoidal power dissipated

$$\Rightarrow \quad I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

For sinusoidal wave form $\quad I_{eff}^2 = \frac{1}{T}\int_0^T \left(I_m^2 \cos^2(\omega t+\phi)\right)dt$

$$\boxed{V_{rms}^2 = \frac{V_m^2}{2}}$$

$$\boxed{\underset{(I_{rms})}{I_{eff}} = \frac{I_m}{\sqrt{2}}}$$

RMS value current (Root Mean Square)

$$\therefore \quad P = \frac{I_m^2}{2} R = \frac{I_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} R$$

$$\boxed{P_{avg} = I_{eff}^2 R}$$

$$\boxed{P = \frac{V_{eff}^2}{R}}$$

$$I_{eff} = I_{rms}$$

$$V_{eff} = V_{rms}$$

For multiple freq. circuits $I_{eff} = \sqrt{I_{1eff}^2 + I_{2eff}^2 + I_{3eff}^2 + \cdots I_{Neff}^2}$

$$I_{eff}^2 = \frac{1}{2}\left(I_{m_1}^2 + I_{m2}^2 + \cdots I_{mN}^2\right)$$

$$\left(I_{eff1}^2 + I_{2eff}^2 + \cdots + I_{eff}\right)$$

$$I_{rms}^2 = I_{eff}^2 = \left(I_{eff1}^2 + I_{2eff} \right.$$

(L, C, R)

For Reactive circuits

(L & C)

Due to ckt. elements

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

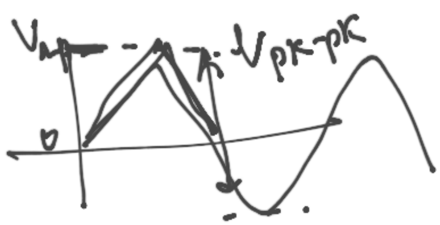
$$P_{avg} = \underbrace{V_{eff} I_{eff}}_{\text{Apparent Power}} \cos(\theta - \phi)$$

$$\cos(\theta - \phi) = \frac{P_{avg}}{V_{eff} I_{eff}} = \text{Power Factor}$$

(PF)

$$\theta - \phi = \text{power factor angle}$$

$PF = 0$ (for L & C only circuits)



Complex Power

$$\underline{V}_{eff} = V_{eff} \angle \theta$$ (amp)

$$\underline{I}_{eff} = i_{eff} \angle \phi$$

$$P_{avg} = V_{eff}\, I_{eff}\, \mathrm{Re}\left(e^{j(\theta-\phi)}\right) \longrightarrow$$ Coming from discussion earlier.

$$= \mathrm{Re}\left[V_{eff}\, I_{eff}\left(e^{j\theta} \cdot e^{-j\phi}\right)\right]$$

$$= \mathrm{Re}\left[V_{eff}\, e^{j\theta}\; I_{eff}\, e^{-j\phi}\right]$$

$$P_{avg} = \mathrm{Re}\left(\underbrace{V_{eff}\; I_{eff}^{*}}_{\text{Complex Power}}\right)$$

Complex conjugate → $S = A + jB$, $S^{*} = A - jB$

Complex Power $= \underline{V}_{eff} * \underline{I}_{eff}^{*} = S$

Complex power

$$\text{(VA)} \leftarrow S = P + jQ$$

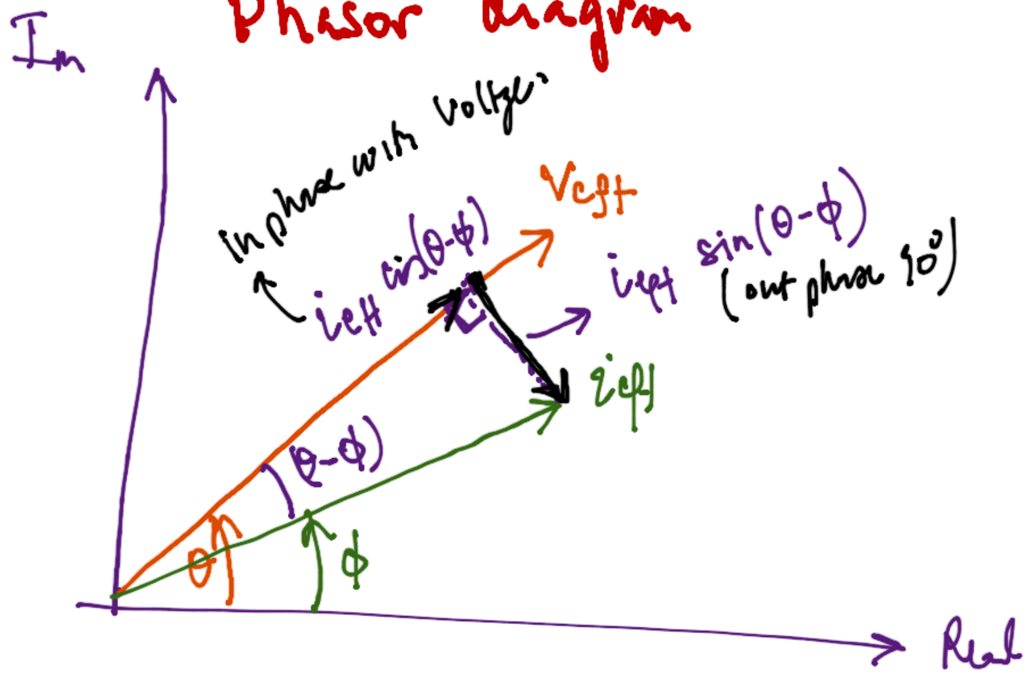
P → Real power (W)

Q → 'Reactive Power'

Units: Volt * Amp Reactive (VAR)

No energy transfer
= Wattless power

Phasor diagram



$V_{eff} = V_{eff} \angle \theta$

$I_{eff} = i_{eff} \angle \phi$

Real Power (W)

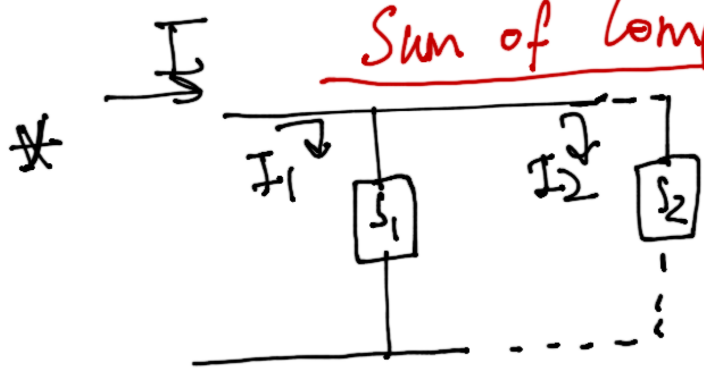
$V_{eff} * i_{eff} \cos(\theta-\phi)$

$V_{eff}\, i_{eff} \sin(\theta-\phi)$

Reactive part (VAR)

(90° out of phase with V_m)

Sum of Complex Power



$\sum$ Complex Power $= S_1 + S_2 + \cdots$

Complex $= V I^* = S$

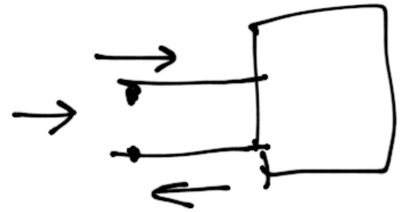
$= V(I_1^* + I_2^* + \cdots)$.

$= V I_1^* + V I_2^* + \cdots$

$= S_1 + S_2$

CHAPTER 13

One Port Network



One port Network

1 port.

Node voltage

Currents

$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 + \cdots + Z_{1N} I_N \\ V_2 = \\ \vdots \\ V_N = Z_{N1} I_1 + \cdots + Z_{NN} I_N \end{cases}$$

$$\underbrace{\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}}_{\substack{V \\ \text{Input}}} = \underbrace{\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ \vdots & & & \\ Z_{N1} & Z_{N2} & & Z_{NN} \end{bmatrix}}_{\substack{\Delta_2 \\ \text{Impedance Matrix}}} \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}}_{\substack{I \\ \text{Current mat}}}$$

$$I = V / \Delta_2$$

$$\frac{1}{\Delta_Z} = \text{Admittance Matrix } Y$$

$$\left\{ \begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned} \right\}$$ Simple case example.

$$V_1 = Z\, I$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad V_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Impedance Parameter (Z)

$$Y = \frac{I}{V}$$

$$I_1 = Y_{11} V_1 + \cdots + Y_{1N} V_N$$

$$I_2 = Y_{21} V_1 + \cdots + Y_{2N} V_N$$

$$\vdots$$

$$I_N = Y_{N1} V_1 + \cdots + Y_{NN} V_N$$

$$\underbrace{\quad}_{I} \qquad Y = \begin{pmatrix} Y_{11} & \cdots & Y_{1N} \\ \vdots & & \\ Y_{N1} & & Y_{NN} \end{pmatrix} \qquad V = \begin{pmatrix} V_1 \\ \vdots \\ V_N \end{pmatrix}$$

Admittance matrix

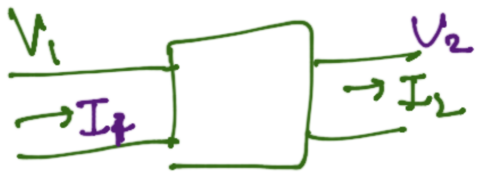
$$Y_{11} = \left.\frac{I}{V}\right|_{V_2 = V_3 = \cdots = V_N = 0}$$

$$\vdots$$

Hybrid

HYBRID PARAMETER & MATRIX

$$\begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{cases}$$



H matrix $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$ → Hybrid parameter

V_1, I_2 → Outputs

I_1, V_2 → Forcing

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = h \text{ matrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \frac{V_1}{I} \Big|_{V=0}$$

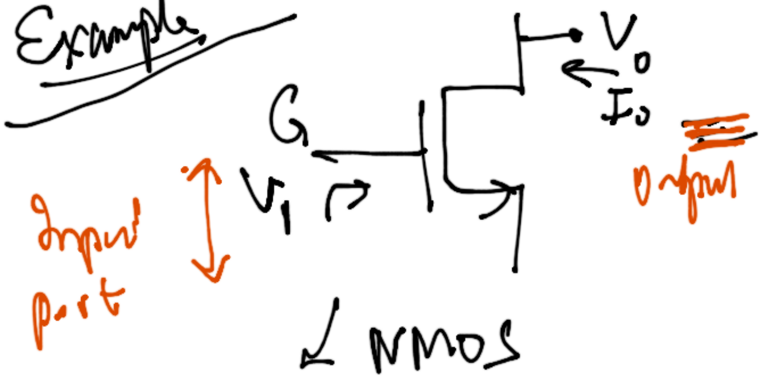
Transmision Matrix

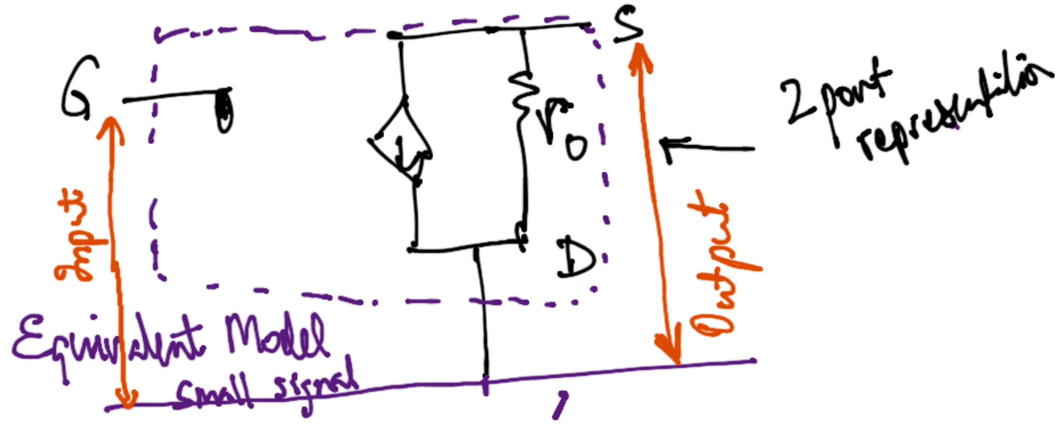
$$V_1 = t_{11} V_2 - t_{12} I_2$$

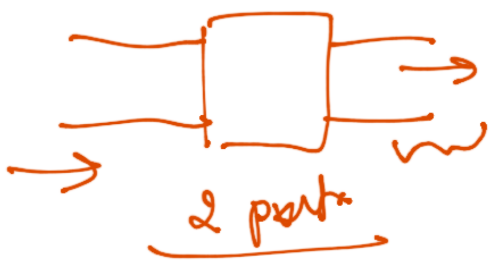
$$I_1 = t_{21} V_2 - t_{22} I_2$$

$$t = \begin{bmatrix} t_{11} & -t_{12} \\ t_{21} & -t_{22} \end{bmatrix}$$ Transmission Parameters.

Example







$$\underbrace{\begin{bmatrix} V_o \\ I_o \end{bmatrix}}_{\text{Outputs}} = \underbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}_{\text{Parameters}} \underbrace{\begin{bmatrix} V_i \\ I_i \end{bmatrix}}_{\text{Inputs}}$$

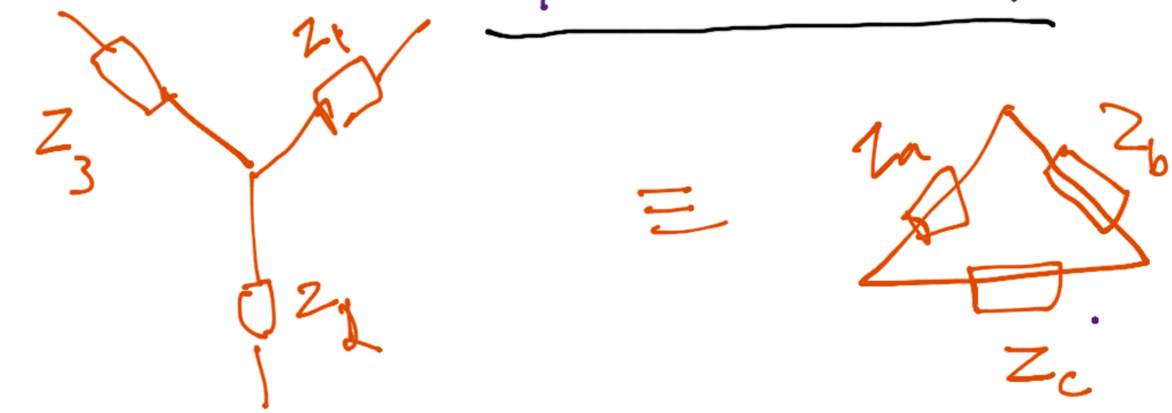
Parameters Defining Transistors

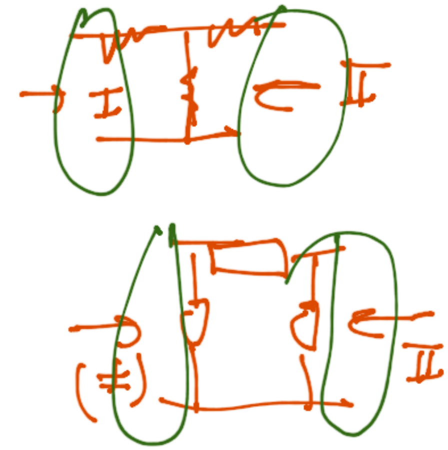
$$\frac{V_o}{V_i} = \text{Gain}$$

$$\left.\frac{V_o}{I_o}\right| = \text{Output resistance}$$

$$\frac{I_o}{V_i} = \text{Trans-conductance} \quad \left(\frac{\partial i_d}{\partial v_{gs}}\right)$$

$Y \leftrightarrow Z$ Conversion





Same results as Impedance