EC5.102: Information and Communication

Module: Kraft inequality

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A quick review

Overview till now and agenda for rest of the classes

- Goal of a communication system: How can a transmitter "speak" with a receiver?
- Digital communication system
- Where are we with respect to block diagram of a digital communication system?
- Rest of the classes...
  - Source coding (Details in the course "Information theory")
  - Channel coding (Details in the course "Information theory", "Introduction to coding theory", and "Advanced coding theory")
  - Modulation (Details in the course "Communication theory")
  - Channels (Details in the courses "Information theory", "Communication theory", and "Wireless communication")

Topics in source coding

Recap of source coding

- Aim: Data compression (called as source encoding)
- Desirable properties/categories of a source code
- Huffman codes
- When do say the source code is the "best"? How do I define "best"?
- Intuitive understanding of source coding theorem:
  - Huffman codes for n-th order extension $\mathcal{X}^n$ of a input source data $\mathcal{X}$
  - Key idea for proving source coding theorem: Typical/non-typical sets
- Source coding theorem: For any rate $R > H(X)$, there exists a sequence of codes of rate R such that the corresponding probability of decoding error approaches zero as n tends to infinity.

Topics in source coding: Kraft inequality

- Suppose we wish to construct an instantaneous code (prefix code)
- Let $\ell_1, \ell_2, \ldots, \ell_m$ be codeword lengths of an instantaneous code.
- Kraft inequality: Any binary instantaneous code with lengths $\ell_1, \ell_2, \ldots, \ell_m$ should satisfy:

$$\sum_{i=1}^{m} 2^{-\ell_i} \leq 1.$$

Conversely, given a set of codeword lengths that satisfy this inequality, there exists an instantaneous code with these codeword lengths.

Proof outline: Kraft inequality

Kraft inequality: Any binary instantaneous code with lengths $\ell_1, \ell_2, \ldots, \ell_m$ should satisfy: $\sum_{i=1}^{m} 2^{-\ell_i} \leq 1$.

- Without loss of generality, suppose $\ell_1 \leq \ell_2 \leq \ldots \leq \ell_m$.
- Binary tree: Branches represent codewords
- Prefix condition: Each codeword eliminates its descendants as possible codewords.
- Focus on the branches at level ℓ_m:
  1. Some of them are codewords
  2. Some are descendants of codewords
  3. Some are neither
- A codeword at level ℓ_i has $2^{\ell_m - \ell_i}$ descendants at level ℓ_m.

$$\sum_{i=1}^{m} 2^{\ell_m - \ell_i} \leq 2^{\ell_m}$$