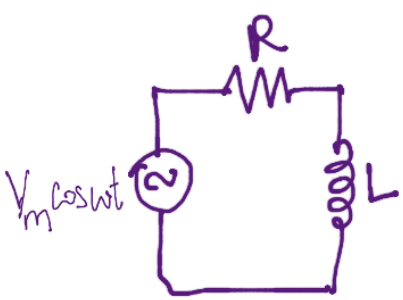
Sinusoid Response

R-L



KVL $\quad L\dfrac{di}{dt} + iR = V_m \cos\omega t$ —— (1)

ω → Input angular freq. (rad/s) — π rad $= 180^\circ$

→ freq. of input sig.

Solution i will be form of $\sin\omega t$ and $\cos\omega t$

$$i = I_1 \cos\omega t + I_2 \sin\omega t$$

Substitute i in (1) and separate $\cos\omega t$ & $\sin\omega t$ terms & $=0$.

$\left(e^{-\alpha t/\tau}\right)$ → decay

⇒ Solve I_1 & I_2 (constants)

$$i = \frac{R V_m}{(R^2+\omega^2L^2)}\cos\omega t + \frac{\omega L V_m}{(R^2+\omega^2L^2)}\sin\omega t$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\longrightarrow\ i = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\left[\underbrace{\frac{R}{\sqrt{R^2+\omega^2L^2}}}_{\cos\theta}\cos\omega t + \underbrace{\frac{\omega L}{\sqrt{R^2+\omega^2L^2}}}_{\sin\theta}\sin\omega t\right]$$

S.S after Nat. response decay

$$\boxed{i = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)} = \frac{V_m}{\sqrt{(R^2+\omega^2L^2)}}\,\mathrm{Re}\left(e^{j(\omega t-\theta)}\right)$$

Observations (L-R)

- i LAGS behind v_s by $\theta = \tan^{-1}\frac{\omega L}{R}$ [if $R \to 0$ lag $= 90°$]

- $i \propto v_s$ $\frac{V_m}{\sqrt{R^2+\omega^2 L^2}}$ ✓

- $j\omega L$ is called INDUCTIVE REACTANCE (X_L)

- Forced resp — Steady state response (here) is time varying but does not decay. (We allow the transient natural response to die

Complex Forcing Function (easy method to solve) $(j = i = \sqrt{-1})$

- Replace $\underbrace{V_m \cos\omega t}_{\text{supply}} \longrightarrow V_m e^{j\omega t}$ (Real)
- Let us assume a solution of form $I_m e^{j(\omega t+\phi)}$ ϕ – phase shift.

$$\rightarrow L\frac{di}{dt} + iR = L I_m j\omega\, e^{j(\omega t+\phi)} + R I_m e^{j(\omega t+\phi)} = V_m e^{j\omega t}$$

$$\Rightarrow I_m (j\omega L + R) e^{j\phi} = V_m \Rightarrow I_m e^{j\phi} = \frac{V_m}{(j\omega L + R)}$$

$$I_m \underbrace{e^{j\phi}}_{(\cos\phi + j\sin\phi)} = \frac{V_m}{R^2+\omega^2L^2}(R - j\omega L)$$

Real LHS = Real RHS

Im LHS = Im RHS

$$I_m \cos\phi = \frac{V_m}{R^2+\omega^2L^2} \cdot R \longrightarrow (1)$$

$$I_m \sin\phi = -\frac{V_m}{R^2+\omega^2L^2} \omega L \longrightarrow (2)$$

$$\phi = \tan^{-1}\frac{(2)}{(1)} = -\tan^{-1}\frac{\omega L}{R}$$

$$I_m = \sqrt{(1)^2 + (2)^2} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}$$

$$\boxed{\begin{aligned} V\cos(\omega t) &\to V_m e^{j\omega t} \\ I\cos(\omega t-\phi) &\to I_m e^{j(\omega t+\phi)} \end{aligned}}$$

Ans: Real part of $I_m e^{j(\omega t-\phi)} \Rightarrow$ $\boxed{i = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)}$

* Superposition theorem: Real forcing function → real part of response

Imaginary forcing function → imaginary part of response.

$$I_m = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \cos\Big(\omega t - \underbrace{\tan^{-1}\frac{\omega L}{R}}_{\phi}\Big) \rightarrow$$

Polar notation : $\Big[\underbrace{I_m}_{\text{Magnitude}} \; \underbrace{\angle\phi}_{\text{Phase difference}}\Big] \equiv \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}} \;\angle -\tan^{-1}\dfrac{\omega L}{R}$

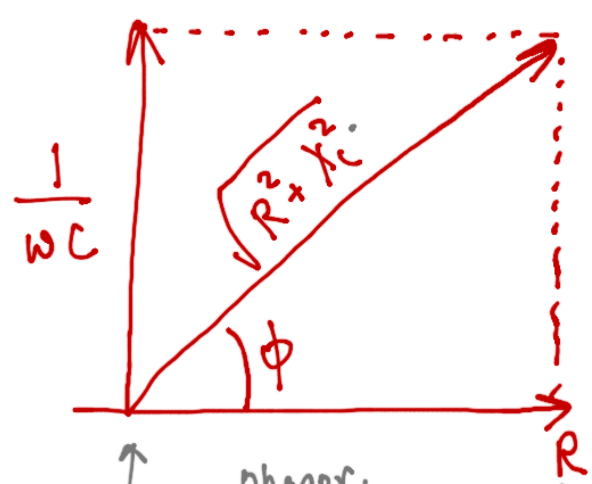
$\omega = \boxed{}$

HW : Solve R-C circuit S.S solution using (Complex forced function)

Show : (i) Current leads the voltage

(ii) $I_m = \dfrac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$ and $\angle\phi = \tan^{-1}\left(\dfrac{1}{\omega RC}\right)$

$X_C = \dfrac{1}{j\omega C}$: Capacitive Reactance



Draw phasor diagram

Phasors

$i(t) = I_m \cos(\omega t + \phi)$: Time domain

$\Downarrow$

$i(t) = R_e\left[I_m e^{j(\omega t + \phi)}\right]$

$\omega \to$ ang. freq. (rad/s)

$\omega = \frac{2\pi}{T} = 2\pi f$, $f \to$ freq (Hz), $T \to$ Time period

$\Downarrow$

$\hookrightarrow \underline{I} = I_m \underline{e}^{j\phi} \Rightarrow \underline{I_m \angle\phi}$: Frequency Domain.

(polar form)

Eg $V = 115 \angle{-45^\circ}$ V, $\omega = 500$ rad/s

$v(t) = 115 \sin(500t - 45^\circ)$

time	phasor
$R \rightarrow$	R
$L \rightarrow$	$j\omega L = X_L$ (Ind. Reactn.)
$C \rightarrow$	$\frac{1}{j\omega C} = \frac{-j}{\omega C} = X_C$ = Cap. Reactn.

Simple solution using $\boxed{\text{phasors}}$

$X_L = j\omega L$

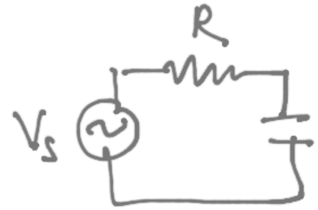
Impedance $Z = \underline{R} + \overbrace{\underline{j\omega L}}^{X_L}$

V supply $= V_s \cos\omega t = \underline{V_s \angle 0^\circ}$

$\therefore I = \frac{V_{supply}}{Z} = \frac{V_s \angle 0}{R + j\omega L}$

$\rightarrow |I| = \frac{V_s}{\sqrt{R^2 + \omega^2 L^2}}$; $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Similarly for R-C

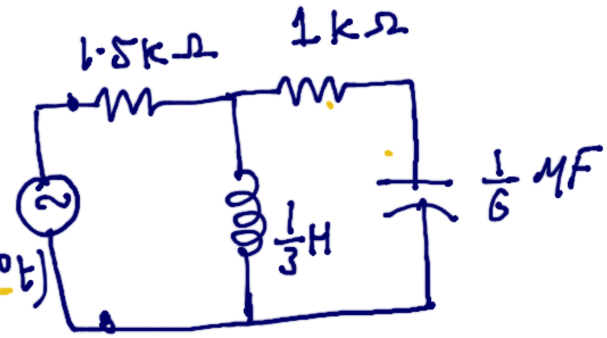


$+\frac{1}{j\omega C}$

$$I = \frac{V_s \angle 0^\circ}{\left\{R + \frac{1}{j\omega C}\right\}} \quad \leftarrow \text{Supply}, \quad \rightarrow Z \ \Omega$$

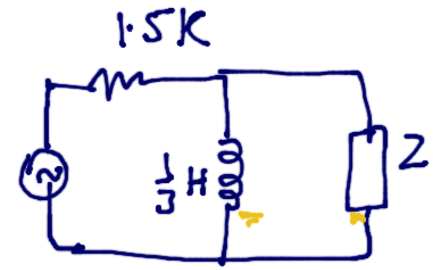
$$\underset{(s.s)}{I} = \frac{V_s}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \angle \tan^{-1} \frac{1}{(\omega RC)}$$

Example (10.7)



$= 40 \sin(3000t)$

$\equiv$



$$Z_1 = \underset{R}{1000} - j\underset{X_C}{2000}$$

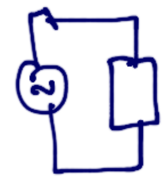
$\equiv$

$\omega = 3000$ rad/s

$$V_s = 40 \cos(3000t - 90^\circ) \rightarrow \phi$$

$$Z_L = j\omega L = j\, 3000 * \frac{1}{3} = j\, 1k\ \Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j\, 3000 * \frac{10^{-6}}{6}} = -j\ 2000\ \Omega$$



$$Z_q = 1.5k + \left(j\underset{X_L}{1000} \,\|\, Z_1\right)$$

$$= \underset{R}{1.5K} + \frac{j1K * (1K - j2K)}{1Kj + 1K - j2K}$$

$$I = \frac{V \angle \phi}{Z_q} = \frac{40\, e^{\phi}}{\frac{(3-j)}{2} \times 1000 \times 1.5K}$$

$$= \frac{40\, e^{\phi}}{K\, e^{\theta}} = \left(\frac{40}{K}\right) e^{(\phi - \theta)}$$

$$Z = 1.5K + \frac{(j+2)K\,(1-j)}{(1-j)\,(1+j)}$$

$$Z = \left(\frac{3-j}{2}\right) K * 1.5K$$