IV. Conclusion

This report shows that we are able to increase the output power of the ^{87}Rb maser by about 5 dBm by using optical pumping at both ends and to make the short-term frequency stability improvement of about a factor of 1.4. The short-term frequency stability might be improved further by using two diode lasers with higher intensity and by optimizing the cell temperature. Our measurements are consistent with the density matrix treatment.

Acknowledgment

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A Simple Wide-Band Sine Wave Quadrature Oscillator

Ralph Hölzel

***Abstract*—A simple sine wave oscillator delivering two signals in quadrature has been developed, covering the frequency range from below 1 Hz to 100 MHz and being voltage-controllable over each frequency decade. The circuit consists of two 90° phase shift stages using active allpass filters and an inverter, all arranged in a feedback loop. Although general application is envisaged, the design is particularly suitable in particle electrorotation studies.**

I. Introduction

Quadrature generators, i.e., generators producing two signals of identical frequency and amplitude but of different phase angle, are used in a variety of applications, as for example in telecommunications f… quadrature mixers and single-sideband generators [1] or for measurement purposes in vector generators or selective voltmeters [2]. They have found further utility in the physical characterization of microscopic particles and biological cells using the phenomenon of electrorotation, whereby rotational torques are induced through application of rotating electric fields of variable frequency [3]–[5]. The availability of wide bandwidth quadrature oscillators of simple design is important for such studies, and it is this consideration that has led to the circuit development described here.

Rectangular quadrature signals (i.e., of 90° phase shift) are readily available up to some tens of MHz using ring oscillators or by dividing the frequency of a square wave by two and combining this with the original signal via logic gates [5]. Sinusoidal voltages can be generated using special commercially available IC's (e.g., Burr Brown 4423 up to 20 kHz). Higher frequencies can be achieved by programming the differential equation of a sinusoidal oscillation with the help of operational amplifiers [6]. For that two integrator stages are connected in series with an inverter, of which the output is fed back to the input of the first stage. However, the upper frequency limit of such an oscillator is still limited by the relatively low bandwidth of the integrators.

Here we describe a circuit that utilizes two allpass filters as 90° phase shifters in series with an inverter operating up to 100 MHz.

II. Design Considerations

Ideal first-order allpass filters (Fig. 1) are characterized by a frequency-dependent phase shift varying from −180° at low frequencies to 0° at high frequencies, given by

$$\varphi = -2 \operatorname{arccot} (2\pi f RC), \tag{1}$$

while the amplification equals unity for $R_1 = R_2$ at all frequencies [7]. Cascading two allpasses and an inverter and feeding back the output to the first stage (Fig. 2) lead to an oscillation [8] at the frequency where $\varphi = -90°$, i.e., at

$$f = (2\pi RC)^{-1}. \tag{2}$$

Equations (1) and (2) only hold for moderately low frequencies, since real operational amplifiers exhibit an additional phase shift increasing with frequency due to parasitic capacitances. Generally, oscillation can only occur if $\varphi_1 + \varphi_2 + \varphi_i = -360°$.

As long as the inverter stage does not add any additional phase error, i.e., $\varphi_i = 180°$, the sum of the allpasses' phase shifts must amount to $\varphi_1 + \varphi_2 = 360° - \varphi_i = 180°$ for oscillation. For identical allpass stages φ_1 and φ_2 are identical, even if they do not work ideally, e.g., at higher frequencies. With φ_1 and $\varphi_2 = 180°$ and $\varphi_1 = \varphi_2$ follows $\varphi_1 = \varphi_2 = 90°$. That means the output signals are always in quadrature provided 1) the inverter's phase error is zero and 2) the allpass stages are identical. Therefore, the bandwidth of the inverter is more important for the circuit's frequency range than the bandwidth of the allpasses.

In selecting suitable operational amplifiers account should be taken of the fact that all stages work at amplifications of unity. Therefore, many high-speed operational amplifiers, that are usually not unity gain stable, cannot be used or only so with some restrictions. To cancel variations in amplification and to maintain low distortions, the inverter's gain should also be controllable. However, inverter gains deviating from unity will correspond to deviating allpass gains and thus lead to an inequality of the output amplitudes.

In order to control the oscillation frequency via a dc voltage, field effect transistors (FET), light dependent resistors (LDR) or capacitance diodes can be used. The latter are less useful for frequencies below 1 kHz because of their maximum capacitance values of about 1 nF and the typical input resistance of the order 1 MΩ of fast operational amplifiers. FET's are available as matched

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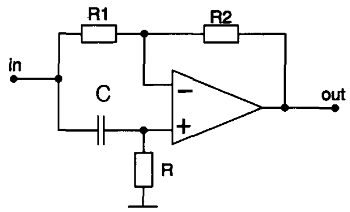

Fig. 1. First-order allpass filter.

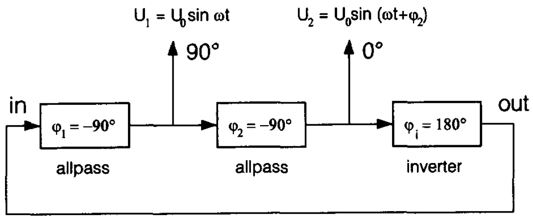

Fig. 2. Block-diagram of the quadrature oscillator.

pairs and thus have the advantage over LDR's in achieving equality of both phase shift stages. Furthermore, the approximately reciprocal dependence of the drain-source-resistance on the gate-source-voltage [9] can be exploited to achieve a linear relationship between controlling voltage and oscillator frequency.

III. Circuitry and Experimental Results

To prevent instability caused by stray effects mainly of the two-pole switch S_1 (Fig. 3) only moderately fast operational amplifiers (Analog Devices AD 847, full power bandwidth 10 MHz) are used in the allpass stages. The RC network at the inverter's input permits a stable function of the fast op amp, which otherwise would need a gain of 4 or more. The oscillation amplitude is limited by the nonlinearity of germanium diodes AA 112 and can be adjusted by V_g via FET BF 256, which works as a voltage-controlled resistor. At frequencies below a few MHz the FET can be replaced by a fixed resistor, since the amplitude remains nearly constant. However, for minimizing signal distortion it is advisable to control the gain automatically via V_g and a PI-regulator, that measures the actual oscillation amplitude.

The frequency can be tuned by V_f, which controls the drain-source-resistances of the FET pair 2 N 5912. Good linearity between controlling voltage and oscillation frequency was achieved over a whole frequency decade as shown in Fig. 4. By switching the frequency-determining capacitors a frequency range of more than 7 decades from below 1 Hz up to 10 MHz could be covered. On examining the use of different types of operational amplifiers in the allpass stages it was found that, as a good measure of the accessible frequency range, the full power bandwidth cited in the semiconductor's datasheet can be used rather than the unity gain or 3 dB bandwidth.

The signal's harmonic content at 100 kHz amounts to -24 dB for the third and less than -36 dB for all other harmonics, if the gain control voltage V_g is kept constant at a value that allows frequency variations over several decades. If V_g is optimized and controlled automatically by a PI-regulator the third harmonic is reduced to -38 dB.

Even higher frequencies can be achived using faster amplifiers and through a reduction of stray capacitances. For this purpose a

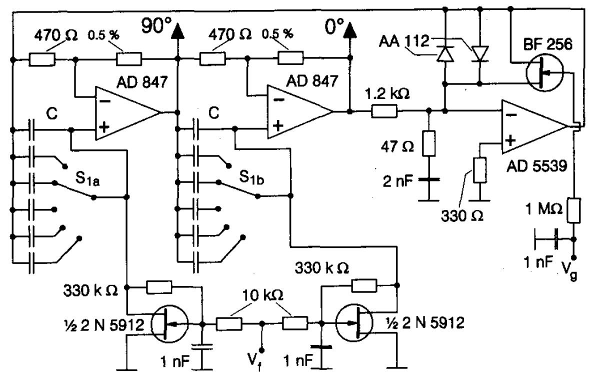

Fig. 3. Circuit diagram of the voltage-controlled sine quadrature oscillator operating up to 10 MHz.

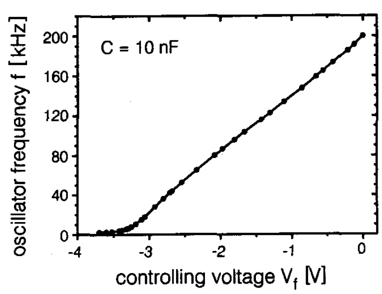

Fig. 4. Dependence of the output frequency of the sine quadrature VCO (Fig. 3) on the controlling dc voltage.

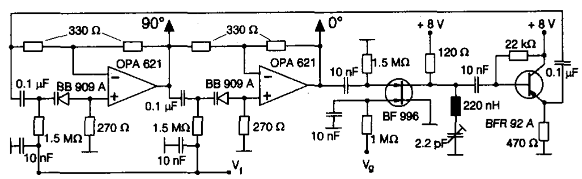

Fig. 5. Circuit diagram of the quadrature VCO for frequencies up to 100 MHz.

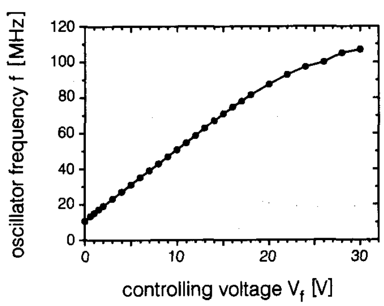

Fig. 6. Dependence of the output frequency of the quadrature VCO (Fig. 5) on the controlling dc voltage.

second circuit was built employing surface mount components (Fig. 5), with the very fast op amp OPA 621 (Burr Brown, full power bandwidth 80 MHz) being employed in the allpass filter stages and with the relatively large FET pair being replaced by two capacitance diodes. A dual-gate MOSFET, followed by a bipolar collector stage, operates as a voltage-controlled inverter, and the LC network at the MOSFET's drain suppresses parasitic oscillations at 230 MHz caused by the op amp (that is usually only stable at gains of at least 2). By these means the frequency range was extended to 100 MHz, although somewhat higher operating frequencies are possible, provided unequal amplitudes of the quadrature outputs can be tolerated. The correlation between tuning voltage and frequency is linear over nearly a frequency decade (Fig. 6), with decreasing slope above 80 MHz.

This quadrature oscillator design has been successfully employed to generate rotating electrical fields in studies of the ac electrokinetics of biological cells [3], [4].

IV. Conclusion

A simple voltage-controlled sine wave oscillator with quadrature output has been developed, which is useful at frequencies from below 1 Hz to about 100 MHz for applications such as the study of particle electrorotation phenomena. If a lower frequency limit is required, this should be achieved by rapid switching of the frequency-determining capacitors with analogue switches [10], rather than by using large electrolytic capacitors. A further expansion of the bandwidth to higher frequencies might be possible using even faster hybrid operational amplifiers or discrete components, but this would be at the expense of the circuit's simplicity.

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An Algorithm for Solving Roberts–von Hippel Equation: Separation of Close Solutions

T. P. Iglesias, A. Seoane, and J. Rivas

***Abstract*—The proximity of the solutions of the transcendental equation tanh $z/z = c$ on the complex plane arising in the determination of the complex dielectric permittivity with the short-circuited line method is studied. A numerical procedure is described which is capable of obtaining the required number of solutions, even those lying very close to each other, from one single initial point. The method is applied to measurements of various organic liquids.**

I. Introduction

The short-circuited line method for the determination of the complex dielectric permittivity [1] requires solving a transcendental equation of the form $(\tanh z)/z = c$ on the complex plane, where c is obtained experimentally. Although this equation possesses infinite solutions, only one of them is physical. In order to eliminate this ambiguity it is necessary either to know in advance an approximate value of the permittivity, to carry out two experiments with samples of different thickness, or to perform measurements at two nearby frequencies [2], [3]. Before deciding which of the solutions is the physical one it is necessary to be able to identify and distinguish from each other all solutions which can be relevant. In practice it is enough to consider only a few of the infinite solutions, with small values of the real part, given the range of dielectric permittivities of usual substances. It is important to identify and distinguish close roots when studying, for instance, the effect that the thickness of the sample has on the uncertainty of the permittivity [4]. It is also necessary when measuring dielectric permittivities with network analyzers, where the impedance of the sample is measured for a continuous range of frequencies and a fixed sample thickness [5]. An analogous situation arises in the time-domain dielectric characterization technique which uses a short-circuited sample.

Several methods for the resolution of the equation above have been developed. One of them is Dakin–Works' approximation [6], which is not applicable for samples with loss tangent $\tan \delta > 0.1$ due to the large errors that can arise, as it is shown by S. O. Nelson *et al.* [7]. There are also graphical methods such as Roberts–von Hippel's [1] or Delbos–Demau's [8]. The latter is used when the sample has constant thickness. Computers have allowed the use of iterative numerical methods, of which the more commonly used is Newton–Raphson's [4], [9]. This method converges from a good initial approximate value to one single root, so that if there were two roots close to each other one of them might be ignored. Gelinas *et al.* [10] also use Netwon's method for the case $Re(c) \leq 0$, since there are no close roots corresponding to this region of the complex plane.

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