Network, Signals & Systems

Introduction

NeSS

- Hybrid course
- Network theory / circuit analysis
- Signals & Systems

What are systems?

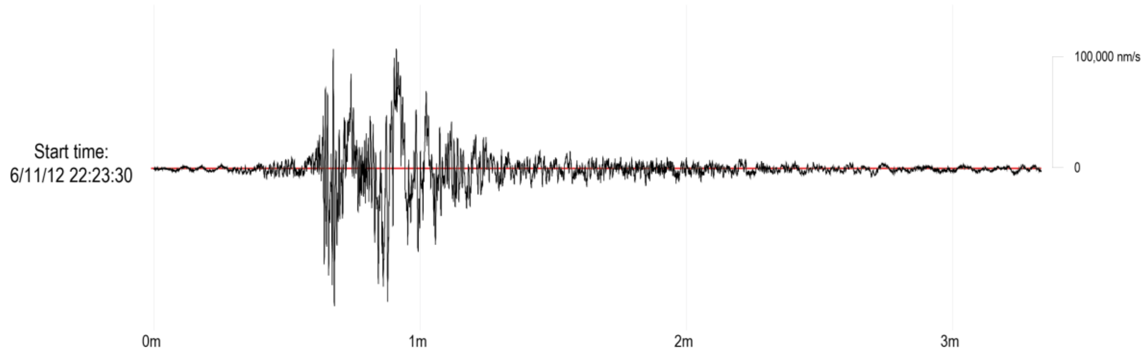
Examples of systems

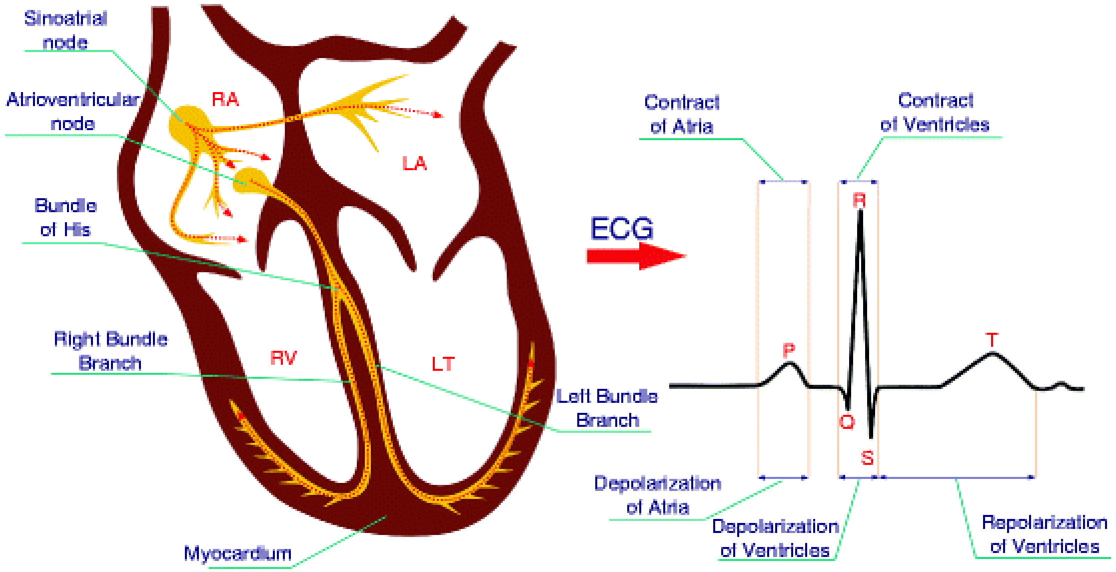
An example of a complex system

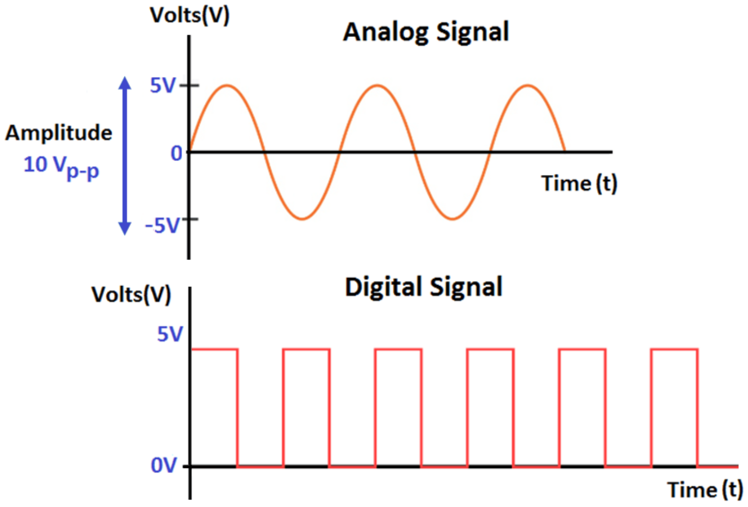
- https://predictabledesigns.com/whats-inside-a-smartphone/
- https://www.electronics-notes.com/articles/connectivity/cellular-mobile-phone/how-cellphone-works-inside-components.php

What are signals?

Examples of signals







Network as an example of system

Study of Signals in NeSS

Study of Systems in NeSS

Scope of NeSS

- Focus on linear systems
- Role of mathematics
- Generality of ECE subjects
- Example – CD & CD player

Signals

- Independent vs dependent variable
- Classification of signals
- Complex numbers overview
- Sinusoidal signal
- Fourier series representation

Classification of signals

- Continuous-time vs discrete-time
- Analog vs Digital
- Deterministic vs Random
- Periodic vs A-periodic

Sinusoidal signal

- Parameters – amplitude, frequency, initial phase

Analogy to vectors

- Vector algebra
- Basis vectors
- Dot product or Inner product
- Orthogonality of vectors

- Signals as infinite dimensional vectors

Fourier series (FS) representation

- Sinusoids as basis – sum of sinusoids
- Frequency & harmonics

- Signals as infinite dimensional vectors

- Inner product definition for signals

- Inner product of sinusoids

Fourier series analysis & synthesis

- Analysis equations – find FS coefficients
- Synthesis equation – reconstruction using FS coefficients
- Example:

★ Square wave example :

$$x(t) = \begin{cases} 3, & 0 < t < 5 \\ -3, & -5 < t < 0 \end{cases} \quad \text{with period } T = 10$$

Fourier series analysis

* Let $x(t)$ be periodic with $T = \frac{2\pi}{\omega_0}$, then its FS coefficients are given by

* $$a_k = \frac{2}{T}\int_{\langle T\rangle} x(t)\cos(k\omega_0 t)\,dt \qquad k = 1, 2, \ldots \infty$$

* $$b_k = \frac{2}{T}\int_{\langle T\rangle} x(t)\sin(k\omega_0 t)\,dt$$

and * $$a_0 = \frac{1}{T}\int_{\langle T\rangle} x(t)\,dt$$ ...... (average value of signal in one period)

we will use ω_0 $(= 2\pi f_0)$ more commonly

Fourier series synthesis

* FS Synthesis : given FS coefficients, synthesize / reconstruct the periodic signal $x(t)$

* partial reconstruction : use $\{a_k, b_k\}$, $k = 0, 1, 2, \ldots R$

$$\hat{x}(t) = a_0 + \sum_{k=1}^{R} a_k \cos(2\pi k f_0 t) + \sum_{k=1}^{R} b_k \sin(2\pi k f_0 t)$$

reconstruction error : $e(t) = x(t) - \hat{x}(t)$

Square wave FS

* Square wave example :

$$x(t) = \begin{cases} 3, & 0 < t < 5 \\ -3, & -5 < t < 0 \end{cases} \quad \text{with period } T = 10$$

$$x(t) = \sum_{k=2r-1}^{r=1\ldots\infty} \frac{12}{k\pi} \sin(k\omega_0 t) = \sum_{k\text{-odd}} \frac{12}{k\pi} \sin\left(\frac{2\pi k t}{10}\right)$$

Does this work ?

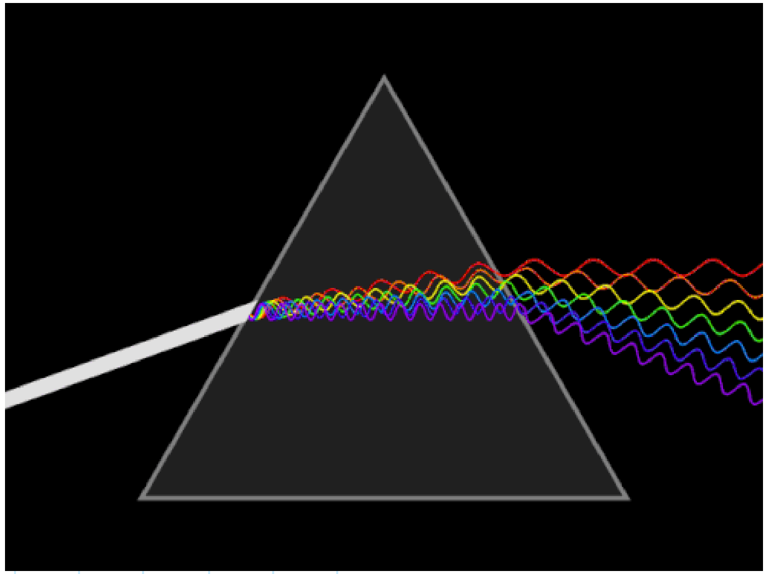
* visualization of partial FS synthesis / reconstruction

https://www.jezzamon.com/fourier/

Fourier series

- Odd & Even signals
- Half-wave symmetry
  - https://www.allaboutcircuits.com/technical-articles/the-effect-of-symmetry-on-the-fourier-coefficients/
- Various forms of FS representation
  - Trigonometric
  - Compact trigonometric
  - Complex exponential

Fourier series



- Complex FS & Trigonometric FS
  - Euler's formula
- Spectrum

* Fourier Spectrum

$$x(t) \xleftrightarrow{FS} \begin{cases} \{a_k, b_k\} \rightarrow \text{real-valued} \\ \{C_k, \theta_k\} \rightarrow \text{real-valued} \\ \{d_k\} \rightarrow \text{complex-valued} \end{cases}$$

$$\left.\begin{array}{l} |d_k| - \text{magnitude spectrum} \\ \measuredangle\, d_k - \text{phase spectrum} \end{array}\right\} \text{Fourier spectrum}$$

Complex Fourier series

- Orthogonality of complex sinusoids
- Complex FS symmetry for real signals
- Dual representation – Time domain vs Frequency domain
- Examples – square wave, sin(t), cos(t), etc.

Fourier series convergence

- When does the summation converge?
- Valid for ALL* periodic signals?

- Dirichlet conditions
  - Absolutely integrable
  - Finite maxima & minima
  - Finite discontinuities

- Points of discontinuity – Gibbs phenomenon

Fourier series properties

- Linearity
- Time shift
- Time reversal
- Time scaling
- Parseval’s relation
- Derivative

- Examples

Systems

- How to describe a system? Representation
- Given input to the system, how to find the output? Analysis
- Given input-output description, how to design a system? Design

- System building using basic blocks – example systems
  - Scalar
  - Delay
  - Integrator/Accumulator
  - Adder

Elementary signals for system analysis

- Unit impulse signal
  - Dirac delta function
  - Intuition using a limiting pulse
- Impulse input to systems
  - Integrator, scalar, delay
- Unit step signal
  - Relation with unit impulse

Elementary signals for system analysis

- Unit impulse & unit step signals
- Shifted and scaled impulses
- Properties of unit impulse
  - Sifting property
- System response to unit impulse

Properties of systems

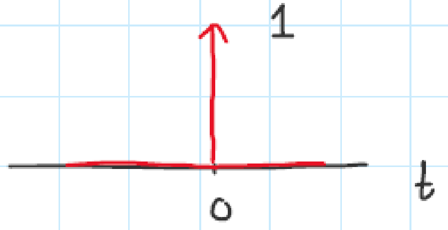
- Memory
- Causality
- Stability
- Linearity (L)
- Time-invariance (TI)

Unit impulse signal

* Unit impulse signal $\delta(t)$: defined as follows

① $\delta(t) = 0 \quad \forall\, t \neq 0$

② $\int_{-\infty}^{\infty} \delta(t) = 1$



* $\delta(t)$ sifting property

① $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

② $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$ .... "sifts" out $x(t_0)$

Linear & Time-invariant (LTI) systems

* Linear systems : H is a linear system

If $x_1(t) \xrightarrow{H} y_1(t)$ & $x_2(t) \xrightarrow{H} y_2(t)$

Then, $\alpha x_1(t) + \beta x_2(t) \xrightarrow{H} \alpha y_1(t) + \beta y_2(t)$

* Time invariant systems :

delay operator : $x(t) \to \boxed{D_\Delta} \to x(t-\Delta)$

H is a time invariant system if :

a $x(t) \to \boxed{D_\Delta} \to \boxed{H} \to y_1(t)$

&

b $x(t) \to \boxed{H} \to \boxed{D_\Delta} \to y_2(t)$

a & b are the same systems (i.e. $y_1(t) = y_2(t)$)

Impulse input to an LTI system & convolution

* Linear & Time - invariant (LTI) systems

here H is an LTI system

① $\delta(t) \longrightarrow \boxed{H} \longrightarrow h(t)$

② $\delta(t-\tau) \longrightarrow \boxed{H} \longrightarrow h(t-\tau)$

③ $x(\tau)\,\delta(t-\tau) \longrightarrow \boxed{H} \longrightarrow x(\tau)\,h(t-\tau)$

④ $x(t) \longrightarrow \boxed{H} \longrightarrow x(t) * h(t) = y(t)$

$$\int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$$

* LTI systems are fully desbribed by their impulse response

H

$$\delta(t) \xrightarrow{H} h(t)$$

$$\Rightarrow \quad x(t) \xrightarrow{H} x(t) * h(t)$$

Convolution operator

For LTI : $\delta(t) \xrightarrow{H} h(t)$ impulse response

for any arbitrary input $\longrightarrow x(t) \xrightarrow{H} \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = y(t)$

τ is dummy variable, can use any other notation *

NOTE

$\delta(t-r)$ & $\delta(r-t)$ are same signals.*

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(r)\, h(t-r)\, dr$$

* Convolution Integral :

$$y(t) = x(t) * h(t)$$

Convolution operator

Convolution properties

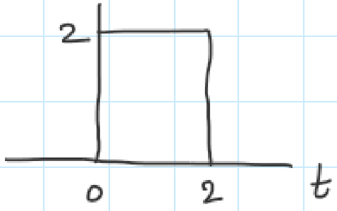
* Properties of convolution operator

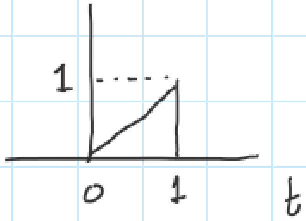
ⓐ Commutative : $x(t) * h(t) = h(t) * x(t)$

ⓑ Associative : $[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$

ⓒ Distributive : $x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$

* Convolution integral example

① consider $x(t) =$ 

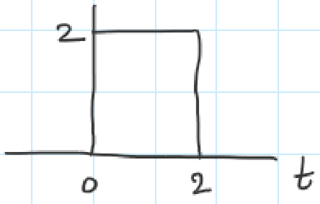
& $h(t) =$ 

Find $y(t) = x(t) * h(t)$

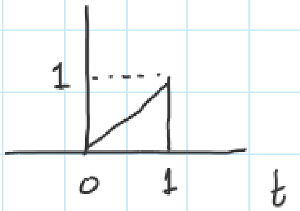
Convolution example

* Convolution integral example

consider $x(t) =$



& $h(t) =$



Find $y(t) = x(t) * h(t)$

→ $$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Steps of convolution :

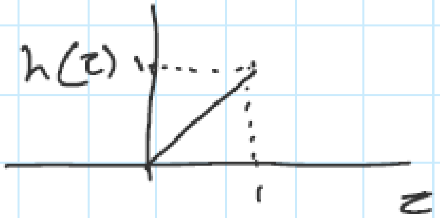
a) flip i.e. $h(-\tau)$

b) shift i.e. $h(t-\tau)$

c) product i.e. $x(\tau) h(t-\tau)$

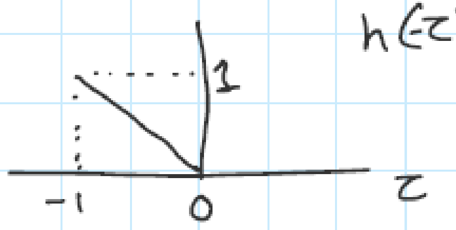
d) integrate i.e. $\int_{-\infty}^{\infty}$

Convolution example continued...



$$h(\tau) = \begin{cases} \tau & 0 \le \tau \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$h(-\tau)$

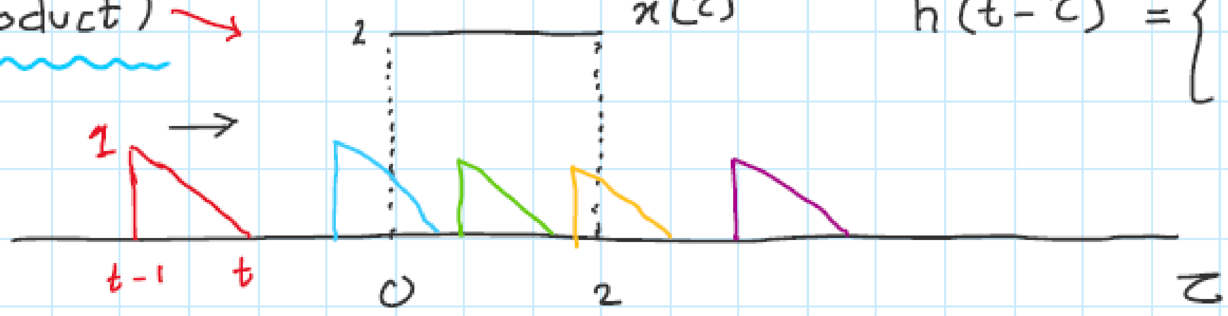


$h(-\tau)$ ... (flip) ⓐ

... (shift) ⓑ

$$h(t-\tau) = \begin{cases} t-\tau, & t-1 \le \tau \le t \\ 0, & \text{otherwise} \end{cases}$$

ⓒ (product)

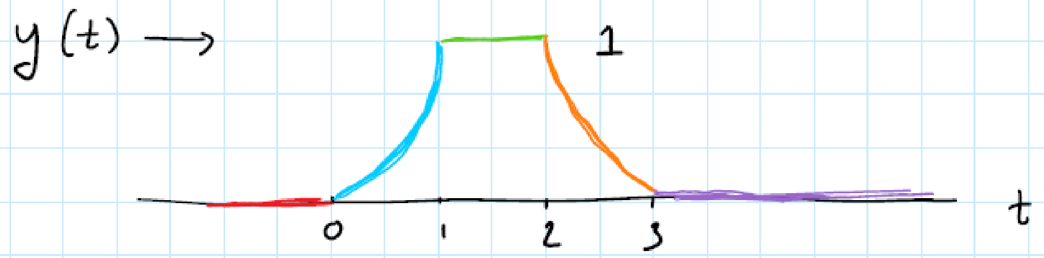


Convolution example continued...

d (integrate) →

$$x(t) * h(t) = \begin{cases} ① \quad = 0 & t < 0 \\ ② \quad = \int_0^t 2(t-\tau)\,d\tau = t^2 & 0 \le t \le 1 \\ ③ \quad = \int_{t-1}^t 2(t-\tau)\,d\tau = 1 & 1 \le t \le 2 \\ ④ \quad = \int_{t-1}^2 2(t-\tau)\,d\tau = -t^2 + 4t - 3 & 2 \le t \le 3 \\ ⑤ \quad = 0 & t > 3 \end{cases}$$

Sketch :



Impulse response & convolution representation

- Impulse response can be found using the given system description
- LTI systems fully described by their impulse response
- LTI systems have convolution representation
- Non LTI systems will not have this representation

System properties – BIBO stability

④ Stable systems

* A system is said to be stable if for bounded inputs the output is also bounded.

i.e. Bounded input bounded output (BIBO) stability

$$x(t) \rightarrow \boxed{H} \rightarrow y(t)$$

i.e. if $|x(t)| \leq B < \infty \quad \forall\, t$, then for BIBO

Stability $\Rightarrow$ $|y(t)| \leq M < \infty \quad \forall\, t$

BIBO stability of LTI systems

* LTI systems with BIBO stability

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

Claim: A system is BIBO stable if & only if $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

i.e. $h(t)$ is absolutely integrable.

Causality of LTI systems

LTI system
eigenfunctions

$$x(t) \longrightarrow \underset{h(t)}{\boxed{LTI}} \longrightarrow y(t)$$

any LTI system

Ex. $e^{st} \longrightarrow$

$$y(t) = e^{st} * h(t) \quad \cdots \text{[convolution]}$$

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{st}\, e^{-s\tau}\, d\tau$$

$$= e^{st} \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

$$= H(s)\, e^{st}$$

$$e^{st} \longrightarrow H(s)\, e^{st} \quad \cdots \text{[shape is preserved]}$$

eigenfunctions for LTI systems

Laplace transform & LTI systems

* Laplace transform (LT)

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$ ···· Analysis equation

$\{e^{st}\}$ – basis signals

s – complex frequency variable $(s = \sigma + j\omega)$

$X(s)$ – complex valued function of s

* LTI systems & LT

eigenfunction

$$e^{st} \longrightarrow \underset{\text{LTI}}{\boxed{h(t)}} \longrightarrow H(s)\, e^{st}$$

$$h(t) \overset{\text{LT}}{\longleftrightarrow} H(s)$$

$h(t)$ – impulse response of LTI system

$H(s)$ – system transfer function

system descriptors *

Laplace transform basis signals

Fourier series for periodic signals - representation in terms of sin() & cos() or $e^{j\omega t}$ harmonics

FS coefficients : $a_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)\,dt$

measure of similarity between the given signal $x(t)$ & basis signal $\cos(k\omega_0 t)$

Laplace transform :

Basis $\rightarrow \{e^{st}\}$ s – complex number

* $s = \sigma + j\omega$ complex number plane

$x(t) \xleftrightarrow{LT} X(s)$

s – complex frequency

Analysis * $X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$ .... definition

LT examples

① $x(t) = e^{-t} u(t)$ right-sided signal

$X(s) = \dfrac{1}{s+1}$ & $Re(s) > -1$

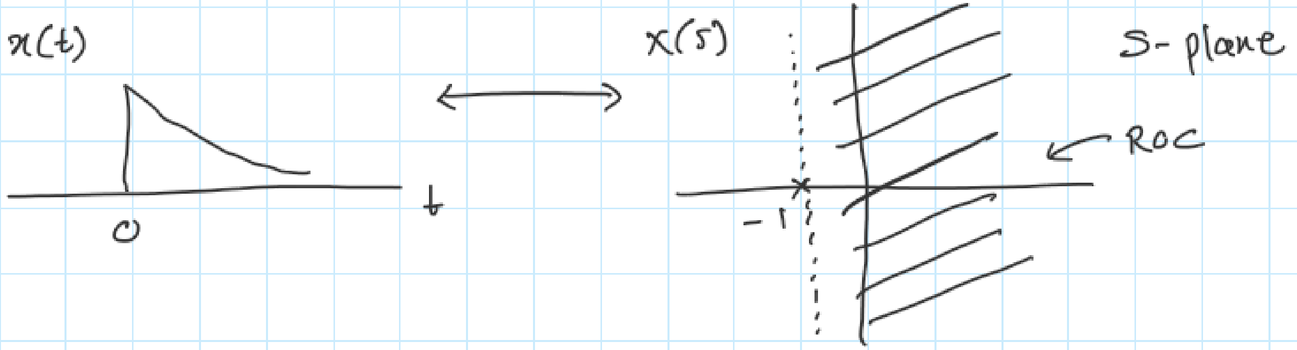
ROC

* Laplace transform always has two parts:

ⓐ expression ⓑ ROC } both must be always specified.

* note: $x(t)$ is right-sided signal &

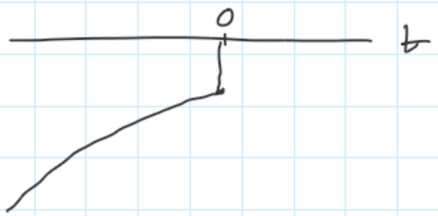
ROC is also right-sided in the s-plane.

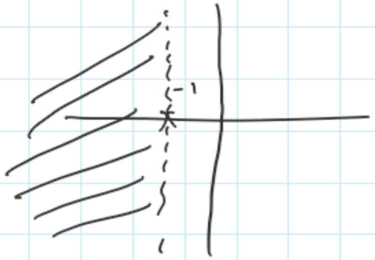


LT examples

(2) $x(t) = -e^{-t}\,u(-t)$ left-sided signal

$$X(s) = \frac{1}{s+1} \quad \& \quad \mathrm{Re}(s) < -1$$





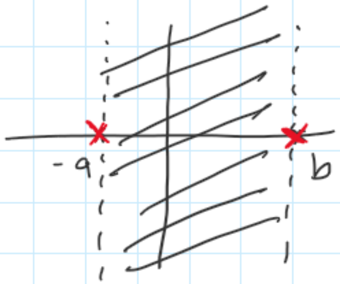
(3) two-sided signal

$a, b \in \mathbb{R}^+$

$$X(s) = \frac{1}{b-s} + \frac{1}{s+a} \qquad -a < \mathrm{Re}(s) < b$$

(a) expression

$$\frac{(a+b)}{(s+a)(b-s)}$$



(b) ROC

LT – poles and zeros

* Rational form of Laplace transform

$$x(s) = \frac{A(s)}{B(s)}$$ polynomials in s variable

roots of the polynomials $A(s)$ & $B(s)$ are important

roots of $A(s)$ i.e. $A(s) = 0$ → zeros

roots of $B(s)$ i.e. $B(s) = 0$ → poles

poles are important to decide ROC

* pole-zero plot

Laplace transform examples

* some standard example signals and their LT

* $e^{-at}u(t) \xleftrightarrow{LT} \frac{1}{s+a}$ & $Re(s) > -a$ $a \in \mathbb{R}$

* $-e^{-at}u(-t) \xleftrightarrow{LT} \frac{1}{s+a}$ & $Re(s) < -a$ $a \in \mathbb{R}$

* $\delta(t) \xleftrightarrow{LT} 1$ & complete s-plane

* $\delta(t-t_0) \xleftrightarrow{LT} e^{-st_0}$ & —— " —— " ——

* $e^{j\omega t}u(t) \xleftrightarrow{LT} \frac{1}{s-j\omega}$ & $Re(s) > 0$ $\omega \in \mathbb{R}$

* $e^{-at}u(t) \xleftrightarrow{LT} \frac{1}{s+a}$ & $Re(s) > -Re(a)$, a complex

Properties of Laplace transform

- Linearity
- Time shift
- Frequency shift
- Time reversal
- Time scaling
- Derivative in time, derivative in frequency
- Integration
- Convolution

Laplace transform – causality & stability

- LTI systems – transfer function H(s)

LTI system analysis using Laplace Transform

we know

$$X(s) \longrightarrow \overset{h(t)}{\boxed{H(s)}} \longrightarrow Y(s) = X(s)H(s)$$

convolution property

we are interested in additional system properties and their effect on impulse response & system function

- Causal system
- Stable system
- Causal & Stable systems

Solve the RC circuit using Laplace transform