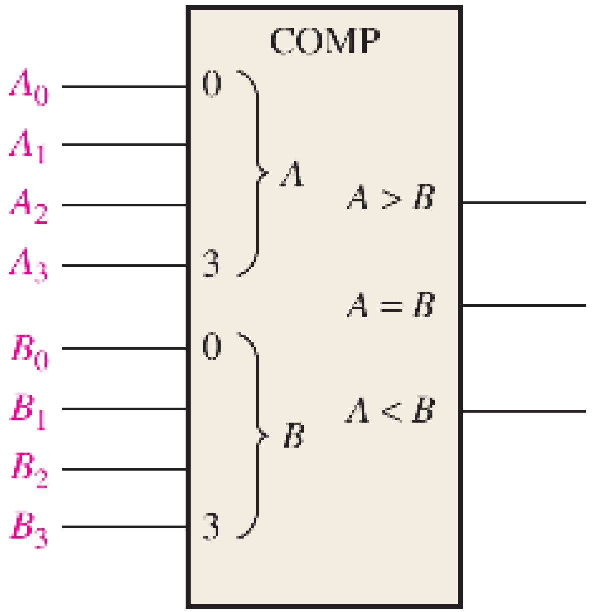
Lecture 15 – Combinational circuits 4

Chapter 5

The Binary comparator

Binary comparator

- The comparison of two numbers is an operation that determines whether one number is greater than, less than, or equal to the other number
- A *magnitude comparator* is a combinational circuit that compares two numbers A and B and determines their relative magnitudes
- The outcome of the comparison is specified by three binary variables that indicate whether $A > B$, $A = B$, or $A < B$
- On the one hand, the circuit for comparing two n-bit numbers has 2^{2n} entries in the truth table and becomes too cumbersome, even with $n = 3$
- On the other hand, as one may suspect, a comparator circuit possesses a certain amount of regularity



Binary comparator

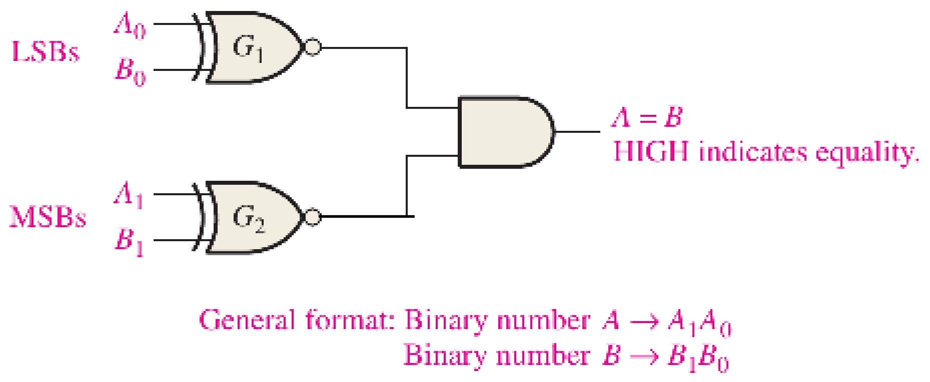
- Consider two numbers, *A* and *B* , with four digits each $A_3A_2A_1A_0$ and $B_3B_2B_1B_0$
- The two numbers are equal if all pairs of significant digits are equal: $A_3 = B_3$, $A_2 = B_2$, $A_1 = B_1$, **and** $A_0 = B_0$
- To check bit-wise equality, we can use the XNOR gate

$$x_i = A_iB_i + A_i'B_i' \text{ for } i = 0, 1, 2, 3$$

- For equality to exist, all x_i variables must be equal to 1, a condition that dictates an AND operation of all variables:

$$(A = B) = x_3x_2x_1x_0$$

Eg: Comparison of 2-bit numbers



Binary comparator

- To determine whether *A* is greater or less than *B*, we inspect the relative magnitudes of pairs of significant digits, starting from the most significant position
- If the two digits of a pair are equal, we compare the next lower significant pair of digits
- The comparison continues until a pair of unequal digits is reached
- If the corresponding digit of *A* is 1 and that of *B* is 0, we conclude that $A > B$. Else, we have $A < B$

Eg: $A_3 A_2 A_1 A_0 = 1001$ *and* $B_3 B_2 B_1 B_0 = 1010$

Binary comparator

- The comparison can be expressed logically by the two Boolean functions:

$$(A > B) = A_3B_3' + x_3A_2B_2' + x_3x_2A_1B_1' + x_3x_2x_1A_0B_0'$$

$$(A < B) = A_3'B_3 + x_3A_2'B_2 + x_3x_2A_1'B_1 + x_3x_2x_1A_0'B_0$$

Binary comparator

$x_i = A_iB_i + A_i'B_i'$ for i = 0, 1, 2, 3

$(A = B) = x_3x_2x_1x_0$

$(A > B)$
$= A_3B_3' + x_3A_2B_2' + x_3x_2A_1B_1'$
$+ x_3x_2x_1A_0B_0'$

$(A < B)$
$= A_3'B_3 + x_3A_2'B_2 + x_3x_2A_1'B_1$
$+ x_3x_2x_1A_0'B_0$

Interesting: Can we prove that only one of (A=B), (A>B) and (A<B) will be “1” at any given time?