Lecture 3 – Binary Representation

Chapter 1

Diminished radix complement

- Given an n-digit number *N* in base *r*, the (*r* - 1)'s complement of *N* , i.e., its diminished radix complement, is defined as $(r^n - 1) - N$

- For decimal numbers, the 9's complement of *N* is $(10^n - 1) - N$
- In this case, $10^n\ -\ 1$ is a number represented by *n* 9s
  - Eg: if *n* = 4, we have 10^4 = 10,000 and r^n-1 = 10^4 - 1 = 9999
  - If n=2, we have 10^2 = 100 and r^n – 1 = 10^2 - 1 = 99

- It follows that the 9's complement of a decimal number is obtained by subtracting each digit from 9
  - Eg: 9's complement of 76 = 99 – 76 = 23
  - 9's complement of 1242 = 9999 – 1242 = 8757
  - 9's complement of 99981 is 99999 – 99981 = 18

Diminished radix complement

- For n-bit binary numbers, the 1's complement of N is $(2^n - 1) - N$.

- Again, $(2^n - 1)$ is a binary number represented by n 1s
  - For example, if $n = 4$, we have $2^4 = (10000)_2$ and $2^4 - 1 = (1111)_2$.
  - If n=2, we have $2^2 = (100)_2$, and $2^2 - 1 = 11$,
- 1's complement of a binary number can be obtained by subtracting each bit from 1
- However, when subtracting binary digits from 1, we can have either 1 - 0 = 1 or 1 - 1 = 0, which causes the bit to change from 0 to 1 or from 1 to 0, respectively
- Therefore, **the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.**
- Examples of 1's complement:

  1's complement of 1011000 = 1111111 – 1011000 = 0100111

  1's complement of 100 = 111 – 100 = 011

Radix complement

- The *r*'s complement of an *n*-digit number *N* in base *r* is defined as $r^n - N$ for $N \neq 0$ and as 0 for $N = 0$
- r's complement can be obtained by adding 1 to the (*r* - 1)'s complement, since $r^n - N = [(r^n - 1) - N] + 1$
- Thus, the 10's complement of decimal 2389 is 7610 + 1 = 7611 and is obtained by adding 1 to the 9's complement value
- The 2's complement of binary 101100 is 010011 + 1 = 010100 and is obtained by adding 1 to the 1's-complement value
- Examples:
  - $(66772)_{10}$
    - 33227+1 = 33228
  - $(10011)_2$
    - 01100 + 1 = 01101

Some notes on complements

- If the original number *N* contains a radix point, the point should be removed temporarily in order to form the *r*'s or (*r* - 1)'s complement

- The radix point is then restored to the complemented number in the same relative position

- Example: 9's complement and 10's complement of $(82.314)_{10}$
  - 9's complement : 17.685
  - 10's complement: 17.686

Some notes on Complements

- **The complement of the complement restores the number to its original value**

- *r*'s complement of *N* is $r^n - N$, so that the complement of the complement is $r^n - (r^n - N) = N$ and is equal to the original number

- *(r-1)*'s complement of *N* is $r^n - 1 - N$, so that the complement of the complement is $(r^n - 1) - (r^n - 1 - N) = N$ and is equal to the original number

Subtraction with Radix complements

- Subtraction using method of borrowing is less efficient when implemented with digital hardware
- Consider subtraction $M - N$ in base *r*
- Here is the algorithm using Radix complement:
  1. Take radix complement of subtrahend *N*: $r^n - N$
  2. **Add** this to *M*: $(r^n - N) + M = r^n + (M - N) = r^n - (N - M)$
  3. If you get a carry in the $(n+1)^{th}$ digit,
     - then the result is positive, discard the carry and you are done
  4. If you ***do not*** get a carry in the $(n+1)^{th}$ digit,
     - then the result is **negative**. Take the radix complement of the number to get the answer, then put a negative sign

Subtraction with Radix complements

Subtraction using 10's complement

- $(4637)_{10} - (2579)_{10}$

  1. 10's complement of 2579 = 7421
  2. 4637 + 7421 = 12058 ($r^n + (M - N)$)
  3. Result after removing the end carry : 2058

- $(2579)_{10} - (4637)_{10}$
  - 10's complement of 4637 = 5363
  - 2579 + 5363 = 7942 ($r^n - (N - M)$)
  - No end carry. Hence, answer is – (10's complement of 7942) = – 2058

Binary subtraction with complements

Perform the following subtractions using 2's complement method:

- $(110001)_2 - (010100)_2$ [$(49)_{10} - (20)_{10}$]
  - 2's complement of 010100 = 101100

$$\begin{array}{r} 110001 \\ +\underline{101100} \\ \underline{1}011101 \end{array}$$

  - Result obtained by dropping end carry : 011101 [$(29)_{10}$]
- $(010110)_2 - (100)_2$ [$(22)_{10} - (4)_{10}$]
  - 2's complement of 000100 = 111100

$$\begin{array}{r} 010110 \\ +\underline{111100} \\ \underline{1}010010 \end{array}$$

  - Result obtained by dropping end carry : 010010 [$(18)_{10}$]

Binary subtraction with complements

Perform the following subtractions using 2's complement method:

- $(1000011)_2 - (1010100)_2$ [$(67)_{10} - (84)_{10}$]

  - 2's complement of 1010100 = 0101100

$$\begin{array}{r} 1000011 \\ +\underline{0101100} \\ 1101111 \end{array}$$

  - No end carry ⇨ the answer is – (2's complement of 1101111) = – 0010001 [$-(17)_{10}$]

Subtraction with Diminished radix complements

- Lets assume we have to perform $M - N$ in base r

- Here is the algorithm using Diminished radix complement:
  1. Take diminished radix complement of N: $r^n - 1 - N$
  2. **Add** this to M: $r^n - 1 - N + M = r^n + (M - N - 1) = (r^n - 1) - (N - M)$
  3. If you get a carry in the (n+1)th digit,
     - then the result is positive, ***add the carry to the result*** and you are done
  4. If you ***do not*** get a carry in the (n+1)th digit,
     - then the result is **negative**. Take the diminished radix complement of the number to get the answer, then put a negative sign

Subtraction with Diminished radix complements

9’s complement subtraction:

- $(76425)_{10} - (28321)_{10}$
  - 9’s complement of 28321 is 71678
  - 76425 + 71678 = 148103; $[\, r^n + (M - N - 1) \,]$

    drop the carry and add 1 to 48103 ; sum is 48104
  - End carry => result is positive

- $(2124)_{10} - (9667)_{10}$
  - 9’s complement of 9667 is 0332
  - 2124 + 0332 = 2456; $[\, r^n - 1 - (N - M) \,]$
  - No end carry => answer is – (9’s complement of 2456) = – 7543

Subtraction with Diminished radix complements

Perform the following subtractions using 1’s complement method:

- $(101011)_2 - (111001)_2$ [$(43)_{10} - (57)_{10}$]
  - 1’s complement of 111001 = 000110

$$\begin{array}{r} 101011 \\ +\ \underline{000110} \\ 110001 \end{array}$$

  - No end carry => answer is – (1’s complement of 110001) = – 1110
- $(1)_2 - (10100)_2$

$$\begin{array}{r} 1 \\ +\ \underline{01011} \\ 01100 \end{array}$$

No end carry => answer is – (1’s complement of 01100) = – 10011

Binary subtraction with complements

Perform the following subtractions using 2's complement method:

- $(110001)_2 - (010100)_2$ [$(49)_{10} - (20)_{10}$]
  - 2's complement of 010100 = 101100

$$\begin{array}{r} 110001 \\ +\underline{101100} \\ \underline{1}011101 \end{array}$$

  - Result obtained by dropping end carry : 011101 [$(29)_{10}$]
- $(010110)_2 - (100)_2$ [$(22)_{10} - (4)_{10}$]
  - 2's complement of 000100 = 111100

$$\begin{array}{r} 010110 \\ +\underline{111100} \\ \underline{1}010010 \end{array}$$

  - Result obtained by dropping end carry : 010010 [$(18)_{10}$]

Binary subtraction with complements

Perform the following subtractions using 2's complement method:

- $(1000011)_2 - (1010100)_2$
  - 2's complement of 1010100 = 0101100

$$\begin{array}{r} 1000011 \\ +\underline{0101100} \\ 1101111 \end{array}$$

  - No end carry $\Rightarrow$ the answer is – (2's complement of 1101111) = – 0010001

Representation of negative numbers

- In ordinary arithmetic, a negative number is indicated by a minus sign and a positive number by a plus sign
- Computers must represent everything with binary digits
- It is customary to represent the sign with a bit placed in the leftmost position of the number
- The convention is to make the sign bit 0 for positive and 1 for negative
- This can be done using:
    1. Signed magnitude representation
    2. Signed complement representation
        1. Signed 1’s complement representation
        2. Signed 2’s complement representation