$$i = \frac{V}{R}\left(1 - e^{-t/(L/R)}\right)$$

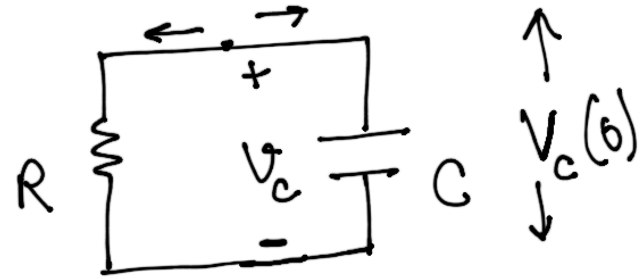
L DC current (constant) = short circuit $L \to 0$

$t \to \infty$

C DC voltage (constant) = no current = open circuit

$t \to \infty$

Source Free R-C circuit



$v_R = v_c$

$[\, i_R = i_c \,]$

$i_c = C\frac{dv}{dt}$

$=$

Any instant $i(t)$ is flowing and $v(t)$ is the voltage across the cap.

KCL $\quad \frac{v}{R} + C\frac{dv}{dt} = 0$ —— (1)

General sol $\quad v(t) = Ae^{+st}$

$$\frac{dv}{dt} = +Ase^{+st}$$

Subs. in (1) $\quad \frac{Ae^{st}}{R} + CAse^{st} = 0$

$$Ae^{st}\left(\frac{1}{R} + Cs\right) = 0$$

$$\frac{1}{R} + Cs = 0$$

$$s = -\frac{1}{CR}$$

$$v(t) = \underset{=}{A}\, e^{-\frac{1}{RC}t}$$

$t = 0$

$$V_c = A e^{0} \Rightarrow A = V_c$$

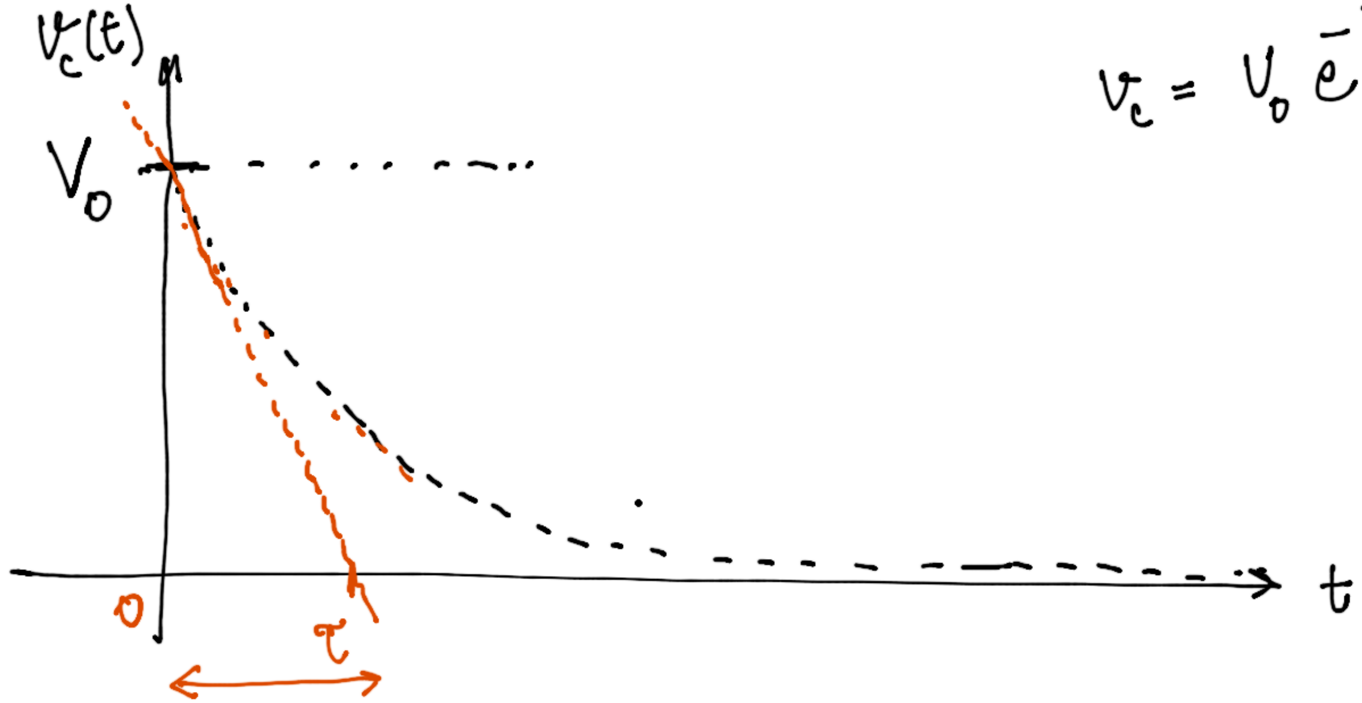
$\downarrow$

Initial voltage on cap.

$$\boxed{v(t) = V_c\, e^{-\frac{t}{RC}}}$$

$$i_c(t) = -\frac{V_c . C}{RC} e^{-t/RC} = -\frac{V_c}{R} e^{-t/RC}$$

$$v_c = V_0 e^{-t/RC}$$



Initial slope

Slope $\quad \dfrac{dv_c}{dt} = -\dfrac{V_0}{RC} e^{-t/RC}$

Slope $(t=0)$ $\quad \dfrac{dv_c}{dt} = -\dfrac{V_0}{RC} = -\dfrac{V_0}{\tau}$ ← Time constant

$$\boxed{\tau = RC}$$

τ_1 — R_1 small — Small C_1 — faster responding

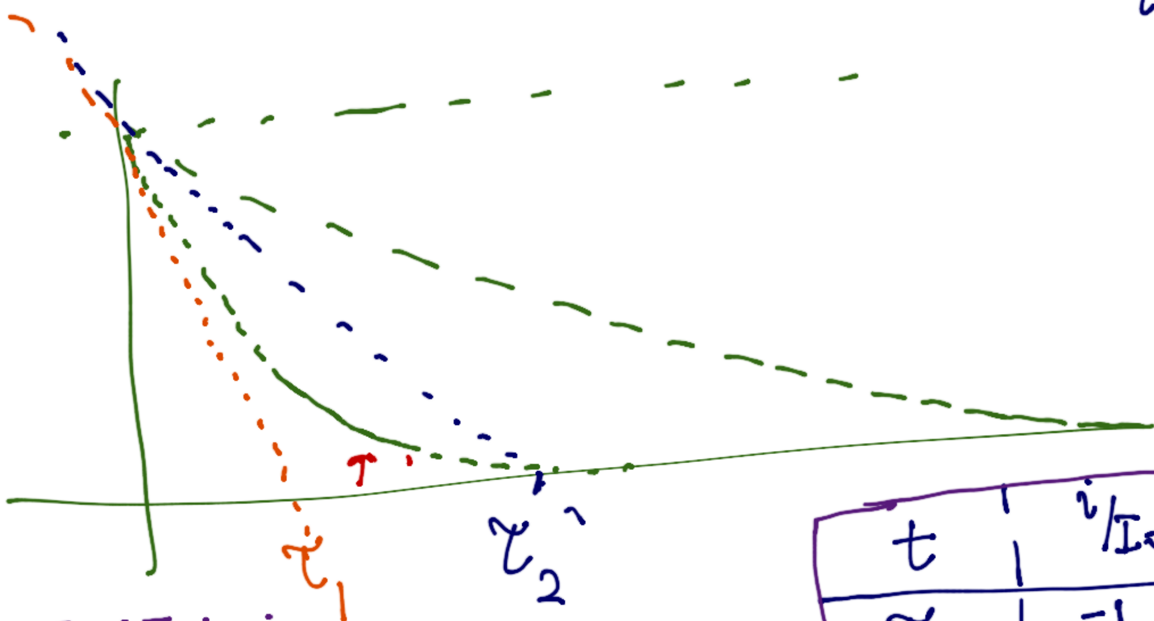
τ_2 — R_2 (large) — large. C_2 — Slower response.

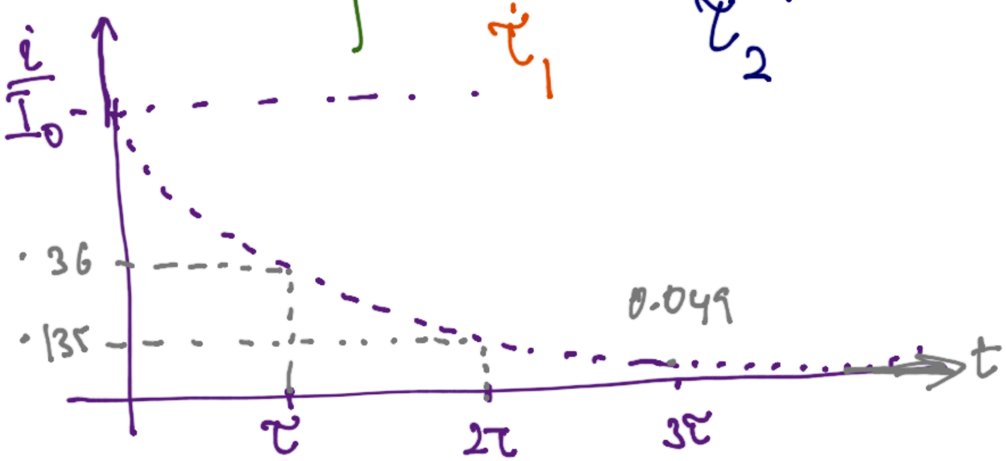
$$i = -\frac{V}{R} e^{-\frac{t}{RC}}$$

$t = 0$ $\quad i(0) = \frac{V}{R} = I_0$

$$i = -I_0 e^{-t/RC}$$

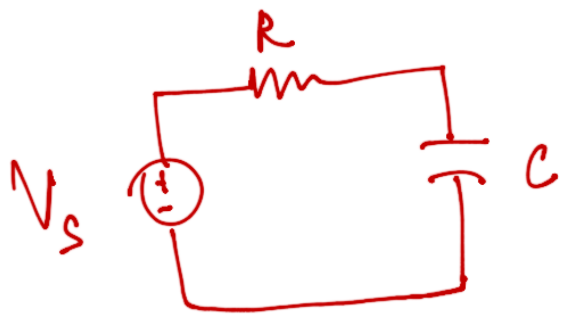
$$\frac{i}{I_0} = -e^{-t/RC}$$





t	i/I_0	%
τ	$e^{-1} = 0.36$	36%
2τ	$e^{-2} = 0.1353$	13%
3τ	$e^{-3} = 0.049$	4.9%
5τ	$e^{-5} = 0.006739$	.6%

Voltage Driven R-C circuit



$v_c(0) = V_{c0}$

(KVL)

$$V_s = iR + v_c$$

$$V_s = iR + \frac{1}{C}\int i\,dt$$

$$C(V_s - iR) = \int_0^t i\,dt$$

$$i = C\frac{dv}{dt}$$

$$V_s = RC\frac{dv_c(t)}{dt} + v_c(t)$$

Assuming $V_c(t) = Ae^{st}$: $V_s = RCAse^{st} + Ae^{st}$

$$V_s = \underbrace{Ae^{st}}(RCs + 1)$$

$$V_s = A e^{st} (RCs+1)$$

$t=0$ $\quad V_s = A (RCs+1)$ $\Big\}$ solved.

Solving by integrating the DE after variable separation

$$V_s = RC \frac{dv_c}{dt} + v_c$$

$$(V_s - v_c) = RC \frac{dv_c}{dt}$$

$$\int_0^{v_c} \frac{dv_c}{(V_s - v_c)} = \int_0^t \frac{dt}{RC} \Rightarrow \left[\ln (V_s - v) \right]_0^{v_c} = \frac{t}{RC}$$

$$+ \ln \frac{V_s - V_c}{V_s - V_{c0}} = -\frac{t}{RC}$$

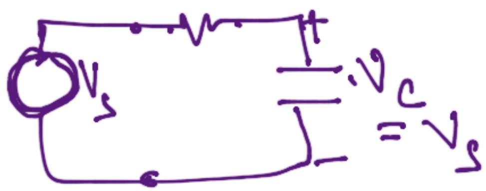
$$V_s - V_c = (V_s - V_{co})\, e^{-t/RC}$$

$$V_c = V_s - (V_s - V_{co})\, e^{-t/RC}$$

If $V_{co} = 0$

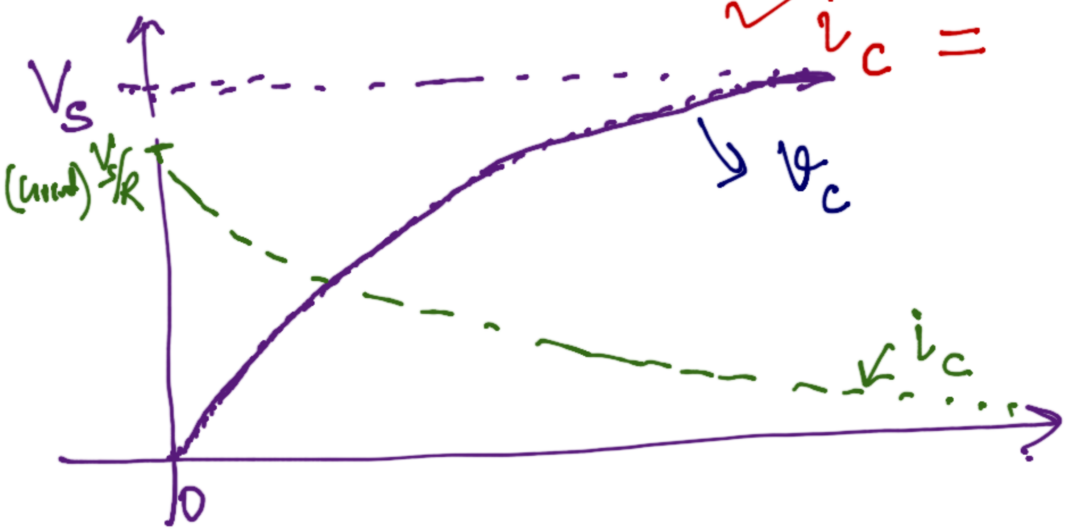
$$\checkmark\ V_c = V_s\left(1 - e^{-t/RC}\right)$$

$$V_c(t=\infty) = V_s$$



$$i_c = C\, V_s \left(\frac{t}{RC}\right) e^{-t/RC}$$

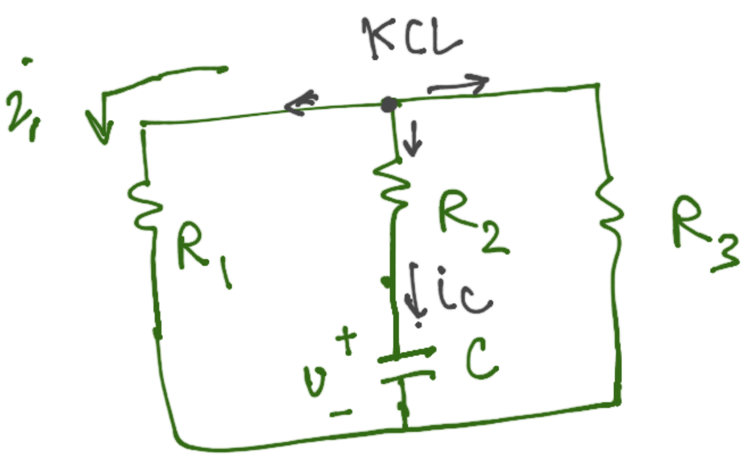
$$\checkmark\ i_c = \frac{V_s}{R}\, e^{-t/RC}$$



Initial condition

v_c at $t=0$

$\downarrow$



$t = 0^-$: just before $t=0$

$t = 0^+$: " after $t=0$

For cap $v_c(0^-) = v_c(0^+)$

ind. $i_L(0^-) = i_L(0^+)$.

i_c

$v(0^+)$

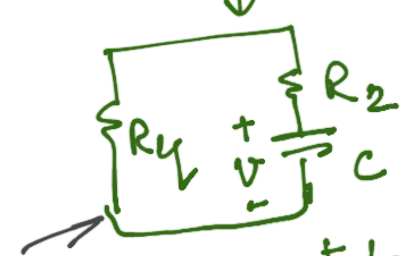
$i_1(0)$

Example 2.5

Given : $v(0^-) = V_0 = v_c(0^+)$

(initial) $v_c(t=0)$

$\downarrow$



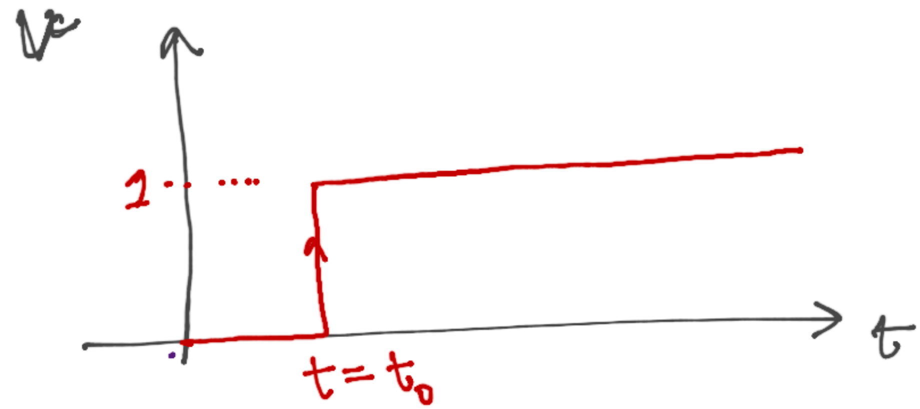
$v_c = A e^{-t/RC}$

$\tau = RC \begin{cases} R = R_{eq} + R_2 \\ C \end{cases}$

$t=0 \quad v = V_0$

Voltage Function

Unit Step



$t < t_0 \qquad v = 0$

$t > t_0 \qquad v = 1$

$$u(t - t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$

$u(t_0^-) = 0$ & $u(t_0^+) = 1 \Rightarrow$ Instant change

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

unit step at zero

Practice 8.7

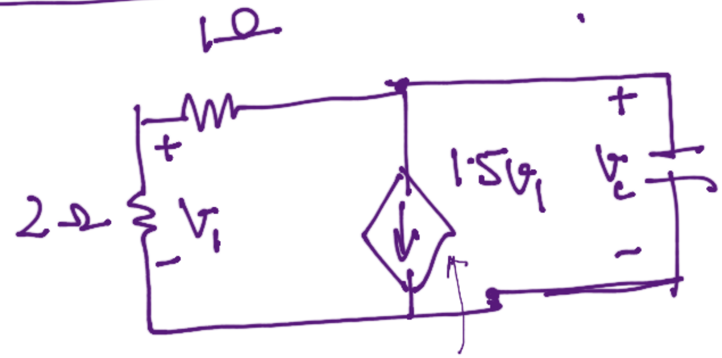


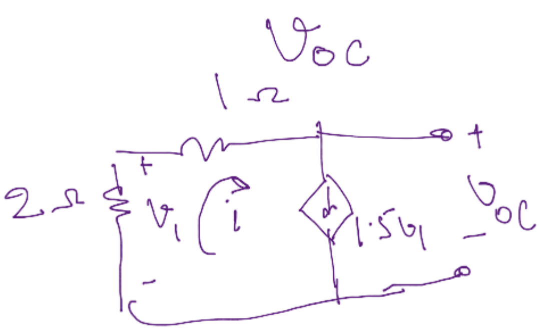
Fig A

Dependent source

If $v_c(0^-) = 11\,V$,

$v_c(t) = ?$

Step 1: To find τ, we need equivalent Resistance connected to capacitor. (Thevenin equivalent)

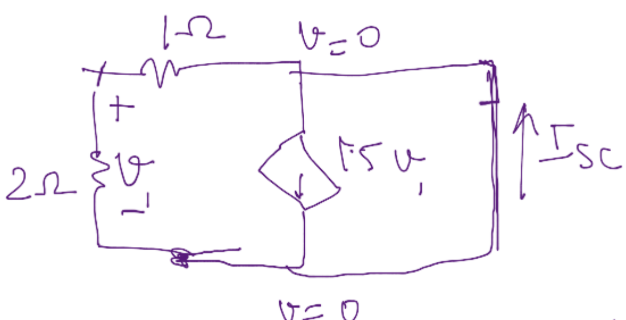


KVL (open ckt V_{oc})

$$-2i - 1i - v_{oc} = 0 \Rightarrow v_{oc} = -3i$$

$$i = 1.5v_1 = -2i$$

$$\Rightarrow v_{oc} = -3\left(\frac{1.5v}{-2}\right) = 2.25v$$

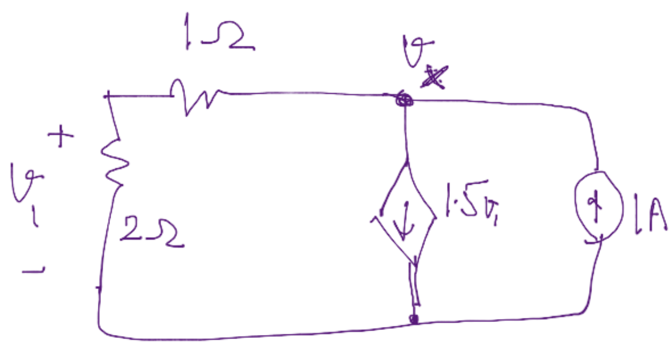


KCL (Short circuit I_{sc})

$$I_{sc} = 1.5v_1 + \frac{0-v_1}{1} = 0.5v_1$$

$$\left.\frac{0-v}{1} + \frac{0-v}{2} = 0\right\} \times \rightarrow \text{Does not satisfy.}$$

Apply 1 Amp current source



KCL

$$1 - 1.5 v_1 - \frac{v_x - v_1}{1} = 0$$

$$1 - 0.5 v_1 - v_x = 0 \quad \text{— (1)}$$

$$\frac{v_x - v_1}{1} = \frac{v_1}{2} \Rightarrow 2v_x = 3v_1 \quad \text{— (2)}$$

(1) $\Rightarrow$

$$1 - 0.5\left(\frac{2v_x}{3}\right) - v_x = 0$$

$$1 - \frac{1}{3}v_x - v_x = 0 = 1 - \frac{4}{3}v_x = 0$$

$$v_x = 3/4 \text{ V.}$$

$$R_{th} = \frac{3/4\text{ V}}{1\text{A}} = 3/4\,\Omega$$

Taking General solution for ckt in figure A : $v_c = A e^{-t/\tau}$

$$\tau = \frac{3}{4}\,\Omega * 2\text{m} = \frac{3}{2} \times 10^{-3} \text{ sec.}$$

$t=0$ $\quad v_c(0) = A = 11\text{V}$ $\quad \therefore$ $\boxed{V_c = 11 e^{-\frac{2t}{0.003}}}$ Decaying fnct v_1