Analog Electronic Circuits (EC2.103) : Assignment-2

Spring 2024, IIIT Hyderabad, Due date : Wed 17^{th} Jan, 2024 (18:00 hrs)

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Instructions:

1. Submit your assignment as a single pdf (Name_RollNo.pdf) at moodle on or before the due date
2. Hand-written/typed (latex/word) submissions are allowed
3. Report should be self explanatory and must carry complete solution - Answers with schematics, SPICE directives, annotated waveforms, inference/discussion on results
4. Post your queries on moodle, discussions are highly encouraged on moodle

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1. **Two capacitor problem**

   For the circuit shown in Fig. 1, switch Sw1 is closed at t=0. Initial condition of capacitors are given as $V_{C_1}(0^-) = V_0$ V and $V_{C_2}(0^-) = 0$ V.

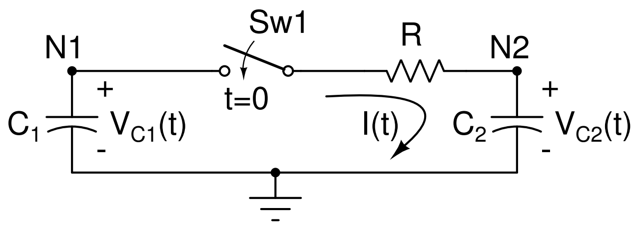

Figure 1

   (a) Derive and plot the expression of $V_{C_1}(t)$, $V_{C_1}(t)$ and current $I(t)$ in the network as a function of time. Give plots and intuitive explanation for the cases- i) $C_1 = C_2$, ii) $C_1 = 10 \times C_2$ and iii) $C_1 = \frac{1}{10}C_2$.

   (b) Consider $C_1 = 10$ nF and $V_{C_1}(0^-) = V_0 = 1$ V and $V_{C_2}(0^-) = 0$ V, $R = 1\ k\Omega$. Using LTSPICE, run transient analysis and plot $V_{C_1}(t)$, $V_{C_2}(t)$ for the cases - 1) $C_1 = C_2$, ii) $C_1 = 10 \times C_2$ and iii) $C_1 = \frac{1}{10}C_2$.

   *(Hint: To give Initial condition to capacitors C_1 & C_2, Go to **Edit** $\rightarrow$ **Spice Directive** and write **.ic V(N1)=1 V(N2)=0** in the command box and click OK. For implementing switch use SW component (voltage controlled switch) and add spice directive **.model SW SW(Ron=1m Roff=1Meg Vt=.5 Vh=0)** (Why?) Give Control voltage to switch as **PULSE(0 1 10u 10n 10n 1000u 2000u)** (why?), run transient for 200 μs.)*

   (c) At time t = 0 , calculate the energy stored (E_0) in the circuit. (hint :- Energy stored in Capacitor $\frac{1}{2} \times CV^2$). From your simulation plots in the previous part (b), find total energy at steady state (t = ∞) and compare with E_0 for all three cases (give a table). Do you have any answer/thoughts for this paradox?

   (d) Consider $C_1 = C_2 = 10$ nF and $V_{C_1}(0^-) = V_0 = 1$ V and $V_{C_2}(0^-) = 0$ V, plot $V_{C_1}(t)$, $V_{C_1}(t)$ for the cases - a) $R = 10\ k\Omega$, b) $R = 10\ \Omega$ and c) $R = 1\ m\Omega$. Qualitatively discuss effect of reducing R value on the settling time of $V_{C_1}(t)$ and $V_{C_2}(t)$

   Suggested reading: Interested students may have a look, no grades for reading

   1) R. C. Levine, "Apparent Nonconservation of Energy in the Discharge of an Ideal Capacitor," in IEEE Transactions on Education, vol. 10, no. 4, pp. 197-202, Dec. 1967, doi: 10.1109/TE.1967.4320288.

2) J. Hoekstra, "A Solution of the Two-Capacitor Problem Through its Similarity to Single-Electron Electronics," in IEEE Open Journal of Circuits and Systems, vol. 1, pp. 13-21, 2020, doi: 10.1109/OJCAS.2020.2977216.

2. **RC circuits as filters**

(a) For the circuit shown in Fig. 2, it is given that R = 20 MΩ and C = 10 pF and $V_C(0^-) = 0$ V (zero initial voltage across capacitor).

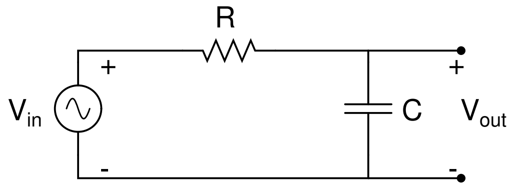

Figure 2

i) As discussed in the lecture, present intuitive explanation for $V_{out}(t)$ as a function of time for a step input $V_{in} = V_0 u(t)$. (V_{out} initial, final type of rise/fall - linear/exponential). Also discuss, how it acts as a **low pass filter**.

ii) Derive the expression for $V_{out}(t)$ and current across the capacitor for the input $V_{in} = V_0 u(t)$. Using LTSPICE simulations, plot the voltage and current with $V_{in} = 5u(t)$ V and verify your theoretical expressions.
(Hint:Run Transient analysis for the Step input signal PWL(0 0 1m 5), Run the transient for 20 ms.)

iii) Write the expression for Transfer Function $\frac{V_{out}(s)}{V_{in}(s)}$. Find the expressions for gain ($|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}|$) and phase ($\phi$) as a function of frequency (ω) and find the $3\,\mathrm{dB}$ cut-off frequency.

iv) Verify your derivation and hand calculations with AC analysis of the circuit by plotting gain and phase w.r.t frequency.
(Hint: Set AC amplitude for Vin voltage source as 1. For AC analysis set Type of Sweep = Decade, Number of points = 1000, Start Frequency = 5 Hz, Stop Frequency = 50 MHz).

(b) For the circuit given in figure 3, repeat all the analysis from part (a) for R = 20 kΩ and C = 10 pF. Please note that this circuit acts as a **high pass filter**. Give intuitive explanation and derive/calculate/simulate accordingly.
(Hint:For AC analysis Set Type of Sweep = Decade, Number of points = 1000, Start Frequency = 5 Hz, Stop Frequency = 50 MHz.

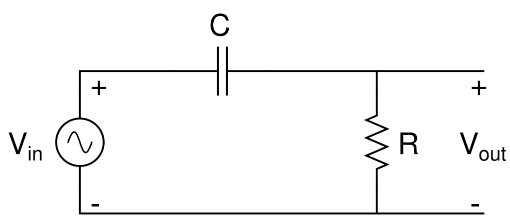

Figure 3

(c) Fig. 4 depicts a scenario, where Z_1 represents the impedance of a probe and Z_2 represents the impedance of an oscilloscope. Consider that the input voltage ($V_{in}(t)$) is a pulse of width $T_b(>> R_i C_i)$ (i=1,2) as described below:

$$V_{in}(t) = \begin{cases} V_1 & t \leq 0 \\ V_2 & 0 \leq t \leq T_b \; (V_2 \geq V_1) \\ V_1 & t \geq T_b \end{cases}$$

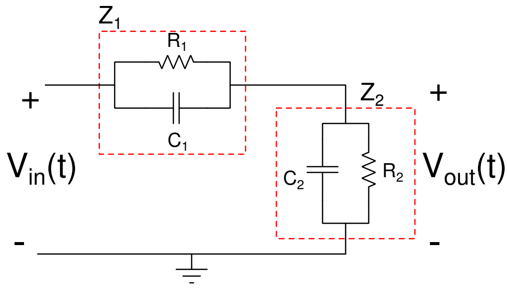

Figure 4

i) Intuitively find and explain the values of $V(C_1)$, $V(C_2)$, $I(R_1)$, $I(C_1)$, $I(R_2)$, $I(C_2)$ at $t = 0^-$ and $t = 0^+$, for the two cases given below:

A. (R_1C_1 not equal to R_2C_2) $R_1 = 10\ M\Omega\ C_1 = 2\ pF$, $R_2 = 5\ M\Omega$, $C_1 = 50\ pF$.

B. (R_1C_1 is equal to R_2C_2) $R_1 = 10\ M\Omega\ C_1 = 2\ pF$, $R_2 = 1\ M\Omega$, $C_1 = 20\ pF$.

ii) Does this circuit allows to pass quick transitions in the input to output (high pass)? What about slow transitions (low pass)? Briefly comment.

iii) Verify your theoretical/intuitive values by simulating the above circuit using LT-SPICE for both cases (A and B). (Plot $V(C_1), V(C_2), I(R_1), I(C_1),\ I(R_2), I(C_2)$ as a function of time).
(Hint:Run Transient analysis for the input pulse signal PULSE(2 5 0 1p 1p 200u), Run the transient for 400 μs).

iv) Derive the transfer function $\frac{V_{out}(s)}{V_{in}(s)}$ for the given circuit. From the transfer function comment on the nature of the circuit - low-pass, high-pass or **all-pass filter**?

v) Give a sinusoidal input(***SINE(0 5 5k)***) to the above circuit with the values of R,C same as in case-B ($R_1 = 10\ M\Omega\ C_1 = 2\ pF$, $R_2 = 1\ M\Omega$, $C_1 = 20\ pF$), perform AC analysis and find the -3 dB bandwidth (BW) of the circuit.

vi) Run transient analysis by varying the frequency of the sinusoidal input and plot the output waveforms.
(HintFor AC analysis Set Type of Sweep = Decade, Number of points = 1000, Start Frequency = 5k Hz, Stop Frequency = 5GHz).

3. Plot Bode magnitude and phase plots for following functions. T is a constant. (References: Lecture notes. For further readings text books on control system - 1) Linear control system by B.S. Manke, 2) Control System by Nagrath Gopal, 3) Control Systems by Ogata)

(a) $H(j\omega) = 1 + j\omega T$

(b) $H(j\omega) = \frac{1}{1+j\omega T}$

(c) $H(j\omega) = \frac{1+j\omega T_1}{1+j\omega T_2}$

(d) $H(j\omega) = \frac{1-j\omega T_1}{1+j\omega T_2}$

(e) $H(s) = \frac{10}{(s+1)(s+2)}$

(f) $H(s) = \frac{(s-1)}{(s+1)(s+2)}$

(g) $H(s) = \frac{2(s+1)}{s^2(s+2)(s+0.5)}$