EEE 554 Random Signal Theory   Fall 2018   MATLAB Project Assignment, **Due date: Friday, December 7, 2018**

Consider the discrete-time (DT) wide sense stationary (WSS) random process (RP)

$$X[n] = A + W[n], \;\; n = 0, 1, \ldots, N-1,$$

where $N = 1024$ samples, $A \in \mathcal{R}$, $A > 0$, is a deterministic number, and $W[n]$ is additive white Gaussian noise with zero mean and variance $\sigma_W^2 = 1$. The data provided in `process.mat` consists of $M = 20$ realizations (or Monte Carlo simulations) of the random process $X[n]$. Some basic commands in MATLAB follow: download the data, separate the realizations, average the realizations (to reduce the noise variance), plot the first realization and plot the average.

```
clear; clf;
load process.mat             % load data to Matlab
process=Received_Data;       % rename the data in process.mat that is called Received_Data
[N, M]=size(process);        % N= number of samples per realization
                             % M = number of realizations
for m=1:M,
  realization{m}=process(:,m);   % Obtain each realization separately
end
%
% Note:  at the end of the "for loop", realization{:} creates a cell of dimension 1xM
%    e.g.,  if we let r4=realization{4};  then r4 is an Nx1 vector of the 4th realization
%
n=0:N-1; x=mean(process,2);  % Average realizations
figure(1);
subplot(211); plot(n, realization{4}); % Plot fourth realization
subplot(212); plot(n, x);         % Plot the average of realizations
```

When MATLAB is used below, provide your code and any plots you deem relevant.

(a) Evaluate the mean η_X, autocorrelation sequence (ACS) $r_X[k]$, and power spectral density (PSD) $S_X(f)$ of the DT WSS random process $X[n]$. The answers should be in terms of the constant A.

(b) Use MATLAB and `subplot.m` to plot one of the realizations of $X[n]$ and $X_{\text{aver}}[n] = (1/M)\sum_{m=1}^{M} X_m[n]$, the average of the realizations, where $M = 20$. Demonstrate mathematically that the variance of $X_{\text{aver}}[n]$ is $\sigma^2 = (1/M)$.

(c) Compute the temporal mean for each of the M realizations and plot the vector of temporal means. Discuss whether the temporal mean remains the same in all realizations.

(d) Compute the ergodic mean corresponding to 10 distinctly different sample values of $X[n]$ and plot the vector of ergodic means. Discuss whether the ergodic mean remains the same in all sample values.

(e) From your replies in parts (c) and (d), is the random process $X[n]$ ergodic in the mean? Explain your answer.

(f) (i) Compute the temporal mean estimate of the ACS $r_X[k]$ of $X[n]$.
(ii) Plot the estimated ACS $\hat{r}_X[k]$ for all k.

(g) (i) Compute the averaged periodogram estimate of the PSD $S_X(f)$ of $X[n]$.
(ii) Plot the estimated $\hat{S}_X(f)$, for $-0.5 < f < 0.5$, periodic with period one.
(iii) What is the effect of varying the averaged periodogram parameters I and L on your results in (i) and (ii)? Justify your answer by providing relevant plots, if needed.
Note that to compute the averaged periodogram, $X[n]$ is divided into I non-overlapping data blocks, each of length L, such that $N = I\,L$. Also, the expected PSD of $X[n]$ is as obtained in part (a).