EEE554 – MATLAB Project Assignment

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a. Answer:

a. Discrete time (DT) wide sense stationary (WSS) random process (RP)

$$X[n] = A + W[n] \quad , n = 0, 1 \ldots . N-1$$

where,

$N = 1024$ samples , $A \in R$, $A > 0$

$W[n]$ is additive white Gaussian noise with

$\mu_w = E[W[n]] = 0$

& $\sigma_w^2 = 1$.

i) mean η_x :-

$$E[X[n]] = E[A + W[n]]$$
$$= E[A] + E[W[n]]$$
$$= A + 0$$
$$= A$$

ii) Autocorrelation sequence (ACS),

$$r_x[k] = E[x[n] \times [n+k]]$$
$$= E[(A + w[n])(A + W[n+k])$$
$$= E[A^2 + A w[n+k] + A w[n] + w[n] w[n+k]]$$
$$= E[A^2] + A \underbrace{E[w(n+k)]}_{=0} + A \underbrace{E[w[n]}_{=0} + E[w[n] \cdot w[n+k]]$$
$$= E[A^2] + \sigma^2 \delta[n - (n+k)]$$

$$\gamma_x[k] = A^2 + (1)\,\delta[-k]$$

$\because E[A^2] = A^2$ & $\delta[-k] = \delta[k]$ & $\sigma^2 = 1$

$$\gamma_x[k] = \delta(k) + A^2$$

Hence, $\gamma_x[k] = A^2 + 1$

iii) Power spectral density (PSD)

$$S_x(f) = \sum_{k=0}^{N-1} \gamma_x[k]\, e^{-j2\pi fk}$$

$$= \sum_{k=0}^{N-1} \left(\delta(k) + A^2\right) e^{-j2\pi fk}$$

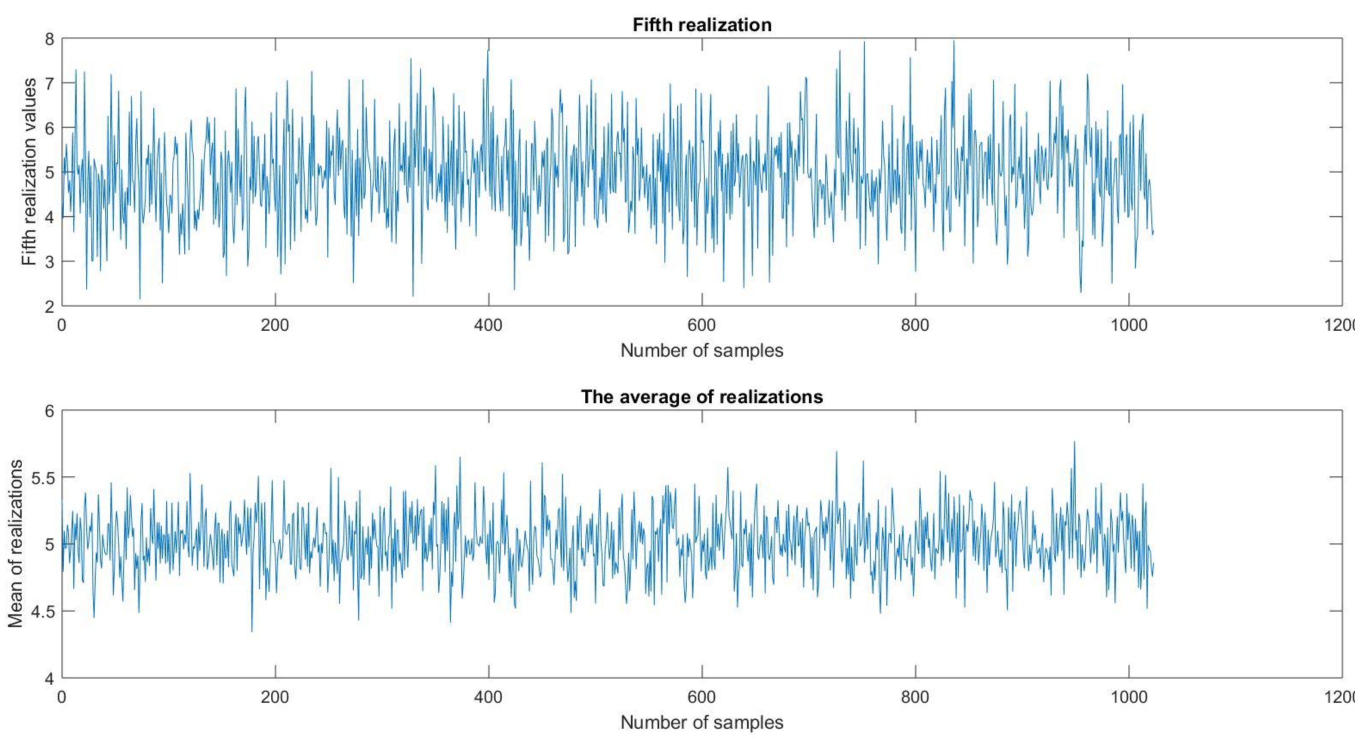
$$S_x(f) = A^2 + 1$$

$-0.5 < f < 0.5$, periodic, period one.

b. MATLAB Code :

```
clear; clf;
load process.mat        % load data to Matlab
process=Received_Data; % rename the data in process.mat that is called Received_Data
[N, M]=size(process);  % N= number of samples per realization

% M = number of realizations
for m=1:M,
realization{m}=process(:,m); % Obtain each realization separately
end

% Note: at the end of the "for loop", realization{:} creates a cell of dimension 1xM
% Here, we have considered r5=realization{5}; hence r5 is an Nx1 vector of the 5th
realization

n=0:N-1;
x=mean(process,2); % Average realizations
figure(1);

subplot(211); plot(n, realization{5}); % Plot fifth realization
title('Fifth realization');
xlabel('Number of samples'); ylabel('Fifth realization values');

subplot(212); plot(n, x); % Plot the average of realizations
title('The average of realizations');
xlabel('Number of samples'); ylabel('Mean of realizations');
```

Plots:



Mathematical demonstration of the variance of X_aver[n]:

b. $X_{aver}[n] = \left(\frac{1}{M}\right) \sum_{m=1}^{M} x_m[n]$

$M = 20$

Variance of

To prove, ^ $X_{aver}[n]$ is $\sigma^2 = (1/M)$, we start with expanding above function.

$$X_{aver}[n] = \left(\frac{1}{M}\right) \left[x_1[n] + x_2[n] + x_3[n] \cdots x_M[n]\right]$$

$$\text{var}(X_{aver}[n]) = \text{Var}\left(\left(\frac{1}{M}\right)\left[x_1[n] + x_2[n] \cdots x_M[n]\right]\right)$$

$$= \frac{1}{M^2} \text{var}(x_1[n] + x_2[n] \cdots x_M[n])$$

... By property of variance

$$= \frac{M\sigma^2}{M^2} \quad \left(\because \text{var}(x_1[n] + x_2[n] \cdots x_M[n]) = \sigma^2 \cdot M\right)$$

$$= \frac{\sigma^2}{M}$$

$\because$ variance $\sigma^2 = 1$

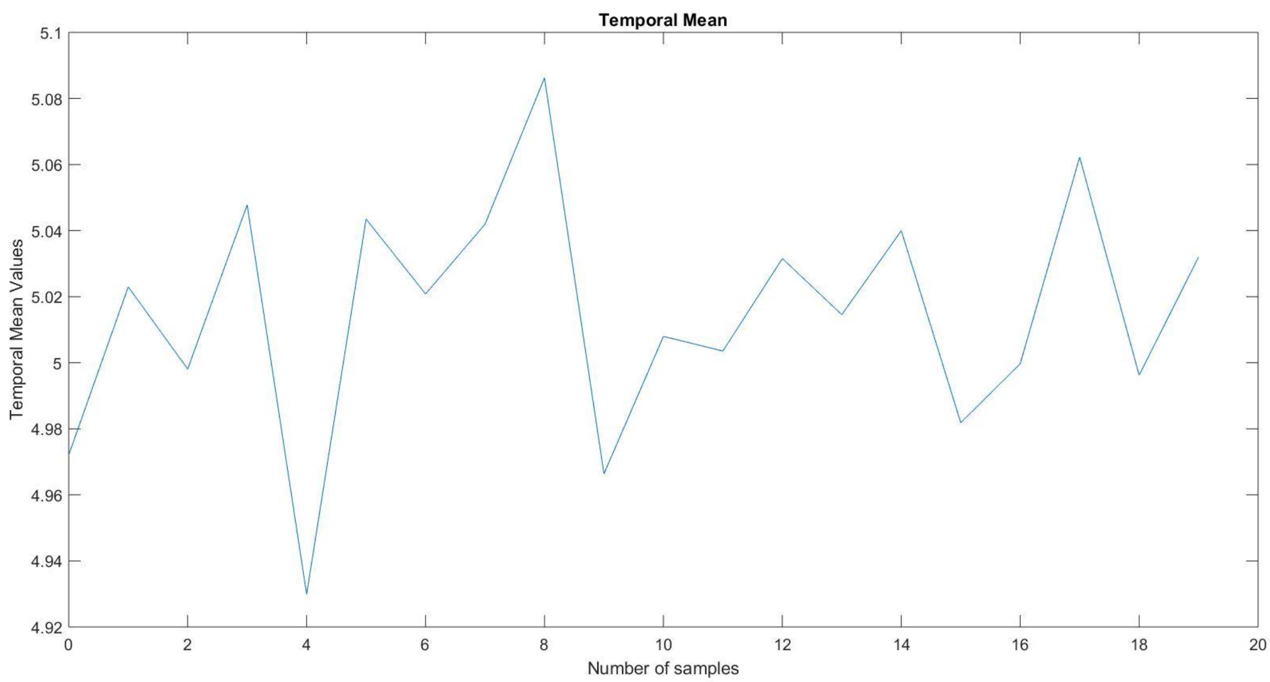
$$\therefore \text{var}(X_{aver}[n]) = \frac{1}{M}$$

Hence, proved.

c. MATLAB Code:

```
clear; clf;
load process.mat % load data to Matlab
process=Received_Data; % rename the data in process.mat that is called Received_Data
[N, M]=size(process); % N= number of samples per realization

% M = number of realizations
for m=1:M,
realization{m}=process(:,m); % Obtain each realization separately
end

Temporal_mean=mean(process); %Average Realizations
Temporal_mean= Temporal_mean'
Final_mean=mean(Temporal_mean) %Mean of all temporal mean

plot(0:m-1, Temporal_mean);
title('Temporal Mean');
xlabel('Number of samples'); ylabel('Temporal Mean Values');
```

Plots:



MATLAB Output:

```
Temporal_mean =    4.9721      5.0230         4.9981         5.0478         4.9299
                   5.0435      5.0208         5.0419         5.0862         4.9664
                   5.0080      5.0035         5.0315         5.0145         5.0400
                   4.9818      4.9997         5.0622         4.9962         5.0320

Final_mean =       5.0150
```

Observation of c:

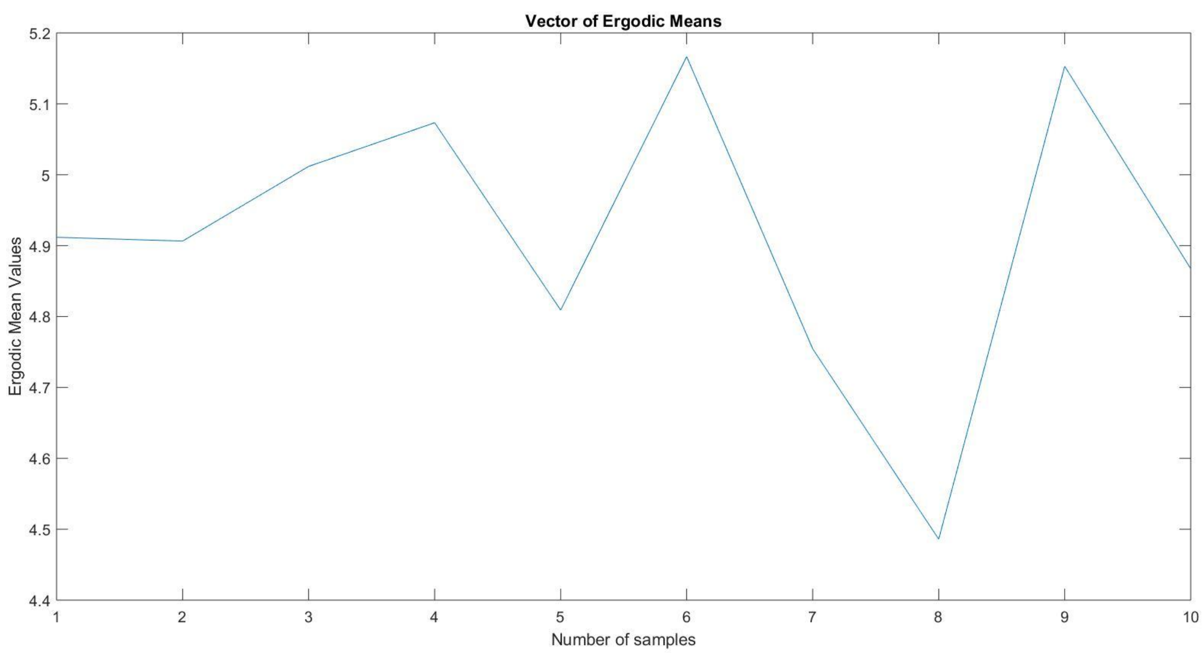
From the above output, we can see that temporal mean varies slightly for all M (=20) realizations. Values varies from 4.9664 to 5.0862. The average of all temporal means is computed which is mentioned as Final_mean. Hence, Temporal mean is different for all realizations.

d. MATLAB Code:

```
clear; clf;
load process.mat % load data to Matlab
process=Received_Data; % rename the data in process.mat that is called Received_Data
[N, M]=size(process); % N= number of samples per realization
% M = number of realizations
for m=1:M,
realization{m}=process(:,m); % Obtain each realization separately
end

reduced_samples=randperm(1024,10); % 10 samples to be selected from total 1024 samples.

for i=1:10
new_mean(i)=mean(process(reduced_samples (i),: )); %Average Realizations
end
new_mean=new_mean'
finmean=mean(new_mean) %Mean of all ergodic mean

plot(1:10, new_mean);
title('Vector of Ergodic Means');
xlabel('Number of samples'); ylabel('Ergodic Mean Values');
```

Plots:

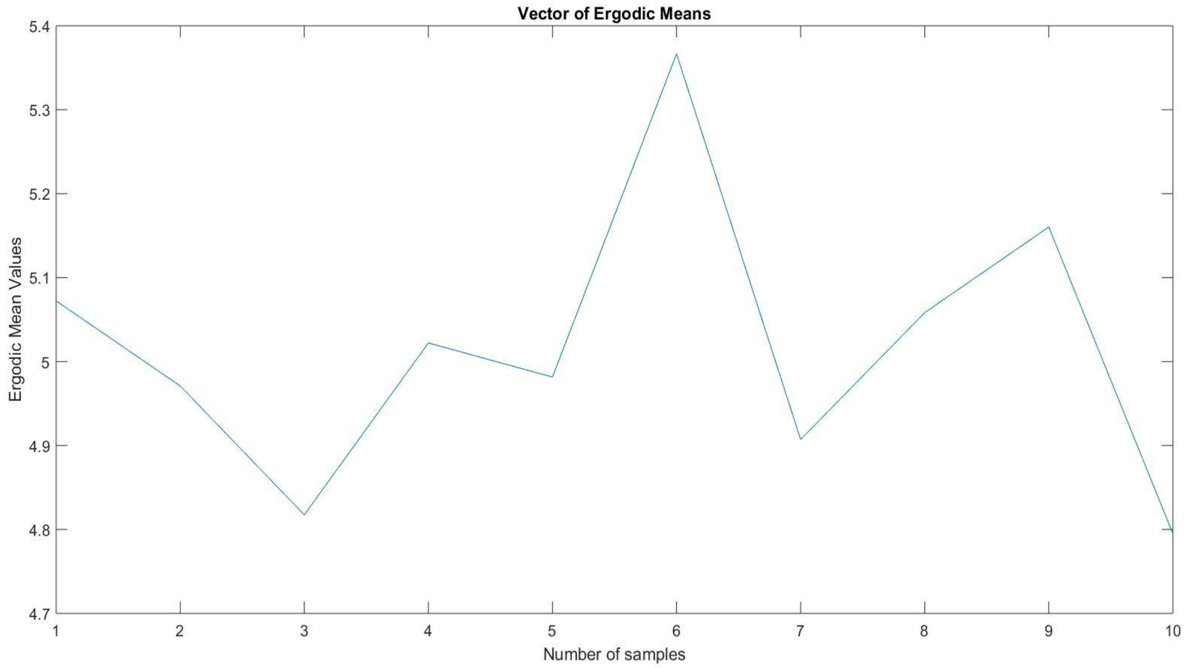
1. Sample 1 :



MATLAB Output:

```
>> Part_d

new_mean = 4.9117       4.9064       5.0117      5.0733       4.8090       5.1665       4.7545
           4.4860       5.1528       4.8674

finmean =    4.9139
```

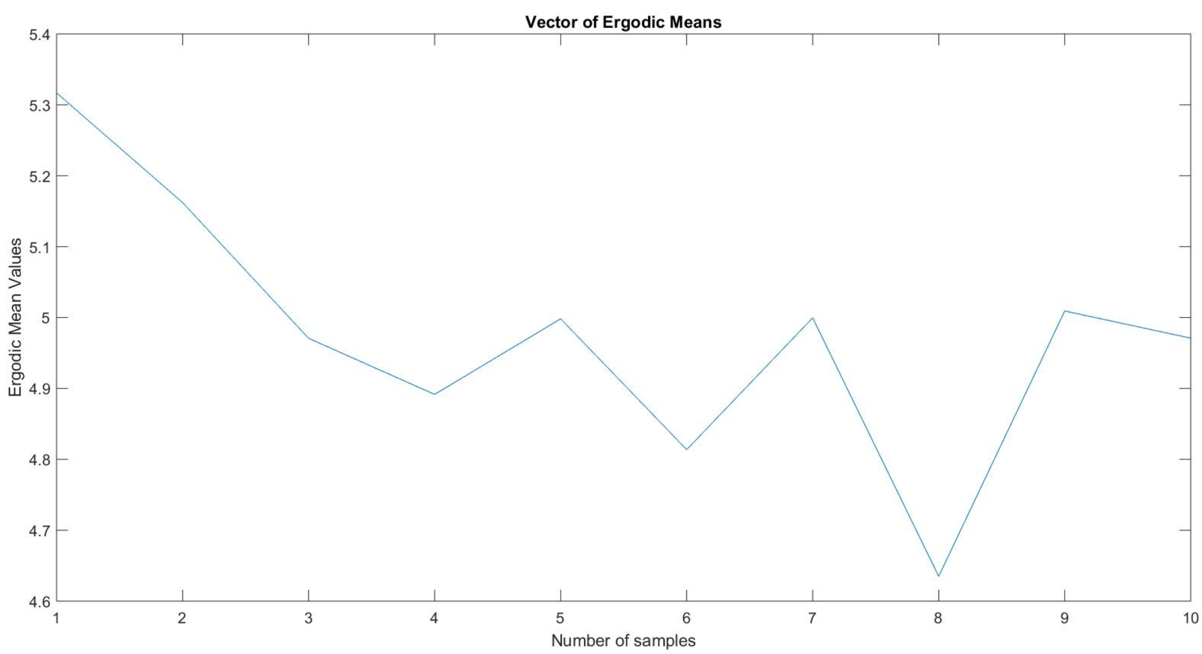
2. Sample 2 :



MATLAB Output:

```
>> Part_d

new_mean =   5.0722      4.9709      4.8174      5.0222       4.9815     5.3666      4.9072
             5.0582      5.1603      4.7951

finmean =   5.0152
```

3. Sample 3 :



MATLAB Output:

```
>> Part_d

new_mean = 5.3168      5.1621     4.9706     4.8916     4.9982     4.8137     4.9996

           4.6350      5.0092     4.9709

finmean =  4.9768
```

Observation of d:

Different 10 samples are taken every time out of total 1024 samples per realization. Above output shows 3 different realizations for distinct 10 randomly selected samples.
It is seen that ergodic mean is slightly different for each different sample in one realization and it is different for all realizations.
Final mean is the average of all ergodic mean in the same realization which is also different for all realizations.
Hence, ergodic mean is different for sample values.

e.

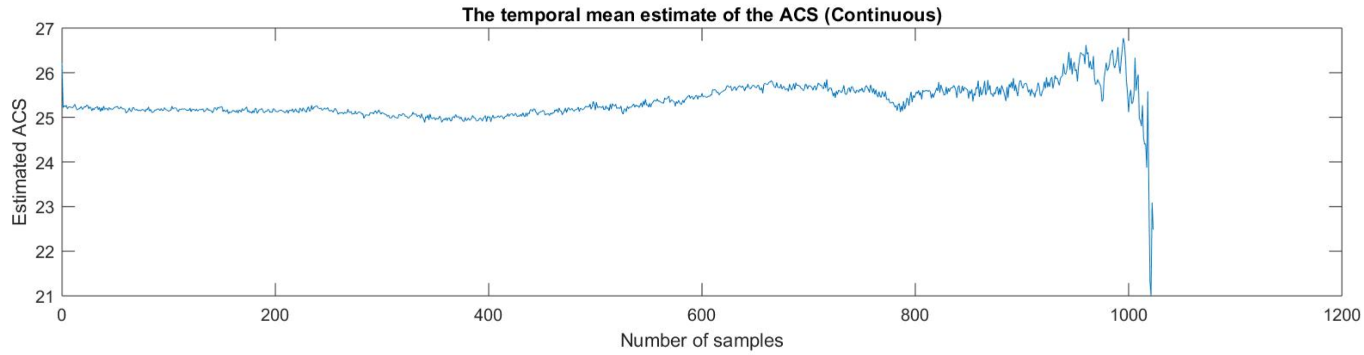
Random process is ergodic in the mean if,
Temporal Mean = Ergodic Mean = Mean of X[n]

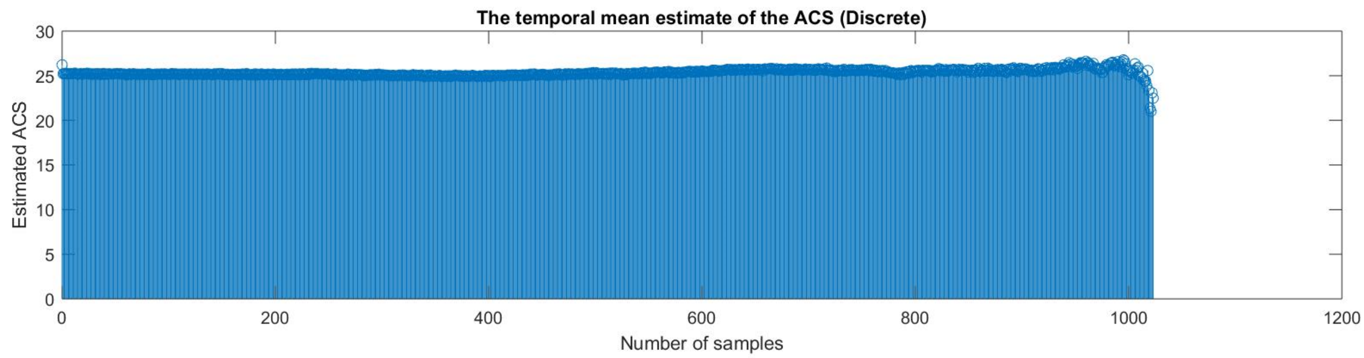
But, from output of part (c) and part (d), we can see that Mean obtained for Temporal , Ergodic and X[n] are different and hence, the random process X[n] is not ergodic in the mean.

f. MATLAB Code:

```matlab
clear; clf;
load process.mat % load data to Matlab
process=Received_Data; % rename the data in process.mat that is called Received_Data
[N, M]=size(process); % N= number of samples per realization

% M = number of realizations
for m=1:M,
    realization{m}=process(:,m); % Obtain each realization separately
end

y(1)=0; %Initialize

for i=1:M
    for k=0:N-1
                 %Temporal mean estimate of the ACS rX[k] of X[n]
        y(i,k+1) = (1/(N-k))* sum(process(1:N-k,i).*process(k+1:N,i));
    end
end

subplot(211);
plot(0:N-1,y(2,1:N)) %Continuous Samples
title('The temporal mean estimate of the ACS (Continuous)');
xlabel('Number of samples'); ylabel('Estimated ACS');

subplot(212);
stem(0:N-1,y(2,1:N)) %Discrete Samples
title('The temporal mean estimate of the ACS (Discrete)');
xlabel('Number of samples'); ylabel('Estimated ACS');
```

Plots:





Observation of f:

i) Temporal mean estimate of the ACS rX[k] of X[n] is obtained in code.
ii) Continuous and discrete outputs are plotted for the estimated ACS rX[k] of X[n].

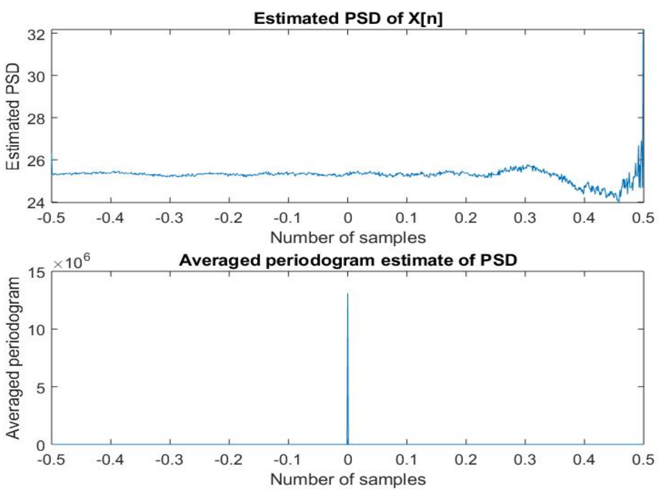
g. MATLAB Code:

```
clear;
clf;
load process.mat % load data to Matlab
process=Received_Data; % rename the data in process.mat that is called Received_Data
[N, M]=size(process); % N= number of samples per realization

% M = number of realizations
for m=1:M
realization{m}=process(:,m); % Obtain each realization separately
end

% The averaged periodogram estimate of the PSD Sx(f) of X[n]
for i=0:N-1
r1(i+1,1)=(1/(N-i))*sum(realization{m}(1:N-i).*realization{m}(1+i :N));
end

I=128; %Various values of I is considered e.g. 1, 132, 128, 256, 512
N=1024; %set FFT size
L = N/I % X[n] is divided into I non-overlapping data blocks, each of length L
r=r1;
Pav1=r1;
f=[0:N-1]'/N-0.5;
for m=1:M
for i=0:I-1
```

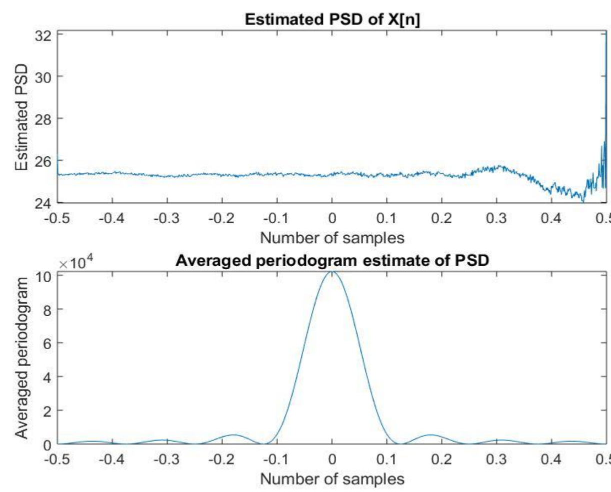
```
value = r((i*L)+1:(i*L)+L);
Pav1=Pav1+(1/(I*L))*[abs(fftshift(fft(value,N))).^2];
end
end
length(Pav1);

subplot(211)
plot(f,r1); title('Estimated PSD of X[n]');
xlabel('Number of samples'); ylabel('Estimated PSD');

subplot(212)
plot(f,Pav1); title('Averaged periodogram estimate of PSD');
xlabel('Number of samples'); ylabel('Averaged periodogram');
```

Plots:

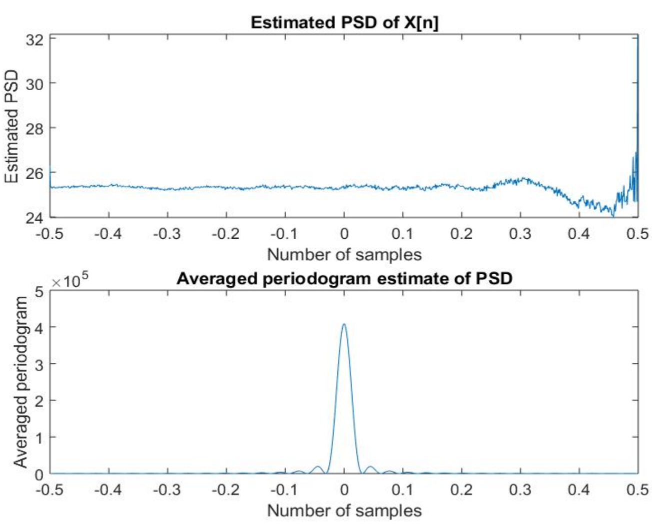
I=1



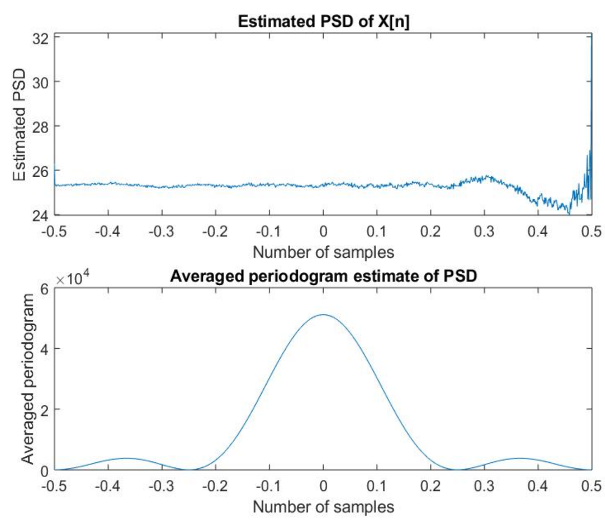
I=128



I=32



I=256



Observation of g:

i) The averaged periodogram estimate of the PSD Sx(f) of X[n] is computed.

ii) The estimated Sx(f) is plotted for $-0.5<f<0.5$, periodic with period one.

iii) By varying the value of I, we get different values of L as L=I/N keeping N=1024. As we increase the value of I, we can observe that the value of averaged periodogram estimate of PSD spreads across the zero and amplitude value decreases.

4 different values of I is taken (I= 1, 32,128, 256) and graphs are plotted as shown above. Hence, as at I=1, we get the sharp peak across 0. And as value of I is increased, the shape of curve is spread over 0.