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Math 31A Lecture Notes: Critical Points

Definition; *A critical point c of function f*: either:

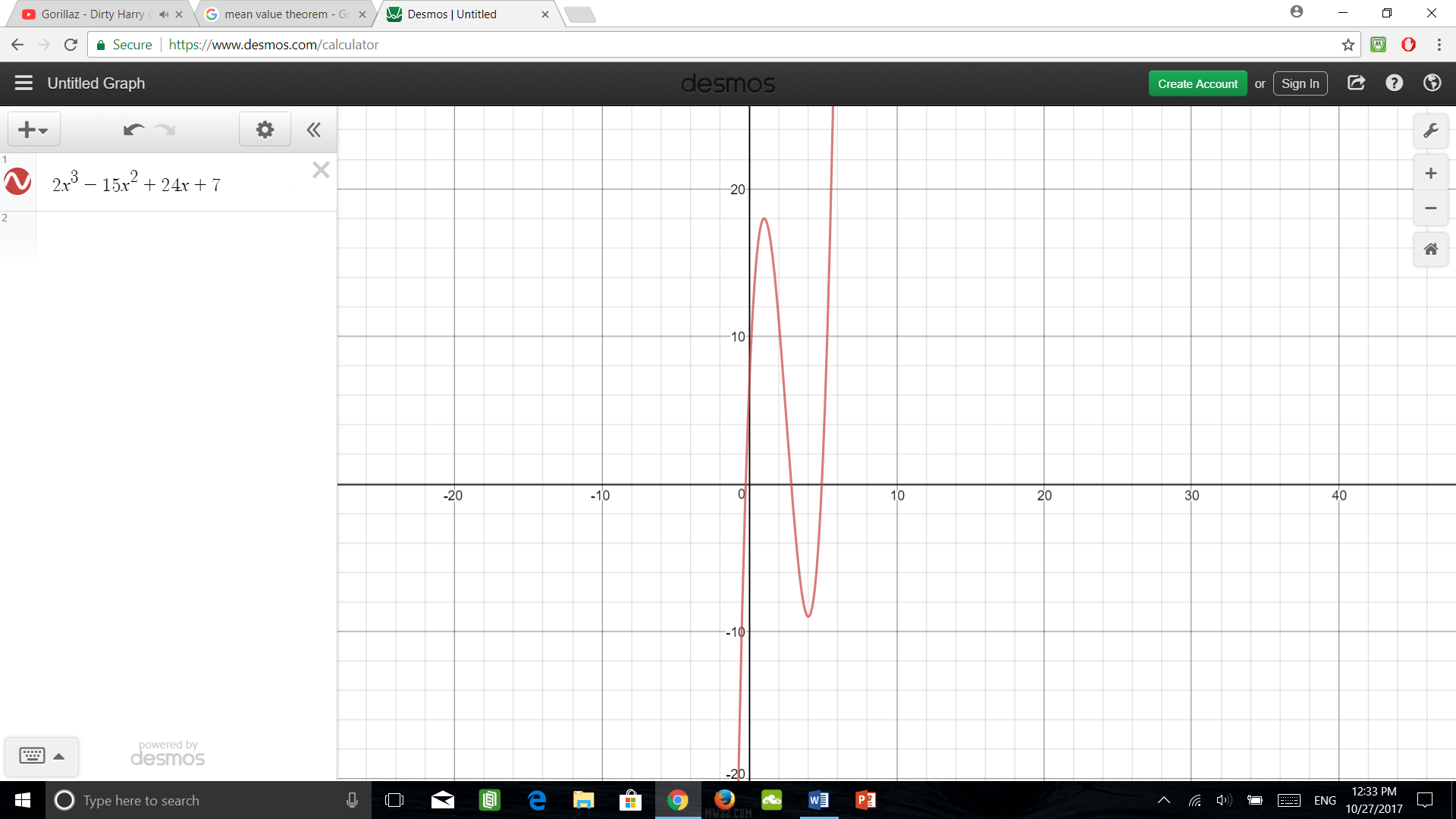
* does not exist

Theorem: If f(c) is a local min/max of f, then c is a critical point.

Theorem: Assume f is continuous on an interval [a, b] and f(c) is the absolute minimum or maximum of f on [a, b]. Then, c is either:

* A critical point

Example 1: Find the extrema of f on [0, 6];

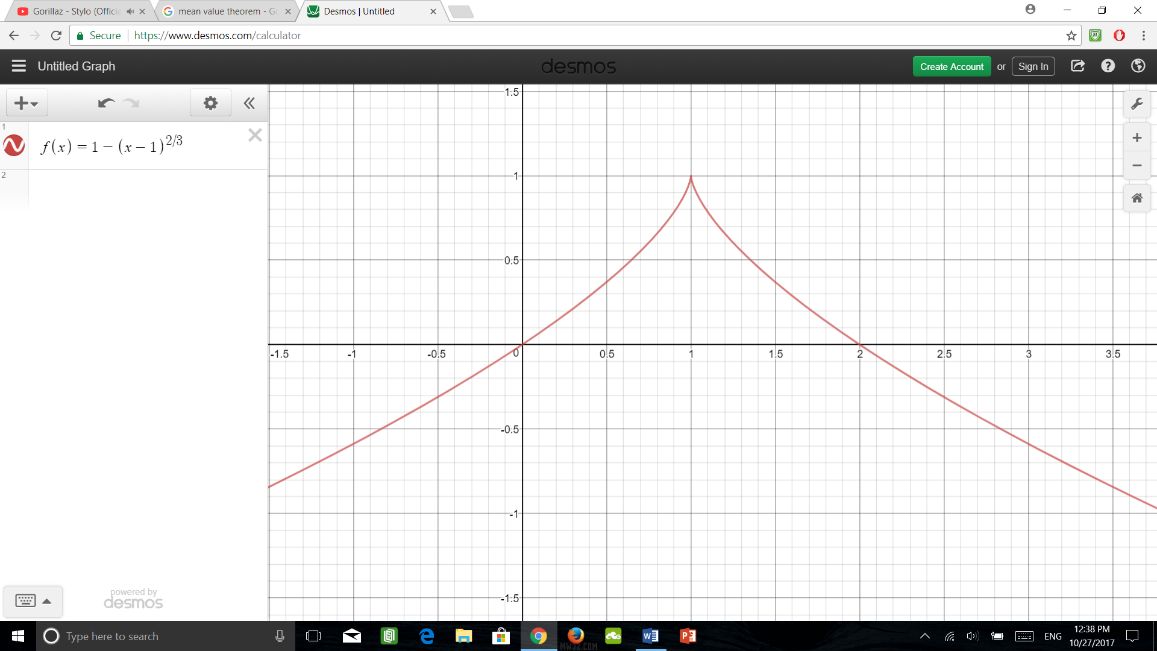


Critical points: c = 1 and c = 4

|  |  |
| --- | --- |
| x | f(x) |
| 1 (critical point) | 18 |
| 4 (critical point) | -9 |
| 0 (end point) | 7 |
| 6 (end point) | 43 |

Example 2: Find the extrema of f on [-1, 2];

Critical points: c = 1

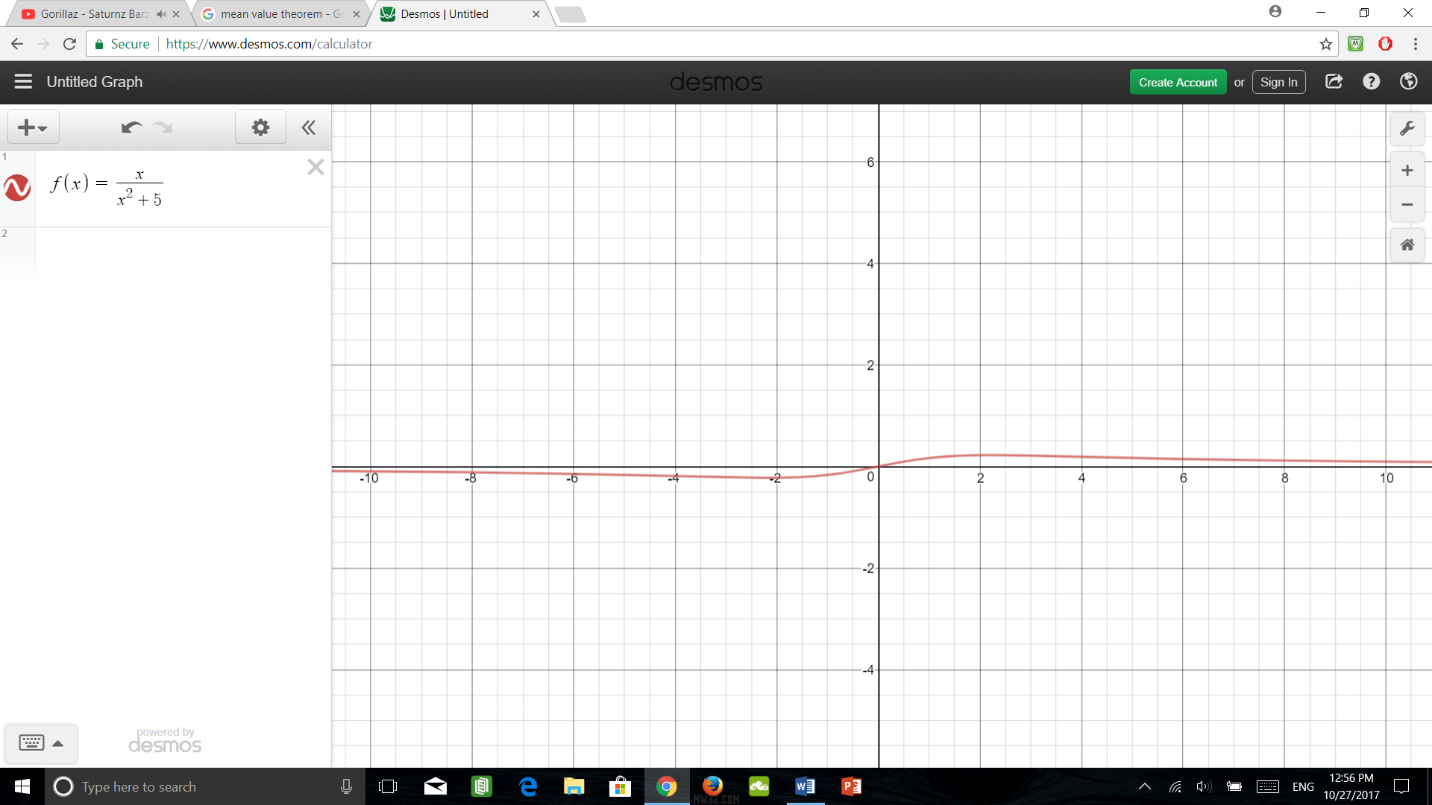


|  |  |
| --- | --- |
| x | f(x) |
| 1 (critical point) | 1 (max) |
| -1 (end point) | -0.59 (min) |
| 2 (end point) | 0 |

Example 3: Find the extreme values of on [4, 8]

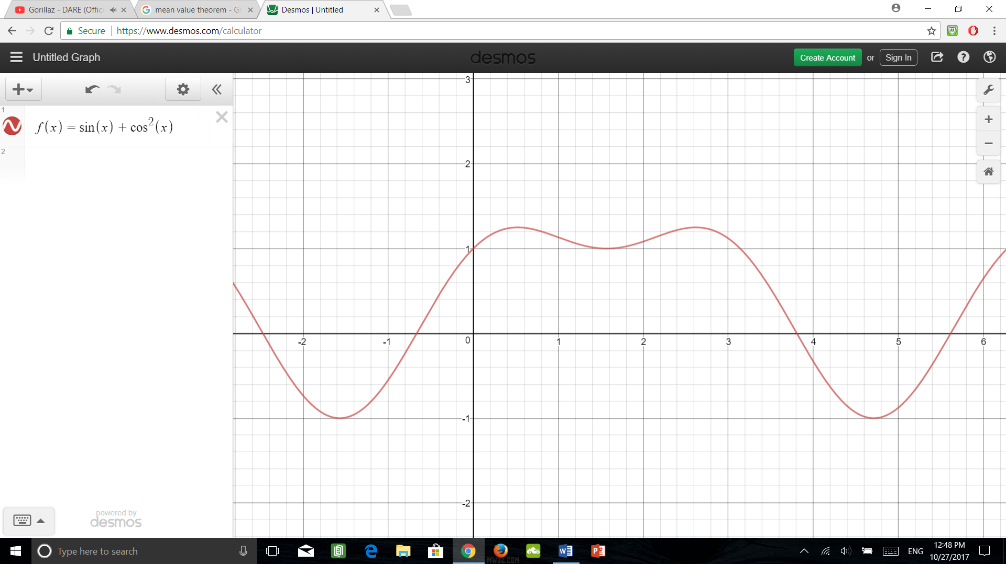
Critical points:

Critical points are outside of the interval.



|  |  |
| --- | --- |
| x | f(x) |
| 4 | 4/21 (min) |
| 8 | 8/69 (max) |

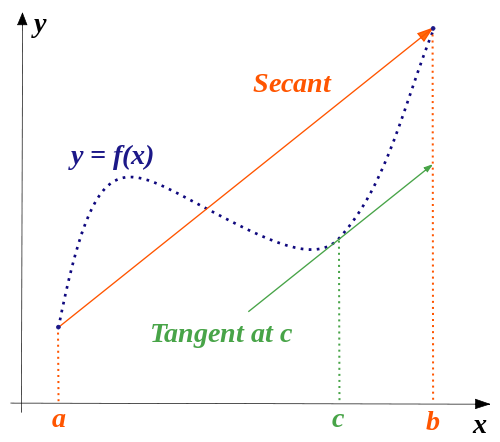
Example 4: Find the min/max of on [0, 2]



OR

|  |  |
| --- | --- |
| x | f(x) |
|  | 1 |
|  | -1 |
|  | 5/4 |
|  | 5/4 |
| 0 | 1 |
| 1 | 1 |

Theorem; *The Mean Value Theorem*: Assume f is continuous in [a, b] and differentiable in (a, b). Then, there is at least one such that:



Clarification: Imagine a car with an average speed of 70 mph. In order to reach the average speed of 70 mph, the car must reach two speeds: one more than 70 mph, one less than 70 mph.

Definition; *Increasing and Decreasing Functions*:

A function f is:

* Increasing on (a, b) if f(x1) < f(x2) for x1 < x­2 at x1, x2 ϵ (a, b)
* Decreasing on (a, b) if f(x1) > f(x2) for x1 < x­2 at x1, x2 ϵ (a, b)

Definition; *Monotonic function*: f is monotonic on (a, b) if f is either inc. or dec. on (a, b).

Theorem: f is differentiable on (a, b) if:

* then f is increasing on (a, b).
* then f is decreasing on (a, b).

Proof: Let

MVT (mean value theorem) gives us for any x1 < x­2

s.t.

> 0 > 0

Hence,