

Definition 1 ($\text{LRA}(G)$). Let $G \triangleq (N, T, P, S)$ be a CFG. $L(G) \triangleq \{z \in T^* \mid S \Rightarrow_P^* z\}$.

- (i) $G' \triangleq (N', T', P', S') = (N \uplus \{S'\}, T \uplus \{\$\}, P \cup \{[S' \rightarrow S\$]\}, S')$.
- (ii) $L(G') \triangleq \{z \in T^* \mid S' \Rightarrow_{P'}^* z\}$. $L(G) = L(G')$.
- (iii) $V \triangleq N \uplus T$. $V' \triangleq N' \uplus T'$.
- (iv) $\beta \xRightarrow{\text{rm } P'} \gamma \stackrel{\text{def}}{\iff} (\beta, \gamma) \in \{(\alpha Az, \alpha \omega z) \mid [A \rightarrow \omega] \in P', \alpha \in V'^*, z \in T'^*\}$.
- (v) $I_{\text{LR}(0)} \triangleq \{[A \rightarrow \alpha \cdot \beta] \mid [A \rightarrow \alpha \beta] \in P'\}$.
- (vi) $\text{Cl}(q) =_\mu q \cup \{[A \rightarrow \varepsilon \cdot \omega] \mid [A \rightarrow \omega] \in P', [B \rightarrow \beta \cdot A\gamma] \in \text{Cl}(q)\}$ for $q \in \mathcal{P}(I_{\text{LR}(0)})$.
- (vii) $q_0 := \text{Cl}(\{[S' \rightarrow \varepsilon \cdot S\$]\})$.
- (viii) $\text{GOTO}(q, X) := \text{Cl}(\{[A \rightarrow \alpha X \cdot \beta] \mid [A \rightarrow \alpha \cdot X\beta] \in q\})$ for $q \in \mathcal{P}(I_{\text{LR}(0)})$ and $X \in V'$.
- (ix) $Q \triangleq \text{PT} \setminus \{\emptyset\}$ where $\text{PT} =_\mu \{q_0\} \cup \{\text{GOTO}(q, X) \mid q \in \text{PT}, X \in V'\}$.
- (x) $\varepsilon : p \rightarrow q \stackrel{\text{def}}{\iff} p \in Q \wedge p = q$. $X\alpha : p \rightarrow q \stackrel{\text{def}}{\iff} p \in Q \wedge \alpha : \text{GOTO}(p, X) \rightarrow q$.
- (xi) **Config** $\triangleq \{(\alpha : p \rightarrow q, z) \mid \alpha \in V'^*, p \in Q, q \in Q, z \in T'^*, \alpha : p \rightarrow q\}$.
- (xii) $\delta(q, X) := \text{GOTO}(q, X)$ for $q \in Q$ and $X \in V'$.
- (xiii) **reduce**(q, t) $:= \{[A \rightarrow \omega] \mid [A \rightarrow \omega \cdot \varepsilon] \in q\}$ for $q \in Q$ and $t \in T'$.
- (xiv) Let \vdash be a binary relation on the set **Config** with two introduction rules:
$$\frac{q'' = \delta(q', t)}{(\alpha : q \rightarrow q', tz) \vdash (\alpha t : q \rightarrow q'', z)} \text{ Shift} \quad \frac{[A \rightarrow \omega] \in \text{reduce}(q', t)}{(\alpha \omega : q \rightarrow q', tz) \vdash (\alpha A : q \rightarrow q'', tz)} \text{ Reduce}(A \rightarrow \omega)$$
- (xv) $q_f := \delta(\delta(q_0, S), \$)$.
- (xvi) $L(\text{LRA}(G)) \triangleq \{z \in T^* \mid (\varepsilon : q_0 \rightarrow q_0, z\$) \vdash^* (S\$: q_0 \rightarrow q_f, \varepsilon)\}$. $L(G) = L(\text{LRA}(G))$.

Theorem 2. Define a function **LA** : $\{(q, [A \rightarrow \omega]) \mid q \in Q, [A \rightarrow \omega \cdot \varepsilon] \in q\} \rightarrow \mathcal{P}(T')$ by

$$\mathbf{LA}(q, [A \rightarrow \omega]) := \left\{ t \in T' \mid S' \xRightarrow{\text{rm } P'}^* \alpha A t z, \alpha \omega : q_0 \rightarrow q, \alpha \in V'^*, z \in T'^* \right\}. \quad (1)$$

Then, overriding **reduce** : $Q \times T' \rightarrow \mathcal{P}(P')$ of the LR(0) parser $\text{LRA}(G)$ with

$$(q, t) \mapsto \{[A \rightarrow \omega] \mid [A \rightarrow \omega \cdot \varepsilon] \in q, t \in \mathbf{LA}(q, [A \rightarrow \omega])\}$$

yields an LALR(1) parser if there are no conflicts among actions.

Theorem 3. Letting $R!x \triangleq \{y \mid (x, y) \in R\}$, define relations **Read** and **Follow** from the set

$$\{(p, A) \mid p \in Q, A \in N', \delta(p, A) \neq \emptyset\}$$

to the set T' inductively as follows:

$$\begin{array}{ll} \frac{\delta(\delta(p, A), t) \neq \emptyset}{t \in \mathbf{Read}!(p, A)} \text{ DR} & \frac{\delta(p, A) = r \quad C \Rightarrow_{P'}^* \varepsilon}{\mathbf{Read}!(r, C) \subseteq \mathbf{Read}!(p, A)} \text{ reads} \\ \frac{t \in \mathbf{Read}!(p, A)}{t \in \mathbf{Follow}!(p, A)} \text{ Read} & \frac{[B \rightarrow \beta \cdot A\gamma] \in p \quad \beta : p' \rightarrow p \quad \gamma \Rightarrow_{P'}^* \varepsilon}{\mathbf{Follow}!(p', B) \subseteq \mathbf{Follow}!(p, A)} \text{ includes} \end{array}$$

Then, the equality

$$\mathbf{LA}(q, [A \rightarrow \omega]) = \{t \in T' \mid p \in Q, \omega : p \rightarrow q, \delta(p, A) \neq \emptyset, t \in \mathbf{Follow}!(p, A)\} \quad (2)$$

holds whenever $q \in Q$ and $[A \rightarrow \omega \cdot \varepsilon] \in q$.