Definition 1 (LRA(G)). *Let $G \triangleq (N, T, P, S)$ be a CFG.*

(i) $G' := (N', T', P', S') \triangleq (N \uplus \{S'\}, T \uplus \{\$\}, P \cup \{[S' \to S\$]\}, S')$.
(ii) $V \triangleq N \uplus T$. $V' \triangleq N' \uplus T'$.
(iii) $\beta \underset{\mathbf{rm}}{\Longrightarrow}_{P'} \gamma \overset{\text{def}}{\Longleftrightarrow} (\beta, \gamma) \in \{(\alpha A z, \alpha\omega z) \mid [A \to \omega] \in P', \alpha \in V'^*, z \in T'^*\}$.
(iv) $I_{\mathrm{LR}(0)} \triangleq \{[A \to \alpha \cdot \beta] \mid [A \to \alpha\beta] \in P'\}$.
(v) $\mathrm{Cl}(q) =_\mu q \cup \{[A \to \varepsilon \cdot \omega] \mid [A \to \omega] \in P', [B \to \beta \cdot A\gamma] \in \mathrm{Cl}(q)\}$ *for* $q \in \mathcal{P}(I_{\mathrm{LR}(0)})$.
(vi) $q_0 := \mathrm{Cl}(\{[S' \to \varepsilon \cdot S\$]\})$.
(vii) $\mathrm{GOTO}(q, X) := \mathrm{Cl}(\{[A \to \alpha X \cdot \beta] \mid [A \to \alpha \cdot X\beta] \in q\})$ *for* $q \in \mathcal{P}(I_{\mathrm{LR}(0)})$ *and* $X \in V'$.
(vii) $Q \triangleq \mathrm{PT} \setminus \{\emptyset\}$ *where* $\mathrm{PT} =_\mu \{q_0\} \cup \{\mathrm{GOTO}(q, X) \mid q \in \mathrm{PT}, X \in V'\}$.
(ix) $\varepsilon : p \to q \overset{\text{def}}{\Longleftrightarrow} p \in Q \wedge p = q$. $X\alpha : p \to q \overset{\text{def}}{\Longleftrightarrow} p \in Q \wedge \alpha : \mathrm{GOTO}(p, X) \to q$.
(x) $\mathbf{Config} \triangleq \{(\alpha : p \to q, z) \mid \alpha \in V'^*, p \in Q, q \in Q, z \in T'^*, \alpha : p \to q\}$.
(xi) $\delta(q, X) := \mathrm{GOTO}(q, X)$ *for* $q \in Q$ *and* $X \in V'$.
(xii) $\mathbf{reduce}(q, t) := \{[A \to \omega] \mid [A \to \omega \cdot \varepsilon] \in q\}$ *for* $q \in Q$ *and* $t \in T'$.
(vii) *Let $\vdash$ be a binary relation on the set* $\mathbf{Config}$ *with two introduction rules:*

$$\frac{q'' = \delta(q', t)}{(\alpha : q \to q', tz) \vdash (\alpha t : q \to q'', z)}\ \text{Shift} \qquad \frac{[A \to \omega] \in \mathbf{reduce}(q', t)}{(\alpha\omega : q \to q', tz) \vdash (\alpha A : q \to q'', tz)}\ \text{Reduce}(A \to \omega)$$

(xiv) $q_f := \delta(\delta(q_0, S), \$)$.
(xv) *The language accepted by* LRA(G) *is defined as*

$$L(\mathrm{LRA}(G)) := \{z \in T^* \mid (\varepsilon : q_0 \to q_0, z\$) \vdash^* (S\$: q_0 \to q_f, \varepsilon)\}.$$

Fact 2. *It is known that $L(G) = L(\mathrm{LRA}(G))$, where*

$$L(G) \triangleq \{z \in T^* \mid S \Rightarrow_P^* z\}$$

is the language generated by G.

Theorem 3. *Define* $\mathbf{LA} : \{(q, [A \to \omega]) \mid q \in Q, [A \to \omega \cdot \varepsilon] \in q\} \to \mathcal{P}(T')$ *by*

$$\mathbf{LA}(q, [A \to \omega]) := \left\{ t \in T' \;\middle|\; S' \underset{\mathbf{rm}}{\Longrightarrow}^*_{P'} \alpha A t z, \alpha\omega : q_0 \to q, \alpha \in V'^*, z \in T'^* \right\}. \tag{1}$$

Then, overriding $\mathbf{reduce} : Q \times T' \to \mathcal{P}(P')$ *of the* LR(0) *parser* LRA(G) *with*

$$(q, t) \mapsto \{[A \to \omega] \mid [A \to \omega \cdot \varepsilon] \in q, t \in \mathbf{LA}(q, [A \to \omega])\}$$

yields an LALR(1) *parser if there are no conflicts, which accepts the same language.*

Theorem 4. *Letting $R \,!\, x \triangleq \{y \mid (x, y) \in R\}$, define relations* $\mathbf{Read}$ *and* $\mathbf{Follow}$ *from the set*

$$\{(p, A) \mid p \in Q, A \in N', \delta(p, A) \neq \emptyset\}$$

to the set T' inductively as follows:

$$\frac{\delta(\delta(p, A), t) \neq \emptyset}{t \in \mathbf{Read} \,!\, (p, A)}\ \text{DR} \qquad \frac{\delta(p, A) = r \quad C \Rightarrow^*_{P'} \varepsilon}{\mathbf{Read} \,!\, (r, C) \subseteq \mathbf{Read} \,!\, (p, A)}\ \text{reads}$$

$$\frac{t \in \mathbf{Read} \,!\, (p, A)}{t \in \mathbf{Follow} \,!\, (p, A)}\ \text{Read} \qquad \frac{[B \to \beta \cdot A\gamma] \in p \quad \beta : p' \to p \quad \gamma \Rightarrow^*_{P'} \varepsilon}{\mathbf{Follow} \,!\, (p', B) \subseteq \mathbf{Follow} \,!\, (p, A)}\ \text{includes}$$

Then, whenever $q \in Q$ and $[A \to \omega \cdot \varepsilon] \in q$, we can compute $\mathbf{LA}(q, [A \to \omega])$ *by*

$$\mathbf{LA}(q, [A \to \omega]) = \{t \in T' \mid p \in Q, \omega : p \to q, \delta(p, A) \neq \emptyset, t \in \mathbf{Follow} \,!\, (p, A)\}. \tag{2}$$