

**Definition 1** ( $\text{LRA}(G)$ ). Let  $G \triangleq (N, T, P, S)$  be a CFG.  $L(G) \triangleq \{z \in T^* \mid S \Rightarrow_P^* z\}$ .

- (i)  $G' \triangleq (N', T', P', S') = (N \uplus \{S'\}, T \uplus \{\$\}, P \cup \{[S' \rightarrow S\$]\}, S')$ .
- (ii)  $L(G') \triangleq \{z \in T^* \mid S' \Rightarrow_{P'}^* z\}$ .  $L(G) = L(G')$ .
- (iii)  $V \triangleq N \uplus T$ .  $V' \triangleq N' \uplus T'$ .
- (iv)  $\beta \xRightarrow[\text{rm } P']{\text{def}} \gamma \iff (\beta, \gamma) \in \{(\alpha Az, \alpha \omega z) \mid [A \rightarrow \omega] \in P', \alpha \in V'^*, z \in T'^*\}$ .
- (v)  $I_{\text{LR}(0)} \triangleq \{[A \rightarrow \alpha \cdot \beta] \mid [A \rightarrow \alpha\beta] \in P'\}$ .
- (vi)  $\text{Cl}(q) =_\mu q \cup \{[A \rightarrow \varepsilon \cdot \omega] \mid [A \rightarrow \omega] \in P', [B \rightarrow \beta \cdot A\gamma] \in \text{Cl}(q)\}$  for  $q \in \wp(I_{\text{LR}(0)})$ .
- (vii)  $q_0 := \text{Cl}(\{[S' \rightarrow \varepsilon \cdot S\$]\})$ .
- (viii)  $\text{GOTO}(q, X) := \text{Cl}(\{[A \rightarrow \alpha X \cdot \beta] \mid [A \rightarrow \alpha \cdot X\beta] \in q\})$  for  $q \in \wp(I_{\text{LR}(0)})$  and  $X \in V'$ .
- (ix)  $Q \triangleq \text{PT} \setminus \{\emptyset\}$  where  $\text{PT} =_\mu \{q_0\} \cup \{\text{GOTO}(q, X) \mid q \in \text{PT}, X \in V'\}$ .
- (x)  $\varepsilon : p \rightarrow q \xRightarrow{\text{def}} p \in Q \wedge p = q$ .  $X\alpha : p \rightarrow q \xRightarrow{\text{def}} p \in Q \wedge \alpha : \text{GOTO}(p, X) \rightarrow q$ .
- (xi) **Config**  $\triangleq \{(\alpha : p \rightarrow q, z) \mid \alpha \in V'^*, p \in Q, q \in Q, z \in T'^*, \alpha : p \rightarrow q\}$ .
- (xii)  $\delta(q, X) := \text{GOTO}(q, X)$  for  $q \in Q$  and  $X \in V'$ .
- (xiii) **reduce**( $q, t$ )  $:= \{[A \rightarrow \omega] \mid [A \rightarrow \omega \cdot \varepsilon] \in q\}$  for  $q \in Q$  and  $t \in T'$ .
- (xiv) Let  $\vdash$  be a binary relation on the set **Config** with two introduction rules:
$$\frac{q'' = \delta(q', t)}{(\alpha : q \rightarrow q', tz) \vdash (\alpha t : q \rightarrow q'', z)} \text{Shift} \quad \frac{[A \rightarrow \omega] \in \text{reduce}(q', t)}{(\alpha \omega : q \rightarrow q', tz) \vdash (\alpha A : q \rightarrow q'', tz)} \text{Reduce}(A \rightarrow \omega)$$
- (xv)  $q_f := \delta(\delta(q_0, S), \$)$ .
- (xvi)  $L(\text{LRA}(G)) \triangleq \{z \in T^* \mid (\varepsilon : q_0 \rightarrow q_0, z\$) \vdash^* (S\$ : q_0 \rightarrow q_f, \varepsilon)\}$ .  $L(G) = L(\text{LRA}(G))$ .

**Theorem 2.** Define  $\mathbf{LA} : \{(q, [A \rightarrow \omega]) \mid q \in Q, [A \rightarrow \omega \cdot \varepsilon] \in q\} \rightarrow \wp(T')$  by

$$\mathbf{LA}(q, [A \rightarrow \omega]) := \left\{ t \in T' \mid S' \xRightarrow[\text{rm } P']{*} \alpha Atz, \alpha \omega : q_0 \rightarrow q, \alpha \in V'^*, z \in T'^* \right\}. \quad (1)$$

Then, overriding **reduce** :  $Q \times T' \rightarrow \wp(P')$  of the LR(0) parser  $\text{LRA}(G)$  with

$$(q, t) \mapsto \{[A \rightarrow \omega] \mid [A \rightarrow \omega \cdot \varepsilon] \in q, t \in \mathbf{LA}(q, [A \rightarrow \omega])\}$$

yields an LALR(1) parser if there are no conflicts.

**Theorem 3.** Letting  $R!x \triangleq \{y \mid (x, y) \in R\}$ , define relations **Read** and **Follow** from the set

$$\{(p, A) \mid p \in Q, A \in N', \delta(p, A) \neq \emptyset\}$$

to the set  $T'$  inductively as follows:

$$\begin{array}{ll} \frac{\delta(\delta(p, A), t) \neq \emptyset}{t \in \mathbf{Read}!(p, A)} \text{DR} & \frac{\delta(p, A) = r \quad C \Rightarrow_{P'}^* \varepsilon}{\mathbf{Read}!(r, C) \subseteq \mathbf{Read}!(p, A)} \text{reads} \\ \frac{t \in \mathbf{Read}!(p, A)}{t \in \mathbf{Follow}!(p, A)} \text{Read} & \frac{[B \rightarrow \beta \cdot A\gamma] \in p \quad \beta : p' \rightarrow p \quad \gamma \Rightarrow_{P'}^* \varepsilon}{\mathbf{Follow}!(p', B) \subseteq \mathbf{Follow}!(p, A)} \text{includes} \end{array}$$

Then, whenever  $q \in Q$  and  $[A \rightarrow \omega \cdot \varepsilon] \in q$ , we can compute  $\mathbf{LA}(q, [A \rightarrow \omega])$  by

$$\mathbf{LA}(q, [A \rightarrow \omega]) = \{t \in T' \mid p \in Q, \omega : p \rightarrow q, \delta(p, A) \neq \emptyset, t \in \mathbf{Follow}!(p, A)\}. \quad (2)$$