

일차논리 메타정리

임기정

1 Generalized Weakening

$$\frac{\Gamma \subseteq \Gamma' \quad \Gamma \vdash \varphi}{\Gamma' [\eta] \vdash \varphi [\eta]}$$

1.1 \forall -case

- (1) $y \notin \text{FV}(\Gamma)$
- (2) $y \notin \text{FV}((\forall x)\varphi)$
- (3) $\Gamma \vdash \varphi[x := y]$
- (4) $\Gamma \subseteq \Gamma'$
- (G) $\Gamma' [\eta] \vdash (\forall x\varphi) [\eta]$

Proof. Let z be a fresh individual variable. Since $\Gamma[y := z] \subseteq \Gamma'[y := z]$,

$$\begin{aligned} & \Gamma' [\eta] \vdash (\forall x\varphi) [\eta] \\ \iff & \Gamma' [\eta] \vdash \forall x (\varphi [\eta]) \\ \iff & \Gamma' [y := z] [z := \eta(y); \eta] \vdash \forall x (\varphi [z := \eta(y); \eta]) \\ \iff & \Gamma' [y := z] [z := \eta(y); \eta] \vdash \varphi [z := \eta(y); \eta] [x := z] \\ \iff & \Gamma' [y := z] [z := \eta(y); \eta] \vdash \varphi [y := z; z := \eta(y); \eta] [x := z] \\ \iff & \Gamma' [y := z] [z := \eta(y); \eta] \vdash \varphi [y := z; z := \eta(y); \eta] [x := y [y := z; z := \eta(y); \eta]] \\ \iff & \Gamma' [y := z] [z := \eta(y); \eta] \vdash \varphi [x := y] [y := z; z := \eta(y); \eta] \\ \iff & \Gamma' [y := z] [y := z; z := \eta(y); \eta] \vdash \varphi [x := y] [y := z; z := \eta(y); \eta] \\ \iff & \begin{cases} \Gamma \subseteq \Gamma' [y := z], \\ \Gamma \vdash \varphi [x := y]. \end{cases} \end{aligned}$$

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