# 일차논리

## *임기정*

### 1 Weakening

(1) $y \notin \mathrm{FV}(\Gamma)$

(2) $y \notin \mathrm{FV}((\forall x)\,\varphi)$

(3) $\Gamma \vdash \varphi[x := y]$

(4) $\Gamma \subseteq \Gamma'$

(G) $\Gamma'[\eta] \vdash (\forall x \varphi)[\eta]$

*Proof.* Since $\Gamma[y := z] \subseteq \Gamma'[y := z]$,

$$
\begin{aligned}
&\Gamma'[\eta] \vdash (\forall x \varphi)[\eta] \\
&\quad \Longleftarrow\ \Gamma'[\eta] \vdash \forall x\,(\varphi[\eta]) \\
&\quad \Longleftarrow\ \Gamma'[y := z][z := \eta(y); y] \vdash \forall x\,(\varphi[z := \eta(y); \eta]) \\
&\quad \Longleftarrow\ \Gamma'[y := z][z := \eta(y); y] \vdash \varphi[z := \eta(y); \eta][x := z] \\
&\quad \Longleftarrow\ \Gamma'[y := z][z := \eta(y); y] \vdash \varphi[y := z; z := \eta(y); \eta][x := z] \\
&\quad \Longleftarrow\ \Gamma'[y := z][z := \eta(y); y] \vdash \varphi[y := z; z := \eta(y); \eta][x := y[y := z; z := \eta(y); \eta]] \\
&\quad \Longleftarrow\ \Gamma'[y := z][z := \eta(y); y] \vdash \varphi[x := y][y := z; z := \eta(y); \eta] \\
&\quad \Longleftarrow\ \Gamma'[y := z][y := z; z := \eta(y); y] \vdash \varphi[x := y][y := z; z := \eta(y); \eta] \\
&\quad \Longleftarrow\ \begin{cases} \Gamma \subseteq \Gamma'[y := z], \\ \Gamma \vdash \varphi[x := y]. \end{cases}
\end{aligned}
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