일차논리 메타정리

임기정

1 Generalized Weakening

Let η be a variable renaming. Then,

$$\frac{\Gamma \subseteq \Gamma' \qquad \Gamma \vdash \varphi}{\Gamma'\,[\eta] \vdash \varphi\,[\eta]}$$

1.1 $\forall$-case

(1) $y \notin \mathrm{FV}\,(\Gamma)$

(2) $y \notin \mathrm{FV}\,((\forall x)\,\varphi)$

(3) $\Gamma \vdash \varphi\,[x := y]$

(4) $\Gamma \subseteq \Gamma'$

(G) $\Gamma'\,[\eta] \vdash (\forall x\varphi)\,[\eta]$

Proof. Let z be a fresh individual variable. Since $\Gamma\,[y := z] \subseteq \Gamma'\,[y := z]$,

$$
\begin{aligned}
&\Gamma'\,[\eta] \vdash (\forall x\varphi)\,[\eta] \\
&\quad \Longleftarrow\ \Gamma'\,[\eta] \vdash \forall x\,(\varphi\,[\eta]) \\
&\quad \Longleftarrow\ \Gamma'\,[y := z]\,[z := \eta\,(y)\,;\eta] \vdash \forall x\,(\varphi\,[z := \eta\,(y)\,;\eta]) \\
&\quad \Longleftarrow\ \Gamma'\,[y := z]\,[z := \eta\,(y)\,;\eta] \vdash \varphi\,[z := \eta\,(y)\,;\eta]\,[x := z] \\
&\quad \Longleftarrow\ \Gamma'\,[y := z]\,[z := \eta\,(y)\,;\eta] \vdash \varphi\,[y := z; z := \eta\,(y)\,;\eta]\,[x := z] \\
&\quad \Longleftarrow\ \Gamma'\,[y := z]\,[z := \eta\,(y)\,;\eta] \vdash \varphi\,[y := z; z := \eta\,(y)\,;\eta]\,[x := y\,[y := z; z := \eta\,(y)\,;\eta]] \\
&\quad \Longleftarrow\ \Gamma'\,[y := z]\,[z := \eta\,(y)\,;\eta] \vdash \varphi\,[x := y]\,[y := z; z := \eta\,(y)\,;\eta] \\
&\quad \Longleftarrow\ \Gamma'\,[y := z]\,[y := z; z := \eta\,(y)\,;\eta] \vdash \varphi\,[x := y]\,[y := z; z := \eta\,(y)\,;\eta] \\
&\quad \Longleftarrow\ \begin{cases} \Gamma \subseteq \Gamma'\,[y := z]\,, \\ \Gamma \vdash \varphi\,[x := y]\,. \end{cases}
\end{aligned}
$$

$\square$