일차논리 메타정리

임기정

1 Generalized Weakening

Let η be a variable renaming – that is, a function which maps variables to variables. Then,

$$\frac{\Gamma \subseteq \Gamma' \qquad \Gamma \vdash \varphi}{\Gamma' [\eta] \vdash \varphi [\eta]}$$

1.1 $\forall$-case

(1) $y \notin \mathrm{FV}(\Gamma)$

(2) $y \notin \mathrm{FV}((\forall x)\, \varphi)$

(3) $\Gamma \vdash \varphi [x := y]$

(4) $\Gamma \subseteq \Gamma'$

(G) $\Gamma' [\eta] \vdash (\forall x \varphi) [\eta]$

Proof. Let z be a fresh variable. Since $\Gamma [y := z] \subseteq \Gamma' [y := z]$,

$$\begin{aligned}
&\Gamma' [\eta] \vdash (\forall x \varphi) [\eta] \\
&\quad \Longleftarrow \ \Gamma' [\eta] \vdash \forall x (\varphi [\eta]) \\
&\quad \Longleftarrow \ \Gamma' [y := z] [z := \eta (y) ; \eta] \vdash \forall x (\varphi [z := \eta (y) ; \eta]) \\
&\quad \Longleftarrow \ \Gamma' [y := z] [z := \eta (y) ; \eta] \vdash \varphi [z := \eta (y) ; \eta] [x := z] \\
&\quad \Longleftarrow \ \Gamma' [y := z] [z := \eta (y) ; \eta] \vdash \varphi [y := z; z := \eta (y) ; \eta] [x := z] \\
&\quad \Longleftarrow \ \Gamma' [y := z] [z := \eta (y) ; \eta] \vdash \varphi [y := z; z := \eta (y) ; \eta] [x := y [y := z; z := \eta (y) ; \eta]] \\
&\quad \Longleftarrow \ \Gamma' [y := z] [z := \eta (y) ; \eta] \vdash \varphi [x := y] [y := z; z := \eta (y) ; \eta] \\
&\quad \Longleftarrow \ \Gamma' [y := z] [y := z; z := \eta (y) ; \eta] \vdash \varphi [x := y] [y := z; z := \eta (y) ; \eta] \\
&\quad \Longleftarrow \ \begin{cases} \Gamma \subseteq \Gamma' [y := z] , \\ \Gamma \vdash \varphi [x := y] . \end{cases}
\end{aligned}$$

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