

Keppelin

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Problem	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Opened															
Akulyat															
Read															
Solved															
Written															
Aarzhantsev															
Read															
Solved															
Written															
KiKOS															
Read															
Solved															
Written															
AC															

## Data structures (1)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null\_type. Time:  $\mathcal{O}(\log N)$

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
    Tree<int> t, t2; t.insert(8);
    auto it = t.insert(10).first;
    assert(it == t.lower_bound(9));
    assert(t.order_of_key(10) == 1);
    assert(t.order_of_key(11) == 2);
    assert(*t.find_by_order(0) == 8);
    t.join(t2); // assuming T< T2 or T> T2, merge t2 into t
}
```

HashMap.h

Description: Hash map with mostly the same API as unordered\_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
    const uint64_t C = 11(4e18 * acos(0)) | 71;
    ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};
__gnu_pbds::gp_hash_table<ll,int,chash> h({}, {}, {}, {}, {}, {1<<16});
```

SegmentTree.h

```
v += (1 << MAXLOG); t[v] = x;
while (v != 1) {v >>= 1; t[v] = min(t[2 * v], t[2 * v + 1]);}

l += (1 << MAXLOG); r += (1 << MAXLOG); int res = inf;
while (l != r) {
    if (l & 1) res = min(res, t[l++]);
    if (!(r & 1)) res = min(res, t[r--]);
    l >>= 1, r >>= 1;
}
res = min(res, t[1]);

// segtree from top:
int tm = (tl + tr) / 2;
```

template.h

37 lines

```
#include <bits/stdc++.h>
using namespace std;

typedef long long ll;

#ifdef DEBUG
#define var(x) cerr << #x << ": " << x << '\n';
#define range(a, b) cerr << #a << ", " << #b << ": "; for (auto _it = a; _it != b; ++_it) cerr << *_it << ' '; cerr << '\n';
#else
#define cerr if (false) cerr
#define var(x)
#define range(a, b)
#endif

#define pii pair<int, int>
#define T(x, i) get<i>(x)
#define F first
#define S second
#define all(v) v.begin(), v.end()
#define forn(i, n) for (int i = 0; i < n; i++)

const int MAXN = 1e6 + 10;
int n;

void solve() {

}

signed main() {
    #ifdef DEBUG
        freopen("input.in", "r", stdin);
        freopen("output.out", "w", stdout);
    #endif
    ios_base::sync_with_stdio(0); cin.tie(0);
    while (cin >> n) solve();
    cerr << "Runtime is: " << clock() * 1.0 / CLOCKS_PER_SEC << endl;
}
```

dsu.h

6 lines

```
if (pos <= tm) upd(2 * v, tl, tm, pos, x); else upd(2 * v + 1,
    tm + 1, tr, pos, x);
t[v] = min(t[2 * v], t[2 * v + 1]);
if (r < tl || l > tr) return 0;
if (l <= tl && tr <= r) return t[v];
```

dsu.h

6 lines

```
// for dcp offline: remember updates and perform rollback
void init() {for (int i = 0; i < n; i++) p[i] = i; sz[i] = 1;}
int get(int x) {if (p[x] == x) return x; return p[x] = get(p[x
    ]);}
```

a = get(a); b = get(b); (a == b) return;

if (sz[a] > sz[b]) swap(a, b); p[a] = b; sz[b] += sz[a];

FenwickTree.h

5 lines

```
for (int i = x + 1; i < MAXN; i += i & -i) f[i] += val;
for (int i = x + 1; i > 0; i -= i & -i) ans += f[i];
int pos = 0; for (int i = (1 << MAXLOG); i > 0; i >>= 1)
if (pos + i < MAXN && f[pos + i] <= x) {
    x -= f[pos + i]; pos += i;}
```

sparsetable.h

7 lines

```
lg[1] = 0; for (int i = 2; i < MAXN; i++) lg[i] = lg[i >> 1] +
    1;
```

```
for (int i = 0; i < n; i++) table[0][i] = a[i];
for (int j = 1; j < MAXLOG; j++) for (int i = 0; i + (1 << j)
    <= n; i++)
table[j][i] = min(table[j - 1][i], table[j - 1][(i + (1 << (j - 1)))]);
```

```
int lvl = lg[r - 1 + 1];
return min(table[lvl][l], table[lvl][r - (1 << lvl) + 1]);
```

disjoint.h d41d8c, 20 lines

```
lg[0] = MAXLOG - 1; for (int i = 1; i < MAXN; i++) lg[i] = lg[i
    >> 1] - 1;
void build_disjoint(int level=0, int tl=0, int tr=(1 << MAXLOG)
    - 1) {
    int tm = (tl + tr) / 2;int cur = inf;
    for (int i = tm; i >= tl; i--) {
        table[level][i] = min(cur, a[i]);
        cur = table[level][i];
    }
    cur = inf;
    for (int i = tm + 1; i <= tr; i++) {
        table[level][i] = min(cur, a[i]);
        cur = table[level][i];
    }
    if (tl == tr) return;
    build_disjoint(level + 1, tl, tm);
    build_disjoint(level + 1, tm + 1, tr);
}
int query_disjoint(int l, int r) {
    int lvl = lg[r ^ 1];
    return min(table[lvl][l], table[lvl][r]);
}
```

hld.h  
**Description:** Heavy-light decomposition. To remove extra log, precache the answer for each path on the prefix, then you query prefixes in O(1) and one segment in O(log).

```
const int N = 1 << 17;
// par - parent, heavy - heavy child, h - depth
int par[N], heavy[N], h[N], sz[N];
int root[N], pos[N];
int n;
vector<int> g[N];

void dfs(int v) {
    sz[v] = 1; int mx = 0;
    for (int i = 0; i < g[v].size(); ++i) {
        int to = g[v][i];
        if (to == par[v]) {
            // remove the edge to the parent
            swap(g[v][i], g[v].back());
            g[v].pop_back(); --i; continue;
        }
        par[to] = v; h[to] = h[v] + 1; sz[v] += sz[to];
        dfs(to);
        if (sz[to] > mx) {
            heavy[v] = to, mx = sz[to];
            swap(g[v][0], g[v][i]); // with this swap we can
                query subtrees too.
        }
    }
}
```

```
// op(l, r) - update the answer on the path from l to r
int path(int u, int v) {
    int sum = 0;
    for (; root[u] != root[v]; v = par[root[v]]) {
```

```
        if (h[root[u]] > h[root[v]]) swap(u, v);
        sum += segtree.get(pos[root[v]], pos[v]);
    }
    if (h[u] > h[v]) swap(u, v);
    sum += segtree.get(pos[u], pos[v]); return sum;
}

int subtree(int v) { return segtree.get(pos[v], pos[v] + sz[v]
    - 1); }

void init() {
    memset(heavy, -1, sizeof(heavy[0]) * n);
    par[0] = -1;
    h[0] = 0;
    dfs(0);
    for (int i = 0, cpos = 0; i < n; i++) {
        if (par[i] == -1 || heavy[par[i]] != i) {
            for (int j = i; j != -1; j = heavy[j]) {
                root[j] = i;
                pos[j] = cpos++;
                segtree.upd(pos[j], val[j]);
            }
        }
    }
}
```

MoQueries.h  
**Description:** Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change step to add/remove the edge (a, c) and remove the initial add call (but keep in).  
**Time:**  $\mathcal{O}(N\sqrt{Q})$

```
void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
vi s(sz(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
iota(all(s), 0);
sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
// also can group by blocks, move right via add, move left sqrt
    steps and then rollback()
for (int qi : s) {
    pii q = Q[qi];
    while (L > q.first) add(--L, 0); while (R < q.second) add(R
        ++, 1);
    while (L < q.first) del(L++, 0); while (R > q.second) del(--R
        , 1);
    res[qi] = calc();}

// for trees; if problem on vertices: also add lca
if (tin[a] > tin[b]) swap(a, b);
if (tin[a] <= tin[b] && tin[b] < tin[a] + sz[a]) l = tin[a], r
    = tin[b];
else l = tout[a], r = tin[b];
```

Graph (2)

2.1 Fundamentals

BellmanFord.h  
**Description:** Calculates shortest paths from s in a graph that might have negative edge weights.  
**Time:**  $\mathcal{O}(VE)$

```
const ll inf = LLONG_MAX;
```

```
vector<pair<int, int>> g[MAXN];
int dist[MAXN];

void bellmanFord(int s) {
    dist[s] = 0; // other dists = INF
    // if possible negative cycles: for (i = 0..n) for(edge)
        relax(edge.to), cycle if relax is success on (n+1)th
        step
    queue<int> q; vector<int> in_queue(n);
    for (q.push(s); q.size(); q.pop()) {
        int v = q.front(); in_queue[v] = 0;
        for (auto [to, w] : g[v]) {
            if (dist[to] > dist[v] + w) {
                dist[to] = dist[v] + w; // if last iteration and
                    detected neg. cycle -> put -inf.
                if (!in_queue[to]) q.push(to);}}}
    // to determine if vertex has negative dist: for (i = 0..n)
        for(edge) if dist[from] = -inf {dist[to] = -inf;}
```

FloydWarshall.h  
**Description:** Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where  $m[i][j]$  = inf if i and j are not adjacent. As output,  $m[i][j]$  is set to the shortest distance between i and j, inf if no path, or -inf if the path goes through a negative-weight cycle.  
**Time:**  $\mathcal{O}(N^3)$

```
const ll inf = 1LL << 62;
void floydWarshall(vector<vector<ll>>& m) {
    int n = sz(m);
    rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
    rep(k,0,n) rep(i,0,n) rep(j,0,n)
        if (m[i][k] != inf && m[k][j] != inf) {
            auto newDist = max(m[i][k] + m[k][j], -inf);
            m[i][j] = min(m[i][j], newDist);
        }
    rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
        if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
}
```

TopoSort.h  
**Description:** disassemble graph. see 2-sat for dfs-based topsort  
**Time:**  $\mathcal{O}(|V| + |E|)$

```
for (auto& li : gr) for (int x : li) indeg[x]++;
queue<int> q; // use priority-queue for lexic. largest ans.
rep(i,0,sz(gr)) if (indeg[i] == 0) q.push(i);
while (!q.empty()) {
    int i = q.front(); // top() for priority queue
    ret.push_back(i);
    q.pop();
    for (int x : gr[i])
        if (--indeg[x] == 0) q.push(x);
}
```

2.2 Network flow

MinCostMaxFlow.h  
**Description:** Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.  
**Time:**  $\mathcal{O}(FE \log(V))$  where F is max flow.  $\mathcal{O}(VE)$  for setpi.

```
#include <bits/extc++.h>

const ll INF = numeric_limits<ll>::max() / 4;

struct MCMF {
    struct edge {
        int from, to, rev;
```

```
    ll cap, cost, flow;
};
int N;
vector<vector<edge>> ed;
vi seen;
vector<ll> dist, pi;
vector<edge*> par;

MCMF(int N) : N(N), ed(N), seen(N), dist(N), pi(N), par(N) {}

void addEdge(int from, int to, ll cap, ll cost) {
    if (from == to) return;
    ed[from].push_back(edge{ from,to,sz(ed[to]),cap,cost,0 });
    ed[to].push_back(edge{ to,from,sz(ed[from])-1,0,-cost,0 });
}

void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;

    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({ 0, s });

    while (!q.empty()) {
        s = q.top().second; q.pop();
        seen[s] = 1; di = dist[s] + pi[s];
        for (edge& e : ed[s]) if (!seen[e.to]) {
            ll val = di - pi[e.to] + e.cost;
            if (e.cap - e.flow > 0 && val < dist[e.to]) {
                dist[e.to] = val;
                par[e.to] = &e;
                if (its[e.to] == q.end())
                    its[e.to] = q.push({ -dist[e.to], e.to });
                else
                    q.modify(its[e.to], { -dist[e.to], e.to });
            }
        }
        rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
    }

    pair<ll, ll> maxflow(int s, int t) {
        ll totflow = 0, totcost = 0;
        while (path(s), seen[t]) {
            ll fl = INF;
            for (edge* x = par[t]; x; x = par[x->from])
                fl = min(fl, x->cap - x->flow);

            totflow += fl;
            for (edge* x = par[t]; x; x = par[x->from]) {
                x->flow += fl;
                ed[x->to][x->rev].flow -= fl;
            }
        }
        rep(i,0,N) for(edge& e : ed[i]) totcost += e.cost * e.flow;
        return {totflow, totcost/2};
    }
}
```

```
// If some costs can be negative, call this before maxflow:
void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
        rep(i,0,N) if (pi[i] != INF)
            for (edge& e : ed[i]) if (e.cap)
                if ((v = pi[i] + e.cost) < pi[e.to])
                    pi[e.to] = v, ch = 1;
}
```

```
    assert(it >= 0); // negative cost cycle
}
};

Dinic.h
Description: Flow algorithm with complexity  $O(VE \log U)$  where  $U = \max|\text{cap}|$ .  $O(\min(E^{1/2}, V^{2/3})E)$  if  $U = 1$ ;  $O(\sqrt{VE})$  for bipartite matching.
d41d8c, 42 lines

struct Dinic {
    struct Edge {
        int to, rev;
        ll c, oc;
        ll flow() { return max(oc - c, 0LL); } // if you need flows
    };
    vi lvl, ptr, q;
    vector<vector<Edge>> adj;
    Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
    void addEdge(int a, int b, ll c, ll rcap = 0) {
        adj[a].push_back({b, sz(adj[b]), c, c});
        adj[b].push_back({a, sz(adj[a]) - 1, rcap, rcap});
    }
    ll dfs(int v, int t, ll f) {
        if (v == t || !f) return f;
        for (int& i = ptr[v]; i < sz(adj[v]); i++) {
            Edge& e = adj[v][i];
            if (lvl[e.to] == lvl[v] + 1)
                if (ll p = dfs(e.to, t, min(f, e.c))) {
                    e.c -= p, adj[e.to][e.rev].c += p;
                    return p;
                }
        }
        return 0;
    }
    ll calc(int s, int t) {
        ll flow = 0; q[0] = s;
        rep(L,0,31) do { // 'int L=30' maybe faster for random data
            lvl = ptr = vi(sz(q));
            int qi = 0, qe = lvl[s] = 1;
            while (qi < qe && !lvl[t]) {
                int v = q[qi++];
                for (Edge e : adj[v])
                    if (!lvl[e.to] && e.c >> (30 - L))
                        q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
            }
            while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
        } while (lvl[t]);
        return flow;
    }
    bool leftOfMinCut(int a) { return lvl[a] != 0; }
};
```

GlobalMinCut.h  
Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.  
Time:  $\mathcal{O}(V^3)$

```
pair<int, vi> globalMinCut(vector<vi> mat) {
    pair<int, vi> best = {INT_MAX, {}};
    int n = sz(mat);
    vector<vi> co(n);
    rep(i,0,n) co[i] = {i};
    rep(ph,1,n) {
        vi w = mat[0];
        size_t s = 0, t = 0;
        rep(it,0,n-ph) { //  $O(V^2) \rightarrow O(E \log V)$  with prio. queue
            w[t] = INT_MIN;
            s = t, t = max_element(all(w)) - w.begin();
            rep(i,0,n) w[i] += mat[t][i];
        }
    }
}
```

```
    }
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), all(co[t]));
    rep(i,0,n) mat[s][i] += mat[t][i];
    rep(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN;
}
return best;
}
```

GomoryHu.h  
Description: Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.  
Time:  $\mathcal{O}(V)$  Flow Computations

```
"PushRelabel.h"
d41d8c, 13 lines

typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
    vector<Edge> tree;
    vi par(N);
    rep(i,1,N) {
        PushRelabel D(N); // Dinic also works
        for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
        tree.push_back({i, par[i], D.calc(i, par[i])});
        rep(j,i+1,N)
            if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
    }
    return tree;
}
```

FlowWithDemands.h  
Description: Flow with demands  $d(e) \leq f(e) \leq c(e)$ . Add new source  $s'$  and sink  $t'$  to the graph.

- $c'((s', v)) = \sum_{u \in V} d((u, v))$  for each edge  $(s', v)$ .
- $c'((v, t')) = \sum_{w \in V} d((v, w))$  for each edge  $(v, t')$ .
- $c'((u, v)) = c((u, v)) - d((u, v))$  for each edge  $(u, v)$  in the old network.
- $c'((t, s)) = 0$

A flow with the value  $d(e)$ , that originally flowed along the path  $s - \dots - u - v - \dots - t$  can now take the new path  $s' - v - \dots - t - s - \dots - u - t'$ .

## 2.3 Matching

kuhn.h  
d41d8c, 24 lines

```
int timer = 1;
bool find(int j, vector<vi>& g, vi& btoa, vi& vis) {
    if (btoa[j] == -1) return 1;
    vis[j] = timer; int di = btoa[j];
    for (int e : g[di])
        if (vis[e] != timer && find(e, g, btoa, vis)) {
            btoa[e] = di;
            return 1;
        }
    return 0;
}

int dfsMatching(vector<vi>& g, vi& btoa) {
    vi vis;
    // do greedy init to speedup
    rep(i,0,sz(g)) {
        timer++;
        for (int j : g[i])
            if (find(j, g, btoa, vis)) {
                btoa[j] = i;
                break;
            }
    }
}
```

```
    return sz(btoa) - (int)count(all(btoa), -1);
}
```

MinimumVertexCover.h

**Description:** Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h"	d41d8c, 20 lines
-----------------	------------------

```
vi cover(vector<vi>& g, int n, int m) {
    vi match(m, -1);
    int res = dfsMatching(g, match);
    vector<bool> lfound(n, true), seen(m);
    for (int it : match) if (it != -1) lfound[it] = false;
    vi q, cover;
    rep(i,0,n) if (lfound[i]) q.push_back(i);
    while (!q.empty()) {
        int i = q.back(); q.pop_back();
        lfound[i] = 1;
        for (int e : g[i]) if (!seen[e] && match[e] != -1) {
            seen[e] = true;
            q.push_back(match[e]);
        }
    }
    rep(i,0,n) if (!lfound[i]) cover.push_back(i);
    rep(i,0,m) if (seen[i]) cover.push_back(n+i);
    assert(sz(cover) == res);
    return cover;
}
```

hungarian.h

**Description:** Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires  $N \leq M$ .  
**Time:**  $\mathcal{O}(N^2M)$

	d41d8c, 31 lines
--	------------------

```
pair<int, vi> hungarian(const vector<vi> &a) {
    if (a.empty()) return {0, {}};
    int n = sz(a) + 1, m = sz(a[0]) + 1;
    vi u(n), v(m), p(m), ans(n - 1);
    rep(i,1,n) {
        p[0] = i;
        int j0 = 0; // add "dummy" worker 0
        vi dist(m, INT_MAX), pre(m, -1);
        vector<bool> done(m + 1);
        do { // dijkstra
            done[j0] = true;
            int i0 = p[j0], j1, delta = INT_MAX;
            rep(j,1,m) if (!done[j]) {
                auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
                if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
                if (dist[j] < delta) delta = dist[j], j1 = j;
            }
            rep(j,0,m) {
                if (done[j]) u[p[j]] += delta, v[j] -= delta;
                else dist[j] -= delta;
            }
            j0 = j1;
        } while (p[j0]);
        while (j0) { // update alternating path
            int j1 = pre[j0];
            p[j0] = p[j1], j0 = j1;
        }
    }
    rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
    return {-v[0], ans}; // min cost
}
```

GeneralMatching.h

**Description:** Matching for general graphs. Fails with probability  $N/mod$ .  
**Time:**  $\mathcal{O}(N^3)$

"../numerical/MatrixInverse-mod.h"	d41d8c, 40 lines
------------------------------------	------------------

```
vector<pii> generalMatching(int N, vector<pii>& ed) {
    vector<vector<ll>> mat(N, vector<ll>(N)), A;
    for (pii pa : ed) {
        int a = pa.first, b = pa.second, r = rand() % mod;
        mat[a][b] = r, mat[b][a] = (mod - r) % mod;
    }

    int r = matInv(A = mat), M = 2*N - r, fi, fj;
    assert(r % 2 == 0);

    if (M != N) do {
        mat.resize(M, vector<ll>(M));
        rep(i,0,N) {
            mat[i].resize(M);
            rep(j,N,M) {
                int r = rand() % mod;
                mat[i][j] = r, mat[j][i] = (mod - r) % mod;
            }
        } while (matInv(A = mat) != M);

    vi has(M, 1); vector<pii> ret;
    rep(it,0,M/2) {
        rep(i,0,M) if (has[i])
            rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
                fi = i; fj = j; goto done;
            } assert(0); done:
        if (fj < N) ret.emplace_back(fi, fj);
        has[fi] = has[fj] = 0;
        rep(sw,0,2) {
            ll a = modpow(A[fi][fj], mod-2);
            rep(i,0,M) if (has[i] && A[i][fj]) {
                ll b = A[i][fj] * a % mod;
                rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
            }
            swap(fi,fj);
        }
    }
    return ret;
}
```

2.4 DFS algorithms

BiconnectedComponents.h

**Description:** Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.  
**Usage:** int eid = 0; ed.resize(N);  
for each edge (a,b) {  
ed[a].emplace\_back(b, eid);  
ed[b].emplace\_back(a, eid++); }  
bicomps([&](const vi& edgelist) {...});  
**Time:**  $\mathcal{O}(E + V)$

	d41d8c, 32 lines
--	------------------

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
    int me = num[at] = ++Time, top = me;
    for (auto [y, e] : ed[at]) if (e != par){
        if (num[y]) {
            top = min(top, num[y]);
            if (num[y] < me)
```

```
        st.push_back(e);
    } else {
        int si = sz(st);
        int up = dfs(y, e, f);
        top = min(top, up);
        if (up == me) {
            st.push_back(e);
            f(vi(st.begin() + si, st.end()));
            st.resize(si);
        }
        else if (up < me) st.push_back(e);
        else { /* e is a bridge */ }
    }
}
return top;
}
```

template<class F>

```
void bicomps(F f) {
    num.assign(sz(ed), 0);
    rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f);
}
```

2sat.h

**Description:** Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem,

	d41d8c, 25 lines
--	------------------

```
vector<int> g[MAXN]; vector<int> gr[MAXN]; vector<int> topsort;
int used[MAXN]; int color[MAXN];
void make_or(int v1, int v2) {
    g[v1 ^ 1].push_back(v2); gr[v2].push_back(v1 ^ 1);
    g[v2 ^ 1].push_back(v1); gr[v1].push_back(v2 ^ 1);
}

void dfs(int v) {
    if (used[v]) return; used[v] = 1;
    for (auto to : g[v]) dfs(to);
    topsort.push_back(v);
}

void assign_color(int v, int c /* increment c globally each dfs iteration */) {
    if (color[v]) return; color[v] = c;
    for (auto to : gr[v]) assign_color(to, c);
}

dfs(/* all */); reverse(topsort.begin(), topsort.end());
assign_color(/*all*/);
```

```
for (int i : vars) {
    if (color[i] == color[i ^ 1]) {}// no solution
    else ans[i] = (color[i] > color[i ^ 1] ? 1 : 0);
}
```

EulerWalk.h

	d41d8c, 15 lines
--	------------------

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
    int n = sz(gr);
    vi D(n), its(n), eu(nedges), ret, s = {src};
    D[src]++; // to allow Euler paths, not just cycles
    while (!s.empty()) {
        int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
        if (it == end){ ret.push_back(x); s.pop_back(); continue; }
        tie(y, e) = gr[x][it++];
        if (!eu[e]) {
            D[x]--, D[y]++;
            eu[e] = 1; s.push_back(y);
        }
    }
    for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
```

```
    return {ret.rbegin(), ret.rend()};
}
```

## 2.5 Coloring

### EdgeColoring.h

**Description:** Given a simple, undirected graph with max degree  $D$ , computes a  $(D + 1)$ -coloring of the edges such that no neighboring edges share a color. ( $D$ -coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

**Time:**  $\mathcal{O}(NM)$

```
vi edgeColoring(int N, vector<pii> eds) {
    vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
    for (pii e : eds) ++cc[e.first], ++cc[e.second];
    int u, v, ncols = *max_element(all(cc)) + 1;
    vector<vi> adj(N, vi(ncols, -1));
    for (pii e : eds) {
        tie(u, v) = e;
        fan[0] = v;
        loc.assign(ncols, 0);
        int at = u, end = u, d, c = free[u], ind = 0, i = 0;
        while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
            loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
        cc[loc[d]] = c;
        for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
            swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
        while (adj[fan[i]][d] != -1) {
            int left = fan[i], right = fan[++i], e = cc[i];
            adj[u][e] = left;
            adj[left][e] = u;
            adj[right][e] = -1;
            free[right] = e;
        }
        adj[u][d] = fan[i];
        adj[fan[i]][d] = u;
        for (int y : {fan[0], u, end})
            for (int& z = free[y] = 0; adj[y][z] != -1; z++);
    }
    rep(i, 0, sz(eds))
        for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
    return ret;
}
```

## 2.6 Heuristics

### MaximalCliques.h

**Description:** Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

**Time:**  $\mathcal{O}\left(3^{n/3}\right)$ , much faster for sparse graphs

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
    if (!P.any()) { if (!X.any()) f(R); return; }
    auto q = (P | X)._Find_first();
    auto cands = P & ~eds[q];
    rep(i, 0, sz(eds)) if (cands[i]) {
        R[i] = 1;
        cliques(eds, f, P & eds[i], X & eds[i], R);
        R[i] = P[i] = 0; X[i] = 1;
    }
}
```

### MaximumClique.h

**Description:** Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

**Time:** Runs in about 1s for n=155 and worst case random graphs (p=.90).  
Runs faster for sparse graphs.

```
d41d8c, 49 lines
typedef vector<bitset<200>> vb;
struct Maxclique {
    double limit=0.025, pk=0;
    struct Vertex { int i, d=0; };
    typedef vector<Vertex> vv;
    vb e;
    vv V;
    vector<vi> C;
    vi qmax, q, S, old;
    void init(vv& r) {
        for (auto& v : r) v.d = 0;
        for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
        sort(all(r), [](auto a, auto b) { return a.d > b.d; });
        int mxD = r[0].d;
        rep(i, 0, sz(r)) r[i].d = min(i, mxD) + 1;
    }
    void expand(vv& R, int lev = 1) {
        S[lev] += S[lev - 1] - old[lev];
        old[lev] = S[lev - 1];
        while (sz(R)) {
            if (sz(q) + R.back().d <= sz(qmax)) return;
            q.push_back(R.back().i);
            vv T;
            for(auto v:R) if (e[R.back().i][v.i]) T.push_back({v.i});
            if (sz(T)) {
                if (S[lev]++ / ++pk < limit) init(T);
                int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
                C[1].clear(), C[2].clear();
                for (auto v : T) {
                    int k = 1;
                    auto f = [&](int i) { return e[v.i][i]; };
                    while (any_of(all(C[k]), f)) k++;
                    if (k > mxk) mxk = k, C[mxk + 1].clear();
                    if (k < mnk) T[j++].i = v.i;
                    C[k].push_back(v.i);
                }
                if (j > 0) T[j - 1].d = 0;
                rep(k, mnk, mxk + 1) for (int i : C[k])
                    T[j].i = i, T[j++].d = k;
                expand(T, lev + 1);
            } else if (sz(q) > sz(qmax)) qmax = q;
            q.pop_back(), R.pop_back();
        }
    }
    vi maxClique() { init(V), expand(V); return qmax; }
    Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {
        rep(i, 0, sz(e)) V.push_back({i});
    }
};
```

### MaximumIndependentSet.h

**Description:** To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertex-Cover.

## 2.7 Trees

### BinaryLifting.h

**Description:** Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.

**Time:** construction  $\mathcal{O}(N \log N)$ , queries  $\mathcal{O}(\log N)$

```
d41d8c, 25 lines
vector<vi> treeJump(vi& P) {
    int on = 1, d = 1;
    while (on < sz(P)) on *= 2, d++;
    vector<vi> jmp(d, P);
```

```
    rep(i, 1, d) rep(j, 0, sz(P))
        jmp[i][j] = jmp[i-1][jmp[i-1][j]];
    return jmp;
}

int jmp(vector<vi>& tbl, int nod, int steps) {
    rep(i, 0, sz(tbl))
        if (steps & (1 << i)) nod = tbl[i][nod];
    return nod;
}

int lca(vector<vi>& tbl, vi& depth, int a, int b) {
    if (depth[a] < depth[b]) swap(a, b);
    a = jmp(tbl, a, depth[a] - depth[b]);
    if (a == b) return a;
    for (int i = sz(tbl); i--;) {
        int c = tbl[i][a], d = tbl[i][b];
        if (c != d) a = c, b = d;
    }
    return tbl[0][a];
}

LCA.h
Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.
Time:  $\mathcal{O}(N \log N + Q)$ 
"../data-structures/RMQ.h" d41d8c, 11 lines
void dfs(int v, int p = -1, int h = 0) {
    tin[v] = path.size();
    path.push_back({d[v], v});
    for (int y : g[v]) if (y != par) {
        dfs(y, v, h + 1);
        path.push_back({d[v], v});
    }
    tout[v] = path.size() - 1;
}
int lca(int a, int b){return min(min(tin[a], tin[b]), max(tout[a], tout[b])).second;}
int dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];}

CompressTree.h
Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most  $|S| - 1$ ) pairwise LCA's and compressing edges. Returns a list of (par, orig-index) representing a tree rooted at 0. The root points to itself.
Time:  $\mathcal{O}(|S| \log |S|)$ 
"LCA.h" d41d8c, 21 lines
typedef vector<pair<int, int>> vpi;
vpi compressTree(LCA& lca, const vi& subset) {
    static vi rev; rev.resize(sz(lca.time));
    vi li = subset, &T = lca.time;
    auto cmp = [&](int a, int b) { return T[a] < T[b]; };
    sort(all(li), cmp);
    int m = sz(li)-1;
    rep(i, 0, m) {
        int a = li[i], b = li[i+1];
        li.push_back(lca.lca(a, b));
    }
    sort(all(li), cmp);
    li.erase(unique(all(li)), li.end());
    rep(i, 0, sz(li)) rev[li[i]] = i;
    vpi ret = {pii(0, li[0])};
    rep(i, 0, sz(li)-1) {
        int a = li[i], b = li[i+1];
        ret.emplace_back(rev[lca.lca(a, b)], b);
    }
    return ret;
}
```

```
}
```

LinkCutTree.h

**Description:** Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.

**Time:** All operations take amortized  $\mathcal{O}(\log N)$ . d41d8c, 90 lines

```
struct Node { // Splay tree. Root's pp contains tree's parent.
    Node *p = 0, *pp = 0, *c[2];
    bool flip = 0;
    Node() { c[0] = c[1] = 0; fix(); }
    void fix() {
        if (c[0]) c[0]->p = this;
        if (c[1]) c[1]->p = this;
        // (+ update sum of subtree elements etc. if wanted)
    }
    void pushFlip() {
        if (!flip) return;
        flip = 0; swap(c[0], c[1]);
        if (c[0]) c[0]->flip ^= 1;
        if (c[1]) c[1]->flip ^= 1;
    }
    int up() { return p ? p->c[1] == this : -1; }
    void rot(int i, int b) {
        int h = i ^ b;
        Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y : x;
        if ((y->p = p)) p->c[up()] = y;
        c[i] = z->c[i ^ 1];
        if (b < 2) {
            x->c[h] = y->c[h ^ 1];
            y->c[h ^ 1] = x;
        }
        z->c[i ^ 1] = this;
        fix(); x->fix(); y->fix();
        if (p) p->fix();
        swap(pp, y->pp);
    }
    void splay() {
        for (pushFlip(); p; ) {
            if (p->p) p->p->pushFlip();
            p->pushFlip(); pushFlip();
            int c1 = up(), c2 = p->up();
            if (c2 == -1) p->rot(c1, 2);
            else p->p->rot(c2, c1 != c2);
        }
    }
    Node* first() {
        pushFlip();
        return c[0] ? c[0]->first() : (splay(), this);
    }
};

struct LinkCut {
    vector<Node> node;
    LinkCut(int N) : node(N) {}

    void link(int u, int v) { // add an edge (u, v)
        assert(!connected(u, v));
        makeRoot(&node[u]);
        node[u].pp = &node[v];
    }
    void cut(int u, int v) { // remove an edge (u, v)
        Node *x = &node[u], *top = &node[v];
        makeRoot(top); x->splay();
        assert(top == (x->pp ? x->c[0]));
        if (x->pp) x->pp = 0;
        else {
            x->c[0] = top->p = 0;
        }
    }
};
```

```
        x->fix();
    }
}
bool connected(int u, int v) { // are u, v in the same tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
}
void makeRoot(Node* u) {
    access(u);
    u->splay();
    if (u->c[0]) {
        u->c[0]->p = 0;
        u->c[0]->flip ^= 1;
        u->c[0]->pp = u;
        u->c[0] = 0;
        u->fix();
    }
}
Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
        pp->splay(); u->pp = 0;
        if (pp->c[1]) {
            pp->c[1]->p = 0; pp->c[1]->pp = pp; }
        pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
}
};
```

DirectedMST.h

**Description:** Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

**Time:**  $\mathcal{O}(E \log V)$

"/data-structures/UnionFindRollback.h" d41d8c, 60 lines

```
struct Edge { int a, b; ll w; };
struct Node {
    Edge key;
    Node *l, *r;
    ll delta;
    void prop() {
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    }
    Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
    if (!a || !b) return a ? b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
    RollbackUF uf(n);
    vector<Node*> heap(n);
    for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node(e));
    ll res = 0;
    vi seen(n, -1), path(n), par(n);
    seen[r] = r;
    vector<Edge> Q(n), in(n, {-1,-1}), comp;
    deque<tuple<int, int, vector<Edge>>> cys;
    rep(s,0,n) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
```

```
            if (!heap[u]) return {-1,{};};
            Edge e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
            Q[qi] = e, path[qi++] = u, seen[u] = s;
            res += e.w, u = uf.find(e.a);
            if (seen[u] == s) {
                Node* cyc = 0;
                int end = qi, time = uf.time();
                do cyc = merge(cyc, heap[w = path[--qi]]);
                while (uf.join(u, w));
                u = uf.find(u), heap[u] = cyc, seen[u] = -1;
                cys.push_front({u, time, {Q[qi], &Q[end]}});
            }
        }
        rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
    }

    for (auto& [u,t,comp] : cys) { // restore sol (optional)
        uf.rollback(t);
        Edge inEdge = in[u];
        for (auto& e : comp) in[uf.find(e.b)] = e;
        in[uf.find(inEdge.b)] = inEdge;
    }
    rep(i,0,n) par[i] = in[i].a;
    return {res, par};
}
```

centroid.h

**Description:** Centroid Decomposition

**Time:**  $\mathcal{O}(N \log N)$  d41d8c, 32 lines

```
int dfs(int v, int p, int sz, int &c) {
    int cnt = 1;
    for (auto to : g[v]) {
        if (used[to] || to == p) continue;
        cnt += dfs(to, v, sz, c);
    }
    if (c == -1 && (cnt * 2 >= sz || p == -1)) c = v;
    return cnt;
}

int find_centroid(int v, int sz, int lg, int p = -1) {
    int c = -1;
    dfs(v, -1, sz, c); // can set new sz
    used[c] = 1;
    // get log n parents in decomposition (if needed)
    cd[c][lg] = c; for (int i = 0; i < lg; i++) cd[c][i] = cd[p][i];

    // solve for all pathes that go through c
    // ie do all vertical paths v -> c & c <- u
    // careful that v and u are from different subtrees of c
    // for (auto to : g[c]) if (!used[to]) {
    //     solve_subtree(to, c);
    //     match_with_previous(to, prev_combined);
    //     merge_results(to, prev_combined);
    // }

    for (auto to : g[c]) {
        if (used[to]) continue;
        find_centroid(to, sz / 2, lg + 1, c);
    }
    return c;
}
```

small2large.h

**Description:** Small to Large optimization. Merge all the sets to the largest one while processing in dfs. if you work not with subtree sizes, but subtree depth, works in linear time.

eulertourtree.h

```
/*      Euler tour tree. Treap with support for parents
*/

struct node {
    int sz, y, push, depth, l, r, p, pc;
    node(int val = 0) {
        y = rng(); sz = 1;
        push = 0; depth = val;
        l = -1; r = -1; p = -1;};

node T[MAXN];

int getsz(int v) {
    if (v == -1) return 0;
    return T[v].sz;
}

void push(int v) {
    if (v == -1) return;
    T[v].depth += T[v].push;
    if (T[v].l != -1) T[T[v].l].push += T[v].push;
    if (T[v].r != -1) T[T[v].r].push += T[v].push;
    T[v].push = 0;
}

void recalc(int v) {
    if (v == -1) return;
    T[v].sz = getsz(T[v].l) + getsz(T[v].r) + 1;
    if (T[v].l != -1) {
        T[T[v].l].p = v;
        T[T[v].l].pc = 0;
    }
    if (T[v].r != -1) {
        T[T[v].r].p = v;
        T[T[v].r].pc = 1;
    }
    T[v].p = -1;
}

int pos(int v) {
    int sz = getsz(T[v].l);
    while (v != -1) {
        int pc = T[v].pc;
        v = T[v].p;
        if (pc) sz += getsz(T[v].l) + 1;
    }
    return sz;
}

pair<int, int> split(int v, int k) {
    if (v == -1) return {-1, -1};
    push(v);
    if (getsz(T[v].l) >= k) {
        auto pa = split(T[v].l, k);
        if (pa.F != -1) T[pa.F].p = -1;
        if (pa.S != -1) T[pa.S].p = -1;
        T[v].l = pa.S;
        recalc(v);
        return {pa.F, v};
    } else {
```

d41d8c, 99 lines

```
        auto pa = split(T[v].r, k - getsz(T[v].l) - 1);
        if (pa.F != -1) T[pa.F].p = -1;
        if (pa.S != -1) T[pa.S].p = -1;
        T[v].r = pa.F;
        recalc(v);
        return {v, pa.S};
    }
}

int merge(int l, int r) {
    push(l);
    push(r);
    recalc(l);
    recalc(r);
    if (l == -1) return r;
    if (r == -1) return l;
    if (T[l].y > T[r].y) {
        T[l].r = merge(T[l].r, r);
        recalc(l);
        return l;
    } else {
        T[r].l = merge(l, T[r].l);
        recalc(r);
        return r;
    }
}

int root;
int get(int v) {
    int to_cut_l = pos(v);
    auto [L, R] = split(root, to_cut_l);
    int to_cut_r = 1;
    auto p = split(R, to_cut_r);
    int ans = T[p.F].ans;
    R = merge(p.F, p.S);
    root = merge(L, R);
    return ans;
}
```

2.8 Math

2.8.1 Number of Spanning Trees

Create an  $N \times N$  matrix  $mat$ , and for each edge  $a \rightarrow b \in G$ , do  $mat[a][b]--$ ,  $mat[b][b]++$  (and  $mat[b][a]--$ ,  $mat[a][a]++$  if  $G$  is undirected). Remove the  $i$ th row and column and take the determinant; this yields the number of directed spanning trees rooted at  $i$  (if  $G$  is undirected, remove any row/column).

2.8.2 Erdős–Gallai theorem

A simple graph with node degrees  $d_1 \geq \dots \geq d_n$  exists iff  $d_1 + \dots + d_n$  is even and for every  $k = 1 \dots n$ ,

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k).$$

2.8.3 Karp’s algorithm

Min mean weight cycle: find  $\max_v \min_k \frac{dist_n(v) - dist_k(v)}{n - k}$  for a fixed start point.

Strings (3)

KMP.h

**Description:**  $pi[x]$  = the length of the longest prefix of  $s$  that ends at  $x$ , other than  $s[0..x]$  itself (abacaba -> 0010123).

d41d8c, 5 lines

```
vector<int> p(s.size());
for (int i = 1; i < s.size(); i++) {
    int g = p[i-1];
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]);
}
```

Zfunc.h

**Description:**  $z[i]$  computes the length of the longest common prefix of  $s[i:]$  and  $s$ , except  $z[0] = 0$ . (abacaba -> 0010301)

**Time:**  $\mathcal{O}(n)$

d41d8c, 5 lines

```
vector<int> z(S.size()); int l = -1, r = -1;
for (int i = 1; i < S.size(); i++) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]]) z[i]++;
    if (i + z[i] > r) l = i, r = i + z[i];
}
```

Manacher.h

**Description:** For each position in a string, computes  $p[0][i]$  = half length of longest even palindrome around pos  $i$ ,  $p[1][i]$  = longest odd (half rounded down).

**Time:**  $\mathcal{O}(N)$

d41d8c, 13 lines

```
array<vi, 2> manacher(const string& s) {
    int n = sz(s);
    array<vi, 2> p = {vi(n+1), vi(n)};
    rep(z, 0, 2) for (int i=0, l=0, r=0; i < n; i++) {
        int t = r-i+!z;
        if (i<r) p[z][i] = min(t, p[z][l+t]);
        int L = i-p[z][i], R = i+p[z][i]-!z;
        while (L>=1 && R+1<n && s[L-1] == s[R+1])
            p[z][i]++, L--, R++;
        if (R>r) l=L, r=R;
    }
    return p;
}
```

MinRotation.h

**Description:** Finds the lexicographically smallest rotation of a string.

**Usage:** rotate(v.begin(), v.begin()+minRotation(v), v.end());

**Time:**  $\mathcal{O}(N)$

d41d8c, 8 lines

```
int minRotation(string s) {
    int a=0, N=sz(s); s += s;
    rep(b, 0, N) rep(k, 0, N) {
        if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
        if (s[a+k] > s[b+k]) {a = b; break;}
    }
    return a;
}
```

SuffixArray.h

**Description:** Builds suffix array for a string.  $sa[i]$  is the starting index of the suffix which is  $i$ ’th in the sorted suffix array. The returned vector is of size  $n + 1$ , and  $sa[0] = n$ . The lcp array contains longest common prefixes for neighbouring strings in the suffix array:  $lcp[i] = lcp(sa[i], sa[i-1])$ ,  $lcp[0] = 0$ . The input string must not contain any zero bytes.

**Time:**  $\mathcal{O}(n \log n)$

d41d8c, 23 lines

```
struct SuffixArray {
    vi sa, lcp;
    SuffixArray(string& s, int lim=256) { // or basic_string<int>
        int n = sz(s) + 1, k = 0, a, b;
        vi x(all(s)), y(n), ws(max(n, lim)), rank(n);
        x.push_back(0), sa = lcp = y, iota(all(sa), 0);
        for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
```

```
p = j, iota(all(y), n - j);
rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
fill(all(ws), 0);
rep(i,0,n) ws[x[i]]++;
rep(i,1,lim) ws[i] += ws[i - 1];
for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
swap(x, y), p = 1, x[sa[0]] = 0;
rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
    (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
}
rep(i,1,n) rank[sa[i]] = i;
for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
    for (k && k--, j = sa[rank[i] - 1];
        s[i + k] == s[j + k]; k++);
}
};
```

SuffixTree.h

**Description:** Ukkonen’s algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

**Time:**  $\mathcal{O}(26N)$

d41d8c, 50 lines

```
struct SuffixTree {
    enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
    int toi(char c) { return c - 'a'; }
    string a; // v = cur node, q = cur position
    int t[N][ALPHA], l[N], r[N], p[N], s[N], v=0, q=0, m=2;

    void ukkadd(int i, int c) { suff:
        if (r[v]<=q) {
            if (t[v][c]==-1) { t[v][c]=m; l[m]=i;
                p[m++]=v; v=s[v]; q=r[v]; goto suff; }
            v=t[v][c]; q=l[v];
        }
        if (q==-1 || c==toi(a[q])) q++; else {
            l[m+1]=i; p[m+1]=m; l[m]=l[v]; r[m]=q;
            p[m]=p[v]; t[m][c]=m+1; t[m][toi(a[q])]=v;
            l[v]=q; p[v]=m; t[p[m]][toi(a[l[m]])]=m;
            v=s[p[m]]; q=l[m];
            while (q<r[m]) { v=t[v][toi(a[q])]; q+=r[v]-l[v]; }
            if (q==r[m]) s[m]=v; else s[m]=m+2;
            q=r[v]-(q-r[m]); m+=2; goto suff;
        }
    }
}
```

```
SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
}
```

```
// example: find longest common substring (uses ALPHA = 28)
pii best;
int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
        mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
        best = max(best, {len, r[node] - len});
    return mask;
}
```

```
}
static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
}
};
```

SuffixAutomaton.h

**Description:** Builds suffix automaton for a string. For a suffix tree give a reversed string and take tree of suflinks. if debugging, use aabab

**Time:**  $\mathcal{O}(n)$

d41d8c, 37 lines

```
struct node {
    int link = -1, p = -1, len = 0;
    char pc = '#'; map<char, int> next;
}; node v[2 * MAXN];

int add_char(int ls, char c) {
    if (v[ls].next.find(c) != v[ls].end()) return v[ls].next[c];
    v[++mx] = node(ls, v[ls].len + 1, c);
    int p = ls;
    for (; p != -1 && v[p].next.find(c) == v[p].end(); p = v[p].link)
        v[p].next[c] = mx;
    if (p == -1) {
        v[mx].link = 0;
        return mx;
    }
    int q = v[p].next[c];
    if (v[q].p == p) {
        v[mx].link = q;
        return mx;
    }
    v[++mx] = node(p, v[p].len + 1, c);
    v[mx].next = v[q].next; v[mx].link = v[q].link;
    v[q].link = v[mx - 1].link = mx;
    for (; p != -1 && v[p].next[c] == q; p = v[p].link)
        v[p].next[c] = mx;
    return mx - 1;
}
```

```
void subautomaton(int x, int tm) {
    if (x == -1 || used[x] == tm) return;

    used[x] = tm;
    //....

    subautomaton(v[x].p, tm);
    subautomaton(v[x].link, tm);
}
```

Hashing.h

**Description:** use base 10 for debug if needed

d41d8c, 7 lines

```
int tpow[MAXN]; int h[MAXN]; int t = 179;
tpow[0] = 1; for (int i = 1; i < MAXN; i++) (tpow[i] = 11l * t
    * tpow[i - 1]) % MOD;
for (int i = 0; i < n; i++) {
    h[i] = (11l * (i == 0 ? 0: h[i - 1]) * t + s[i] - 'a' + 1) %
        MOD;
}
int get_hash(int l, int r) {
    if (!l) return h[r];
    return (11l * MOD + hpow[r] - (11l * hpow[l - 1] * tpow[r - 1
        + 1]) % MOD) % MOD;
}
```

AhoCorasick.h

**Description:** Lazy dp / bfs. suflink ups for caab:  $caab \rightarrow aab \rightarrow ab \rightarrow a \rightarrow root$ . two main formulas  $go[v][c] = go[suflink(v)][c]$ ,  $suflink[v] = go[suflink(parent[v]), pchar[v]]$ .

d41d8c, 13 lines

```
struct V{ int parent; int pchar; int link; int go[26]; /*for
    fast answer int term_link;*/ };

queue<int> q; q.push(0);
for (q.push(0); !q.empty(); q.pop()) {
    V v = /*cast*/ q.front();
    for (/* char c */) {
        // can also do while(go[p][c] == -1) p = suflink[p]
        // but doesnt build explicit automaton
        suflink[v] = go[suflink[parent[v]]][pchar[v]];
        if (go[v][c] == -1) go[v][c] = go[suflink[v]][c];
        else q.push(go[v][c]);
    }
}
```

PalindomTree.h

**Description:** Builds palindrome tree for a string.

d41d8c, 26 lines

```
int n, last, sz;
// link[v] - longest palindrom suffix of v

void init() {
    s[n++] = -1;
    link[0] = 1;
    len[1] = -1;
    sz = 2;
}

int get_link(int v) {
    while (s[n-len[v]-2] != s[n-1])
        v = link[v];
    return v;
}

void add_char(int c) {
    s[n++] = c;
    last = get_link(last);
    if (!to[last][c]) {
        len[sz] = len[last] + 2;
        link[sz] = to[get_link(link[last])][c];
        to[last][c] = sz++;
    }
    last = to[last][c];
}
```

DP (4)

aliensTrick.h

**Description:** having (n, k) problem, try binsearching some penalty for ”each k” so that optimal ”k” is k

d41d8c, 16 lines

```
ll aliens() {
    ll L = -1e13, R = 1e13;
    while (L + 1 < R) {
        ll mid = (L + R) >> 1;
        pair<ll, int> X = check(mid); // returns (ans, k)
        if (X.second > k) {
            R = mid;
        }
        else {
            L = mid;
        }
    }
    pair<ll, int> res = check(R);
}
```



```
// note res.second is not exactly R, we hope that answer
    for R is the same
return res.first - k * R;
```

DivideAndConquerDP.h

Description: Given  $opt[i][j] \leq opt[i][j+1]$ , solve in  $\text{nmlog}$  d41d8c, 12 lines

```
void solve(int layer, int l, int r, int L = 0, int R = n) {
    if (l > r) return;
    int m = (l + r) >> 1;
    opt[layer][m] = L;
    for (int k = L; k <= R; ++k) {
        if (cost(layer, m, k) < cost(layer, m, opt[layer][m])) {
            opt[layer][m] = k;
        }
    }
    solve(layer, l, m - 1, L, opt[layer][m]);
    solve(layer, m + 1, r, opt[layer][m], R);
}
```

FastKnapsack.h

Description:  $s \sqrt{s} / 64$  d41d8c, 18 lines

```
int costs[MAXN];
bitset<MAXW> knapsack() {
    sort(costs, costs + n);
    vector<int> items;
    for (int i = 0; i < n; i++) {
        int cnt = 1;
        while (i + 1 < n && costs[i + 1] == costs[i]) i++, cnt++;
        for (int j = 1; j <= cnt; j *= 2) {
            items.push_back(j * costs[i]);
            cnt -= j;
        }
        if (cnt > 0) items.push_back(cnt * costs[i]);
    }
    bitset<MAXW> dp;
    dp[0] = 1;
    for (int item : items) dp |= dp << item;
    return dp;
}
```

Knuth.h

Description: when we know  $opt_{i,j-1} \leq opt_{i,j} \leq opt_{i+1,j}$ ,  $O(n^2)$  d41d8c, 13 lines

```
void knuth() { //
    // .. calc for len 1
    for (int len = 2; len <= n; ++len) {
        for (int i = 0; i + len - 1 <= n; ++i) {
            int j = i + len - 1;
            for (int k = opt[i][j - 1]; k <= opt[i + 1][j]; ++k) {
                if (cost(i, j, k) < cost(i, j, opt[i][j])) {
                    opt[i][j] = k;
                }
            }
        }
    }
}
```

LIS.h

Description: Compute indices for the longest increasing subsequence.  
Time:  $O(N \log N)$  d41d8c, 17 lines

```
template<class I> vi lis(const vector<I>& S) {
    if (S.empty()) return {};
    vi prev(sz(S));
    typedef pair<I, int> p;
```

```
vector<p> res;
rep(i,0,sz(S)) {
    // change 0 -> i for longest non-decreasing subsequence
    auto it = lower_bound(all(res), p{S[i], 0});
    if (it == res.end()) res.emplace_back(), it = res.end()-1;
    *it = {S[i], i};
    prev[i] = it == res.begin() ? 0 : (it-1)->second;
}
int L = sz(res), cur = res.back().second;
vi ans(L);
while (L--) ans[L] = cur, cur = prev[cur];
return ans;
}
```

sos.h

Description: sum over subsets,  $O(2^n \cdot n)$  d41d8c, 22 lines

```
int a[1 << MAXLOG];
ll sos() { // takes 1 << n elements
    for (int i = 0; i < n; i++)
        for (int mask = 0; mask < (1 << n); mask++)
            if (mask & (1 << i))
                a[mask] += a[mask ^ (1 << i)];
}

// having an array of sums of subsets, find the original array
ll rev_sos() {
    for (int mask = 0; mask < (1 << n); mask++) {
        for (int i = 0; i < n; i++) {
            dp[mask][i] = (i == 0 ? 0 : dp[mask][i - 1]);
            if (mask & (1 << i))
                dp[mask][i] += (i == 0 ? ans[mask ^ (1 << i)] : dp[mask ^ (1 << i)][i - 1]);
        }
        ans[mask] = given_sum[mask] - dp[mask][n - 1];
        for (int i = 0; i < n; i++) {
            dp[mask][i] += ans[mask];
        }
    }
}
```

submasks.h

Description:  $O(3^n)$  d41d8c, 3 lines

```
for (int mask = 0; mask < (1 << n); mask++)
    for (int sub = mask; sub > 0; sub = (sub - 1) & mask)
        // do something, if needed add 0-mask
```

cht-merge.h

d41d8c, 98 lines

```
struct Line {
    int k, m;

    int operator ()(int x) {return k * x + m;}
    friend Line operator + (Line a, Line b) {return Line{a.k + b.k, a.m + b.m};}
    friend bool operator == (Line a, Line b) {return a.k == b.k && a.m == b.m;}
    friend bool operator < (Line a, Line b) {return a.k < b.k || a.k == b.k && a.m < b.m;}
};
struct Lines {
    vector<pair<int, Line>> lines;
    Lines() {
        lines = {make_pair(-INF, Line{0, 0})};
    }
    int get(int x) {
        // or keep track of pointer
```

```
    auto it = upper_bound(lines.begin(), lines.end(), make_pair(x, Line{}), [&](pair<int, Line> A, pair<int, Line> B) {
        {
            return A.first < B.first;
        }
    });
    --it;
    return x * it->second.k + it->second.m;
}

void add(Line line) {
    while (true) {
        if (lines.size() == 0) {
            lines.push_back({-INF, line});
            break;
        }
        auto [lx, last] = lines.back();
        assert(line.k >= last.k);
        int dk = line.k - last.k;
        int dm = line.m - last.m;
        if (dk == 0) {
            if (dm <= 0) {
                return;
            }
            lines.pop_back();
            continue;
        }
        // dk * x + dm > 0
        // x > -dm / dk
        int nlx;
        if ((-dm) % dk == 0) {
            nlx = -dm / dk + 1;
        } else {
            nlx = (-dm + dk - 1) / dk;
        }
        if (nlx <= lx) {
            lines.pop_back();
        } else {
            lines.push_back({nlx, line});
            break;
        }
    }
}

Lines merge_sum(Lines& other) {
    int aptr = 0, bptr = 0;
    Lines res;
    while (aptr + 1 < lines.size() || bptr + 1 < other.lines.size()) {
        if (aptr + 1 == lines.size()) {
            bptr++;
            res.lines.push_back({other.lines[bptr].F, lines[aptr].S + other.lines[bptr].S});
        } else if (bptr + 1 == other.lines.size()) {
            aptr++;
            res.lines.push_back({lines[aptr].F, lines[aptr].S + other.lines[bptr].S});
        } else if (lines[aptr+1].F > other.lines[bptr+1].F) {
            bptr++;
            res.lines.push_back({other.lines[bptr].F, lines[aptr].S + other.lines[bptr].S});
        } else if (lines[aptr+1].F < other.lines[bptr+1].F) {
            aptr++;
            res.lines.push_back({lines[aptr].F, lines[aptr].S + other.lines[bptr].S});
        } else {
            aptr++;
            bptr++;
            res.lines.push_back({lines[aptr].F, lines[aptr].S + other.lines[bptr].S});
        }
    }
}
```

```
        return res;
    }
}
Lines merge_max(Lines& other) {
    int aptr = 0, bptr = 0;
    Lines res;
    while (aptr + 1 < lines.size() || bptr + 1 < other.lines.size()) {
        if (aptr + 1 == lines.size()) {
            bptr++;
            res.add(other.lines[bptr].S);
        } else if (bptr+1 == other.lines.size()) {
            aptr++;
            res.add(lines[aptr].S);
        } else if (lines[aptr+1].S < other.lines[bptr+1].S) {
            aptr++;
            res.add(lines[aptr].S);
        } else {
            bptr++;
            res.add(other.lines[bptr].S);
        }
    }
}
```

LineContainer.h

**Description:** Container where you can add lines of the form  $kx+m$ , and query maximum values at points  $x$ . Useful for dynamic programming (“convex hull trick”).  
**Time:**  $\mathcal{O}(\log N)$

```
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};
```

```
// max, first * x + second
vector<pair<ld, pair<int, int>>> cht(vector<pair<int, int>>& ln)
{
    sort(ln.begin(), ln.end(), [&](const auto& t1, const auto& t2)
    {
        return make_pair(t1.first, -t1.second) < make_pair(t2.first, -t2.second);
    });
};
```

```
vector<pair<ld, pair<int, int>>> ch;
for (const auto& line : ln) {
    while (!ch.empty()) {
        auto lst = ch.back().second;
        if (lst.first == line.first) break;
        ld x = (ld)(lst.second - line.second) / (line.first - lst.first);
        if (x > ch.back().first) { ch.emplace_back(x, line);
            break; }
        ch.pop_back();
    }
    if (ch.empty()) { ch.emplace_back(make_pair(-INF, line));
        continue; }
}
return ch;
}
```

LiChao.h

**Description:** Push lines to a segment tree and query the maximum value at a point.

```
struct Line {
    ld k = 0, b = -inf; // (check if you need -inf^2)
    ld operator() (ld x) { return k * x + b; }
} a[maxn * 4];
void insert(int l, int r, Line s, int v=0) {
    if (l + 1 == r) {
        if (s(l) > a[v](l)) a[v] = s;
        return;
    }
    int m = (l + r) / 2;
    if (a[v].k > s.k) swap(a[v], s);
    if (a[v](m) < s(m)) {
        swap(a[v], s);
        insert(l, m, s, 2 * v + 1);
    }
    else insert(m, r, s, 2 * v + 2);
}
ld query(int l, int r, int x, int v=0) {
    if (l + 1 == r) return a[v](x);
    int m = (l + r) / 2;
    if (x < m) return max(a[v](x), query(l, m, x, 2 * v + 1));
    else return max(a[v](x), query(m, r, x, 2 * v + 2));
}
```

Number theory (5)

5.1 Modular arithmetic

ModularArithmetic.h

```
Description: Operators for modular arithmetic.
const ll mod = 17; // change to something else
struct xet {
    int val;
    explicit operator int() const { return val; }
    xet(ll x = 0) { val = (x >= -mod && x < mod ? x : x % mod);
        if (val < 0) val += mod; }
    xet(ll a, ll b) { *this += a; *this /= b; }
    xet& operator+=(xet const &b) { val += b.val; if (val >= mod)
        val -= mod; return *this; }
    xet& operator-=(xet const &b) { val -= b.val; if (val < 0)
        val += mod; return *this; }
    xet& operator*=(xet const &b) { val = (val * (ll)b.val) % mod
        ; return *this; }
    friend xet mypow(xet a, ll n) {
        xet res = 1;
        while (n) { if (n & 1) res *= a; a *= a; n >>= 1; }
    }
```

```
        return res;
    }
}
friend xet inv(xet a) { return mypow(a, mod - 2); }
xet& operator/=(xet const& b) { return *this *= inv(b); }
friend xet operator+(xet a, const xet &b) { return a += b; }
friend xet operator-(xet a, const xet &b) { return a -= b; }
friend xet operator-(xet a) { return 0 - a; }
friend xet operator*(xet a, const xet &b) { return a *= b; }
friend xet operator/(xet a, const xet &b) { return a /= b; }
friend bool operator==(xet const &a, xet const &b) { return a.val == b.val; }
friend bool operator!=(xet const &a, xet const &b) { return a.val != b.val; }
friend bool operator<(xet const &a, xet const &b) { return a.val < b.val; }

friend istream& operator>>(istream& stream, xet &a) { return stream >> a.val; }
friend ostream& operator<<(ostream& stream, const xet &a) { return stream << a.val; }
};
```

ModInverse.h

**Description:** Pre-computation of modular inverses. Assumes  $\text{LIM} \leq \text{mod}$  and that mod is a prime.

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

```
const ll mod = 1000000007; // faster if const

ll modpow(ll b, ll e) {
    ll ans = 1;
    for (; e; b = b * b % mod, e /= 2)
        if (e & 1) ans = ans * b % mod;
    return ans;
}
```

ModLog.h

**Description:** Returns the smallest  $x > 0$  s.t.  $a^x = b \pmod m$ , or  $-1$  if no such  $x$  exists.  $\text{modLog}(a,1,m)$  can be used to calculate the order of  $a$ .  
**Time:**  $\mathcal{O}(\sqrt{m})$

```
ll modLog(ll a, ll b, ll m) {
    ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
    unordered_map<ll, ll> A;
    while (j <= n && (e = f = e * a % m) != b % m)
        A[e * b % m] = j++;
    if (e == b % m) return j;
    if (__gcd(m, e) == __gcd(m, b))
        rep(i,2,n+2) if (A.count(e = e * f % m))
            return n * i - A[e];
    return -1;
}
```

ModSum.h

**Description:** Sums of mod'ed arithmetic progressions.  
 $\text{modsum}(\text{to}, c, k, m) = \sum_{i=0}^{\text{to}-1} (ki + c) \% m$ .  $\text{divsum}$  is similar but for floored division.  
**Time:**  $\log(m)$ , with a large constant.

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
    ull res = k / m * sumsq(to) + c / m * to;
```

```
k %= m; c %= m;
if (!k) return res;
ull to2 = (to * k + c) / m;
return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}

ll modsum(ull to, ll c, ll k, ll m) {
    c = ((c % m) + m) % m;
    k = ((k % m) + m) % m;
    return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

**ModMulLL.h**  
**Description:** Calculate  $a \cdot b \bmod c$  (or  $a^b \bmod c$ ) for  $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$ .  
**Time:**  $\mathcal{O}(1)$  for modmul,  $\mathcal{O}(\log b)$  for modpow

d41d8c, 11 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
    ll ret = a * b - M * ull(1.L / M * a * b);
    return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
    ull ans = 1;
    for (; e; b = modmul(b, b, mod), e /= 2)
        if (e & 1) ans = modmul(ans, b, mod);
    return ans;
}
```

**ModSqrt.h**  
**Description:** Tonelli-Shanks algorithm for modular square roots. Finds  $x$  s.t.  $x^2 = a \pmod p$  ( $-x$  gives the other solution).  
**Time:**  $\mathcal{O}(\log^2 p)$  worst case,  $\mathcal{O}(\log p)$  for most  $p$

"ModPow.h" d41d8c, 24 lines

```
ll sqrt(ll a, ll p) {
    a %= p; if (a < 0) a += p;
    if (a == 0) return 0;
    assert(modpow(a, (p-1)/2, p) == 1); // else no solution
    if (p % 4 == 3) return modpow(a, (p+1)/4, p);
    // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
    ll s = p - 1, n = 2;
    int r = 0, m;
    while (s % 2 == 0)
        ++r, s /= 2;
    while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
    ll x = modpow(a, (s + 1) / 2, p);
    ll b = modpow(a, s, p), g = modpow(n, s, p);
    for (; r = m) {
        ll t = b;
        for (m = 0; m < r && t != 1; ++m)
            t = t * t % p;
        if (m == 0) return x;
        ll gs = modpow(g, 1LL << (r - m - 1), p);
        g = gs * gs % p;
        x = x * gs % p;
        b = b * g % p;
    }
}
```

5.2 Primality

**FastEratosthenes.h**  
**Description:** Prime sieve for generating all primes smaller than LIM.  
**Time:** LIM=1e9  $\approx$  1.5s

d41d8c, 20 lines

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
    const int S = (int)round(sqrt(LIM)), R = LIM / 2;
    vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)*1.1));
```

```
vector<pii> cp;
for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
    cp.push_back({i, i * i / 2});
    for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
}
for (int L = 1; L <= R; L += S) {
    array<bool, S> block{};
    for (auto &[p, idx] : cp)
        for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
    rep(i,0,min(S, R - L))
        if (!block[i]) pr.push_back((L + i) * 2 + 1);
}
for (int i : pr) isPrime[i] = 1;
return pr;
}
```

**MillerRabin.h**  
**Description:** Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to  $7 \cdot 10^{18}$ ; for larger numbers, use Python and extend A randomly.  
**Time:** 7 times the complexity of  $a^b \bmod c$ .

"ModMulLL.h" d41d8c, 12 lines

```
bool isPrime(ull n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n-1), d = n >> s;
    for (ull a : A) { // ^ count trailing zeroes
        ull p = modpow(a%n, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--)
            p = modmul(p, p, n);
        if (p != n-1 && i != s) return 0;
    }
    return 1;
}
```

**Factor.h**  
**Description:** Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).  
**Time:**  $\mathcal{O}(n^{1/4})$ , less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" d41d8c, 18 lines

```
ull pollard(ull n) {
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    auto f = [&](ull x) { return modmul(x, x, n) + i; };
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) x = ++i, y = f(x);
        if ((q = modmul(prd, max(x,y) - min(x,y), n)) prd = q;
        x = f(x), y = f(f(y));
    }
    return __gcd(prd, n);
}
vector<ull> factor(ull n) {
    if (n == 1) return {};
    if (isPrime(n)) return {n};
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), all(r));
    return l;
}
```

5.3 Divisibility

**euclid.h**  
**Description:** Finds two integers  $x$  and  $y$ , such that  $ax + by = \gcd(a, b)$ . If you just need gcd, use the built in `__gcd` instead. If  $a$  and  $b$  are coprime, then  $x$  is the inverse of  $a \pmod b$ .

d41d8c, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
```

```
ll d = euclid(b, a % b, y, x);
return y -= a/b * x, d;
}

CRT.h
Description: Chinese Remainder Theorem.
crt(a, m, b, n) computes x such that x ≡ a (mod m), x ≡ b (mod n). If |a| < m and |b| < n, x will obey 0 ≤ x < lcm(m, n). Assumes mn < 2^62.
Time: log(n)
"euclid.h" d41d8c, 7 lines
ll crt(ll a, ll m, ll b, ll n) {
    if (n > m) swap(a, b), swap(m, n);
    ll x, y, g = euclid(m, n, x, y);
    assert((a - b) % g == 0); // else no solution
    x = (b - a) % n * x % n / g * m + a;
    return x < 0 ? x + m*n/g : x;
}
```

5.3.1 Bézout’s identity

For  $a \neq, b \neq 0$ , then  $d = \gcd(a, b)$  is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If  $(x, y)$  is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

**phiFunction.h**  
**Description:** Euler’s  $\phi$  function is defined as  $\phi(n) := \#$  of positive integers  $\leq n$  that are coprime with  $n$ .  $\phi(1) = 1$ ,  $p$  prime  $\Rightarrow \phi(p^k) = (p - 1)p^{k-1}$ ,  $m, n$  coprime  $\Rightarrow \phi(mn) = \phi(m)\phi(n)$ . If  $n = p_1^{k_1}p_2^{k_2}...p_r^{k_r}$  then  $\phi(n) = (p_1 - 1)p_1^{k_1-1}...(p_r - 1)p_r^{k_r-1}$ .  $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$ .  $\sum_{d|n} \phi(d) = n$ ,  $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$ .  
**Euler’s thm:**  $a, n$  coprime  $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$ .  
**Fermat’s little thm:**  $p$  prime  $\Rightarrow a^{p-1} \equiv 1 \pmod p \forall a$ .

d41d8c, 8 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
    rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
    for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
        for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}
```

5.4 Fractions

**ContinuedFractions.h**  
**Description:** Given  $N$  and a real number  $x \geq 0$ , finds the closest rational approximation  $p/q$  with  $p, q \leq N$ . It will obey  $|p/q - x| \leq 1/qN$ . For consecutive convergents,  $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$ .  $(p_k/q_k)$  alternates between  $> x$  and  $< x$ . If  $x$  is rational,  $y$  eventually becomes  $\infty$ ; if  $x$  is the root of a degree 2 polynomial the  $a$ ’s eventually become cyclic.  
**Time:**  $\mathcal{O}(\log N)$

d41d8c, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
    ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
    for (;) {
        ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
            a = (ll)floor(y), b = min(a, lim),
            NP = b*P + LP, NQ = b*Q + LQ;
        if (a > b) {
```

```
// If b > a/2, we have a semi-convergent that gives us a
// better approximation; if b = a/2, we *may* have one.
// Return {P, Q} here for a more canonical approximation.
return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
    make_pair(NP, NQ) : make_pair(P, Q);
}
if (abs(y = 1/(y - (d)a)) > 3*N) {
    return {NP, NQ};
}
LP = P; P = NP;
LQ = Q; Q = NQ;
}
```

FracBinarySearch.h

**Description:** Given  $f$  and  $N$ , finds the smallest fraction  $p/q \in [0,1]$  such that  $f(p/q)$  is true, and  $p, q \leq N$ . You may want to throw an exception from  $f$  if it finds an exact solution, in which case  $N$  can be removed.  
**Usage:** fracBS([])(Frac f) { return f.p>=3\*f.q; }, 10); // {1,3}  
**Time:**  $\mathcal{O}(\log(N))$

d41d8c, 25 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
    bool dir = 1, A = 1, B = 1;
    Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
    if (f(lo)) return lo;
    assert(f(hi));
    while (A || B) {
        ll adv = 0, step = 1; // move hi if dir, else lo
        for (int si = 0; step; (step *= 2) >>= si) {
            adv += step;
            Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
            if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
                adv -= step; si = 2;
            }
        }
        hi.p += lo.p * adv;
        hi.q += lo.q * adv;
        dir = !dir;
        swap(lo, hi);
        A = B; B = !adv;
    }
    return dir ? hi : lo;
}
```

5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$

with  $m > n > 0, k > 0, m \perp n$ , and either  $m$  or  $n$  even.

5.6 Primes

$p = 962592769$  is such that  $2^{21} \mid p - 1$ , which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power  $p^a$ , except for  $p = 2, a > 2$ , and there are  $\phi(\phi(p^a))$  many. For  $p = 2, a > 2$ , the group  $\mathbb{Z}_{2^a}^\times$  is instead isomorphic to  $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$ .

5.7 Estimates

$\sum_{d|n} d = O(n \log \log n).$

The number of divisors of  $n$  is at most around 100 for  $n < 5e4$ , 500 for  $n < 1e7$ , 2000 for  $n < 1e10$ , 200 000 for  $n < 1e19$ .

5.8 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d)g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$  (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n)g(d)$

$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m)g(\lfloor \frac{n}{m} \rfloor)$

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

$n$	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$n$	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$n$	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

**Description:** Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.  
**Time:**  $\mathcal{O}(n)$

d41d8c, 6 lines

```
int permToInt(vi& v) {
    int use = 0, i = 0, r = 0;
    for (int x:v) r = r * ++i + __builtin_popcount(use & ~(1<<x)),
        use |= 1 << x; // (note: minus, not ~!)
    return r;
}
```

6.1.2 Cycles

Let  $g_S(n)$  be the number of  $n$ -permutations whose cycle lengths all belong to the set  $S$ . Then

$$\sum_{n=0}^\infty g_S(n) \frac{x^n}{n!} = \exp \left( \sum_{n \in S} \frac{x^n}{n} \right)$$

6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

6.1.4 Burnside’s lemma

Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g.x = x$ ).

If  $f(n)$  counts “configurations” (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$													
$n$	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim 2\text{e}5$	$\sim 2\text{e}8$

6.2.2 Lucas’ Theorem

Let  $n, m$  be non-negative integers and  $p$  a prime. Write  $n = n_k p^k + \dots + n_1 p + n_0$  and  $m = m_k p^k + \dots + m_1 p + m_0$ . Then  $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$ .

6.2.3 Binomials

multinomial.h

**Description:** Computes  $\binom{k_1 + \dots + k_n}{k_1, k_2, \dots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \dots k_n!}$ .

d41d8c, 6 lines

```
ll multinomial(vi& v) {
    ll c = 1, m = v.empty() ? 1 : v[0];
    rep(i, 1, sz(v)) rep(j, 0, v[i])
        c = c * ++m / (j+1);
    return c;
}
```

6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is  $B(t) = \frac{t}{e^t - 1}$  (FFT-able).

$B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^n n^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^\infty f(i) &= \int_m^\infty f(x)dx - \sum_{k=1}^\infty \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^\infty f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

6.3.2 Stirling numbers of the first kind

Number of permutations on  $n$  items with  $k$  cycles.

$$\begin{aligned} c(n,k) &= c(n-1,k-1) + (n-1)c(n-1,k), \quad c(0,0) = 1 \\ \sum_{k=0}^n c(n,k)x^k &= x(x+1)\dots(x+n-1) \end{aligned}$$

$$\begin{aligned} c(8,k) &= 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1 \\ c(n,2) &= 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots \end{aligned}$$

6.3.3 Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$   $j$ :s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$   $j$ :s s.t.  $\pi(j) \geq j$ ,  $k$   $j$ :s s.t.  $\pi(j) > j$ .

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

6.3.5 Bell numbers

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$ . For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.3.6 Labeled unrooted trees

- # on  $n$  vertices:  $n^{n-2}$
- # on  $k$  existing trees of size  $n_i$ :  $n_1 n_2 \dots n_k n^{k-2}$
- # with degrees  $d_i$ :  $(n-2)! / ((d_1-1)! \dots (d_n-1)!)$

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an  $n \times n$  grid.
- strings with  $n$  pairs of parenthesis, correctly nested.
- binary trees with with  $n+1$  leaves (0 or 2 children).
- ordered trees with  $n+1$  vertices.
- ways a convex polygon with  $n+2$  sides can be cut into triangles by connecting vertices with straight lines.
- permutations of  $[n]$  with no 3-term increasing subseq.

Mathematics (7)

7.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by  $x = -b/2a$ .

$$\begin{aligned} ax + by &= e & x &= \frac{ed - bf}{ad - bc} \\ cx + dy &= f & y &= \frac{af - ec}{ad - bc} \end{aligned} \Rightarrow$$

In general, given an equation  $Ax = b$ , the solution to a variable  $x_i$  is given by

$$x_i = \frac{\det A'_i}{\det A}$$

where  $A'_i$  is  $A$  with the  $i$ 'th column replaced by  $b$ .

7.2 Recurrences

If  $a_n = c_1 a_{n-1} + \dots + c_k a_{n-k}$ , and  $r_1, \dots, r_k$  are distinct roots of  $x^k - c_1 x^{k-1} - \dots - c_k$ , there are  $d_1, \dots, d_k$  s.t.

$$a_n = d_1 r_1^n + \dots + d_k r_k^n.$$

Non-distinct roots  $r$  become polynomial factors, e.g.  $a_n = (d_1 n + d_2) r^n$ .

7.3 Trigonometry

$$\begin{aligned} \sin(v+w) &= \sin v \cos w + \cos v \sin w \\ \cos(v+w) &= \cos v \cos w - \sin v \sin w \end{aligned}$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$

$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W) \tan(v-w)/2 = (V-W) \tan(v+w)/2$$

where  $V, W$  are lengths of sides opposite angles  $v, w$ .

$$\begin{aligned} a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi) \end{aligned}$$

where  $r = \sqrt{a^2 + b^2}$ ,  $\phi = \text{atan2}(b, a)$ .

7.4 Geometry

7.4.1 Triangles

Side lengths:  $a, b, c$

$$\text{Semiperimeter: } p = \frac{a+b+c}{2}$$

$$\text{Area: } A = \sqrt{p(p-a)(p-b)(p-c)}$$

$$\text{Circumradius: } R = \frac{abc}{4A}$$

$$\text{Inradius: } r = \frac{A}{p}$$

Length of median (divides triangle into two equal-area triangles):  $m_a = \frac{1}{2} \sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[ 1 - \left( \frac{a}{b+c} \right)^2 \right]}$$

$$\text{Law of sines: } \frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$$

$$\text{Law of cosines: } a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$\text{Law of tangents: } \frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$$

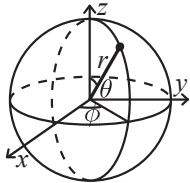
7.4.2 Quadrilaterals

With side lengths  $a, b, c, d$ , diagonals  $e, f$ , diagonals angle  $\theta$ , area  $A$  and magic flux  $F = b^2 + d^2 - a^2 - c^2$ :

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is  $180^\circ$ ,  $ef = ac + bd$ , and  $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$ .

7.4.3 Spherical coordinates



$$x = r \sin \theta \cos \phi$$
$$y = r \sin \theta \sin \phi$$
$$z = r \cos \theta$$

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \operatorname{acos}(z/\sqrt{x^2 + y^2 + z^2})$$
$$\phi = \operatorname{atan2}(y, x)$$

7.5 Derivatives/Integrals

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan x = 1 + \tan^2 x$$
$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a}$$
$$\int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \operatorname{erf}(x)$$
$$\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

7.6 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n+1)(n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$
$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

7.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$
$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

Polynomial

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

7.8 Probability theory

Let  $X$  be a discrete random variable with probability  $p_X(x)$  of assuming the value  $x$ . It will then have an expected value (mean)  $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$  and variance  $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$  where  $\sigma$  is the standard deviation. If  $X$  is instead continuous it will have a probability density function  $f_X(x)$  and the sums above will instead be integrals with  $p_X(x)$  replaced by  $f_X(x)$ .

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent  $X$  and  $Y$ ,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

7.8.1 Discrete distributions

Binomial distribution

The number of successes in  $n$  independent yes/no experiments, each which yields success with probability  $p$  is  $\operatorname{Bin}(n, p)$ ,  $n = 1, 2, \dots$ ,  $0 \leq p \leq 1$ .

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \sigma^2 = np(1-p)$$

$\operatorname{Bin}(n, p)$  is approximately  $\operatorname{Po}(np)$  for small  $p$ .

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability  $p$  is  $\operatorname{Fs}(p)$ ,  $0 \leq p \leq 1$ .

$$p(k) = p(1-p)^{k-1}, k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time  $t$  if these events occur with a known average rate  $\kappa$  and independently of the time since the last event is  $\operatorname{Po}(\lambda)$ ,  $\lambda = t\kappa$ .

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

7.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between  $a$  and  $b$  and 0 elsewhere it is  $\operatorname{U}(a, b)$ ,  $a < b$ .

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is  $\operatorname{Exp}(\lambda)$ ,  $\lambda > 0$ .

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean  $\mu$  and variance  $\sigma^2$  are well described by  $\mathcal{N}(\mu, \sigma^2)$ ,  $\sigma > 0$ .

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If  $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$  and  $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$  then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

Numerical (8)

8.1 Polynomials and recurrences

Polynomial.h

d41d8c, 17 lines

```
struct Poly {
    vector<double> a;
    double operator() (double x) const {
        double val = 0;
        for (int i = sz(a); i--;) (val *= x) += a[i];
        return val;
    }
    void diff() {
        rep(i,1,sz(a)) a[i-1] = i*a[i];
        a.pop_back();
    }
    void divroot(double x0) {
        double b = a.back(), c; a.back() = 0;
        for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
        a.pop_back();
    }
};
```

PolyRoots.h

**Description:** Finds the real roots to a polynomial.

**Usage:** polyRoots({{2,-3,1}},-1e9,1e9) // solve x<sup>2</sup>-3x+2 = 0

**Time:**  $\mathcal{O}(n^2 \log(1/\epsilon))$

"Polynomial.h"41d8c, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
    if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
    vector<double> ret;
    Poly der = p;
    der.diff();
    auto dr = polyRoots(der, xmin, xmax);
    dr.push_back(xmin-1);
    dr.push_back(xmax+1);
    sort(all(dr));
    rep(i,0,sz(dr)-1) {
        double l = dr[i], h = dr[i+1];
        bool sign = p(l) > 0;
        if (sign ^ (p(h) > 0)) {
            rep(it,0,60) { // while (h - l > 1e-8)
                double m = (l + h) / 2, f = p(m);
                if ((f <= 0) ^ sign) l = m;
                else h = m;
            }
            ret.push_back((l + h) / 2);
        }
    }
    return ret;
}
```

PolyInterpolate.h

**Description:** Given  $n$  points  $(x[i], y[i])$ , computes an  $n$ -1-degree polynomial  $p$  that passes through them:  $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$ . For numerical precision, pick  $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$ .

**Time:**  $\mathcal{O}(n^2)$

d41d8c, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    rep(k,0,n-1) rep(i,k+1,n)
        y[i] = (y[i] - y[k]) / (x[i] - x[k]);
    double last = 0; temp[0] = 1;
    rep(k,0,n) rep(i,0,n) {
        res[i] += y[k] * temp[i];
        swap(last, temp[i]);
        temp[i] -= last * x[k];
    }
    return res;
}
```

BerlekampMassey.h

**Description:** Recovers any  $n$ -order linear recurrence relation from the first  $2n$  terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size  $\leq n$ .

**Usage:** berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}

**Time:**  $\mathcal{O}(N^2)$

"../number-theory/ModPow.h"41d8c, 20 lines

```
vector<ll> berlekampMassey(vector<ll> s) {
    int n = sz(s), L = 0, m = 0;
    vector<ll> C(n), B(n), T;
    C[0] = B[0] = 1;

    ll b = 1;
    rep(i,0,n) { ++m;
        ll d = s[i] % mod;
        rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
        if (!d) continue;
        T = C; ll coef = d * modpow(b, mod-2) % mod;
```

LinearRecurrence.h

**Description:** Generates the  $k$ 'th term of an  $n$ -order linear recurrence  $S[i] = \sum_j S[i-j-1]tr[j]$ , given  $S[0 \dots \geq n-1]$  and  $tr[0 \dots n-1]$ . Faster than matrix multiplication. Useful together with Berlekamp-Massey.

**Usage:** linearRec({0, 1}, {1, 1}, k) //  $k$ 'th Fibonacci number

**Time:**  $\mathcal{O}(n^2 \log k)$

d41d8c, 26 lines

```
rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
if (2 * L > i) continue;
L = i + 1 - L; B = T; b = d; m = 0;
}

C.resize(L + 1); C.erase(C.begin());
for (ll& x : C) x = (mod - x) % mod;
return C;
}
```

GoldenSectionSearch.h

**Description:** Finds the argument minimizing the function  $f$  in the interval  $[a,b]$  assuming  $f$  is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is  $eps$ . Works equally well for maximization with a small change in the code. See Ternary-Search.h in the Various chapter for a discrete version.

**Usage:** double func(double x) { return 4+x+.3\*x\*x; }

double xmin = gss(-1000,1000,func);

**Time:**  $\mathcal{O}(\log((b-a)/\epsilon))$

d41d8c, 14 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
    int n = sz(tr);

    auto combine = [&](Poly a, Poly b) {
        Poly res(n * 2 + 1);
        rep(i,0,n+1) rep(j,0,n+1)
            res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
        for (int i = 2 * n; i > n; --i) rep(j,0,n)
            res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
        res.resize(n + 1);
        return res;
    };

    Poly pol(n + 1), e(pol);
    pol[0] = e[1] = 1;

    for (++k; k; k /= 2) {
        if (k % 2) pol = combine(pol, e);
        e = combine(e, e);
    }

    ll res = 0;
    rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
    return res;
}
```

8.2 Optimization

GoldenSectionSearch.h

**Description:** Finds the argument minimizing the function  $f$  in the interval  $[a,b]$  assuming  $f$  is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is  $eps$ . Works equally well for maximization with a small change in the code. See Ternary-Search.h in the Various chapter for a discrete version.

**Usage:** double func(double x) { return 4+x+.3\*x\*x; }

double xmin = gss(-1000,1000,func);

**Time:**  $\mathcal{O}(\log((b-a)/\epsilon))$

d41d8c, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
    double r = (sqrt(5)-1)/2, eps = 1e-7;
    double x1 = b - r*(b-a), x2 = a + r*(b-a);
    double f1 = f(x1), f2 = f(x2);
    while (b-a > eps)
        if (f1 < f2) { //change to > to find maximum
            b = x2; x2 = x1; f2 = f1;
            x1 = b - r*(b-a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r*(b-a); f2 = f(x2);
        }
    return a;
}
```

HillClimbing.h

**Description:** Poor man's optimization for unimodal functions.

d41d8c, 14 lines

```
typedef array<double, 2> P;

template<class F> pair<double, P> hillClimb(P start, F f) {
    pair<double, P> cur(f(start), start);
    for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
        rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) {
            P p = cur.second;
            p[0] += dx*jmp;
            p[1] += dy*jmp;
            cur = min(cur, make_pair(f(p), p));
        }
    }
    return cur;
}
```

Integrate.h

**Description:** Simple integration of a function over an interval using Simpson's rule. The error should be proportional to  $h^4$ , although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

d41d8c, 7 lines

```
template<class F>
double quad(double a, double b, F f, const int n = 1000) {
    double h = (b - a) / 2 / n, v = f(a) + f(b);
    rep(i,1,n*2)
        v += f(a + i*h) * (i&1 ? 4 : 2);
    return v * h / 3;
}
```

IntegrateAdaptive.h

**Description:** Fast integration using an adaptive Simpson's rule.

**Usage:** double sphereVolume = quad(-1, 1, [](double x) { return quad(-1, 1, [&](double y) { return quad(-1, 1, [&](double z) { return x\*x + y\*y + z\*z < 1; }}});});

d41d8c, 15 lines

```
typedef double d;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
    d c = (a + b) / 2;
    d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
    if (abs(T - S) <= 15 * eps || b - a < 1e-10)
        return T + (T - S) / 15;
    return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
}

template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
    return rec(f, a, b, eps, S(a, b));
}
```

Simplex.h

**Description:** Solves a general linear maximization problem: maximize  $c^T x$  subject to  $Ax \leq b, x \geq 0$ . Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of  $c^T x$  otherwise. The input vector is set to an optimal  $x$  (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that  $x = 0$  is viable.

**Usage:** vvd A = {{1,-1}, {-1,1}, {-1,-2}};

vd b = {1,1,-4}, c = {-1,-1}, x;

T val = LPSolver(A, b, c).solve(x);

**Time:**  $\mathcal{O}(NM * \#pivots)$ , where a pivot may be e.g. an edge relaxation.  $\mathcal{O}(2^n)$  in the general case.

d41d8c, 68 lines

```
typedef double T; // long double, Rational, double + modP>...
typedef vector<T> vd;
```

```
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
    int m, n;
    vi N, B;
    vvd D;

    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
        rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
        rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m+1][n] = 1;
    }

    void pivot(int r, int s) {
        T *a = D[r].data(), inv = 1 / a[s];
        rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
            T *b = D[i].data(), inv2 = b[s] * inv;
            rep(j,0,n+2) b[j] -= a[j] * inv2;
            b[s] = a[s] * inv2;
        }
        rep(j,0,n+2) if (j != s) D[r][j] *= inv;
        rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }

    bool simplex(int phase) {
        int x = m + phase - 1;
        for (;;) {
            int s = -1;
            rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
            if (D[x][s] >= -eps) return true;
            int r = -1;
            rep(i,0,m) {
                if (D[i][s] <= eps) continue;
                if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                    < MP(D[r][n+1] / D[r][s], B[r])) r = i;
            }
            if (r == -1) return false;
            pivot(r, s);
        }
    }

    T solve(vd &x) {
        int r = 0;
        rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
        if (D[r][n+1] < -eps) {
            pivot(r, n);
            if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
            rep(i,0,m) if (B[i] == -1) {
                int s = 0;
                rep(j,1,n+1) ltj(D[i]);
                pivot(i, s);
            }
        }
        bool ok = simplex(1); x = vd(n);
        rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
        return ok ? D[m][n+1] : inf;
    }
};
```

8.3 Matrices

```
Matrix.h
Description: Basic operations on square matrices.
Usage: Matrix<int, 3> A;
A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};
vector<int> vec = {1,2,3};
vec = (A^N) * vec;

vector<vector<int>> multiply(const vector<vector<int>>& a,
    const vector<vector<int>>& b) {
    int n = a.size(); int m = a[0].size(); int k = b[0].size();
    assert(b.size() == m);
    vector<vector<int>> v(k, vector<int>(m)); vector<vector<int>>
        res(n, vector<int>(k));
    for (int i = 0; i < m; i++) for (int j = 0; j < k; j++) v[j][i] = b[i][j];
    // if using mod, can do MOD^2 by if-ing and taking 1 final mod
    for (int i = 0; i < n; i++) for (int j = 0; j < k; j++) for (
        int l = 0; l < m; l++) res[i][k] += a[i][l] * v[j][l];
}
```

```
Determinant.h
Description: Calculates determinant of a matrix. Destroys the matrix.
Time: O(N^3)

double det(vector<vector<double>>& a) {
    int n = sz(a); double res = 1;
    rep(i,0,n) {
        int b = i;
        rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1;
        res *= a[i][i];
        if (res == 0) return 0;
        rep(j,i+1,n) {
            double v = a[j][i] / a[i][i];
            if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
        }
    }
    return res;
}
```

```
IntDeterminant.h
Description: Calculates determinant using modular arithmetics. Modulos
can also be removed to get a pure-integer version.
Time: O(N^3)

const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
    int n = sz(a); ll ans = 1;
    rep(i,0,n) {
        rep(j,i+1,n) {
            while (a[j][i] != 0) { // gcd step
                ll t = a[i][i] / a[j][i];
                if (t) rep(k,i,n)
                    a[i][k] = (a[i][k] - a[j][k] * t) % mod;
                swap(a[i], a[j]);
                ans *= -1;
            }
        }
        ans = ans * a[i][i] % mod;
        if (!ans) return 0;
    }
    return (ans + mod) % mod;
}
```

```
SolveLinear.h
Description: Solves A * x = b. If there are multiple solutions, an arbitrary
one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: O(n^2m)

typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m);
    vi col(m); iota(all(col), 0);

    rep(i,0,n) {
        double v, bv = 0;
        rep(r,i,n) rep(c,i,m)
            if ((v = fabs(A[r][c])) > bv)
                br = r, bc = c, bv = v;
        if (bv <= eps) {
            rep(j,i,n) if (fabs(b[j]) > eps) return -1;
            break;
        }
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) swap(A[j][i], A[j][bc]);
        bv = 1/A[i][i];
        rep(j,i+1,n) {
            double fac = A[j][i] * bv;
            b[j] -= fac * b[i];
            rep(k,i+1,m) A[j][k] -= fac*A[i][k];
        }
        rank++;
    }

    x.assign(m, 0);
    for (int i = rank; i--;) {
        b[i] /= A[i][i];
        x[col[i]] = b[i];
        rep(j,0,i) b[j] -= A[j][i] * b[i];
    }
    return rank; // (multiple solutions if rank < m)
}
```

```
SolveLinear2.h
Description: To get all uniquely determined values of x back from Solve-
Linear, make the following changes:

"SolveLinear.h"

rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
    rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
    x[col[i]] = b[i] / A[i][i];
fail:; }
```

```
SolveLinearBinary.h
Description: Solves Ax = b over F2. If there are multiple solutions, one is
returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: O(n^2m)

typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
    int n = sz(A), rank = 0, br;
    assert(m <= sz(x));
    vi col(m); iota(all(col), 0);
    rep(i,0,n) {
        for (br=i; br<n; ++br) if (A[br].any()) break;
```



```
    if (br == n) {
        rep(j,i,n) if(b[j]) return -1;
        break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
        A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
        b[j] ^= b[i];
        A[j] ^= A[i];
    }
    rank++;
}

x = bs();
for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
}
return rank; // (multiple solutions if rank < m)
}
```

d41d8c, 35 lines

MatrixInverse.h

**Description:** Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set  $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$  where  $A^{-1}$  starts as the inverse of A mod p, and k is doubled in each step.  
**Time:**  $\mathcal{O}(n^3)$

```
int matInv(vector<vector<double>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<double>> tmp(n, vector<double>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;

    rep(i,0,n) {
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n)
            if (fabs(A[j][k]) > fabs(A[r][c]))
                r = j, c = k;
        if (fabs(A[r][c]) < 1e-12) return i;
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j,0,n)
            swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]);
        double v = A[i][i];
        rep(j,i+1,n) {
            double f = A[j][i] / v;
            A[j][i] = 0;
            rep(k,i+1,n) A[j][k] -= f*A[i][k];
            rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
        }
        rep(j,i+1,n) A[i][j] /= v;
        rep(j,0,n) tmp[i][j] /= v;
        A[i][i] = 1;
    }

    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        double v = A[j][i];
        rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
    }

    rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
    return n;
}
```

Tridiagonal.h

**Description:**  $x = \text{tridiagonal}(d, p, q, b)$  solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i, 1 \leq i \leq n,$$

where  $a_0, a_{n+1}, b_i, c_i$  and  $d_i$  are known.  $a$  can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, \dots, -1, 1\}, \{0, c_1, c_2, \dots, c_n\}, \{b_1, b_2, \dots, b_n, 0\}, \{a_0, d_1, d_2, \dots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.  
If  $|d_i| > |p_i| + |q_{i-1}|$  for all  $i$ , or  $|d_i| > |p_{i-1}| + |q_i|$ , or the matrix is positive definite, the algorithm is numerically stable and neither tr nor the check for diag[i] == 0 is needed.  
**Time:**  $\mathcal{O}(N)$

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
    int n = sz(b); vi tr(n);
    rep(i,0,n-1) {
        if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
            b[i+1] -= b[i] * diag[i+1] / super[i];
            if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
            diag[i+1] = sub[i]; tr[++i] = 1;
        } else {
            diag[i+1] -= super[i]*sub[i]/diag[i];
            b[i+1] -= b[i]*sub[i]/diag[i];
        }
    }
    for (int i = n; i--;) {
        if (tr[i]) {
            swap(b[i], b[i-1]);
            diag[i-1] = diag[i];
            b[i] /= super[i-1];
        } else {
            b[i] /= diag[i];
            if (i) b[i-1] -= b[i]*super[i-1];
        }
    }
    return b;
}
```

8.4 Fourier transforms

FastFourierTransform.h

**Description:**  $\text{fft}(a)$  computes  $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$  for all  $k$ . N must be a power of 2. Useful for convolution:  $\text{conv}(a, b) = c$ , where  $c[x] = \sum a[i]b[x-i]$ . For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if  $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$  (in practice  $10^{16}$ ; higher for random inputs). Otherwise, use NTT/FFTMod.  
**Time:**  $\mathcal{O}(N \log N)$  with  $N = |A| + |B|$  ( $\sim 18$  for  $N = 2^{22}$ )

```
typedef ll cx; // or complex<double>
cx g = 31; // for mod = 998244353, n = 2 ** 23, cx(cos(2 * PI / (1 << MAXLOG)), sin(2 * PI / (1 << MAXLOG))) for doubles
cx gr = modinv(g, mod); // or -g

// input data, output data, size of input/output view, unity
// root, start of input view, step of input view, start of
// output view
```

```
inline void fft(vector<cx> &a, vector<cx> &ans, int n, cx z,
    int abg, int ast, int ansbg) {
    if (n == 1) {
        ans[ansbg] = a[abg];
        return;
    }
    fft(a, ans, n / 2, (z * z) % mod, abg, ast * 2, ansbg);
    fft(a, ans, n / 2, (z * z) % mod, abg + ast, ast * 2, ansbg +
        n / 2);
    cx x = 1;
    for (int i = 0; i < n / 2; i++) {
        cx ans1 = ans[ansbg + i];
        cx ans2 = ans[ansbg + i + n / 2];
        ans[ansbg + i] = (ans1 + x * ans2) % mod;
        ans[ansbg + i + n / 2] = (ans1 - (x * ans2) % mod) + mod)
            % mod;
        x = (x * z) % mod;
    }
}

void multiply(vector<cx> &a, vector<cx> &b, vector<cx> &ans) {
    fft(a, a_res, 1 << MAXLOG, g, 0, 1, 0);
    fft(b, b_res, 1 << MAXLOG, g, 0, 1, 0);
    for (int i = 0; i < 1 << MAXLOG; i++) {
        c[i] = (a_res[i] * b_res[i]) % mod;
    }
    fft(c, ans, 1 << MAXLOG, gr, 0, 1, 0);
    cx mp = modinv(1 << MAXLOG, mod);
    for (int i = 0; i < 1 << MAXLOG; i++) {
        ans[i] = (ans[i] * mp) % mod;
    }
    // also can use g in ifft + reverse(ans.begin() + 1, ans.end
    ());
    // use when g != 31, log != 23
}
```

FastSubsetTransform.h

**Description:** Transform to a basis with fast convolutions of the form  $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$ , where  $\oplus$  is one of AND, OR, XOR. The size of  $a$  must be a power of two.  
**Time:**  $\mathcal{O}(N \log N)$

```
void FST(vi& a, bool inv) {
    for (int n = sz(a), step = 1; step < n; step *= 2) {
        for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
            int &u = a[j], &v = a[j + step]; tie(u, v) =
                inv ? pii(v - u, u) : pii(v, u + v); // AND
                inv ? pii(v, u - v) : pii(u + v, u); // OR
                pii(u + v, u - v); // XOR
        }
        if (inv) for (int& x : a) x /= sz(a); // XOR only
    }
    vi conv(vi a, vi b) {
        FST(a, 0); FST(b, 0);
        rep(i,0,sz(a)) a[i] *= b[i];
        FST(a, 1); return a;
    }
}
```

Geometry (9)

9.1 Geometric primitives

Point.h

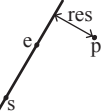
**Description:** Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
```

```
template<class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T x=0, T y=0) : x(x), y(y) {}
    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
    P operator+(P p) const { return P(x+p.x, y+p.y); }
    P operator-(P p) const { return P(x-p.x, y-p.y); }
    P operator*(T d) const { return P(x*d, y*d); }
    P operator/(T d) const { return P(x/d, y/d); }
    T dot(P p) const { return x*p.x + y*p.y; }
    T cross(P p) const { return x*p.y - y*p.x; }
    T cross(P a, P b) const { return (a-*this).cross(b-*this); }
    T dist2() const { return x*x + y*y; }
    double dist() const { return sqrt((double)dist2()); }
    // angle to x-axis in interval [-pi, pi]
    double angle() const { return atan2(y, x); }
    P unit() const { return *this/dist(); } // makes dist()==1
    P perp() const { return P(-y, x); } // rotates +90 degrees
    P normal() const { return perp().unit(); }
    // returns point rotated 'a' radians ccw around the origin
    P rotate(double a) const {
        return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
    friend ostream& operator<<(ostream& os, P p) {
        return os << "(" << p.x << ", " << p.y << ")"; }
};
```

lineDistance.h

Description:  
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.



Point.h

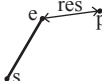
d41d8c, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
    return (double) (b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description:  
Returns the shortest distance between point p and the line segment from point s to e.

Usage: Point<double> a, b(2,2), p(1,1);  
bool onSegment = segDist(a,b,p) < 1e-10;

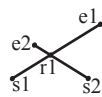


Point.h

d41d8c, 6 lines

SegmentIntersection.h

Description:  
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



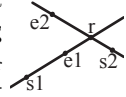
```
Usage: vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
"Point.h", "OnSegment.h"
d41d8c, 13 lines

template<class P> vector<P> segInter(P a, P b, P c, P d) {
    auto oa = c.cross(d, a), ob = c.cross(d, b),
        oc = a.cross(b, c), od = a.cross(b, d);
    // Checks if intersection is single non-endpoint point.
    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
        return {(a * ob - b * oa) / (ob - oa)};
    set<P> s;
    if (onSegment(c, d, a)) s.insert(a);
    if (onSegment(c, d, b)) s.insert(b);
    if (onSegment(a, b, c)) s.insert(c);
    if (onSegment(a, b, d)) s.insert(d);
    return {all(s)};
}
```

lineIntersection.h

Description:  
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.

Usage: auto res = lineInter(s1,e1,s2,e2);  
if (res.first == 1)  
cout << "intersection point at " << res.second << endl;



Point.h

d41d8c, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
    auto d = (e1 - s1).cross(e2 - s2);
    if (d == 0) // if parallel
        return {-(s1.cross(e1, s2) == 0), P(0, 0)};
    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
    return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

Description:  
Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: bool left = sideOf(p1,p2,q)==1;

Point.h

d41d8c, 9 lines

OnSegment.h

Description:  
Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

Point.h

d41d8c, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
    return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h

Description:  
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

Point.h

d41d8c, 6 lines

Angle.h

Description:  
A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage: vector<Angle> v = {w[0], w[0].t360() ...}; // sorted  
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }  
// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i

Point.h

d41d8c, 35 lines

```
struct Angle {
    int x, y;
    int t;
    Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
    Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
    int half() const {
        assert(x || y);
        return y < 0 || (y == 0 && x < 0);
    }
    Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
    Angle t180() const { return {-x, -y, t + half()}; }
    Angle t360() const { return {x, y, t + 1}; }
};

bool operator<(Angle a, Angle b) {
    // add a.dist2() and b.dist2() to also compare distances
    return make_tuple(a.t, a.half(), a.y * (11)b.x) <
        make_tuple(b.t, b.half(), a.x * (11)b.y);
}
```

Angle.h

Usage:  
// Given two points, this calculates the smallest angle between them, i.e., the angle that covers the defined line segment.  
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {  
 if (b < a) swap(a, b);  
 return (b < a.t180() ?  
 make\_pair(a, b) : make\_pair(b, a.t360()));  
}  
Angle operator+(Angle a, Angle b) { // point a + vector b  
 Angle r(a.x + b.x, a.y + b.y, a.t);  
 if (a.t180() < r) r.t--;  
 return r.t180() < a ? r.t360() : r;  
}  
Angle angleDiff(Angle a, Angle b) { // angle b - angle a  
 int tu = b.t - a.t; a.t = b.t;  
 return {a.x\*b.x + a.y\*b.y, a.x\*b.y - a.y\*b.x, tu - (b < a)};  
}

9.2 Circles

CircleIntersection.h

Description:  
Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

Point.h

d41d8c, 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
    if (a == b) { assert(r1 != r2); return false; }
    P vec = b - a;
    double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
```

```
        p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
    if (sum*sum < d2 || dif*dif > d2) return false;
    P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
    *out = {mid + per, mid - per};
    return true;
}
```

CircleTangents.h

**Description:** Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

```
"Point.h" d41d8c, 13 lines
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
    P d = c2 - c1;
    double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
    if (d2 == 0 || h2 < 0) return {};
    vector<pair<P, P>> out;
    for (double sign : {-1, 1}) {
        P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
        out.push_back({c1 + v * r1, c2 + v * r2});
    }
    if (h2 == 0) out.pop_back();
    return out;
}
```

CirclePolygonIntersection.h

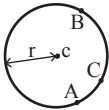
**Description:** Returns the area of the intersection of a circle with a ccw polygon. **Time:**  $\mathcal{O}(n)$

```
"../content/xepelin-geometry/Point.h" d41d8c, 19 lines
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
    auto tri = [&](P p, P q) {
        auto r2 = r * r / 2;
        P d = q - p;
        auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
        auto det = a * a - b;
        if (det <= 0) return arg(p, q) * r2;
        auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
        if (t < 0 || 1 <= s) return arg(p, q) * r2;
        P u = p + d * s, v = p + d * t;
        return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
    };
    auto sum = 0.0;
    rep(i,0,sz(ps))
        sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
    return sum;
}
```

circumcircle.h

**Description:**

The circumcircle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



```
"Point.h" d41d8c, 9 lines
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
    return (B-A).dist()*(C-B).dist()*(A-C).dist() /
        abs((B-A).cross(C-A))/2;
}
```

```
P ccCenter(const P& A, const P& B, const P& C) {
    P b = C-A, c = B-A;
    return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

**Description:** Computes the minimum circle that encloses a set of points. **Time:** expected  $\mathcal{O}(n)$

```
"circumcircle.h" d41d8c, 17 lines
pair<P, double> mec(vector<P> ps) {
    shuffle(all(ps), mt19937(time(0)));
    P o = ps[0];
    double r = 0, EPS = 1 + 1e-8;
    rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
        o = ps[i], r = 0;
        rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
            o = (ps[i] + ps[j]) / 2;
            r = (o - ps[i]).dist();
            rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
                o = ccCenter(ps[i], ps[j], ps[k]);
                r = (o - ps[i]).dist();
            }
        }
    }
    return {o, r};
}
```

9.3 Polygons

InsidePolygon.h

**Description:** Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow. **Usage:** vector<P> v = {P{4,4}, P{1,2}, P{2,1}}; bool in = inPolygon(v, P{3, 3}, false); **Time:**  $\mathcal{O}(n)$

```
"Point.h", "OnSegment.h", "SegmentDistance.h" d41d8c, 11 lines
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
    int cnt = 0, n = sz(p);
    rep(i,0,n) {
        P q = p[(i + 1) % n];
        if (onSegment(p[i], q, a)) return !strict;
        //or: if (segDist(p[i], q, a) <= eps) return !strict;
        cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
    }
    return cnt;
}
```

PolygonArea.h

**Description:** Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

```
"Point.h" d41d8c, 6 lines
template<class T>
T polygonArea2(vector<Point<T>>& v) {
    T a = v.back().cross(v[0]);
    rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
    return a;
}
```

PolygonCenter.h

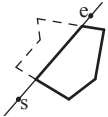
**Description:** Returns the center of mass for a polygon. **Time:**  $\mathcal{O}(n)$

```
"Point.h" d41d8c, 9 lines
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
    P res(0, 0); double A = 0;
```

```
    for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
        res = res + (v[i] + v[j]) * v[j].cross(v[i]);
        A += v[j].cross(v[i]);
    }
    return res / A / 3;
}
```

PolygonCut.h

**Description:** Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away. **Usage:** vector<P> p = ...; p = polygonCut(p, P(0,0), P(1,0));



```
"Point.h", "LineIntersection.h" d41d8c, 13 lines
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
    vector<P> res;
    rep(i,0,sz(poly)) {
        P cur = poly[i], prev = i ? poly[i-1] : poly.back();
        bool side = s.cross(e, cur) < 0;
        if (side != (s.cross(e, prev) < 0))
            res.push_back(lineInter(s, e, cur, prev).second);
        if (side)
            res.push_back(cur);
    }
    return res;
}
```

ConvexHull.h

**Description:** Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull. **Time:**  $\mathcal{O}(n \log n)$



```
"Point.h" d41d8c, 13 lines
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
    if (sz(pts) <= 1) return pts;
    sort(all(pts));
    vector<P> h(sz(pts)+1);
    int s = 0, t = 0;
    for (int it = 2; it--; s = --t, reverse(all(pts)))
        for (P p : pts) {
            while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
            h[t++] = p;
        }
    return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

HullDiameter.h

**Description:** Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points). **Time:**  $\mathcal{O}(n)$

```
"Point.h" d41d8c, 12 lines
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
    int n = sz(S), j = n < 2 ? 0 : 1;
    pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
    rep(i,0,j)
        for (; j = (j + 1) % n) {
            res = max(res, {{S[i] - S[j]}.dist2(), {S[i], S[j]}});
            if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
                break;
        }
    return res.second;
}
```

PointInsideHull.h

**Description:** Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

**Time:**  $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h"	d41d8c, 14 lines
--------------------------------------	------------------

typedef Point<ll> P;

```
bool inHull(const vector<P>& l, P p, bool strict = true) {
    int a = 1, b = sz(l) - 1, r = !strict;
    if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
    if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
    if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
    }
    return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

**Description:** Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon:  $\bullet(-1, -1)$  if no collision,  $\bullet(i, -1)$  if touching the corner  $i$ ,  $\bullet(i, i)$  if along side  $(i, i + 1)$ ,  $\bullet(i, j)$  if crossing sides  $(i, i + 1)$  and  $(j, j + 1)$ . In the last case, if a corner  $i$  is crossed, this is treated as happening on side  $(i, i + 1)$ . The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.

**Time:**  $\mathcal{O}(\log n)$

"Point.h"	d41d8c, 39 lines
-----------	------------------

```
#define cmp(i, j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
    int n = sz(poly), lo = 0, hi = n;
    if (extr(0)) return 0;
    while (lo + 1 < hi) {
        int m = (lo + hi) / 2;
        if (extr(m)) return m;
        int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
        (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
    }
    return lo;
}
```

```
#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
    int endA = extrVertex(poly, (a - b).perp());
    int endB = extrVertex(poly, (b - a).perp());
    if (cmpL(endA) < 0 || cmpL(endB) > 0)
        return {-1, -1};
    array<int, 2> res;
    rep(i, 0, 2) {
        int lo = endB, hi = endA, n = sz(poly);
        while ((lo + 1) % n != hi) {
            int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
            (cmpL(m) == cmpL(endB) ? lo : hi) = m;
        }
        res[i] = (lo + !cmpL(hi)) % n;
        swap(endA, endB);
    }
    if (res[0] == res[1]) return {res[0], -1};
    if (!cmpL(res[0]) && !cmpL(res[1]))
        switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
            case 0: return {res[0], res[0]};
            case 2: return {res[1], res[1]};
        }
```

```
}
return res;
}
```

9.4 Misc. Point Set Problems

ClosestPair.h

**Description:** Finds the closest pair of points.

**Time:**  $\mathcal{O}(n \log n)$

"Point.h"	d41d8c, 17 lines
-----------	------------------

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
    assert(sz(v) > 1);
    set<P> S;
    sort(all(v), [](P a, P b) { return a.y < b.y; });
    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
    int j = 0;
    for (P p : v) {
        P d{1 + (ll)sqrt(ret.first), 0};
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
        for (; lo != hi; ++lo)
            ret = min(ret, {( *lo - p).dist2(), { *lo, p }});
        S.insert(p);
    }
    return ret.second;
}
```

kdTree.h

**Description:** KD-tree (2d, can be extended to 3d)

"Point.h"	d41d8c, 63 lines
-----------	------------------

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
    P pt; // if this is a leaf, the single point in it
    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
    Node *first = 0, *second = 0;

    T distance(const P& p) { // min squared distance to a point
        T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
        T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
        return (P(x, y) - p).dist2();
    }

    Node(vector<P>&& vp) : pt(vp[0]) {
        for (P p : vp) {
            x0 = min(x0, p.x); x1 = max(x1, p.x);
            y0 = min(y0, p.y); y1 = max(y1, p.y);
        }
        if (vp.size() > 1) {
            // split on x if width >= height (not ideal...)
            sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
            // divide by taking half the array for each child (not
            // best performance with many duplicates in the middle)
            int half = sz(vp)/2;
            first = new Node({vp.begin(), vp.begin() + half});
            second = new Node({vp.begin() + half, vp.end()});
        }
    }
};

struct KDTree {
    Node* root;
    KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}
};
```

```
pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
        // uncomment if we should not find the point itself:
        // if (p == node->pt) return {INF, P()};
        return make_pair((p - node->pt).dist2(), node->pt);
    }
```

```
Node *f = node->first, *s = node->second;
T bfirst = f->distance(p), bsec = s->distance(p);
if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);
```

```
// search closest side first, other side if needed
auto best = search(f, p);
if (bsec < best.first)
    best = min(best, search(s, p));
return best;
}
```

```
// find nearest point to a point, and its squared distance
// (requires an arbitrary operator< for Point)
pair<T, P> nearest(const P& p) {
    return search(root, p);
}
};
```

FastDelaunay.h

**Description:** Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order  $\{t[0][0], t[0][1], t[0][2], t[1][0], \dots\}$ , all counter-clockwise.

**Time:**  $\mathcal{O}(n \log n)$

"Point.h"	d41d8c, 88 lines
-----------	------------------

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t ll1; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX, LLONG_MAX); // not equal to any other point
```

```
struct Quad {
    Q rot, o; P p = arb; bool mark;
    P& F() { return r()->p; }
    Q& r() { return rot->rot; }
    Q prev() { return rot->o->rot; }
    Q next() { return r()->prev(); }
} *H;
```

```
bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    ll1 p2 = p.dist2(), A = a.dist2()-p2,
        B = b.dist2()-p2, C = c.dist2()-p2;
    return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
```

```
Q makeEdge(P orig, P dest) {
    Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
    H = r->o; r->r()->r() = r;
    rep(i, 0, 4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest;
    return r;
}
```

```
void splice(Q a, Q b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}

Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next());
    splice(q->r(), b);
    return q;
}
```

```
pair<Q,Q> rec(const vector<P>& s) {
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
        if (sz(s) == 2) return { a, a->r() };
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]);
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
    }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
    Q A, B, ra, rb;
    int half = sz(s) / 2;
    tie(ra, A) = rec({all(s) - half});
    tie(B, rb) = rec({sz(s) - half + all(s)});
    while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
        (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
    Q base = connect(B->r(), A);
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
        Q t = e->dir; \
        splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); \
        e->o = H; H = e; e = t; \
    }
    for (;;) {
        DEL(LC, base->r(), o); DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
            base = connect(RC, base->r());
        else
            base = connect(base->r(), LC->r());
    }
    return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
    sort(all(pts)); assert(unique(all(pts)) == pts.end());
    if (sz(pts) < 2) return {};
    Q e = rec(pts).first;
    vector<Q> q = {e};
    int qi = 0;
    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
    q.push_back(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear();
    while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
    return pts;
}
```

sortPoints.h  
Description: Sort set of points around some point.  
Time:  $\mathcal{O}(n\log n)$

d41d8c, 6 lines

```
sort(points.begin(), points.end(), [&](const Point& a, const
    Point& b) {
    Point v1 = a - center, v2 = b - center;
    int sidel = sideOf(center, center + Point(1, 0), a);
    int side2 = sideOf(center, center + Point(1, 0), b);
    return {sidel, -v1.cross(v2), v1.dist2()} < {side2, 0, v2.
        dist2()};
});
```

halfplanes.h  
Description: Halfplanes intersection.  
Time:  $\mathcal{O}(n\log n)$

d41d8c, 96 lines

```
namespace hpi {
    const ld eps = 1e-8;
    typedef pair<ld, ld> pi;

    bool z(ld x) { return fabs(x) < eps; }

    ld ccw(pi a, pi b, pi c) {
        return (b.fr - a.fr) * (c.sc - a.sc) - (b.sc - a.sc) * (c.
            fr - a.fr);
    }

    struct line {
        ld a, b, c;

        bool operator<(const line &l) const {
            bool f1 = pi(a, b) > pi(0, 0);
            bool f2 = pi(l.a, l.b) > pi(0, 0);
            if (f1 != f2) return f1 > f2;
            ld t = ccw(pi(0, 0), pi(a, b), pi(l.a, l.b));
            return z(t) ? c * hypot(l.a, l.b) < l.c * hypot(a, b) : t
                > 0;
        }

        pi sl() { return pi(a, b); }
    };

    pi cross(line a, line b) {
        ld det = a.a * b.b - b.a * a.b;
        return pi((a.c * b.b - a.b * b.c) / det, (a.a * b.c - a.c *
            b.a) / det);
    }

    bool bad(line a, line b, line c) {
        if (ccw(pi(0, 0), a.sl(), b.sl()) <= 0) return false;
        pi cs = cross(a, b);
        return cs.first * c.a + cs.second * c.b >= c.c;
    }

    bool solve(vector<line>& v, vector<pi>* solution) { // ax +
        by <= c;
        sort(v.begin(), v.end());
        deque<line> d;
        for (auto &i: v) {
            if (!d.empty() && z(ccw(pi(0, 0), d.back().sl(), i.sl())))
                continue;
            while (d.size() >= 2 && bad(d[d.size() - 2], d.back(), i)
                ) d.pop_back();
            while (d.size() >= 2 && bad(i, d[0], d[1])) d.pop_front()
                ;
            d.push_back(i);
        }
        while (d.size() > 2 && bad(d[d.size() - 2], d.back(), d[0])
            ) d.pop_back();
        while (d.size() > 2 && bad(d.back(), d[0], d[1])) d.
            pop_front();
        if (solution != nullptr) solution->clear();
        for (int i = 0; i < d.size(); i++) {
            line cur = d[i], nxt = d[(i + 1) % d.size()];
            if (ccw(pi(0, 0), cur.sl(), nxt.sl()) <= eps) return
                false;
            if (solution != nullptr) solution->emplace_back(cross(cur
                , nxt));
        }
        v = vector<line>(d.begin(), d.end());
        return true;
    }
}
```

```
}
}

// halfplane of d(p, v1) \le d(p, v2), ax + by <= c
hpi::line nearest(const vect<ld>& v1, const vect<ld>& v2) {
    return hpi::line{
        (ld)((v2.x - v1.x) * 2), (ld)((v2.y - v1.y) * 2),
        (ld)(v2.x * v2.x + v2.y * v2.y - v1.y * v1.y - v1.x * v1.x)
    };
}

// halfplane on the left side of v1v2, ax + by <= c
hpi::line left_side(const vect<ld>& v1, const vect<ld>& v2) {
    return hpi::line{
        (ld)(v2.y - v1.y), (ld)(v1.x - v2.x),
        (ld)(v1.x * v2.y - v2.x * v1.y)
    };
}

const int X = 1e9;

// points should be different !!!
vector<vector<hpi::line>> voronoi(const vector<vect<ld>>& v) {
    vector<vector<hpi::line>> res;
    res.reserve(v.size());
    for (int i = 0; i < (int)v.size(); i++) {
        vector<hpi::line> lines;
        lines.reserve((int)v.size() + 3);
        lines.emplace_back(hpi::line{-1, 0, X});
        lines.emplace_back(hpi::line{1, 0, X});
        lines.emplace_back(hpi::line{0, -1, X});
        lines.emplace_back(hpi::line{0, 1, X});
        for (int j = 0; j < (int)v.size(); j++) {
            if (j != i) {
                lines.emplace_back(nearest(v[i], v[j]));
            }
        }
        hpi::solve(lines, nullptr);
        res.emplace_back(lines);
    }
    return res;
}
```

halfplaneLinear.h  
Description: Halfplane intersection; get one point of intersection  
Time:  $\mathcal{O}(n)$

d41d8c, 75 lines

```
template <typename T>
bool intersection(const line<T>& l1, const line<T>& l2, vect<ld
    >& p) {
    auto pr = l1.a * l2.b - l1.b * l2.a;
    if (abs(pr) == 0) { return false; }
    auto prx = l1.b * l2.c - l1.c * l2.b;
    auto pry = l1.c * l2.a - l1.a * l2.c;
    p.x = (ld)prx / pr;
    p.y = (ld)pry / pr;
    return true;
}

// ax + by + c >= 0
template <typename T>
bool checkPlaneInt(vector<line<T>> l, vect<ld>& A) {
    shuffle(l.begin(), l.end(), rnd);
    auto f = [&](int i, const vect<ld>& a) {
        return a.x * l[i].a + a.y * l[i].b + l[i].c;
    };
    auto some_point = [&](int i) {
        if (abs(l[i].a) > abs(l[i].b)) {
            return vect<ld>(-(ld)l[i].c / l[i].a, 0.0);
        }
    };
}
```

```
    } else {
        return vect<ld>(0.0, -(ld)l[i].c / l[i].b);
    }
};
A = some_point(0);
for (int i = 1; i < (int)l.size(); i++) {
    if (f(i, A) < -eps) {
        bool has_mn = false;
        bool has_mx = false;
        vect<ld> mn, mx;
        A = some_point(i);
        for (int j = 0; j < i; j++) {
            auto vj = l[j].normal();
            auto vi = l[i].normal();
            auto vec = (vj ^ vi);
            if (abs(vec) < eps) {
                auto p = some_point(i);
                if ((vj * vi) < -eps && f(j, p) < -eps) {
                    return false;
                }
            }
        }
    } else {
        vect<ld> cur;
        intersection(l[i], l[j], cur);
        if (vec < 0) {
            if (!has_mx || f(j, mx) < 0) {
                mx = cur;
            }
            has_mx = true;
            if (has_mn && f(j, mn) < -eps) {
                return false;
            }
        }
    }
}
}
}
if (has_mx && has_mn) {
    if (make_pair(mx.y, mx.x) > make_pair(mn.y, mn.x)) {
        A = mx;
    } else {
        A = mn;
    }
}
} else if (has_mx) A = mx;
else if (has_mn) A = mn;
}
}
return true;
}
```

triangulation.h

Description: Triangulation of polygon.

Time: idk

---

*// polygon must be given counterclockwise (can be checked with signed area), points must be different !!!*

```
vector<tuple<int, int, int>> triangulation(const vector<vect<ll>> &v) {
    vector<tuple<int, int, int>> ans;
    int n = v.size();
    vector<bool> used(n, false);
    queue<int> all;
    for (int i = 0; i < n; i++) all.push(i);
```

```
for (int iter = 0; iter < n - 2; iter++) {
    while (true) {
        if (all.empty()) return ans;
        int p = all.front();
        all.pop();
        if (used[p]) continue;
        int x = (p + n - 1) % n;
        while (used[x]) x = (x + n - 1) % n;
        int y = p;
        int z = (p + 1) % n;
        while (used[z]) z = (z + 1) % n;
        if (((v[x] - v[y]) ^ (v[z] - v[y])) >= 0) continue;
        bool bad = false;
        for (int i = 0; i < n; i++) {
            if (!used[i] && i != x && i != y && i != z) {
                if (area(v[x], v[y], v[z]) ==
                    area(v[x], v[y], v[i]) + area(v[x], v[z], v[i]) +
                    area(v[z], v[y], v[i])) {
                    bad = true;
                    break;
                }
            }
        }
        if (bad) continue;
        ans.emplace_back(x, y, z);
        used[y] = true;
        all.push(x);
        all.push(z);
        break;
    }
}
return ans;
}
```

### 9.5 3D PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

---

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilst) {
    double v = 0;
    for (auto i : trilst) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
    return v / 6;
}
```

### Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

---

```
template<class T> struct Point3D {
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
    bool operator<(R p) const {
        return tie(x, y, z) < tie(p.x, p.y, p.z);
    }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z);
    }
    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
    P operator*(T d) const { return P(x*d, y*d, z*d); }
    P operator/(T d) const { return P(x/d, y/d, z/d); }
    T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
    P cross(R p) const {
        return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
    }
    T dist2() const { return x*x + y*y + z*z; }
    double dist() const { return sqrt((double)dist2()); }
```

```
//Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
double phi() const { return atan2(y, x); }
//Zenith angle (latitude) to the z-axis in interval [0, pi]
double theta() const { return atan2(sqrt(x*x+y*y),z); }
P unit() const { return *this/(T)dist(); } //makes dist()=1
//returns unit vector normal to *this and p
P normal(P p) const { return cross(p).unit(); }
//returns point rotated 'angle' radians ccw around axis
P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u.dot(u)*(1-c) + (*this)*c - cross(u)*s;
}
};
```

### 3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. \*No four points must be coplanar\*, or else random results will be returned. All faces will point outwards.

Time:  $\mathcal{O}(n^2)$

---

```
typedef Point3D<double> P3;

struct PR {
    void ins(int x) { (a == -1 ? a : b) = x; }
    void rem(int x) { (a == x ? a : b) = -1; }
    int cnt() { return (a != -1) + (b != -1); }
    int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
    assert(sz(A) >= 4);
    vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
    vector<F> FS;
    auto mf = [&](int i, int j, int k, int l) {
        P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
        if (q.dot(A[l]) > q.dot(A[i]))
            q = q * -1;
        F f{q, i, j, k};
        E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
        FS.push_back(f);
    };
    rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
        mf(i, j, k, 6 - i - j - k);

    rep(i,4,sz(A)) {
        rep(j,0,sz(FS)) {
            F f = FS[j];
            if (f.q.dot(A[i]) > f.q.dot(A[f.a])) {
                E(a,b).rem(f.c);
                E(a,c).rem(f.b);
                E(b,c).rem(f.a);
                swap(FS[j--], FS.back());
                FS.pop_back();
            }
        }
        int nw = sz(FS);
        rep(j,0,nw) {
            F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
            C(a, b, c); C(a, c, b); C(b, c, a);
        }
    }
    for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
        A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
    return FS;
};
```

sphericalDistance.h  
**Description:** Returns the shortest distance on the sphere with radius  $r$  between the points with azimuthal angles (longitude)  $f_1$  ( $\phi_1$ ) and  $f_2$  ( $\phi_2$ ) from  $x$  axis and zenith angles (latitude)  $t_1$  ( $\theta_1$ ) and  $t_2$  ( $\theta_2$ ) from  $z$  axis ( $0 =$  north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows.  $dx \cdot radius$  is then the difference between the two points in the  $x$  direction and  $d \cdot radius$  is the total distance between the points.

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz);
    return radius*2*asin(d/2);
}
```

## Various (10)

### 10.1 Intervals

IntervalContainer.h  
**Description:** Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).  
**Time:**  $\mathcal{O}(\log N)$

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
    if (L == R) return is.end();
    auto it = is.lower_bound({L, R}), before = it;
    while (it != is.end() && it->first <= R) {
        R = max(R, it->second);
        before = it = is.erase(it);
    }
    if (it != is.begin() && (--it)->second >= L) {
        L = min(L, it->first);
        R = max(R, it->second);
        is.erase(it);
    }
    return is.insert(before, {L,R});
}
```

```
void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second;
    if (it->first == L) is.erase(it);
    else (int&)it->second = L;
    if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h  
**Description:** Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add || R.empty(). Returns empty set on failure (or if G is empty).  
**Time:**  $\mathcal{O}(N \log N)$

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
    vi S(sz(I)), R;
    iota(all(S), 0);
    sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
    T cur = G.first;
    int at = 0;
    while (cur < G.second) { // (A)
        pair<T, int> mx = make_pair(cur, -1);
```

```
        while (at < sz(I) && I[S[at]].first <= cur) {
            mx = max(mx, make_pair(I[S[at]].second, S[at]));
            at++;
        }
        if (mx.second == -1) return {};
        cur = mx.first;
        R.push_back(mx.second);
    }
    return R;
}
```

ConstantIntervals.h  
**Description:** Split a monotone function on  $[from, to]$  into a minimal set of half-open intervals on which it has the same value. Runs a callback  $g$  for each such interval.  
**Usage:** `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`  
**Time:**  $\mathcal{O}(k \log \frac{n}{k})$

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
    if (p == q) return;
    if (from == to) {
        g(i, to, p);
        i = to; p = q;
    } else {
        int mid = (from + to) >> 1;
        rec(from, mid, f, g, i, p, f(mid));
        rec(mid+1, to, f, g, i, p, q);
    }
}

template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
    if (to <= from) return;
    int i = from; auto p = f(i), q = f(to-1);
    rec(from, to-1, f, g, i, p, q);
    g(i, to, q);
}
```

### 10.2 Debugging tricks

`-Wl,-stack_size -Wl,1000000` local stack size.

`signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions).

`_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).

`feenableexcept(29);` kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

### 10.3 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

#### 10.3.1 Bit hacks

$x \ \& \ -x$  is the least bit in  $x$ .

`for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of  $m$  (except  $m$  itself).

`c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after  $x$  with the same number of bits set.

`rep(b,0,K) rep(i,0,(1 << K))`  
`if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

`#pragma GCC target ("avx2")` can double performance of `10.3.3` `FastMod` causes crashes on old machines.

FastMod.h  
**Description:** Compute  $a \% b$  about 5 times faster than usual, where  $b$  is constant but not known at compile time. Returns a value congruent to  $a \pmod b$  in the range  $[0, 2b)$ .

```
typedef unsigned long long ull;
struct FastMod {
    ull b, m;
    integer overflows (but is really slow) b) {}
    ull reduce(ull a) { // a % b + (0 or b)
        return a - (ull)((__uint128_t(m) * a) >> 64) * b;
    }
};
```

FastInput.h  
**Description:** Read an integer from stdin. Usage requires your program to pipe in input from file.  
**Usage:** `./a.out < input.txt`  
**Time:** About 5x as fast as `cin/scanf`.

```
inline char gc() { // like getchar()
    static char buf[1 << 16];
    static size_t bc, be;
    if (bc >= be) {
        buf[0] = 0, bc = 0;
        be = fread(buf, 1, sizeof(buf), stdin);
    }
    return buf[bc++]; // returns 0 on EOF
}

int readInt() {
    int a, c;
    while ((a = gc()) < 40);
    if (a == '-') return -readInt();
    while ((c = gc()) >= 48) a = a * 10 + c - 48;
    return a - 48;
}
```

BumpAllocator.h  
**Description:** When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
    static size_t i = sizeof buf;
    assert(s < i);
    return (void*)&buf[i -= s];
}
void operator delete(void*) {}
```

SmallPtr.h

Description: A 32-bit pointer that points into BumpAllocator memory.

"BumpAllocator.h" d41d8c, 10 lines

template<class T> struct ptr {  
 unsigned ind;  
 ptr(T\* p = 0) : ind(p ? unsigned((char\*)p - buf) : 0) {  
 assert(ind < sizeof buf);  
 }  
 T& operator\*() const { return \*(T\*)(buf + ind); }  
 T\* operator->() const { return &\*&this; }  
 T& operator[](int a) const { return (&\*&this)[a]; }  
 explicit operator bool() const { return ind; }  
};

BumpAllocatorSTL.h

Description: BumpAllocator for STL containers.

Usage: vector<vector<int, small<int>>> ed(N); d41d8c, 14 lines

char buf[450 << 20] alignas(16);  
size\_t buf\_ind = sizeof buf;

template<class T> struct small {  
 typedef T value\_type;  
 small() {}  
 template<class U> small(const U&) {}  
 T\* allocate(size\_t n) {  
 buf\_ind -= n \* sizeof(T);  
 buf\_ind &= 0 - alignof(T);  
 return (T\*)(buf + buf\_ind);  
 }  
 void deallocate(T\*, size\_t) {}  
};

pragma.h

d41d8c, 2 lines

#pragma GCC optimize("O3,unroll-loops")  
#pragma GCC target ("avx2,bmi,bmi2,lzcnt,popcnt")

SIMD.h

Description: Cheat sheet of SSE/AVX intrinsics, for doing arithmetic on several numbers at once. Can provide a constant factor improvement of about 4, orthogonal to loop unrolling. Operations follow the pattern "\_mm(256)?\_name\_(si(128|256)|epi(8|16|32|64)|pd|ps)". Not all are described here; grep for \_mm\_ in /usr/lib/gcc/\*/4.9/include/ for more. If AVX is unsupported, try 128-bit operations, "emmintrin.h" and #define \_SSE\_ and \_MMX\_ before including it. For aligned memory use \_mm\_malloc(size, 32) or int buf[N] alignas(32), but prefer loadu/storeu.

d41d8c, 43 lines

#pragma GCC target ("avx2") // or sse4.1  
#include "emmintrin.h"

typedef \_\_m256i mi;  
#define L(x) \_mm256\_loadu\_si256((mi\*)&(x))

```
// High-level/specific methods:  
// load(u)?_si256, store(u)?_si256, setzero_si256, _mm_malloc  
// blendv_(epi8|ps|pd) (z?y:x), movemask_epi8 (hibits of bytes)  
// i32gather_epi32(addr, x, 4): map addr[] over 32-b parts of x  
// sad_epu8: sum of absolute differences of u8, outputs 4xi64  
// maddubs_epi16: dot product of unsigned i7's, outputs 16xi15  
// madd_epi16: dot product of signed i16's, outputs 8xi32  
// extractf128_si256(, i) (256->128), cvtsi128_si32 (128->lo32)  
// permute2f128_si256(x,x,1) swaps 128-bit lanes  
// shuffle_epi32(x, 3*64+2*16+1*4+0) == x for each lane  
// shuffle_epi8(x, y) takes a vector instead of an mm  
  
// Methods that work with most data types (append e.g. _epi32):  
// set1, blend (i8?x:y), add, adds (sat.), mullo, sub, and/or,
```

```
// andnot, abs, min, max, sign(1,x), cmp(gt|eq), unpack(lo|hi)  
  
int sumi32(mi m) { union {int v[8]; mi m;} u; u.m = m;  
    int ret = 0; rep(i,0,8) ret += u.v[i]; return ret; }  
mi zero() { return _mm256_setzero_si256(); }  
mi one() { return _mm256_set1_epi32(-1); }  
bool all_zero(mi m) { return _mm256_testz_si256(m, m); }  
bool all_one(mi m) { return _mm256_testc_si256(m, one()); }  
  
ll example_filteredDotProduct(int n, short* a, short* b) {  
    int i = 0; ll r = 0;  
    mi zero = _mm256_setzero_si256(), acc = zero;  
    while (i + 16 <= n) {  
        mi va = L(a[i]), vb = L(b[i]); i += 16;  
        va = _mm256_and_si256(_mm256_cmpgt_epi16(vb, va), va);  
        mi vp = _mm256_madd_epi16(va, vb);  
        acc = _mm256_add_epi64(_mm256_unpacklo_epi32(vp, zero),  
                               _mm256_add_epi64(acc, _mm256_unpackhi_epi32(vp, zero)));  
    }  
    union {ll v[4]; mi m;} u; u.m = acc; rep(i,0,4) r += u.v[i];  
    for (;i<n;++i) if (a[i] < b[i]) r += a[i]*b[i]; //<- equiv  
    return r;  
}
```

troubleshoot.txt

52 lines

Pre-submit:  
Write a few simple test cases if sample is not enough.  
Are time limits close? If so, generate max cases.  
Is the memory usage fine?  
Could anything overflow?  
Make sure to submit the right file.

Wrong answer:  
Print your solution! Print debug output, as well.  
Are you clearing all data structures between test cases?  
Can your algorithm handle the whole range of input?  
Read the full problem statement again.  
Do you handle all corner cases correctly?  
Have you understood the problem correctly?  
Any uninitialized variables?  
Any overflows?  
Confusing N and M, i and j, etc.?  
Are you sure your algorithm works?  
What special cases have you not thought of?  
Are you sure the STL functions you use work as you think?  
Add some assertions, maybe resubmit.  
Create some testcases to run your algorithm on.  
Go through the algorithm for a simple case.  
Go through this list again.  
Explain your algorithm to a teammate.  
Ask the teammate to look at your code.  
Go for a small walk, e.g. to the toilet.  
Is your output format correct? (including whitespace)  
Rewrite your solution from the start or let a teammate do it.

Runtime error:  
Have you tested all corner cases locally?  
Any uninitialized variables?  
Are you reading or writing outside the range of any vector?  
Any assertions that might fail?  
Any possible division by 0? (mod 0 for example)  
Any possible infinite recursion?  
Invalidated pointers or iterators?  
Are you using too much memory?  
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:  
Do you have any possible infinite loops?

What is the complexity of your algorithm?  
Are you copying a lot of unnecessary data? (References)  
How big is the input and output? (consider scanf)  
Avoid vector, map. (use arrays/unordered\_map)  
What do your teammates think about your algorithm?  
  
Memory limit exceeded:  
What is the max amount of memory your algorithm should need?  
Are you clearing all data structures between test cases?



# Techniques (A)

techniques.txt	161 lines
Recursion	
Divide and conquer	
Finding interesting points in N log N	
Algorithm analysis	
Master theorem	
Amortized time complexity	
Greedy algorithm	
Scheduling	
Max contiguous subvector sum	
Invariants	
Huffman encoding	
Graph theory	
Dynamic graphs (extra book-keeping)	
Breadth first search	
Depth first search	
* Normal trees / DFS trees	
Dijkstra's algorithm	
MST: Prim's algorithm	
Bellman-Ford	
Konig's theorem and vertex cover	
Min-cost max flow	
Lovasz toggle	
Matrix tree theorem	
Maximal matching, general graphs	
Hopcroft-Karp	
Hall's marriage theorem	
Graphical sequences	
Floyd-Warshall	
Euler cycles	
Flow networks	
* Augmenting paths	
* Edmonds-Karp	
Bipartite matching	
Min. path cover	
Topological sorting	
Strongly connected components	
2-SAT	
Cut vertices, cut-edges and biconnected components	
Edge coloring	
* Trees	
Vertex coloring	
* Bipartite graphs (=> trees)	
* 3^n (special case of set cover)	
Diameter and centroid	
K'th shortest path	
Shortest cycle	
Dynamic programming	
Knapsack	
Coin change	
Longest common subsequence	
Longest increasing subsequence	
Number of paths in a dag	
Shortest path in a dag	
Dynprog over intervals	
Dynprog over subsets	
Dynprog over probabilities	
Dynprog over trees	
3^n set cover	
Divide and conquer	
Knuth optimization	
Convex hull optimizations	
RMQ (sparse table a.k.a 2^k-jumps)	
Bitonic cycle	
Log partitioning (loop over most restricted)	
Combinatorics	

Computation of binomial coefficients
Pigeon-hole principle
Inclusion/exclusion
Catalan number
Pick's theorem
Number theory
Integer parts
Divisibility
Euclidean algorithm
Modular arithmetic
* Modular multiplication
* Modular inverses
* Modular exponentiation by squaring
Chinese remainder theorem
Fermat's little theorem
Euler's theorem
Phi function
Frobenius number
Quadratic reciprocity
Pollard-Rho
Miller-Rabin
Hensel lifting
Vieta root jumping
Game theory
Combinatorial games
Game trees
Mini-max
Nim
Games on graphs
Games on graphs with loops
Grundy numbers
Bipartite games without repetition
General games without repetition
Alpha-beta pruning
Probability theory
Optimization
Binary search
Ternary search
Unimodality and convex functions
Binary search on derivative
Numerical methods
Numeric integration
Newton's method
Root-finding with binary/ternary search
Golden section search
Matrices
Gaussian elimination
Exponentiation by squaring
Sorting
Radix sort
Geometry
Coordinates and vectors
* Cross product
* Scalar product
Convex hull
Polygon cut
Closest pair
Coordinate-compression
Quadtrees
KD-trees
All segment-segment intersection
Sweeping
Discretization (convert to events and sweep)
Angle sweeping
Line sweeping
Discrete second derivatives
Strings
Longest common substring
Palindrome subsequences

Knuth-Morris-Pratt
Tries
Rolling polynomial hashes
Suffix array
Suffix tree
Aho-Corasick
Manacher's algorithm
Letter position lists
Combinatorial search
Meet in the middle
Brute-force with pruning
Best-first (A*)
Bidirectional search
Iterative deepening DFS / A*
Data structures
LCA (2^k-jumps in trees in general)
Pull/push-technique on trees
Heavy-light decomposition
Centroid decomposition
Lazy propagation
Self-balancing trees
Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
Monotone queues / monotone stacks / sliding queues
Sliding queue using 2 stacks
Persistent segment tree
Answering for a group of elements and having a lower bound on the answer for each element, maybe the max of lower bounds is the answer?