Assignment 6

Problem 1

b). $L = \{\, 0^n \# 0^{2n} \# 0^{3n} \; : \; n \geq 0 \,\}$.

Let's prove L is not context free. Towards a contradiction, assume L is context free. By the pumping lemma for context-free languages (PL4CFL), there exists a pumping length $p > 0$ such that any word w, $|w| \geq p$ can be split into 5 pieces $w = uvxyz$, $|vy| > 0$, $|vxy| \leq p$ such that for any integer $i \geq 0$, the word $w' = uv^ixy^iz$ belongs to language L. Consider the word $w = 0^p \# 0^{2p} \# 0^{3p}$ which satisfies the conditions of the PL4CFL. Let's analyze all possible partitions of w into 5 pieces. Recall that $|vxy| \leq p$ so the substring vxy cannot span more than two consecutive sequences of zeroes.

- Case 1: vxy is within the leftmost sequence of zeroes. Clealy, if we pump it down, say $i = 0$, we'll decrease the number of zeroes in the leftmost sequence without modifying the number of zeroes on the second and third sequences. Therefore, the resulting word will not be in the language.
- Case 2: vx is within the leftmost sequence of zeroes but y includes the first $\#$ (and possibly some zeroes from the second sequence). Clearly, if we pump it up, say $i = 2$, we'll obtain a word $w' = uv^2xy^2z$ which contains three symbols $\#$, and thus w' is not in the language.
- Case 3: v is within the leftmost sequence of zeroes, x contains the leftmost symbol $\#$ and y comprises only symbols from the middle sequence of zeroes. Clearly, if we pump it down, say $i = 0$, the number of zeroes in the first and second sequence will decrease but the number of zeroes in the third sequence will not. Therefore, the word will not be in the language.
- Case 4: vxy is completely within the middle or rightmost sequence of zeroes. These cases are analogous to case 1.
- Case 5: x or y include the rightmost symbol $\#$. This case is analogous to cases 3 or 2 (respectively).

Since for all possible partitions the resulting pumped word w' is not in language L, then we obtain a contradiction. Therefore, L is not context free.

P2

B is context free.

$B = \{0^n 0^{2n} \# 0^{3n}\} \Rightarrow B = \{0^{3n} \# 0^{3n}\}$

CFG:

- S -> ASA | #
- A -> 000

P3

2.31 $B = \{w | w$ is a palindrome over $\{0,1\}$ and w contains an equal number of 0s and 1s$\}$

For a contradiction assume that B is context free. Therefore, B has a pumping length p. Take $s = 0^p 1^{2p} 0^p \in B$ with $|s| > p$. Therefore, there exists $uvxyz$ such that (1) $uv^i xy^i z \in B$ for all $i \geq 0$, (2) $|vy| > 0$ and (3) $|vxy| \leq p$. We will now proceed by cases to show that no matter the values of $uvxyz$ we choose, we will reach a contradiction.

Case 1: vxy consists of only 1s. Then $uv^2xy^2z \notin B$, since it will no longer have the same number of 0s and 1s.
Case 2: vxy contains at least one 0. Then $uv^2xy^2z \notin B$, since it will no longer be a palindrome. This is because from (3), vxy can only contain symbols from the starting 0s or the final 0s, but not both. Thus, after pumping s will have a different number 0s before and after the 1s.

From (2) these are all the cases. In each case we contradict (1). Therefore, B is not context free.

P4

2.32 $C = \{w \in \{0,1,2,3,4\}^* |$ in w the number of 1s equals the number of 2s and the number of 3s equals the number of 4s $\}$

For a contradiction assume that C is context free. Therefore, C has a pumping length p. Take $s = 1^p 3^p 2^p 4^p \in C$ with $|s| > p$. Therefore, there exists $uvxyz$ such that (1) $uv^i xy^i z \in C$ for all $i \geq 0$, (2) $|vy| > 0$ and (3) $|vxy| \leq p$. We will now proceed by cases to show that no matter the values of $uvxyz$ we choose, we will reach a contradiction.

Case 1: vxy contains a 1. Then $uv^2xy^2z \notin C$, since it will no longer have the same number of 1s and 2s. This is because from (3), vxy cannot contain any 2s.
Case 2: vxy contains a 2. Then $uv^2xy^2z \notin C$, since it will no longer have the same number of 1s and 2s. This is because from (3), vxy cannot contain any 1s.
Case 1: vxy contains a 3. Then $uv^2xy^2z \notin C$, since it will no longer have the same number of 3s and 4s. This is because from (3), vxy cannot contain any 4s.
Case 1: vxy contains a 4. Then $uv^2xy^2z \notin C$, since it will no longer have the same number of 3s and 4s. This is because from (3), vxy cannot contain any 3s.

From (2) we see that these are all the cases. In each one we contradict (1). Therefore, C is not context free.