S. Al-Gahtani E. Devour Z. Dong

CHAPTER 1: BASIC CONCEPTS

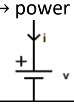
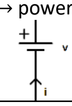
STANDARD SI PREFIXES

10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{0}	10^{3}	10^{6}	10^{9}	10^{12}
pico (p)	nano (n)	micro (μ)	milli (m)		kilo (k)	mega (M)	giga (G)	tera (T)

BASIC ELECTRIC QUANTITIES

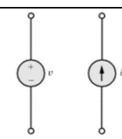
Quantity	Relationship	Unit
Current	$i(t) = \frac{dq(t)}{dt}$	Amperes (A)
Electric Charge	$q(t) = \int_{-\infty}^{t} i(x)\,dx$	Coulombs (C)
Voltage	$v = \frac{dw}{dq}$	Volts (V)
Power	$p = vi = \frac{dw}{dt}$	Watts (W)
Energy	$\Delta w = \int_{t_1}^{t_2} p\,dt = \int_{t_1}^{t_2} vi\,dt$	Joules (J)

THE PASSIVE SIGN CONVENTION

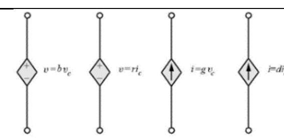
$vi = +p \rightarrow$ power is absorbed	$vi = -p \rightarrow$ power is Supplied
	

$\xrightarrow{\text{Tellegen's Theorem}} \Sigma p = 0\ W$ in an electrical network

ELECTRIC SOURCES



Independent Source: two terminal element that maintain a specified quantity (v or i).



Dependent or Controlled Sources: generate a quantity (v or i) determined by another quantity at specified location in the circuit.

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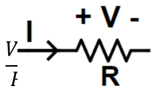
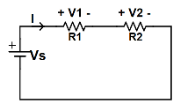
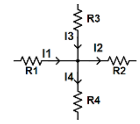
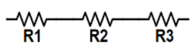
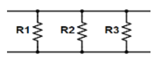
CHAPTER 2: RESISTIVE CIRCUIT

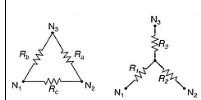
Ohm's Law $V = IR$

$$\xrightarrow{\text{based on Ohm's law}} P = VI = I^2R = \frac{V}{I}$$

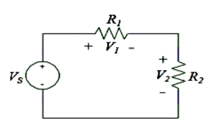


Kirchhoff's Laws	
Kirchhoff's Voltage Law (KVL)	Kirchhoff's Current Law (KCL)
$\sum_{j=1}^{N} v_j\,(t) = 0$ around any loop N: # of voltages in a loop	$\sum_{j=1}^{N} i_j\,(t) = 0$ @ any node, N: # of branches at a node
EX: Single-loop circuit $\sum_{j=1}^{3} V_j = 0 \Rightarrow -V_s + V_1 + V_2 = 0$ 	EX: Single-node circuit $\sum_{j=1}^{4} I_j = 0 \Rightarrow I_1 - I_2 + I_3 - I_4 = 0$ 

The Equivalent Resistance	
Series	Parallel
 $R_s = R_1 + R_2 + R_3 + \cdots$	 $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

Y-Δ Transformation		
$R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$ $R_b = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2}$ $R_c = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}$		$R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$ $R_2 = \frac{R_aR_c}{R_a + R_b + R_c}$ $R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$
$if\ R_a = R_b = R_c\ \&\ R_1 = R_2 = R_3 \Rightarrow R_y = \frac{1}{3}R_\Delta\ \ \&\ \ R_\Delta = 3R_y$		

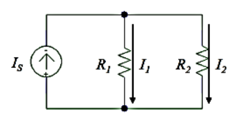
Voltage Division Rule (VDR)



$$V_1 = V_s\frac{R_1}{R_1 + R_2}$$

$$V_2 = V_s\frac{R_2}{R_1 + R_2}$$

Current Division Rule (CDR)



$$I_1 = I_s\frac{R_2}{R_1 + R_2}$$

$$I_2 = I_s\frac{R_1}{R_1 + R_2}$$

Circuit with Dependent Source(s)

Procedures:

STEP 1 Treat the dependent source as an independent source
STEP 2 Apply KVL and/or KCL
STEP 3 Specify its relationship to its controlling quantity

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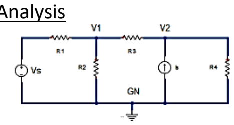
CHAPTER 3: NODAL & LOOP ANALYSIS TECHNIQUES

Nodal Analysis for an N-node Circuit

Procedures:

STEP 1 Determine # of nodes (N) and choose a reference node (GN).
STEP 2 Apply KCL to non GN nodes.
STEP 3 Express currents in terms of node voltages.
STEP 4 Solve for node voltages from the resulting N-1 eqn.

Example of Nodal Analysis



N=3 @ V1,V2 and GN
N-1= 3-1=2 linear eqn

KCL @ V1 node: $\frac{V_1 - V_s}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} = 0$

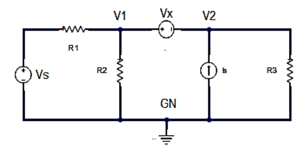
KCL @ V2 node: $\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_4} - I_s = 0$

Nodal Analysis with Supernode

Supernode: a voltage source connected between two voltage nodes.
Procedures:

STEP 1 Determine # of nodes (N) and choose a reference node (GN).
STEP 2 Wright a constraint eqn. for voltage source(s) (Nv) that forms supernode(s).
STEP 3 Apply KCL to non GN nodes.
STEP 4 Express currents in terms of node voltages.
STEP 5 Solve for node voltages from the resulting N-1-Nv eqn.

Example of Nodal Analysis with a Supernode



N=3 @ V_1, V_2 and GN

Nv=1 (1 voltage source)

N-1-Nv=3-1-1= 1 linear eqn.

$V_x = V_1 - V_2$ (constraint eqn.)

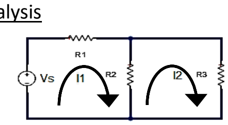
KCL @ supernode: $\frac{V_1 - V_s}{R_1} + \frac{V_1}{R_2} + \frac{V_2}{R_3} - I_s = 0$

Loop Analysis for an N-loop Circuit

Procedures:

STEP 1 Determine # of meshes (M)
STEP 2 Apply KVL to each mesh.
STEP 3 Express voltages in terms of mesh currents.
STEP 4 Solve for mesh currents from the resulting M eqn.

Example of Loop Analysis



M=2 Loops for I_1 and I_2
M=2 linear eqn

KVL around Loop 1: $-V_s + R_1 I_1 + R_2\,(I_1 - I_2) = 0$

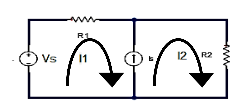
KVL around Loop 2: $R_2\,(I_2 - I_1) + R_3 I_2 = 0$

Loop Analysis for an N-loop Circuit with Supermesh

Supermesh: a current source connected between two current loops.
Procedures:

STEP 1 Determine # of meshes (M)
STEP 2 Wright a constraint eqn. for current source(s) (Mc) that forms supermesh(s).
STEP 3 Apply KVL to each mesh.
STEP 4 Express voltages in terms of mesh currents.
STEP 5 Solve for mesh currents from the resulting M-Mc eqn.

Example of Loop Analysis



M=2 Loos for I_1 and I_2

Mc =1 (1 current source)

M-Mc=2-1=1 linear eqn.

$I_s = I_2 - I_1$ (constraint eqn.)

KVL around Loop 1: $-V_s + R_1 I_1 + R_2\, I_2 = 0$

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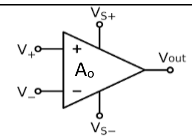
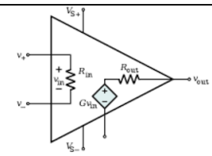
CHAPTER 4: OPERATIONAL AMPLIFIERS (Op-amp)

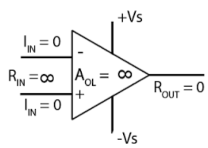
Op-amp

Model		Circuit	
	V_+ : noninverting input voltage V_- : inverting input voltage A_o (G): op-amp gain (Typically: 10^4, 10^6) V_{out}: output voltage $V_{out} = A_o(V_+ - V_-)$		Op-amp is charactrized by: - Hign R_{in} - Low R_{out} - Very high A_o

Ideal Op-amp

$V_+ = V_-$

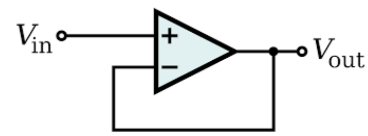
$I_+ = I_- = 0$



Unity Gain Buffer

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{R_{in}}{A_o R_{in} + R_{out}}}$$

$$A_o \gg 1 \ \ and \ \ R_{in} \gg R_{out} \Longrightarrow \frac{V_{out}}{V_{in}} \approx 1$$



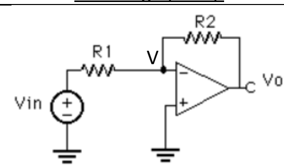
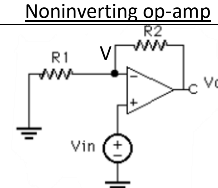
Solving Op-amp Circuit

Procedures:

STEP 1 Use the Ideal op-amp model

STEP 2 Apply nodal analysis to the resulting circuit.

STEP 3 Solve for V_{out} (or V_o) the gain A_v.

Inverting op-amp	Noninverting op-amp
	
STEP 1 $V_+ = V_- = V = 0V$	STEP 1 $V_+ = V_- = V = V_{in}$
STEP 2 Apply nodal analysis @ V $\frac{0 - V_{in}}{R_1} + \frac{0 - V_o}{R_2} = 0$	STEP 2 Apply nodal analysis @ V $\frac{V_{in}}{R_1} + \frac{V_{in} - V_o}{R_2} = 0$
STEP 3 $V_o = -V_{in}\frac{R_2}{R_1}$ $A_v = \frac{V_o}{V_{in}} = -\frac{R_2}{R_1}$	STEP 3 $V_o = V_{in}\frac{R_1 + R_2}{R_1}$ $A_v = \frac{V_o}{V_{in}} = \frac{R_1 + R_2}{R_1}$

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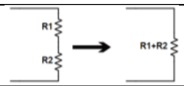
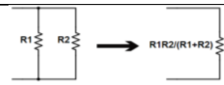
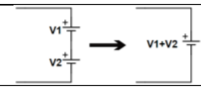
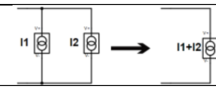
CHAPTER 5: ADDITIONAL ANALYSIS TECHNIQUES

Equivalence

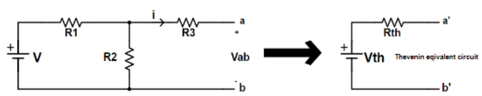
Thevenin (th) Theorem

V_{th} is the open circuit voltage between A and B.

$$i = 0A \rightarrow V_{th} = V_{ab} = V\frac{R_2}{R_1 + R_2}$$

R_{th} is the resistance seen at AB where voltage source(s) is shorted and current source(s) is open.

$$R_{th} = \frac{R_1R_2}{R_1 + R_2} + R_3$$

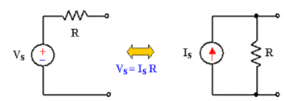


Linearity Property

Homogeneity (Scaling)	Additivity
$V = IR \rightarrow kV = kIR$	$V_1 = I_1R$ & $V_2 = I_2R \rightarrow V_1 + V_2 = (I_1 + I_2)R$

Source Transformation

Interchange between Thevenin and Norton equivalent circuits.



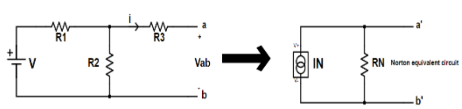
Norton (N) Theorem

I_N is the open circuit voltage V_{AB} divide by R_{th}.

$$I_N = \frac{V_{ab}}{R_N}$$

$R_N = R_{th}$

$$R_N = R_{th} = \frac{R_1R_2}{R_1 + R_2} + R_3$$

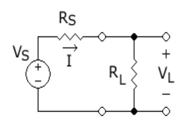


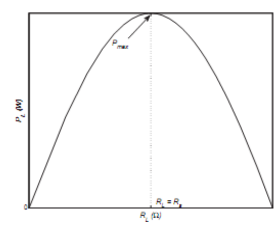
Maximum Power Transfer

$$P_L = \frac{V_L^2}{R_L}$$

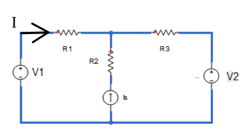
$$V_L = V_s\frac{R_L}{R_L + R_s}$$

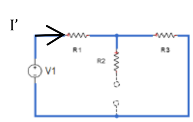
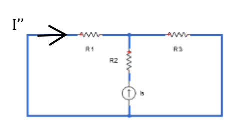
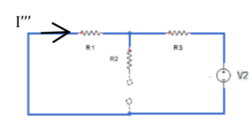
$$R_L = R_s \rightarrow P_{Lmax} = \frac{V_s^2}{R_s} = \frac{V_s^2}{R_L}$$





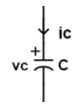
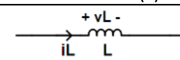
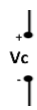
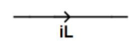
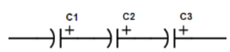
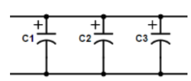
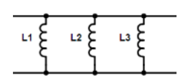
Superposition Theorem



STEP1: Pick a source and remove other: - voltage source: short circuit - current source: open circuit			
STEP 2: Determine individual solution	$I' = \frac{V_1}{R_1 + R_3}$	$I'' = -I_s\frac{R_3}{R_1 + R_3}$	$I''' = \frac{-V_2}{R_1 + R_3}$
STEP 3: Summate the individual solutions	$I = I' + I'' + I'''$		

- # of sources = # of individual solution
- Not valid for nonlinear quantities i.e. power (P)

CHAPTER 6: CAPACITANCE and INDUCTANCE

	Capacitance (C)	Inductance (L)
Symbol		
Unit	Farad (F)	Henry (H)
Main Relationships	$i_c(t) = C\frac{dv_c(t)}{dt}$ $p(t) = Cv_c(t)\frac{dv_c(t)}{dt}$ $w_c(t) = \frac{1}{2}Cv_c^2(t)$	$v_L(t) = L\frac{di_L(t)}{dt}$ $p(t) = Li_L(t)\frac{di_L(t)}{dt}$ $w_L(t) = \frac{1}{2}Li_L^2(t)$
DC Steady State	Open circuit: $i_c(t) = 0$ 	Short circuit: $v_L(t) = 0$ 
Series Combination	$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$ 	$L_s = L_1 + L_2 + L_3 + \cdots$ 
Parallel Combination	$C_p = C_1 + C_2 + C_3 + \cdots$ 	$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots$ 

- Note: $v_c(t)$ and $i_L(t)$ cannot change instantaneously

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CHAPTER 7: 1ST AND 2ND ORDER TRANSIENT CIRCUITS

1st Order Circuits

1st order differential equation $\longrightarrow \frac{dx(t)}{dt} + ax(t) = f(t)$

Solution: $x(t) = x_p(t) + x_c(t) = k_1 + k_2 e^{\frac{-t}{\tau}}, \tau = \frac{1}{a}$

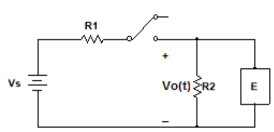
Particular/forced response $\longrightarrow x_p(t) = k_1$

Complementary/natural response $\longrightarrow x_c(t) = k_2 e^{-at}$

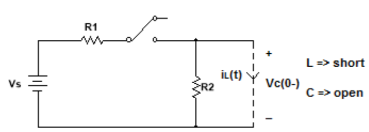
- Solution Procedures:

Step 1: Assume $v_o(t) = k_1 + k_2 e^{\frac{-t}{\tau}}$

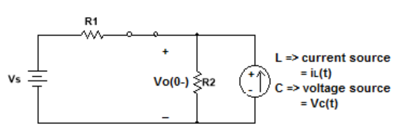
$$v_o(0^+) = k_1 + k_2, x(\infty) = k_1, \tau = \frac{L}{R_{eq}} = CR_{eq}$$



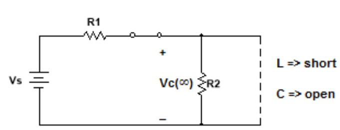
Step 2: Draw circuit before switch changes ($t = 0^-$), Solve $i_L(0^-)$ or $v_C(0^-)$.



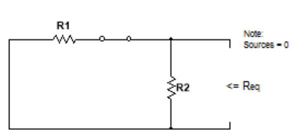
Step 3: Draw circuit after switch changes ($t = 0^+$), Solve $v_o\ (0^+)$.



Step 4: Draw circuit after switch changed ($t = \infty \approx 5\tau$), Solve $v_o(\infty)$.



Step 5: Draw circuit after switch changed ($t = \infty \approx 5\tau$), Solve R_{eq} with respect to storage element.



2nd Order Circuits

2nd order differential equation$\longrightarrow \frac{dx^2(t)}{dt^2} + a_1 \frac{dx(t)}{dt} + a_2 x(t) = 0,$

$a_1 = 2\zeta$ and $a_2 = \omega_o^2$

ζ = damping ratio

ω_o = undamped natural frequency

- Solution Procedures:

Step 1: Write differential equation that describes circuit.

$$\frac{dx^2(t)}{dt^2} + a_1 \frac{dx(t)}{dt} + a_2 x(t) = 0$$

Step 2: Derive characteristic equation.

$$s^2 + 2\zeta\omega_o s + \omega_o^2 = 0$$

Step 3: Determine roots of the characteristic equation.

Step 4: Determine damping condition:

Case 1 – roots are real and unequal

$\zeta > 1$ = *overdamped*

=> $x(t) = k_1 e^{-(\zeta\omega_o - \sqrt{\zeta^2-1})t} + k_2 e^{-(\zeta\omega_o + \sqrt{\zeta^2-1})t}$

Case 2 – roots are real and equal

$\zeta = 1$ = *critically damped*

=> $x(t) = B_1 e^{-\zeta\omega_o t} + B_2 t e^{-\zeta\omega_o t}$

Case 3 – roots are complex

$\zeta < 1$ = *underdamped*

=> $x(t) = e^{-\zeta\omega_o t}\left(A_1 \cos\left(\omega_o\sqrt{1-\zeta^2}t\right) + A_2 \sin\left(\omega_o\sqrt{1-\zeta^2}t\right)\right)$

Step 5: Two initial conditions (given or derived) are used to obtain the two unknown coefficients in the response equation.

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Chapter 8: AC Steady-State Analysis

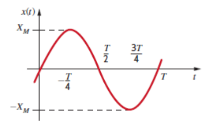
Sinusoids

$x(t) = X_m \sin(\omega t)$

X_m = peak value/amplitude

ω = radian frequency (f) (rad/s)

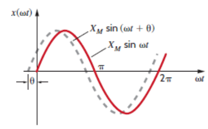
f = $\frac{1}{T}$ (Hz), T = time (s)



$x_1(t) = X_1 \sin(\omega t + \theta_1)$

$\theta_1 = 0°$

$x_2(t) = X_2 \sin(\omega t + \theta_2)$



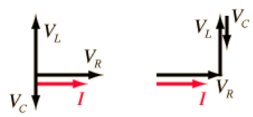
$\therefore x_1(t)$ "*leads*" $x_2(t)$ *by* θ_2

or $x_2(t)$ "*lags*" $x_1(t)$ *by* θ_2, for $\theta_1 \neq \theta_2$

If $\theta_1 = \theta_2$, $x_1(t)$ *and* $x_2(t)$ *are* "*in phase*".

Phase Diagramed

Resistor	Inductor	Capacitor
I is *in phase* with V	I *lags* V by 90°	I *leads* V by 90°



Waveform transforms

$\cos(\omega t) = \sin(\omega t + 90°)$

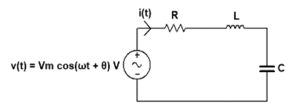
$\sin(\omega t) = \cos(\omega t - 90°)$

$-\cos(\omega t) = \cos(\omega t \pm 180°)$

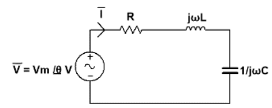
$-\sin(\omega t) = \sin(\omega t \pm 180°)$

Domains

Time Domain



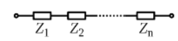
Frequency Domain



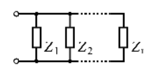
Impedance (Z) and Admittance (Y)

$\bar{Z} = \frac{\bar{V}}{\bar{I}} = R \pm jX = \frac{V_m}{I_m}\underline{/\theta_v - \theta_i} = Z\,\underline{/\theta_Z}$ (Ω)

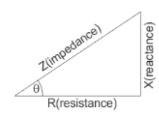
$\bar{Z}_s = \bar{Z}_1 + \bar{Z}_2 + \cdots + \bar{Z}_n$



$\frac{1}{\bar{Z}_p} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \cdots + \frac{1}{\bar{Z}_n}$



$Z = \sqrt{R^2 + X^2},\ \theta_Z = \tan^{-1}\frac{X}{R}$



$R = Z\cos\theta_Z, X = Z\sin\theta_Z$

$\bar{Y} = \frac{1}{\bar{Z}} = G + jB$ (S)

$\frac{1}{\bar{Y}_s} = \frac{1}{\bar{Y}_1} + \frac{1}{\bar{Y}_2} + \cdots + \frac{1}{\bar{Y}_n}$

$\bar{Y}_p = \bar{Y}_1 + \bar{Y}_2 + \cdots + \bar{Y}_n$

AC Analysis

KVL: $\bar{V}_s = \bar{I}_1\bar{Z}_1 + \bar{V}_1$

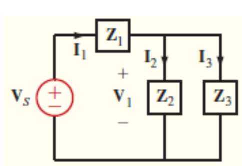
KCL: $\bar{I}_1 = \bar{I}_2 + \bar{I}_3$

Voltage Division:

$$\bar{V}_1 = \frac{\bar{V}_s(\bar{Z}_2||\bar{Z}_3)}{\bar{Z}_1 + (\bar{Z}_2||\bar{Z}_3)}$$

Current Division:

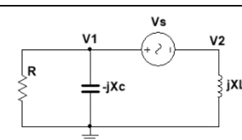
$$\bar{I}_2 = \frac{\bar{I}_1(\bar{Z}_3)}{\bar{Z}_2 + \bar{Z}_3}$$



Nodal Analysis:

1) $\bar{V}_1 - \bar{V}_2 = \bar{V}_s$

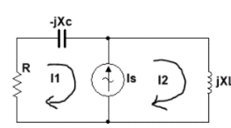
2) $\frac{\bar{V}_1}{R} + \frac{\bar{V}_1}{-jX_C} + \frac{\bar{V}_2}{jX_L} = 0$



Loop Analysis:

1) $\bar{I}_2 - \bar{I}_1 = \bar{I}_s$

2) $\bar{I}_1(R - jX_C) + \bar{I}_2(jX_L) = \bar{V}_s$



Other Analysis Techniques: Superposition, Source Exchange, Thévenin's Theorem, Norton's Theorem

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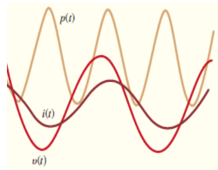
CHAPTER 9: STEADY-STATE POWER ANALYSIS

Instantaneous power

p(t) = v(t)i(t) = VmIm cos(ωt + θv) cos(ωt + θi) W

=> p(t) = $\left(\frac{VmIm}{2}\cos(\theta v - \theta i)\right) + \left(\frac{VmIm}{2}\cos(2\omega t + \theta v + \theta i)\right)$

constant *double frequency*



Average Power

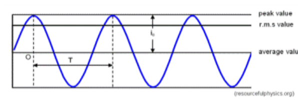
$$P_{AVG} = \frac{V_m I_m}{2}\cos(\theta v - \theta i)$$

$\therefore P_R = \frac{1}{2} V_m I_m$ W, and $P_L = P_C = 0$ W

Effective/RMS Values

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

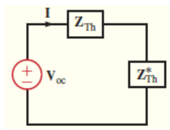
Sinusoid: $I_{RMS} = \frac{I_m}{\sqrt{2}} \therefore P_R = I_{RMS}^2 R = \frac{V_{RMS}^2}{R}$



Maximum Average Power Transfer

$\bar{Z}_L = \bar{Z}_{TH}^*$ (i.e. $\bar{Z}_{TH} = R + jX \Rightarrow \bar{Z}_L = R - jX$)

$$P_{max} = \frac{1}{2} I^2 R_{TH} = \frac{V_{TH}^2}{8R_{TH}}$$



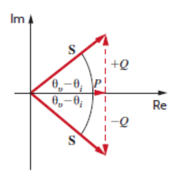
Complex Power

$\bar{S} = \bar{V}_{RMS}\bar{I}_{RMS}^* = V_{RMS}I_{RMS}\,\underline{/(\theta v - \theta i)}$ (VA) = $\underline{S/\theta s}$ = P + jQ

$P = Re(\bar{S}) = V_{RMS}I_{RMS}\cos(\theta v - \theta i)$ (W)

$Q = Im(\bar{S}) = V_{RMS}I_{RMS}\sin(\theta v - \theta i)$ (VAR)

(Note: +Q = inductor, -Q = capacitor)



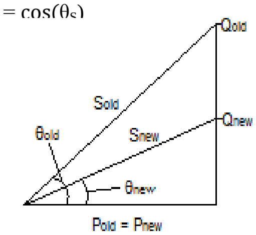
Power Factor

$$pf = \frac{P}{S} = \frac{P}{V_{RMS}I_{RMS}} = \cos(\theta v - \theta i) = \cos(\theta_S)$$

- Power Factor Correction

$$Q_{cap} = Q_{new} - Q_{old} = -j\omega C V_{RMS}^2$$

$$\therefore C = \frac{Q_{cap}}{\omega V_{RMS}^2}$$



Single-Phase Three-Wire Circuits

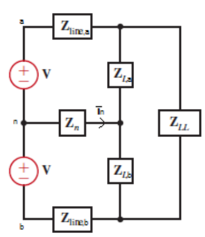
(Typical AC network in households)

$\bar{V}_{an} = \bar{V}_{nb}$

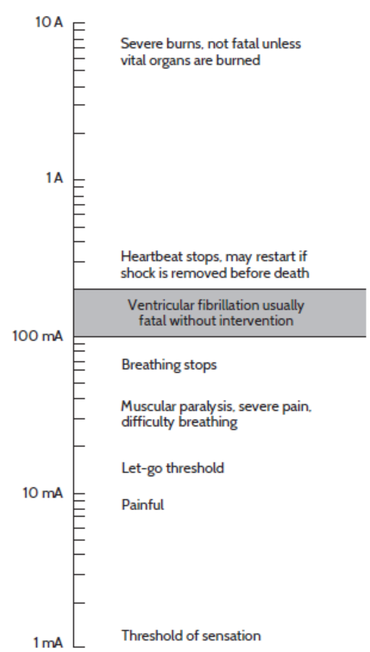
If $\bar{Z}_{line,a} = \bar{Z}_{line,b}$

& $\bar{Z}_{L,1} = \bar{Z}_{L,2} = \bar{Z}_{LL}$

=> $\bar{I}_n = 0$



Safety Considerations



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CHAPTER 10: MAGNETICALLY COUPLED NETWORKS

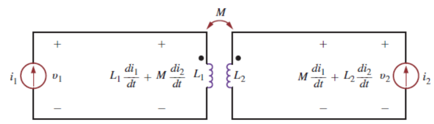
Mutual Inductance

Dot notation – current $i_1(t)$ and $i_2(t)$:

BOTH entering or BOTH leaving => +M,

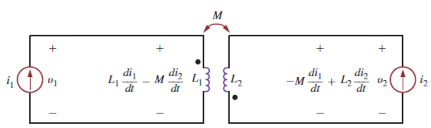
otherwise => –M

Examples:



$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$$

$$v_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt}$$



$$v_1(t) = L_1\frac{di_1(t)}{dt} - M\frac{di_2(t)}{dt}$$

$$v_2(t) = -M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt}$$

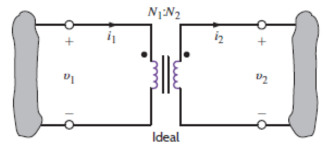
Energy Analysis

$$w(t) = \frac{1}{2}L_1[i_1(t)]^2 + \frac{1}{2}L_2[i_2(t)]^2 \pm Mi_1(t)i_2(t)$$

If $M \le \sqrt{L_1L_2}$, instantaneous energy stored is nonnegative.

Ideal Transformer

$$n = \frac{N_2}{N_1}$$

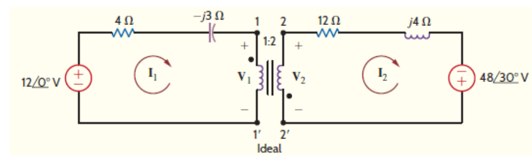


$$\frac{\bar{V}_1}{\bar{V}_2} = \frac{1}{n}$$

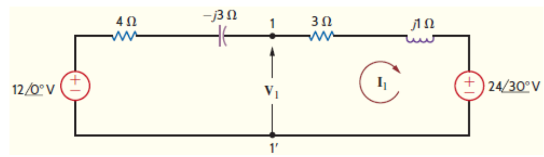
$$\frac{\bar{I}_1}{\bar{I}_2} = n$$

Impedance Reflection:

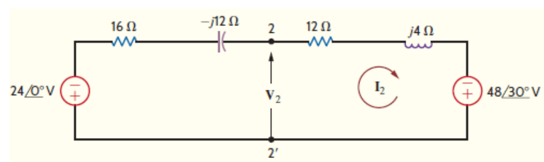
Example:



Primary: $\bar{Z}_{1,eq} = \frac{\bar{Z}_2}{n^2}$ *and* $\bar{V}_{1,eq} = \frac{\bar{V}_{s,2}}{n}$



Secondary: $\bar{Z}_{2,eq} = n^2\bar{Z}_1$ *and* $\bar{V}_{2,eq} = \frac{\bar{V}_{s,1}}{n}$



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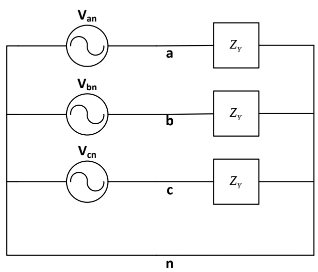
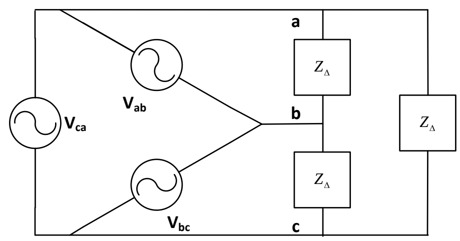
CHAPTER 11: POLYPHASERS CIRCUITS

Balanced three-phase (3ϕ) quantities	
Frequency domain Representation	Time domain Representation
$A_{an} = A_{peak} \angle \theta$ $A_{bn} = A_{peak} \angle (\theta - 120°)$ $A_{cn} = A_{peak} \angle (\theta + 120°)$	$A_{an} = A_{peak} \cos(\theta)$ $A_{bn} = A_{peak} \cos(\theta - 120°)$ $A_{cn} = A_{peak} \cos(\theta + 120°)$
A: voltage (V) or current (I)	

Voltage, current, and impedance relationships for Y-Δ configurations	
Y connection	Δ connection
	
$V_{LL} = \sqrt{3} V_{LN} \angle \theta + 30^\circ$ (LL: line to line) (LN: line to neutral)	$V_L = V_P \angle \theta$ (L: line to line) (P: phase voltage)
$I_L = I_P \angle \theta$	$I_L = \sqrt{3} I_P \angle \theta + 30^\circ$
$I_n = I_a + I_b + I_c = 0$	$V_{ab} + V_{bc} + V_{ca} = 0$
$I_{LN} = \dfrac{V_{LN}}{Z_Y}$	$I_{LL} = \dfrac{V_{LL}}{Z_\Delta}$
$Z_Y = \dfrac{Z_\Delta}{3}$	$Z_\Delta = 3 Z_Y$

Power relationships in 3ϕ circuits				
Real power	Reactive power	Complex power	Instantaneous power	Power factor
$P_T = 3P_P$ $= \sqrt{3} V_L I_L \cos\theta$ P_P: 1ϕ real power P_T: 3ϕ real power L: line	$Q_T = 3Q_P$ $= \sqrt{3} V_L I_L \sin\theta$ Q_P: 1ϕ reactive power Q_T: 3ϕ reactive power L: line	$S_T = P_T + jQ_T$ S_T: 3ϕ complex power	$p(t) = 3\dfrac{V_m I_m}{2} \cos\theta \ W$ V_m: maximum voltage I_m: maximum current	$pf = \dfrac{P_T}{S_T}$ $= cos(\theta_V - \theta_I)$ θ_V: voltage phase angel θ_I: current phase angel

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CHAPTER 12: VARIABLE-FREQUENCY NETWORK PERFORMANCE

Network transfer functions

Input	output	Transfer function
Voltage	Voltage	Voltage gain $G_v(jw)$
Current	Voltage	Transimpedance $Z(jw)$
Current	Current	Current gain $G_i(jw)$
Voltage	Current	Transadmittance $Y(jw)$

Network function

$$H(jw) = H(s) = \frac{N(s)}{D(s)} = K\,\frac{(s-z_1)\,(s-z_2)\dots(s-z_m)}{(s-p_1)\,(s-p_2)\dots(s-p_n)}$$

K: constant	z_m: zeros of $H(s)$	p_n: poles of $H(s)$

$s = z_1 \; or \; z_2 \dots, z_m \rightarrow H(s) = 0$

$s = p_1 \; or \; p_2 \dots, p_n \rightarrow H(s) = \infty$

Bode plot Basics

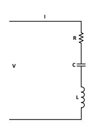
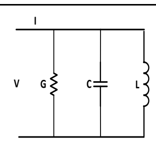
$$\mathrm{H(jw)} = \mathrm{K}\,\frac{(\mathrm{jw}-\mathrm{z}_1)\,(\mathrm{jw}-\mathrm{z}_2)\dots}{(\mathrm{jw}-\mathrm{p}_1)\,(\mathrm{jw}-\mathrm{p}_2)\dots} \quad \text{(standard form)}$$

$$\text{Decibles(dB)} = 20\log\frac{V_{out}}{V_{in}} = 10\log\left(\frac{V_{out}}{V_{in}}\right)^2 = 10\log\frac{P_{out}}{P_{in}}$$

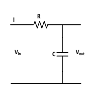
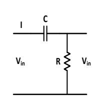
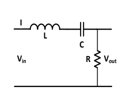
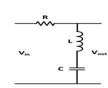
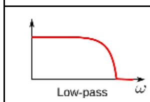
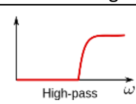
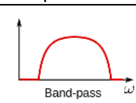
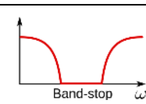
Bode plots	1. Magnitude ($20\log\lvert H(jw)\rvert$ dB) vs w (rad/s)	2. phase angle ($\angle H(jw)$) vs. w (rad/s)
Magnitude in dB	$20\log\lvert H(jw)\rvert = 20\log\lvert K\rvert + 20\log\lvert jw-z_1\rvert + 20\log\lvert jw-z_2\rvert + \cdots - 20\log\lvert jw-p_1\rvert - 20\log\lvert jw-p_2\rvert - \cdots$	
Phase angel	$\angle H(jw) = \angle K + \angle(jw-z_1) + \angle(jw-z_2) + \cdots - \angle(jw-p_1) - \angle(jw-p_2) - \cdots$	
Sketch bode plot	STEP 1 Rewrite the transfer function (TF) in Standard form. STEP 2 Separate the TF into its constituent parts and find corner frequency (w_r) i.e. $z_1, z_2,\; p_1,\; p_2$ STEP 3 Draw the magnitude plot @ each w_r by +20dB/dec for a zero and -20dB/dec for a pole. STEP 4 Calculate the phase angle @ each w_r and draw the resulting phase plot.	

Resonance

Circuit topology

Series resonance	Parallel resonance
	
Impedance	**Admittance**
$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$	$Y(j\omega) = G + j\omega C + \frac{1}{j\omega L}$
Quality factor	
$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$	$Q = \frac{R}{\omega_0 L} = \frac{1}{G\omega_0 L} = R\omega_0 C = \frac{\omega_0 C}{G}$
Resonant frequency $\omega_0 = \frac{1}{\sqrt{LC}}$	
Bandwidth	
$\omega_{HI} - \omega_{LO} = \frac{R}{L}$	$\omega_{HI} - \omega_{LO} = \frac{1}{RC}$

Passive Filters

High-pass filter	Low-pass filter	Band-pass filter	Band-reject filter
			
Magnitude response			
			
Transfer functions			
$G_v(j\omega) = \frac{1}{1+j\omega RC}$	$G_v(j\omega) = \frac{j\omega RC}{1+j\omega RC}$	$G_v(j\omega) = \frac{R}{1+j(\omega L - 1/\omega C)}$	
Magnitude			
$M(\omega) = \frac{1}{\sqrt{1+(\omega\tau)^2}}$ Time constant $\tau = RC$	$M(\omega) = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$	$M(\omega) = \frac{RC\omega}{\sqrt{(RC\omega)^2 + (\omega^2 LC - 1)^2}}$	
Phase angle		**Bandwidth**	
$\phi(\omega) = -\tan^{-1}\omega\tau$	$\phi(\omega) = \frac{\pi}{2} - \tan^{-1}\omega\tau$	$BW = \frac{R}{L}$	

Scaling a circuit

	Resistor	Inductor	Capacitor
Magnitude scaling	$R' \rightarrow K_M R$	$L' \rightarrow K_M L$	$C' \rightarrow \frac{C}{K_M}$
Frequency scaling	$R' \rightarrow R$	$L' \rightarrow \frac{L}{K_F}$	$C' \rightarrow \frac{C}{K_F}$

CHAPTER 13: THE LAPLACE TRANSFORM

Basics

Laplace transform	$\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)e^{-st}\,dt$
Inverse Laplace transform	$\mathcal{L}^{-1}[F(s)] = f(t) = \frac{1}{2\pi j}\int_{\sigma-jw}^{\sigma+jw} F(s)e^{st}dt$
$s = \sigma + jw$	
Unit impulse	$\int_{t_1}^{t_2} f(t)\,\delta(t-t_0) = \begin{cases} f(t_0) & t_1 < t_0 < t_2 \\ 0 & t_0 < t_1, t_0 > t_2 \end{cases}$

Laplace Transform Pairs

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\frac{t^n e^{-at}}{n!}$	$\frac{1}{(s+a)^{n+1}}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$

Laplace Transform Properties

Property	$f(t)$	$F(s)$
Magnitude scaling	$Af(t)$	$AF(s)$
Addition/ subtraction	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
Time scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right),\ a>0$
Time shifting	$f(t-t_0)u(t-t_0),\ t \geq 0$	$e^{-t_0 s}F(s)$
	$f(t)u(t-t_0),\ t \geq 0$	$e^{-t_0 s}L\left[f(t+t_0)\right]$
Frequency shifting	$e^{-at}f(t)$	$F(s+a)$
Differentiation	$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f(0) \cdots - s^0 f(0)$
Multiplication by t	$tf(t)$	$-\frac{d^n f(s)}{ds^n}$
	$t^n f(t)$	$(-1)^n \frac{d^n f(s)}{ds^n}$
division by t	$\frac{f(t)}{t}$	$\int_s^\infty F(\lambda)d\lambda$
Integration	$\int_0^t f(\lambda)d\lambda$	$\frac{1}{s}F(s)$
Convolution	$\int_0^t f_1(\lambda)\,f_2(t-\lambda)d\lambda$	$F_1(s)F_2(s)$
Initial-value theorems	$\lim_{t\to 0} f(t)$	$\lim_{s\to\infty} sF(s)$
Final-value theorems	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$

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CHAPTER 14: APPLICATION OF THE LAPLACE TRANSFORM TO CIRCUIT ANALYSIS

Circuit Element Models

Resistor (R) in s-domain	Inductor (L) in s-domain	Capacitor (C) in s-domain
$R \to R \quad V(s) = RI(s)$	$L \to sL \begin{cases} V(s) = sLI(s) - Li(0) \\ I(s) = \dfrac{V(s)}{sL} + \dfrac{i(0)}{s} \end{cases}$	$C \to \dfrac{1}{sC} \begin{cases} V(s) = \dfrac{I(s)}{sC} + \dfrac{v(0)}{s} \\ I(s) = sCV(s) - Cv(0) \end{cases}$

Laplace Circuit Solution

Procedures:

STEP 1 Solve for initial capacitor voltages and inductor currents.

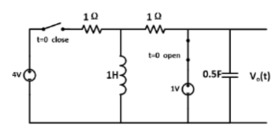
STEP 2 Draw the circuit at t<0, replace the capacitor with an open circuit and inductor with a short circuit.

STEP 3 Draw an s-domain circuit by substituting an s-domain representation for all circuit elements.

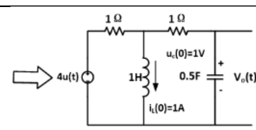
STEP 4 Use circuit analysis to solve for the appropriate voltage and/or current.

STEP 5 Perform the inverse Laplace transform to convert the voltage and/or current back to the time domain.

Example



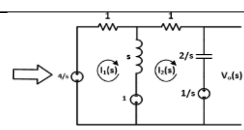
- Solve for initial conditions



$V_c(0) = 1\text{V}$

$I_L(0) = 1\text{A}$

- Convert to s-domain



1- Apply mesh analysis to find I_1 and I_2

$$(s+1)I_1(s) - sI_2(s) = \frac{4}{s} + 1$$

$$-sI_1(s) + \left(s + \frac{2}{s} + 1\right)I_2(s) = \frac{-1}{s} - 1$$

$$I_1(s) = \frac{4s^2 + 6s + 8}{s(2s^2 + 3s + 2)}$$

$$I_2(s) = \frac{2s - 1}{2s^2 + 3s + 2}$$

2- Solve for $V_o(s)$

$$V_o(s) = \frac{2}{s} I_2(2) + \frac{1}{s} = \frac{s + 3.5}{s^2 + 1.5s + 1}$$

3- Convert to time domain

$$v_o(t) = \left[4.29e^{-0.75t} \cos\left(\frac{\sqrt{7}}{4} t - 76.5^\circ \right) \right] u(t)\ V$$

Steady-State Respone

$\mathbf{Y}(s) = \mathbf{H}(s)\mathbf{X}(s)$

Transient comes from the poles of $\mathbf{H}(s)$ & Steady-state portion comes from the poles of $\mathbf{X}(s)$

The steady-state response: X_M is the maximum of $\mathbf{X}(s)$

$$y_{ss}(t) = X_M \left| H(j\omega_0) \right| \cos\left[\omega_0 t + \phi(j\omega_0) \right]$$

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CHAPTER 15: FOURIER ANALYSIS TECHNIQUES

Fourier Series

$$f(t) = f(t+nT_0)\ \ n = \pm1, \pm2, \pm3\cdots \Leftrightarrow f(t) = a_0 + \sum_{n=1}^{\infty} D_n \cos(n\omega_0 t + \theta_n)$$

Exponential Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} f(t) e^{-jn\omega_0 t} dt$$

Trigonometric Fourier series

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\cos n\omega_0 t dt \qquad b_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\sin n\omega_0 t dt$$

Types Trigonometric Fourier series

Even-function symmetry

$$f(t) = f(-t)$$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\cos n\omega_0 t dt$$

$$b_n = 0$$

$$a_0 = \frac{2}{T_0}\int_0^{T_0/2} f(t) dt$$

Odd-function symmetry

$$f(t) = -f(-t)$$

$$a_n = 0 \quad for\ all\ n > 0$$

$$b_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\sin n\omega_0 t dt$$

$$a_0 = 0$$

Half-wave symmetry

$$f(t) = -f\left(t - \frac{T_0}{2}\right)$$

n even

$$a_n = 0$$

$$b_n = 0$$

n odd

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\cos n\omega_0 t dt$$

$$b_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\sin n\omega_0 t dt$$

$$a_0 = 0$$

Average Power

$$P = V_{DC} I_{DC} + \sum_{n=1}^{\infty} \frac{V_n I_n}{2}\cos(\theta_{v_n} - \theta_{i_n})$$

Fourier Transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \qquad f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{-j\omega t} d\omega$$

Fourier Transform Pairs

$f(t)$	$F(\omega)$
$F^{-1}[F(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$	$F[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$
$\delta(t-a)$	$e^{-j\omega a}$
A	$2\pi A\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos\omega_0 t$	$\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$
$\sin\omega_0 t$	$j\pi\delta(\omega+\omega_0)-j\pi\delta(\omega-\omega_0)$
$e^{-at}u(t),\ a>0$	$\frac{1}{a+j\omega}$
$e^{-\lvert a\rvert t},\ a>0$	$\frac{2a}{a^2+\omega^2}$
$e^{-at}\cos u_0 t u(t),\ a>0$	$\frac{j\omega+a}{(j\omega+a)^2+\omega_0^2}$
$e^{-at}\sin u_0 t u(t),\ a>0$	$\frac{\omega_0}{(j\omega+a)^2+\omega_0^2}$

Fourier Transform Properties

Linearity	$Af(t)$	$AF(\omega)$
Addition/subtraction	$f_1(t)\pm f_2(t)$	$F_1(\omega)\pm F_2(\omega)$
Time-scaling	$f(at)$	$\frac{1}{a}F\left(\frac{\omega}{a}\right),\ a>0$
Time-shifting	$f(t-t_0)$	$e^{-jwt_0}F(\omega)$
modulation	$e^{jwt_0}f(t)$	$F(\omega-\omega_0)$
differentiation	$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$
Multiplication by t	$t^n f(t)$	$(j)^n \frac{d^n F(\omega)}{d\omega^n}$
convolution	$\int_{-\infty}^{\infty} f_1(x) f_2(t-x) dx$	$F_1(\omega)F_2(\omega)$
	$f_1(t) f_2(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(x)F_2(\omega-x) dx$
Parseval's Theorem	$\int_{-\infty}^{\infty} f^2(t) dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert F(\omega)\rvert^2 d\omega$

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CHAPTER 15: FOURIER ANALYSIS TECHNIQUES

Fourier series $f(t) = f(t+nT_0)\ n = \pm1, \pm2, \pm3\cdots \Leftrightarrow f(t) = a_0 + \sum_{n=1}^{\infty} D_n \cos(n\omega_0 t + \theta_n)$ where $\omega_0 = \frac{2\pi}{T_0}$

Exponential Fourier series	Trigonometric Fourier series
$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$	$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$
$c_n = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} f(t) e^{-jn\omega_0 t} dt$	$a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\cos n\omega_0 t dt \quad b_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\sin n\omega_0 t dt$

Special situation for trigonometric Fourier series

Even-function symmetry $f(t) = f(-t)$	Odd-function symmetry $f(t) = -f(-t)$	Half-wave symmetry $f(t) = -f\left(t - \frac{T_0}{2}\right)$	
		n even	n odd
$a_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\cos n\omega_0 t dt$	$a_n = 0 \quad for\ all\ n > 0$	$a_n = 0$	$a_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\cos n\omega_0 t dt$
$b_n = 0$	$b_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\sin n\omega_0 t dt$	$b_n = 0$	$b_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\sin n\omega_0 t dt$
$a_0 = \frac{2}{T_0}\int_0^{T_0/2} f(t) dt$	$a_0 = 0$	$a_0 = 0$	

Average power $P = V_{DC}I_{DC} + \sum_{n=1}^{\infty} \frac{V_n I_n}{2}\cos(\theta_{v_n} - \theta_{i_n})$

Fourier transform pair

$f(t)$	$F(\omega)$
$F^{-1}[F(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$	$F[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$
$\delta(t-a)$	$e^{-j\omega a}$
A	$2\pi A\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos\omega_0 t$	$\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$
$\sin\omega_0 t$	$j\pi\delta(\omega + \omega_0) - j\pi\delta(\omega - \omega_0)$
$e^{-at}u(t),\ a > 0$	$\frac{1}{a + j\omega}$
$e^{-\lvert a\rvert t},\ a > 0$	$\frac{2a}{a^2 + \omega^2}$
$e^{-at}\cos u_0 t u(t),\ a > 0$	$\frac{j\omega + a}{(j\omega + a)^2 + \omega_0^2}$
$e^{-at}\sin u_0 t u(t),\ a > 0$	$\frac{\omega_0}{(j\omega + a)^2 + \omega_0^2}$

Property of Fourier's transform

Property	$f(t)$	$F(\omega)$
Linearity	$Af(t)$	$AF(\omega)$
Addition/subtraction	$f_1(t) \pm f_2(t)$	$F_1(\omega) \pm F_2(\omega)$
Time-scaling	$f(at)$	$\frac{1}{a}F\left(\frac{\omega}{a}\right),\ a > 0$
Time-shifting	$f(t - t_0)$	$e^{-jwt_0}F(\omega)$
modulation	$e^{jwt_0} f(t)$	$F(\omega - \omega_0)$
differentiation	$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$
Multiplication by t	$t^n f(t)$	$(j)^n \frac{d^n F(\omega)}{d\omega^n}$
convolution	$\int_{-\infty}^{\infty} f_1(x) f_2(t - x) dx$	$F_1(\omega)F_2(\omega)$
	$f_1(t) f_2(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(x)F_2(\omega - x) dx$

Fourier transform for an aperiodic function $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$ $\quad f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{-j\omega t} d\omega$

Parseval's Theorem $\int_{-\infty}^{\infty} f^2(t) dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert F(\omega)\rvert^2 d\omega$