COMP-4200

Formal Languages

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LAB SECTION:	001

Homework #5

Problem 1

30 points

Exercise 2.6. Give context-free grammars (CFGs) generating the following languages.

1. The set of strings over the alphabet $\Sigma = \{0, 1\}$ with more a's than b's.

$$S \to SaSaSaS \mid SaSbSaS \mid SbSaSaS \mid \epsilon$$

2. The complement of the language $\{a^n b^n \mid n \geq 0\}$

$$\begin{aligned} S &\to CbCaX \mid A \mid B \\ A &\to aAb \mid Ab \mid b \\ B &\to aBb \mid aB \mid a \\ C &\to a \mid b \end{aligned}$$

Problem 2

15 points

Exercise 2.9. Give context-free grammars (CFGs) generating the following language:

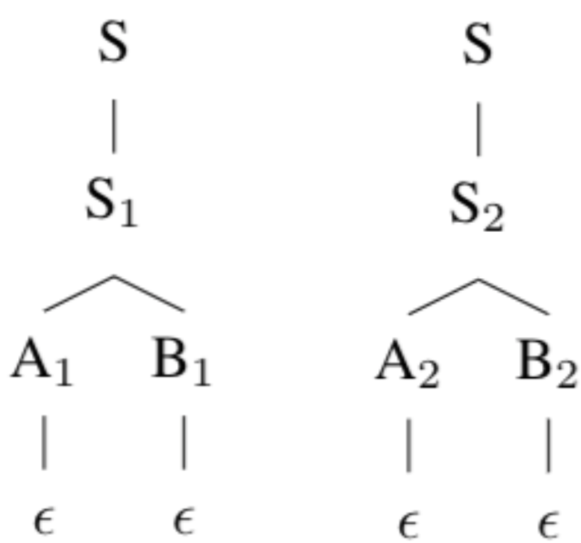
$A = \{a^i b^j c^k \mid i = j \; or \; j = k \; where \; i, j, k \geq 0\}$

CFG for A:

$$S \to S_1 \mid S_2$$
$$S_1 \to A_1 B_2$$
$$A_1 \to aA_1b \mid \epsilon$$
$$B_1 \to cB_1 \mid \epsilon$$
$$S_2 \to A_2 B_2$$
$$A_2 \to aA_2 \mid \epsilon$$
$$B_2 \to bB_2c \mid \epsilon$$

Is your grammar ambiguous? Why or why not? If yes, please provide an example of two different leftmost derivations that generate the same string.

→ This grammar is ambiguous. Generally, any string $a^i b^j c^k$ with $i = j = k$ can be derived ambiguously in this grammar. By considering the empty string ϵ, we are able to come up with two different left-most derivations that generate the same string.



Problem 3

15 points

Exercise 2.14. Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

Please provide all intermediate steps with comments on how you transform from the grammar from one version to another (these steps are critical for your work to be graded).

$$A \to BAB \mid B \mid \epsilon$$
$$B \to 00 \mid \epsilon$$

STEP ONE:

Ensure that the start variable doesn't appear on the RHS of any rule by adding a new start variable "S" as well as the new transition $S \to A$.

$$S \to A$$
$$A \to BAB \mid B \mid \epsilon$$
$$B \to 00 \mid \epsilon$$

STEP TWO:

Eliminate ϵ rules. This includes rules that go to ϵ that doesn't involve the start variable.

$$S \to A \mid \epsilon$$
$$A \to BAB \mid B \mid AB \mid BB \mid BA$$
$$B \to 00$$

STEP THREE:

Eliminate unit rules.

$$S \to BAB \mid 00 \mid AB \mid BB \mid BA \mid \epsilon$$
$$A \to BAB \mid 00 \mid AB \mid BB \mid BA$$
$$B \to 00$$

STEP FOUR:

Ensure that there isn't more than one terminal on the RHS by converting any necessary terminals into variables.

$$S \to BAB \mid CC \mid AB \mid BB \mid BA \mid \epsilon$$
$$A \to BAB \mid CC \mid AB \mid BB \mid BA$$
$$B \to CC$$
$$C \to 0$$

STEP FIVE:

Ensure that there is exactly two variables on the RHS of any rule.

$$S \to XB \mid CC \mid AB \mid BB \mid BA \mid \epsilon$$
$$A \to XB \mid CC \mid AB \mid BB \mid BA$$
$$B \to CC$$
$$C \to 0$$
$$X \to BA$$