COMP-4200

Formal Languages

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DATE:	November 8, 2021
LAB SECTION:	001

Homework #6

Problem 1

20 points

Problem **2.30** (page 157) in the textbook.

Use Pumping Lemma for CFLs to prove the following language is not context-free.

$$A = \{\ 0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\ \}$$

Note: Alphabet $\Sigma = \{0, \#\}$ That is, # is a character in a string just like 0 or 1 in other languages discussed in this course.

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→ To find a contradiction, we will assume that A is context-free and let p be the pumping length for A given by the pumping lemma.

→ We will take a string $s = 0^p \# 0^{2p} \# 0^{3p} \in A$ with $|s| > p$, such that now A has a pumping length p. There exists $uvxyz$ such that (1) $uv^ixy^iz \in A$ for all $i \geq 0$, (2) $|vy| > 0$ and (3) $|vxy| \leq p$. It will not matter how many times we choose to divide s into uvxyz, a contradiction will occur. We will use cases to proceed with showing that A is not context-free:

CASE 1: vxy is within the leftmost sequence of zeroes. If we were to pump it down to $i = 1$, we will decrease the number of zeroes in the leftmost sequence without altering the number of zeroes on the second and third sequence, therefore the resulting string will not be in the language.

CASE 2: vx is within the leftmost sequence of zeroes but y includes the first # (and possibly some zeroes from the second sequence). If we were to pump it up to $i = 2$, we will get a string $s_0 = uv^2xy^2z$ which contains three #s, thus s_0 is not in the language.

CASE 3: v is within the leftmost sequence of zeroes, x contains the leftmost #, and y includes only symbols from the middle sequence of zeroes. If we were to pump it down to $i = 0$, then the number of zeroes in the first and second sequence will decrease but the number of zeroes in the third sequence will not. Hence the string will not be int he language.

CASE 4: vxy is completely within the middle or rightmost sequence of zeroes. This case is equivalent to case 1.

CASE 5: x or y include the rightmost #. This case is similar to cases 2 and 3.

→ Hence, from obtaining a contradiction resulting in s not being within the language, we're able to conclude that A is not context-free.

Problem 2

20 points

Is the following language *B* context-free? If yes, show a context-free grammar (CFG) that generates *B*. If no, please prove it using Pumping Lemma for CFLs.

$$B = \{\ 0^n 0^{2n} \# 0^{3n} \mid n \geq 0\ \}$$

Note: *B* is the same as *A* in Problem 1, except that there is one less # in each string of *B* (compared to the strings in *A*.

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Pumping Lemma for Context-Free Languages

1. **Assume that B is context-free and let p be the pumping length for B given by the pumping lemma**
2. **Take a string "s" such that** $s = 0^p 0^{2p} \# 0^{3p} = 0^{3p} \# 0^{3p}$
3. **Divide "s" into parts** $uvxyz$

Context-Free Grammar for B:

$S \rightarrow 0S0 \mid \# \mid \epsilon$

Problem 3

20 points

Problem **2.31** (page 157) in the textbook.

Let B be the language of all palindromes over {0,1} containing equal numbers of 0s and 1s. Show that *B* is not context-free.

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→ To find a contradiction, we will suppose that B is context-free and let p be the pumping length for B given by the pumping lemma.

→ We will take a string $s = 0^p1^{2p}0^p \in B$ with $|s| > p$, such that now B has a pumping length p. There exists $uvxyz$ such that (1) $uv^ixy^iz \in B$ for all $i \geq 0$, (2) $|vy| > 0$ and (3) $|vxy| \leq p$. It will not matter how many times we choose to divide s into uvxyz, a contradiction will occur. We will use cases to proceed with showing that B is not context-free:

CASE 1: vxy consists of only 1s. Then $uv^2xy^2z \notin B$, since it will no longer have the same number of 0s and 1s.

CASE 2: vxy contains at least one 0. Then $uv^2xy^2z \notin B$, since it will no longer be a palindromic string. This is because from (3), vxy can only contain symbols from the starting 0s or the ending 0s but not both. Hence, after pumping "s" will have a different number 0s before and after the 1s.

→ Each case will violate (1) from the pumping lemma and thus is a contradiction of our assumption from earlier, therefore B is not context-free.

Problem 4

20 points

Problem **2.32** (page 157) in the textbook.

Let $\Sigma = \{0, 1, 2, 3, 4\}$ and $C = \{w \in \Sigma^* \mid$ in w, the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s$\}$. Show that C is not context-free.

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→ To find a contradiction, we will assume that C is context-free and let p be the pumping length for C given by the pumping lemma.

→ We will take a string $s = 1^p2^p3^p4^p \in C$ with $|s| > p$, such that now C has a pumping length p. There exists $uvxyz$ such that (1) $uv^ixy^iz \in C$ for all $i \geq 0$, (2) $|vy| > 0$ and (3) $|vxy| \leq p$. It will not matter how many times we choose to divide s into uvxyz, a contradiction will occur. We will use cases to proceed with showing that C is not context-free:

CASE 1: vxy contains a 1. Then $uv^2xy^2z \notin C$, since it will no longer have the same number of 1s and 2s. This is because from (3), vxy cannot contain any 2s.

CASE 2: vxy contains a 2. Then $uv^2xy^2z \notin C$, since it will no longer have the same number of 1s and 2s. This is because from (3), vxy cannot contain any 1s.

CASE 3: vxy contains a 3. Then $uv^2xy^2z \notin C$, since it will no longer have the same number of 3s and 4s. This is because from (3), vxy cannot contain any 4s.

CASE 4: vxy contains a 4. Then $uv^2xy^2z \notin C$, since it will no longer have the same number of 3s and 4s. This is because from (3), vxy cannot contain any 3s.

→ Hence, each case will contradict (1) from the pumping lemma, therefore C is not context-free.