EXPERIMENT 13
Electrical Measurements: Filters

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The objectives of this session are to:

- Use NI ELVIS II+ system to investigate networks excited with variable-frequency sinusoidal signals
- Provide a brief introduction to basic capabilities of the NI ELVIS prototype environment
- Become familiar with the manipulation of the virtual instruments, such as the function generator, Bode analyzer, and oscilloscope within the NI ELVIS II+ system.

I. Introduction

The ac steady-state behavior of circuits depends on the frequency of the sinusoidal input. The impedances of all of the elements in the circuit, except for resistors, are dependent on the frequency of the excitation. The impedance of a capacitor is

$$\overline{Z}_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = -jX_C, \; X_C \equiv \frac{1}{\omega C}$$

Notice that X_C is infinite for $\omega = 0$ (zero frequency or dc) and reduces to a small value for very large values of frequency. Therefore, the capacitor behaves in the steady-state analysis of a circuit as an open-circuit to dc and as a short-circuit for very large frequencies. The impedance of an inductor is

$$\overline{Z}_L = j\omega L = jX_L, \quad X_L \equiv \omega L$$

Note that X_L is zero for zero frequency and approaches infinity as the frequency becomes larger and larger. Therefore, the inductor behaves in the steady-state analysis of a circuit as a short-circuit to dc and becomes an open-circuit for very high frequencies.

Combinations of inductors, capacitors and resistors connected together to form a circuit can produce interesting behavior as a function of frequency. As the frequency of the excitation (input) to a circuit increases, the resistance of the resistors stays the same, the impedance of the inductors increase, and the impedance of the capacitors decrease. Three such circuits that are classified as simple frequency filters will be experimentally examined in this experiment. Each filter will be tested by applying a fixed amplitude sinusoidal voltage to the input of the filter and measuring the filter output as the **frequency** of the input sinusoidal voltage is varied. The phasor output voltage of the filter, divided by the phasor input voltage, is defined as the voltage transfer function of the filter. Since this voltage ratio will depend on the frequency of the input voltage, the transfer function is a function of frequency, i.e., the transfer function is defined as

$$G(f) \equiv \frac{V_{out}}{V_{in}}$$

which is a function of frequency f. The magnitude of the transfer function, i.e., the magnitude of the output voltage divided by the magnitude of the input voltage, will be plotted for each filter as a function of the frequency of the input sinusoidal voltage.

The analysis of a frequency selective circuit is nothing more than a study of the ac steady-state solution of the circuit at several values of frequency. Most of the time this analysis is performed with the frequency f, or $\omega = 2\pi f$, treated as a variable. A solution is obtained in the form of an expression $G(f)$, or $G(\omega)$, that is a function of frequency. Then this expression is evaluated for several values of frequency to determine how the circuit behaves.

A Low-pass Filter

The first circuit, shown in Figure 1, is a type of low pass filter. To illustrate how frequency-domain analysis is used to analyze the low pass filter, the transfer function is derived under the condition that $R_L << R$.

$$V_{out} = \frac{V_{in}(R)}{R_L + R + j\omega L} \approx \frac{V_{in}(R)}{R + j\omega L}$$

$$G(\omega) = \frac{V_{out}}{V_{in}} = \frac{R}{R + j\omega L}$$

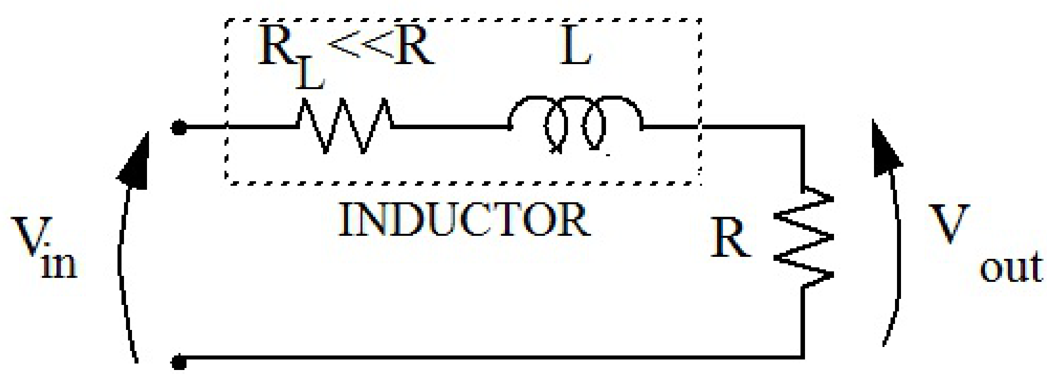

Figure 1: Low-pass Filter

Notice that

$$G(0) = \frac{R}{R + j0} = 1$$

and

$$G(\infty) = \frac{R}{R + j\infty} = 0$$

The circuit is called a low-pass filter because the output voltage is equal to the input voltage for low frequencies and the output voltage approaches 0 as the frequency is increased. (i.e., low frequency inputs are allowed to “pass” while high frequency inputs are “blocked”).

A High-pass Filter

The second circuit, shown in Figure 2, has a voltage transfer function that equals one for high frequencies, but approaches zero as the frequency is decreased, reaching zero when the frequency is zero. The circuit is called a high-pass filter because the output voltage is equal to the input voltage for high frequencies and the output voltage approaches 0 as the frequency is decreased. (i.e., low frequency inputs are "blocked" while high frequency inputs are allowed to "pass").

$$G(\omega) = \frac{R}{R + \frac{1}{j\omega C}}$$

$$G(0) = \frac{R}{R + \frac{1}{j0}} = 0$$

$$G(\infty) = \frac{R}{R + \frac{1}{j\infty}} = 1$$

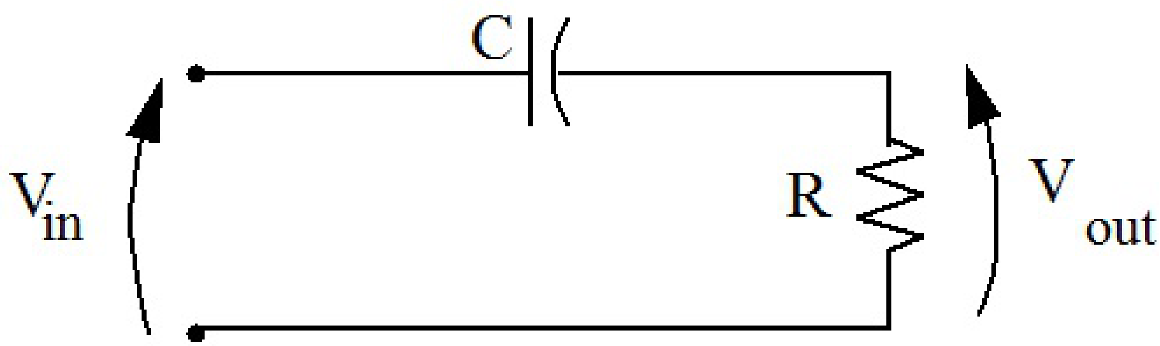

Figure 2: High-pass Filter

A Band-pass Filter

The third circuit, shown in Figure 3, is a band-pass filter. An analysis of this circuit shows that: 1) at low frequencies the impedance of the capacitor is large; 2) at high frequencies the impedance of the inductor is large, and 3) at some mid frequency, ωL and $-1/\omega C$ will cancel (when $\omega = 1/\sqrt{LC}$), making the output voltage equal to the input voltage. Therefore, the filter will attenuate low- and high-frequency signals and pass the signals in a band of frequencies in a middle range. This mid range band of frequencies is called the pass-band of the band-pass filter.

$$G(\omega) = \frac{R}{R + \frac{1}{j\omega C} + j\omega L}$$

$$G(0) = \frac{R}{R + \frac{1}{j0} + j0} = 0$$

$$G(\infty) = \frac{R}{R + \frac{1}{j\infty} + j\infty} = 0$$

$$G(\frac{1}{\sqrt{LC}}) = \frac{R}{R + \frac{\sqrt{LC}}{jC} + \frac{jL}{\sqrt{LC}}} = \frac{R}{R + \frac{-jLC + jLC}{C\sqrt{LC}}} = 1$$

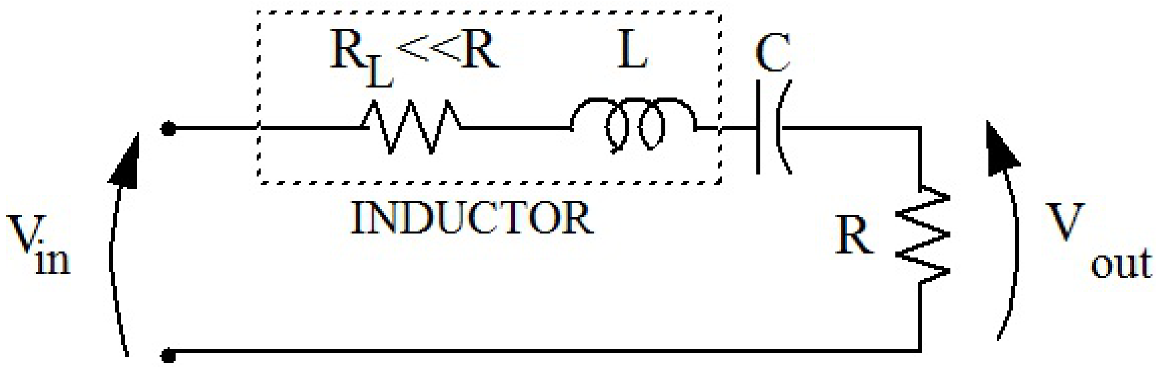

Figure 3: Band-pass Filter

Filters are used to select signals that are in a desired frequency range and reject those that are not desired. For example, a radio receiver uses a band-pass filter to separate the desired radio station from all of the other stations. To change stations the pass-band is moved to the frequency range of the new station. Telephones use a filter that will reject 60 Hz signals that are picked up from the power system. Without this "notch filter" the telephone would have a 60 Hz buzz at all times.

Terminology related to filters

It is often convenient to define standard quantities that will simplify the analysis and improve the understanding of filter circuits. A particular frequency that is often defined for filters is the cutoff frequency, f_c – the frequency value that will make the output magnitude equal to 0.707 times the input magnitude:

$$|G(f_c)| = \frac{1}{\sqrt{2}} = \left|\frac{1}{1 \pm j1}\right| \approx 0.707$$

A reduction from 1 to 0.707 in the magnitude of $G(f)$ is a reduction of 3dB ($20log_{10}1 - 20log_{10}0.707 = 3$db) and causes the power, which is the square of the voltage, to reduce from P to P/2. Therefore, the cutoff frequency is also called the **3dB frequency** and the **half-power frequency**.

II. Exercise 1: R-L low-pass filter study

R-L Impedance Analysis

The NI ELVIS prototyping boarding and instrument launcher have a very unique tool available for analyzing the effect of frequency of an impedance. On your desktop, open the NI Instrument Launcher and click on **Impedance Analyzer**. Next, on the NI prototyping board, wire up the series R-L circuit shown in Figure 4 with R = 1 kΩ and L = 0.4 H.

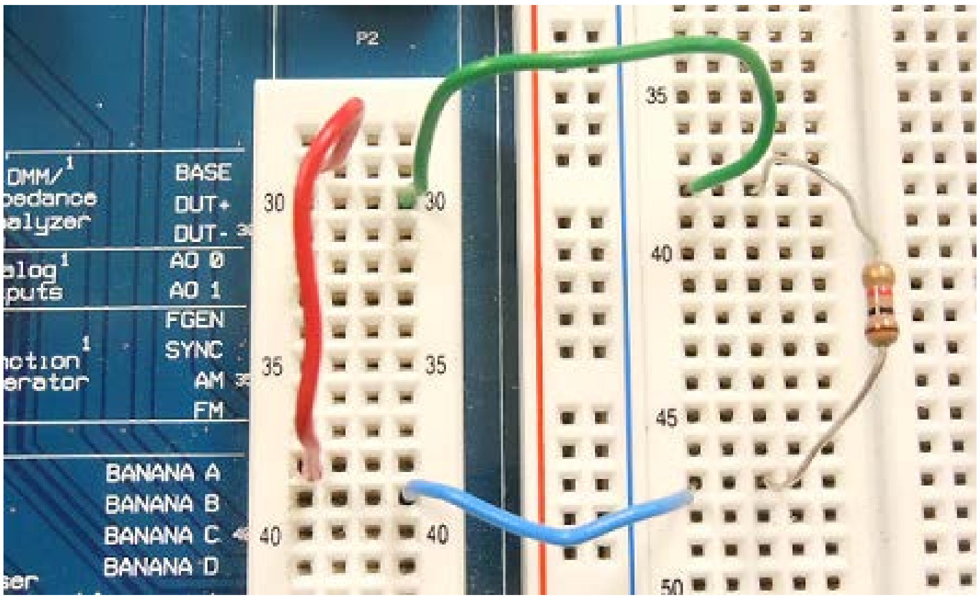

Figure 4: Connections for for Low-pass Filter Impedance Analysis

The series R-L connects to **DUT+** and **DUT-**. The inductor has to be connected through the banana jacks (refer to a previous lab manual if you need help remembering how).

With the prototyping board on, set the **Measurement Frequency** to 1 kHz and hit **Run**. Change the **Visible Section** to the quadrant where the resultant phasor is displayed. Include a screenshot of the Impedance Analyzer's front panel in your lab manual. Press **Stop**. Repeat the above with the frequency set at 200 Hz. What did and did not change about the impedance? Why or why not is this what you would expect?

Analog R-L Filter Analysis using Function Generator and Oscilloscope

This section provides an introduction to using NI ELVIS function generator and oscilloscope for ac characterization of a simple R-L low-pass filter. We will use the simple R-L low-pass filter as shown in Figure 1 with R = 1 kΩ and L = 0.4 H. We will construct the circuit using NI ELVIS system as shown in Figure 5.

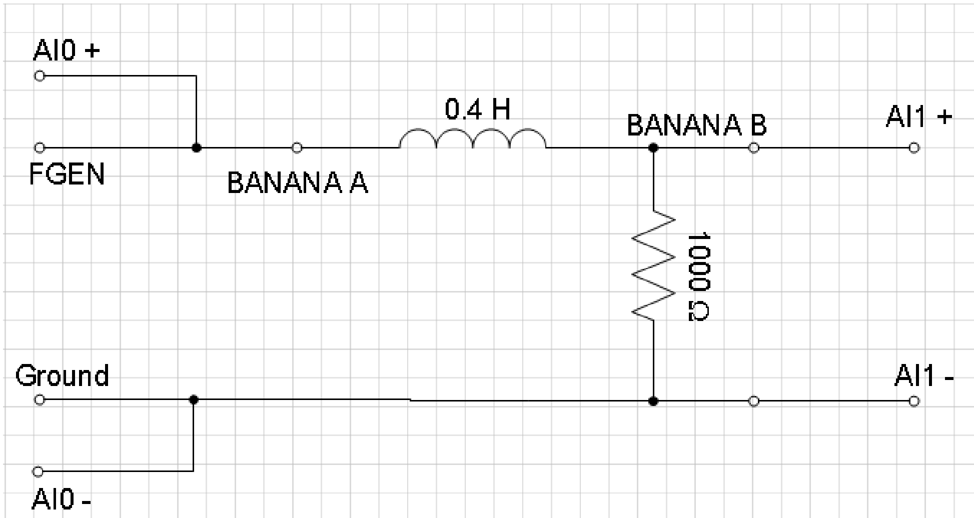

Figure 5: R-L Filter Connectivity for NI ELVIS Workstation

Carry out the analysis in the following steps:

1. The input signal is obtained from “FGEN" and “GROUND" pins on the prototype board. Connect the 1 kΩ resistor via the breadboard. The 0.4 H inductor will need to be connected via ELVIS’s banana jacks. The input and output signals will also need to be measured via NI’s desktop osciollscope so they will need to be connected to two of the AI ports (e.g., AI0 and AI1). Figure 5 shows an example of how the electrical connections could be wired.

2. On your desktop, open the NI ELVIS instrument launcher. From the NI ELVIS launcher, open the controls for the function generator and adjust them to generate a sine wave with frequency of 100 Hz, $V_{pp} = 10$ V, and no DC Offset applied. Set the Signal Route to the “Prototyping board”. The front panel should appear as shown in Figure 6.

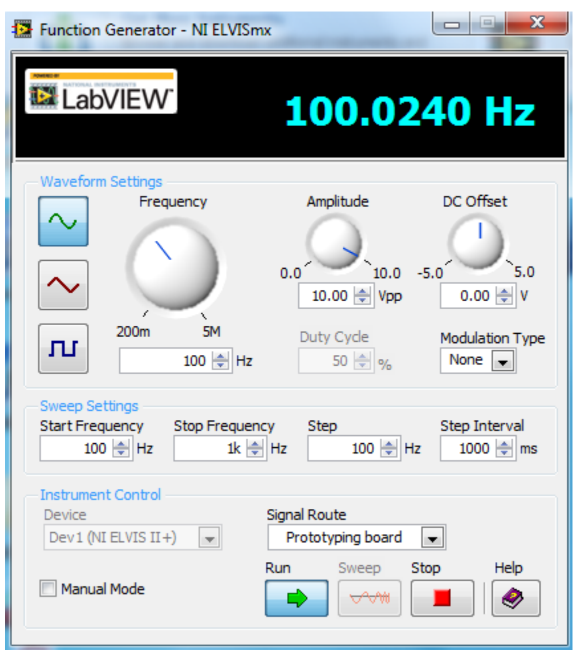

Figure 6: Function Generator Set to Produce a Sine Wave

3. Now, from the NI ELVIS instrument launcher, select the Oscilloscope to open the o-scope front panel. Adjust the o-scope's source, trigger, etc. so that your input and output signals can be accurately measured. Remember that the input to the R-L circuit is connected to "FGEN" and "GROUND" on the breadboard, and this input should also be connected to one of the analog input (AI) ports. S elect the appropriate AI in the Source pull down list under Channel 0 and Channel 1 Settings so that the input and output signals can be clearly displayed. You may also have to adjust the type of trigger, the Volts/Div, and/or the Time/Div settings. The Autoscale button may be helpful in getting you started with some of these settings. You can also change the frequency of the input signal on the FGEN front panel to see the corresponding changes on the oscilloscope. At the bottom of the screen, you can see the RMS, frequency, and amplitude (peak-to-peak) of the signals.

4. By varying the input frequency to the filter, we can obtain the cutoff frequency of the filter using the oscilloscope measurements. Vary the input frequency until
$$V_{out,pp} = \frac{V_{in,pp}}{\sqrt{2}}$$
Once the above relationship is true, your operating frequency is the cutoff frequency of the filter, f_C.
Note: as you adjust the frequency of your input, the input's peak-to-peak value is likely to change slightly. This is expected for most nonideal sources. Just make sure you satisfy the above equation.
Note: as you change the frequency (and, therefore, the output voltage magnitude), you will have to continue to change your Volts/Div and Time/Div settings to make sure the osciolloscope makes accurate measurements.

5. Once you are operating at the filter's cutoff frequency, take a screenshot of your function generator's and oscilloscope's front panels. Include these in your report. Make sure the osciolloscope's measurements are clearly legible. Do not forget to include the circuit schematic (Figure 1) in your report with input and output voltages labeled.

Analog RL Filter Analysis Using Bode Analyzer

A Bode plot presents the frequency analysis of a given circuit. Magnitude response is plotted as circuit gain in decibels as a function of log frequency. Phase response is plotted as the phase difference between input and output signals on a linear scale as a function of log frequency. NI ELVIS has a Bode analyzer which facilitates automatic Bode plot generation of a given circuit. To obtain the Magnitude and Phase response of the R-L filter, follow the following steps:

1. Make sure the function generator and osciolloscope have been stopped.
2. Retain the circuit configuration in Figure 5, and, from the NI ELVIS instrument launcher, select **Bode Analyzer**. The initial Bode analyzer front panel should appear as shown in Figure 7.

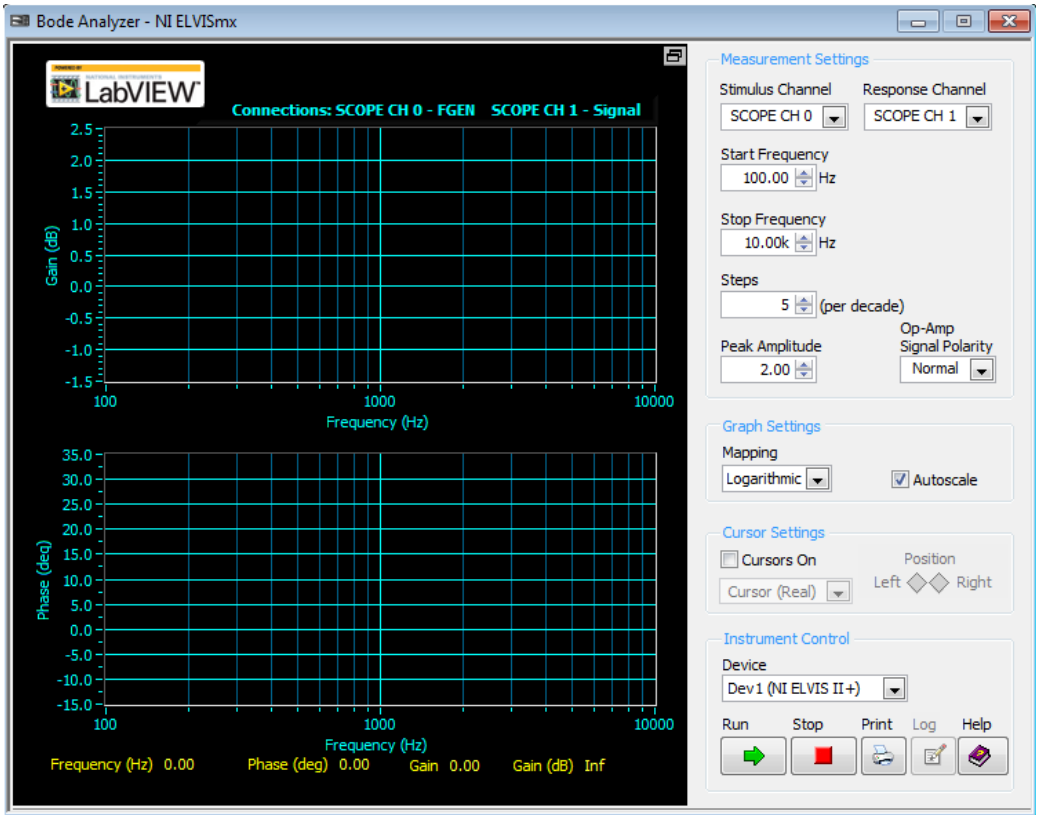

Figure 7: Uninitialized NI ELVIS Bode Analyzer Front Panel

3. Bode analyzer controls the input signal to the circuit only from the FGEN port on the breadboard. Note when using Bode analyzer, there will be no output from the FGEN BNC on the workstation. The **Stimulus Channel** should be connected to the source (the AI port which FGEN and ground are connected to), and the **Response Channel** should be connected to your output signal (the AI port which is across the output). Bode analyzer provides the flexibility to automatically scan the input signal frequency over a range specified by **Start/Stop frequency** values. The steps used during this frequency scan can also be set to a specific value.
4. For analyzing the R-L low pass filter, let's make the following settings on the Bode analyzer front panel:
   - **Start Frequency** to 100 Hz
   - **Stop Frequency** to 35 kHz
   - **Steps** to 25 per decade
   - **Peak Amplitude** to 5 V
   - Select **Autoscale**
   - Click on **Run**

Once the analysis is complete, the output should appear as shown in Figure 8. In Figure 8, the cursor has been placed to measure the 3dB frequency. This can be achieved by selecting the **Cursors On** and dragging the cursor using the left mouse button on the plot to the desired position. The cursor can also be shifted to the desired position using the two diamond shaped buttons in the Cursor Position. Due to limited resolution, you will probably not be able to place the cursor on -3dB exactly. Place it as close as possible and include a screenshot of the bode analyzer's front panel. Your cutoff frequencies ought to be very close to one another.

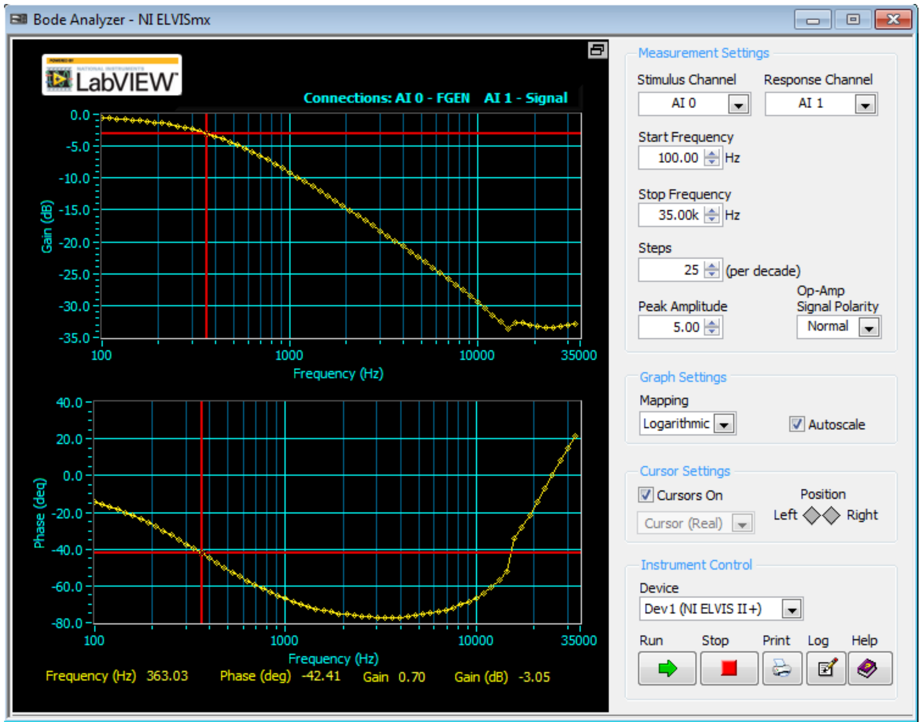

Figure 8: Bode Analyzer Output with 3dB Frequency Shown by Using Cursors

III. Exercise 2: R-C high-pass filter study

Change the inductance L to a capacitance C = 0.1 µF to set-up the high-pass filter. Repeat what has been done in Section 2 and 3, except, for the impedance analysis, compare the R-C impedance at 1 kHz and 2.5 kHz (make sure you **Stop** the impedance analyzer in between runs). Answer the same questions and include all necessary figures and screenshots in your report.

IV. Exercise 3: R-L-C bandpass filter study

Construct the band-pass filter of Figure 3 using C = 0.1 µF, L = 0.4 H, and R = 1 kΩ. Repeat what has been done in Sections 2 and 3, except, for the impedance analysis, compare the R-L-C impedance at 1 kHz and 745 Hz (make sure you **Stop** the impedance analyzer in between runs). Answer the following questions rather than the ones asked in the previous sections:

What is unusual about the R-L-C impdedance at 745 Hz?

What phenomena is being approximated?

Since the band-pass filter has two cutoff frequencies, you will need multiple screenshots of your oscilloscope, function generator, and bode analyzer front panels (one for each cutoff frequency). Answer all questions and include all necessary figures and screenshots in your report.

Clean-up your workstation before leaving.