4200 - Formal Languages: Final Exam

Fall 2019

Dec 11, 2019 at 4:00 - 6:30pm

Student:

Directions: The test is open-book and open-note, but using electronic devices (e.g. phone, tablets or computers) during the exam is NOT allowed. For each problem, show your work completely. Give reasons for all answers.
There are **7 problems** for a total of **85 points**.

Problem 1

10 points
Alphabet $\Sigma = \{0, 1\}$.

$$A = \{0, 1\}^*$$

$$B = L(\alpha)$$

where α is a regular expression: $\alpha = 0^* \cup 1^*$

$$C = A \cap \overline{B}$$

For each of the languages (A, B, $\overline{B}$, and C), write down five example strings in it. Construct a deterministic finite automaton (DFA) that recognizes the language C.

Problem 2

10 points

Define a nondeterministic finite automaton (NFA) M such that $L(M) = L(\beta)$ where β is a regular expression:

$$\beta = (1 \cup \epsilon)^* \cup 11^* \epsilon (0 \cup 1) \cup \epsilon$$

Note: for grading purposes, please do NOT simplify the given regular expression before constructing the NFA.

Problem 3

10 points

Write down a context-free grammar for the following language D. Please use S as the start variable.

$$D = \{\ 1^{x+2y}0^x1^y \mid x, y \geq 0\ \}$$

Problem 4

20 points

Is there a *decidable* Turing machine that accepts the following language D? (i.e. a Turing machine that halts for every input).

$$D = \{\ 1^{x+2y}0^x1^y \mid x, y \geq 0\ \}$$

- If yes, please describe in English how this Turing machine works (as if you were trying to convince a friend that this algorithm works). In addition to the English description, transition diagrams are optional (if you think it helps explaining), but not required.
- If no, please explain why.

Problem 5

10 points

Is the language D in Problem 4 regular or not?

$$D = \{\ 1^{x+2y}0^x1^y \mid x, y \geq 0\ \}$$

If yes, please show that via a DFA/NFA or a regular expression. If not, please prove it via pumping lemma for regular languages.

Problem 6

10 points

$$E = \{\ 1^{x+2y}0^x1^y1^y \mid x, y \geq 0\ \}$$

Please answer Yes or No.

- Is the language E regular?
- Is the language E context-free?
- Is the language E Turing-decidable?
- Is the language E Turing-recognizable?

Problem 7

15 points

$$G = \{\ 1^{x+2y}0^x1^y0^y \mid x, y \geq 0\ \}$$

Please answer Yes or No.

- Is the language G regular?
- Is the language G context-free?
- Is the language G Turing-decidable?
- Is the language G Turing-recognizable?

If you think the language G is not context-free, please prove it via pumping lemma for context-free languages.