

4200 - Formal Languages: Final Exam
Fall 2020

May 1, 2020 at 8:00 - 10:30am

Student:

Directions: The test is open-book and open-note, but using electronic devices (e.g. phone, tablets or computers) during the exam is NOT allowed. For each problem, show your work completely. Give reasons for all answers.

There are **7 problems** for a total of **85 points**.

Problem 1

10 points

$$A = \{0, 1, 2\}^*$$

$$B = \{0, 1\}^*$$

$$C = A - B$$

For each of the languages (A , B , and C), write down five example strings in it. Define a deterministic finite automaton (DFA) that recognizes the language C .

Problem 2

10 points

Define a regular expression that recognizes the set A that passwords that satisfy the following criteria:

- Minimum password length: 2
- Maximum password length: 4
- Each character in the password must be one of the following LaTeX: $\{ a, b, c, A, B, C, 1, 2, 3 \}$
- Password must contain at least one upper-case letter
- Password must contain at least one number

Answer:

$$\Sigma = \{a, b, c, A, B, C, 1, 2, 3\} \quad (1)$$

$$U = \{A, B, C\} \quad (2)$$

$$N = \{1, 2, 3\} \quad (3)$$

$$\alpha = \quad (4)$$

$$NU \cup UN \quad (5)$$

$$\cup \Sigma NU \cup \Sigma UN \cup U \Sigma N \cup U N \Sigma \cup N \Sigma U \cup N U \Sigma \quad (6)$$

Problem 3

15 points

Given a language $E = \{1^n 0^m 0^{2n+1} \mid n, m \geq 0\}$. Please answer the following questions:

1. Is the above language regular or non-regular?
2. Is the above language context-free or not?
3. Is the above language Turing-recognizable or not?
4. Based on your answers to the above questions, please answer one of the following:
 - a. If you answer E is *regular* and *context-free*, please provide an NFA or a regular expression that recognizes E .
 - b. If you answer E is *regular* and NOT *context-free*, please provide a proof (showing E is not context-free) using Pumping Lemma for Context-free Languages.
 - c. If you answer E is NOT *regular* and *context-free*, please provide a proof (showing E is not regular) using Pumping Lemma for Regular Languages.
 - d. If you answer E is NOT *regular* and NOT *context-free*, please provide a proof (showing E is not context-free) using Pumping Lemma for Context-free Languages.

Problem 4

15 points

Given a language $G = \{1^x 0^y 1^x 0^y \mid x, y \geq 0\} \cup \{1^k 0^n 1^k 0^v \mid k, v, n \geq 0\}$.

Please answer the following questions:

1. Is the above language regular or non-regular?
2. Is the above language context-free or not?
3. Is the above language Turing-recognizable or not?
4. Based on your answers to the above two questions, please answer one of the following:
 - a. If you answer G is *regular* and *context-free*, please provide an NFA or a regular expression that recognizes G .
 - b. If you answer G is *regular* and NOT *context-free*, please provide a proof (showing G is not context-free) using Pumping Lemma for Context-free Languages.
 - c. If you answer G is NOT *regular* and *context-free*, please provide a proof (showing G is not regular) using Pumping Lemma for Regular Languages.
 - d. If you answer G is NOT *regular* and NOT *context-free*, please provide a proof (showing G is not context-free) using Pumping Lemma for Context-free Languages.

Problem 5

15 points

Given a language $H = \{1^x 0^y 1^x 0^y \mid x, y \geq 0\} \cap \{1^k 0^n 1^k 0^v \mid k, v, n \geq 0\}$.

Please answer the following questions:

1. Is the above language regular or non-regular?
2. Is the above language context-free or not?
3. Is the above language Turing-recognizable or not?
4. Based on your answer to question 1 above, please answer one of the following:
 - a. If you answer H is *regular*, please provide an NFA or a regular expression that recognizes H .
 - b. If you answer H is NOT *regular*, please provide a proof (showing H is not regular) using Pumping Lemma for Regular Languages.