COMP-4200

Formal Languages

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LAB SECTION:	001

Homework #1

Problem 1

0.1 Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

f. {n| n is an integer and n = n + 1} . . .
→ **ANSWER: The set of all integers that are equal to one added to n, but no integer is equal to its successor — an empty set.**

e. {w| w is a string of 0s and 1s and w equals the reverse of w} . . .
→ **ANSWER: The set of all strings containing 0's and 1's and every string is a palindrome — a set of palindromic bit strings.**

0.6 Let X be the set {1, 2, 3, 4, 5} and Y be the set {6, 7, 8, 9, 10}. The unary function f : X - !Y and the binary function g : X Å Y - !Y are described in the following tables.

d. What are the range and domain of g? . . .
→ **ANSWER: Range of g = Y = {6,7,8,9,10} ... Domain of g = X * Y = { (1, 6), (1, 7), (1,8), (1, 9), (1, 10), (2, 6), (2, 7), (2, 8), (2, 9), (2, 10), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (4, 6), (4,7),(4, 8), (4, 9), (4, 10), (5, 6), (5, 7), (5, 8), (5, 9), (5, 10) }**

e. What is the value of g(4, f(4))? . . .
→ **ANSWER: g(4, f(4)) = g(4, 7) = 8**

Problem 2

Prove the following formulas by mathematical induction. For each solution, please specify your (1) base case, (2) induction hypothesis, and (3) the inductive step.

1. For all $n \in \mathbb{Z}^+$:

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$$

I. Base Case: Assume n = 1 and that the LHS and RHS will hold

$$\sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{1+1} = \frac{1}{2}$$

→ Hence, the base case hold for LHS and RHS for n = 1

II. Induction Step: Now, we will suppose this will hold for n = k+1

$$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+2)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{k+2}$$

→ Hence, the inductive step holds for n = k + 1, for all n that exists within $\mathbf{Z}^+$

III. Conclusion: By induction, the formula is proven.

2. For all $n \in \mathbb{Z}^+$:

$$1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

* Base Case: Take $n=1$

$\Rightarrow 1^3 = \left[\frac{1(1+1)}{2}\right]^2 = \left[\frac{1 \cdot 2}{2}\right]^2$

$\Rightarrow 1 = 1$

Hence, the base case holds

* Inductive Step: Take $n = K+1$

$= 1^3 + 2^3 + 3^3 + \dots + (K+1)^3$

$= 1^3 + 2^3 + 3^3 + \dots + K^3 + (K+1)^3$

$= \left[\frac{K(K+1)}{2}\right]^2 + (K+1)^3$

$= \frac{K^2(K+1)^2}{4} + (K+1)^3$

$= (K+1)^2\left[\frac{K^2 + 4(K+1)}{4}\right]$

$= (K+1)^2\left[\frac{K^2 + 4K + 4}{4}\right]$

$= (K+1)^2\left[\frac{K^2 + 2K + 2K + 4}{4}\right]$

$= (K+1)^2\left[\frac{K(K+2) + 2(K+2)}{4}\right]$

$= (K+1)^2\left[\frac{(K+2)(K+2)}{4}\right] = (K+1)^2\left(\frac{K+2}{2}\right)^2$

$= \left[\frac{(K+1)(K+2)}{2}\right]^2 =$ RHS $\therefore$ Hence proven!