Combinatorics
Assignment 8

November 11, 2017

Exercise 13

The given pair of graphs is not isomorphic, because u_2 in the first graph has a degree of four and is connected to u_3 which has a degree of four, we can not find two vertices in graph two that are connected and both have degree four.

Exercise 14

d) The number of paths of length 5 between two different vertices in K_4 is ..., because ...

Exercise 15

c) For the given graph G we have that

- $\kappa(G) = 2$, because when removing b and l, the graph is disconnected. This can't be done with only one node.
- $\lambda(G) = 2$, because when removing the edge a to b and k to l, this can't be done with only one edge.
- $\min_{v \in V} \deg(v) = 3$,
- And in $\kappa(G) \leq \lambda(G) \leq \min_{v \in V} \deg(v)$ the second inequality is strict.

d) For the given graph G we have that

- $\kappa(G) = 4$, because we can only seperate nodes b, c or e. These nodes can only be seperated by deleting four other nodes.
- $\lambda(G) = 4$, because the degree of each node is 4, this way we need 4 edges removed for a split in the graph.
- $\min_{v \in V} \deg(v) = 4$,
- And in $\kappa(G) \leq \lambda(G) \leq \min_{v \in V} \deg(v)$ no inequalities are strict.

Exercise 16

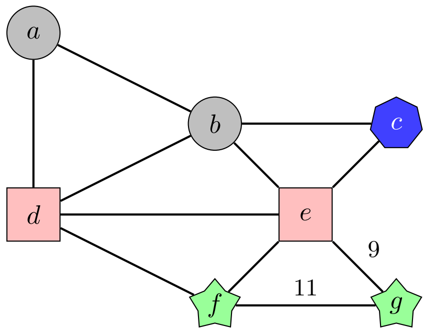
The graph has no Euler circuit, because vertex a has two ways out and one way in, the means vertex a must be the starting point of the Euler circuit, otherwise we can't make a circuit. But when re-entering vertex a we have to leave it a second time for the Euler circuit to be complete, this way we can not make an Euler circuit.

However, it does have an Euler path, for instance a to b to d to b to c to d to c to a to d.

Exercise 17

Either

> Yes, such a graph exists, namely ...
>
> To get you started using tikz, this is an example based upon `http://texample.net/tikz/examples/prims-algorithm/` but without animations and with more different shapes.



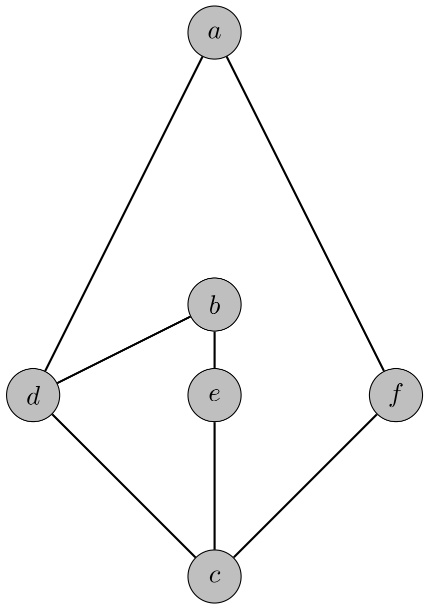
> This graph has no Hamiltion circuit, because ...

or

> No, such a graph doesn't exist, because ...

Exercise 18

This graph is planar, because here is an example:



Exercise 19

This graph (is / is not) homeomorphic to $K_{3,3}$, because ...

Exercise 20

This graph has chromatic number 3, because we can use 3 colors to color this graph by these three sets: f,b,d - a,d - g. Two colors is not possible because a, g and f are connected to each other.

Exercise 21

a) It (is / is not) possible to decrease the chromatic number by removing a single vertex and all edges incident with it, because ...

b) It (is / is not) possible to decrease the chromatic number by removing a single vertex and all edges incident with it, because ...

Exercise 22

a) The following simple graph is nonplanar and has exactly one cut edge: ...

- It is simple because ...
- It has only one cut edge, namely ..., because ...
- It is nonplanar because ...
- It has a minimal number of vertices because ...

b) The chromatic number of this graph is ..., because ...