

Assignment 9

I

```
lower <- 0
upper <- length(A)

while (A[lower + (upper-lower)/2] is not i)
  if (A[lower + (upper-lower)/2] < i)
    do lower <- lower + (upper-lower)/2
  else
    do upper <- lower + (upper-lower)/2

return lower + (upper-lower)/2
```

This algorithm is correct because it will half the array at every iteration. When the middle of the array is smaller than i , the lower bound will jump to the middle of the array. The opposite happens when the array element is larger than i . This way the algorithm will pinpoint the element with value i .

II

1.

Marieke

Guess: $O(2^n)$

Induction: $n = 1 \rightarrow 2^n = 2 = T(1)$

Substitution:

$$\begin{aligned} T(n) &= 2(T(n-1)) + 1 \\ &= 2(2^{n-1}) + 1 \\ &= 2^n + 1 \\ &= O(2^n) \end{aligned}$$

Joost:

David

Guess: $O(5 \cdot \log(n))$

Induction: $n = 1 \rightarrow 5 \cdot \log(n) + n = 1 = T(1)$

Substitution:

$$T(n) = 5 \cdot \log(n) + n$$

III

max-income (lower, upper, I)

```
if (lower == upper)
    return I[lower]
else
    income =  $\sum_{k=0}^n I[k]$ 
    if (income < max-income(lower, upper-1, I))
        income <- max-income(lower, upper-1, I)
    if (income < max-income(lower+1, upper, I))
        income <- max-income(lower+1, upper, I)
    return income
```

Complexity: $O(2^n)$

For every recursive call, the function is executed twice.

IV

Steps:

1. If $n = 1$, add one block to the left-bottom corner.
2. Else, divide the total space into four equal spaces, namely the left-top, right-top, left-bottom and right-bottom. Recursively solve these spaces. After solving, turn the left-top corner 90 degrees to the right and the right-bottom corner 90 degrees to the left. Continue with step 3.
3. Add one block in the middle of the created image.

Complexity: