Combinatorics
Assignment 1

November 11, 2017

Exercise 9

a) There are 999/7 = 142 (142,7 rounded down) numbers between 0 and 999 divisible by 7. There are 99/7 = 14 numbers between 0 and 100 divisible by 7. So there are 142 - 14 = 128 numbers between 100 and 999 divisible by 7.

d) There are 999/4 = 249 (249,75 rounded down) numbers between 0 and 999 divisible by 4. There are 99/4 = 24 numbers between 0 and 99 divisible by 4. In total, there are 249-24 = 225 numbers between 100 and 999 divisible by 4. There are 999-100 = 899 numbers between 100 and 999 of which 225 are divisible by 4. That means that there are 899-225 = 674 numbers between 100 and 999 not divisible by 4.

g) There are 999/3 = 333 numbers between 0 and 999 divisible by 3. There are 99/3 = 33 numbers between 0 and 100 divisible by 3. So there are 333-33 = 300 numbers between 100 and 999 divisible by 3. There are 999/12 = 83 numbers between 0 and 999 that are divisible by 12 (all numbers that are divisible by 3 and 4 are divisible by 12). There are 99/12 = 8 numbers between 0 and 100 that are divisible by 12. In total, there are 83-8 = 75 numbers between 100 and 999 divisible by 12. The amount of numbers between 100 and 999 that are divisible by 3 but not by 4 are 300-75 = 225.

Exercise 10

Of all numbers smaller than n, we must link only one to 1 in the second set. For n, we can choose what to link to, 0 or 1. Because we must link only one of the elements between 1, n-1 to 1 of the second set, we can make n-1 functions for that. For the last element n we have to choose where to link it to. So for each of the n-1 functions, there are two options to link n to, 0 or 1. In total, we can make 2 * (n-1) functions.

Exercise 11

a) We provide an algorithm for constructing such a row:

Task 1 First we pick 5 people out of the remaining 9, this can be done in 9 * 8 * 7 * 6 * 5 = 15.120 ways.

Task 2 Then we remove the doubles, because the 5 people we chose can be in different orders (We can pick for example person A, B, C, D and E but we can also pick person B, A, C, D and E. Those are effectively the same set of people). Therefor divide by 5 * 4 * 3 * 2 * 1 = 120. In total we now have 15.120 / 120 = 126 ways to pick 5 people out of 9.

Task 3 After that, we have 6 people left and we need to choose the order in which they will be on the photograph. For that we can make 6 * 5 * 4 * 3 * 2 * 1 = 720 possible ways.

Task 4 We now use the multiplication law to multiply the two possible ways together, which makes 126 * 720 = 90.720 possible ways.

b) We provide an algorithm for constructing such a row:

Task 1 First we pick 4 people out of the remaining 8, this can be done in 8 * 7 * 6 * 5 = 1.680 ways.

Task 2 Then we remove the doubles, because the 4 people we chose can be in different orders. Therefor divide by 4 * 3 * 2 * 1 = 24. In total we now have 1.680 / 24 = 70 ways to pick 4 people out of 8.

Task 3 After that, we have 6 people left and we need to choose the order in which they will be on the photograph. For that we can make 6 * 5 * 4 * 3 * 2 * 1 = 720 possible ways.

Task 4 We now use the multiplication law to multiply the two possible ways together, which makes 70 * 720 = 50.400 possible ways.

c) We provide an algorithm for constructing such a row:

Task 1 First we pick 5 people out of the remaining 8, this can be done in 8 * 7 * 6 * 5 * 4 = 6.720 ways.

Task 2 Then we remove the doubles, because the 5 people we chose can be in different orders. Therefor divide by 5 * 4 * 3 * 2 * 1 = 120. In total we now have 6.720 / 120 = 56 ways to pick 5 people out of 8.

Task 3 We now have only the amount of ways we can pick 6 people with either the bride or the groom being one of them. To calculate the amount of ways we can pick 6 people with the bride or the groom being one of them we multiply 56 by 2: 56 * 2 = 112.

Task 5 Now we have to arrange the amount of people so we multiply by 6 * 5 * 4 * 3 * 2 * 1 = 720: 112 * 720 = 80.640.

Exercise 12

Let the people be called A, B, C, D and E. Let B and C be friends of A and D and E be enemies of A. Then, in order to prove that it is not necessary to have three mutual friends or enemies, B and C can not be friends, so B and C are enemies. The same thing applies to D and E only they have to be friends. If we let B and E be enemies and D and C friends and we let E and C be enemies and D and B be friends, there is no group of three mutual enemies or friends.

Exercise 13

a) If we put all 7, 77, 777, ... in boxes of the remainder modulo 19, after a 19 digit 77777... number, we will certainly have one box with two numbers in it (because of the pigeonhole principle). After 20 numbers, the number with the most digits is 20 characters long (namely 77777777777777777777). When we subtract a smaller number from this 20 digit number, we will get a number with 20 digits so a number with only 7's and 0's below 54 characters exists.

b) Take n = 77777777777777777700 which can be factored into eighteen 7's and two 0's, so this number has 20 characters.