Matrixrekenen

I

a. $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$R_3 := R_3 - R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right)$$

$R_1 := R_1 - 2 \cdot R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 2 & -2 \\ 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right)$$

$R_2 := R_2 - 2 \cdot R_1$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 2 & -2 \\ 0 & 0 & -5 & -2 & -3 & 4 \\ 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right)$$

$R_2 := -\frac{1}{5} R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 2 & -3 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{3}{5} & -\frac{4}{5} \\ 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right)$$

$R_1 := R_1 - 3 \cdot R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} \\ 0 & 0 & 1 & \frac{2}{5} & \frac{3}{5} & -\frac{4}{5} \\ 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right)$$

$R_2 \longleftrightarrow R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & \frac{2}{5} & \frac{3}{5} & -\frac{4}{5} \end{array}\right)$$

Inverse: $\begin{pmatrix} -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} \\ 0 & -1 & 1 \\ \frac{2}{5} & \frac{3}{5} & -\frac{4}{5} \end{pmatrix}$

b

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$$

$$+ 2 \cdot \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix}$$

$$= 1 \cdot ((0 \cdot 1) - (1 \cdot 1)) - 2 \cdot ((2 \cdot 1) - (3 \cdot 1))$$

$$+ 2 \cdot ((2 \cdot 1) + (3 \cdot 0))$$

$$= (1 \cdot -1) - (2 \cdot \cancel{5}\, -1) + (2 \cdot 2)$$

$= 5$

$$C \quad \begin{pmatrix} -\frac{1}{5} & \frac{1}{5} & \frac{2}{5} \\ 0 & -1 & 1 \\ \frac{2}{5} & \frac{3}{5} & -\frac{4}{5} \end{pmatrix} \cdot \left(\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 2 & 0 & 1 & 0 \\ 2 & 1 & 1 & 10 \end{array}\right) =$$

$$C \cdot (A|B)$$

$$C \cdot (A|B) = (C \cdot A | C \cdot B) = (I | C \cdot B)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -10 \end{array}\right)$$

2. A matrix is invertible if an inverse exists. An inverse does not exists if a matrix in echelon form has less than n pivots. This is the case if the rows in the matrix are not linearly independend.

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$ has no inverse $(R_2 - 2 \cdot R_1 = 0)$

$$C + D = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 0 & 6 \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

C and D are invertible but $C+D$ is not invertible.

3. $\beta = \{(2,1), (5,3)\}$
$C = \{(1,-1), (1,-2)\}$

$$\beta = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} a' \\ b' \end{pmatrix}_C = \begin{pmatrix} a \\ b \end{pmatrix}_\beta$$

$$\begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix} \cdot \begin{pmatrix} a' \\ b' \end{pmatrix}_C = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}_\beta \cdot \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{-2+1} \cdot \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \frac{1}{-1} \cdot \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= -1 \cdot \begin{pmatrix} -2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 13 \\ -3 & -8 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix}$$

$$P = \begin{pmatrix} 5 & 13 \\ -3 & -8 \end{pmatrix}$$

$$\begin{pmatrix} a' \\ b' \end{pmatrix}_\beta = \begin{pmatrix} a \\ b \end{pmatrix}_G$$

$$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{6-5} \cdot \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

$$= 1 \cdot \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 13 \\ -3 & -5 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix}$$

$$Q = \begin{pmatrix} 8 & 13 \\ -3 & -5 \end{pmatrix}$$

$$P \cdot Q = \begin{pmatrix} 5 & 13 \\ -3 & -8 \end{pmatrix} \cdot \begin{pmatrix} 8 & 13 \\ -3 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$Q \cdot P = \begin{pmatrix} 8 & 13 \\ -3 & -5 \end{pmatrix} \cdot \begin{pmatrix} 5 & 13 \\ -3 & -8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

d. $\underline{g} : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$$\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}_C$$

$$\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}_C \cdot \begin{pmatrix} 8 & 13 \\ -3 & -5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ 5 & 8 \end{pmatrix}_\beta$$