Calculus and Probability
Assignment 2

x
x
Group 6

September 1, 2018

Exercise 6

a)

$$\begin{aligned}\lim_{x\to-\infty}\frac{x^3+2x^2+2}{3x^3+x+4} &= \lim_{x\to-\infty}\frac{1+\frac{2}{x}+\frac{2}{x^3}}{3+\frac{1}{x^2}+\frac{4}{x^3}}\\ &= \frac{1}{3}\end{aligned}$$

$\frac{1}{3}$

b)

$$\begin{aligned}\lim_{x\to\infty}\frac{2x+1}{x^2+x} &= \lim_{x\to\infty}\frac{\frac{2}{x}+\frac{1}{x^2}}{1+\frac{1}{x}}\\ &= 0\end{aligned}$$

0

Exercise 7

a)

$$\begin{aligned}f'(a) &= \lim_{h\to0}\frac{2(a+h)+3-2a-3}{h}\\ &= \lim_{h\to0}\frac{2a+2h+3-2a-3}{h}\\ &= \lim_{h\to0}\frac{2h}{h}\\ &= \lim_{h\to0}\frac{2}{1}\\ &= \frac{2}{1}\\ &= 2\end{aligned}$$

$f'(a)=2$

b)

$$
\begin{aligned}
f'(a) &= \lim_{h\to 0} \frac{\frac{5(a+h)-7}{4(a+h)+3} - \frac{5a-7}{4a+3}}{h} \\
&= \lim_{h\to 0} \frac{\frac{5a+5h-7}{4a+4h+3} - \frac{5a-7}{4a+3}}{h} \\
&= \lim_{h\to 0} \frac{\frac{(5a+5h-7)*(4a+3)}{(4a+4h+3)*(4a+3)} - \frac{(5a-7)*(4a+4h+3)}{(4a+3)*(4a+4h+3)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{20a^2+20ah-28a+15a+15h-21}{(4a+4h+3)*(4a+3)} - \frac{20a^2+20ah+15a-28a-28h-21}{(4a+3)*(4a+4h+3)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{20a^2+20ah-28a+15a+15h-21-20a^2-20ah-15a+28a+28h+21}{(4a+4h+3)*(4a+3)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{43h}{(4a+4h+3)*(4a+3)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{43h}{16a^2+16ah+12a+12a+12h+9}}{h} \\
&= \lim_{h\to 0} \frac{\frac{43h}{16a^2+16ah+24a+12h+9}}{h} \\
&= \lim_{h\to 0} \frac{43}{16a^2+16ah+24a+12h+9} \\
&= \frac{43}{16a^2+24a+9}
\end{aligned}
$$

$f'(a) = \frac{43}{16a^2+24a+9}$

Exercise 8

a) We need the differential of $f(x) = \frac{1}{1+\frac{1}{x}} = \frac{x}{x+1}$.

$$
\begin{aligned}
f'(x) &= \frac{(x+1)-x}{(x+1)^2} \\
&= \frac{1}{(x+1)^2}
\end{aligned}
$$

When entering $x = 2$, $f'(2) = \frac{1}{(2+1)^2} = \frac{1}{3^2} = \frac{1}{9}$. But $f(2) = \frac{2}{2+1} = \frac{2}{3}$. We thus come up with the line $t(x) = \frac{1}{9}x + \frac{4}{9}$.
$t(x) = \frac{1}{9}x + \frac{4}{9}$.

b) We need already have the differential of $f(x)$, $f'(x) = \frac{1}{(1+x)^2}$. We need to calculate the limit of both $f(x)$ and $f'(x)$:

$\lim_{x\to\infty} f'(x) = \frac{1}{(x+1)^2} = \frac{1}{\infty} = 0$
$\lim_{x\to\infty} f(x) = \frac{x}{x+1} = \frac{x}{x} = 1$

We could thus say that $y = ax + b$ and $1 = 0x + b$ thus $b = 1$. The tangent line is $t(x) = 1$.
$t(x) = 1$

Exercise 9

a)

$$
\begin{aligned}
f(x) &= e^{tan(x)} \\
f'(x) &= \frac{1}{cos(x)^2} * e^{tan(x)}
\end{aligned}
$$

$f'(x) = \frac{1}{cos(x)^2} * e^{tan(x)}$

b)

$$
\begin{aligned}
f(x) &= -\ln\left(cos(x)\right) \\
f'(x) &= -\frac{1}{cos(x)} * -sin(x) \\
&= \frac{sin(x)}{cos(x)} \\
&= tan(x)
\end{aligned}
$$

$f(x) = tan(x)$

Exercise 10

a)

$$
\begin{aligned}
f(x) &= e^{x^{e^x}} \\
f'(x) &= e^{x^{e^x}} * x^{e^x} \\
&=
\end{aligned}
$$

Answer 10a

b)

$$
\begin{aligned}
f(x) &= \sqrt{x-2} \\
&= (x-2)^{\frac{1}{2}} \\
f'(x) &= \frac{1}{2} * (x-2)^{-\frac{1}{2}} \\
&= \frac{1}{2} * \frac{1}{\sqrt{x-2}} \\
&= \frac{1}{2\sqrt{x-2}}
\end{aligned}
$$

$$
\begin{aligned}
y &= \sqrt{x-2} \\
y^2 &= x-2 \\
y^2+2 &= x \\
f^{-1}(x) &= x^2+2
\end{aligned}
$$

$$
\begin{aligned}
(f^{-1})'(x) &= \frac{1}{2\sqrt{x^2+2-2}} \\
&= \frac{1}{2\sqrt{x^2}} \\
&= \frac{1}{2x} \\
&= \frac{1}{2}x
\end{aligned}
$$

$(f^{-1})'(x) = \frac{1}{2}x$

Answer Form Assignment 2

Name	x
Student Number	x

Question	**Answer**
6a (1pt)	$\frac{1}{3}$
6b (1pt)	0
7a (1pt)	$f'(a) = 2$
7b (1pt)	$f'(a) = \frac{43}{16a^2+24a+9}$
8a (1pt)	$t(x) = \frac{1}{9}x + \frac{4}{9}.$
8b (1pt)	$t(x) = 1$
9a (1pt)	$f'(x) = \frac{1}{cos(x)^2} * e^{tan(x)}$
9b (1pt)	$f(x) = tan(x)$
10a (1pt)	Answer 10a
10b (1pt)	$(f^{-1})'(x) = \frac{1}{2}x$