

# Calculus and Probability

## Assignment 1

x  
x  
Group 6

September 1, 2018

### Exercise 6

a)

$$\begin{aligned}0 &= x - x^3 \\x &= x^3 \\x &\in \{-1, 0, 1\}\end{aligned}$$

$$x \in \{-1, 0, 1\}$$

b)

$$\begin{aligned}0 &< x - x^3 \\x &< x^3 \\0 &< x < 1 \text{ and } x < -1\end{aligned}$$

$$0 < x < 1 \text{ and } x < -1$$

### Exercise 7

$$\begin{aligned}y &= a + (b - a)x \\y &= a + xb - xa\end{aligned}$$

When choosing  $a = b$ , we can run over all values thus  $y$  has no restrictions.  
 $y \in \mathbb{R}$

### Exercise 8

- a) This function is odd because for all  $x$  and  $y = -x$ ,  $3x - x^3 = -(3x - x^3)$ . The function is odd.
- b) This function is even because for all  $x$  and  $y = -x$ ,  $f(x) = f(y)$ . The function is even.

## Exercise 9

a) Domain:

$$\begin{aligned} 7 - x^2 &\geq 0 \\ 7 &\geq x^2 \\ |\sqrt{7}| &\geq x \\ x &\leq |\sqrt{7}| \end{aligned}$$

Range:

The maximum of the function is at  $x = 0$ , thus  $1 + \sqrt{7}$ . The minimum of the function is at  $\sqrt{7 - x^2} = 0$  and thus  $f(x) = 1$ . Domain:  $D(f) = -\sqrt{7} \leq x \leq \sqrt{7}$ , Range:  $R(f) = 1 \leq y \leq 1 + \sqrt{7}$

b) Domain:

The only restriction for  $x$  is  $x \neq 0$ .

Range:

$f(x)$  will never equal 0 or anything lower than 0. This means  $f(x) > 0$ . Domain:  $D(f) = x \neq 0$ , Range:  $R(f) = y > 0$

## Exercise 10

a)

$$\begin{aligned} y &= \frac{ax + b}{cx + d} \\ y * (cx + d) &= ax + b \\ ycx + yd &= ax + b \\ ycx - ax &= b - yd \\ x * (yc - a) &= b - yd \\ x &= \frac{b - yd}{yc - a} \end{aligned}$$

$$x = \frac{b - yd}{yc - a}$$

b) Always. Always.

## Exercise 11

a)

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{x - 2}{x^2 + x - 6} &= \lim_{x \rightarrow 2} \frac{x - 2}{(x - 2)(x + 3)} \\ &= \lim_{x \rightarrow 2} \frac{1}{x + 3} \\ &= \frac{1}{2 + 3} \\ &= \frac{1}{5} \end{aligned}$$

$$\frac{1}{5}$$

b)

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 + x - 2} &= \lim_{x \rightarrow 2} \frac{(x-3)(x-1)}{(x-1)(x-2)} \\ &= \lim_{x \rightarrow 2} \frac{(x-3)}{(x-2)} \\ &= \frac{(1-3)}{(1-2)} \\ &= \frac{-2}{-1} \\ &= 2\end{aligned}$$

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## Answer Form Assignment 1

Name	x
Student Number	x

Question	Answer
6a (1pt)	$x \in \{-1, 0, 1\}$
6b (1pt)	$0 < x < 1$ and $x < -1$
7 (1pt)	$y \in \mathbb{R}$
8a (0.5pt)	The function is odd.
8b (0.5pt)	The function is even.
9a (1pt)	Domain: $D(f) = -\sqrt{7} \leq x \leq \sqrt{7}$ , Range: $R(f) = 1 \leq y \leq 1 + \sqrt{7}$
9b (1pt)	Domain: $D(f) = x \neq 0$ , Range: $R(f) = y > 0$
10a (1pt)	$x = \frac{b-yd}{yc-a}$
10b (1pt)	Always.
11a (1pt)	$\frac{1}{5}$
11b (1pt)	2