Beweren en Bewijzen Leertaak 9

28 april 2017

Opgave 1

a) Mijn studentnummer is dus ik maak de even afleidingsbomen.

b) (2) Stelling: $P \to Q, Q \to R, P \quad \vdash \quad R$

Afleidingsboom:

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{hyp}
\UnaryInfC{$P \to Q, Q \to R, P \vdash Q \to R$}
\AxiomC{}
\RightLabel{hyp}
\UnaryInfC{$P \to Q, Q \to R, P \vdash P \to Q$}
\AxiomC{}
\RightLabel{hyp}
\UnaryInfC{$P \to Q, Q \to R, P \vdash P$}
\RightLabel{$\to E$}
\BinaryInfC{$P \to Q, Q \to R, P \vdash Q$}
\RightLabel{$\to E$}
\BinaryInfC{$P \to Q, Q \to R, P \vdash R$}
\end{prooftree}
$$

(4) Stelling: $P \to (Q \to R) \quad \vdash \quad (P \wedge Q) \to R$

Afkorting(en): $\Sigma = P \to (Q \to R)$

Afleidingsboom:

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{hyp}
\UnaryInfC{$\Sigma, (P \wedge Q) \vdash P \to (Q \to R)$}
\AxiomC{}
\RightLabel{hyp}
\UnaryInfC{$\Sigma, (P \wedge Q) \vdash P \wedge Q$}
\RightLabel{$\wedge E1$}
\UnaryInfC{$\Sigma, (P \wedge Q) \vdash P$}
\RightLabel{$\to E$}
\BinaryInfC{$\Sigma, (P \wedge Q) \vdash Q \to R$}
\AxiomC{}
\RightLabel{hyp}
\UnaryInfC{$\Sigma, (P \wedge Q) \vdash P \wedge Q$}
\RightLabel{$\wedge E2$}
\UnaryInfC{$\Sigma, (P \wedge Q) \vdash Q$}
\RightLabel{$\to E$}
\BinaryInfC{$\Sigma, (P \wedge Q) \vdash R$}
\RightLabel{$\to I$}
\UnaryInfC{$\Sigma \vdash (P \wedge Q) \to R$}
\end{prooftree}
$$

(6) Stelling: $P \wedge (Q \vee R) \quad \vdash \quad (P \wedge Q) \vee (P \wedge R)$

Afkorting(en): $\Sigma = P \wedge (Q \vee R))$ Afleidingsboom:

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{hyp}
\UnaryInfC{$\Sigma \vdash P \wedge (Q \vee R)$}
\RightLabel{$\wedge E2$}
\UnaryInfC{$\Sigma \vdash Q \vee R$}
\AxiomC{}
\RightLabel{hyp}
\UnaryInfC{$\Sigma, Q \vdash P \wedge (Q \vee R)$}
\RightLabel{$\wedge E1$}
\UnaryInfC{$\Sigma, Q \vdash P$}
\AxiomC{}
\RightLabel{hyp}
\UnaryInfC{$\Sigma, Q \vdash Q$}
\RightLabel{$\wedge I$}
\BinaryInfC{$\Sigma, Q \vdash P \wedge Q$}
\RightLabel{$\vee I1$}
\UnaryInfC{$\Sigma, Q \vdash (P \wedge Q) \vee (P \wedge R)$}
\AxiomC{}
\RightLabel{hyp}
\UnaryInfC{$\Sigma, Q \vdash P \wedge (Q \vee R)$}
\RightLabel{$\wedge E1$}
\UnaryInfC{$\Sigma, R \vdash P$}
\AxiomC{}
\RightLabel{hyp}
\UnaryInfC{$\Sigma, R \vdash R$}
\RightLabel{$\wedge I$}
\BinaryInfC{$\Sigma, R \vdash P \wedge R$}
\RightLabel{$\vee I2$}
\UnaryInfC{$\Sigma, R \vdash (P \wedge Q) \vee (P \wedge R)$}
\RightLabel{$\vee E$}
\TrinaryInfC{$\Sigma \vdash (P \wedge Q) \vee (P \wedge R)$}
\end{prooftree}
$$

(8) Stelling: $P \to R, Q \to R \quad \vdash \quad (P \vee Q) \to R$

Afkorting(en): $\Sigma = P \to R, Q \to R)$

Afleidingsboom:

$$\dfrac{\dfrac{\dfrac{}{\Sigma, P \lor Q \vdash P \lor Q}\,hyp \quad \dfrac{\dfrac{}{\Sigma, P \lor Q, P \vdash P}\,hyp \quad \dfrac{}{\Sigma, P \lor Q, P \vdash P \to R}\,hyp}{\Sigma, P \lor Q, P \vdash R} \to E \quad \dfrac{\dfrac{}{\Sigma, P \lor Q, Q \vdash Q}\,hyp \quad \dfrac{}{\Sigma, P \lor Q, Q \vdash Q \to R}\,hyp}{\Sigma, P \lor Q, Q \vdash R} \to E}{\Sigma, P \lor Q \vdash R} \lor E}{\Sigma \vdash (P \lor Q) \to R} \to I$$

c)

Stelling	#hyp	$\# \to E$	$\# \to I$	$\# \land E1$	$\# \land E2$	$\# \land I$	$\# \lor E$	$\# \lor I1$	$\# \lor I2$	#totaal
(2)	3	2	0	0	0	0	0	0	0	5

Stelling	#hyp	$\# \to E$	$\# \to I$	$\# \land E1$	$\# \land E2$	$\# \land I$	$\# \lor E$	$\# \lor I1$	$\# \lor I2$	#totaal
(4)	3	2	1	1	1	0	0	0	0	8

Stelling	#hyp	$\# \to E$	$\# \to I$	$\# \land E1$	$\# \land E2$	$\# \land I$	$\# \lor E$	$\# \lor I1$	$\# \lor I2$	#totaal
(6)	5	0	0	2	1	2	1	1	1	13

Stelling	#hyp	$\# \to E$	$\# \to I$	$\# \land E1$	$\# \land E2$	$\# \land I$	$\# \lor E$	$\# \lor I1$	$\# \lor I2$	#totaal
(8)	5	2	0	0	0	0	1	0	0	8

Opgave 2

Dit is het alternatieve bewijs voor het feit dat uit de aannames volgt dat Kees gelukkig is.

Stelling: $\Sigma \quad \vdash \quad KG$

Afkorting(en): $\Sigma = JV \land PK, JV \to ML, PK \to WT, KT \lor \neg KT, WT \to \neg KT, KT \land ML \to KG, \neg KT \to KG$

Afleidingsboom:

$$\dfrac{\dfrac{}{\Sigma \vdash KT \lor \neg KT}\,hyp \quad \dfrac{\dfrac{}{\Sigma, KT \vdash KT \land ML \to KG}\,hyp \quad \dfrac{\dfrac{}{\Sigma, KT \vdash KT}\,hyp \quad \dfrac{\dfrac{}{\Sigma, KT \vdash JV \to ML}\,hyp \quad \dfrac{\dfrac{}{\Sigma, KT \vdash JV \land PK}\,hyp}{\Sigma, KT \vdash JV} \land E1}{\Sigma, KT \vdash ML} \to E}{\Sigma, KT \vdash KT \land ML} \land I}{\Sigma, KT \vdash KG} \to E \quad Tak3}{\Sigma \quad \vdash \quad KG} \lor E$$

Tak3:

$$\dfrac{\dfrac{\dfrac{}{\Sigma, \neg KT \vdash \neg KT \to KG}\,hyp \quad \dfrac{}{\Sigma, \neg KT \vdash \neg KT}\,hyp}{\Sigma, \neg KT \vdash KG} \to E}{\Sigma \quad \vdash \quad KG} \lor E$$

Opgave 3

a) Dit is de definitie van het onderdeel `Knop` in propositielogica:

b) En dit is het bewijs van de correctheidsstelling.

Stelling: $\Sigma \quad \vdash \quad \text{KnopIn} \leftrightarrow \text{Rinkel}$

Afkorting(en): $\Sigma = \text{SpanningA}, \text{SpanningB} \leftrightarrow \text{SpanningC}, \text{Rinkel} \leftrightarrow \text{SpanningC}, \text{SpanningA} \rightarrow (\text{KnopIn} \leftrightarrow \text{SpanningB})$

Afleidingsboom:

$$\dfrac{Tak1 \qquad \dfrac{\dfrac{}{\Sigma, \text{Rinkel} \vdash \text{KnopIn} \leftrightarrow \text{SpanningB}}\, hyp \qquad \dfrac{\dfrac{}{\Sigma, \text{Rinkel} \vdash \text{SpanningB} \leftrightarrow \text{SpanningC}}\, hyp \qquad Tak2}{\Sigma, \text{Rinkel} \vdash \text{SpanningB}} \leftrightarrow E2}{\Sigma, \text{Rinkel} \vdash \text{KnopIn}} \leftrightarrow E2}{\Sigma \quad \vdash \quad \text{KnopIn} \leftrightarrow \text{Rinkel}} \leftrightarrow I$$

Tak1:

$$\dfrac{\dfrac{}{\Sigma, \text{KnopIn} \vdash \text{Rinkel} \leftrightarrow \text{SpanningC}}\, hyp \qquad \dfrac{\dfrac{}{\Sigma, \text{KnopIn} \vdash \text{SpanningB} \leftrightarrow \text{SpanningC}}\, hyp \qquad \dfrac{Tak3 \qquad \dfrac{}{\Sigma, \text{KnopIn} \vdash \text{KnopIn}}\, hyp}{\Sigma, \text{KnopIn} \vdash \text{SpanningB}} \leftrightarrow E1}{\Sigma, \text{KnopIn} \vdash \text{SpanningC}} \leftrightarrow E2}{\Sigma, \text{KnopIn} \vdash \text{Rinkel}} \leftrightarrow E1$$

Tak2:

$$\dfrac{\dfrac{}{\Sigma, \text{Rinkel} \vdash \text{Rinkel} \leftrightarrow \text{SpanningC}}\, hyp \qquad \dfrac{}{\Sigma, \text{Rinkel} \vdash \text{Rinkel}}\, hyp}{\Sigma, \text{Rinkel} \vdash \text{SpanningC}} \leftrightarrow E1$$

Tak3:

$$\dfrac{\dfrac{}{\Sigma, \text{Rinkel} \vdash \text{SpanningA}}\, hyp \qquad \dfrac{}{\Sigma, \text{Rinkel} \vdash \text{SpanningA} \rightarrow (\text{KnopIn} \leftrightarrow \text{SpanningB})}\, hyp}{\Sigma, \text{Rinkel} \vdash \text{KnopIn} \leftrightarrow \text{SpanningB}} \leftrightarrow E$$