Beweren en Bewijzen Leertaak 11

14 mei 2017

Opgave 1

1. Stelling: $\forall x, \forall y, R\ x\ y \quad \vdash \quad \forall z, R\ z\ z$

   Afkorting(en): $\Sigma := \forall x, \forall y, R\ x\ y$

   Afleidingsboom:

$$
\cfrac{\cfrac{\cfrac{\cfrac{\rule{3cm}{0.4pt}}{\Sigma \vdash \forall x, \forall y, R\ x\ y}\ hyp}{\Sigma \vdash \forall y, R\ a\ y}\ \forall E}{\Sigma \vdash R\ a\ a}\ \forall E}{\Sigma \vdash \forall z, R\ z\ z}\ \forall I
$$

2. Stelling: $\forall x, Px \to Qx, \exists y, Py \quad \vdash \quad \exists x, Qx$

   Afkorting(en): $\Sigma := \forall x, Px \to Qx, \exists y, Py$

   Afleidingsboom:

$$
\cfrac{\cfrac{\rule{2cm}{0.4pt}}{\Sigma \vdash \exists y, Py}\ hyp \qquad \cfrac{\cfrac{\cfrac{\cfrac{\rule{3cm}{0.4pt}}{\Sigma, Py \vdash \forall x, Px \to Qx}\ hyp}{\Sigma, Py \vdash Py \to Qy}\ \forall E \qquad \cfrac{\rule{2cm}{0.4pt}}{\Sigma, Py \vdash Py}\ hyp}{\Sigma, Py \vdash Qy}\ \to E}{\Sigma, Py \vdash \exists x, Qx}\ \exists I}{\Sigma \vdash \exists x, Qx}\ \exists E
$$

3. Stelling: $\exists x, Px \vee Qx \quad \vdash \quad (\exists x, Qx) \vee (\exists x, Px)$

   Afkorting(en): $\Sigma := \exists x, Px \vee Qx$

   Afleidingsboom:

$$
\cfrac{\cfrac{\rule{2cm}{0.4pt}}{\Sigma \vdash \exists x, Px \vee Qx}\ hyp \qquad \cfrac{\cfrac{\rule{2cm}{0.4pt}}{\Sigma, Py \vee Qy \vdash Py \vee Qy}\ hyp \qquad \cfrac{\cfrac{\cfrac{\rule{3cm}{0.4pt}}{\Sigma, Py \vee Qy, Py \vdash Py}\ hyp}{\Sigma, Py \vee Qy, Py \vdash \exists x, Px}\ \exists I}{\Sigma, Py \vee Qy, Py \vdash (\exists x, Qx) \vee (\exists x, Px)}\ \vee I2 \qquad Tak3}{\Sigma, Py \vee Qy \vdash (\exists x, Qx) \vee (\exists x, Px)}\ \vee E}{\Sigma \vdash (\exists x, Qx) \vee (\exists x, Px)}\ \exists E
$$

Met Tak 3 =

$$\cfrac{\cfrac{\overline{\Sigma, Py \vee Qy, Qy \vdash Qy}\ hyp}{\Sigma, Py \vee Qy, Qy \vdash \exists x, Qx}\ \exists I}{\Sigma, Py \vee Qy, Qy \vdash (\exists x, Qx) \vee (\exists x, Px)}\ \vee I1$$

4. Stelling: $\exists x, Rxx \wedge Px \quad \vdash \quad \neg(\forall x, Px \rightarrow \neg(\exists y, Rxy))$

   Afkorting(en): $\Sigma := \exists x, Rxx \wedge Px$

   Afleidingsboom:

$$\cfrac{\cfrac{}{\Sigma \vdash \exists x, Rxx \wedge Px}\ hyp \quad \cfrac{\cfrac{\cfrac{\overline{\Sigma, Raa \wedge Pa, \forall x, Px \rightarrow \neg(\exists y, Rxy) \vdash Raa \wedge Pa}\ hyp}{\Sigma, Raa \wedge Pa, \forall x, Px \rightarrow \neg(\exists y, Rxy) \vdash Raa}\ \wedge E1}{\Sigma, Raa \wedge Pa, \forall x, Px \rightarrow \neg(\exists y, Rxy) \vdash \exists y, Ray}\ \exists I \quad Tak2}{\cfrac{\Sigma, Raa \wedge Pa \vdash \neg(\forall x, Px \rightarrow \neg(\exists y, Rxy))}{}}\ \neg I}{\Sigma \vdash \neg(\forall x, Px \rightarrow \neg(\exists y, Rxy))}\ \exists E$$

   Met Tak 2 =

$$\cfrac{\cfrac{\overline{\Sigma, Raa \wedge Pa, \forall x, Px \rightarrow \neg(\exists y, Rxy) \vdash \forall x, Px \rightarrow \neg(\exists y, Rxy)}\ hyp}{\Sigma, Raa \wedge Pa, \forall x, Px \rightarrow \neg(\exists y, Rxy) \vdash Pa \rightarrow \neg(\exists y, Ray)}\ \forall E \quad \cfrac{\overline{\Sigma, Raa \wedge Pa, \forall x, Px \rightarrow \neg(\exists y, Rxy) \vdash Raa \wedge Pa}\ hyp}{\Sigma, Raa \wedge Pa, \forall x, Px \rightarrow \neg(\exists y, Rxy) \vdash Pa}\ \wedge E2}{\Sigma, Raa \wedge Pa, \forall x, Px \rightarrow \neg(\exists y, Rxy) \vdash \neg(\exists y, Ray)}\ hyp$$

5. Stelling: $\vdash \quad \neg(\forall x, Px \vee (Qx \rightarrow Sx)) \rightarrow \neg(\forall x, Sx)$

   Afkorting(en): $\Sigma := \neg(\forall x, Px \vee (Qx \rightarrow Sx))$

   Afleidingsboom:

$$\cfrac{\cfrac{\cfrac{}{\Sigma, \forall x, Sx \vdash \neg(\forall x, Px \vee (Qx \rightarrow Sx))}\ hyp \quad \cfrac{\cfrac{\cfrac{\overline{\Sigma, \forall x, Sx \vdash \forall x, Sx}\ hyp}{\Sigma, \forall x, Sx \vdash Sb}\ \forall E}{\Sigma, \forall x, Sx \vdash \exists x, Sx}\ \exists I \quad \cfrac{\cfrac{\Sigma, \forall x, Sx, Sa \vdash Pa \vee \neg Pa}{}\ LEM \quad Tak2 \quad Tak3}{\Sigma, \forall x, Sx, Sa \vdash \forall x, Px \vee (Qx \rightarrow Sx)}\ \vee E}{\Sigma, \forall x, Sx \vdash \forall x, Px \vee (Qx \rightarrow Sx)}\ \exists E}{\Sigma \vdash \neg(\forall x, Sx)}\ \neg I}{\vdash \neg(\forall x, Px \vee (Qx \rightarrow Sx)) \rightarrow \neg(\forall x, Sx)}\ \rightarrow I$$

   Met Tak 2 =

$$\cfrac{\cfrac{\cfrac{\overline{\Sigma, \forall x, Sx, Pa \vdash Pa}\ hyp}{\Sigma, \forall x, Sx, Pa \vdash Pa \vee (Qa \rightarrow Sa)}\ \vee I1}{\Sigma, \forall x, Sx, Pa \vdash \forall x, Px \vee (Qx \rightarrow Sx)}\ \forall I}{}$$

   En Tak 3 =

$$\cfrac{\cfrac{\cfrac{\cfrac{\overline{\Sigma, \forall x, Sx, \neg Pa, Qa \vdash \forall x, Sx}\ hyp}{\Sigma, \forall x, Sx, \neg Pa, Qa \vdash Sa}\ \forall E}{\Sigma, \forall x, Sx, \neg Pa \vdash Qa \rightarrow Sa}\ \rightarrow I}{\Sigma, \forall x, Sx, \neg Pa \vdash Pa \vee (Qa \rightarrow Sa)}\ \vee I2}{\Sigma, \forall x, Sx, \neg Pa \vdash \forall x, Px \vee (Qx \rightarrow Sx)}\ \forall E$$

6. Stelling: $\vdash$ $(\forall x, Px \to (\forall y, Ryx)) \to \neg(\exists x, Px \land \neg Rxx)$

   Afkorting(en): $\Sigma := \forall x, Px \to (\forall y, Ryx)$

   Afleidingsboom:

$$
\cfrac{
\cfrac{
\cfrac{}{\Sigma, \exists x, Px \land \neg Rxx \vdash \forall x, Px \to (\forall y, Ryx)} \; hyp \quad Tak2
}{\Sigma \vdash \neg(\exists x, Px \land \neg Rxx)} \; \neg I
}{(\forall x, Px \to (\forall y, Ryx)) \to \neg(\exists x, Px \land \neg Rxx)} \; \to I
$$

   Met Tak 2 =

$$
\cfrac{
\cfrac{}{\Sigma, \exists x, Px \land \neg Rxx \vdash \exists x, Px \land \neg Rxx} \; hyp \quad
\cfrac{
\cfrac{}{\Sigma, \exists x, Px \land \neg Rxx, Pa \land \neg Raa \vdash \neg Raa} \; hyp \quad Tak2B
}{\Sigma, \exists x, Px \land \neg Rxx, Pa \land \neg Raa \vdash \neg(\forall x, Px \to (\forall y, Ryx))} \; \neg E
}{\Sigma, \exists x, Px \land \neg Rxx \vdash \neg(\forall x, Px \to (\forall y, Ryx))} \; \exists E
$$

   Met Tak 2B =

$$
\cfrac{
\cfrac{
\cfrac{}{\Sigma, \exists x, Px \land \neg Rxx, Pa \land \neg Raa \vdash \forall x, Px \to (\forall y, Ryx)} \; hyp
}{\Sigma, \exists x, Px \land \neg Rxx, Pa \land \neg Raa \vdash Pa \to (\forall y, Rya)} \; \forall E \quad
\cfrac{
\cfrac{}{\Sigma, \exists x, Px \land \neg Rxx, Pa \land \neg Raa \vdash Pa \land \neg Raa} \; hyp
}{\Sigma, \exists x, Px \land \neg Rxx, Pa \land \neg Raa \vdash Pa} \; \land E1
}{
\cfrac{
\cfrac{\Sigma, \exists x, Px \land \neg Rxx, Pa \land \neg Raa \vdash \forall y, Rya}{\Sigma, \exists x, Px \land \neg Rxx, Pa \land \neg Raa \vdash Raa} \; \forall E
}{}
} \; \to E
$$

7. Stelling: $\forall x, Px \lor Rxx, \forall x, Px \to (\exists y, Rxy \land Ryx)$ $\vdash$ $\forall x, \exists y, Rxy$

   Afkorting(en): $\Sigma := \forall x, Px \lor Rxx, \forall x, Px \to (\exists y, Rxy \land Ryx)$

   Afleidingsboom:

$$
\cfrac{
\cfrac{
\cfrac{}{\Sigma \vdash \forall x, Px \lor Rxx} \; hyp
}{\Sigma \vdash Pa \lor Raa} \; \forall E \quad
\cfrac{
\cfrac{
\cfrac{}{\Sigma, Pa \vdash \forall x, Px \to \exists y, Rxy \land Ryx} \; hyp
}{\Sigma, Pa \vdash Pa \to \exists y, Ray \land Rya} \; \forall E \quad
\cfrac{}{\Sigma, Pa \vdash Pa} \; hyp
}{
\cfrac{\Sigma, Pa \vdash \exists y, Ray \land Rya}{\Sigma, Pa \vdash \exists y, Ray} \; \exists E \quad Tak2 \quad Tak3
} \; \to E
}{
\cfrac{\Sigma \vdash \exists y, Ray}{\Sigma \vdash \forall x, \exists y, Rxy} \; \forall I
} \; \lor E
$$

   Met Tak 2 =

$$
\cfrac{
\cfrac{
\cfrac{}{\Sigma, Pa, Rab \land Rba \vdash Rab \land Rba} \; hyp
}{\Sigma, Pa, Rab \land Rba \vdash Rab} \; \land E1
}{\Sigma, Pa, Rab \land Rba \vdash \exists y, Ray} \; \exists I
$$

   En Tak 3 =

$$
\cfrac{
\cfrac{}{\Sigma, Raa \vdash Raa} \; hyp
}{\Sigma, Raa \vdash \exists y, Ray} \; \exists I
$$

Opgave 2

a) Het is mij wel gelukt om het bewijs te vinden voor de stelling over de al dan niet getrouwde mensen.

Opgave 3

a) Het is mij wel gelukt om het bewijs te vinden voor de stelling over de responsiecolleges.

b) Mijn formalisatie van deze stelling is:

`Theorem minimaalTweeAndereVakken:`

$\exists x : S, \exists y : S, x! = y \wedge HVxBB \wedge HVyBB \wedge (\exists v1 : V, \exists v2 : V, v1! = v2 \wedge v1! = BB \wedge v2! = BB \wedge HVxv1 \wedge HVxv2 \wedge HVyv1 \wedge HVyv2)$

Opgave 4

a) Het is mij wel gelukt om het bewijs te vinden voor `Taak11_pred001.v`.

b) Het is mij wel gelukt om het bewijs te vinden voor `Taak11_pred009.v`.

c) Het is mij wel gelukt om het bewijs te vinden voor `Taak11_pred020.v`.

d) Het is mij wel gelukt om het bewijs te vinden voor `Taak11_pred031.v`.

e) Het is mij wel gelukt om het bewijs te vinden voor `Taak11_pred042.v`.