1. i

$$\begin{pmatrix} 5 & 0 & 3 & 4 \\ 1 & 2 & 4 & 3 \\ 3 & 4 & 2 & 2 \\ 1 & 3 & 3 & 1 \end{pmatrix}$$

$R_2 := R_2 - R_4$

$$\begin{pmatrix} 5 & 0 & 3 & 4 \\ 0 & -1 & 1 & 2 \\ 3 & 4 & 2 & 2 \\ 1 & 3 & 3 & 1 \end{pmatrix}$$

$R_1 := R_1 - R_4$

$$\begin{pmatrix} 4 & -3 & 0 & 3 \\ 0 & -1 & 1 & 2 \\ 3 & 4 & 2 & 2 \\ 1 & 3 & 3 & 1 \end{pmatrix}$$

$R_4 := R_4 + 3 \cdot R_2$

$$\begin{pmatrix} 4 & -3 & 0 & 3 \\ 0 & -1 & 1 & 2 \\ 3 & 4 & 2 & 2 \\ 1 & 0 & 6 & 7 \end{pmatrix}$$

$R_3 := R_3 + 4R_2$

$$\begin{pmatrix} 4 & -3 & 0 & 3 \\ 0 & -1 & 1 & 2 \\ 3 & 0 & 6 & 10 \\ 1 & 0 & 6 & 7 \end{pmatrix}$$

$R_3 := R_3 - R_4$

$$\begin{pmatrix} 4 & -3 & 0 & 3 \\ 0 & -1 & 1 & 2 \\ 2 & 0 & 0 & 3 \\ 1 & 0 & 6 & 7 \end{pmatrix}$$

$R_1 := R_1 - 2 \cdot R_3$

$$\begin{pmatrix} 0 & -3 & 0 & -3 \\ 0 & -1 & 1 & 2 \\ 2 & 0 & 0 & 3 \\ 1 & 0 & 6 & 7 \end{pmatrix}$$

$R_3 := R_3 - 2 \cdot R_4$

$$\begin{pmatrix} 0 & -3 & 0 & -3 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & -12 & -11 \\ 1 & 0 & 6 & 7 \end{pmatrix}$$

$R_2 := R_2 - \frac{1}{3} \cdot R_1$

$$\begin{pmatrix} 0 & -3 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -12 & -11 \\ 1 & 0 & 6 & 7 \end{pmatrix}$$

$R_3 := R_3 + 12 \cdot R_2$

$$\begin{pmatrix} 0 & -3 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 6 & 7 \end{pmatrix}$$

Switch Rows

$$\begin{pmatrix} 1 & 0 & 6 & 7 \\ 0 & -3 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Echelon form ~~thus independend.~~ and
pivots = # rows thus independend

ii

$$\begin{pmatrix} 1 & 5 & 4 \\ 6 & 0 & 3 \\ -14 & 20 & 7 \end{pmatrix}$$

$R_3 := R_3 - 4 \cdot R_1$

$$\begin{pmatrix} 1 & 5 & 4 \\ 6 & 0 & 3 \\ -18 & 0 & -9 \end{pmatrix}$$

$R_3 := R_3 + 3 \cdot R_2$

$$\begin{pmatrix} 1 & 5 & 4 \\ 6 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

$R_2 := R_2 - 6 \cdot R_1$

$$\begin{pmatrix} 1 & 5 & 4 \\ 0 & -30 & -33 \\ 0 & 0 & 0 \end{pmatrix}$$

Echelon form and # pivots ≠ # rows
thus dependend.

2.

$v = v_1 + v_2 + v_3$

$$= \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -6 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 6 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 6 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$$

3. i, ii,

i is a subspace because x, $3x$ ~~can be zero~~, and $3x$ can be 0 and are closed under addition and multiplication.

4) ii is a subspace because x and $x+y$ can be zero and are closed under addition, and multiplication.

iii is not a subspace because x and $x+1$ can't be 0 at the same time.

iv is a subspace because $a \cdot v$ and $b \cdot w$ are closed under the propeties of a subspace thus $a \cdot v + a \cdot w$ is too.

v is a subspace because any two numbers out of $\mathbb{N}$ are closed under the ~~propeties~~ of addition and multiplication propeties. $\mathbb{N}$ also has 0.

4

~~This~~ Maps are linear if and only if:

$v, v' \in V$

$$f(v+v') = f(v) + f(v')$$

and

$a \in R$

$$f(a \cdot v) = a \cdot f(v)$$

i

$$f(x_1 + x_2, y_1 + y_2, z_1 + z_2) =$$

$$= ((y_1 + y_2) + (z_1 + z_2), 2(x_1 + x_2) + (z_1 + z_2), 3(x_1 + x_2) - (y_1 + y_2) + (z_1 + z_2))$$

$$= (y_1 + y_2 + z_1 + z_2, 2x_1 + 2x_2 + z_1 + z_2, 3x_1 + 3x_2 - y_1 - y_2 + z_1 + z_2)$$

$$= (y_1 + z_1, 2x_1 + z_1, 3x_1 - y_1 + z_1) + (y_2 + z_2, 2x_2 + z_2, 3x_2 - y_2 + z_2)$$

$$= f(x_1, y_1, z_1) + f(x_2, y_2, z_2)$$

and

$$f(a\cdot(x_0, oy, z)) - f(ax, ay, az) =$$

$$= (ay + az, 2ax + az, 3ax - ay + az)$$

$$= (a(y+z), a(2x+z), a(3x-y+z))$$

$$= a \cdot (y+z, 2x+z, 3x-y+z)$$

$$= a \cdot f(x, y, z)$$

f is closed under the properties of linear maps

ii $$f(\cancel{a(x_1+} x_1+x_2, y_1+y_2, z_1+z_2) =$$

$$(a(x_1+x_2)+b(y_1+y_2), c(x_1+x_2)+(z_1+z_2), d(x_1+x_2))$$

$$= (ax_1+ax_2+by_1+by_2, cx_1+cx_2+z_1+z_2, dx_1+dx_2)$$

$$= (ax_1+by_1, cx_1+z_1, dx_1) + (ax_2+by_2, cx_2+z_2, dx_2)$$

$$= f(x_1, y_1, z_1) + f(x_2, y_2, z_2)$$

and

ii $f(l \cdot (x, y, z)) = f(lx, ly, lz)$

$= (alx + bly, clx + lz, dlx)$

$= (l(ax+by), l(cx+z), l(dx))$

$= l \cdot (ax+by, cx+z, dx)$

$= l \cdot f(x, y, z)$

f is closed under the properties of linear maps

5

i $f(x_1 + x_2, y_1 + y_2, z_1 + z_2) =$

$((x_1 + x_2) + (z_1 + z_2), (y_1 + y_2) + (x_1 + x_2)(z_1 + z_2))$

$= (x_1 + x_2 + z_1 + z_2, y_1 + y_2 + x_1 z_1 + x_1 z_2 + x_2 z_1 + x_2 z_2)$

$= (x_1 + z_1, y_1 + x_1 z_1) +$
$(x_2 + z_2, y_2 + x_2 z_2) +$
$(0, x_1 z_2, x_2 z_1)$

$= f(x_1, y_1, z_1) + f(x_2, y_2, z_2) + (0, x_1 z_2 + x_2 z_1)$

$\neq f(x_1, y_1, z_1) + f(x_2, y_2, z_2)$

ii $f(a \cdot (x, y)) = f(ax, ay)$

$= (ax, ay + 3)$

$= a \cdot (x, y + \frac{3}{a})$

$\neq a \cdot f(x, y)$