Calculus and Probability
Assignment 4

x
x
Group 6

September 1, 2018

Exercise 6

a) First we will calculate the roots of f.

$$\begin{aligned} f(x) &= e^x sin(x) \\ e^x sin(x) &= 0 \\ sin(x) &= 0 \\ x &= k * \pi \end{aligned}$$

Because $e^x \neq 0$ and $sin(x) = 0$ at every $k * \pi$ with $k \in Z$. Now we will calculate the y-intersect:

$$\begin{aligned} f(0) &= e^0 sin(0) \\ f(0) &= 1 * 0 \\ f(0) &= 0 \end{aligned}$$

Root: $x = k * \pi$. Y-intersect: $y = 0$.

b) We first have to calculate the derivative f':

$$\begin{aligned} f(x) &= e^x sin(x) \\ f'(x) &= e^x cos(x) + e^x sin(x) \\ &= e^x (cos(x) + sin(x)) \\ f''(x) &= e^x * (-sin(x) + cos(x)) + e^x * (cos(x) + sin(x)) \\ &= e^x * (cos(x) + cos(x) + sin(x) - sin(x)) \\ &= e^x * (2cos(x)) \\ &= 2e^x cos(x) \end{aligned}$$

Now we only need to find $f'(x) = 0$ and $f''(x) = 0$:

$$\begin{aligned}
e^x(sin(x) + cos(x)) &= 0 \\
sin(x) + cos(x) &= 0 \\
cos(x) &= -sin(x) \\
1 &= \frac{-sin(x)}{cos(x)} && = 1 \\
-tan(x) &= 1 \\
tan(x) &= -1 \\
x &= \frac{3}{4}\pi + 2k\pi
\end{aligned}$$

And the zeros of f'':

$$\begin{aligned}
2e^x cos(x) &= 0 \\
2cos(x) &= 0 \\
cos(x) &= 0 \\
x &= \frac{1}{2}\pi + k\pi
\end{aligned}$$

For $f' = 0$: $x = \frac{3}{4}\pi + 2k\pi$. For $f'' = 0$: $x = \frac{1}{2}\pi + k\pi$.

Exercise 7

a) We will calculate both derivatives:

$$\begin{aligned}
f(x) &= cos(4y - xy) \\
f'(x) &= -sin(4y - xy) * -y \\
&= ysin(4y - xy) \\
f(y) &= cos(4y - xy) \\
f'(y) &= -sin(4y - xy) * (4 - x) \\
&= -4sin(4y - xy) + xsin(4y - xy)
\end{aligned}$$

X-derivative: $f'(x) = ysin(4y - xy)$. Y-derivative: $f'(y) = -4sin(4y - xy) + xsin(4y - xy)$.

b) We will calculate both derivatives:

$$\begin{aligned}
f(x) &= e^{\frac{x}{y}} \\
&= e^{\frac{1}{y}x} \\
f'(x) &= e^{\frac{1}{y}x} * \frac{1}{y} \\
&= \frac{1}{y}e^{\frac{1}{y}x} \\
f(y) &= e^{\frac{1}{y}x} \\
f'(y) &= e^{\frac{1}{y}x} * -x\frac{1}{y^2}
\end{aligned}$$

X-derivative: $f'(x) = \frac{1}{y}e^{\frac{1}{y}x}$. Y-derivative: $f'(y) = e^{\frac{1}{y}x} * -x\frac{1}{y^2}$.

Exercise 8

a)

$$\begin{aligned}\int_1^2 (3\sqrt{x} + \frac{3}{x^2})dx &= 2x\sqrt{x} - \frac{3}{x}]_1^2 \\ &= (2 * 2\sqrt{2} - \frac{3}{2}) - (2 * 1\sqrt{1} - \frac{3}{1}) \\ &= (4\sqrt{2} - 1\frac{1}{2}) - (2 - 3) \\ &= 4\sqrt{2} - 1\frac{1}{2} + 1 \\ &= 4\sqrt{2} - \frac{1}{2}\end{aligned}$$

$\int_1^2 (3\sqrt{x} + \frac{3}{x^2})dx = 4\sqrt{2} - \frac{1}{2}$.

b)

$$\begin{aligned}\int_{-1}^{1} \frac{-5}{\sqrt{1-x^2}}dx &= \int_{-1}^{1} -5 * \frac{1}{\sqrt{1-x^2}}dx \\ &= \int_{-1}^{1} -5 * (1-x^2)^{-\frac{1}{2}}dx \\ &= -5 * (\frac{1}{(1-x^2)^{\frac{1}{2}}})]_{-1}^{1} \\ &= -5 * ((\frac{1}{(1-1^2)^{\frac{1}{2}}}) - (\frac{1}{(1-(-1)^2)^{\frac{1}{2}}})) \\ &= -5 * ((\frac{1}{(1-1)^{\frac{1}{2}}}) - (\frac{1}{(1-1)^{\frac{1}{2}}})) \\ &= 0\end{aligned}$$

$\int_{-1}^{1} \frac{-5}{\sqrt{1-x^2}}dx = 0$.

Exercise 9

a)

$$\begin{aligned}\int_{-\infty}^{-\frac{\pi}{2}} \frac{xcos(x) - sin(x)}{x^2} &= \frac{sin(x)}{x}]_{-\infty}^{-\frac{\pi}{2}} \\ &= (\frac{sin(-\frac{\pi}{2})}{-\frac{\pi}{2}}) - (\lim_{x\to-\infty} \frac{sin(x)}{x}) \\ &= (\frac{-1}{-\frac{\pi}{2}}) - 0 \\ &= \frac{-1}{-\frac{\pi}{2}} \\ &= \frac{2}{\pi}\end{aligned}$$

$\int_{-\infty}^{-\frac{\pi}{2}} \frac{xcos(x)-sin(x)}{x^2} = \frac{2}{\pi}$.

b)

$$\begin{aligned}
\int_1^{\infty} \frac{1-ln(x)}{x^2}dx &= \frac{ln(x)}{x}]_1^{\infty} \\
&= (\lim_{x\to\infty} \frac{ln(x)}{x}) - (\frac{ln(1)}{1}) \\
&= (\lim_{x\to\infty} \frac{ln(x)}{x}) - (\frac{0}{1}) \\
&= (\lim_{x\to\infty} \frac{ln(x)}{x}) - 0 \\
&= \lim_{x\to\infty} \frac{ln(x)}{x} \\
&= \lim x \to \infty \frac{\frac{1}{x}}{1} \\
&= \lim x \to \infty \frac{1}{x} \\
&= 0
\end{aligned}$$

$int_1^{\infty} \frac{1-ln(x)}{x^2}dx = 0.$

Exercise 10

a) We need to solve $\lim_{x\to 0} \frac{cos(mx)-cos(nx)}{x^2}$. For this we can apply L'Hopitals rule because:

$$\begin{aligned}
\lim_{x\to 0} cos(mx) - cos(nx) &= 1 - 1 \\
&= 0 \\
\lim_{x\to 0} x^2 &= 0
\end{aligned}$$

This means that:

$$\lim_{x\to 0} \frac{cos(mx) - cos(nx)}{x^2} = \lim_{x\to 0} \frac{-msin(mx) + nsin(nx)}{2x}$$

We can apply L'Hopitals rule again:

$$\begin{aligned}
\lim_{x\to 0} \frac{-msin(mx) + nsin(nx)}{2x} &= \lim_{x\to 0} \frac{-m^2cos(mx) + n^2cos(nx)}{2} \\
&= \lim_{x\to 0} \frac{1}{2} * (-m^2cos(mx) + n^2cos(nx)) \\
&= \frac{1}{2} \lim_{x\to 0} -m^2cos(mx) + n^2cos(nx) \\
&= \frac{1}{2} * (-m^2 * 1 + n^2 * 1) \\
&= \frac{1}{2}(n^2 - m^2)
\end{aligned}$$

$\lim_{x\to 0} \frac{cos(mx)-cos(nx)}{x^2} = \frac{1}{2}(n^2 - m^2).$

b)

$$\begin{aligned}
\lim_{x\to\infty} x^{\frac{ln2}{1+lnx}} &= \lim_{x\to\infty} e^{ln(x^{\frac{ln2}{1+lnx}})} \\
&= \lim_{x\to\infty} e^{\frac{ln2}{1+lnx}*ln(x)} \\
&= \lim_{x\to\infty} e^{\frac{ln2*lnx}{1+lnx}} \\
&= \lim_{x\to\infty} e^{ln2*\frac{lnx}{1+lnx}} \\
&= \lim_{x\to\infty} 2^{\frac{lnx}{1+lnx}} \\
&= 2^1 \\
&= 2
\end{aligned}$$

$\lim_{x\to\infty} x^{\frac{ln2}{1+lnx}} = 2.$

Answer Form Assignment 4

Name	x
Student Number	x
Group	Group 6

Question	**Answer**
6a (1pt)	Root: $x = k * \pi$. Y-intersect: $y = 0$.
6b (1pt)	For $f' = 0$: $x = \frac{3}{4}\pi + 2k\pi$. For $f'' = 0$: $x = \frac{1}{2}\pi + k\pi$.
7a (1pt)	X-derivative: $f'(x) = ysin(4y - xy)$. Y-derivative: $f'(y) = -4sin(4y - xy) + xsin(4y - xy)$.
7b (1pt)	X-derivative: $f'(x) = \frac{1}{y}e^{\frac{1}{y}x}$. Y-derivative: $f'(y) = e^{\frac{1}{y}x} * -x\frac{1}{y^2}$.
8a (1pt)	$\int_1^2 (3\sqrt{x} + \frac{3}{x^2})dx = 4\sqrt{2} - \frac{1}{2}$.
8b (1pt)	$\int_{-1}^{1} \frac{-5}{\sqrt{1-x^2}}dx = 0$.
9a (1pt)	$\int_{-\infty}^{-\frac{\pi}{2}} \frac{xcos(x)-sin(x)}{x^2} = \frac{2}{\pi}$.
9b (1pt)	$int_1^\infty \frac{1-ln(x)}{x^2}dx = 0$.
10a (1pt)	$\lim_{x\to 0} \frac{cos(mx)-cos(nx)}{x^2} = \frac{1}{2}(n^2 - m^2)$.
10b (1pt)	$\lim_{x\to\infty} x^{\frac{ln2}{1+lnx}} = 2$.