Matrixrekenen

1.

a. $u = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ $v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $w = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\langle u, u \rangle = (1 \cdot 1) + (1 \cdot 1) + (-1 \cdot -1)$
$= 3$

$\langle v, v \rangle = (1 \cdot 1) + (2 \cdot 2) + (3 \cdot 3)$
$= 14$

$\langle w, w \rangle = (0 \cdot 0) + (0 \cdot 0) + (1 \cdot 1)$
$= 1$

$\langle u, v \rangle = (1 \cdot 1) + (1 \cdot 2) + (-1 \cdot 3)$
$= 0$

$\langle v, w \rangle = (1 \cdot 0) + (2 \cdot 0) + (3 \cdot 1)$
$= 3$

b. $\| u \| = \sqrt{1^2 + 1^2 + (-1)^2}$
$= \cancel{\sqrt{3}}$
$\sqrt{3}$

$\| v \| = \sqrt{1^2 + 2^2 + 3^2}$
$= \sqrt{14}$

$$\|w\| = \sqrt{0^2 + 0^2 + 1^2}$$
$$= \sqrt{1}$$
$$= 1$$

c. $$d(u, v) = \sqrt{(1-1)^2 + (1-2)^2 + (-1-3)^2}$$
$$= \sqrt{0 + 1 + 16}$$
$$= \sqrt{17}$$

$$d(v, w) = \sqrt{(1-0)^2 + (2-0)^2 + (3-1)^2}$$
$$= \sqrt{1 + 4 + 4}$$
$$= \sqrt{9}$$
$$= 3$$

d. $u' = au$

$$u' = \frac{1}{\|u\|} \cdot u$$
$$= \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$$

$$v' = bv$$

$$= \frac{1}{\|v\|} v$$

$$= \frac{1}{\sqrt{14}} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{pmatrix}$$

$$w' = cw$$

$$= \frac{1}{\|w\|} w$$

$$= \frac{1}{1} w$$

$$= w$$

$$= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

2.

a.

$$u = \begin{pmatrix} 2 \\ 2 \\ 2+\lambda \end{pmatrix} \qquad v = \begin{pmatrix} 1 \\ 1 \\ 1-\lambda \end{pmatrix}$$

For the two vectors to be independent, $(2+\lambda)$ and $(1-\lambda)$ must be coprime

$\cancel{\lambda > 0} \quad \to \begin{cases} \lambda > 0 \\ \lambda < 0 \end{cases}$

all othe values are permitted.

For the two vectors to be orthogonal $\langle u, v \rangle$ must be 0.

$$(2 \cdot 1) + (2 \cdot 1) + ((2+\lambda)(1-\lambda)) = 0$$
$$2 + 2 + (-\lambda^2 - \lambda + 2) = 0$$
$$-\lambda^2 - \lambda + 6 = 0$$
$$\lambda^2 + \lambda - 6 = 0$$
$$(\lambda+3)(\lambda-2) = 0$$
$$\lambda = -3 \quad \text{or} \quad \lambda = 2$$

b

$\lambda = 1$

$$u = \begin{Bmatrix} 2 \\ 2 \\ 3 \end{Bmatrix} \qquad v = \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

We need $\{u, v'\}$

$$v' = v - \frac{\langle v, u \rangle}{\langle u, u \rangle} u$$

$$= \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \frac{4}{17} \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{8}{17} \\ \frac{8}{17} \\ \frac{12}{17} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{9}{17} \\ \frac{9}{17} \\ -\frac{12}{17} \end{pmatrix}$$

Now, u and v' are orthogonal

3.

a. $\{v_1, v_2, v_3\}$

We need: $\{v_1', v_2', v_3'\}$

$$v_2' = v_2 - \frac{\langle v_2, v_1 \rangle}{\langle v_1, v_1 \rangle} \cdot v_1$$

$$= \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} - \frac{1}{2} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 1 \\ 0 \end{pmatrix} = v_2'$$

$$v_3' = v_3 - \frac{\langle v_3, v_1 \rangle}{\langle v_1, v_1 \rangle} \cdot v_1$$

$$V_3' = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} - \frac{0}{2}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{1\frac{1}{2}}\begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{1\frac{1}{2}}\begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -\frac{3}{8} \\ \frac{3}{8} \\ \frac{3}{4} \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{8} \\ -\frac{3}{8} \\ \frac{1}{4} \\ 1 \end{pmatrix}$$

Set: $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{3}{8} \\ -\frac{3}{8} \\ \frac{1}{4} \\ 1 \end{pmatrix} \right\}$

b)

$$v_1^n = \frac{1}{\|v_1\|} v_1$$

$$= \frac{1}{\sqrt{2}} v_1$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{pmatrix}$$

$$v_2^n = \frac{1}{\|v_2\|} v_2$$

$$= \frac{1}{\sqrt{1\frac{1}{2}}} v_2$$

$$= \cancel{\frac{1}{2\sqrt{1\frac{1}{2}}}} \begin{pmatrix} -\frac{1}{2\sqrt{1\frac{1}{2}}} \\ \frac{1}{2\sqrt{1\frac{1}{2}}} \\ \frac{1}{\sqrt{1\frac{1}{2}}} \\ 0 \end{pmatrix}$$

d

$$v_3^n = \frac{1}{\| v_3 \|} v_3$$

$$= \frac{1}{\sqrt{\frac{13}{64}}} v_3$$

$$= \begin{pmatrix} \frac{3}{\sqrt{86}} \\ \frac{3}{\sqrt{86}} \\ \sqrt{\frac{2}{43}} \\ 4 \cdot \sqrt{\frac{2}{43}} \end{pmatrix}$$