Matrixrekenen

assignment 4

1.

i

$$\begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$R_2 := R_2 + R_1$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

$R_1 := R_1 + R_3$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

$R_2 := R_2 + R_3$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$R_2 := \frac{1}{2} \cdot R_2$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$R_1 := R_1 - R_2$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$R_3 := R_3 - R_2$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This proves that ~~this~~ these ~~matri~~ vectors are linearly independed.

~~Dit is de standaard basis~~ dus

This is the standard basis so these ~~[illegible]~~ vectors are spanning

ii
$$\begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$R_1 := R_1 + R_2$

$$\begin{pmatrix} 0 & 1 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$R_1 := R_1 + R_3$

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

R_1 can be constructed with a linear-combination of the othe vectors so these vectors are not linear independed (and so also not a basis) iii $\cancel{\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 2 \\ 0 & 0 \end{pmatrix}}$ $\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 2 \\ 0 & 0 \end{pmatrix}$

These vectors are linear- independend (given in echelon form).

The vectors are spanning because each point in U can be made by:

$$x-y \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + y \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \\ 0 \end{pmatrix}.$$

2. $A \cdot B \cdot C =$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 4 & -3 \\ 2 & 0 \end{pmatrix} \cdot C =$$

$$\begin{pmatrix} 1 \cdot 4 + 1 \cdot 2 & 1 \cdot -3 + 1 \cdot 0 \\ 0 \cdot 4 + 1 \cdot 2 & 0 \cdot -3 + 1 \cdot 0 \\ 1 \cdot 4 + 0 \cdot 2 & 1 \cdot -3 + 0 \cdot 0 \end{pmatrix} \cdot C =$$

$$\begin{pmatrix} 6 & -3 \\ 2 & 0 \\ 4 & -3 \end{pmatrix} \cdot C =$$

$$\begin{pmatrix} 6 & -3 \\ 2 & 0 \\ 4 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 3 \\ 2 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 6 \cdot 1 + -3 \cdot 2 & 6 \cdot 3 + -3 \cdot 0 & 6 \cdot 3 + -3 \cdot 1 \\ 2 \cdot 1 + 0 \cdot 2 & 2 \cdot 3 + 0 \cdot 0 & 2 \cdot 3 + 0 \cdot 1 \\ 4 \cdot 1 + -3 \cdot 2 & 4 \cdot 3 + -3 \cdot 0 & 4 \cdot 3 + -3 \cdot 1 \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 18 & 15 \\ 2 & 6 & 6 \\ -2 & 12 & 9 \end{pmatrix}$$

$C \cdot A \cdot B =$

$$\begin{pmatrix} 1 & 3 & 3 \\ 2 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot B =$$

$$\begin{pmatrix} 1 \cdot 1 + 3 \cdot 0 + 3 \cdot 1 & 1 \cdot 1 + 1 \cdot 3 + 0 \cdot 3 \\ 2 \cdot 1 + 0 \cdot 0 + 1 \cdot 1 & 2 \cdot 1 + 0 \cdot 1 + 1 \cdot 0 \end{pmatrix} \cdot B =$$

$$\begin{pmatrix} 4 & 4 \\ 3 & 2 \end{pmatrix} \cdot B =$$

$$\begin{pmatrix} 4 & 4 \\ 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 4 & -3 \\ 2 & 0 \end{pmatrix} =$$

$$\begin{pmatrix} 4 \cdot 4 + 4 \cdot 2 & 4 \cdot -3 + 4 \cdot 0 \\ 3 \cdot 4 + 2 \cdot 2 & 3 \cdot -3 + 2 \cdot 0 \end{pmatrix} =$$

$$\begin{pmatrix} 24 & -12 \\ 16 & -9 \end{pmatrix}$$

3.

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1 + c_2 - c_3 \\ c_1 - c_2 + c_3 \\ -c_1 + c_2 + c_3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

$\downarrow$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 1 & -1 & 1 & 2 \\ -1 & 1 & 1 & 1 \end{array}\right)$$

$R_3 := R_3 + R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 1 & -1 & 1 & 2 \\ 0 & 2 & 0 & 4 \end{array}\right)$$

$R_3 := \frac{1}{2} R_3$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & 2 \end{array}\right)$$

$R_2 := R_2 + R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 2 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \end{array}\right)$$

$R_2 := \frac{1}{2} R_2$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 1 & 0 & 0 & 2\frac{1}{2} \\ 0 & 1 & 0 & 2 \end{array}\right)$$

$$R_1 := R_1 - R_2 - R_3$$

$$\left(\begin{array}{ccc|c} 0 & 0 & -1 & -1\frac{1}{2} \\ 1 & 0 & 0 & 2\frac{1}{2} \\ 0 & 1 & 0 & 2 \end{array}\right)$$

$$R_1 := -1 R_1$$

$$\left(\begin{array}{ccc|c} 0 & 0 & 1 & 1\frac{1}{2} \\ 1 & 0 & 0 & 2\frac{1}{2} \\ 0 & 1 & 0 & 2 \end{array}\right)$$

$$[v]_B = \begin{pmatrix} 2\frac{1}{2} \\ 2 \\ 1\frac{1}{2} \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1 + c_2 + c_3 \\ 0 + c_2 + c_3 \\ 0 + 0 + c_3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$[v]_C = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

w:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1 + c_2 - c_3 \\ c_1 - c_2 + c_3 \\ -c_1 + c_2 + c_3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

$\downarrow$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{array}\right)$$

$R_3 := R_3 + R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 0 & 2 & 0 & 2 \end{array}\right)$$

$R_1 := R_1 + R_2$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 1 & -1 & 1 & 1 \\ 0 & 2 & 0 & 2 \end{array}\right)$$

$$c_2 = 1$$

$$c_1 = 1$$

$$1 = c_1 - c_2 + c_3$$

$$= c_3$$

$$\Downarrow$$

$$c_3 = 1$$

$$[w]_B = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1 + c_2 + c_3 \\ 0 + c_2 + c_3 \\ 0 + 0 + c_3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$c_3 = 1$$

$$\cancel{1 - c_2 = c_2}$$

$$c_2 = \cancel{c_3} \; 1 - c_3$$

$$= 0$$

$$c_1 = 1 - c_2 - c_3$$

$$= 1 - 0 - 1$$

$$= 0$$

$$[w]_C = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

4

a.

$$\begin{pmatrix} 1 & 1 & 0 & 2 \\ 2 & 3 & 1 & 6 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

b. $f((x_1, x_2, x_3, x_4)) = (4x_1 + x_2 + 3x_3 - 5x_4, x_1 + 3x_2 + 5x_3 + 2x_4)$

c.

$$f\left(\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$f\left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

~~f~~