Combinatorics
Assignment 7

November 11, 2017

Exercise 12

R is symmetric Let u and v be vertices in G such that $u\,R\,v$. Then, this means that there is an undirected edge from u to v. This edge is undirected thus this edge can also be written as an edge from v to u. The relation is thus symmetric.

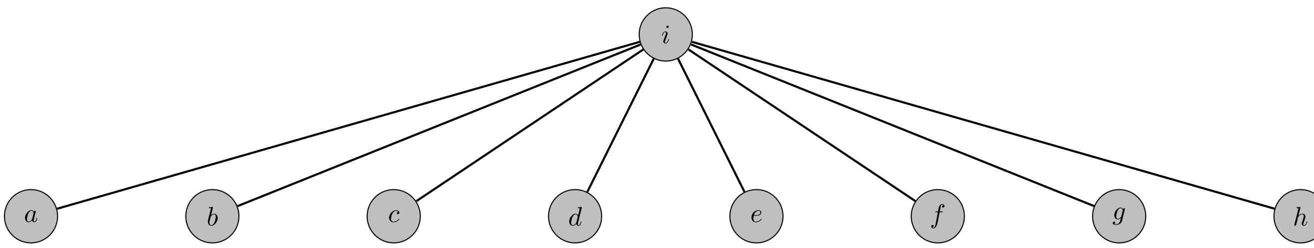
R is reflexive Let u be a vertex in G. Then, by definition of G, there exists an edge from u to u. This means that R is a reflexive relation on G.

Exercise 13

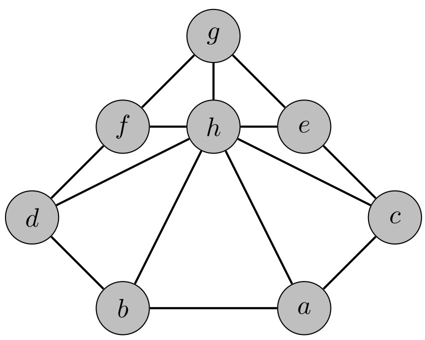
The statements that need to be executed before S_6 are the statements connected with a directed edge ending in S_6. These statements are: S_1, S_2, S_3 and S_4.

Exercise 14

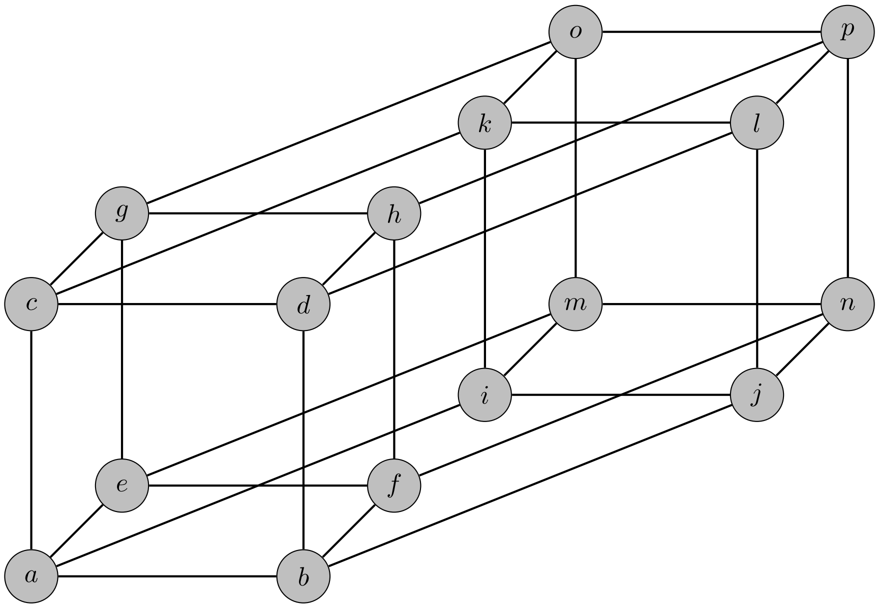
b) This is $K_{1,8}$:



e) This is W_7:



f) This is Q_4:



Exercise 15

a) The graph K_n is bipartite for some $n \in \mathbb{N}$, namely $n = 1$ and $n = 2$, because after $n = 2$ there is always a triangle in the graph of K_n.

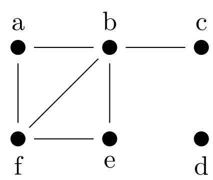
b) The graph C_n is bipartite for some $n \in \mathbb{N}$, namely for all $n \in \mathbb{N}$ for which n is even, because if n is not even, when splitting the graph into V_1 and V_2 and coloring the nodes blue and red, we eventually come to a node for which the neighbors of that node are red and blue, this last node makes it so that this graph is not bipartite. ...

c) The graph W_n is bipartite for no $n \in \mathbb{N}$ because the node in the middle of the graph W_n is always connected to all other nodes, when making this node red all other nodes need to be blue but these nodes are also connected to each other.

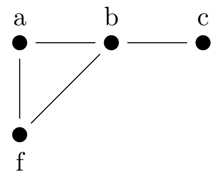
d) The graph Q_n is bipartite for all $n \in \mathbb{N}$ because Q_n is represented by bitstrings of length n, the only vertices that are connected are the ones that different one bit. We can thus color the bitstrings with an even number of zeros red, and the other ones blue. In this way only blue and red vertices are connected.

Exercise 16

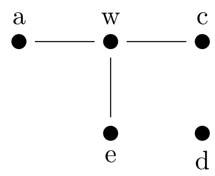
Let G be the graph:



a) The subgraph induced by the vertices a, b, c and f is



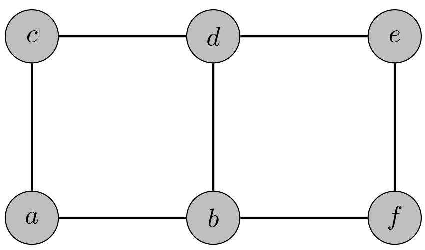
b) The new graph G_1 obtained from G by contracting the edge connecting b and f is



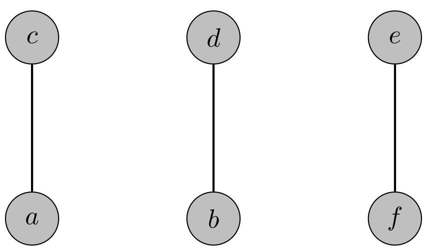
Exercise 17

a) The sequence 5, 4, 3, 2, 1, 0 is not graphic because the degree 5 can not be satisfied in a simple graph because there are only four nodes left (with degree one or higher). We need to draw or a self loop or a double edge.

e) The sequence 3, 3, 2, 2, 2, 2 is graphic as you can see in this graph



f) The sequence 1, 1, 1, 1, 1, 1 is graphic as you can see in this graph



Exercise 18

A graph is n-regular if the amount of neighbors of all vertices of the graph is equal to n. So if we take a bipartite n-regular graph, we can state that there are u vertices in V_1. The amount of edges going out of V_1 is then equal to $u * n$. Now, the amount of vertices going in to V_2 is also $u * n$. Because each vertex v has n edges connected to it, the amount of vertices in V_2 is $\frac{u*v}{n} = u$. This means $|\,V_1\,| = |\,V_2\,|$.

Exercise 19

a) This is a planar graph.
The number of edges is 13.

The in-degree of b is 1.
The out-degree of b is 3.

b) Yes, only W_3 is a complete wheel graph.

c) This is true because, when a graph contains only one edge and one vertex, this statement holds. When adding a lot of self loops, this statement holds because M increases with 2 each time a self loop is added to the vertex just as the formula increases 2 each time an edge is added.

d) This graph is undirected and symmetric because the adjency matrix is symmetric too.