Calculus and Probability
Assignment 3

x
x
Group 6

September 1, 2018

Exercise 6

a)

$$
\begin{aligned}
f(x) &= arccos(cos\ x^2) \\
f'(x) &= \frac{-1}{\sqrt{1-(cos\ x^2)^2}} * -sin\ x^2 * 2x \\
&= \frac{-1}{\sqrt{1-(cos\ x^2)^2}} * -2x\ sin\ x^2 \\
&= \frac{2x\ sin\ x^2}{\sqrt{1-(cos\ x^2)^2}}
\end{aligned}
$$

$f'(x) = \frac{2x\ sin\ x^2}{\sqrt{1-(cos\ x^2)^2}}$

Exercise 7

a)

$$
\begin{aligned}
h(x) &= \frac{x^2}{1+e^{-x}} \\
&= \frac{f(x)}{g(x)}
\end{aligned}
$$

We will first check if L'Hopital's rule can be applied.

$$
\begin{aligned}
\lim_{x\to\infty} f(x) &= \lim_{x\to\infty} x^2 \\
&= \infty \\
\lim_{x\to\infty} g(x) &= \lim_{x\to\infty} 1+e^{-x} \\
&= \lim_{x\to\infty} 1+\frac{1}{e^x} \\
&= 1
\end{aligned}
$$

The limits are not the same so we can't apply L'Hopital's rule. We must thus use the normal limit rules.

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^2}{1+e^{-x}} &= \lim_{x\to\infty} \frac{x^2}{1+\frac{1}{e^x}} \\
&= \lim_{x\to\infty} \frac{x^2}{1+0} \\
&= \lim_{x\to\infty} \frac{x^2}{1} \\
&= \lim_{x\to\infty} x^2 \\
&= \infty
\end{aligned}$$

$\lim_{x\to\infty} \frac{x^2}{1+e^{-x}} = \infty$

b)

$$\lim_{x\to 0} \frac{sin(x) + Ax + Bx^3}{x^5} = \frac{1}{C}$$

We can use L'Hopital's rule because:

$$\begin{aligned}
\lim_{x\to 0} sin(x) + Ax + Bx^3 &= 0 + A*0 + B*0 \\
&= 0 \\
\lim_{x\to 0} x^5 &= 0
\end{aligned}$$

We can now make $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$.

$$\lim_{x\to 0} \frac{f'(x)}{g'(x)} = \lim_{x\to 0} \frac{cos(x) + A + 3Bx^2}{5x^4}$$

Answer 7b

Exercise 8

a)

$$\begin{aligned}
g^{(0)}(x) &= cos(3x) \\
g^{(1)}(x) &= 3 * -sin(3x) \\
g^{(2)}(x) &= 9 * -cos(3x) \\
g^{(3)}(x) &= 27 * sin(3x) \\
g^{(2015)}(x) &= 3^{2015} * sin(3x)
\end{aligned}$$

$g^{(2015)}(x) = 3^{2015} * sin(3x)$

Exercise 9

a) The roots of f are at (-1, 0) and (3, 0).

$$\begin{aligned} f(x) &= 0 \\ (x+1)^2(x-3) &= 0 \\ (x+1)^2 &= 0 \text{ or } (x-3) = 0 \\ (x+1)^2 &= 0 \\ (x+1) &= 0 \\ x &= -1 \\ (x-3) &= 0 \\ x &= 3 \end{aligned}$$

The y-intercept is at (0, -3).

$$\begin{aligned} f(0) &= (0+1)^2 * (0-3) \\ &= 1^2 * -3 \\ &= -3 \end{aligned}$$

The roots of f are at (-1, 0) and (3, 0). The y-intercept is at (0, -3).

b)

$$\begin{aligned} \lim_{x\to-\infty} f(x) &= \lim_{x\to-\infty} (x+1)^2(x-3) \\ &= \lim_{x\to-\infty} x^2 * x \\ &= \lim_{x\to-\infty} x^3 \\ &= -\infty \\ \lim_{x\to\infty} f(x) &= \lim_{x\to\infty} (x+1)^2(x-3) \\ &= \lim_{x\to\infty} x^2 * x \\ &= \lim_{x\to\infty} x^3 \\ &= \infty \end{aligned}$$

$\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$.

c)

$$\begin{aligned} f(x) &= (x+1)^2(x-3) \\ &= (x^2+2x+1) * (x-3) \\ &= x^3 + 2x^2 + x - 3x^2 - 6x - 3 \\ &= x^3 - x^2 - 5x - 3 \\ f'(x) &= 3x^2 - 2x - 5 \\ f'(x) &= 0 \\ 3x^2 - 2x - 5 &= 0 \end{aligned}$$

$a = 3$, $b = -2$, $c = -5$.

$$\begin{aligned}
D &= b^2 - 4ac \\
&= (-2)^2 - 4 * 3 * -5 \\
&= 4 + 60 \\
&= 64 \\
x_1 &= \frac{-b + \sqrt{D}}{2a} \\
&= \frac{2+8}{6} \\
&= \frac{10}{6} \\
&= \frac{5}{3} \\
&= 1\frac{2}{3} \\
x_2 &= \frac{-b - \sqrt{D}}{2a} \\
&= \frac{2-8}{6} \\
&= \frac{-6}{6} \\
&= -1
\end{aligned}$$

We now know that the minima and maxima are at $x = 1\frac{2}{3}$ and $x = -1$.

$$\begin{aligned}
f(\frac{5}{3}) &= \frac{5}{3}^3 - \frac{5}{3}^2 - 5 * \frac{5}{3} - 3 \\
&= \frac{5^3}{3^3} - \frac{5^2}{3^2} - \frac{-25}{3} - 3 \\
&= \frac{125}{27} - \frac{25}{9} - \frac{25}{3} - 3 \\
&= \frac{125}{27} - \frac{75}{27} - \frac{225}{27} - \frac{81}{27} \\
&= \frac{125 - 75 - 225 - 81}{27} \\
&= \frac{-256}{27} \\
f(-1) &= (-1)^3 - (-1)^2 - 5 * -1 - 3 \\
&= -1 - 1 + 5 - 3 \\
&= 0
\end{aligned}$$

The minima and maxima are at ($1\frac{2}{3}$, $\frac{-256}{27}$) and (-1, 0).

d) We will calculate $f''(x)$.

$$\begin{aligned}
f'(x) &= 3x^2 - 2x - 5 \\
f''(x) &= 6x - 2 \\
f''(x) &= 0 \\
6x - 2 &= 0 \\
6x &= 2 \\
x &= \frac{1}{3}
\end{aligned}$$

The point of inflection is at $x = \frac{1}{3}$. The right side of this point is convex, the left side is concave.

Exercise 10

a)

$$\begin{aligned}
h'(x) &= cos^3(x) \\
&= (1 - sin^2(x)) * cos(x) \\
&= cos(x) - cos(x)sin^2(x) \\
h(x) &= sin(x) - \frac{sin^3(x)}{3} + C
\end{aligned}$$

$h(x) = sin(x) - \frac{sin^3(x)}{3} + C.$

b) For f_1, we will use the normal integration rules, for f_2 the $sin^2(x) + cos^2(x) = 1$ rule can be used and for f_3 we will use the previous exercise.

$$\begin{aligned}
f_1 &= -\frac{1}{2}cos^2(x) + C \\
f_2 &= \frac{1}{4}(-cos(2x) - 1) + C \\
f_3 &= \frac{sin^2(x)}{2} + C
\end{aligned}$$

$f_1 = -\frac{1}{2}cos^2(x) + C$, $f_2 = \frac{1}{4}(-cos(2x) - 1) + C$ and $f_3 = \frac{sin^2(x)}{2} + C$.

Answer Form Assignment 3

Name	x
Student Number	x

Question	**Answer**
6 (1pt)	$f'(x) = \frac{2x\ sin\ x^2}{\sqrt{1-(cos\ x^2)^2}}$
7a (1pt)	$\lim_{x\to\infty} \frac{x^2}{1+e^{-x}} = \infty$
7b (1pt)	Answer 7b
8a (1pt)	$g^{(2015)}(x) = 3^{2015} * sin(3x)$
9a (1pt)	The roots of f are at (-1, 0) and (3, 0). The y-intercept is at (0, -3).
9b (1pt)	$\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$.
9c (1pt)	The minima and maxima are at $(1\frac{2}{3}, \frac{-256}{27})$ and (-1, 0).
9d (1pt)	The point of inflection is at $x = \frac{1}{3}$. The right side of this point is convex, the left side is concave.
10a (1pt)	$h(x) = sin(x) - \frac{sin^3(x)}{3} + C$.
10b (1pt)	$f_1 = -\frac{1}{2}cos^2(x) + C$, $f_2 = \frac{1}{4}(-cos(2x) - 1) + C$ and $f_3 = \frac{sin^2(x)}{2} + C$.