Combinatorics
Assignment 2

November 11, 2017

Exercise 10

There are three different ways of forming a committee with six members and more women then men. The first is a committee with 6 women, then there are C(15,6) possible combinations. The second way is to have one men en five women in the committee, then there are C(15,5) * C(10,1) possible combinations. The last way of forming a committee with 6 members is with 4 women and 2 men, this makes C(15,4) * C(10,2) possible combinations.

In total, this makes: C(15,6) + C(15,5) * C(10,1) + C(15,4) * C(10,2) possible combinations.

Exercise 11

There are sixteen different possibilities of how the medals are rewarded:
6 gold: $\binom{6}{6}$
5 gold: $\binom{6}{5}$
4 gold: $\binom{6}{4}$
3 gold: $\binom{6}{3}$
2 gold, 4 bronze: $\binom{6}{2}\binom{4}{4}$
2 gold, 3 bronze: $\binom{6}{2}\binom{4}{3}$
2 gold, 2 bronze: $\binom{6}{2}\binom{4}{2}$
2 gold, 1 bronze: $\binom{6}{2}\binom{4}{1}$
1 gold, 5 silver: $\binom{6}{1}\binom{5}{5}$
1 gold, 4 silver: $\binom{6}{1}\binom{5}{4}$
1 gold, 3 silver: $\binom{6}{1}\binom{5}{3}$
1 gold, 2 silver: $\binom{6}{1}\binom{5}{2}$
1 gold, 1 silver, 4 bronze: $\binom{6}{1}\binom{5}{1}\binom{4}{4}$
1 gold, 1 silver, 3 bronze: $\binom{6}{1}\binom{5}{1}\binom{4}{3}$
1 gold, 1 silver, 2 bronze: $\binom{6}{1}\binom{5}{1}\binom{4}{2}$
1 gold, 1 silver, 1 bronze: $\binom{6}{1}\binom{5}{1}\binom{4}{1}$

In total this makes: 873 possible combinations.

Exercise 12

The 9^{th} row of Pascal's triangle is:

$$1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1$$

Using Pascal's identity we can derive from this that the 10^{th} row is:

$$1 \quad 9 \quad 36 \quad 84 \quad 126 \quad 126 \quad 84 \quad 36 \quad 9 \quad 1$$

Exercise 13

We prove the identity $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$, whenever n, r, and k are nonnegative integers with $r \leq n$ and $k \leq r$

a) using a double counting proof:

Left-hand side First, we pick r elements from box N with n elements and put them in another box called box R. Now we can pick k elements from box R and place them in box K. We now have n-r-k elements in box N, r-k elements in box R and k elements in box K.

Right-hand side First, we pick k elements from box N with n elements and put them in another box called box K. Now we can pick r elements from the same box N and put them in box R. We now have n-r-k elements in box N, r-k elements in box R and k elements in box K.

b) using an algebraic proof:

$$\begin{aligned}
\binom{n}{r}\binom{r}{k} &= \frac{n!}{r!(n-r)!} * \frac{r!}{k!(r-k)!} \\
&= \frac{n! * \frac{r!}{k!(r-k)!}}{r!(n-r)!} \\
&= \frac{n!r! * \frac{1}{k!(r-k)!}}{r!(n-r)!} \\
&= \frac{n! * \frac{1}{k!(r-k)!}}{(n-r)!} \\
&= n! * \frac{1}{k!(r-k)!} * \frac{1}{(n-r)!} \\
&= n! * \frac{1}{k!(r-k)!(n-r)!} \\
&= \frac{n!}{k!} * \frac{1}{(r-k)!(n-r)!} \\
&= \frac{n!}{k!(n-k)!} * \frac{(n-k)!}{(r-k)!(n-r)!} \\
&= \binom{n}{k} * \frac{(n-k)!}{(r-k)!(n-r)!} \\
&= \binom{n}{k} * \frac{(n-k)!}{(r-k)!((n-k)-(r-k))!} \\
&= \binom{n}{k}\binom{n-k}{r-k}
\end{aligned}$$

Exercise 14

The expression $\left(1+\sqrt{2}\right)^n + \left(1-\sqrt{2}\right)^n$ is a natural number if n is an element of N because when n is even, the squareroot of 2 becomes a real number and the second part of the formula becomes positive, when n is uneven, the second part of the formula becomes negative and will equal the first part of the formula.

Exercise 15

a) (i)

$$\binom{123456}{123457} = 0$$

(ii)

$$\binom{3\pi}{2\pi} = undefined$$

(iii)

$$\binom{-\sqrt{5}}{4} = 6.687435$$

b) We prove the identity

$$\sum_{i=0}^{n} \binom{n}{i} = 2^n$$

using a combinatorial proof:
Let S be a set with n elements in it, then the Powerset P of S is a set containing all subsets of S. In the course Mathematical Structures, we learned that the amount of elements in P of S is equal to 2^n.
When making combinations of elements with the left side of the formula, we are effectively creating subsets of elements (with the elements we chose). This lets us make a bijective function from the set S', containing all possible combinations of set S and P of S, containing all possible subsets of S. We make this function by letting each element of S' point to each element of S with the same elements in it. This function is bijective because the two sets contain the same subsets of elements.

c) The coefficient of x^5y^8 in the polynomial $(3x + 4y)^{12}$ is $1287 * 243 * 65536 = 20495794176$ because $\binom{13}{8} * (3x)^5 + (4y)^8 = 1287 * (3x)^5 * (4y)^8 = 1287 * 243x^5 * 65536y^8$.