Combinatorics
Assignment 3

November 11, 2017

Exercise 11

We can place the digit 9 on six places in a number with six digits. If 9 is in the number, there are a couple of number combinations that make up 13 when added together:
0-0-0-0-4
0-0-0-1-3
0-0-0-2-2
0-0-1-1-2
0-1-1-1-1

This can be done in 5 ways. For each of these, we can calculate how many possible rearrangements there are:
0-0-0-0-4 in $\frac{6!}{4!1!1!} = 30$
0-0-0-1-3 in $\frac{6!}{3!1!1!1!} = 120$
0-0-0-2-2 in $\frac{6!}{3!2!1!} = 60$
0-0-1-1-2 in $\frac{6!}{2!2!1!1!} = 180$
0-1-1-1-1 in $\frac{6!}{4!1!1!} = 30$
In total, this makes: $30 + 120 + 60 + 180 + 30 = 420$ possible numbers.

Exercise 12

For each of the five players, we make one box in which we can put the cards for each player. These are distinguishable objects and distinguishable cards. For each player we pick 7 cards. We make one box for the cards that remain after picking, this is the trash box. The formula for this equation sounds:

$$\frac{n!}{n1! * n2! * n3! * ... * nk!} = \frac{52!}{7! * 7! * 7! * 7! * 7! * 17!}$$

Exercise 13

a) A dozen indistinguishable copies of the same book can be placed on four distinguishable shelves in C(15,3) = 455 ways, because this is the same problem as the balls in the boxes.We can make a line with * as books and — as separater for shelves. This way we can make a string with three — and twelve *. We only have to pick three positions out of fifteen total positions for the three —.

b) A dozen books where no two books are the same can be placed on four distinguishable shelves where the positions of the books matter in $C(15, 3) * 12!$ ways, because if we first make a bitstring of twelve 0's and three 1's where the 1's represent the borders between each shelve (so if the bitstring is: 010000100010000, the first shelf has one book, the second one has four books, etcetera...). This can be done in C(15,3) ways. Then we will choose the book to put on the first 0, this can be done in 12 ways, the next 0 can be done in 11 ways, etcetera. In total, this makes C(15,3) * 12! ways.

Exercise 14

a) The number of ways we can distribute five balls into three boxes if each box must have at least one ball in it if both the balls and boxes are labeled is $C(5,3) * 3! * C(3,1) * C(3,1) = 540$, because if we first pick three balls out of five, and put those three balls each in one different box, we already have fulfilled the task of putting a minimal of one ball in each box, this can be done in $(C(5,3) * 3!$ ways. Then we have to pick one out of three boxes for both other balls, this can be done in $C(3,1) * C(3,1)$ ways.

b) The number of ways we can distribute five balls into three boxes if each box must have at least one ball in it if the balls are labeled, but the boxes are unlabeled is $S(5,3) = 25$ because we have to make partitions of the set 1,2,3,4,5. This can be done using the Sirling number S(5,3) .

c) The number of ways we can distribute five balls into three boxes if each box must have at least one ball in it if the balls are unlabeled, but the boxes are labeled is $C(7,2) = 21$, because we can make a sting with two - and five * representing balls.

d) The number of ways we can distribute five balls into three boxes if each box must have at least one ball in it if both the balls and boxes are unlabeled is 2, because these are the possibilities:
3,1,1
2,2,1

Exercise 15

The number of different terms in the expansion of $(x_1+x_2+\cdots+x_m)^n$ after all terms with identical sets of exponents are added is n+1, because every exponent between n and 0 (both included) will be present after working out all the math.

Exercise 16

This experiment is of the type where we have to divide distinguishable objects over indistinguishable boxes, so we need to make partitions, firstly we can calculate all partitions if all purses have a minimal of one bill, then the amount of partitions is: $S(7,3) = 301$. Then we need to calculate the amount of combinations when one purse is empty, we can calculate this by taking the Stirling number of $S(7,2) = 63$. In total this makes: $301 + 63 = 364$ possible combinations.

Exercise 17

a) This experiment is of the type rearrangement, so $\frac{9!}{4!3!2!} = 1260$ possible rearrangements.

b) We provide an algorithm for creating such a medium sundae:

   Task 1 Choose three different flavors, this can be done in $C(18,3) = 816$ ways.

   Task 2 Choose two different sauces, this can be done in $C(7,2) = 21$ ways.

   Task 3 Choose two different sauces, this can be done in $C(5,2) = 10$ ways.

   In total, there are: $816 * 21 * 10 = 171360$ ways.

c) We provide an algorithm for creating such a surjective function:

   Task 1 First, we shuffle the row of the set 0,1,2,3,4,5, this can be done in $6! = 720$ ways.

Task 2 Then we take the first number and link it to a, the second one to b, etc.

Task 3 We can link the last number in the row to exactly one of the five elements in the set a,b,c,d,e. This can be done in 5 ways.

In total, this makes $6! * 5 = 3600$ different ways.