



Fuzzy logic and unmanned Surface Vehicles

Implementing collision avoidance in Python

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Abstract

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The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

Keywords: COLREGS, Unmanned Surface Vessels, Fuzzy Logic

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Todo list

List of Abbreviations

COLREGs	Convention on the International Regulations for Preventing Collisions at Sea, 1972
FMF	Fuzzy Membership Function
OPRA	Oriented Point Reasoning Algebra
SA	Situational Awareness
FIS	Fuzzy Inference System

1 Introduction

Maritime shipping can be considered as one of the pillars of modern economy. In fact International Chamber of Shipping [1] state that 90% of the worlds total trade is handled by maritime shipping. This sums up to roughly 1 600 000 seafarers serving on international trading merchant ships worldwide. However, the recent rapid development of sensor technology and artificial intelligence could potentially reduce the operational costs of such vessels, by developing Unmanned Surface Vehicles (USV). The number of persons needed to operate vessels could thereby be reduced significantly. Furthermore, a majority of the accidents reported between 2011 and 2016 can be linked to human erroneous action [2]. USVs could, thus potentially decrease both the operational cost and number of accidents.

It is therefore of utmost interest to overcome the challenges faced when developing USVs. One such challenge is to develop algorithms that handle collision avoidance between vessels according to the rules of navigation. Various approaches have already been tried to some extent. This thesis will further examine the use of fuzzy logic in a collision avoidance for USVs.

1.1 Purpose of this thesis

The basis for this thesis lies in research done by Perera, Carvalho, and Soares [3]. However, this implementation tries to further test previous findings by recreating the previous implementation in python instead of MatLab and testing the algorithm in more challenging simulation scenarios than before.

1.2 Disposition

This first chapter will provide some general information about the thesis along with background information of the topic as well as the purpose of the thesis. Additional background information of USVs as well as their advantages and challenges are presented in Chapter 2. Chapter 3 explains the International Regulations for Preventing Collisions at Sea (COLREGs) and describes a few examples of collision situations. Previous approaches to automation of COLREGs rules are presented in chapter 4 before the theory for the fuzzy logic approach used in this thesis is thoroughly explained in chapter 5. The implementation of the fuzzy logic based Autonomous Navigation System and the simulation framework is described in Chapter 6. Chapter 7 presents the simulation scenarios as well as results and evaluations of the simulations. The results from chapter 7 are discussed in chapter 8. Finally, a conclusion is provided in chapter 9.

2 Unmanned Surface Vehicles

Unmanned Surface Vehicles (USV) or Autonomous Surface Crafts (ASC) are vehicles operating the seas without a crew on-board. USVs encompass both fully autonomous vehicles, from now on referred to as ASCs, and semi-autonomous vehicles. Development of USVs has been ongoing for the last two decades [4]. However, the majority of the USVs developed are of the semi-autonomous type [5], [6], meaning that they depend on human intervention to some extent usually by a supervisor located on shore. Although semi-autonomous USVs greatly increase the safety of the operating personnel [5], they do not completely remove the need for human interaction. Supervision of several semi-autonomous vehicles can admittedly be handled by a single person, which significantly decreases the number of person-hours needed to accomplish a specific mission [4]. The person-hours needed for surveillance could, however, be removed completely by an ASC. It is, therefore, of great interest to overcome the challenges associated with ASCs.

Yuh, Marani, and Blidberg [7] mention that roughly two-thirds of the earth's surface is covered by water, with an average depth of the oceans being 3688 m [8]. Thereby, adding up to a vast amount of explorable areas of which 95 % is yet to be seen by human eyes [9]. Utilization of autonomous vehicles could notably facilitate the exploration of these, yet unknown, areas. Although ASCs are situated on the surface of the ocean they can greatly increase the efficiency of Unmanned Underwater Vehicles (UUV) by acting as a gateway between UUVs and services, such as GPS, not easily available in underwater environments [5].

Liu, Zhang, Yu, *et al.* [5] have, furthermore, compiled a list of potential applications of USVs, along with previous research on the topics. The list is divided into five major categories: scientific research, environmental missions, ocean resource exploration, military use, and other applications.

Jokioinen [10] predict semi-autonomous remote-controlled ships to be in commercial use already before the end of this decade. These systems can be used to guide vessels into harbours and other densely congested locations, while full automation can be used on the high seas.

USVs have, apart from the obvious decrease in man hours, a few other advantages over traditional manned vessels. The absence of humans on board means that facilities and resources such as canteens, manned bridges, showers etc. are no longer needed. The weight saved increases the manoeuvrability and deployability and can also be used to increase the payload the vessel is able to carry. USVs can, furthermore, conduct longer and more hazardous voyages than manned vessels since the personnel is located safely on land. This has the additional benefit of decreasing the operational costs[5], [10].

Even though USVs bring many advantages to the maritime industry, there are still quite a few challenges that need to be solved. The sensor technology needed exists already, the challenges lie in combining the different sensor outputs. Additionally, algorithms that generate decisions based on the sensor data, need to be developed [10].

USVs should, furthermore, be able to operate in a highly variable environment, all over the world, in terms of climate, traffic density and communication channels available. Algorithms and systems developed should, therefore, ideally be usable in all situations that might arise in this environment. This introduces new challenges both hardware and software wise [5]. The limited scope of this thesis narrows the research to only the software related challenges with a focus on path planning, especially collision avoidance. Complete SA is also assumed.

All vessels operating on water are bound to follow the international maritime "Rules of the Road" called the Convention on the International Regulations for Preventing Collisions at Sea, 1972 (COLREGs) explained in section 3. This is also true for USVs, since they should be able to operate in situations that comprise both other USVs and manned vessels that follow the COLREGs rules. Furthermore, they should be able to act in a safe manner in situations where other vessels, for some reason, are not complying with the COLREGs rules. Hence, it is crucial for the USVs to follow COLREGs to

ensure safe operations.

COLREGs were originally introduced in 1972, to reduce the amount of collisions at sea. They are written to be interpreted by humans. Situations where multiple rules apply and contradict each other might therefore arise. These situations are usually solved with the use of good seamanship and human deduction. These aspects pose challenges when translating the COLREGs rules into computer understandable code [11].

Finally, USVs must be at least as safe as existing vessels. Currently human error seems to be the most common cause of incidents at sea [12]–[14] and 56 % of all major collision include COLREGs violations[15]. USVs could greatly reduce errors of this kind , however, new risks will arise and proper risk analysis is therefore crucial. This raises the question whose liable for an accident involving USV:s [10]? Parallels can be drawn to a recent fatal accident involving an autonomous car and a pedestrian. The car was in autonomous mode when the accident occurred. However, the safety driver present in the car was not focused on monitoring the car [16]. Should liability be placed on the safety driver or the car manufacturer?

3 COLREGs

The Convention on the International Regulations for Preventing Collisions at Sea, 1972 (COLREGs) were adopted 1972 and entered into force 1977 in its original form. COLREGs consist of 38 rules, grouped into five different sections and is developed by the International Maritime Organization (IMO). The original objective of COLREGs was to ensure traffic separation between maritime vessels in an increasingly populated environment back in 1972 and it is, therefore, sometimes referred to as the Rules of the Road for maritime vessels. COLREGs have since then been in use and are still mandatory to adhere to on international waters. However, a great deal has changed since the COLREGs initially entered into force and COLREGs has, therefore, received amendments several times [17].

National regulations might differ from the COLREGs to some extent. However, COLREGs mention that ‘Nothing in these Rules shall interfere with the operation of special rules made by an appropriate authority for roadsteads, harbours, rivers, lakes or inland waterways connected with the high seas and navigable by seagoing vessels. Such special rules shall conform as closely as possible to these Rules.’ [18]

The following sections will present the different parts of the COLREGs regulations, with emphasis on the parts related to manoeuvring a USV in international waters. Information in this subsection is taken from the official COLREGs regulations [18] if not otherwise specified. Part D and E will not be discussed further since they are not of interest for the scope of this thesis. Moreover, rules from other section that are not of interest for the scope of this thesis will not be mentioned.

Part A consist of three rules and address general conditions when and to which vessels COLREGs apply. This includes all vessels navigating on high seas and all waters connected therewith. Additionally, it specifies that special rules regarding roadsteads, harbours, rivers, lakes or inland waterways

connected with the high seas and navigable by seagoing vessels shall conform as closely as possible to these Rules (*Rule1*). Furthermore, words such as vessel, power-driven vessel, sailing vessel, length, breadth, and other words used in the regulations are defined (*Rule3*). Finally, it is stated that the COLREGs rules do not in any way free the vessel, owner, master or crew from responsibility to follow the rules and act according to the ordinary practice of seamen (*Rule2*).

Part B is split up into three different sections. The first section consists of rules 4-10 and applies in any condition of visibility. Section II (rules 11-18) apply to vessels in sight of each other, while the last section consists of just one rule that specifies reduced visibility operations. It states that vessels should proceed at speeds appropriate to the circumstances and use radar to determine collision risks. Speed should be reduced to a minimum if a vessel can hear another vessels fog horn apparently forward of her beam (*Rule19*).

Section I addresses how vessels shall operate in order to ensure proper situational awareness with emphasis on collision risk detection. It is mandatory for all vessels to at all times maintain proper look-out by sight and hearing and all other available means to ensure the best possible situational awareness and to determine if a risk of collision exists (*Rule5*). This includes the use of radar equipment although caution should be exercised not to trust insufficient data. A collision is considered to be imminent when two vessels maintains constant compass bearings to each other over a prolonged time. However, a constant bearing is only a sufficient condition for a collision risk, not a necessary one. Close range, large vessels or a tow might pose a collision risk without a constant bearing. Moreover, it is stated that any doubt whether a risk of collision exists shall be treated as if a risk exists (*Rule7*). Vessels shall, furthermore, maintain a speed so that they can take proper and effective action to avoid a collision and stop at a distance appropriate to the prevailing circumstances and conditions (*Rule 6*). Any actions made, in order to to avoid collisions, shall be taken according to the rules and large enough so that they are easily observable by other vessels. They shall additionally as far as possible be conducted in ample time. Small alterations should, in other words, be avoided. Course alterations might also be accompanied by a lowering of speed if considered necessary to ensure that

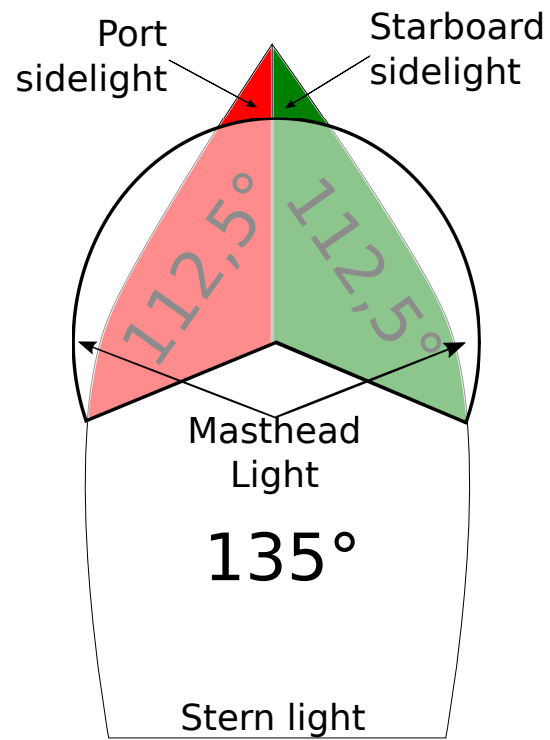


FIGURE 3.1: Mandatory navigation lights

the vessels pass each other at safe distance (*Rule 8*).

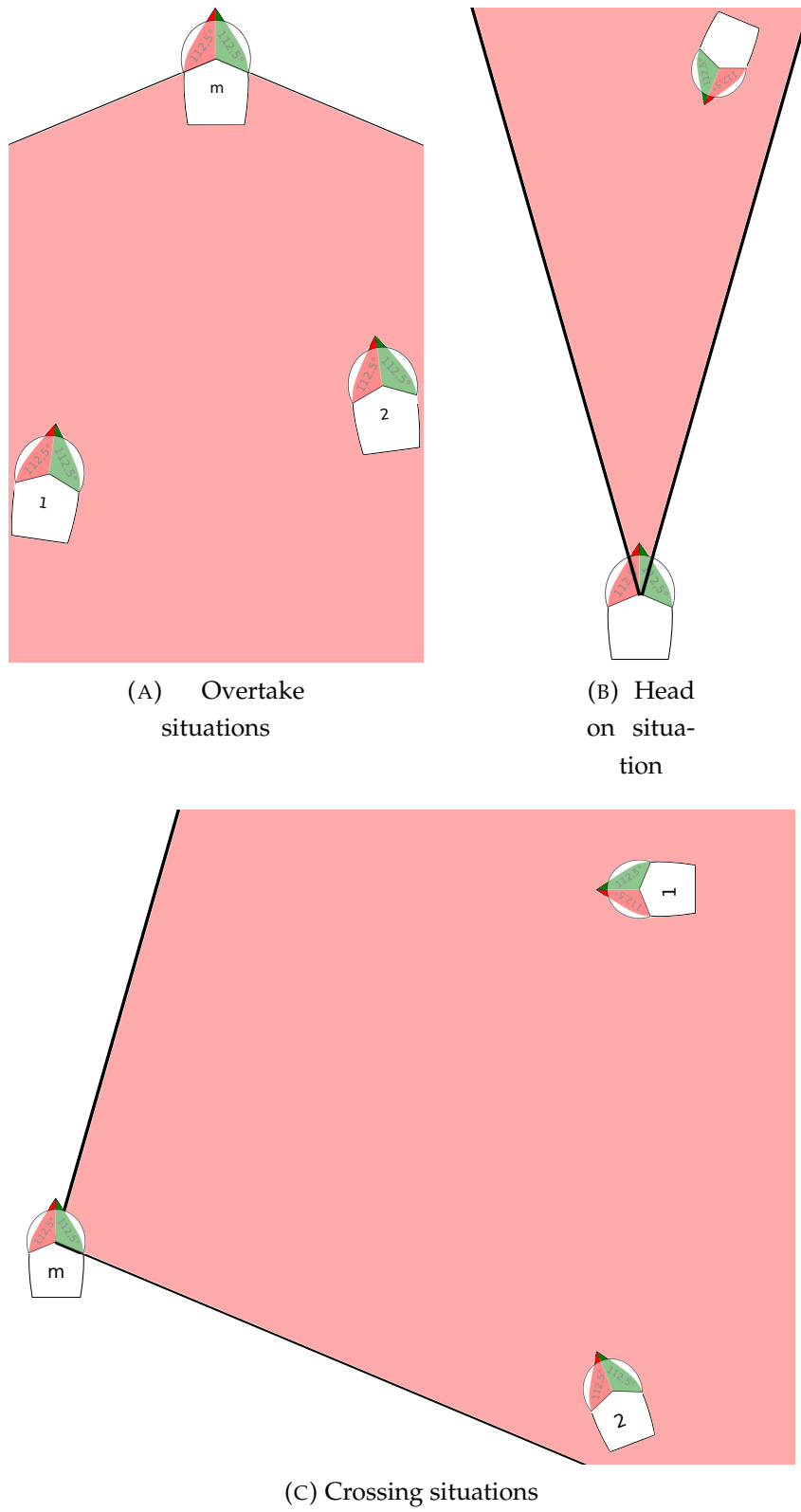


FIGURE 3.2: COLREGs collision scenarios

Section II defines three different scenarios, involving two vessels, and specifies the action to take for both vessels. The scenarios are a head-on, a crossing and an overtaking situation. A vessel overtaking another vessel shall keep out of the way of the stand on vessel. A vessel is considered to overtake another vessel when approaching it from a direction more than 22.5° abaft the stand on vessels beam, i.e with a relative bearing between 112.5° and 247.5° from the stand on vessel. This sector is the same as the white stern light in figure 3.1 and the light red sector in figure 3.2a (*Rule 13*).

A head-on situation, on the other hand, is when a vessel sees another vessel ahead or nearly ahead, i.e. it can see the masthead lights in line and/or both sidelights, shown in figure 3.1. A rule of thumb commonly used is $\pm 5^\circ$ relative bearing, shown as the light red sector in figure 3.2b. The give way vessel shall, in this case, alter its course to starboard (*Rule 14*). Finally, it is stated that vessel coming from starboard, i.e right, has the right of way in a crossing situation. This is usually interpreted as target vessels in the sector represented by the main vessels starboard light, green light in figure 3.1 or the light red sector in figure 3.2c. The give way vessel shall if possible avoid crossing ahead of the other vessel and, therefore, alter its course to starboard (*Rule 15*). All actions by the give way vessels shall be taken as early as possible (*Rule 16*), while the stand on vessels shall keep its course and speed, provided that the give way vessel acts according to the regulations. However, the stand on vessel might take action to best avoid a collision if it finds that the actions by the give way vessels are insufficient to prevent a collision (*Rule 17*). Responsibilities between vessels of different types are also addressed. For instance, a power-driven vessel shall give way for vessels engaged in fishing, vessels not under command, vessels restricted in her ability to move, and finally fishing vessels (*Rule 18*).

The three situations described above acts as basis for all collision avoidance decisions on the high sea. Some situations might at first seem counter-intuitive, for instance, boat 2 in figure 3.2c. COLREG rules state that vessel *m* shall change course to starboard and pass behind vessel 2 even-though vessel 2 is currently behind vessel *m*. However, it is crucial to remember rule 7 and the conditions needed for a collision risk. A collision is in this case only imminent if vessel 2 has a much higher speed than vessel *m* so

that their relative bearings remains the same over time. It is in that case intuitive for vessel m to alter course to starboard and possibly decelerate thereby allowing vessel 2 to pass in front of her.

Moreover, many situations include multiple target vessels and regarding every target as an isolated situation might give contradictory decisions. A holistic approach to the situation must, therefore, be taken. This is called good seamanship and is one of the greater challenges when developing a COLREGs compliant algorithm. Good seamanship is even more important when taking into account vessels of different kind, terrain and economics. A light motorboat might for instance gladly give way to a large tanker on a tight schedule even-though the tanker is obliged to give way. This thesis will concentrate on rules that could be axiomatizable for the implementation and contemplate others in the discussion in chapter 8

4 Automation of COLREGS

USVs have been in development since at least 1993 when MIT started its Sea Grant College Program, Autonomous Surface Craft (ASCs) [4]. Many different approaches and algorithms have since then been tried out. This section will briefly look into some of the COLREGs compliant approaches, before finally presenting the approach which this thesis is based on.

Larson, Bruch, Halterman, *et al.* [19] uses a two-level two-dimensional obstacle map to create a near-field reactive control technique. One layer for a near-field model and the other for a far-field model. COLREGs rules are incorporated into the solution by utilizing a rule-based approach in the path planner. The solution is made with computational efficiency in mind and the provided trajectory is, therefore, not optimal. It is also stated that the sensors and processing algorithm need more work. The paper does not mention why the specific approach was chosen. However, the authors have previously worked on unmanned ground vehicle for 25 years and mention that they have transitioned some of their work into the USV word.

Another approach, developed by Benjamin, Leonard, Curcio, *et al.* [11] and Benjamin and Curcio [20] utilizes multi-objective optimization and interval programming, within a behaviour based architecture. Behaviour-based architecture was chosen since the model can be split into several independent modules which are easier to maintain and develop, than a single complex world model. Interval programming helps solve scenarios that involve multiple rules and situations where rules compete with the mission objective. The solution has been tested with kayak-based autonomous surface crafts and initial tests show promising results, but the solution cannot yet be called COLREGS compliant.

Naeem, Irwin, and Yang [21] propose a path-planner that uses line of sight guidance between way-points. Obstacles are avoided by introducing a starboard heading bias to the line of sight heading until clear of conflict.

The solution is supposed to be straight forward yet effective and provide an onboard detection and avoidance system. Which is something that, according to the authors, many current solutions lack. The paper compares the trajectory, produced by the proposed on-the-fly algorithm, to that of a modified off-line DPSS strategy. Multiple simulations show that both algorithms yield COLREGs compliant results. However, a simulation with multiple dynamic targets was not conducted.

Spatio temporal reasoning has also been considered as a way to translate the COLREG rules into computer understandable rules. Kreutzmann, Wolter, Dylla, *et al.* [22] and Wolter, Dylla, and Kreutzmann [23] suggest the use of Oriented Point Reasoning Algebra (OPRA) with m granularity to represent vessels position and relative moving direction. A qualitative spatio-temporal reasoning toolbox is used to convert the geometric scenario consisting of the vessels Cartesian coordinates into a qualitative scene description based on OPRA relations. The same toolbox can then generate all possible scenarios that are spatially or temporally possible. Model checking can then be conducted on the observations to check whether the behaviour of the involved vessels is COLREGs compliant. Spatio Temporal was chosen in order to bridge the gap between primitive logic concepts and abstract navigation regulations, thereby facilitating formalization of the navigation rules.

Lee, Kwon, and Joh [24] combine fuzzy logic with modified virtual force fields to create a COLREGs compliant algorithm. The fuzzy logic rule-set used consists of about two hundred rules, which might be computational challenging. The authors do, however, mention that the rules-set might be streamlined. Fuzzy logic is also used Perera, Carvalho, and Soares [3] which, furthermore uses Bayesian networks to solve cases with multiple target vessels. The presented simulations consist of one vessel utilizing the algorithm to avoid three dummy vessels that are keeping their course and speed. The rule-set is in this paper also quite extensive. Fuzzy logic has been used in several decision-making systems before, where rules written for humans need to be interpreted by computers [3].

Both fuzzy logic and spatio temporal reasoning with model checking was considered to be used as the foundation for this thesis. However, fuzzy

logic was finally chosen due to the fact that the qualitative representation of a scenario in spatio temporal reasoning is limited the definitions of the calculus used, in this case OPRA.

5 COLREGs modeling using fuzzy logic

The concept of fuzzy sets was first mentioned by Zadeh [25] in 1964 as a way of dealing with sets of objects, where an objects membership to a set can be represented by a value in the real interval $[0, 1]$. Compared to classic set theory where membership is restricted to the two values of one and zero. Fuzzy logic enables one to define sets such as "old men" or "vessels on reciprocal course" [25], which facilitates the process of converting human abstract reasoning into computer understandable logic.

5.1 Fuzzy sets

The fuzzy set "all old men" can be defined in the following way. Let the *universe set* \mathbf{X} be the set of ages $[0, 100]$. The set of old men can then be written as $\tilde{A} = \{a \in \mathbf{X} | a \text{ Is old}\}$ [26]

However, the set requirement 'Is old' is still undefined and therefore vague. Defining the threshold of being old at an age of 60 does divide the universal set \mathbf{X} into subsets of 'old' and 'not old', but the division does not distinguish between humans aged 1 and 59. Fuzzy set theory introduces the concept of membership functions, to solve this problem. A Fuzzy Membership Function (FMF) describes the grade of membership of a in \tilde{A} . This function is often written as $\mu_{\tilde{A}}(a)$. Figure 5.1a shows a very simple linear FMF for the example $\tilde{A} = \{\text{'Is old'}\}$, with age on the X-axis and membership grade in Y. It can then be read that, for instance, an age of 90 gives a membership value of 0.8 for the fuzzy set \tilde{A} or $\mu_{\tilde{A}}(90) = 0.8$. Similar fuzzy sets and FMFs can then be defined for 'Is young': $\tilde{C} = \{c \in \mathbf{X} | c \text{ Is young}\}$ and 'Is middle-aged': $\tilde{B} = \{b \in \mathbf{X} | b \text{ Is middle-aged}\}$. The FMFs can then be

plotted on the same graph which results in figure 5.1b [27]. This enables an age to be a member of two fuzzy set simultaneously. For example the age 60 is part of both the 'Is old' and 'Is middle age' fuzzy sets, with membership values of 0.2 and 0.8 respectively.

A fuzzy set can then be written as an array of tuples consisting of the objects and its associated membership value: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x) | x \in X)\}$ [28], which gives:

$\tilde{A} = \{(0,0), (10,0), \dots, (50,0), (60,0.2), (70,0.4), (80,0.6), (90,0.8), (100,1)\}$ for the 'Is old' fuzzy set \tilde{A} in figure 5.1a [27].

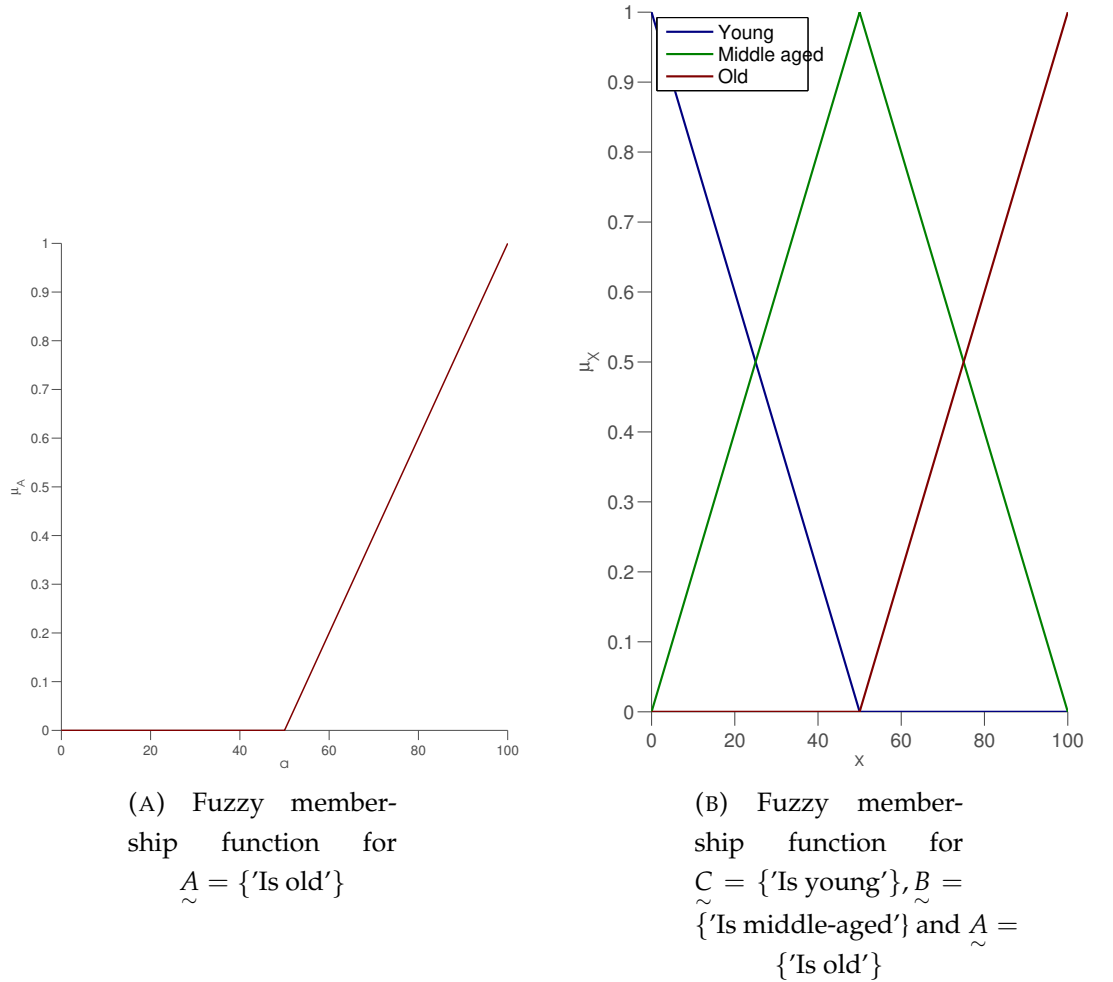


FIGURE 5.1: Graphical representations of a FMFs

5.2 Fuzzy logic

Fuzzy sets and set theory can, in the same manner as classical boolean sets, be used to define logical expressions. Fuzzy logic allows for half-truths, i.e. truth values in the interval $[0,1]$. Whereas boolean logic is restricted to truth values of one and zero. The truth value T of a fuzzy preposition P is, therefore a mapping from $[0,1]$ to the universe T , as can be seen in equation 5.1 [27].

$$T : u \in U \rightarrow (0,1) \quad (5.1)$$

The truth value of proposition P is therefore given by the membership grade $\mu_{\tilde{A}}(x)$ of x in \tilde{A}

Many of the same operators and connectives used in classical logic, does also, apply to fuzzy logic [27]. The following section will present the rules necessary for the scope of the thesis and examples using the sets '*Is old*' \tilde{A} and '*Is middle-aged*' \tilde{B} defined in section 5.1.

Negation is in fuzzy logic defined as $T(\tilde{\bar{P}}) = 1 - T(\tilde{P})$. This can be used to calculate the membership value of the age 90 in the negation to the '*Is old*' fuzzy set \tilde{A} in the following way: $\mu_{\tilde{\bar{A}}}(90) = 1 - \mu_{\tilde{A}}(90) = 1 - 0.8 = 0.2$. With the result that an age of 90 has a membership value of 0.2 in the '*Is not old*' fuzzy set $\tilde{\bar{A}}$

A disjunction ($\tilde{P} \vee \tilde{Q} : x$ is \tilde{A} or \tilde{B}) can be defined as $T(\tilde{P} \vee \tilde{Q}) = \max(T(\tilde{P}), T(\tilde{Q}))$. An example using the age sets could be '*is old or middle-aged*' for the age of 60, which gives $\max(\mu_{\tilde{A}}(60), \mu_{\tilde{B}}(60)) = \max(0.2, 0.8) = 0.8$.

Conjunction ($\tilde{P} \wedge \tilde{Q} : x$ is \tilde{A} and \tilde{B}) follows the same pattern as disjunction but with min instead of max: $T(\tilde{P} \wedge \tilde{Q}) = \min(T(\tilde{P}), T(\tilde{Q}))$. An example using the same values as above will thus be $\min(\mu_{\tilde{A}}(60), \mu_{\tilde{B}}(60)) = \min(0.2, 0.8) = 0.2$.

Finally, **implication** ($\tilde{P} \rightarrow \tilde{Q} : x$ is \tilde{A} , then x is \tilde{B}) is slightly more difficult to define in fuzzy logic. This is because implication can be defined in multiple ways in classical logic, all of which are equivalent. However, these are not equivalent in fuzzy logic since the law of absorption of negation does

not hold in fuzzy logic. Classical material implication would define it as: $T(\underset{\sim}{P} \rightarrow \underset{\sim}{Q}) = T(\underset{\sim}{P} \vee \underset{\sim}{Q}) = \max(T(\underset{\sim}{P}), T(\underset{\sim}{Q}))$ [29]. However, implication can also be written in rule based form $\underset{\sim}{P} \rightarrow \underset{\sim}{Q}$ is, IF x is $\underset{\sim}{A}$, THEN y is $\underset{\sim}{B}$ which is equivalent to the fuzzy relation $R = (\underset{\sim}{A} \times \underset{\sim}{B}) \cup (\overline{\underset{\sim}{A}} \times \underset{\sim}{Y})$ in the same way as in classical logic. To which Zadeh has defined the following membership function [29]:

$$\mu_{\underset{\sim}{R}}(x, y) = \max[\mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(y), (1 - \mu_{\underset{\sim}{A}}(x))] \quad (5.2)$$

Although, fuzzy logic has multiple other implication operators and choosing the right one is often context dependent. This thesis uses Mamdani's implication (equation 5.3), since it is one of the most common ones used in control systems due to its simplicity [27]. However, Mamdani's implication does not strictly extend the material implication since $0 \rightarrow 0$, though it is still useful in rule-based systems where rules with a false antecedent should be disregarded.

$$\mu_{\underset{\sim}{R}}(x, y) = \min[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)] \quad (5.3)$$

5.3 Fuzzy (rule-based) systems

Fuzzy logic can be used to model complex systems described in natural language, originally written to be interpreted by humans. Such knowledge can often be written as rules in the following form [27].

$$\text{IF premise (antecedent), THEN conclusion (consequent)} \quad (5.4)$$

Combining multiple rules enables one to describe complex systems in a relatively simple structure. Rules can, furthermore, contain multiple antecedents and consequents. However, this raises the question of how multiple antecedents, as shown in rule 5.5, can be decomposed into a single antecedent and the rules aggregated into a single consequent [27].

$$\text{IF } x \text{ is } \underset{\sim}{A}_1 \text{ and } \underset{\sim}{A}_2 \dots \text{and } \underset{\sim}{A}_L \text{ THEN } y \text{ is } \underset{\sim}{B}_s \quad (5.5)$$

Conjunctive antecedents can be rewritten as a new fuzzy set

$$A_{\underset{\sim}{S}} = A_{\underset{\sim}{1}} \cap A_{\underset{\sim}{2}} \cap \dots \cap A_{\underset{\sim}{L}}$$

with the membership function

$$\mu_{A_{\underset{\sim}{S}}}(x) = \min \left[\mu_{A_{\underset{\sim}{1}}}(x), \mu_{A_{\underset{\sim}{2}}}(x), \dots, \mu_{A_{\underset{\sim}{L}}}(x) \right]$$

Rule 5.5 can then be rewritten as

$$\text{IF } A_{\underset{\sim}{S}} \text{ THEN } B_{\underset{\sim}{S}}$$

Disjunctive antecedents can similarly be written as

$$A_{\underset{\sim}{S}} = A_{\underset{\sim}{1}} \cup A_{\underset{\sim}{2}} \cup \dots \cup A_{\underset{\sim}{L}}$$

$$\mu_{A_{\underset{\sim}{S}}}(x) = \max \left[\mu_{A_{\underset{\sim}{1}}}(x), \mu_{A_{\underset{\sim}{2}}}(x), \dots, \mu_{A_{\underset{\sim}{L}}}(x) \right]$$

The same principle can be applied to find the overall consequent when multiple rules apply. Conjunctive rules where both consequents must be applied can be found by the fuzzy intersection of all the rule consequents. Whereas disjunctive rules use the fuzzy union of the rule consequents.

5.4 A fuzzy logic model for COLREGs

Antecedents consequents and rules are needed in order to make a fuzzy system for a COLREGs compliant auto pilot. The model used in this thesis is created by Perera, Carvalho, and Soares [3] and has four antecedents, two consequents and an extensive rule set of nearly 200 rules. The antecedents used are relative bearing to the target vessel, relative course of the target vessel, distance between the vessels, and the ratio between the vessels speed. The main vessels relative bearings are divided into ten sets. These resulting sectors represent the different sets in the relative course universe [0, 360]. The relative courses of the target vessel are, likewise, divided into eight sectors, which represent sets in the relative course universe [0, 360]. The distance universe consists of three sets, representing radii from the main

vessel, called R_A , R_B and R_{VD} . R_{VD} is the vessel domain into which other vessel shall not enter. R_A and R_B depicts the area around the main vessel in which other vessels are detected. R_A is used when the main vessel is the give way vessel and R_B when it is the stand on vessel. A graphical representation of the bearing, range and course sectors can be seen in figure 5.2. The bearing sets are marked as sectors (I-X) around the main vessel, course set as sectors a-h around the target vessel and finally range as circles around the main vessel. The final antecedent, speed ratio $= \frac{V_{target}}{V_{main}}$, consists of three sets: < 1 , ≈ 1 and > 1 . Visualizations of the antecedent FMFs, as well as their distribution in their corresponding universes, can be seen in figure 5.3 and consequent FMS in figure 5.4.

Finally, a rules set is needed to connect the antecedents with the consequents. The rule set used for this thesis is based on the rules, seen in table A.1 developed by Perera, Carvalho, and Soares [3]. A few rules have furthermore been added to handle overtake scenarios. These can be seen in table A.2 Each row in the tables depict a rule in the FIS system. The table consists of four columns. The first column represents the main vessels relative bearing to the target vessel. The columns values go from I-X which, represent the ten sectors around the main vessel in figure 5.2 and the FMFs seen in figure 5.3a. Column number represents the target vessels relative course given in values from a-h which correspond to the eight sectors around the target vessel in figure 5.2 and the FMF in figure 5.3b. Columns three and four are both split up into two sub-columns. The main column represents range intervals. Ranges are defined from the fuzzy sets R_A , R_B and R_{VD} mentioned earlier. The third column represents ranges $R_{VD} - R_A$ and the fourth $R_A - R_B$. The first sub-column displays the rules speed ratio and the second consequents of the rule.

An analysis and optimization of the values used to specify the FMFs and the rules in the rule set are unfortunately outside of the scope of this thesis. The values are therefore used as specified in previous research [3], with the exception of a few added rules to ensure collisions avoidance when one vessel is overtaking another. It can, therefore, be seen as a modelling of the COLREG rules an not an axiomatic solution.

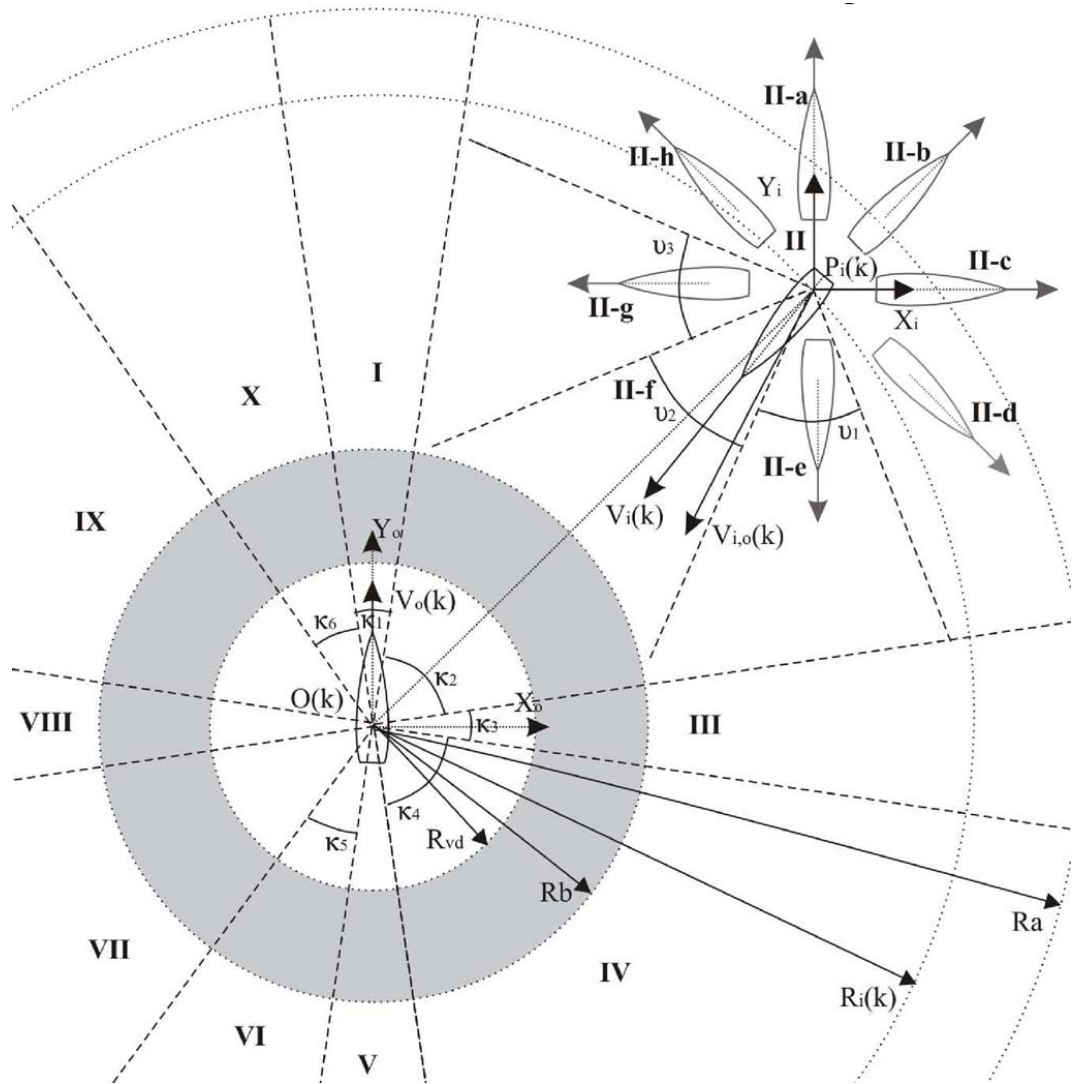
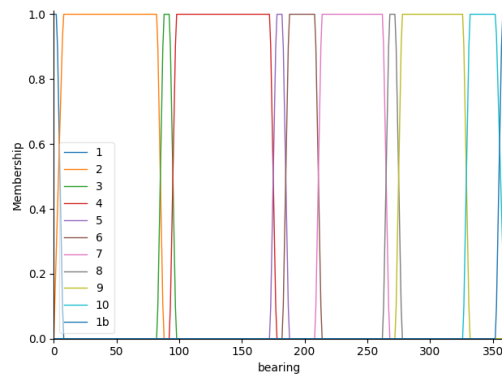
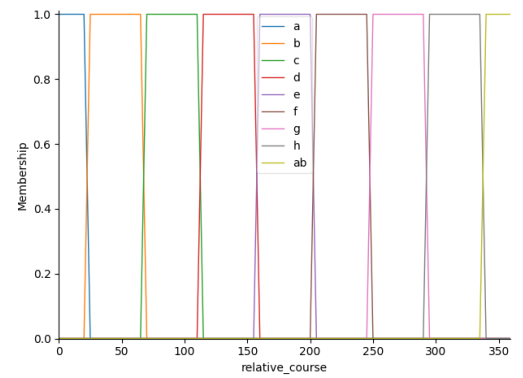


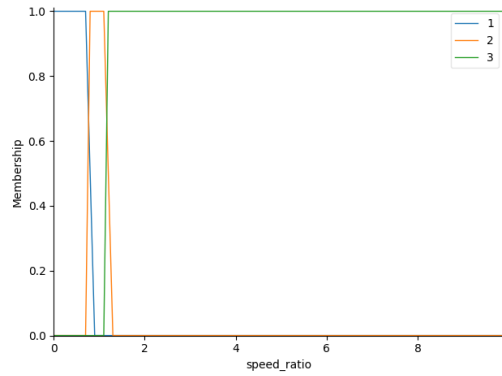
FIGURE 5.2: Mathematical representation of the inputs to a FIS for a two vessel collision situation[3]



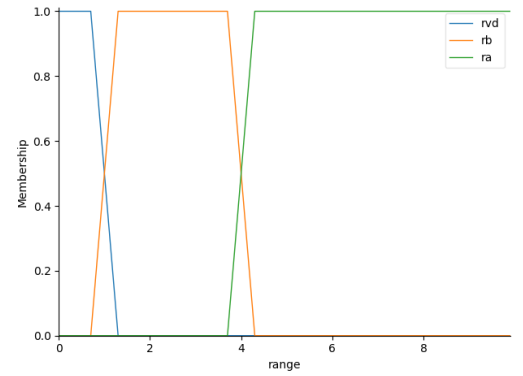
(A) Relative bearing of the target vessel from the main vessel



(B) Relative course of the target vessel to the main vessel



(C) Speed ratio target vessel/main vessel



(D) Distance between the main vessel and the target vessel

FIGURE 5.3: Antecedent FMFs

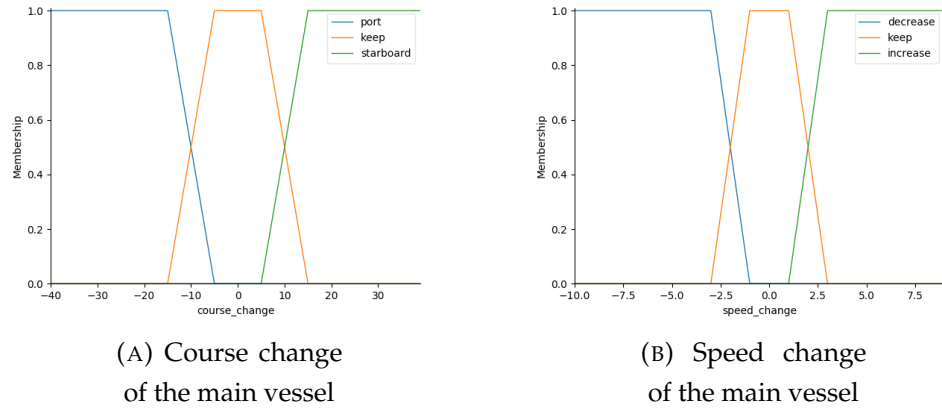


FIGURE 5.4: Consequent FMFs

5.5 Mamdani's fuzzy inference method

This thesis will, as mentioned in section 5.2, use the Mamdani implication operator in its Fuzzy Inference System (FIS). Mamdani's method was chosen since it is the most common one in practice literature [27].

The following subsections will explain Mamdani's method by going through an example based on the model presented in section 5.4. The example uses a scenario where two vessels are located in a 2 dimensional 10*10 NM Cartesian coordinate system.

Vessel A starts at coordinate (0; -4.5) with heading 0, speed 5 kts and rate of turn 2 degrees per second.

Vessel B starts at coordinate (-4.5; -4.5) with heading 203, speed 10 kts and rate of turn 2 degrees per second. The initial state of the scenario is depicted in figure 5.5

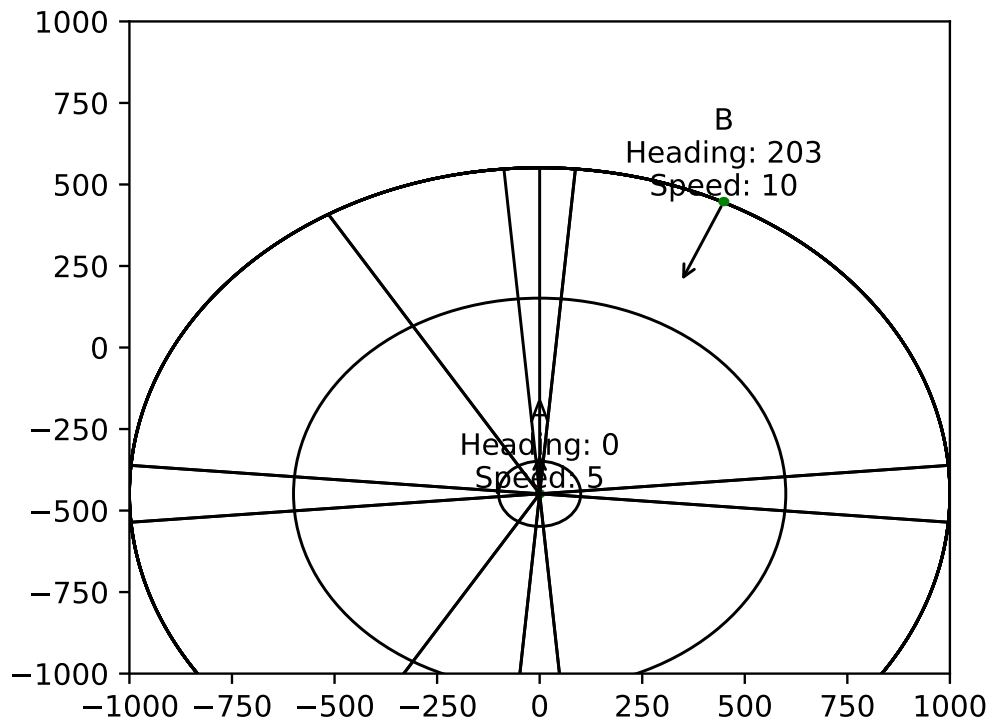


FIGURE 5.5: Initial state of the example scenario

A FIS can be said to consist of six steps [30]:

1. Rule determination
2. Input fuzzification
3. Combination of antecedents
4. Obtain consequences
5. Aggregate consequences
6. Defuzzification

This example will use a subset of the rules used in the real system. The rules used are:

Rule 1 IF target in sector II AND relative course is in sector f AND speed ratio is greater than 1 AND range is R_vd OR R_b OR R_a
THEN change course to starboard AND decrease speed.

Rule 2 IF target in sector II AND relative course is in sector e AND speed ratio is greater than 1 AND range is R_vd OR R_b OR R_a
THEN keep course AND increase speed.

Next, the crisp values, gained when applying the model presented in section 5.4 on the scenario in figure 5.5, are fuzzified. The target vessel is located in sector II with a relative bearing of 26.7° , which results in a membership value of 1 in sector 2. This is visualized in figure 5.6a by the red line starting from the x value 26.7 meeting the orange curve representing sector II at a y value of 1.

The same principle is applied to the three other inputs: relative course, distance, and speed ratio. Distance and speed ratio will also, in this case, yield membership values of 1, for the fuzzy sets distance R_a and speed ratio >1 . However, the relative course of 203 is located in the fuzzy area between sector 2-e and 2-f which results in membership values of 0.4 and 0.6 respectively, as shown in figure 5.6b by the red arrows. The horizontal line meets the purple e curve at Y value 0.4 and the brown curve at Y value 0.6.

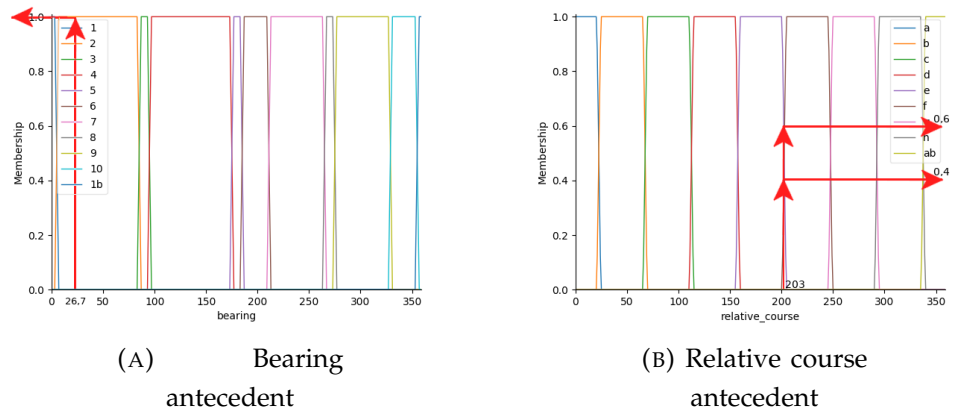


FIGURE 5.6: Fuzzified antecedent FMFs for the example scenario

The membership values gained for the individual antecedents can then be combined using the AND and OR operators specified in the rule set. This results in the following calculations.

$$\text{Rule1} : \min(1; 0.6; 1; \max(0; 0; 1)) = 0.6 \quad (5.6)$$

$$\text{Rule2} : \min(1; 0.4; 1; \max(0; 0; 1)) = 0.4 \quad (5.7)$$

These values represent the grade to which the given scenario satisfies the antecedents of the rules. Mamdani's inference method can then be used on each rule to obtain the consequent value by taking the minimum of the antecedent value, from the previous step, and the consequent. The result is the original consequent FMF with its top cut of at the antecedent value for the rule, calculated in the previous step. The result can be seen in figure 5.7a as the green area under the starboard curve and in figure 5.7b as the blue area under the decrease curve since rule 1 has change course to starboard and decrease speed as consequents. The same procedure applies to rule two which has keep course and increase speed as consequents. The coloured areas are therefore under the orange keep curve in figure 5.8a and the green increase curve in figure 5.8b

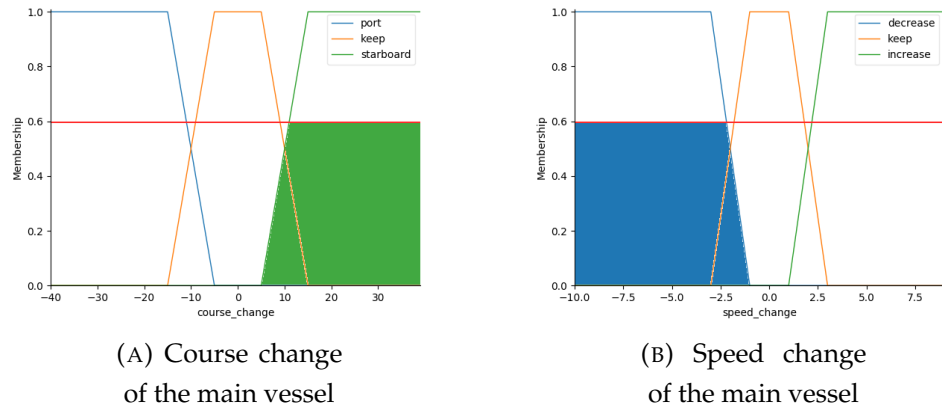


FIGURE 5.7: Consequent FMFs for rule 1 after applying Mamdani's inference method

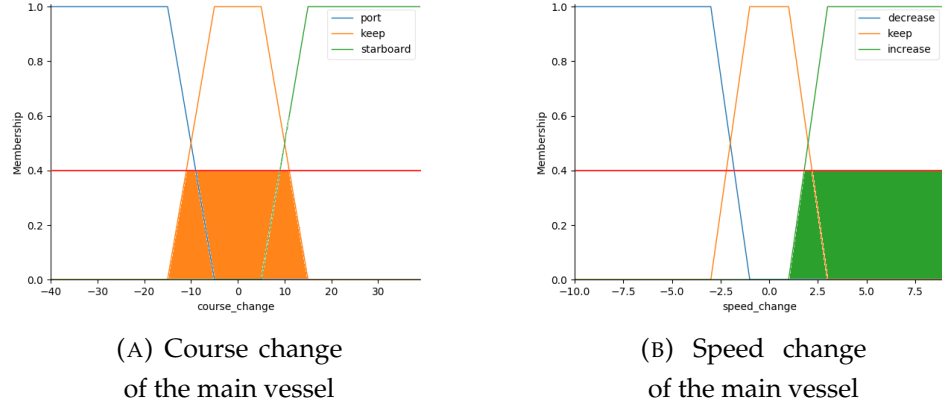


FIGURE 5.8: Consequent FMFs for rule 2 after applying Mamdani's inference method

The result is two consequent curves for each rule, one for course change and one for speed change. These need to be combined into two final curves to be able to calculate the final course and speed corrections. This is achieved by combining each rule consequent by their maximum values. In this case the green and orange areas in figures 5.7a and 5.8a and the blue and green areas in figures 5.7b and 5.8b. The two resulting areas are presented in figure 5.9 in red.

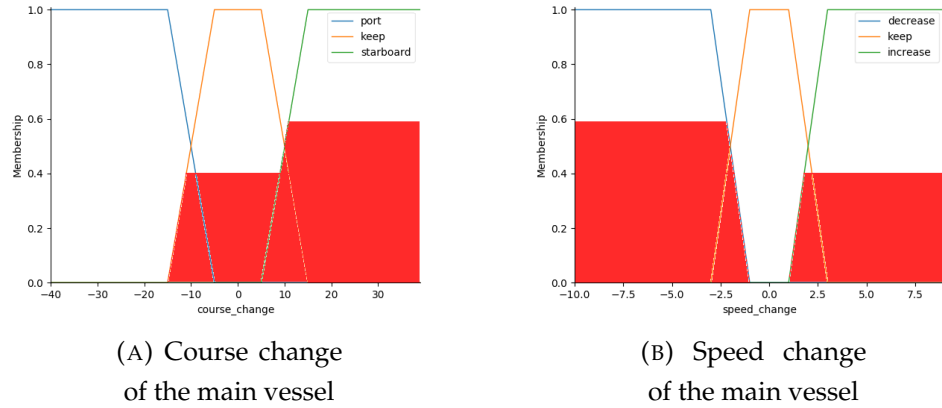


FIGURE 5.9: Combined Consequent FMFs for the example scenario

Finally, the fuzzy sets gained from the previous step need to be converted into a crisp numerical value that can be used by the auto pilot. This process is called defuzzification and is in this case achieved by calculating the geometrical centre value along the x-axis for the FMFs in figure 5.9. The results are a $+15.5^\circ$ course change and a -1.7 knot speed change.

6 Implementation

The objective of this thesis was to implement and evaluate a collision avoidance algorithm for USVs. Several related approaches were analysed before the fuzzy logic approach presented by Perera, Carvalho, and Soares [3], was chosen to be implemented. The solution presented in the original paper is implemented in MATLAB, while the solution presented here is implemented in Python. Python was chosen due to the writer's previous knowledge of the language as well as the availability of fuzzy logic python libraries. The library used in this implementation is called SciKit-Fuzzy [31].

6.1 The fuzzy inference system

Scikit fuzzy provides a simple application programming interface to setup a FIS. This section will describe the process setting up a simple FIS that calculates salary based on age and the number of previous jobs. Fuzzy sets for age are set up in the same way as in 5.1, but with age ranges more appropriate for the example. The sets are therefore young = 18 - 35, middle aged = 30 - 50, and old 45-65. Additionally, job and salary sets are introduced. Line 4-6 in listing 1 initializes the universes for the different fuzzy sets. These are 18-100 for age and 0-10 for jobs, both with a step of 1. Age and jobs act as antecedents in the FIS and the initialization call is therefore to `ctrl.Antecedent`. Salary goes from 1500-10000 with steps of 500 and acts as consequent and is therefore initialized with `ctrl.Consequent`. The numbers used are made up for the sake of the example.

Next, the membership functions are initialized in lines 7-15. The age sets are defined on lines 7-9 as described above. Three different job sets are defined: *few* (<3), *medium* (2-6), and *many* (>5). The salary sets are *low* (1500 - 2500), *medium* (2000-7000), and *high* (>6000). All sets are initialized

using `fuzz.trimf` which produces a triangular membership function. The resulting FMs are visualized in figure 6.1

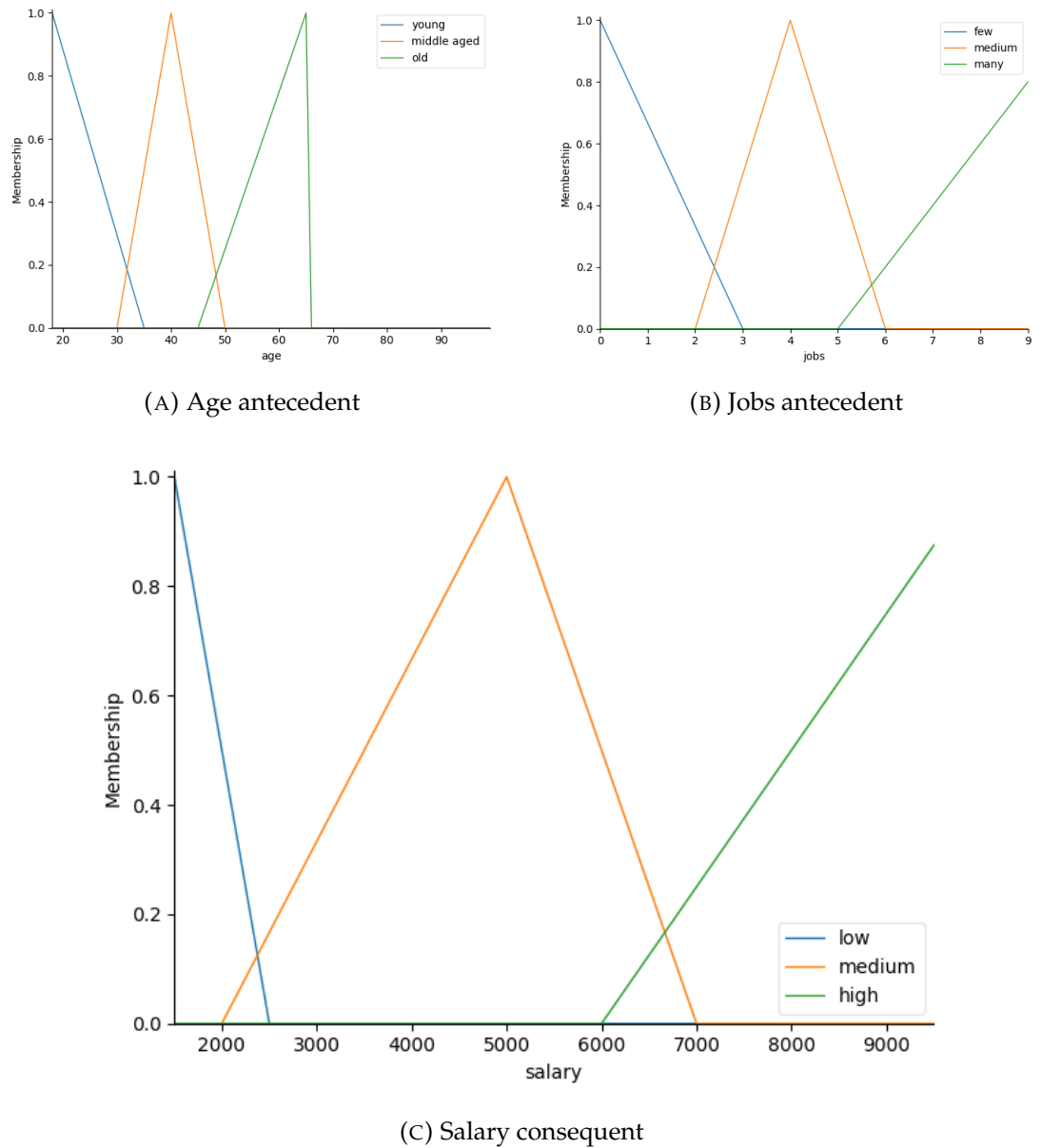


FIGURE 6.1: FMSs used in example FIS

Next, the following rules are defined, on lines 16 -21, to connect the antecedents with the consequents:

Rule 1 IF young OR few jobs THEN salary is low

Rule 2 IF middle-aged AND few jobs THEN salary is low

Rule 3 IF middle-aged OR medium amount of jobs THEN salary is medium

Rule 4 IF middle-aged AND many jobs THEN salary is high

Rule 5 IF old OR many jobs THEN salary is high

Finally, the rules are passed to the Control System on line 25 and `ctrl.ControlSystemSimulation` is called to complete the FIS initialization. The system is now ready to take input and calculate output based on the rule set specified. An example using `age = 35` and `jobs = 1` is input on lines 25-28 and the output printed on lines 30-32. The example outputs a salary of 4157.

```

1  import numpy as np
2  import skfuzzy as fuzz
3  from skfuzzy import control as ctrl

4  age = ctrl.Antecedent(np.arange(18, 100, 1), 'age')
5  jobs = ctrl.Antecedent(np.arange(0, 10, 1), 'jobs')
6  salary = ctrl.Consequent(np.arange(1500, 10000, 500), 'salary')

7  age['young'] = fuzz.trimf(age.universe, [18, 18, 35])
8  age['middle aged'] = fuzz.trimf(age.universe, [30, 40, 50])
9  age['old'] = fuzz.trimf(age.universe, [45, 65, 65])

10 jobs['few'] = fuzz.trimf(jobs.universe, [0, 0, 2])
11 jobs['medium'] = fuzz.trimf(jobs.universe, [1, 4, 6])
12 jobs['many'] = fuzz.trimf(jobs.universe, [5, 10, 10])

13 salary['low'] = fuzz.trimf(salary.universe, [1500, 1500, 2500])
14 salary['medium'] = fuzz.trimf(salary.universe, [2000, 5000, 7000])
15 salary['high'] = fuzz.trimf(salary.universe, [6000, 10000, 10000])
16 salary.view()
17 rules = []
18 rules.append(ctrl.Rule(age['young'] | jobs['few'], salary['low']))
19 rules.append(ctrl.Rule(age['middle aged'] & jobs['many'], salary['high']))
20 rules.append(ctrl.Rule(age['middle aged'] | jobs['medium'], salary['medium']))
21 rules.append(ctrl.Rule(age['middle aged'] & jobs['few'], salary['low']))
22 rules.append(ctrl.Rule(age['old'] | jobs['many'], salary['high']))

23 navigation_ctrl = ctrl.ControlSystem(rules)

24 fis = ctrl.ControlSystemSimulation(navigation_ctrl)

25 age = 35
26 jobs = 1
27 fis.input['age'] = age
28 fis.input['jobs'] = jobs
29 fis.compute()
30 print("Age: " + str(age))
31 print("Jobs: " + str(jobs))
32 print(fis.output['salary'])

```

LISTING 1: FIS initialization

6.2 Architecture

This section will present the high-level structure of the implementation with the help of the class diagram seen in figure 6.2. Each class and their interactions will be briefly presented.

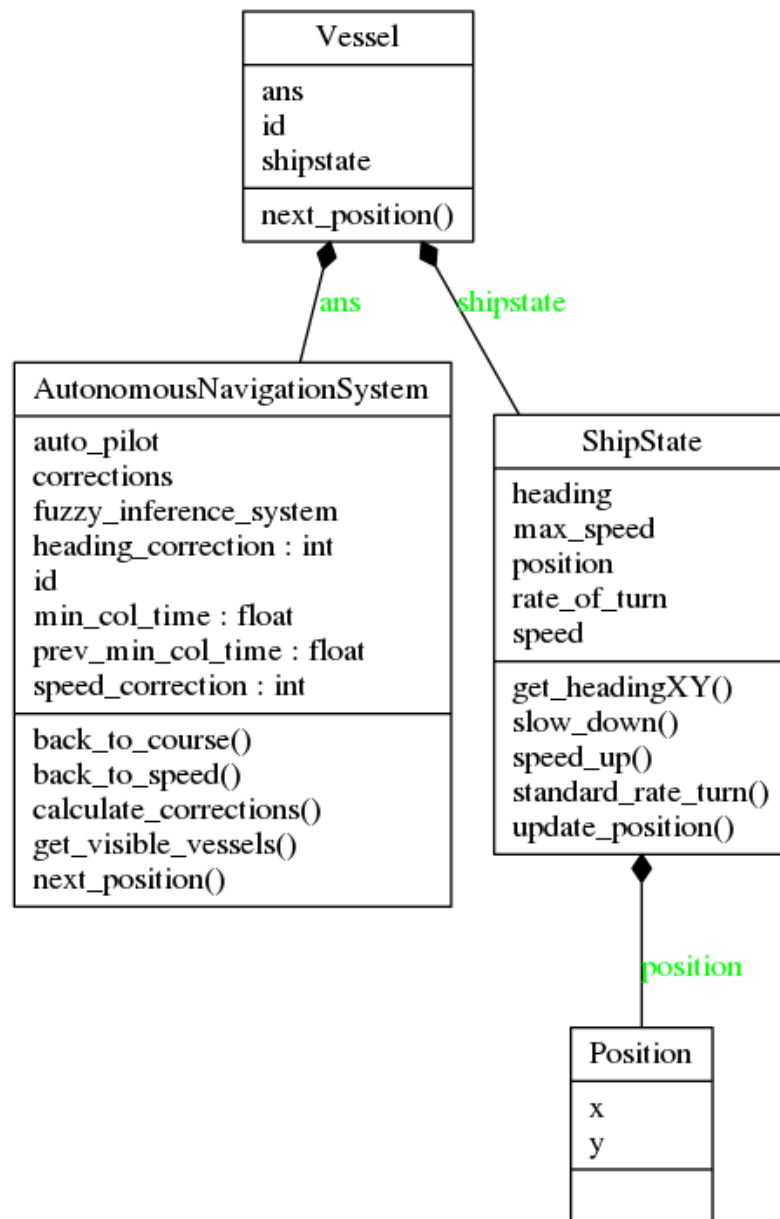


FIGURE 6.2: Class diagram

6.2.1 Classes

Each simulation scenario consists of at least one vessel. These vessels are represented by the *Vessel* class. The current state of the vessel is represented by the *ShipState* class which also contains the *Position* class. Furthermore,

each vessel needs a navigation system, which is represented by the *AutonomousNavigationSystem* class. The classes will be more thoroughly presented in the following subsections.

Vessel

Vessel is the main class and the class with which the simulation script interacts. It contains, apart from the previously mentioned *AutonomousNavigationSystem* and *ShipState* an id and a method to calculate its position in the next time frame. A new vessel object is created with the following call:

```
Vessel(id, heading, position_x, position_y, speed, max_speed,  
      rate_of_turn, fuzzy_inference_system, auto_pilot)
```

This constructor call specifies the ID for the vessel as well as its initial position, speed and heading. Furthermore, it defines the vessels maximum speed and rate of turn. The two final parameters specify the fuzzy inference system to use and whether the vessel shall use the navigation system. Setting the final boolean to false creates a rogue vessel that will just keep its initial speed and heading, thereby not complying with the COLREG rules. The *Vessel* class has only one method, which calculates the vessels position in the next time frame after applying possible corrections to heading and speed.

Position

The position class simply holds the vessels current coordinates in the Cartesian coordinate system used.

ShipState

ShipState holds information about the state of the vessel in the current time frame. This includes the vessels current position, heading and speed. The simulation does not distinguish between course and heading since no drift is simulated. Furthermore, limits such as maximum allowed speed and

```

1 rel_course = observed_vessel.shipstate.heading - shipstate.heading
2 if rel_course < 0:
3     rel_course = 360 + rel_course

```

LISTING 2: Relative course calculation

standard rate of turn are specified in this class. Finally, the class holds methods to change the ships heading by the specified standard rate of turn or speed by 1 kt, for the next time frame.

AutonomousNavigationSystem

The AutonomousNavigationSystem class from now on referred to as ANS is what separates an autonomous vessel from an ordinary vessel. The ANS combines the information from the ShipState class with situational awareness information provided by a separate Situational Awareness (SA) module, in order to calculate needed corrections to speed and course.

The SA is in this simulation case represented by a service that holds all information regarding the current scenario. A real system would have a SA module that reads and processes information from different sensors, such as LiDAR, cameras, on board the vessel. Information needed about a target vessel is its heading, speed, and position. These are used to calculate the four different inputs to the FIS system. Listing 3 shows the method used to calculate the relative bearing fed into the FIS system. Compass bearing is first calculated from the two provided coordinate pairs after which the result is converted into a relative bearing. Relative course is then calculated as the observed vessels heading - the own vessels heading, as shown in listing 2. Distance can be obtained by using the Pythagorean theorem on the differences between the vessels in the X and Y axis. Finally, speed ratio is defined as $\frac{\text{Speed of the observed vessel}}{\text{Speed of the own vessel}}$.

These inputs are fed into the FIS system, for each target vessels, and the recommended corrections presented by FIS are stored in an array. The expected time until collision for each target vessel is also calculated. Knowing the distance between the vessels, their relative velocity is needed to calculate the time. Equation 6.1 shows the relative velocity calculation, based on

the law of Cosines. V stands for velocity, θ for heading, the subscript m for own vessel and t for target vessel.

$$V_r = \sqrt{V_m^2 + V_t^2 - 2V_m V_t \cos(\theta_m - \theta_t)} \quad (6.1)$$

The previous calculations result in speed and heading correction suggestions, as well as an expected time until collision, for each target vessel. However, these recommendations might contradict each other and a way to prioritize the corrections in order of urgency is therefore needed. This is in this implementation solved in a simple matter by calculating the weighted arithmetic mean of the corrections using the expected time until collision as weight. The resulting corrections are then stored in ShipState as target heading.

Finally the heading and speed of the vessel is updated in the following manner. The vessels is steered towards the proposed heading change by a maximum of the vessels defined maximum rate of turn. The speed is similarly changed towards the proposed speed by a maximum of one knot. No proposed correction by the FIS means that the vessel is currently not in a scenario that satisfies a rule in the rule set. However, a previous correction suggestion might still not be completed due to the vessels limited rate of turn, acceleration or deceleration. The vessel will ,therefore, continue to change its heading and speed towards the saved target speed until they are the same. The target heading is then gradually changed towards the original heading until the vessel is back on its original heading. The process is visualized in Figure 6.3.

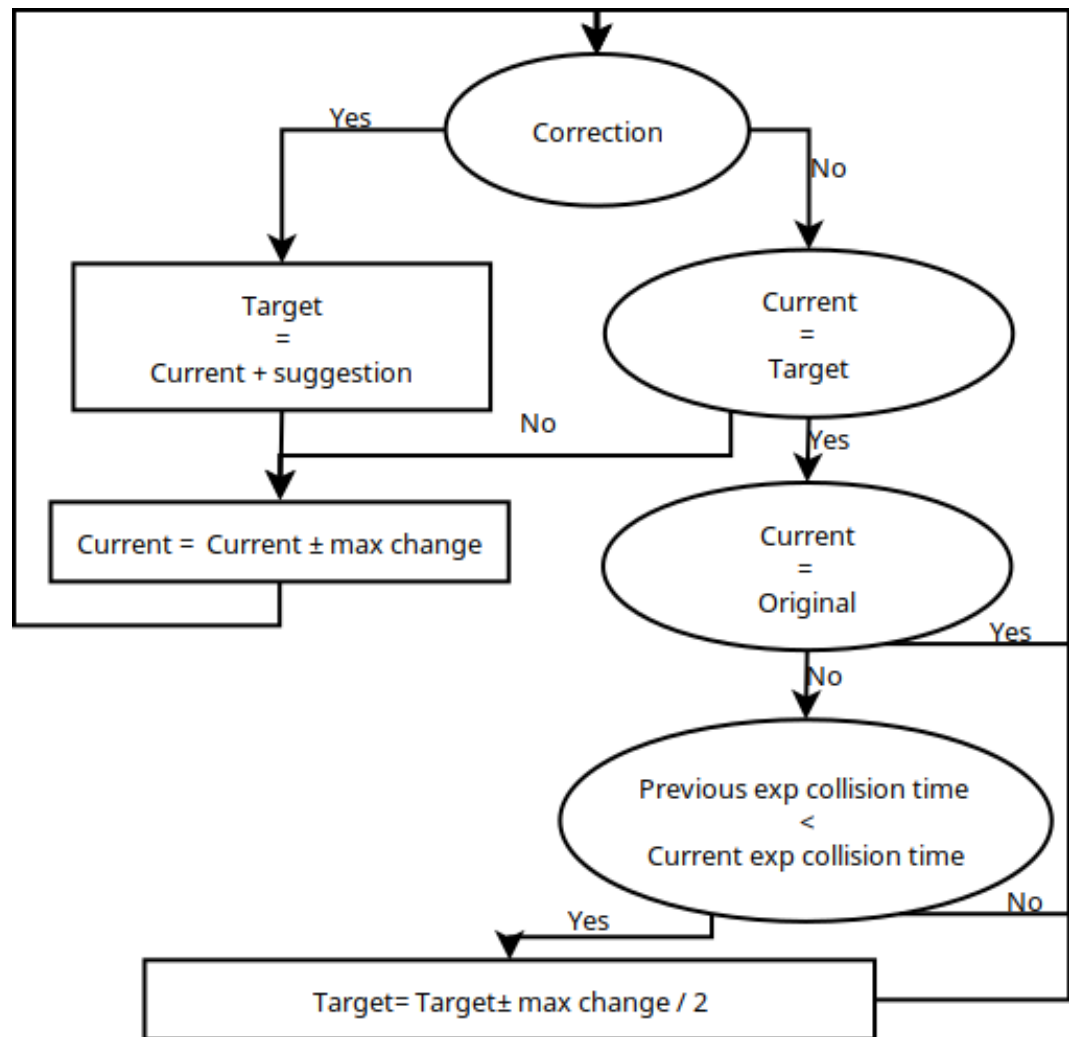


FIGURE 6.3: Flow chart explaining application of heading and speed corrections

6.3 Simulation

The goal of the implementation is to simulate a real-world situation with respect to time, speed, acceleration, and rate of turn while neglecting environmental factors such as weather. The simulation is therefore limited to two dimensions in a Cartesian coordinate system. The interval between time frames in the simulation is set to correspond to one second in the real world. Each iteration of the main simulation loop must, therefore, update

the vessels SA by scanning the environment for target vessels. The information gained is then passed to the FIS which generates the course and speed corrections used to update heading and speed as explained in section 6.2.1.

The efficiency and quality of the output of the FIS system, is to a large degree dependent on the FMFs. Designing correct FMF and rules to use in a FIS is an extensive mathematical project on its own. The FMFs used in this thesis are, therefore, based on parameters previously developed by Perera, Carvalho, and Soares [32]. All FMFs are in the form of isosceles trapezoids, i.e. trapezoids with legs of equal length. The values used is represented in the tables in 6.1. The values in the tables from and to columns represent the x values for the midpoint of the trapezoid legs. The fuzziness of the FMF is then defined as the range of the projection of the leg on the x axis. The fuzziness for the FMFs are 5° for relative bearing and course as well as course change; 0.3NM for range ; 0.5 kts for speed ratio and 1 kts for speed change.

TABLE 6.1: Parameters for the FMFs used

TABLE 6.2: Relative bearing FMF intervals

Identifier	from [°]	to [°]
I	355	5
II	5	85
III	85	95
IV	95	175
V	175	185
VI	185	211
VII	211	265
VIII	265	275
XI	275	329
X	329	355

TABLE 6.4: Range FMF intervals

Identifier	from [NM]	to [NM]
R_{VD}	0	1
R_B	1	4
R_A	4	10

TABLE 6.6: Course change FMF intervals

Identifier	from [°]	to [°]
P	-40	-10
Keep	-10	10
SB	10	40

TABLE 6.3: Relative course FMF intervals

Identifier	from [°]	to [°]
a	337.5	22.5
b	22.5	67.5
c	67.5	112.5
d	112.5	157.5
e	157.5	202.5
f	202.5	247.5
g	247.5	292.5
h	292.5	337.5

TABLE 6.5: Speed factor FMF intervals

Identifier	from	to
< 1	0	0.8
≈ 1	0.8	1.2
> 1	1.2	5

TABLE 6.7: Speed change FMF intervals

Identifier	from [kts]	to [kts]
Decrease	-10	-2
Keep	-2	2
Increase	2	10

7 Evaluation

Four scenarios are defined in order to test the ANS in multi vessel situations. However, a simple crossing scenario is first shown to test the algorithm in a basic COLREGs situation. Ideas for the other scenarios are from ACTS Plus Project Consortium [33]–[36], with a few minor modifications. All scenarios are set in high visibility on the high sea.

7.1 Crossing scenario

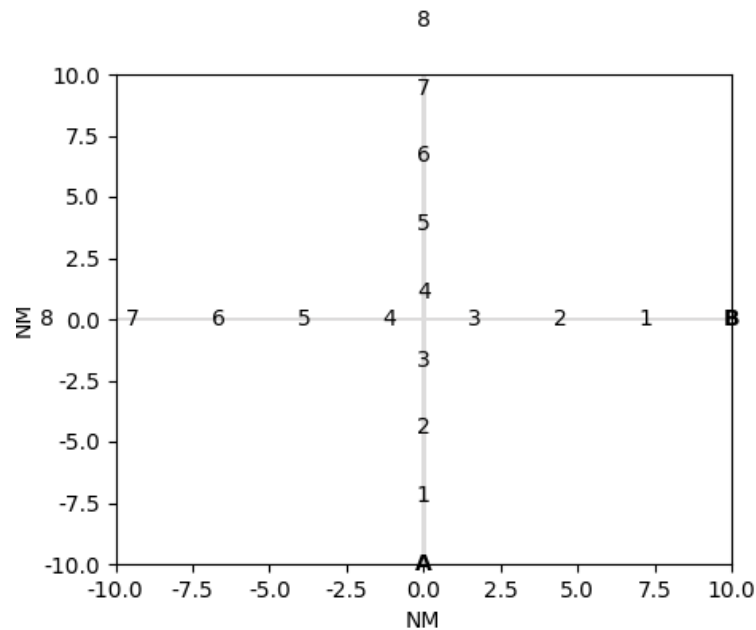


FIGURE 7.1: Crossing scenario

The first scenario depicts a simple crossing situation where two vessels are on perpendicular courses. The vessels start approximately 14 NM from each other with a speed of 10 kts as seen in figure 7.1. The vessels max speed is

set to 12 kts and maximum rate of turn $3^\circ/\text{s}$. The two vessels are to collide in origo if no corrections are made to either course or speed. COLREGs *rule 15* stipulate that vessel (A) that has the other vessel on its starboard side in a crossing situation should alter its course to starboard, thereby, avoiding passing in front of the other vessel (B).

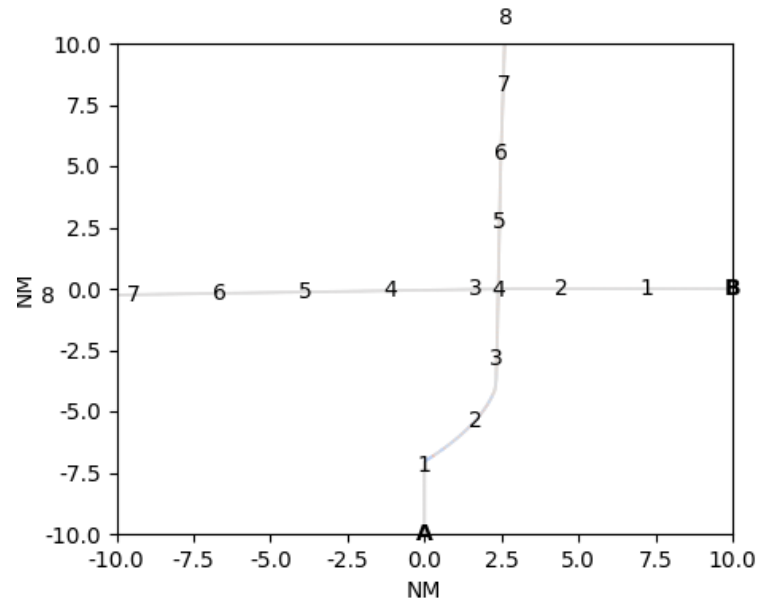


FIGURE 7.2: Crossing scenario

Figure 7.2 shows the scenario when both boats are guided by the FIS system. The numbers on the vessels tracks are printed every thousand seconds, which equal 16 minutes and 40 seconds, and help to compare the vessels positions at given time frames. These numbers will from here on be referred to with bold numbers.

It can be seen that both vessels continue along their original paths until vessel A reaches **1**, at which point the distance to vessel B has shrunk to 10NM and the algorithm becomes aware of the target vessel. Vessel A will at this point initiate a starboard turn of 24° to pass behind vessel B as specified in COLREGs. Vessel A will then follow headings suggested by the ANS until **2.7** where vessel B is no longer considered a threat, i.e. no rule in the FIS applies, after which it steers back to its original heading. Vessel B

will during the whole scenario be the stand on vessel and, therefore, keep its course and speed.

The ANS suggestions are exactly as described in the COLREGS documents for this case. The manoeuvre is initiated at a distance of 10 NM, which can be regarded as ample time (*Rule 8*). The correction is, furthermore, large and therefore clearly visible to the other vessels involved.

7.2 Overtaking and head-on scenario

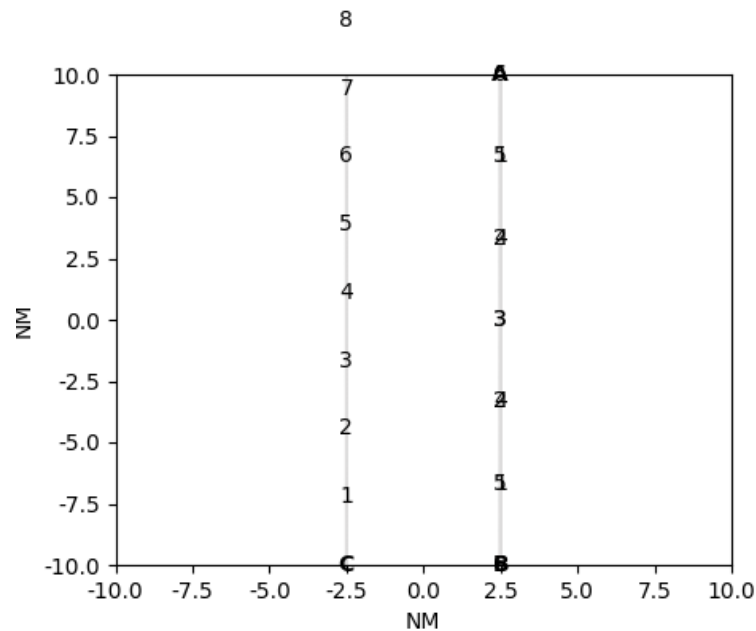


FIGURE 7.3: Overtaking and head-on scenario [36]

This scenario defines a three vessel scenario, where two vessels are meeting on reciprocal courses. One of the vessels is, furthermore, overtaking a third vessel on the stand on vessels starboard side. A visualization of the scenario is shown in figure 7.3. Vessels A and B starts on reciprocal tracks 20 NM from each other, while B and C start abreast 5 NM from each other. A, B, and C has a max speed of 15 kts. A and B start at 12 kts while C is slightly slower at 10 kts. Max rate of turn of all vessels is set to $3^\circ/\text{second}$.

Vessels A and B are therefore obliged to alter their courses to starboard to prevent a head-on collision (*Rule 14*). Moreover, vessel B shall keep out

of the way of *C* and in no circumstance alter its course so that it becomes a crossing vessel to *C* (*Rule 13*). All corrections shall furthermore be ample, so that it is recognizable by the other vessel, and taken as early as possible (*Rule 16*) [36].

ACTS Plus Project Consortium [36] suggest that vessel *A* should alter course to starboard and pass ahead of vessel *C*, to avoid a collision.

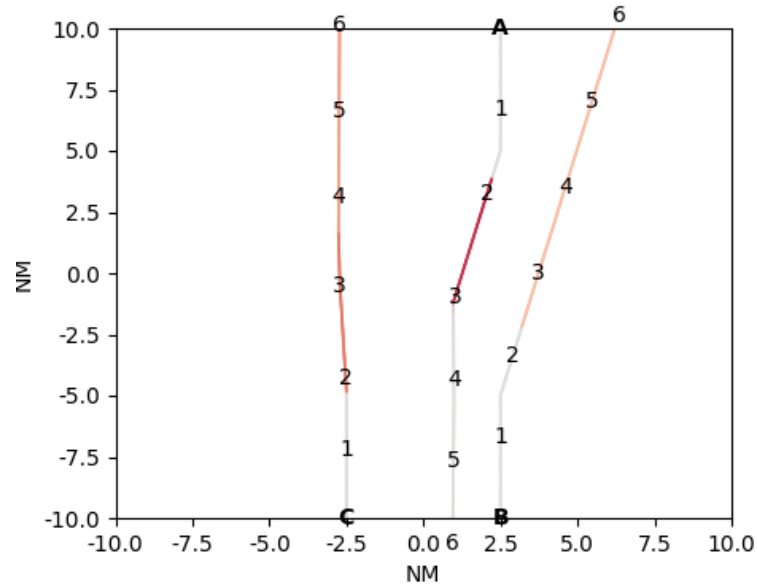


FIGURE 7.4: Overtaking and head-on scenario

The same initial scenario with the ANS enabled can be seen in figure 7.4. Vessels *A* and *B* are the first vessels to make a alteration to their initial state. Both initiate starboard turns at 1.5 to avoid colliding head-on. Vessel *B* continues on that course for the rest of the scenario, while vessel *A* steers back to its original course at 3 when it has passed vessel *C*. Vessels *C* and *A* will at 1.8 accelerate since they both have each other in sector *II* with a relative course in sector *e* and are therefore in a head-on situation.

The ANS simulation does not result in the same manoeuvres as recommended by ACTS Plus Project Consortium [36], since vessel *A* is passing between the two other vessels instead of ahead of vessel *C*. This might to some extent be due to the spacing between the vessels on the X-axis. A 5 NM spacing was chosen since the R_b radius is set to 4 NM. The vessels where able to

pass each other at safe distance, but the solution involves more vessels making alterations to their courses than the recommended one. The speed alterations could also be considered unnecessary. The speed alterations comes directly from the rule-set and can therefore probably be improved by fine tuning the rules. However, the amount of alterations, stems from the fact that vessel A starts its first alterations at **1.5**, when vessel C is still outside of its range and therefore unknown to vessel A. Vessel A will become aware of vessel C at **1.8**, when vessel C has already passed from sector I to II, and vessel A will, therefore, accelerate instead of turning further to the right.

The solution presented in figure 7.4, although different from the one recommended by ACTS Plus Project Consortium [36] succeed in keeping the vessel from colliding. All course changes are additionally, correct according to the COLREG rules. The speed changes are however, accelerations instead of decelerations as described in *rule 8*.

7.3 Overtaking and crossing scenario 1

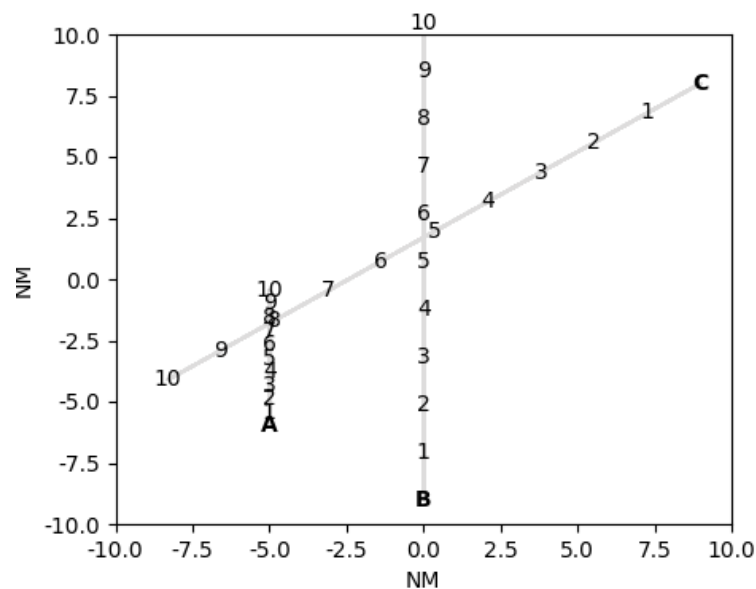


FIGURE 7.5: Overtaking and crossing scenario 1 [35]

The following three scenarios both depict scenario where vessels *A* and *B* are in an overtaking situation while *C* crosses both their paths. *C* crosses *A* and *B*'s paths with a 45° angle in the first scenario (figure 7.5). *A* starts at (-5, -6), *B* at (0, -9) and *C* in (9, 8). *A* and *B* has an initial heading of 0° while *C* start with a heading of 235° . The speeds of the vessels is set to ensure that *C* will collide with both vessels if no corrections are applied. Moreover, the speed of *B* must be greater than the speed of *A* since *B* is overtaking *A*. The vessels initial speed are, therefore: *A* = 2kts, *B* = 7kts and *C* = 7.6kts. The maximum speeds of the vessels are 10, 15 and 20 kts respectively. Max rate of turn of all vessels is set to 3° /second.

Rule 13 and 16 apply to this scenario in the same way as the previous one, with the exception that the vessels involved in the overtaking situation is vessel *A* and *B* instead of *B* and *C*. This implies that *B* shall keep out of way of *A* (Rule 13), while *A* shall keep its course and speed (Rule 17).

Additionally, both *A* and *B* are crossing *C*'s path with a risk of collision. This means that *A* and *B* should alter their courses to starboard and avoid passing in front of *C* (Rule 15). Vessel *C* shall, meanwhile, keep its course and speed. This results in contradictory obligations for vessel *A*, where it should keep its course and speed for *B* and simultaneously keep out of the way for vessel *C*.

ACTS Plus Project Consortium [35] suggest the following manoeuvres for vessel *A* and *B* in accordance with the ordinary practice of seamen: Both vessels might alter course to starboard and, thereby, pass behind vessel *C*. Alternatively vessel *A* may reduce speed or make a 360° turn to port, while vessel *B* reduces speed, makes a 360° to starboard or alters its course to starboard.

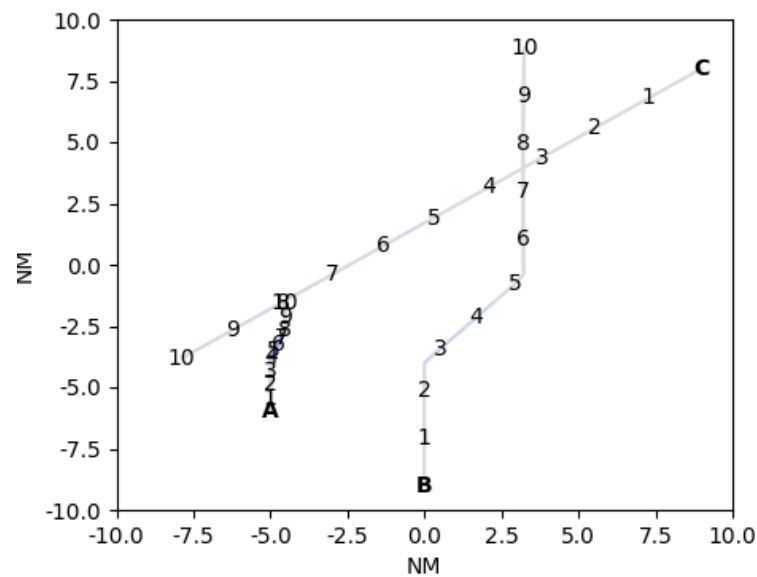


FIGURE 7.6: Overtaking and crossing scenario 1

The recommendations by the ANS matches the ordinary practice of seamen quite well for this scenario. Vessel *A* chooses to slow down and alter its course slightly to starboard, while vessel *B* alters its course to starboard. Thus vessel *A* made two corrections instead of one as recommended. The vessels manage to keep a safe distance and none of the stand on vessels are forced to alter their courses. Vessel *C* keeps its original heading and course during the whole simulation. A visualization can be seen in figure 7.6.

7.4 Overtaking and crossing scenario 2

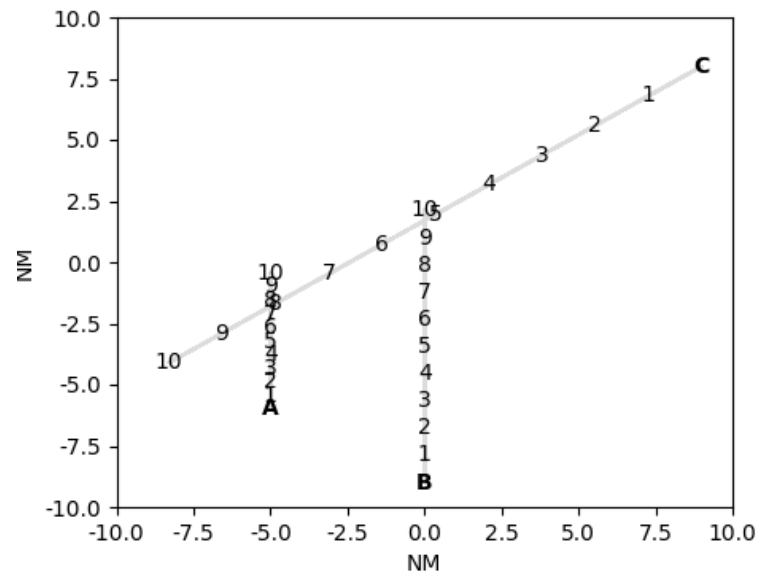


FIGURE 7.7: Overtaking and crossing scenario 2 [34]

The next scenario, visualized in figure 7.7, differs from the previous only in that B and C are not at risk of collision since B's speed is decreased to 4 kts. This means that B has no obligations to alter its course to starboard to avoid C as in the previous scenario. All other rules from the previous scenario do apply and vessel A is, therefore, still in the contradictory situation where it shall both keep course and speed for vessel B and alter course to starboard to avoid vessel C.

The following actions by vessel A solve the situation in accordance with the ordinary practice of seamen. It might either slow down, make a 360 degree turn to port or alter course to starboard and pass in front of vessel B if time permits.

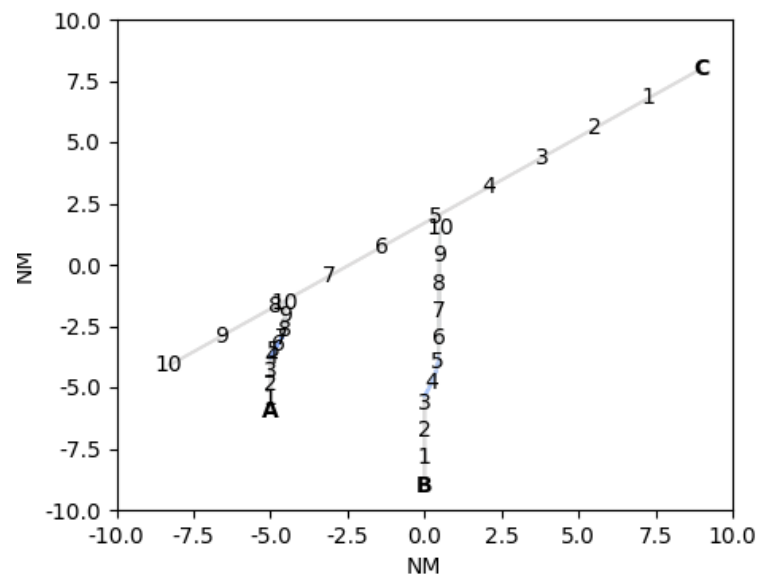


FIGURE 7.8: Overtaking and crossing scenario 2

This scenario is almost identical to the previous with the exception that there is no risk of collision between vessel *B* and *C*. These vessels should therefore, ideally not make any alterations to their courses. However, vessel *A* has to avoid vessel *C*, which is coming from starboard. Figure 7.8 shows how vessel *A* both alters course and slows down as in the previous scenario, which causes vessel *B* to also alter its course to starboard to avoid vessel *A*. This manoeuvre by vessel *B* is completely unnecessary since vessel *A*'s speed is significantly lower than *B*'s and a collision is, therefore, not imminent even though vessel *A* turns towards vessel *B*.

7.5 Overtaking and crossing scenario 3

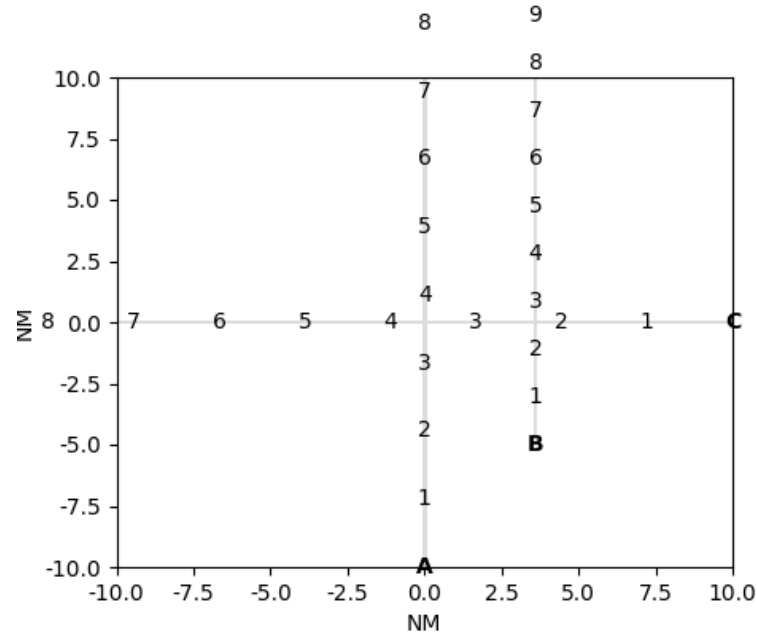


FIGURE 7.9: Overtaking and crossing scenario 3 [33]

The last scenario depicts a similar scenario as the previous two, with two vessels in an overtaking situation while a third cross their paths. The scenario is visualized in figure 7.9. Vessel *A* starts at (0, -10), vessel *B* at (3.6, 5) and *C* at (10, 0). The two vessels involved in the overtaking situation, that is *A* and *B*, start with a heading of 0 ° while *C* starts with a 270 ° heading in order to cross the two other vessels paths perpendicularly. *A* and *C* start at a speed of 10 kts while *B* starts at 15 kts. All vessels have as maximum speed of 20 kts and a maximum rate of turn of 3 °/s. The same rules apply as in the two previous rules, but the manoeuvring space for vessels *A* and *B* is slightly more limited due to the angle which *C* approaches on.

Vessel *A* is also in this scenario in a situation where two rules contradict each other. It shall keep course and speed for vessel *B* while simultaneously avoiding vessel *C*. Vessel *C* shall keep course and speed for both vessel *B* and *A*. Finally vessel *B* shall keep out of the way of vessel *A* and alter course to starboard to avoid vessel *C*.

One of the following actions is recommended to avoid collisions [33]. Both vessels might alter their course to starboard and pass behind vessel C to avoid collision. The action must however be initiated by vessel B. Alternatively vessel A might either reduce speed or make a 360 degree turn. The turn must be initiated early if made to starboard. Vessel B must at the same time also reduce speed, make a 360 °turn to starboard or alter course to starboard to avoid colliding with vessel C

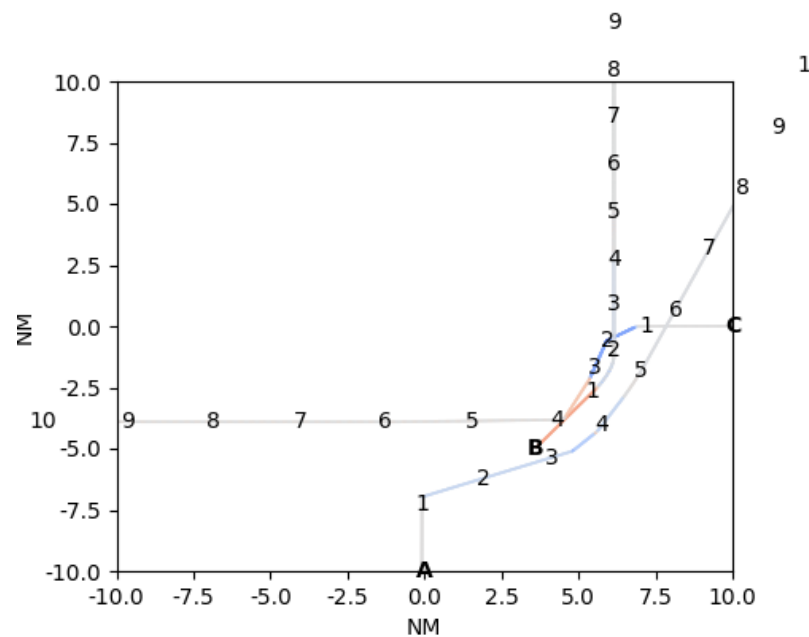


FIGURE 7.10: Overtaking and crossing scenario 3

The ANS simulation starts out as described above with vessels A and B altering courses to starboard to pass behind vessel C. Additionally Vessel A slows down while vessel B accelerates. The two vessels avoid each other exactly as described by ACTS Plus Project Consortium [33], apart from the extra speed changes. However, the alterations made by vessel B to avoid vessel C and A make the distance between vessel B and A shrink to under R_b , which forces vessel C to alter its course to starboard and decrease its speed. This causes vessels B to steer towards vessel C, thereby increasing the collision risk between the two vessels and breaking the COLREG rules. It could be argued that the initial placement of the vessels does not correspond to a normal situation since vessel B and C start at a distance less than

R_a and therefore have limited manoeuvring space.

8 Discussion

The goal of this thesis is to further examine the work of Perera, Carvalho, and Soares [3], regarding the use of fuzzy logic to facilitate collision avoidance amongst USVs. The previous research presents successful results in basic multi vessel situations at sea. However, only one of the vessels featured in the scenarios of that paper uses FIS to navigate. The other vessels are dummy vessels that keep their speed and course for the whole scenario. The contribution of this thesis would, therefore, be to test the usage of a FIS based ANS in scenarios where multiple vessels navigate with the use of FIS based ANS. Such scenarios put the vessels in situations where they are simultaneously supposed to give way and stand on and thereby has to prioritize the actions suggested by the ANS. An average weighted on expected collision time is used to prioritize the FIS recommended actions, rather than Bayesian networks, as suggested by Perera, Carvalho, and Soares [3].

The results of the simulations presented in section 7 show that the FIS based ANS manages to prevent collision in all of the scenarios. Although the corrections made by the vessels were not always in accordance with the ordinary practice of seamen. For instance, the scenarios depicted in figures 7.4 and 7.10 show vessels accelerating although the COLREG rules only mention decreasing the speed as a proper response in case of an impending collision. This can to some extent be blamed on the configuration of the FIS, i.e the antecedents and consequents, as well as the rules, specified. The previously mentioned example stems from a rule with acceleration as a consequent. The FIS configuration is in other words crucial to the quality of the ANS system. The possibility to simply define rules was one of the reasons why fuzzy logic was initially chosen for this thesis since such a system would enable people with navigation expertise to verify the rules even though they do not understand the underlying technology. This would potentially facilitate the process to certify the algorithm as safe. However, a

large rule-set and multiple inputs might quickly become difficult to comprehend since the change in one rule might have implications on have another vessel acts in a certain scenario.

9 Conclusion

9.1 Further work

A Fuzzy Inference system rule set

TABLE A.1: Rule set for a COLREGs compliant fuzzy inference system [3]

Bearing	Course	Range($R_{VD} - R_A$)		Range($R_A - R_B$)	
		Speed ratio	Decision	Speed ratio	Decision
I	d	<1		<1	
		≈ 1		≈ 1	
		>1		>1	
	e	<1	SB	<1	SB
		≈ 1	SB	≈ 1	SB
		>1	SB	>1	SB
	f	<1		<1	
		≈ 1		≈ 1	
		>1		>1	
II	e	<1		<1	
		≈ 1	Acc	≈ 1	Acc
		>1	Acc	>1	Acc
	f	<1		<1	
		≈ 1	SB, Dec	≈ 1	SB, Dec
		>1	SB, Dec	>1	SB, Dec
	g	<1		<1	
		≈ 1	SB	≈ 1	SB
		>1	SB	>1	SB
III	f	<1		<1	
		≈ 1	Acc	≈ 1	Acc
		>1	Acc	>1	Acc
SB= Starboard		P= Port		Empty decision = No change	
Acc = Accelerate				Dec = Decelerate	

Bearing	Course	Range($R_{VD} - R_A$)		Range($R_A - R_B$)	
		Speed ratio	Decision	Speed ratio	Decision
IV	g	<1		<1	
		≈ 1	Dec	≈ 1	Dec
		>1	Dec	>1	Dec
	h	<1		<1	
		≈ 1	Dec	≈ 1	Dec
		>1	Dec	>1	Dec
	g	<1		<1	
		≈ 1	Acc	≈ 1	Acc
		>1	Acc	>1	Acc
	h	<1		<1	
		≈ 1	P, Dec	≈ 1	P, Dec
		>1	P, Dec	>1	P, Dec
V	a	<1		<1	
		≈ 1	P, Dec	≈ 1	P, Dec
		>1	P, Dec	>1	P, Dec
	h	<1		<1	
		≈ 1		≈ 1	
		>1		>1	
	a	<1		<1	
		≈ 1	P	≈ 1	P
		>1	P	>1	P
	b	<1		<1	
		≈ 1		≈ 1	
		>1		>1	
VI	a	<1		<1	
		≈ 1	Dec	≈ 1	
		>1	Dec	>1	
	b	<1		<1	
		≈ 1	Dec	≈ 1	
		>1	Dec	>1	
		<1		<1	
		≈ 1		≈ 1	
		>1		>1	

SB= Starboard

P= Port

Empty decision = No change

Acc = Accelerate

Dec = Decelerate

Bearing	Course	Range($R_{VD} - R_A$)		Range($R_A - R_B$)	
		Speed ratio	Decision	Speed ratio	Decision
VII	c	<1		<1	
		≈ 1	Acc	≈ 1	
		>1	Acc	>1	
	a	<1		<1	
		≈ 1	SB	≈ 1	
		>1	SB	>1	
	b	<1		<1	
		≈ 1	SB, Dec	≈ 1	
		>1	SB, Dec	>1	
VIII	c	<1		<1	
		≈ 1	Dec	≈ 1	
		>1	Dec	>1	
	b	<1		<1	
		≈ 1	Dec	≈ 1	
		>1	Dec	>1	
	c	<1		<1	
		≈ 1	Dec	≈ 1	
		>1	Dec	>1	
IX	d	<1		<1	
		≈ 1	Acc	≈ 1	
		>1	Acc	>1	
	c	<1		<1	
		≈ 1	P	≈ 1	
		>1	P	>1	
	d	<1		<1	
		≈ 1	P, Dec	≈ 1	
		>1	P, Dec	>1	
	e	<1		<1	
		≈ 1	Acc	≈ 1	
		>1	Acc	>1	

SB= Starboard

P= Port

Empty decision = No change

Acc = Accelerate

Dec = Decelerate

Bearing	Course	Range($R_{VD} - R_A$)		Range($R_A - R_B$)	
		Speed ratio	Decision	Speed ratio	Decision
X	c	<1		<1	
		≈ 1	Dec	≈ 1	
		>1	Dec	>1	
	d	<1		<1	
		≈ 1	Dec	≈ 1	
		>1	Dec	>1	
	e	<1		<1	
		≈ 1	Acc	≈ 1	
		>1	Acc	>1	
SB= Starboard		P= Port		Empty decision = No change	
Acc = Accelerate				Dec = Decelerate	

TABLE A.2: Rules regarding COLREG rule 15 added to the rule set developed by Perera, Carvalho, and Soares [3]

Bearing	Course	Range($R_{VD} - R_A$)		Range($R_A - R_B$)	
		Speed ratio	Decision	Speed ratio	Decision
I	a	<1	SB	<1	SB
		≈ 1		≈ 1	
		>1		>1	
	h	<1	SB	<1	SB
		≈ 1		≈ 1	
		>1		>1	
	b	<1	SB	<1	SB
		≈ 1		≈ 1	
		>1		>1	
II	a	<1	P	<1	P
		≈ 1		≈ 1	
		>1		>1	
X	a	<1	SB	<1	SB
SB= Starboard		P= Port		Empty decision = No change	
Acc = Accelerate				Dec = Decelerate	

Bearing	Course	Range($R_{VD} - R_A$)		Range($R_A - R_B$)	
		Speed ratio	Decision	Speed ratio	Decision
		≈ 1		≈ 1	
		>1		>1	
SB = Starboard		P = Port		Empty decision = No change	
Acc = Accelerate				Dec = Decelerate	

B Python Code

```

1  def theta_to_nav(theta):
2      heading = theta + 90
3      heading = heading - theta * 2
4      if heading > 360:
5          heading = heading - 360
6      elif heading < 0:
7          heading = heading + 360
8      return heading

9  def compass_to_relative(compass, heading):
10     x = compass - heading
11     if x < 0:
12         return x + 360
13     else:
14         return x

15  def calculate_initial_compass_bearing(pointA, pointB):
16     if (type(pointA) != tuple) or (type(pointB) != tuple):
17         raise TypeError("Only tuples are supported as arguments")
18     if pointB[0] > pointA[0]:
19         x_diff = abs(pointA[0] - pointB[0])
20     else:
21         x_diff = -abs(pointA[0] - pointB[0])

22     if pointB[1] > pointA[1]:
23         y_diff = abs(pointA[1] - pointB[1])
24     else:
25         y_diff = -abs(pointA[1] - pointB[1])

26     initial_bearing = math.atan2(y_diff, x_diff)

27     initial_bearing = math.degrees(initial_bearing)
28     initial_bearing = theta_to_nav(initial_bearing)
29     compass_bearing = (initial_bearing + 360) % 360
30     return compass_bearing

31  def cartesian_coords_to_compass(main_vessel, observed_vessel):
32     main_observed = calculate_initial_compass_bearing(
33         (main_vessel.position.x
34          main_vessel.position.y),
35         (observed_vessel.position.x,
36          observed_vessel.position.y))

37     return main_observed

38  def cartesian_coords_to_relative(main_vessel, observed_vessel):
39     main_observed = cartesian_coords_to_compass(main_vessel,
40         observed_vessel)

41     main_observed = compass_to_relative(main_observed,
42         main_vessel.heading)

43     return main_observed

```

LISTING 3: Methods used to calculate the relative bearing from a ship to another from their heading and Cartesian coordinates

Swedish summary

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