Utilizing fuzzy logic to control umnanned surface vessel according to maritime navigation rules

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Abstract

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The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too…

Acknowledgements

The acknowledgments and the people to thank go here, don't forget to include your project advisor…

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List of Abbreviations

COLREGs	Convention on the International Regulations for Preventing Collisions at Sea, 1972
FMF	**F**uzzy **M**embership **F**unction

1 Introduction

[1] [2] [3]

2 Unmanned Surface Vehicles

Unmanned Surface Vehicles (USV) or Autonomous Surface Crafts (ASC) are vehicles operating the seas without a crew on-board. USVs encompass both fully autonomous vehicles, from now on referred to as ASCs, and semi-autonomous vehicles. Development of USVs has been ongoing for the last two decades [4]. However, the majority of the USVs developed are of the semi-autonomous type [5], [6], meaning that they depend on human intervention to some extent usually by a supervisor located on shore. Although semi-autonomous USVs greatly increase the safety of the operating personnel [5], they do not completely remove the need for human interaction. Supervision of several semi-autonomous vehicles can be admittedly be handled by a single person, which significantly decreases the number of person-hours needed to accomplish a specific mission [4]. The person-hours needed for surveillance could, however, be removed completely by a ASC. It is, therefore, of great interest to overcome the challenges associated with ASCs.

2.1 Usage

Yuh, Marani, and Blidberg [7] mention that roughly two-thirds of the earth's surface is covered by water, with an average depth of the oceans being 3688 m [8]. Thereby, adding up to a vast amount of explorable areas of which 95 % is yet to be seen by human eyes [9]. Utilization of autonomous vehicles could notably facilitate the exploration of these, yet unknown, areas. Although ASCs are situated on the surface of the ocean they can greatly increase the efficiency of Unmanned Underwater Vehicles (UUV) by acting as a gateway between UUVs and services, such as GPS, not easily available in underwater environments [5].

Liu, Zhang, Yu, *et al.* [5] have, furthermore, compiled a list of potential applications of USVs, along with previous research on the topics. The list is divided into five major categories: scientific research, environmental missions, ocean resource exploration, military use, and other applications.

2.2 Advantages

USVs have, apart from the obvious decrease in man hours, a few advantages over traditional manned vessels. The absence of humans on board means that facilities and resources such as canteens, manned bridges, showers etc. are no longer needed. The weight saved increases the manoeuvrability and deployability and can also be used to increase the payload the vessel is able to carry. USVs can, furthermore, conduct longer and more hazardous voyages than manned vessels since the personnel is located safely on land. This has the additional benefit of decreasing the operational costs [5].

2.3 Challenges

Even though USVs bring many advantages to the maritime industry, there are still quite a few challenges that need to be solved. The following section presents some of these challenges and their relation to the scope of this thesis.

USVs should be able to operate in a highly variable environment, all over the world, in terms of climate, traffic density and communication channels available. Algorithms and systems developed should, therefore, ideally be usable in all situations that might arise in this environment. This introduces challenges both hardware and software wise [5]. The limited scope of this thesis narrows the research to only the software related challenges with a focus on path planning, especially collision avoidance.

All vessels operating on water are bound to follow the international maritime "Rules of the Road" called the Convention on the International Regulations for Preventing Collisions at Sea, 1972 (COLREGs) explained in section 3. This is also true for USVs, since they should be able to operate in situations that comprise both other USVs and manned vessels that follow the COLREGs rules. Furthermore, they should be able to act in a safe manner in situations where other vessels, for some reason, are not complying with the COLREGs rules. Hence, it is crucial for the USVs to follow COLREGs to ensure safe operations.

COLREGs were originally introduced in 1972, to reduce the amount of collisions at sea. They are written to be interpreted by humans. Situations where multiple rules apply and contradict each other might therefore arise. These situations are usually solved with the use of good seamanship and human deduction. These aspects pose challenges when translating the COLREGs rules into computer understandable code [10].

3 COLREG

The Convention on the International Regulations for Preventing Collisions at Sea, 1972 (COLREGs) consist of 38 rules, grouped into five different sections, designed by the International Maritime Organization (IMO). The rules were adopted 1972 and entered into force 1977. COLREGs were designed to ensure traffic separation between vessels in an increasingly populated environment back in 1972 and it is, therefore, sometimes referred to as the Rules of the Road for maritime vessels. COLREGs have since been in use and mandatory to adhere to on international waters, although it has received amended several times [11].

National regulations might differ from the COLREGs to some extent. However, COLREGs mention that 'Nothing in these Rules shall interfere with the operation of special rules made by an appropriate authority for roadsteads, harbours, rivers, lakes or inland waterways connected with the high seas and navigable by seagoing vessels. Such special rules shall conform as closely as possible to these Rules.' [12]

3.1 Parts

This subsection will present the different parts of the COLREGs regulations, with emphasis on the parts related to manoeuvring a USV in international waters. Information in this subsection is taken from the official COLREGs regulations [12] if not otherwise specified.

3.1.1 Part A General

Rule 1 covers which vessels that are affected by the COLREGs regulations. This includes all vessels navigating on high seas and all waters connected therewith. Additionally, it specifies that special rules regarding roadsteads, harbours, rivers, lakes or inland waterways connected with the high seas and navigable by seagoing vessels shall conform as closely as possible to these Rules.

Rule 2 states that the COLREGs rules do not in any way free the vessel, owner, master or crew from responsibility to follow the rules and act according to the ordinary practice of seamen.

Rule 3 Defines the meaning of words used in the regulations, such as: vessel, power-driven vessel, sailing vessel, length, and breadth.

3.1.2 Part B Steering and Sailing

Rule 4 simply states that all rules in Part B Section I (rules 4-10) apply in any condition of visibility
Rule 5 states that all vessels shall at all times maintain proper look-out by sight and hearing and all other available means to ensure the best possible situational awareness.
Rule 6 states that vessels shall maintain a speed so that they can take proper and effective action to avoid a collision and stop within a distance appropriate to the prevailing circumstances and conditions.
Rule 7 states that all vessels shall use all means possible to determine if a risk of collision exists. This includes the use of radar equipment, however, caution should be exercised not to trust insufficient data. Moreover, Rule 7 states that any doubt whether a risk of collision exists shall be treated as if a risk exists. Finally, it specifies that a constant compass bearing between two vessels means that the two vessels are on collision course with each other. However, a constant bearing is only a sufficient condition for a risk of collision, not a necessary one. Close range, large vessels or a tow might pose a collision risk, without a constant bearing.
Rule 8 concerns actions to avoid collision. Actions shall, for instance, be taken according to the rules and as far as possible be conducted in ample time and large enough so that they are easily observable by other vessels. Small alterations should, in other words, be avoided. Course alterations might, moreover, be accompanied by a lowering of speed, if necessary to ensure in the vessels passing each other at safe distance.
Rule 9 concerns navigation in narrow channels and, therefore, not of interest for the scope of this thesis.
Rule 10 Concerns Traffic Separation Schemes ruled by IMO. These are traffic route systems developed for particularly congested shipping areas [13] and, therefore, not of interest for the scope of this thesis.
Rule 11 simply states that all rules in Part B Section II (rules 11-18) apply to vessels in sight of each other.
Rule 12 concerns sailing vessels and, therefore, not of interest for the scope of this thesis.
Rule 13 specifies the action to be taken by a vessel (give way vessel) when overtaking another vessel (stand on vessel). The give way vessel shall keep out of the way of the stand on vessel. A vessel is considered to overtake another vessel when approaching the other stand on vessel from a direction more than 22.5 °abaft the stand on vessels beam, i.e with a relative bearing between 112,5 °and 247,5 °from the stand on vessel.
Rule 14 defines a head-on situation as well as actions to be taken in case of a head-on situation. A head-on situation is when a vessel sees another vessel ahead or nearly ahead, i.e. it can see the masthead light in line or both sidelights. The give way vessel shall, in this case, alter its course to starboard.
Rule 15 states that vessel coming from starboard, i.e right, has the right of way in a crossing situation. The give way vessel shall if possible avoid crossing ahead of the other vessel and, therefore, alter its course to starboard.

Rule 16 states that give way vessel shall take action as early as possible.
Rule 17 states that stand on vessels shall keep its course and speed, provided that the give way vessel acts according to the regulations. However, the stand on vessel might take action to best avoid a collision if it finds that the actions by the give way vessels are insufficient to prevent a collision.
Rule 18 specifies responsibilities between vessels of different types. For instance, a power-driven vessel shall give way for a vessel engaged in fishing..
Rule 19 specifies reduced visibility operations. Vessels should proceed at speeds appropriate to the circumstances and use radar to determine collision risks. Speed should be reduced to a minimum if a vessel can hear another vessels fog horn apparently forward of her beam.

3.1.3 Part C Lights and Shapes

Part C specifies the lights and shapes a vessel shall exhibit, and when they should be used. These are therefore not of interest for the scope of this thesis, with the exception of *rule 21 and 22*, which specify the arcs that specific lights should cover as well as the distance the should be visible from.
Masthead lights cover a 225 °arc from 247,5 °to 112,5 °and should be visible from at least 2-6 miles, depending on the length of the vessel.
Sidelights are green on the starboard side and red on port. They cover a 112,5 arc from 0 relative bearing to 112,5 and 247,5 respectively. They should be visible from at least 1-3 miles, depending on the length of the vessel.

3.1.4 Part D Sound and Light signals

Part D specifies sound and light signals and is not of interest for the scope of this thesis.

3.1.5 Part E Exemptions

Is not of interest for the scope of this thesis.

4 Related research

USVs have been in development since at least 1993 when MIT started its Sea Grant College Program, Autonomous Surface Craft (ASCs) [4]. Different approaches have been tried over the years. This section will look into some of the COLREGs compliant approaches as well as their pros and cons.

Lee, Kwon, and Joh [14] combine fuzzy logic with modified virtual force fields to create a COLREGs compliant algorithm. The fuzzy logic rule-set used consists of about two hundred rules, which might be computational challenging. The authors do, however, mention that the rules-set might be streamlined. Benjamin, Leonard, Curcio, *et al.* [10] and Benjamin and Curcio [15] have developed a promising solution based on multi-objective optimization and interval programming, within a behaviour based architecture for capturing COLREGS rules. The solution has, furthermore, been tested with kayak-based autonomous surface crafts. Initial tests show promising results, but the solution cannot yet be called COLREGS compliant. Larson, Bruch, Halterman, *et al.* [16] uses a two-level two-dimensional obstacle map to create a near-field reactive control technique. COLREGs rules are incorporated into the solution by utilizing a rule-based approach in the path planner. The solution is made with computational efficiency in mind and the provided trajectory is, therefore, not optimal. It is also stated that the sensors and processing algorithm need more work. Zhuang, Su, Liao, *et al.* [17] have developed a solution based on relative coordinates and an evolutionary path planner. The solution has been tested with simple simulations of two vessels crossing and approaching head-on. Naeem, Irwin, and Yang [18] propose a path-planer that uses line of sight guidance between way-points. Obstacles are avoided by introducing a starboard heading bias to the line 5of sight heading until clear of conflict. The paper compares the trajectory, produced by the proposed on-the-fly algorithm, to that of a modified off-line DPSS strategy. Multiple simulations show that both algorithms yield COLREGs compliant results. However, a simulation with multiple dynamic targets was not conducted. Kuwata, Wolf, Zarzhitsky, *et al.* [19] has conducted at-sea testing of a velocity obstacle (VO) based local path planner, with quite promising results. .

5 Fuzzy set theory and logic

The concept of fuzzy sets was first mentioned by Zadeh [20] in 1964 as a way of dealing with sets of objects, where an objects membership to a set can be represented by a value in the real interval [0, 1]. Compared to classic set theory where membership is restricted to the two values of one and zero. Fuzzy logic enables one to define sets such as "old men" or "vessels on reciprocal course" [20], which facilitates the process of converting human abstract reasoning into computer understandable logic.

5.1 Fuzzy sets

The fuzzy set "all old men" can be defined in the following way. Let the *universe set* $\mathbf{X}$ be the set of ages [0, 100]. The set of old men can then be written as $\underset{\sim}{A} = \{a \in \mathbf{X} | a \text{ is old}\}$ [21]

However, the set requirement 'is old' is still undefined and therefore vague. Defining the threshold of being old at an age of 60 does divide the universal set $\mathbf{X}$ into subsets of 'old' and 'not old', but the division does not distinguish between humans aged 1 and 59. Fuzzy set theory introduces the concept of membership functions, to solve this problem. A Fuzzy Membership Function (FMF) describes the grade of membership of a in $\underset{\sim}{A}$. This function is often written as $\mu_{\underset{\sim}{A}}(a)$. Figure 5.1 shows a very simple linear FMF for the example $\underset{\sim}{A} = \{\text{'is old'}\}$, where for instance $\mu_{\underset{\sim}{A}}(10) = 0.1$. Similar FMFs can be defined for young and middle-aged which results in the graph seen in figure 5.2 [22].

A fuzzy set can then be written as an array of tuples consisting of the objects and its associated membership value: $\underset{\sim}{A} = \left\{(x, \mu_{\underset{\sim}{A}}(x) | x \in X)\right\}$ [23], which gives $\underset{\sim}{A} = \{(0,1), (10,0.9), (20,0.8), \ldots, (90,0.1), (100,0)\}$ for the example $\underset{\sim}{A}$={'is young'} in figure 5.1 [22].

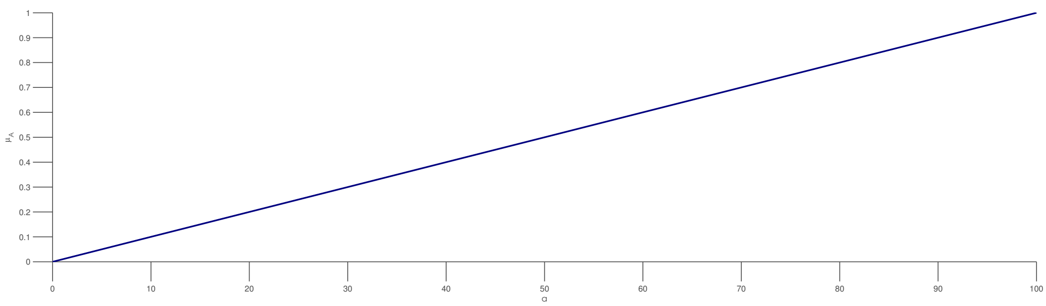

FIGURE 5.1: Fuzzy membership function for $\underset{\sim}{A} = \{\text{'Is old'}\}$

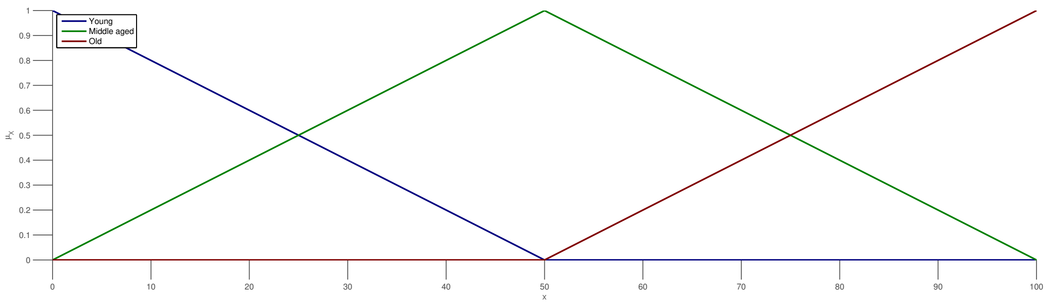

FIGURE 5.2: Fuzzy membership function for $\underset{\sim}{A} = \{\text{'Is young'}\}, \underset{\sim}{B} = \{\text{'Is middle-aged'}\}$ and $\underset{\sim}{C} = \{\text{'Is old'}\}$

5.2 Fuzzy logic

Fuzzy sets and set theory can, in the same manner as classical boolean sets, be used to define logical expressions. Fuzzy logic allows for half-truths, i.e. truth values in the interval [0,1]. Whereas boolean logic is restricted to truth values of one and zero. The truth value T of a fuzzy preposition P is, therefore a mapping from [0,1] to the universe T, as can be seen in 5.1 [22].

$$T : u \in U \rightarrow (0, 1) \tag{5.1}$$

The truth value of proposition P is therefore given by the membership grade $\mu_{\underset{\sim}{A}}(x)$ of x in $\underset{\sim}{A}$

Many of the same operators and connectives used in classical logic, does also apply to fuzzy logic. This thesis will only present the rules necessary for the scope of the thesis, which are the following [22]:

Negation

$$T(\overline{\underset{\sim}{P}}) = 1 - T(\underset{\sim}{P})) \tag{5.2}$$

Disjunction

$$\underset{\sim}{P} \vee \underset{\sim}{Q} : x \text{ is } \underset{\sim}{A} \text{ or } \underset{\sim}{B} \quad T(\underset{\sim}{P} \vee \underset{\sim}{Q}) = max(T(\underset{\sim}{P}), T(\underset{\sim}{Q})) \tag{5.3}$$

Conjunction

$$\underset{\sim}{P} \wedge \underset{\sim}{Q} : x \text{ is } \underset{\sim}{A} \text{ and } \underset{\sim}{B} \quad T(\underset{\sim}{P} \wedge \underset{\sim}{Q}) = min(T(\underset{\sim}{P}), T(\underset{\sim}{Q})) \tag{5.4}$$

Implication

$$\underset{\sim}{P} \rightarrow \underset{\sim}{Q} : x \text{ is } \underset{\sim}{A}, \text{ then } x \text{ is } \underset{\sim}{B} \tag{5.5}$$

$$T(\underset{\sim}{P} \rightarrow \underset{\sim}{Q}) = T(\overline{\underset{\sim}{P}} \vee \underset{\sim}{Q}) = max(T(\overline{\underset{\sim}{P}}), T(\underset{\sim}{Q}))$$

Implication can also be written in rule-based form

$$\underset{\sim}{P} \rightarrow \underset{\sim}{Q} \text{ is, IF } x \text{ is } \underset{\sim}{A} \text{ , THEN } t \text{ is } \underset{\sim}{B}$$

$$\Leftrightarrow$$

$$R = (\underset{\sim}{A} \times \underset{\sim}{B}) \cup (\overline{\underset{\sim}{A}} \times \underset{\sim}{Y})$$

With the membership function:

$$\mu_{\underset{\sim}{R}}(x, y) = max[\mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(y), (1 - \mu_{\underset{\sim}{A}}(x))] \tag{5.6}$$

Equation 5.6 is equivalent to the material implication used in traditional boolean logic [24]. However, fuzzy logic has multiple implication operators [22]. The work in this thesis will use Mamdani's implication, see equation 5.7.

$$\mu_{\underset{\sim}{R}}(x, y) = min[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x))] \tag{5.7}$$

5.3 Fuzzy (rule-based) systems

Fuzzy logic can be used to model complex systems described in natural language, originally written to be interpreted by humans. Such knowledge can often be written as rules in the following form [22].

$$\text{IF premise (antecedent), THEN conclusion (consequent)} \tag{5.8}$$

Combining multiple rules enables one to describe complex systems in a relatively simple structure. Rules can, furthermore, contain multiple antecedents and consequents. However, this raises the question of how multiple antecedents, as shown in rule 5.9, can be decomposed into a single antecedent and the rules aggregated into a single consequent [22].

$$\text{IF } x \text{ is } \underset{\sim}{A_1} \text{ and } \underset{\sim}{A_2} \ldots \text{and } \underset{\sim}{A_L} \text{ THEN } y \text{ is } \underset{\sim}{B_s} \tag{5.9}$$

Conjunctive antecedents can be rewritten as a new fuzzy set

$$\underset{\sim}{A_s} = \underset{\sim}{A_1} \cap \underset{\sim}{A_2} \cap \cdots \cap \underset{\sim}{A_L}$$

with the membership function

$$\mu_{\underset{\sim}{A_S}}(x) = min\left[\mu_{\underset{\sim}{A_1}}(x), \mu_{\underset{\sim}{A_2}}(x), \ldots, \mu_{\underset{\sim}{A_3}}(x)\right]$$

Rule 5.9 can then be rewritten as

$$\text{IF } \underset{\sim}{A_S} \text{ THEN } \underset{\sim}{B_S}$$

Disjunctive antecedents can similarly be written as

$$\underset{\sim}{A_s} = \underset{\sim}{A_1} \cup \underset{\sim}{A_2} \cup \cdots \cup \underset{\sim}{A_L}$$

$$\mu_{\underset{\sim}{A_S}}(x) = max\left[\mu_{\underset{\sim}{A_1}}(x), \mu_{\underset{\sim}{A_2}}(x), \ldots, \mu_{\underset{\sim}{A_3}}(x)\right]$$

The same principle can be applied to find the overall consequent when multiple rules apply. Conjunctive rules where both consequents must be applied can be found by the fuzzy intersection of all the rule consequents. Whereas disjunctive rules use the fuzzy union of the rule consequents.

5.3.1 Mamdani's fuzzy inference method

Introduction

Evaluate antecedent

Obtain conclusion

Aggregate conclusions

Defuzzify

Summary

5.4 Fuzzy logic model for COLREGs

6 Scikit-Fuzzy

[25] [26]

7 Implementation

8 Scenarios

A Appendix A

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