

# ACM-ICPC Team Reference Document

## Tula State University (Fursov, Perezyabov, Vasin)

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```

#include<...>
using namespace std;
#define fast cin.tie(0); cout.tie(0); ios::sync_with_stdio
(0);
#define all(x) x.begin(), x.end()
#define rall(x) x.rbegin(), x.rend()
#define ff first
#define ss second
#define nl "\n"
#define sp " "
#define yes "YES"
#define no "NO"
#define bool_out(x) (x ? yes : no)
#define ll long long
#define int ll
const int inf = 1e18;
const int mod = 1e9 + 7;
const int pow_mod = 1e9 + 6;
void solve() {}
signed main() {
    int t = 1;
    while (t--)
        solve();
}

```

## 1.2 Cpp Includes

```
#include <stdio>
#include <stdlib>
#include <cmath>
#include <climits>
#include <cstring>
#include <iostream>
#include <iomanip>
#include <fstream>
#include <algorithm>
#include <vector>
#include <set>
#include <unordered_set>
#include <map>
#include <unordered_map>
#include <queue>
#include <stack>
#include <string>
#include <list>
#include <bitset>
#include <sstream>
#include <functional>
#include <complex>
#include <random>
```

## 1.3 Py Template

```
from math import ceil, log, sqrt, floor
def solve():
    pass
def main():
    t = 1
    t = int(input())
    for _ in range(t):
        solve()
```

# 2 Data Structures

## 2.1 Disjoint Set Union

```
struct dsu {
    vector<int> p, size;
    dsu(int n) {
        p.assign(n, 0); size.assign(n, 0);
        for (int i = 0; i < n; i++) {
            p[i] = i;
            size[i] = 1;
        }
    }
    int get(int v) {
        if (p[v] != v) p[v] = get(p[v]);
        return p[v];
    }
    void unite(int u, int v) {
        auto x = get(u), y = get(v);
        if (size[x] > size[y]) swap(x, y);
        p[x] = y;
        size[x] += size[y];
    }
};
```

## 2.2 Segtree Sum

```
struct sum_tree {
    vector<int> tree;
    int size;
    void init(int n) {
        size = 1;
        while (size < n) size <<= 1;
        tree.assign(2 * size - 1, 0);
    }
};
```

```
void build(vector<int> &a, int x, int lx, int rx) {
    if (rx - lx == 1) {
        if (lx < a.size())
            tree[x] = a[lx];
        return;
    }
    int m = (lx + rx) / 2;
    build(a, 2 * x + 1, lx, m);
    build(a, 2 * x + 2, m, rx);
    tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
}

void build(vector<int> &a) {
    init(a.size());
    build(a, 0, 0, size);
}

void set(int i, int v, int x, int lx, int rx) {
    if (rx - lx == 1) {
        tree[x] = v;
        return;
    }
    int m = (lx + rx) / 2;
    if (i < m) set(i, v, 2 * x + 1, lx, m);
    else set(i, v, 2 * x + 2, m, rx);
    tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
}

void set(int i, int v) {
    set(i, v, 0, 0, size);
}

int sum(int l, int r, int x, int lx, int rx) {
    if (rx <= l || r <= lx) return 0;
    if (l <= lx && rx <= r) return tree[x];
    int m = (lx + rx) / 2;
    int sum1 = sum(l, r, 2 * x + 1, lx, m);
    int sum2 = sum(l, r, 2 * x + 2, m, rx);
    return sum1 + sum2;
}

int sum(int l, int r) {
    return sum(l, r, 0, 0, size);
}
};
```

## 2.3 Segtree Min Count

```
struct min_count_tree {
    struct node {
        int min;
        int count;
    };
    node combine(node a, node b) {
        if (a.min < b.min) return a;
        if (a.min > b.min) return b;
        return { a.min, a.count + b.count };
    }
    const node zero = { inf, 0 };
    vector<node> tree;
    int size;
    void init(int n) {
        size = 1;
        while (size < n) size <<= 1;
        tree.assign(2 * size - 1, { 0, 0 });
    }
    void build(vector<int> &a, int x, int lx, int rx) {
        if (rx - lx == 1) {
            if (lx < a.size())
                tree[x] = { a[lx], 1 };
            return;
        }
        int m = (lx + rx) / 2;
        build(a, 2 * x + 1, lx, m);
        build(a, 2 * x + 2, m, rx);
        tree[x] = combine(tree[2 * x + 1], tree[2 * x + 2]);
    }
    void build(vector<int> &a) {
        init(a.size());
        build(a, 0, 0, size);
    }
    void set(int i, int v, int x, int lx, int rx) {
        if (rx - lx == 1) {
            tree[x] = { v, 1 };
            return;
        }
        int m = (lx + rx) / 2;
        if (i < m) set(i, v, 2 * x + 1, lx, m);
        else set(i, v, 2 * x + 2, m, rx);
    }
};
```

```

    tree[x] = combine(tree[2 * x + 1], tree[2 * x + 2]);
}
void set(int i, int v) {
    set(i, v, 0, 0, size);
}
node calc(int l, int r, int x, int lx, int rx) {
    if (rx <= l || r <= lx) return zero;
    if (l <= lx && rx <= r) return tree[x];
    int m = (lx + rx) / 2;
    node calc1 = calc(l, r, 2 * x + 1, lx, m);
    node calc2 = calc(l, r, 2 * x + 2, m, rx);
    return combine(calc1, calc2);
}
node calc(int l, int r) {
    return calc(l, r, 0, 0, size);
}
};

```

## 2.4 Segtree First Above

```

struct first_above_tree {
    //tree_max
    int first_above(int v, int x, int lx, int rx) {
        if (tree[x] < v) return -1;
        if (rx - lx == 1) return lx;
        int m = (lx + rx) / 2;
        int res = first_above(v, 2 * x + 1, lx, m);
        if (res == -1) res = first_above(v, 2 * x + 2, m, rx);
        return res;
    }
    int first_above(int v) {
        return first_above(v, 0, 0, size);
    }
};

```

## 2.5 Segtree First Above Left

```

struct first_above_left_tree {
    //tree_max
    int first_above(int v, int l, int x, int lx, int rx) {
        if (tree[x] < v || rx <= l) return -1;
        if (rx - lx == 1) return lx;
        int m = (lx + rx) / 2;
        int res = first_above(v, l, 2 * x + 1, lx, m);
        if (res == -1) res = first_above(v, l, 2 * x + 2, m, rx);
        return res;
    }
    int first_above(int v, int l) {
        return first_above(v, l, 0, 0, size);
    }
};

```

## 2.6 Segtree K Ones

```

struct k_ones_tree {
    //tree_sum
    int find(int k, int x, int lx, int rx) {
        if (rx - lx == 1) return lx;
        int m = (rx + lx) / 2;
        if (k < tree[2 * x + 1]) return find(k, 2 * x + 1, lx, m);
        else return find(k - tree[2 * x + 1], 2 * x + 2, m, rx);
    }
    int find(int k) {
        return find(k, 0, 0, size);
    }
};

```

## 2.7 Segtree Intersecting Segments

```

struct sum_tree {};
signed main() {
    sum_tree tr;
    int n; cin >> n;
    tr.init(2 * n);
    vector<int> pos(n, -1), ans(n, 0), A(2 * n);
    for (int i = 0; i < 2 * n; i++) cin >> A[i];
    for (int i = 0; i < 2 * n; i++) {
        int a = A[i] - 1;
        if (pos[a] == -1) {
            pos[a] = i;
            tr.set(pos[a], 1);
        }
        else {
            ans[a] = tr.sum(pos[a] + 1, i);
            tr.set(pos[a], 0);
            pos[a] = 0;
        }
    }
    pos.assign(n, -1); reverse(A.begin(), A.end());
    for (int i = 0; i < 2 * n; i++) {
        int a = A[i] - 1;
        if (pos[a] == -1) {
            pos[a] = i;
            tr.set(pos[a], 1);
        }
        else {
            ans[a] += tr.sum(pos[a] + 1, i);
            tr.set(pos[a], 0);
            pos[a] = 0;
        }
    }
    for (int i = 0; i < n; i++)
        cout << ans[i] << " ";
}

```

## 2.8 Segtree Max Sum

```

struct max_sum_tree {
    //tree_min_count
    struct node {
        long long seg, pref, suf, sum;
    };
    node one_element(int x) {
        return {
            max(x, 0LL), //seg
            max(x, 0LL), //pref
            max(x, 0LL), //suf
            x //sum
        };
    }
    node combine(node a, node b) {
        return {
            /*seg*/ max(a.seg, max(b.seg, a.suf + b.pref)),
            /*pref*/ max(a.pref, a.sum + b.pref),
            /*suf*/ max(b.suf, b.sum + a.suf),
            /*sum*/ a.sum + b.sum
        };
    }
    const node zero = { 0, 0, 0, 0 };
};

```

## 2.9 Segrtree Nested Segments

```

struct tree_sum {};
signed main() {
    tree_sum tree;
    int n, m;
    cin >> n;
    tree.init(2 * n);
    vector<int> pos(n, -1), otv(n, 0);
    for (int i = 0; i < 2 * n; i++) {
        int a; cin >> a; --a;
        if (pos[a] == -1) {

```

```

        pos[a] = i;
    }
    else {
        otv[a] = tree.sum(pos[a], i);
        tree.set(pos[a], 1);
    }
}
for (int i = 0; i < n; i++) {
    cout << otv[i] << " ";
}
}

```

## 2.10 Segment Tree With Lazy Propagation

```

// mass assignment
struct lazy_seg_tree {
    vector<int> tree, lazy;
    int size;
    init(int n) {
        size = 1;
        while (size < n) size <<= 1;
        tree.assign(2 * size - 1, 0);
        lazy.assign(2 * size - 1, 0);
    }
    void build(vector<int> &a, int x, int lx, int rx) {
        if (rx - lx == 1) {
            if (lx < a.size())
                tree[x] = a[lx];
            return;
        }
        int m = (lx + rx) / 2;
        build(a, 2 * x + 1, lx, m);
        build(a, 2 * x + 2, m, rx);
        tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
    }
    void build(vector<int> &a) {
        init(a.size());
        build(a, 0, 0, size);
    }
    void push(int x) {
        tree[2 * x + 1] = lazy[x];
        lazy[2 * x + 1] = lazy[x];
        tree[2 * x + 2] = lazy[x];
        lazy[2 * x + 2] = lazy[x];
        lazy[x] = -1;
    }
    void update(int v, int l, int r, int x, int lx, int rx) {
        if (rx <= l && r <= lx) return;
        if (l <= lx && rx <= r) {
            push(x);
            tree[x] = v;
            lazy[x] = v;
            return;
        }
        int m = (lx + rx) / 2;
        tree[x] = v;
        lazy[x] = v;
        update(v, l, r, 2 * x + 1, lx, m);
        update(v, l, r, 2 * x + 2, m, rx);
    }
    void update(int v, int l, int r) {
        update(v, l, r, 0, 0, size);
    }
    int get(int i, int x, int lx, int rx) {
        if (rx - lx == 1) return tree[x];
        int m = (lx + rx) / 2;
        if (i < m) get(i, 2 * x + 1, lx, m);
        else get(i, 2 * x + 2, m, rx);
    }
    int get(int i) {
        return get(i, 0, 0, size);
    }
};

```

## 2.11 Segtree Seg Adding

```

struct seg_adding_tree {

```

```

// sum_tree + difference array
void add(int i, int v, int x, int lx, int rx) {
    if (rx - lx == 1) {
        tree[x] += v;
        return;
    }
    int m = (lx + rx) / 2;
    if (i < m) add(i, v, 2 * x + 1, lx, m);
    else add(i, v, 2 * x + 2, m, rx);
    tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
}
void add(int l, int r, int v) {
    add(l, v, 0, 0, size);
    add(r, -v, 0, 0, size);
    return;
}
long long get(int i) {
    return sum(0, i + 1);
}
};

```

## 2.12 Segtree Inversions

```

struct sum_tree { };
signed main() {
    sum_tree tree;
    int n; cin >> n;
    tree.init(n);
    for (int i = 0; i < n; i++) {
        int a; cin >> a;
        cout << tr.sum(a, n) << endl;
        tr.set(a - 1, 1);
    }
}

```

## 2.13 Segtree Inversions II

```

struct inversion_tree {
    // sum_tree
    int find(int k, int x, int lx, int rx) {
        if (rx == lx + 1) {
            return lx;
        }
        int m = (rx + lx) / 2;
        if (k < tree[2 * x + 2]) return find(k, 2 * x + 2, m, rx);
        else return find(k - tree[2 * x + 2], 2 * x + 1, lx, m);
    }
    int find(int k) {
        return find(k, 0, 0, size);
    }
};
signed main() {
    inversion_tree tree;
    int n, m;
    cin >> n;
    vector<int> A(n), P(n), E(n, 1);
    tree.build(E);
    for (int i = 0; i < n; i++) {
        cin >> A[i];
    }
    int pos = 0;
    for (int i = n - 1; i >= 0; i--) {
        pos = tree.find(A[i]);
        P[i] = pos + 1;
        tree.set(pos, 0);
    }
    for (int i = 0; i < n; i++) {
        cout << P[i] << " ";
    }
}

```

## 3 Graphs

### 3.1 Articulation Point

```
vector<vector<int>> g;
vector<bool> used;
int timer = 0;
vector<int> tin, fup;
set<int> result;

void dfs(int v, int p = -1) {
    used[v] = true;
    tin[v] = fup[v] = timer++;
    int children = 0;

    for (size_t i = 0; i < g[v].size(); ++i) {
        int to = g[v][i];
        if (to == p) continue;
        if (used[to]) fup[v] = min(fup[v], tin[to]);
        else {
            dfs(to, v);
            fup[v] = min(fup[v], fup[to]);
            if (fup[to] >= tin[v] && p != -1) result.insert(v);
            children++;
        }
    }

    if (p == -1 && children > 1) result.insert(v);
}

signed main() {
    int n, m, k;
    cin >> n >> m;

    g.resize(n);
    used.assign(n, false);
    tin.resize(n);
    fup.resize(n);

    for (int i = 0; i < m; i++) {
        int first, second;
        cin >> first >> second;
        first--; second--;
        g[first].push_back(second);
        g[second].push_back(first);
    }

    for (int i = 0; i < n; i++)
        dfs(i);

    for (auto it = result.begin(); it != result.end(); it++)
        cout << *it + 1 << " ";
}
```

### 3.2 Bfs

```
vector<vector<int>> v;
vector<int> u;
queue<int> q;

void bfs(int i, int n) {
    q.push(i);
    u[i] = 1;

    while (!q.empty()) {
        int j = q.front(); q.pop();

        for (auto &x : v[j]) {
            if (!u[x]) {
                u[x] = 1;
                q.push(x);
            }
        }

        cout << j << " ";
    }
}

signed main() {
    int n, m;
```

```
    cin >> n >> m;

    v.resize(n);

    for (int i = 0; i < m; i++) {
        int x, y;
        cin >> x >> y;
        v[--x].push_back(--y);
        v[y].push_back(x);
    }

    for (int i = 0; i < n; i++) {
        u.assign(n, 0);
        bfs(i, n);
        cout << '\n';
    }
}
```

### 3.3 Bridges

```
vector<vector<int>> g;
vector<bool> used;
int timer = 0;
vector<int> tin, fup;
vector<pair<int, int>> result;

void dfs(int v, int p = -1) {
    used[v] = true;
    tin[v] = fup[v] = timer++;

    for (int i = 0; i < g[v].size(); i++) {
        int to = g[v][i];
        if (to == p) continue;
        if (used[to]) fup[v] = min(fup[v], tin[to]);
        else {
            dfs(to, v);
            fup[v] = min(fup[v], fup[to]);
            if (fup[to] > tin[v] && count(all(g[v]), to) == 1)
                result.push_back({ min(v, to), max(to, v) });
        }
    }
}

void find_bridges(int n) {
    timer = 0;
    for (int i = 0; i < n; i++) {
        if (!used[i]) dfs(i);
    }
}

signed main() {
    int n;
    cin >> n;

    g.resize(n);
    used.assign(n, false);
    tin.resize(n);
    fup.resize(n);
    cin.ignore();

    for (int i = 0; i < n; i++) {
        int current = 0, count = 0;
        cin >> current >> count;
        for (int j = 0; j < count; j++) {
            int temp = 0;
            cin >> temp;
            g[current].push_back(temp);
        }
    }

    find_bridges(n);

    if (result.size()) {
        sort(all(result));
        for (int i = 0; i < result.size(); i++) {
            cout << result[i].first << " " << result[i].second << endl;
        }
    }
    else cout << "Empty" << endl;
}
```

### 3.4 Components Of Strong Connectivity

```
vector<vector<int>> g, gr;
vector<char> used;
vector<int> order, component;

void dfs1(int v) {
    used[v] = true;
    for (size_t i = 0; i < g[v].size(); ++i)
        if (!used[g[v][i]]) dfs1(g[v][i]);
    order.push_back(v);
}

void dfs2(int v) {
    used[v] = true;
    component.push_back(v);
    for (size_t i = 0; i < gr[v].size(); ++i)
        if (!used[gr[v][i]]) dfs2(gr[v][i]);
}

signed main() {
    int n;
    cin >> n;

    for (int i = 0; i < n; i++) {
        int a = 0, b = 0;
        cin >> a >> b;
        g[a].push_back(b);
        gr[b].push_back(a);
    }

    used.assign(n, false);
    for (int i = 0; i < n; ++i)
        if (!used[i]) dfs1(i);

    used.assign(n, false);
    for (int i = 0; i < n; ++i) {
        int v = order[n - 1 - i];
        if (!used[v]) {
            dfs2(v);
            for (int j = 0; j < component.size(); j++)
                cout << component[j] << " ";
            cout << '\n';
            component.clear();
        }
    }
}
```

### 3.5 Connected Component

```
void dfs(vector<vector<int>> &mass, vector<bool> &used, int
    vertex) {
    used[vertex] = true;
    for (int i = 0; i < mass[vertex].size(); i++)
        if (used[mass[vertex][i]] == false)
            dfs(mass, used, mass[vertex][i]);
}

signed main() {
    int n = 0, m = 0, second = 0, first = 0, result = 0;
    cin >> n >> m;
    vector<vector<int>> mass(n);
    vector<bool> used(n, false);

    for (int i = 0; i < m; i++) {
        cin >> first >> second;
        first--;
        second--;
        mass[first].push_back(second);
        mass[second].push_back(first);
    }

    for (int i = 0; i < n; i++) {
        if (!used[i]) {
            dfs(mass, used, i);
            result++;
        }
    }

    cout << result << '\n';
}
```

### 3.6 Cycles

```
int cycle_start = -1, cycle_end = 0;
vector<int> p;

bool dfs(vector<vector<int>> g, vector<bool> used, vector<
    int> color, int vertex) {
    color[vertex] = 1;
    for (int i = 0; i < g[vertex].size(); i++) {
        int to = g[vertex][i];
        if (color[to] == 0) {
            if (dfs(g, used, color, to)) {
                p[to] = vertex;
                return true;
            }
        }
        else if (color[to] == 1) {
            cycle_start = to;
            cycle_end = vertex;
            return true;
        }
    }
    color[vertex] = 2;
    return false;
}

signed main() {
    int n = 0, m = 0, second = 0, first = 0, req = 0;
    cin >> m >> n;
    vector<vector<int>> mass(n);
    vector<bool> used(n, false);
    vector<int> color(n, 0);
    vector<int> cycle;

    p.assign(n, -1);

    for (int i = 0; i < m; i++) {
        cin >> first >> second;
        first--;
        second--;
        mass[first].push_back(second);
    }

    for (int i = 0; i < n; i++)
        if (dfs(mass, used, color, i))
            break;

    if (cycle_start == -1)
        cout << "No" << endl;
    else {
        cout << "Yes" << endl;

        cycle.push_back(cycle_start);
        for (int v = cycle_end; v != cycle_start; v = p[v])
            cycle.push_back(v);
        cycle.push_back(cycle_start);

        reverse(cycle.begin(), cycle.end());

        for (int i = 0; i < cycle.size(); i++)
            cout << cycle[i] + 1 << " ";
        cout << endl;
    }
}
```

### 3.7 Eulerian Cycle Path

```
signed main() {
    int n = 0;
    cin >> n;
    vector<vector<int>> g(n, vector<int>(n));
    vector<int> deg(n);

    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            deg[i] += g[i][j];

    int first = 0;
    while (!deg[first]) ++first;
    int v1 = -1, v2 = -1;
    bool bad = false;
```

```

for (int i = 0; i < n; ++i)
    if (deg[i] & 1)
        if (v1 == -1) v1 = i;
        else if (v2 == -1) v2 = i;
        else bad = true;

if (v1 != -1) {
    ++g[v1][v2];
    ++g[v2][v1];
}

stack<int> st;
st.push(first);

vector<int> res;

while (!st.empty()) {
    int v = st.top();
    int i = 0;
    for (i = 0; i < n; ++i)
        if (g[v][i]) break;
    if (i == n) {
        res.push_back(v);
        st.pop();
    }
    else {
        --g[v][i];
        --g[i][v];
        st.push(i);
    }
}

if (v1 != -1) {
    for (size_t i = 0; i + 1 < res.size(); ++i) {
        if (res[i] == v1 && res[i + 1] == v2 || res[i] ==
            v2 && res[i + 1] == v1) {
            vector<int> res2;
            for (size_t j = i + 1; j < res.size(); ++j)
                res2.push_back(res[j]);
            for (size_t j = 1; j <= i; ++j) res2.push_back(
                res[j]);
            res = res2;
            break;
        }
    }
}

for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)
        if (g[i][j]) bad = true;

if (bad)
    cout << -1 << endl;
else
    for (size_t i = 0; i < res.size(); ++i)
        printf("%d ", res[i] + 1);
}

```

### 3.8 Dijkstra

```

signed main() {
    int inf = 1e18;
    int n = 0, m = 0;
    cin >> n >> m;
    vector<vector<pair<int, int>>> g(n);

    for (int i = 0; i < m; i++) {
        int to = 0, from = 0, len = 0;
        cin >> from >> to >> len;
        g[from - 1].push_back({ to - 1, len });
        g[to - 1].push_back({ from - 1, len });
    }

    vector<int> d(n, inf);
    vector<int> p(n);
    int from = 0, to = 0;
    d[0] = 0;
    priority_queue<pair<int, int>> q;
    q.push({ 0, 0 });

    while (!q.empty()) {
        int v = q.top().second;
        int cur_d = -q.top().first;
        q.pop();

```

```

        if (cur_d > d[v]) continue;
        for (int i = 0; i < g[v].size(); i++) {
            int to = g[v][i].first;
            int length = g[v][i].second;
            if (d[v] + length < d[to]) {
                d[to] = d[v] + length;
                p[to] = v;
                q.push({ -d[to], to });
            }
        }
    }

    if (d[n - 1] == inf) {
        cout << -1 << endl;
        return 0;
    }
    if (!d[n - 1]) {
        cout << 0 << endl;
        return 0;
    }
    vector<int> way;
    for (int v = n - 1; v != 0; v = p[v])
        way.push_back(v + 1);
    way.push_back(1);

    for (int i = way.size() - 1; i >= 0; i--)
        cout << way[i] << " ";
}

```

### 3.9 Prim

```

int main() {
    map<int, vector<pair<int, int>>> mass;
    vector<int> check;
    vector<int> result;
    vector<pair<int, int>> way;
    int n = 0, m = 0, third = 0, temp = 10e5, top = 10e5,
        sum = 0, count = 0, first = 0, second = 0, parent
        = 0, child = 0;
    cin >> n >> m;

    check.resize(n - 1);
    result.push_back(0);

    for (int i = 0; i < n - 1; i++)
        check[i] = i + 1;

    for (int i = 0; i < m; i++) {
        cin >> first >> second >> third;
        first--;
        second--;
        mass[first].push_back(make_pair(second, third));
        mass[second].push_back(make_pair(first, third));
    }

    while (!check.empty()) {
        for (int i = 0; i < result.size(); i++) {
            for (int j = 0; j < mass[result[i]].size(); j++)
                {
                    if (mass[result[i]][j].second < temp && find(
                        check.begin(), check.end(), mass[result[
                            i]][j].first) != check.end()) {
                        temp = mass[result[i]][j].second;
                        top = mass[result[i]][j].first;
                        parent = result[i];
                    }
                }
        }

        result.push_back(top);
        for (int k = 0; k < check.size(); k++) {
            if (check[k] == top) {
                count = k;
                break;
            }
        }

        check.erase(check.begin() + count);
        sum += temp;
        count = 0;
        temp = 10e5;
        top = 10e5;
    }
}

```

```
    cout << sum << endl;
}
```

### 3.10 Topological Sort

```
vector<bool> used;
vector<int> ans;
vector<vector<int>>> g;

void dfs(int v) {
    used[v] = true;
    for (size_t i = 0; i < g[v].size(); ++i) {
        int to = g[v][i];
        if (!used[to]) dfs(to);
    }
    ans.push_back(v);
}

void topological_sort(int n) {
    used.assign(n, false);
    ans.clear();
    for (int i = 0; i < n; ++i)
        if (!used[i]) dfs(i);
    reverse(ans.begin(), ans.end());
}

signed main() {
    int n; // число вершин
    cin >> n;
    used.assign(n, false);
    g.resize(n);
    topological_sort(n);
}
```

### 3.11 Dfs With Timestamps

```
vector<vector<int>>> adj;
vector<int> tIn, tOut, color;
int dfs_timer = 0;

void dfs(int v) {
    tIn[v] = dfs_timer++;
    color[v] = 1;
    for (int u : adj[v])
        if (color[u] == 0)
            dfs(u);
    color[v] = 2;
    tOut[v] = dfs_timer++;
}
```

### 3.12 Bellman Ford Algorithm

```
struct Edge {
    int a, b, cost;
};

int n, m, v; // v - starting vertex
vector<Edge> e;

/* Finds SSSP with negative edge weights.
 * Possible optimization: check if anything changed in a
 * relaxation step. If not - you can break early.
 * To find a negative cycle: perform one more relaxation
 * step. If anything changes - a negative cycle exists.
 */
void solve() {
    vector<int> d(n, oo);
    d[v] = 0;
    for (int i = 0; i < n - 1; ++i)
        for (int j = 0; j < m; ++j)
            if (d[e[j].a] < oo)
                d[e[j].b] = min(d[e[j].b], d[e[j].a] + e[j].cost);
}
```

### 3.13 Lowest Common Ancestor

```
int n, l; // l == logN (usually about ~20)
vector<vector<int>>> adj;

int timer;
vector<int> tin, tout;
vector<vector<int>>> up;

void dfs(int v, int p) {
    tin[v] = ++timer;
    up[v][0] = p;
    // wUp[v][0] = weight[v][u]; // <- path weight sum to 2^i-th ancestor
    for (int i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];
    // wUp[v][i] = wUp[v][i-1] + wUp[up[v][i-1]][i-1];

    for (int u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }

    tout[v] = ++timer;
}

bool isAncestor(int u, int v) {
    return tin[u] <= tin[v] && tout[v] <= tout[u];
}

int lca(int u, int v) {
    if (isAncestor(u, v))
        return u;
    if (isAncestor(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!isAncestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}

void preprocess(int root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<int>(l + 1));
    dfs(root, root);
}
```

### 3.14 Bipartite Graph

```
class BipartiteGraph {
private:
    vector<int> _left, _right;
    vector<vector<int>>> _adjList;
    vector<int> _matchR, _matchL;
    vector<bool> _used;

    bool _kuhn(int v) {
        if (_used[v]) return false;
        _used[v] = true;
        FOR(i, 0, (int) _adjList[v].size()) {
            int to = _adjList[v][i] - _left.size();
            if (_matchR[to] == -1 || !_kuhn(_matchR[to])) {
                _matchR[to] = v;
                _matchL[v] = to;
                return true;
            }
        }
        return false;
    }

    void _addReverseEdges() {
        FOR(i, 0, (int) _right.size()) {
            if (_matchR[i] != -1) {
                _adjList[_left.size() + i].pb(_matchR[i]);
            }
        }
    }

    void _dfs(int p) {
        if (_used[p]) return;
    }
}
```



```

        _used[p] = true;
        for (auto x : _adjList[p]) {
            _dfs(x);
        }
    }
    vector<pii> _buildMM() {
        vector<pair<int, int> > res;
        FOR(i, 0, (int) _right.size()) {
            if (_matchR[i] != -1) {
                res.push_back(make_pair(_matchR[i], i));
            }
        }
        return res;
    }
public:
    void addLeft(int x) {
        _left.pb(x);
        _adjList.pb({});
        _matchL.pb(-1);
        _used.pb(false);
    }
    void addRight(int x) {
        _right.pb(x);
        _adjList.pb({});
        _matchR.pb(-1);
        _used.pb(false);
    }
    void addForwardEdge(int l, int r) {
        _adjList[l].pb(r + _left.size());
    }
    void addMatchEdge(int l, int r) {
        if (l != -1) _matchL[l] = r;
        if (r != -1) _matchR[r] = l;
    }
    // Maximum Matching
    vector<pii> mm() {
        _matchR = vector<int>(_right.size(), -1);
        _matchL = vector<int>(_left.size(), -1);
        // ^ these two can be deleted if performing MM on
        // already partially matched graph
        _used = vector<bool>(_left.size() + _right.size(),
            false);

        bool path_found;
        do {
            fill(_used.begin(), _used.end(), false);
            path_found = false;
            FOR(i, 0, (int) _left.size()) {
                if (_matchL[i] < 0 && !_used[i]) {
                    path_found |= _kuhn(i);
                }
            }
        } while (path_found);

        return _buildMM();
    }

    // Minimum Edge Cover
    // Algo: Find MM, add unmatched vertices greedily.
    vector<pii> mec() {
        auto ans = mm();
        FOR(i, 0, (int) _left.size()) {
            if (_matchL[i] != -1) {
                for (auto x : _adjList[i]) {
                    int ridx = x - _left.size();
                    if (_matchR[ridx] == -1) {
                        ans.pb({ i, ridx });
                        _matchR[ridx] = i;
                    }
                }
            }
        }
        FOR(i, 0, (int) _left.size()) {
            if (_matchL[i] == -1 && (int) _adjList[i].size()
                > 0) {
                int ridx = _adjList[i][0] - _left.size();
                _matchL[i] = ridx;
                ans.pb({ i, ridx });
            }
        }
        return ans;
    }

    // Minimum Vertex Cover

```

```

    // Algo: Find MM. Run DFS from unmatched vertices from
    // the left part.
    // MVC is composed of unvisited LEFT and visited RIGHT
    // vertices.
    pair<vector<int>, vector<int>> mvc(bool runMM = true) {
        if (runMM) mm();
        _addReverseEdges();
        fill(_used.begin(), _used.end(), false);
        FOR(i, 0, (int) _left.size()) {
            if (_matchL[i] == -1) {
                _dfs(i);
            }
        }
        vector<int> left, right;
        FOR(i, 0, (int) _left.size()) {
            if (!_used[i]) left.pb(i);
        }
        FOR(i, 0, (int) _right.size()) {
            if (_used[i + (int) _left.size()]) right.pb(i);
        }
        return { left, right };
    }

    // Maximal Independent Vertex Set
    // Algo: Find complement of MVC.
    pair<vector<int>, vector<int>> mivs(bool runMM = true) {
        auto m = mvc(runMM);
        vector<bool> containsL(_left.size(), false),
            containsR(_right.size(), false);
        for (auto x : m.first) containsL[x] = true;
        for (auto x : m.second) containsR[x] = true;
        vector<int> left, right;
        FOR(i, 0, (int) _left.size()) {
            if (!containsL[i]) left.pb(i);
        }
        FOR(i, 0, (int) _right.size()) {
            if (!containsR[i]) right.pb(i);
        }
        return { left, right };
    }
};

```

### 3.15 Max Flow With Dinic

```

struct Edge {
    int f, c;
    int to;
    pii revIdx;
    int dir;
    int idx;
};

int n, m;
vector<Edge> adjList[MAX_N];
int level[MAX_N];

void addEdge(int a, int b, int c, int i, int dir) {
    int idx = adjList[a].size();
    int revIdx = adjList[b].size();
    adjList[a].pb({ 0, c, b, {b, revIdx}, dir, i });
    adjList[b].pb({ 0, 0, a, {a, idx}, dir, i });
}

bool bfs(int s, int t) {
    FOR(i, 0, n) level[i] = -1;
    level[s] = 0;
    queue<int> Q;
    Q.push(s);
    while (!Q.empty()) {
        auto t = Q.front(); Q.pop();
        for (auto x : adjList[t]) {
            if (level[x.to] < 0 && x.f < x.c) {
                level[x.to] = level[t] + 1;
                Q.push(x.to);
            }
        }
    }
    return level[t] >= 0;
}

int send(int u, int f, int t, vector<int> &edgeIdx) {
    if (u == t) return f;
    for (; edgeIdx[u] < adjList[u].size(); edgeIdx[u]++) {

```

```

    auto &e = adjList[u][edgeIdx[u]];
    if (level[e.to] == level[u] + 1 && e.f < e.c) {
        int curr_flow = min(f, e.c - e.f);
        int next_flow = send(e.to, curr_flow, t, edgeIdx)
        ;
        if (next_flow > 0) {
            e.f += next_flow;
            adjList[e.revIdx.first][e.revIdx.second].f -=
                next_flow;
            return next_flow;
        }
    }
    return 0;
}

int maxFlow(int s, int t) {
    int f = 0;
    while (bfs(s, t)) {
        vector<int> edgeIdx(n, 0);
        while (int extra = send(s, oo, t, edgeIdx)) {
            f += extra;
        }
    }
    return f;
}

void init() {
    cin >> n >> m;
    FOR(i, 0, m) {
        int a, b, c;
        cin >> a >> b >> c;
        a--; b--;
        addEdge(a, b, c, i, 1);
        addEdge(b, a, c, i, -1);
    }
}

```

## 3.16 Floyd's Algorithm

```

//floyd
for (int k = 0; k < n; k++) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (d[i][k] == inf || d[k][j] == inf) continue;
            d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
        }
    }
}

```

## 3.17 Kuhn

```

int n, k;
vector < vector<int> > g;
vector<int> mt;
vector<bool> used;
bool kuhn(int v) {
    if (used[v]) return false;
    used[v] = true;
    for (auto i : g[v]) {
        int to = i;
        if (mt[to] == -1 || kuhn(mt[to])) {
            mt[to] = v;
            return true;
        }
    }
    return false;
}

signed main() {
    cin >> n >> m >> k;
    g.resize(n);
    for (int i = 0; i < k; i++) {
        int temp1, temp2;
        cin >> temp1 >> temp2;
        g[temp1 - 1].push_back(temp2 - 1);
    }
    mt.assign(m, -1);
    for (int v = 0; v < n; ++v) {

```

```

        used.assign(n, false);
        kuhn(v);
    }
}

```

# 4 Geometry

## 4.1 Graham

```

struct point {
    int x, y;
};
point operator-(point a, point b) {
    return {
        a.x - b.x,
        a.y - b.y
    };
};
bool operator==(point a, point b) {
    return (a.x == b.x) && (a.y == b.y);
}
int operator^(point a, point b) {
    return a.x * b.y - a.y * b.x;
};

bool comp(point &a, point &b) {
    return ((a ^ b) > 0) || ((a ^ b) == 0 && a.x * a.x + a.y
        * a.y > b.x * b.x + b.y * b.y);
};

vector<point> graham(vector<point> points) {
    point p0 = points[0];
    for (point p : points)
        if (p.y < p0.y || (p.y == p0.y && p.x > p0.x)) p0 =
            p;

    for (point &p : points) {
        p.x -= p0.x;
        p.y -= p0.y;
    }

    sort(all(points), comp);

    vector<point> hull;
    for (point p : points) {
        while (hull.size() >= 2 && ((p - hull.back()) ^ (
            hull[hull.size() - 2] - hull.back())) <= 0)
            hull.pop_back();
        hull.push_back(p);
    }

    for (point &p : hull) {
        p.x += p0.x;
        p.y += p0.y;
    }

    return hull;
}

```

## 4.2 2d Vector

```

template <typename T>
struct vec {
    T x, y;
    vec() : x(0), y(0) { }
    vec(T _x, T _y) : x(_x), y(_y) { }

    vec operator+(const vec &b) {
        return vec<T>(x + b.x, y + b.y);
    }
    vec operator-(const vec &b) {
        return vec<T>(x - b.x, y - b.y);
    }
    vec operator*(T c) {
        return vec(x * c, y * c);
    }
}

```

```

T operator*(const vec &b) {
    return x * b.x + y * b.y;
}
T operator^(const vec &b) {
    return x * b.y - y * b.x;
}
bool operator<(const vec &other) const {
    if (x == other.x) return y < other.y;
    return x < other.x;
}
bool operator==(const vec &other) const {
    return x == other.x && y == other.y;
}
bool operator!=(const vec &other) const {
    return !(*this == other);
}
friend ostream &operator<<(ostream &out, const vec &v) {
    return out << "(" << v.x << ", " << v.y << ")";
}
friend istream &operator>>(istream &in, vec<T> &v) {
    return in >> v.x >> v.y;
}
T norm() { // squared length
    return (*this) * (*this);
}
ld len() {
    return sqrt(norm());
}
ld angle(const vec &other) { // angle between this and
    // other vector
    return acosl((*this) * other / len() / other.len());
}
vec perp() {
    return vec(-y, x);
}
};

```

### 4.3 Line

```

template <typename T>
struct line { // expressed as two vectors
    vec<T> start, dir;
    line() {}
    line(vec<T> a, vec<T> b) : start(a), dir(b - a) {}

    vec<ld> intersect(line l) {
        ld t = ld((l.start - start) ^ l.dir) / (dir ^ l.dir);
        // For segment-segment intersection this should be
        // in range [0, 1]
        vec<ld> res(start.x, start.y);
        vec<ld> dirld(dir.x, dir.y);
        return res + dirld * t;
    }
};

```

### 4.4 Circle Line Intersection

```

// ax + by + c = 0, radius is at (0, 0)
double r, a, b, c;

// If the center is not at (0, 0), fix the constant c to
// translate everything so that center is at (0, 0)
double x0 = -a * c / (a * a + b * b), y0 = -b * c / (a * a
    + b * b);
if (c*c > r*r*(a*a+b*b)+eps)
    puts ("no points");
else if (abs(c*c - r*r*(a*a+b*b)) < eps) {
    puts ("1 point");
    cout << x0 << ' ' << y0 << '\n';
}
else {
    double d = r*r - c*c/(a*a+b*b);
    double mult = sqrt(d / (a*a+b*b));
    double ax, ay, bx, by;
    ax = x0 + b * mult;
    bx = x0 - b * mult;
    ay = y0 - a * mult;
    by = y0 + a * mult;
    puts ("2 points");
}

```

```

cout << ax << ' ' << ay << '\n' << bx << ' ' << by << '\n';
}

```

### 4.5 7zip Cord

```

ll dfs(vector<vector<int>> &Map, int i, int j, vector<
    vector<bool>> &used, vector<int> &Xvalue, vector<int>
    &Yvalue) {
    used[i][j] = true;
    bool flag = false;
    ll sum = Xvalue[i] * Yvalue[j];
    int a[] = { 0, -1, 1, 0 };
    int b[] = { -1, 0, 0, 1 };
    for (int h = 0; h < 4; h++)
        if (Map[i + a[h]][j + b[h]] == 0 && !used[i + a[h]][
            j + b[h]]) {
            flag = true;
            sum += dfs(Map, i + a[h], j + b[h], used, Xvalue,
                Yvalue);
        }
    if (!flag) {
        return Xvalue[i] * Yvalue[j];
    }
    return sum;
}

int main() {
    int w, h, n;
    cin >> w >> h >> n;
    set<int> x, y;
    unordered_map<int, int> X, Y;
    vector<vector<int>> lines;
    vector<int> Xvalue, Yvalue;
    x.insert(0);
    y.insert(0);
    x.insert(w);
    y.insert(h);
    for (int i = 0; i < n; i++) {
        int x1, y1, x2, y2;
        cin >> x1 >> y1 >> x2 >> y2;
        if (x1 < 0)
            x1 = 0;
        if (x1 > w)
            x1 = w;
        if (y1 < 0)
            y1 = 0;
        if (y1 > h)
            y1 = h;
        if (x2 < 0)
            x2 = 0;
        if (x2 > w)
            x2 = w;
        if (y2 < 0)
            y2 = 0;
        if (y2 > h)
            y2 = h;
        lines.push_back({ x1, y1, x2, y2 });
        x.insert(x1);
        x.insert(x2);
        y.insert(y1);
        y.insert(y2);
    }
    int index = 0;
    for (auto _x : x) {
        X[_x] = index;
        index += 2;
    }
    index = 0;
    for (auto _y : y) {
        Y[_y] = index;
        index += 2;
    }
    int prev = 0;
    for (auto _x = ++x.begin(); _x != x.end(); _x++) {
        Xvalue.push_back(0);
        Xvalue.push_back(*_x - prev);
        prev = *_x;
    }
    Xvalue.push_back(0);
    prev = 0;
    for (auto _y = ++y.begin(); _y != y.end(); _y++) {
        Yvalue.push_back(0);
        Yvalue.push_back(*_y - prev);
    }
}

```

```

    prev = *_y;
}
Yvalue.push_back(0);
int Xs = Xvalue.size();
int Ys = Yvalue.size();
vector<vector<int>> Map(Xs, vector<int>(Ys, 0));
for (int i = 0; i < Xs; i++) {
    Map[i][0] = 1;
    Map[i][Ys - 1] = 1;
}
for (int i = 0; i < Ys; i++) {
    Map[0][i] = 1;
    Map[Xs - 1][i] = 1;
}
for (int i = 0; i < n; i++) {
    if (lines[i][0] == lines[i][2]) {
        int x = X[lines[i][0]];
        int y1 = Y[lines[i][1]];
        int y2 = Y[lines[i][3]];
        if (y1 > y2)
            y1 ^= y2 ^= y1 ^= y2;
        for (int i = y1; i <= y2; i++)
            Map[x][i] = 1;
    }
    else {
        int y = Y[lines[i][1]];
        int x1 = X[lines[i][0]];
        int x2 = X[lines[i][2]];
        if (x1 > x2)
            x1 ^= x2 ^= x1 ^= x2;
        for (int i = x1; i <= x2; i++)
            Map[i][y] = 1;
    }
}
vector<ll> s;
vector<vector<bool>> used(Xs, vector<bool>(Ys, false));
for (int i = 1; i < Xs - 1; i++) {
    for (int j = 1; j < Ys - 1; j++) {
        if (Map[i][j] == 0 && !used[i][j])
            s.push_back(dfs(Map, i, j, used, Xvalue,
                           Yvalue));
    }
}
sort(s.rbegin(), s.rend());
for (auto _s : s)
    cout << _s << "\n";
}

```

## 4.6 Formulae

### Triangles.

*Radius of circumscribed circle:*

$$R = \frac{abc}{4S}.$$

*Radius of inscribed circle:*

$$r = \frac{S}{p}.$$

*Side via medians:*

$$a = \frac{2}{3}\sqrt{2(m_b^2 + m_c^2) - m_a^2}.$$

*Median via sides:*

$$m_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}.$$

*Bisector via sides:*

$$l_a = \frac{2\sqrt{bcp(p-a)}}{b+c}.$$

*Bisector via two sides and angle:*

$$l_a = \frac{2bc \cos \frac{\alpha}{2}}{b+c}.$$

### Right triangles.

Let  $a$ ,  $b$  and  $c$  - cathets and hypotenuse,  $h$  - height to hypotenuse, dividing  $c$  to  $c_a$  and  $c_b$ .

Then

$$h^2 = c_a \cdot c_b,$$

$$a^2 = c_a \cdot c,$$

$$b^2 = c_b \cdot c.$$

### Quadrangles.

*Sides of circumscribed quadrangle:*

$$a + c = b + d.$$

*Square of circumscribed quadrangle:*

$$S = \frac{Pr}{2} = pr.$$

*Angles of inscribed quadrangle:*

$$\alpha + \gamma = \beta + \delta = 180^\circ.$$

*Square of inscribed quadrangle:*

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}.$$

### Circles.

*Intersection of circle and line:*

$$\begin{cases} (x - x_0)^2 + (y - y_0)^2 = R^2 \\ y = ax + b \end{cases}$$

$$(x - x_0)^2 + (ax + b - y_0)^2 = R^2$$

$$(1 + a^2)x^2 + (2a(b - y_0) - 2x_0)x + (x_0^2 + (b - y_0)^2 - R^2) = 0$$

Intersection points are solution of equation. If discriminant  $D < 0$  then there are no intersection points. If discriminant  $D = 0$  then there is one intersection point. If discriminant  $D > 0$  then there are two intersection points.

*Intersection of circle and circle:*

$$\begin{cases} (x - x_0)^2 + (y - y_0)^2 = R_0^2 \\ (x - x_1)^2 + (y - y_1)^2 = R_1^2 \end{cases}$$

$$2(x_0 - x_1)x + 2(y_0 - y_1)y = (R_1^2 - R_0^2) + (x_0^2 - x_1^2) + (y_0^2 - y_1^2)$$

$$y = \frac{1}{2} \frac{(R_1^2 - R_0^2) + (x_0^2 - x_1^2) + (y_0^2 - y_1^2)}{y_0 - y_1} - \frac{x_0 - x_1}{y_0 - y_1} x$$

Task comes to intersection of circle and line.

## 5 Algebra

### 5.1 Combinations

```

int c(int n, int k) {
    int result = 1;

    for (int i = 1; i <= k; i++) {
        result *= n - i + 1;
        result /= i;
    }

    return result;
}

const int N = 20;
vector<vector<int>> C(N + 1, vector<int>(N + 1, 1));

for (int i = 1; i < N + 1; i++)
    for (int j = 1; j < N + 1; j++)
        C[i][j] = C[i - 1][j] + C[i][j - 1];

```

### 5.2 Eratosthenes

```

template<class T>
class prime {
private:
    std::vector<unsigned>pr;
    std::vector<T>lp;
public:
    prime() {};

```

```

prime(unsigned limit) {
    lp.resize(++limit, 0);
    pr.clear();
    pr.push_back(1);
    for (unsigned i = 2; i < limit; ++i) {
        register unsigned max_index = lp[i];
        if (max_index == 0) {
            max_index = pr.size();
            pr.push_back(i);
        }
        register unsigned d;
        for (unsigned j = 1; j <= max_index && (d = i * pr[j]) < limit; ++j)
            lp[d] = j;
    }
}

bool is_prime(unsigned number) {
    return number < lp.size() && !lp[number];
}

unsigned sequence_number(unsigned prime_number) {
    if (!is_prime(prime_number))
        return 0;
    return std::lower_bound(pr.begin(), pr.end(),
        prime_number) - pr.begin();
}

unsigned return_prime(unsigned sequence_number) {
    if (sequence_number && sequence_number < pr.size())
        return pr[sequence_number];
    return 0;
}

unsigned least_divisor(unsigned number) {
    if (number >= lp.size())
        return 0;
    if (is_prime(number))
        return number;
    return pr[lp[number]];
}

unsigned limit() {
    return lp.size() - 1;
}

std::vector<unsigned> factorize(unsigned number) {
    std::vector<unsigned> v;
    if (number < lp.size()) {
        while (!is_prime(number)) {
            v.push_back(pr[lp[number]]);
            number /= pr[lp[number]];
        }
        v.push_back(number);
    }
    return v;
}
};
//

```

```

signed main()
{
    int t = 1;
    cin >> t;
    int n = 1e7;
    vector<int> p(1e7 + 1, 0);
    vector<int> mass;
    for (int i = 2; i * i <= n; i++) {
        if (p[i] == 1) continue;
        if (i * i <= n) {
            for (int j = i * i; j <= n; j += i) {
                p[j] = 1;
            }
        }
    }
    for (int i = 2; i <= 1e7; ++i) {
        if (p[i] == 0) { // если 0 - число простое
            mass.push_back(i);
        }
    }
    while (t--)
    {
        int l = 0, r = 0;
        cin >> l >> r;
        int lx = lower_bound(mass.begin(), mass.end(), l) -
            mass.begin();
        int rx = lower_bound(mass.begin(), mass.end(), r +
            1) - mass.begin();
        cout << rx - lx << endl;
    }
}
//

```

```

const int SQRT_MAXN = 10000; //
// корни из максимального значения N
const int S = 1e7 + 1;
bool nprime[SQRT_MAXN], bl[S];
int primes[SQRT_MAXN], cnt;

signed main() {

    int t = 1;
    cin >> t;
    int n = 1e7 + 1;
    int nsqrt = (int)sqrt(n + .0);
    for (int i = 2; i <= nsqrt; ++i)
        if (!nprime[i]) {
            primes[cnt++] = i;
            if (i * 111 * i <= nsqrt)
                for (int j = i * i; j <= nsqrt; j += i)
                    nprime[j] = true;
        }

    int result = 0;
    vector<int> mass;
    for (int k = 0, maxk = n / S; k <= maxk; ++k) {
        memset(bl, 0, sizeof bl);
        int start = k * S;
        for (int i = 0; i < cnt; ++i) {
            int start_idx = (start + primes[i] - 1) / primes[i];
            int j = max(start_idx, (long long)2) * primes[i] - start;
            for (; j < S; j += primes[i])
                bl[j] = true;
        }
        if (k == 0)
            bl[0] = bl[1] = true;
        for (int i = 0; i < S && start + i <= n; ++i)
            if (!bl[i])
                mass.push_back(i);
    }
    while (t--) {
        int l = 0, r = 0;
        cin >> l >> r;
        int lx = lower_bound(mass.begin(), mass.end(), l) -
            mass.begin();
        int rx = lower_bound(mass.begin(), mass.end(), r +
            1) - mass.begin();
        cout << rx - lx << endl;
    }
}

```

## 5.3 Fft

```

const int fft_mod = 7340033; // 7 * 2^20 + 1
const int fft_root = 5; // 5 ^ (2^20) == 1 mod 7340033
const int fft_root_1 = 4404020; // 5 * 4404020 == 1 mod 7340033
const int fft_pw = 1 << 20; // 2 ^ 20

vector<int> fft(vector<int> a, bool invert = 0) {
    int n = a.size();

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if (i < j) swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <= 1) {
        int root_len = invert ? fft_root_1 : fft_root;

        for (int i = len; i < fft_pw; i <= 1)
            root_len = (root_len * root_len) % fft_mod;

        for (int i = 0; i < n; i += len) {
            int root = 1;
            for (int j = 0; j < len / 2; j++) {
                int u = a[i + j], v = a[i + j + len / 2] *
                    root % fft_mod;
                a[i + j] = (u + v) % fft_mod;
                a[i + j + len / 2] = (u - v + fft_mod) %
                    fft_mod;
                root = (root * root_len) % fft_mod;
            }
        }
    }
}

```

```

    }
}

if (invert) {
    int _n = 1;
    for (int i = 1; i <= fft_mod - 2; i++) _n = (_n * n)
        % fft_mod;
    for (int i = 0; i < n; i++) a[i] = (a[i] * _n) %
        fft_mod;
}

signed fast_fourier_transform() {
    int n, m, s;
    cin >> n >> m;
    vector<int> a(n), b(m);

    for (int i = 0; i < n; i++) cin >> a[i];
    for (int i = 0; i < m; i++) cin >> b[i];
    s = 1; while (s < n + m - 1) s <= 1;
    a.resize(s); b.resize(s);

    vector<int> fa = fft(a), fb = fft(b), fc(fa.size());
    for (int i = 0; i < fa.size(); i++) fc[i] = fa[i] * fb[i];

    vector<int> c = fft(fc, 1);
    for (int i = 0; i < s; i++) cout << a[i] << ' '; cout <<
        '\n';
    for (int i = 0; i < s; i++) cout << b[i] << ' '; cout <<
        '\n';
    for (int i = 0; i < s; i++) cout << c[i] << ' '; cout <<
        '\n';
}

```

## 5.4 Fibonacci

```

signed fibonacci() {
    int n = 0, m = 0;
    cin >> n >> m;
    vector<vector<int>> mass(2, vector<int>(2));

    mass[0][0] = 0;
    mass[0][1] = 1;
    mass[1][0] = 1;
    mass[1][1] = 1;

    if (n == 1) {
        cout << 1 << endl;
        return 0;
    }
    if (n == 2) {
        cout << 1 << endl;
        return 0;
    }
    if (n == 3) {
        cout << 2 << endl;
        return 0;
    }

    vector<vector<int>> powed = fast_pow(mass, n - 3, m);

    int result = 0;
    for (int i = 0; i < 2; i++) {
        for (int j = 0; j < 2; j++)
            result += powed[i][j];
    }

    cout << result % m << endl;
}

```

## 5.5 Gcd

```

// simple gcd
int gcd(int a, int b) {
    while (b) {
        a %= b;
        swap(a, b);
    }
    return a;
}

```

```

}

// euclidean algorithm
int gcd(int a, int b, int &x, int &y) {
    if (a == 0) {
        x = 0; y = 1;
        return b;
    }

    int x1, y1;
    int d = gcd(b % a, a, x1, y1);

    x = y1 - (b / a) * x1;
    y = x1;

    return d;
}

```

## 5.6 Extended Euclidean Algorithm

```

// ax + by = gcd(a,b)
void solve_eq(int a, int b, int &x, int &y, int &g) {
    if (b == 0) {
        x = 1;
        y = 0;
        g = a;
        return;
    }

    int xx, yy;
    solve_eq(b, a % b, xx, yy, g);

    x = yy;
    y = xx - yy * (a / b);
}

// ax + by = c
bool solve_eq(int a, int b, int c, int &x, int &y, int &g)
{
    solve_eq(a, b, x, y, g);

    if (c % g != 0)
        return false;

    x *= c / g; y *= c / g;

    return true;
}

// finds a solution (x, y) so that x >= 0 and x is minimal
bool solve_eq_non_neg_x(int a, int b, int c, int &x, int &y,
    int &g) {
    if (!solve_eq(a, b, c, x, y, g))
        return false;

    int k = x * g / b;
    x = x - k * b / g;
    y = y + k * a / g;

    if (x < 0) {
        x += b / g;
        y -= a / g;
    }

    return true;
}

```

## 5.7 Euler Totient Function

```

// number of numbers x < n so that gcd(x, n) = 1
int phi(int n) {
    if (n == 1)
        return 1;

    // f = vector<pair<prime, count>>
    auto f = factorize(n);

    int res = n;
}

```

```

for(auto p : f) {
    res = res - res/p.first;
}

return res;
}

```

## 5.8 Factorization

```

vector<int> factorization(int n) {
    vector<int> result;
    for (int i = 2; i * i <= n; i++)
        while (n % i == 0) {
            result.push_back(i);
            n /= i;
        }
    if (n != 1)
        result.push_back(n);
    return result;
}

```

## 5.9 Binary Mult Pow

```

int binmult(int a, int b) {
    int res = 0;
    while (b) {
        if (b & 1)
            res += a;
        a *= 2;
        b >>= 1;
    }
    return res;
}

```

```

int binpow(int a, int n) {
    int res = 1;
    while (n) {
        if (n & 1)
            res *= a;
        a *= a;
        n >>= 1;
    }
    return res;
}

```

## 5.10 Matrices

```

vector<vector<int>> matrix_production(vector<vector<int>> &
a, vector<vector<int>> &b, int mod=0) {
    vector<vector<int>> result(a.size(), vector<int>(b[0].
size()));

    for (int i = 0; i < a.size(); i++) {
        for (int j = 0; j < b[0].size(); j++) {
            for (int k = 0; k < b.size(); k++) {
                if (mod) result[i][j] = (result + a[i][k] * b[
k][j] % mod) % mod;
                else result[i][j] += a[i][k] * b[k][j];
            }
        }
    }

    return result;
}

// recursive pow
vector<vector<int>> fast_pow(vector<vector<int>> &a, int n,
int mod) {
    if (n == 0) {
        vector<vector<int>> temp(a.size(), vector<int>(a[0].
size()));
        for (int i = 0; i < a.size(); i++) {
            temp[i][i] = 1;
        }
        return temp;
    }
}

```

```

if (n % 2 == 1) {
    vector<vector<int>> temp = fast_pow(a, n - 1, mod);
    return matrix_production(temp, a, mod);
}
else {
    vector<vector<int>> b = fast_pow(a, n / 2, mod);
    return matrix_production(b, b, mod);
}
}

// iterative pow
vector<vector<int>> fast_pow(vector<vector<int>> &a, int n,
int mod = 0) {
    vector<vector<int>> res(n, vector<int>(n, 0));
    for (int i = 0; i < n; i++) res[i] = 1;

    while (n) {
        if (n & 1) res = matrix_production(res, a, mod);
        a = matrix_production(a, a, mod);
        n >>= 1;
    }

    return res;
}

```

## 5.11 Catalan

```

//Catalan(n) = (1 / (n + 1)) * C[2n][n]
//Catalan(n) = Sum[i=0...n-1](Catalan(i)*Catalan(n-1-i))

```

## 5.12 Formulae

### Combinations.

$$C_n^k = \frac{n!}{(n-k)!k!}$$

$$C_n^0 + C_n^1 + \dots + C_n^n = 2^n$$

$$C_{n+1}^{k+1} = C_n^{k+1} + C_n^k$$

$$C_n^k = \frac{n}{k} C_{n-1}^{k-1}$$

### Striling approximation.

$$n! \approx \sqrt{2\pi n} \frac{n^n}{e^n}$$

### Euler's theorem.

$$a^{\phi(m)} \equiv 1 \pmod{m}, \gcd(a, m) = 1$$

### Ferma's little theorem.

$$a^{p-1} \equiv 1 \pmod{p}, \gcd(a, p) = 1, p - \text{prime}.$$

### Catalan number.

$$C_0 = 0, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}$$

$$C_n = \frac{2(2n-1)}{n+1} C_{n-1}$$

## 6 Strings

### 6.1 Manaker

```

signed manaker() {
    string s;
    cin >> s;
    int n = s.length();
}

```

```

vector<int> d1(n);
int l = 0, r = -1;

for (int i = 0; i < n; ++i) {
    int k = i > r ? 1 : min(d1[l + r - i], r - i + 1);
    while (i + k < n && i - k >= 0 && s[i + k] == s[i - k])
        ++k;
    d1[i] = k;
    if (i + k - 1 > r)
        l = i - k + 1, r = i + k - 1;
}

vector<int> d2(n);
l = 0, r = -1;

for (int i = 0; i < n; ++i) {
    int k = i > r ? 0 : min(d2[l + r - i + 1], r - i + 1);
    while (i + k < n && i - k - 1 >= 0 && s[i + k] == s[i - k - 1])
        ++k;
    d2[i] = k;
    if (i + k - 1 > r)
        l = i - k, r = i + k - 1;
}

int sum = 0;
for (int i = 0; i < n; ++i) {
    sum += ((d1[i] > 1) ? d1[i] - 1 : 0) + d2[i];
}
cout << sum << '\n';
}

```

## 6.2 Suffixarray

```

void count_sort(vector<int> &p, vector<int> &c) {
    int n = p.size();
    vector<int> cnt(n), p_new(n), pos(n);

    for (auto x : c) cnt[x]++;

    pos[0] = 0;
    for (int i = 1; i < n; ++i)
        pos[i] = pos[i - 1] + cnt[i - 1];

    for (auto x : p) {
        int i = c[x];
        p_new[pos[i]] = x;
        pos[i]++;
    }

    p = p_new;
}

signed suffix_array() {
    string str;
    cin >> str;
    str += "&";
    int len = str.length();
    vector<int> p(len), c(len);
    vector<pair<char, int>> a(len);

    for (int i = 0; i < len; ++i)
        a[i] = { str[i], i };

    sort(a.begin(), a.end());

    for (int i = 0; i < len; ++i)
        p[i] = a[i].second;

    c[p[0]] = 0;
    for (int i = 1; i < len; ++i)
        if (a[i].first == a[i - 1].first)
            c[p[i]] = c[p[i - 1]];
        else
            c[p[i]] = c[p[i - 1]] + 1;

    int k = 0;
    while ((1 << k) < len) {
        for (int i = 0; i < len; ++i)
            p[i] = (p[i] - (1 << k) + len) % len;

        count_sort(p, c);
    }
}

```

```

vector<int> c_new(len);
c_new[p[0]] = 0;

for (int i = 1; i < len; ++i) {
    pair<int, int> prev = { c[p[i - 1]], c[(p[i - 1] + (1 << k)) % len] };
    pair<int, int> now = { c[p[i]], c[(p[i] + (1 << k)) % len] };
    if (now == prev)
        c_new[p[i]] = c_new[p[i - 1]];
    else
        c_new[p[i]] = c_new[p[i - 1]] + 1;
}

c = c_new;
k++;
}

for (int i = 0; i < len; ++i)
    cout << p[i] << " ";
}

```

## 6.3 Z Function

```

signed z_func() {
    string s = "";
    cin >> s;
    vector<int> z(s.size());

    for (int i = 1, l = 0, r = 0; i < s.size(); ++i) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - l]);

        while (z[i] + i < s.size() && s[z[i]] == s[z[i] + i])
            z[i]++;

        if (z[i] + i - 1 > r) {
            l = i;
            r = z[i] + i - 1;
        }
    }
}

```

# 7 Dynamic Programming

## 7.1 Backpack

```

vector<int> result;

void getResult(int k, int s, vector<vector<int>> &dp, vector<int> &mass) {
    if (dp[k][s] == 0)
        return;
    if (dp[k][s - 1] == dp[k][s])
        getResult(k, s - 1, dp, mass);
    else {
        getResult(k - mass[s - 1], s - 1, dp, mass);
        result.push_back(s);
    }
}

signed backpack() {
    int n, w;
    cin >> n >> w;
    vector<int> m(n + 1), c(n + 1);

    int min_w = 0;
    for (int i = 0; i < n; ++i) {
        cin >> m[i] >> c[i];
        min_w += m[i];
    }
    min_w = min(min_w, w); // чтобынебылом1

    vector<vector<int>> dp(min_w + 1, vector<int>(n + 1, 0));

    for (int k = 1; k < n + 1; k++) {
        for (int i = 0; i < min_w + 1; i++) {

```



```

// циклдоcount <= (min(kolvo_predm[k возможно(
// сделать- 1)], min_w / c[i]) + 1) - согранич.
// количествомпредметов
for (int count = 0; count <= 1; count++) {
// еслинеограниченноколичествопредметовза0(nW
// )
// dp[i][k] = dp[i][k - 1]
// if (m[k] <= i) {
// dp[i][k] = max(dp[i][k], dp[i][j - m[k]] +
// c[k - 1]);
// }
// if (i - m[k - 1] * count >= 0) {
// dp[i][k] = max(dp[i][k], dp[i - m[k - 1] *
// count][k - 1] + c[k - 1] * count);
// }
// }
// }
}
}

cout << dp[min_w][n] << endl;
getResult(min_w, n, dp, m);
for (int i = 0; i < result.size(); i++) {
if (result[i] != 0) {
cout << result[i] << ' ';
}
}
}
}

```

## 7.2 Coins

```

signed coins() {
int sum = 0, n = 0;
cin >> sum >> n;
vector<int> coin(n), res(sum + 1, 1e9);

res[0] = 0;
for (int i = 0; i < n; i++) cin >> coin[i];

for (int i = 1; i <= sum; i++) {
for (int j = 0; j < n; j++) {
if (i - coin[j] >= 0)
res[i] = min(res[i], res[i - coin[j]] + 1);
}
}

if (res[sum] == 1e9)
cout << '0';
else
cout << res[sum];
}

```

## 7.3 Increasing Sequence

```

signed increasing_sequence() {
int n = 0;
cin >> n;
vector<int> mass(n, 0), dp(n + 1, 10e8), path, pos(n),
prev(n);

for (int i = 0; i < n; i++) cin >> mass[i];
dp[0] = -10e8;
pos[0] = -1;
int len = 0;

for (int i = 0; i < n; i++) {
int j = upper_bound(dp.begin(), dp.end(), mass[i]) -
dp.begin();
if (dp[j - 1] < mass[i] && mass[i] < dp[j]) {
dp[j] = mass[i];
pos[j] = i;
prev[i] = pos[j - 1];
len = max(len, j);
}
}

cout << len << endl;
int p = pos[len];
while (p != -1) {
path.push_back(mass[p]);
p = prev[p];
}
}

```

```

}
reverse(path.begin(), path.end());
for (int i = 0; i < path.size(); i++) {
cout << path[i] << ' ';
}
cout << '\n';
}
}

```

## 7.4 Palindromes

```

signed palindromes() {
string s = "";
cin >> s;
vector<vector<int>> dp(s.length(), vector<int>(s.length
(), 0));
vector<vector<int>> pal(s.length(), vector<int>(s.
length(), 1));

for (int i = s.length() - 1; i >= 0; i--) {
for (int j = i; j < s.length(); j++)
if (i == j) {
dp[i][j] = 1;
}
else {
pal[i][j] = pal[i + 1][j - 1] & (s[i] == s[j])
dp[i][j] = dp[i][j - 1] + dp[i + 1][j] - dp[i
+ 1][j - 1] + pal[i][j];
}
}
}
}

```

## 7.5 Pyramid

```

signed pyramid() {
int n = 0, m = 0, result = 0;
cin >> n;
vector<vector<int>> mass(n + 1, vector<int>(n + 1, 0));

mass[0][0] = 1;

for (int i = 1; i < n + 1; i++) {
for (int j = 1; j < n + 1; j++) {
if (j > i)
continue;
for (int m = 0; m < j; m++)
mass[i][j] += mass[i - j][m];
}
}

for (int i = 1; i < n + 1; i++) {
result += mass[n][i];
}
cout << result << endl;
}

```

## 7.6 Ribbon

```

signed ribbon() {
int n;
cin >> n;
int count_a = 0, count_b = 0, count_c = 0;
vector<int> path(3), dp(n + 1, -10e8), mass(n + 1, -1);

cin >> path[0] >> path[1] >> path[2];
dp[0] = 0;

for (int i = 1; i <= n; i++) {
for (int j = 0; j < 3; j++) {
if (i - path[j] >= 0) {
dp[i] = max(dp[i], dp[i - path[j]] + 1);
if (dp[i] == (dp[i - path[j]] + 1))
mass[i] = path[j];
}
}
}
}

```

```

if (dp[n] != -10e8)
    cout << dp[n] << endl;
else {
    cout << 0 << endl;
    cout << 0 << " " << 0 << " " << 0 << endl;
    return 0;
}
for (int i = n; i >= 1; i--) {
    if (n <= 0)
        break;
    if (mass[i] == path[0])
        count_a++;
    if (mass[i] == path[1])
        count_b++;
    if (mass[i] == path[2])
        count_c++;

    n -= mass[i];
    i -= mass[i];
}
cout << count_a << " " << count_b << " " << count_c << '\n';
}

```

## 7.7 Route

```

signed route() {
    int n = 0;
    cin >> n;
    int last_i = 300, last_j = 300;
    char temp = ' ';

    vector<vector<int>> array(n, vector<int>(n));
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            cin >> temp;
            array[i][j] = int(temp) - 48;
        }
    }

    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (i == 0 && j == 0)
                continue;
            if (i == 0) {
                array[i][j] = array[i][j - 1] + array[i][j];
                continue;
            }
            if (j == 0) {
                array[i][j] = array[i - 1][j] + array[i][j];
                continue;
            }
            array[i][j] = min(array[i][j] + array[i][j - 1],
                             array[i][j] + array[i - 1][j]);
        }
    }

    for (int i = n - 1; i >= 0; i--) {
        for (int j = n - 1; j >= 0; j--) {
            if (i > last_i || j > last_j)
                continue;
            if (i == n - 1 && j == n - 1) {
                array[i][j] = -1;
                last_i = i;
                last_j = j;
            }
            if (i == 0) {
                array[i][j] = -1;
                last_i = i;
                last_j = j;
                continue;
            }
            if (j == 0 && (array[i][j + 1] == -1 || array[i + 1][j] == -1)) {
                array[i][j] = -1;
                last_i = i;
                last_j = j;
                continue;
            }
            if (array[i - 1][j] < array[i][j - 1]) {
                array[i - 1][j] = -1;
                last_i = i;
                last_j = j;
                break;
            }
        }
    }
}

```

```

    }
    else {
        array[i][j - 1] = -1;
        last_i = i;
        last_j = j;
    }
}

}

for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
        if (i == 0 && j == 0) {
            cout << "#";
            continue;
        }
        if (array[i][j] == -1)
            cout << "#";
        else
            cout << "-";
    }
    cout << '\n';
}
}

```

## 7.8 Upstairs

```

signed upstairs() {
    int n; cin >> n;
    vector<int> coin(n, 0);
    vector<int> dp(n + 1, 0);

    for (int i = 0; i < n; i++) cin >> coin[i];

    for (int i = 1; i <= n; i++) {
        dp[i] = dp[i - 1];
        if (i >= 2)
            dp[i] = max(dp[i], dp[i - 2]);
        dp[i] += coin[i - 1];
    }

    cout << dp[n];
}

```

## 8 Misc

### 8.1 Ternary Search

```

double phi = 1 + (1 + sqrt(5)) / 2;

// continuous ternary search
double cont_ternary_search(double l, double r) {
    double m1 = 1 + (r - l) / phi, m2 = r - (r - l) / phi;
    double f1 = f(m1), f2 = f(m2);
    int count = 200;
    while (count--) {
        if (f1 < f2) {
            r = m2;
            m2 = m1;
            f2 = f1;
            m1 = 1 + (r - l) / phi;
            f1 = f(m1);
        }
        else {
            l = m1;
            m1 = m2;
            f1 = f2;
            m2 = r - (r - l) / phi;
            f2 = f(m2);
        }
    }
    return f((l + r) / 2);
}

// discrete ternary search
double discr_ternary_search(int l, int r) {
    int m1 = 1 + (r - l) / 3, m2 = r - (r - l) / 3;
    while (r - l > 2) {

```

```
    if (f(m1) < f(m2))
        r = m2;
    else
        l = m1;
    m1 = l + (r - l) / 3;
    m2 = r - (r - l) / 3;
}
return min(f(l), min(f(l + 1), f(r)));
}
```