

ACM-ICPC Team Reference Document

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1 Templates

1.1 C++ Template

```
#include <bits/stdc++.h>

using namespace std;

// DEFINES
#define precision(x) cout << fixed << setprecision(x);
#define fast cin.tie(0); ios::sync_with_stdio(0)
#define all(x) x.begin(), x.end()
#define rall(x) x.rbegin(), x.rend()
#define ff first
#define ss second
// #define nl endl
#define nl "\n"
#define sp " "
#define yes "Yes"
#define no "No"
#define int long long

// CONSTANTS
const int INF = 1e18;
// const int MOD = 1e9 + 7;
// const int MOD = 998244353;

// FSTREAMS
ifstream in("input.txt");
ofstream out("output.txt");

// RANDOM
const int RMIN = 1, RMAX = 1e9;

random_device rdev;
mt19937_64 reng(rdev());
uniform_int_distribution<mt19937_64::result_type> dist(RMIN
, RMAX);

// GLOBALS

// SOLUTION
void solve() {
}
```

```
// PREPROCESSING
void prepr() { }

// ENTRANCE
signed main() {
    precision(15);
    fast;
    prepr();
    int t = 1;
    cin >> t;
    while (t--) solve();
}
```

1.2 C++ Include

```
#include <iostream>
#include <iomanip>
#include <fstream>
#include <random>
#include <cmath>
#include <algorithm>
#include <string>
#include <vector>
#include <set>
#include <unordered_set>
#include <map>
#include <unordered_map>
#include <queue>
#include <deque>
#include <stack>
#include <list>
#include <bitset>
```

1.3 Py Template

```
from math import sqrt, ceil, floor, gcd
from random import randint
import sys
```

```
def inpt():
    return sys.stdin.readline().strip()
```

```
input = inpt
```

```
INF = int(1e18)
# MOD = int(1e9 + 7)
# MOD = 998244353
```

```
def solve():
    pass
```

```
t = 1
t = int(input())
for _ in range(t):
    solve()
```

2 Data Structures

2.1 Disjoint Set Union

```
// Theme: Disjoint Set Union
```

```
struct dsu {
    vector<int> p, size;

    dsu(int n) {
        p.assign(n, 0); size.assign(n, 0);
        for (int i = 0; i < n; i++) {
            p[i] = i;
            size[i] = 1;
        }
    }
};
```

```
    }
}

int get(int v) {
    if (p[v] != v) p[v] = get(p[v]);
    return p[v];
}

void unite(int u, int v) {
    auto x = get(u), y = get(v);
    if (x == y) return;
    if (size[x] > size[y]) swap(x, y);
    p[x] = y; size[y] += size[x];
}
};
```

2.2 Segment Tree

```
// Theme: Segment Tree
```

```
struct segtree {
    int size;
    vector<int> tree;

    void init(int n) {
        size = 1;
        while (size < n) size <= 1;
        tree.assign(2 * size - 1, 0);
    }

    void build(vector<int> &a, int x, int lx, int rx) {
        if (rx - lx == 1) {
            if (lx < a.size()) tree[x] = a[lx];
            return;
        }
        int m = (lx + rx) / 2;
        build(a, 2 * x + 1, lx, m);
        build(a, 2 * x + 2, m, rx);
        tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
    }

    void build(vector<int> &a) {
        init(a.size());
        build(a, 0, 0, size);
    }

    // Complexity: O(log(n))
    void set(int i, int v, int x, int lx, int rx) {
        if (rx - lx == 1) {
            tree[x] = v;
            return;
        }
        int m = (lx + rx) / 2;
        if (i < m) set(i, v, 2 * x + 1, lx, m);
        else set(i, v, 2 * x + 2, m, rx);
        tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
    }

    void set(int i, int v) {
        set(i, v, 0, 0, size);
    }

    // Complexity: O(log(n))
    int sum(int l, int r, int x, int lx, int rx) {
        if (l <= lx && rx <= r) return tree[x];
        if (l >= rx || r <= lx) return 0;
        int m = (lx + rx) / 2;
        return sum(l, r, 2 * x + 1, lx, m) +
            sum(l, r, 2 * x + 2, m, rx);
    }

    int sum(int l, int r) {
        return sum(l, r, 0, 0, size);
    }
};
```

2.3 Segment Tree Propagate

```
// Theme: Segment Tree With Propagation
```

```
struct segtree_prop {
    int size;
    vector<int> tree;

    void init(int n) {
```

```

    size = 1;
    while (size < n) size <= 1;
    tree.assign(2 * size - 1, 0);
}

void build(vector<int> &a, int x, int lx, int rx) {
    if (rx - lx == 1) {
        if (lx < a.size()) tree[x] = a[lx];
        return;
    }
    int m = (lx + rx) / 2;
    build(a, 2 * x + 1, lx, m);
    build(a, 2 * x + 2, m, rx);
    tree[x] = tree[2 * x + 1] + tree[2 * x + 2];
}

void build(vector<int> &a) {
    init(a.size());
    build(a, 0, 0, size);
}

void push(int x, int lx, int rx) {
    if (rx - lx == 1) return;
    tree[2 * x + 1] += tree[x];
    tree[2 * x + 2] += tree[x];
    tree[x] = 0;
}

// Complexity: O(log(n))
void add(int v, int l, int r, int x, int lx, int rx) {
    push(x, lx, rx);
    if (rx <= l || r <= lx) return;
    if (l <= lx && rx <= r) {
        tree[x] += v;
        return;
    }
    int m = (lx + rx) / 2;
    add(v, l, r, 2 * x + 1, lx, m);
    add(v, l, r, 2 * x + 2, m, rx);
}

void add(int v, int l, int r) {
    add(v, l, r, 0, 0, size);
}

// Complexity: O(log(n))
int get(int i, int x, int lx, int rx) {
    push(x, lx, rx);
    if (rx - lx == 1) return tree[x];
    int m = (lx + rx) / 2;
    if (i < m) return get(i, 2 * x + 1, lx, m);
    else return get(i, 2 * x + 2, m, rx);
}

int get(int i) {
    return get(i, 0, 0, size);
}
};

```

2.4 Treap

// Theme: Treap (Tree + Heap)

```

// Node
struct node {
    int key, priority;
    shared_ptr<node> left, right;

    node(int key, int priority = INF) :
        key(key),
        priority(priority == INF ?
            reng() : priority) {}
};

// Treap
struct treap {
    shared_ptr<node> root;

    treap() {}

    treap(int root_key, int root_priority = INF) {
        root = shared_ptr<node>(new node(root_key,
            root_priority));
    }

    treap(shared_ptr<node> rt) {
        root = shared_ptr<node>(rt);
    }
};

```

```

treap(const treap &tr) {
    root = shared_ptr<node>(tr.root);
}

// Complexity: O(log(N))
pair<treap, treap> split(int k) {
    auto res = split(root, k);
    return { treap(res.ff), treap(res.ss) };
}

pair<shared_ptr<node>, shared_ptr<node>> split(
    shared_ptr<node> rt, int k) {
    if (!rt) return { nullptr, nullptr };
    else if (rt->key < k) {
        auto [rt1, rt2] = split(rt->right, k);
        rt->right = rt1;
        return { rt, rt2 };
    }
    else {
        auto [rt1, rt2] = split(rt->left, k);
        rt->left = rt2;
        return { rt1, rt };
    }
}

// Complexity: O(log(N))
treap merge(const treap &tr) {
    root = shared_ptr<node>(merge(root, tr.root));
    return *this;
}

shared_ptr<node> merge(shared_ptr<node> rt1, shared_ptr<
    node> rt2) {
    if (!rt1) return rt2;
    if (!rt2) return rt1;
    if (rt1->priority < rt2->priority) {
        rt1->right = merge(rt1->right, rt2);
        return rt1;
    }
    else {
        rt2->left = merge(rt1, rt2->left);
        return rt2;
    }
}
};

```

2.5 Treap K

// Theme: Treap With Segments

```

// Node
struct node_k {
    int key, priority, size;
    shared_ptr<node_k> left, right;

    node_k(int key, int priority = INF) :
        key(key),
        priority(priority == INF ?
            reng() : priority),
        size(1) {}

    friend int sz(shared_ptr<node_k> nd) {
        return (nd ? nd->size : 0);
    }

    void upd() {
        size = sz(left) + sz(right) + 1;
    }
};

// Treap
struct treap_k {
    shared_ptr<node_k> root;

    treap_k() {}

    treap_k(int root_key, int root_priority = INF) {
        root = shared_ptr<node_k>(new node_k(root_key,
            root_priority));
    }

    treap_k(shared_ptr<node_k> rt) {
        root = shared_ptr<node_k>(rt);
    }
};

```

```

treap_k(const treap_k &tr) {
    root = shared_ptr<node_k>(tr.root);
}

// Complexity: O(log(N))
pair<treap_k, treap_k> split_k(int k) {
    auto res = split_k(root, k);
    return { treap_k(res.ff), treap_k(res.ss) };
}

pair<shared_ptr<node_k>, shared_ptr<node_k>> split_k(
    shared_ptr<node_k> rt, int k) {
    if (!rt) return { nullptr, nullptr };
    else if (sz(rt) <= k) return { rt, nullptr };
    else if (sz(rt->left) + 1 <= k) {
        auto [rt1, rt2] = split_k(rt->right, k - sz(rt->
            left) - 1);
        rt->right = rt1;
        rt->upd();
        return { rt, rt2 };
    }
    else {
        auto [rt1, rt2] = split_k(rt->left, k);
        rt->left = rt2;
        rt->upd();
        return { rt1, rt };
    }
}

// Complexity: O(log(N))
treap_k merge_k(const treap_k &tr) {
    root = shared_ptr<node_k>(merge_k(root, tr.root));
    return *this;
}

shared_ptr<node_k> merge_k(shared_ptr<node_k> rt1,
    shared_ptr<node_k> rt2) {
    if (!rt1) return rt2;
    if (!rt2) return rt1;
    if (rt1->priority < rt2->priority) {
        rt1->right = merge_k(rt1->right, rt2);
        rt1->upd();
        return rt1;
    }
    else {
        rt2->left = merge_k(rt1, rt2->left);
        rt2->upd();
        return rt2;
    }
}
};

```

3 Algebra

3.1 Primes Sieve

```

// Theme: Prime Numbers

// Algorithm: Eratosthenes Sieve
// Complexity: O(N*log(log(N)))

// = 0 - Prime,
// != 0 - Lowest Prime Divisor
auto get_sieve(int n) {
    vector<int> sieve(n); // Sieve
    sieve[0] = sieve[1] = 1;

    for (int i = 2; i * i < n; i++)
        if (!sieve[i])
            for (int j = i * i; j < n; j += i)
                sieve[j] = i;

    return sieve;
}

// Algorithm: Prime Numbers With Sieve
// Complexity: O(N*log(log(N)))

auto get_primes(int n) {
    vector<int> primes, sieve = get_sieve(n);

    for (int i = 2; i < sieve.size(); i++)

```

```

        if (!sieve[i])
            primes.push_back(i);

    return primes;
}

// Algorithm: Linear Algorithm
// Complexity: O(N)

// lp[i] = Lowest Prime Divisor
auto get_sieve_primes(int n, vector<int> &primes) {
    vector<int> lp(n);
    lp[0] = lp[1] = 1;

    for (int i = 2; i < n; i++) {
        if (!lp[i]) {
            lp[i] = i;
            primes.push_back(i);
        }
        for (int j = 0; j < primes.size() &&
            primes[j] <= lp[i] &&
            i * primes[j] < n; j++)
            lp[i * primes[j]] = primes[j];
    }

    return lp;
}

```

3.2 Factorization

```

// Theme: Factorization

// Algorithm: Trivial Algorithm
// Complexity: O(sqrt(N))

auto factors(int n) {
    vector<int> factors;

    for (int i = 2; i * i <= n; i++) {
        if (n % i) continue;
        while (n % i == 0) n /= i;
        factors.push_back(i);
    }

    if (n != 1)
        factors.push_back(n);

    return factors;
}

// Algorithm: Factorization With Sieve
// Complexity: O(N*log(log(N)))

auto factors_sieve(int n) {
    vector<int> factors,
        sieve = get_sieve(n + 1);

    while (sieve[n]) {
        factors.push_back(sieve[n]);
        n /= sieve[n];
    }

    if (n != 1)
        factors.push_back(n);

    return factors;
}

// Algorithm: Factorization With Primes
// Complexity: O(sqrt(N)/log(sqrt(N)))

auto factors_primes(int n) {
    vector<int> factors,
        primes = get_primes(n + 1);

    for (auto &i : primes) {
        if (i * i > n) break;
        if (n % i) continue;
        while (n % i == 0) n /= i;
        factors.push_back(i);
    }

    if (n != 1)
        factors.push_back(n);
}

```

```

    return factors;
}

// Algorithm: Fermat Test
// Complexity:  $O(K \cdot \log(N))$ 

bool fermat(int n) {
    if (n == 2) return true;

    uniform_int_distribution<int> distA(2, n - 1);

    for (int i = 0; i < 1000; i++) {
        int a = distA(reng);
        if (gcd(a, n) != 1 ||
            binpow(a, n - 1, n) != 1)
            return false;
    }

    return true;
}

// Algorithm: Pollard Rho Algorithm
// Complexity:  $O(N^{1/4})$ 

int f(int x, int c, int n) {
    return ((x * x) % n + c) % n;
}

int pollard_rho(int n) {
    if (n % 2 == 0) return 2;

    uniform_int_distribution<int> distC(1, n), distX(1, n);

    int c = distC(reng), x = distX(reng);
    int y = x;

    int g = 1;

    while (g == 1) {
        x = f(x, c, n);
        y = f(f(y, c, n), c, n);
        g = gcd(abs(x - y), n);
    }

    return g;
}

// Algorithm: Pollard Rho Factorization + Fermat Test
// Complexity:  $O(N^{1/4} \cdot \log(N))$ 

void factors_pollard_rho(int n, vector<int> &factors) {
    if (n == 1) return;

    if (fermat(n)) {
        factors.push_back(n);
        return;
    }

    int d = pollard_rho(n);

    factors_pollard_rho(d, factors);
    factors_pollard_rho(n / d, factors);
}

```

3.3 Euler Totient Function

```
// Theme: Euler Totient Function
// Algorithm: Euler Product Formula
// Complexity: O(sqrt(N))

// phi = n(1 - 1 / p1) ... (1 - 1 / pi)
int phi(int n) {
    if (n == 1) return 1;

    auto f = factors(n);

    int res = n;
    for (auto &p : f)
        res -= res / p;

    return res;
}
```

3.4 Greatest Common Divisor

```
// Theme: Greatest Common Divisor

// Algorithm: Simple Euclidean Algorithm
// Complexity:  $O(\log(N))$ 

int gcd(int a, int b) {
    while (b) {
        a %= b;
        swap(a, b);
    }
    return a;
}

// Algorithm: Extended Euclidean Algorithm
// Complexity:  $O(\log(N))$ 

// d = gcd(a, b)
// x * a + y * b = d
// returns {d, x, y}
vector<int> euclid(int a, int b) {
    if (!a) return { b, 0, 1 };
    auto v = euclid(b % a, a);
    int d = v[0], x = v[1], y = v[2];
    return { d, y - (b / a) * x, x };
}
```

3.5 Binary Operations

```
// Theme: Binary Operations

// Algorithm: Binary Multiplication
// Complexity:  $O(\log(b))$ 

int binmul(int a, int b, int p = 0) {
    int res = 0;
    while (b) {
        if (b & 1) res = p ? (res + a) % p : (res + a);
        a = p ? (a + a) % p : (a + a);
        b >>= 1;
    }
    return res;
}

// Algorithm: Binary Exponentiation
// Complexity:  $O(\log(b))$ 

int binpow(int a, int b, int p = 0) {
    int res = 1;
    while (b) {
        if (b & 1) res = p ? (res * a) % p : (res * a);
        a = p ? (a * a) % p : (a * a);
        b >>= 1;
    }
    return res;
}
```

3.6 Matrices

[illegible]

```

    return res;
}

// Algorithm: Matrix-Vector Multiplication
// Complexity: O(N*M)

auto m_prod(matrix<int> &a, row<int> &b, int p = 0) {
    int n = a.size(), m = b.size();

    row<int> res(n);

    for (int i = 0; i < n; i++)
        for (int j = 0; j < m; j++)
            res[i] = p ? (res[i] + a[i][j] * b[j] % p) % p
            : (res[i] + a[i][j] * b[j]);

    return res;
}

// Algorithm: Fast Matrix Exponentiation
// Complexity: O(N^3*log(K))

auto m_binpow(matrix<int> a, int x, int p = 0) {
    int n = a.size();

    matrix<int> res(n, row<int>(n));
    for (int i = 0; i < n; i++) res[i][i] = 1;

    while (x) {
        if (x & 1) res = m_prod(res, a, p);
        a = m_prod(a, a, p);
        x >>= 1;
    }

    return res;
}

```

3.7 Fibonacci

```

// Theme: Fibonacci Sequence

// Algorithm: Fibonacci Numbers With Matrix Exponentiation
// Complexity: O(log(N))

int fibonacci(int n) {
    row<int> first_two = { 1, 0 };
    if (n <= 2) return first_two[2 - n];

    matrix<int> fib(2, row<int>(2, 0));
    fib[0][0] = 1; fib[0][1] = 1;
    fib[1][0] = 1; fib[1][1] = 0;

    fib = m_binpow(fib, n - 2);

    row<int> last_two = m_prod(fib, first_two);

    return last_two[0];
}

```

3.8 Baby Step Giant Step

```

// Theme: Discrete Logarithm

// Algorithm: Baby-Step Giant-Step Algorithm
// Complexity: O(sqrt(p)*log(p))

//  $a^x \equiv b \pmod{p}$ ,  $(a, p) = 1$ 
//  $a^{i * m + j} \equiv b \pmod{p}$ ,  $m = \text{ceil}(\sqrt{p})$ 
//  $a^{i * m} \equiv b * a^{-j} \pmod{p}$ 
int baby_giant_step(int a, int b, int p) {
    //  $a^{-1} \equiv a^{p-2} \pmod{p}$ 
    int m = ceil(sqrt(p)), _a = binpow(a, p - 2, p);

    //  $s[b * a^{-j}] = j$ 
    unordered_map<int, int> s;
    for (int j = 0, t = b; j < m; j++, t = t * _a % p) s[t] = j;

    for (int i = 0; i < m; i++) {
        //  $s.find(a^{i * m})$ 
        auto f = s.find(binpow(a, i * m, p));
    }
}

```

```

        //  $i * m + j$ 
        if (f != s.end()) return i * m + f->ss;
    }

    return -1;
}

```

3.9 Combinations

```

// Theme: Combination Number

// Algorithm: Online Multiplication-Division
// Complexity: O(k)

//  $C_n^k$  - from n by k
int C(int n, int k) {
    int res = 1;

    for (int i = 1; i <= k; i++) {
        res *= n - k + i;
        res /= i;
    }

    return res;
}

// Algorithm: Pascal Triangle Preprocessing
// Complexity: O(N^2)

auto pascal(int n) {
    //  $C[i][j] = C_{i+j}^i$ 
    vector<vector<int>> C(n + 1, vector<int>(n + 1, 1));
    for (int i = 1; i < n + 1; i++)
        for (int j = 1; j < n + 1; j++)
            C[i][j] = C[i - 1][j] + C[i][j - 1];

    return C;
}

```

3.10 Permutation

```

// Theme: Permutations

// Algorithm: Next Lexicological Permutation
// Complexity: O(N)

bool perm(vector<int> &v) {
    int n = v.size();

    for (int i = n - 1; i >= 1; i--) {
        if (v[i - 1] < v[i]) {
            reverse(v.begin() + i, v.end());

            int j = distance(v.begin(),
                upper_bound(v.begin() + i, v.end(), v[i - 1]));

            swap(v[i - 1], v[j]);
            return true;
        }
    }

    return false;
}

```

3.11 Fast Fourier Transform

```

// Theme: Fast Fourier Transform

// Algorithm: Fast Fourier Transform (Complex)
// Complexity: O(N*log(N))

using cd = complex<double>;
const double PI = acos(-1);

auto fft(vector<cd> a, bool invert = 0) {
    //  $n = 2^x$ 
    int n = a.size();
}

```

```

// Bit-Reversal Permutation (0000, 1000, 0100, 1100,
// 0010, ...)
for (int i = 1, j = 0; i < n; i++) {
    int bit = n >> 1;
    for (; j >= bit; bit >>= 1) j -= bit;
    j += bit;
    if (i < j) swap(a[i], a[j]);
}

for (int len = 2; len <= n; len <= 1) {
    // Complex Root Of One
    double ang = 2 * PI / len * (invert ? -1 : 1);
    cd lroot(cos(ang), sin(ang));

    for (int i = 0; i < n; i += len) {
        cd root(1);
        for (int j = 0; j < len / 2; j++) {
            cd u = a[i + j], v = a[i + j + len / 2] * root;
            a[i + j] = (u + v);
            a[i + j + len / 2] = (u - v);
            root = (root * lroot);
        }
    }
}

if (invert) {
    for (int i = 0; i < n; i++) a[i] /= n;
}

return a;
}

// Module (7340033 = 7 * (2 ^ 20) + 1)
// Primiive Root (5 ^ (2 ^ 20) == 1 mod 7340033)
// Inverse Primitive Root (5 * 4404020 == 1 mod 7340033)
// Maximum Degree Of Two (2 ^ 20)

const int mod = 7340033;
const int proot = 5;
const int proot_1 = 4404020;
const int pw = 1 << 20;

// Algorithm: Discrete Fourier Transform (Inverse Roots)
// Complexity: O(N*log(N))

auto fft(vector<int> a, bool invert = 0) {
    // n = 2 ^ x
    int n = a.size();

    // Bit-Reversal Permutation (0000, 1000, 0100, 1100,
    // 0010, ...)
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if (i < j) swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <= 1) {
        // Primitive Root Or Inverse Root (Inverse
        // Transform)
        int lroot = invert ? proot_1 : proot;

        // Current Primitive Root
        lroot = binpow(lroot, pw / len, mod);

        for (int i = 0; i < n; i += len) {
            int root = 1;
            for (int j = 0; j < len / 2; j++) {
                int u = a[i + j], v = a[i + j + len / 2] *
                    root % mod;
                a[i + j] = (u + v) % mod;
                a[i + j + len / 2] = (u - v + mod) % mod;
                root = (root * lroot) % mod;
            }
        }
    }

    if (invert) {
        int _n = binpow(n, mod - 2, mod);
        for (int i = 0; i < n; i++) a[i] = (a[i] * _n) % mod;
    }

    return a;
}

```

```

}

// Algorithm: Discrete Fourier Transform
// Complexity: O(N*log(N))

auto fft(vector<int> &a, bool invert = 0) {
    // n = 2 ^ x
    int n = a.size();

    // Bit-Reversal Permutation (0000, 1000, 0100, 1100,
    // 0010, ...)
    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if (i < j) swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <= 1) {
        // Current Primitive Root
        int lroot = binpow(proot, pw / len, mod);

        for (int i = 0; i < n; i += len) {
            int root = 1;
            for (int j = 0; j < len / 2; j++) {
                int u = a[i + j], v = a[i + j + len / 2] *
                    root % mod;
                a[i + j] = (u + v) % mod;
                a[i + j + len / 2] = (u - v + mod) % mod;
                root = (root * lroot) % mod;
            }
        }
    }

    if (invert) {
        reverse(a.begin() + 1, a.end());
        int _n = binpow(n, mod - 2, mod);
        for (int i = 0; i < n; i++) a[i] = (a[i] * _n) % mod;
    }

    return a;
}

```

3.12 Primitive Roots

```

// Module (7 == 3 * (2 ^ 1) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 1} (6)
// Inverse Root {2 ^ 1} (6)
// Degree Of Two (2)

// const int mod = 7
// const int proot = 6
// const int proot_1 = 6
// const int pw = 1 << 1

// Module (13 == 3 * (2 ^ 2) + 1)
// Primitive Root (2)
// Primitive Root {2 ^ 2} (8)
// Inverse Root {2 ^ 2} (5)
// Degree Of Two (4)

// const int mod = 13
// const int proot = 8
// const int proot_1 = 5
// const int pw = 1 << 2

// Module (19 == 9 * (2 ^ 1) + 1)
// Primitive Root (2)
// Primitive Root {2 ^ 1} (18)
// Inverse Root {2 ^ 1} (18)
// Degree Of Two (2)

// const int mod = 19
// const int proot = 18
// const int proot_1 = 18
// const int pw = 1 << 1

```

```

// Module (37 == 9 * (2 ^ 2) + 1)
// Primitive Root (2)
// Primitive Root {2 ^ 2} (31)
// Inverse Root {2 ^ 2} (6)
// Degree Of Two (4)

// const int mod = 37
// const int proot = 31
// const int proot_1 = 6
// const int pw = 1 << 2

// Module (73 == 9 * (2 ^ 3) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 3} (10)
// Inverse Root {2 ^ 3} (22)
// Degree Of Two (8)

// const int mod = 73
// const int proot = 10
// const int proot_1 = 22
// const int pw = 1 << 3

// Module (97 == 3 * (2 ^ 5) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 5} (28)
// Inverse Root {2 ^ 5} (52)
// Degree Of Two (32)

// const int mod = 97
// const int proot = 28
// const int proot_1 = 52
// const int pw = 1 << 5

// Module (109 == 27 * (2 ^ 2) + 1)
// Primitive Root (6)
// Primitive Root {2 ^ 2} (33)
// Inverse Root {2 ^ 2} (76)
// Degree Of Two (4)

// const int mod = 109
// const int proot = 33
// const int proot_1 = 76
// const int pw = 1 << 2

// Module (163 == 81 * (2 ^ 1) + 1)
// Primitive Root (2)
// Primitive Root {2 ^ 1} (162)
// Inverse Root {2 ^ 1} (162)
// Degree Of Two (2)

// const int mod = 163
// const int proot = 162
// const int proot_1 = 162
// const int pw = 1 << 1

// Module (193 == 3 * (2 ^ 6) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 6} (125)
// Inverse Root {2 ^ 6} (105)
// Degree Of Two (64)

// const int mod = 193
// const int proot = 125
// const int proot_1 = 105
// const int pw = 1 << 6

// Module (433 == 27 * (2 ^ 4) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 4} (238)
// Inverse Root {2 ^ 4} (282)
// Degree Of Two (16)

// const int mod = 433
// const int proot = 238
// const int proot_1 = 282

```

```

// const int pw = 1 << 4

// Module (487 == 243 * (2 ^ 1) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 1} (486)
// Inverse Root {2 ^ 1} (486)
// Degree Of Two (2)

// const int mod = 487
// const int proot = 486
// const int proot_1 = 486
// const int pw = 1 << 1

// Module (577 == 9 * (2 ^ 6) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 6} (557)
// Inverse Root {2 ^ 6} (375)
// Degree Of Two (64)

// const int mod = 577
// const int proot = 557
// const int proot_1 = 375
// const int pw = 1 << 6

// Module (769 == 3 * (2 ^ 8) + 1)
// Primitive Root (11)
// Primitive Root {2 ^ 8} (562)
// Inverse Root {2 ^ 8} (26)
// Degree Of Two (256)

// const int mod = 769
// const int proot = 562
// const int proot_1 = 26
// const int pw = 1 << 8

// Module (1153 == 9 * (2 ^ 7) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 7} (1096)
// Inverse Root {2 ^ 7} (445)
// Degree Of Two (128)

// const int mod = 1153
// const int proot = 1096
// const int proot_1 = 445
// const int pw = 1 << 7

// Module (1297 == 81 * (2 ^ 4) + 1)
// Primitive Root (10)
// Primitive Root {2 ^ 4} (355)
// Inverse Root {2 ^ 4} (464)
// Degree Of Two (16)

// const int mod = 1297
// const int proot = 355
// const int proot_1 = 464
// const int pw = 1 << 4

// Module (1459 == 729 * (2 ^ 1) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 1} (1458)
// Inverse Root {2 ^ 1} (1458)
// Degree Of Two (2)

// const int mod = 1459
// const int proot = 1458
// const int proot_1 = 1458
// const int pw = 1 << 1

// Module (2593 == 81 * (2 ^ 5) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 5} (1997)
// Inverse Root {2 ^ 5} (335)
// Degree Of Two (32)

```



```
// const int mod = 2593
// const int proot = 1997
// const int proot_1 = 335
// const int pw = 1 << 5
```

```
// Module (2917 == 729 * (2 ^ 2) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 2} (2863)
// Inverse Root {2 ^ 2} (54)
// Degree Of Two (4)
```

```
// const int mod = 2917
// const int proot = 2863
// const int proot_1 = 54
// const int pw = 1 << 2
```

```
// Module (3457 == 27 * (2 ^ 7) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 7} (540)
// Inverse Root {2 ^ 7} (685)
// Degree Of Two (128)
```

```
// const int mod = 3457
// const int proot = 540
// const int proot_1 = 685
// const int pw = 1 << 7
```

```
// Module (3889 == 243 * (2 ^ 4) + 1)
// Primitive Root (11)
// Primitive Root {2 ^ 4} (1925)
// Inverse Root {2 ^ 4} (1396)
// Degree Of Two (16)
```

```
// const int mod = 3889
// const int proot = 1925
// const int proot_1 = 1396
// const int pw = 1 << 4
```

```
// Module (10369 == 81 * (2 ^ 7) + 1)
// Primitive Root (13)
// Primitive Root {2 ^ 7} (5758)
// Inverse Root {2 ^ 7} (6762)
// Degree Of Two (128)
```

```
// const int mod = 10369
// const int proot = 5758
// const int proot_1 = 6762
// const int pw = 1 << 7
```

```
// Module (12289 == 3 * (2 ^ 12) + 1)
// Primitive Root (11)
// Primitive Root {2 ^ 12} (1331)
// Inverse Root {2 ^ 12} (7968)
// Degree Of Two (4096)
```

```
// const int mod = 12289
// const int proot = 1331
// const int proot_1 = 7968
// const int pw = 1 << 12
```

```
// Module (17497 == 2187 * (2 ^ 3) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 3} (14518)
// Inverse Root {2 ^ 3} (7565)
// Degree Of Two (8)
```

```
// const int mod = 17497
// const int proot = 14518
// const int proot_1 = 7565
// const int pw = 1 << 3
```

```
// Module (18433 == 9 * (2 ^ 11) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 11} (17660)
```

```
// Inverse Root {2 ^ 11} (18123)
// Degree Of Two (2048)
```

```
// const int mod = 18433
// const int proot = 17660
// const int proot_1 = 18123
// const int pw = 1 << 11
```

```
// Module (39367 == 19683 * (2 ^ 1) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 1} (39366)
// Inverse Root {2 ^ 1} (39366)
// Degree Of Two (2)
```

```
// const int mod = 39367
// const int proot = 39366
// const int proot_1 = 39366
// const int pw = 1 << 1
```

```
// Module (52489 == 6561 * (2 ^ 3) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 3} (37459)
// Inverse Root {2 ^ 3} (37783)
// Degree Of Two (8)
```

```
// const int mod = 52489
// const int proot = 37459
// const int proot_1 = 37783
// const int pw = 1 << 3
```

```
// Module (65537 == 1 * (2 ^ 16) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 16} (3)
// Inverse Root {2 ^ 16} (21846)
// Degree Of Two (65536)
```

```
// const int mod = 65537
// const int proot = 3
// const int proot_1 = 21846
// const int pw = 1 << 16
```

```
// Module (139969 == 2187 * (2 ^ 6) + 1)
// Primitive Root (13)
// Primitive Root {2 ^ 6} (8104)
// Inverse Root {2 ^ 6} (40191)
// Degree Of Two (64)
```

```
// const int mod = 139969
// const int proot = 8104
// const int proot_1 = 40191
// const int pw = 1 << 6
```

```
// Module (147457 == 9 * (2 ^ 14) + 1)
// Primitive Root (10)
// Primitive Root {2 ^ 14} (94083)
// Inverse Root {2 ^ 14} (163)
// Degree Of Two (16384)
```

```
// const int mod = 147457
// const int proot = 94083
// const int proot_1 = 163
// const int pw = 1 << 14
```

```
// Module (209953 == 6561 * (2 ^ 5) + 1)
// Primitive Root (10)
// Primitive Root {2 ^ 5} (198463)
// Inverse Root {2 ^ 5} (179931)
// Degree Of Two (32)
```

```
// const int mod = 209953
// const int proot = 198463
// const int proot_1 = 179931
// const int pw = 1 << 5
```

```
// Module (331777 == 81 * (2 ^ 12) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 12} (100795)
// Inverse Root {2 ^ 12} (281060)
// Degree Of Two (4096)
```

```
// const int mod = 331777
// const int proot = 100795
// const int proot_1 = 281060
// const int pw = 1 << 12
```

```
// Module (472393 == 59049 * (2 ^ 3) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 3} (407677)
// Inverse Root {2 ^ 3} (406705)
// Degree Of Two (8)
```

```
// const int mod = 472393
// const int proot = 407677
// const int proot_1 = 406705
// const int pw = 1 << 3
```

```
// Module (629857 == 19683 * (2 ^ 5) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 5} (255372)
// Inverse Root {2 ^ 5} (55433)
// Degree Of Two (32)
```

```
// const int mod = 629857
// const int proot = 255372
// const int proot_1 = 55433
// const int pw = 1 << 5
```

```
// Module (746497 == 729 * (2 ^ 10) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 10} (371573)
// Inverse Root {2 ^ 10} (21163)
// Degree Of Two (1024)
```

```
// const int mod = 746497
// const int proot = 371573
// const int proot_1 = 21163
// const int pw = 1 << 10
```

```
// Module (786433 == 3 * (2 ^ 18) + 1)
// Primitive Root (10)
// Primitive Root {2 ^ 18} (1000)
// Inverse Root {2 ^ 18} (710149)
// Degree Of Two (262144)
```

```
// const int mod = 786433
// const int proot = 1000
// const int proot_1 = 710149
// const int pw = 1 << 18
```

```
// Module (839809 == 6561 * (2 ^ 7) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 7} (500841)
// Inverse Root {2 ^ 7} (2262)
// Degree Of Two (128)
```

```
// const int mod = 839809
// const int proot = 500841
// const int proot_1 = 2262
// const int pw = 1 << 7
```

```
// Module (995329 == 243 * (2 ^ 12) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 12} (712513)
// Inverse Root {2 ^ 12} (946681)
// Degree Of Two (4096)
```

```
// const int mod = 995329
// const int proot = 712513
// const int proot_1 = 946681
// const int pw = 1 << 12
```

```
// Module (1179649 == 9 * (2 ^ 17) + 1)
// Primitive Root (19)
// Primitive Root {2 ^ 17} (612074)
// Inverse Root {2 ^ 17} (1093705)
// Degree Of Two (131072)
```

```
// const int mod = 1179649
// const int proot = 612074
// const int proot_1 = 1093705
// const int pw = 1 << 17
```

```
// Module (1492993 == 729 * (2 ^ 11) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 11} (143225)
// Inverse Root {2 ^ 11} (1126252)
// Degree Of Two (2048)
```

```
// const int mod = 1492993
// const int proot = 143225
// const int proot_1 = 1126252
// const int pw = 1 << 11
```

```
// Module (1769473 == 27 * (2 ^ 16) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 16} (374254)
// Inverse Root {2 ^ 16} (643391)
// Degree Of Two (65536)
```

```
// const int mod = 1769473
// const int proot = 374254
// const int proot_1 = 643391
// const int pw = 1 << 16
```

```
// Module (199065 == 24883 * (2 ^ 3) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 3} (18290)
// Inverse Root {2 ^ 3} (115385)
// Degree Of Two (8)
```

```
// const int mod = 199065
// const int proot = 18290
// const int proot_1 = 115385
// const int pw = 1 << 3
```

```
// Module (2654209 == 81 * (2 ^ 15) + 1)
// Primitive Root (11)
// Primitive Root {2 ^ 15} (1985530)
// Inverse Root {2 ^ 15} (2369076)
// Degree Of Two (32768)
```

```
// const int mod = 2654209
// const int proot = 1985530
// const int proot_1 = 2369076
// const int pw = 1 << 15
```

```
// Module (5038849 == 19683 * (2 ^ 8) + 1)
// Primitive Root (29)
// Primitive Root {2 ^ 8} (4318906)
// Inverse Root {2 ^ 8} (2727143)
// Degree Of Two (256)
```

```
// const int mod = 5038849
// const int proot = 4318906
// const int proot_1 = 2727143
// const int pw = 1 << 8
```

```
// Module (5308417 == 81 * (2 ^ 16) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 16} (3305774)
// Inverse Root {2 ^ 16} (3708247)
// Degree Of Two (65536)
```

```
// const int mod = 5308417
```

```

// const int proot = 3305774
// const int proot_1 = 3708247
// const int pw = 1 << 16

// Module (8503057 == 531441 * (2 ^ 4) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 4} (4589209)
// Inverse Root {2 ^ 4} (2906831)
// Degree Of Two (16)

// const int mod = 8503057
// const int proot = 4589209
// const int proot_1 = 2906831
// const int pw = 1 << 4

// Module (11337409 == 177147 * (2 ^ 6) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 6} (3744116)
// Inverse Root {2 ^ 6} (9616850)
// Degree Of Two (64)

// const int mod = 11337409
// const int proot = 3744116
// const int proot_1 = 9616850
// const int pw = 1 << 6

// Module (14155777 == 27 * (2 ^ 19) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 19} (2742784)
// Inverse Root {2 ^ 19} (1606624)
// Degree Of Two (524288)

// const int mod = 14155777
// const int proot = 2742784
// const int proot_1 = 1606624
// const int pw = 1 << 19

// Module (19131877 == 4782969 * (2 ^ 2) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 2} (19127503)
// Inverse Root {2 ^ 2} (4374)
// Degree Of Two (4)

// const int mod = 19131877
// const int proot = 19127503
// const int proot_1 = 4374
// const int pw = 1 << 2

// Module (28311553 == 27 * (2 ^ 20) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 20} (4493789)
// Inverse Root {2 ^ 20} (13207632)
// Degree Of Two (1048576)

// const int mod = 28311553
// const int proot = 4493789
// const int proot_1 = 13207632
// const int pw = 1 << 20

// Module (57395629 == 14348907 * (2 ^ 2) + 1)
// Primitive Root (10)
// Primitive Root {2 ^ 2} (19864209)
// Inverse Root {2 ^ 2} (37531420)
// Degree Of Two (4)

// const int mod = 57395629
// const int proot = 19864209
// const int proot_1 = 37531420
// const int pw = 1 << 2

// Module (63700993 == 243 * (2 ^ 18) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 18} (48698706)
// Inverse Root {2 ^ 18} (16386043)

```

```

// Degree Of Two (262144)

// const int mod = 63700993
// const int proot = 48698706
// const int proot_1 = 16386043
// const int pw = 1 << 18

// Module (71663617 == 2187 * (2 ^ 15) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 15} (37080182)
// Inverse Root {2 ^ 15} (7507216)
// Degree Of Two (32768)

// const int mod = 71663617
// const int proot = 37080182
// const int proot_1 = 7507216
// const int pw = 1 << 15

// Module (86093443 == 43046721 * (2 ^ 1) + 1)
// Primitive Root (2)
// Primitive Root {2 ^ 1} (86093442)
// Inverse Root {2 ^ 1} (86093442)
// Degree Of Two (2)

// const int mod = 86093443
// const int proot = 86093442
// const int proot_1 = 86093442
// const int pw = 1 << 1

// Module (102036673 == 1594323 * (2 ^ 6) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 6} (50805973)
// Inverse Root {2 ^ 6} (42074539)
// Degree Of Two (64)

// const int mod = 102036673
// const int proot = 50805973
// const int proot_1 = 42074539
// const int pw = 1 << 6

// Module (113246209 == 27 * (2 ^ 22) + 1)
// Primitive Root (7)
// Primitive Root {2 ^ 22} (58671006)
// Inverse Root {2 ^ 22} (62639419)
// Degree Of Two (4194304)

// const int mod = 113246209
// const int proot = 58671006
// const int proot_1 = 62639419
// const int pw = 1 << 22

// Module (120932353 == 59049 * (2 ^ 11) + 1)
// Primitive Root (5)
// Primitive Root {2 ^ 11} (40826043)
// Inverse Root {2 ^ 11} (93710416)
// Degree Of Two (2048)

// const int mod = 120932353
// const int proot = 40826043
// const int proot_1 = 93710416
// const int pw = 1 << 11

// Module (998244353 == 119 * (2 ^ 23) + 1)
// Primitive Root (3)
// Primitive Root {2 ^ 23} (15311432)
// Inverse Root {2 ^ 23} (469870224)
// Degree Of Two (8388608)

// const int mod = 998244353
// const int proot = 15311432
// const int proot_1 = 469870224
// const int pw = 1 << 23

// Module (1000000007 == 500000003 * (2 ^ 1) + 1)

```

```
// Primitive Root (5)
// Primitive Root {2 ^ 1} (1000000006)
// Inverse Root {2 ^ 1} (1000000006)
// Degree Of Two (2)
```

```
// const int mod = 1000000007
// const int proot = 1000000006
// const int proot_1 = 1000000006
// const int pw = 1 << 1
```

3.13 Number Decomposition

```
// Theme: Integer Numbers Decomposition With Composite
Module
```

```
// Module
// m = (p1 ^ m1) * (p2 ^ m2) * ... * (pn ^ mn)
int m;
// Prime Divisors Of Module
vector<int> p;
```

```
struct num {
    // GCD(x, m) = 1
    int x;
    // Powers Of Primes
    vector<int> a;

    num() : x(0), a(vector<int>(p.size())) { }

    // n = (p1 ^ a1) * (p2 ^ a2) * ... * (pn ^ an) * x
    num(int n) : x(0), a(vector<int>(p.size())) {
        if (!n) return;
        for (int i = 0; i < p.size(); i++) {
            int ai = 0;
            while (n % p[i] == 0) {
                n /= p[i];
                ai++;
            }
            a[i] = ai;
        }
        x = n;
    }

    num operator*(const num &nm) {
        vector<int> new_a(p.size());
        for (int i = 0; i < p.size(); i++)
            new_a[i] = a[i] + nm.a[i];
        num res; res.a = new_a;
        res.x = x * nm.x % m;
        return res;
    }

    num operator/(const num &nm) {
        vector<int> new_a(p.size());
        for (int i = 0; i < p.size(); i++)
            new_a[i] = a[i] - nm.a[i];
        num res; res.a = new_a;
        int g = euclid(nm.x, m)[1];
        g += m; g %= m;
        res.x = x * g % m;
        return res;
    }

    int toint() {
        int res = x;
        for (int i = 0; i < p.size(); i++)
            res = res * binpow(p[i], a[i], m) % m;
        return res;
    }
};
```

3.14 Formulae

Combinations.

$$C_n^k = \frac{n!}{(n-k)!k!}$$

$$C_n^0 + C_n^1 + \dots + C_n^n = 2^n$$

$$C_{n+1}^{k+1} = C_n^{k+1} + C_n^k$$

$$C_n^k = \frac{n}{k} C_{n-1}^{k-1}$$

Striling approximation.

$$n! \approx \sqrt{2\pi n} \frac{n^n}{e^n}$$

Euler's theorem.

$$a^{\phi(m)} \equiv 1 \pmod{m}, \gcd(a, m) = 1$$

Ferma's little theorem.

$$a^{p-1} \equiv 1 \pmod{p}, \gcd(a, p) = 1, p - \text{prime.}$$

Catalan number.

$$C_0 = 0, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}$$

$$C_n = \frac{2(2n-1)}{n+1} C_{n-1}$$

$$C_n = \frac{(2n)!}{n!(n+1)!}$$

Arithmetic progression.

$$S_n = \frac{a_1 + a_n}{2} n = \frac{2a_1 + d(n-1)}{2} n$$

Geometric progression.

$$S_n = \frac{b_1(1-q^n)}{1-q} n$$

Infinitely decreasing geometric progression.

$$S_n = \frac{b_1}{1-q} n$$

Sums.

$$\sum_{i=1}^n i = \frac{n(n+1)}{2},$$

$$\sum_{i=1}^n i^2 = \frac{n(2n+1)(n+1)}{6},$$

$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4},$$

$$\sum_{i=1}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30},$$

$$\sum_{i=a}^b c^i = \frac{c^{b+1} - c^a}{c-1}, c \neq 1.$$

4 Geometry

4.1 Vector

```
// Theme: Mathematical 3-D Vector
```

```
template <typename T>
struct vec {
    T x, y, z;
    vec(T x = 0, T y = 0, T z = 0) : x(x), y(y), z(z) { }
    vec<T> operator+(const vec<T> &v) const {
        return vec<T>(x + v.x, y + v.y, z + v.z);
    }
    vec<T> operator-(const vec<T> &v) const {
        return vec<T>(x - v.x, y - v.y, z - v.z);
    }
    vec<T> operator*(T k) const {
        return vec<T>(k * x, k * y, k * z);
    }
    friend vec<T> operator*(T k, const vec<T> &v) {
        return vec<T>(v.x * k, v.y * k, v.z * k);
    }
    vec<T> operator/(T k) {
        return vec<T>(x / k, y / k, z / k);
    }
    T operator*(const vec<T> &v) const {
        return x * v.x + y * v.y + z * v.z;
    }
    vec<T> operator^(const vec<T> &v) const {
```

```

        return { y * v.z - z * v.y, z * v.x - x * v.z, x * v
            .y - y * v.x };
    }
    auto operator<=>(const vec<T> &v) const = default;
    bool operator==(const vec<T> &v) const = default;
    T norm() const {
        return x * x + y * y + z * z;
    }
    double abs() const {
        return sqrt(norm());
    }
    double cos(const vec<T> &v) const {
        return ((*this) * v) / (abs() * v.abs());
    }
    friend ostream &operator<<(ostream &out, const vec<T> &v)
    {
        return out << v.x << sp << v.y << sp << v.z;
    }
    friend istream &operator>>(istream &in, vec<T> &v) {
        return in >> v.x >> v.y >> v.z;
    }
};

```

4.2 Planimetry

// Theme: Planimetry Objects

```

// Point
template <typename T>
struct point {
    T x, y;

    point() : x(0), y(0) { }
    point(T x, T y) : x(x), y(y) { }
};

```

```

// Rectangle
template <typename T>
struct rectangle {
    point<T> ld, ru;

    rectangle(const point<T> &ld, const point<T> &ru) :
        ld(ld), ru(ru) { }
};

```

4.3 Graham

// Theme: Convex Hull

// Algorithm: Graham Algorithm
// Complexity: $O(N \cdot \log(N))$

```

auto graham(const vector<vec<int>> &points) {
    vec<int> p0 = points[0];

    for (auto p : points)
        if (p.y < p0.y ||
            p.y == p0.y && p.x > p0.x)
            p0 = p;

    for (auto &p : points) {
        p.x -= p0.x;
        p.y -= p0.y;
    }

    sort(all(points), [] (vec<int> &p1, vec<int> &p2) {
        return (p1 ^ p2).z > 0 ||
            (p1 ^ p2).z == 0 && p1.norm() > p2.norm(); });

    vector<vec<int>> hull;
    for (auto &p : points) {
        while (hull.size() >= 2 &&
            (((p - hull.back()) ^ (hull[hull.size() - 1] - hull[
                hull.size() - 2])).z >= 0)
            hull.pop_back();
        hull.push_back(p);
    }

    for (auto &p : hull) {
        p.x += p0.x;
        p.y += p0.y;
    }
}

```

```

    }

    return hull;
}

```

4.4 Formulae

Triangles.

Radius of circumscribed circle:

$$R = \frac{abc}{4S}.$$

Radius of inscribed circle:

$$r = \frac{S}{p}.$$

Side via medians:

$$a = \frac{2}{3} \sqrt{2(m_b^2 + m_c^2) - m_a^2}.$$

Median via sides:

$$m_a = \frac{1}{2} \sqrt{2(b^2 + c^2) - a^2}.$$

Bisector via sides:

$$l_a = \frac{2\sqrt{bcp(p-a)}}{b+c}.$$

Bisector via two sides and angle:

$$l_a = \frac{2bc \cos \frac{\alpha}{2}}{b+c}.$$

Bisector via two sides and divided side:

$$l_a = \sqrt{bc - a_b a_c}.$$

Right triangles.

a, b - cathets, c - hypotenuse.

h - height to hypotenuse, divides c to c_a and

$$c_b.$$

$$\begin{cases} h^2 = c_a \cdot c_b, \\ a^2 = c_a \cdot c, \\ b^2 = c_b \cdot c. \end{cases}$$

Quadrangles.

Sides of circumscribed quadrangle:

$$a + c = b + d.$$

Square of circumscribed quadrangle:

$$S = \frac{Pr}{2} = pr.$$

Angles of inscribed quadrangle:

$$\alpha + \gamma = \beta + \delta = 180^\circ.$$

Square of inscribed quadrangle:

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}.$$

Circles.

Intersection of circle and line:

$$\begin{cases} (x - x_0)^2 + (y - y_0)^2 = R^2 \\ y = ax + b \end{cases}$$

Task comes to solution of $\alpha x^2 + \beta x + \gamma = 0$, where

$$\begin{cases} \alpha = (1 + a^2), \\ \beta = (2a(b - y_0) - 2x_0), \\ \gamma = (x_0^2 + (b - y_0)^2 - R^2). \end{cases}$$

Intersection of circle and circle:

$$\begin{cases} (x - x_0)^2 + (y - y_0)^2 = R_0^2 \\ (x - x_1)^2 + (y - y_1)^2 = R_1^2 \end{cases}$$

$$y = \frac{1}{2} \frac{(R_1^2 - R_0^2) + (x_0^2 - x_1^2) + (y_0^2 - y_1^2)}{y_0 - y_1} - \frac{x_0 - x_1}{y_0 - y_1} x$$

Task comes to intersection of circle and line.

5 Stringology

5.1 Z Function

```
// Theme: Z-Function

// Algorithm: Linear Algorithm
// Complexity: O(N)

auto z_func(const string &s) {
    int n = s.size();
    vector<int> z(n);

    for (int i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r) z[i] = min(r - i + 1, z[i - l]);

        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i]++;

        if (i + z[i] - 1 > r) {
            l = i;
            r = i + z[i] - 1;
        }
    }

    return z;
}
```

5.2 Manacher

```
// Theme: Palindromes

// Algorithm: Manacher Algorithm
// Complexity: O(N)

int manacher(const string &s) {
    int n = s.size();
    vector<int> d1(n), d2(n);

    for (int i = 0, l = 0, r = -1; i < n; i++) {
        int k = i > r ? 1 : min(d1[l + r - i], r - i + 1);
        while (i + k < n && i - k >= 0 && s[i + k] == s[i - k]) k++;
        d1[i] = k;
        if (i + k - 1 > r) {
            l = i - k + 1;
            r = i + k - 1;
        }
    }

    for (int i = 0, l = 0, r = -1; i < n; i++) {
        int k = i > r ? 0 : min(d2[l + r - i + 1], r - i + 1);
        while (i + k < n && i - k - 1 >= 0 && s[i + k] == s[i - k - 1]) k++;
        d2[i] = k;
        if (i + k - 1 > r) {
            l = i - k;
            r = i + k - 1;
        }
    }

    int res = 0;
    for (int i = 0; i < n; i++) {
        res += d1[i] + d2[i];
    }

    return res;
}
```

5.3 Trie

```
// Theme: Trie

// Algorithm: Aho-Corasick
// Complexity: O(N)

struct trie {
    // Vertex
    struct vertex {
```

```
        vector<int> next;
        bool leaf;
    };

    // Alphabet size
    static const int N = 26;
    // Maximum Vertex Number
    static const int MX = 2e5 + 1;

    // Vertices Vector
    vector<vertex> t;
    int sz;

    trie(): sz(1) {
        t.resize(MX);
        t[0].next.assign(N, -1);
    }

    void add_str(const string &s) {
        int v = 0;
        for (int i = 0; i < s.length(); i++) {
            char c = s[i] - 'a';
            if (t[v].next[c] == -1) {
                t[sz].next.assign(N, -1);
                t[v].next[c] = sz++;
            }
            v = t[v].next[c];
        }
        t[v].leaf = true;
    }
};
```

5.4 Prefix Function

```
// Theme: Prefix function

// Algorithm: Knuth-Morris-Pratt Algorithm
// Complexity: O(N)

auto pref_func(const string &s) {
    int n = s.size();
    vector<int> pi(n);

    for (int i = 1; i < n; i++) {
        int j = pi[i - 1];

        while (j > 0 && s[i] != s[j]) j = pi[j - 1];

        if (s[i] == s[j]) j++;

        pi[i] = j;
    }

    return pi;
}
```

5.5 Suffix Array

```
// Theme: Suffix array

// Algorithm: Binary Algorithm With Count Sort
// Complexity: O(N*log(N))

void count_sort(vector<int> &p, vector<int> &c) {
    int n = p.size();
    vector<int> cnt(n), p_new(n), pos(n);

    for (auto &x : c) cnt[x]++;

    pos[0] = 0;
    for (int i = 1; i < n; i++)
        pos[i] = pos[i - 1] + cnt[i - 1];

    for (auto &x : p) {
        int i = c[x];
        p_new[pos[i]] = x;
        pos[i]++;
    }

    p = p_new;
}
```

```

auto suffix_array(const string &str) {
    string s = str + '$';
    int n = s.size();

    vector<int> p(n), c(n);
    vector<pair<char, int>> a(n);

    for (int i = 0; i < n; i++) a[i] = { str[i], i };

    sort(a.begin(), a.end());

    for (int i = 0; i < n; i++) p[i] = a[i].second;

    c[p[0]] = 0;
    for (int i = 1; i < n; i++)
        c[p[i]] = c[p[i - 1]] + (a[i].first != a[i - 1].first);

    int k = 0;
    while ((1 << k) < n) {
        for (int i = 0; i < n; i++)
            p[i] = (p[i] - (1 << k) + n) % n;

        count_sort(p, c);

        vector<int> c_new(n);

        c_new[p[0]] = 0;
        for (int i = 1; i < n; i++) {
            pair<int, int> prev = { c[p[i - 1]], c[(p[i - 1] + (1 << k)) % n] };
            pair<int, int> now = { c[p[i]], c[(p[i] + (1 << k)) % n] };
            c_new[p[i]] = c_new[p[i - 1]] + (now != prev);
        }

        c = c_new;
        k++;
    }

    return p;
}

```

6 Dynamic Programming

6.1 Increasing Subsequence

```

// Theme: Longest Increasing Subsequence

// Algorithm: Binary Search Algorithm
// Complexity: O(N*log(N))

auto inc_subseq(const vector<int> &a) {
    int n = a.size();
    vector<int> dp(n + 1, INF), pos(n + 1), prev(n), subseq;

    int len = 0;
    dp[0] = -INF;
    pos[0] = -1;

    for (int i = 0; i < n; i++) {
        int j = distance(dp.begin(), upper_bound(all(dp), a[i]));
        if (dp[j - 1] < a[i] && a[i] < dp[j]) {
            dp[j] = a[i];
            pos[j] = i;
            prev[i] = pos[j - 1];
            len = max(len, j);
        }
    }

    int p = pos[len];
    while (p != -1) {
        subseq.push_back(a[p]);
        p = prev[p];
    }
    reverse(subseq.begin(), subseq.end());

    return subseq;
}

```

7 Graphs

7.1 Graph Implementation

```

// Theme: Graph Implementation

// Adjacency List (Unoriented)
// Adjacency List (Unoriented)

int sz;

vector<vector<int>> graph;

graph.assign(n, {});

for (int i = 0; i < n; i++) {
    int u, v; cin >> u >> v; --u --v;
    graph[u].push_back(v);
    graph[v].push_back(u);
}

// Adjacency List (Oriented)
// Adjacency List (Oriented)

int sz;

vector<vector<int>> graph;
vector<vector<int>> rgraph;

graph.assign(n, {});
rgraph.assign(n, {});

for (int i = 0; i < n; i++) {
    int u, v; cin >> u >> v; --u --v;
    graph[u].push_back(v);
    rgraph[v].push_back(u);
}

// Edges List (Unoriented)
// Edges List (Unoriented)

int sz;

vector<pair<int, int>> edges;
vector<vector<int>> graph;

graph.assign(n, {});

for (int i = 0; i < n; i++) {
    int u, v; cin >> u >> v; --u; --v;
    edges.push_back({ u, v });
    graph[u].push_back(i);
    graph[v].push_back(i);
}

// Edges List + Structure (Unoriented)
// Edges List + Structure (Unoriented)

struct edge {
    int u, v, w;
    edge(int u, int v, int w = 0) : u(u), v(v), w(w) {}
};

int sz;

vector<edge> edges;
vector<vector<int>> graph;

graph.assign(n, {});

for (int i = 0; i < n; i++) {
    int u, v, w; cin >> u >> v >> w; --u; --v;
    edges.push_back({ u, v, w });
    graph[u].push_back(i);
    graph[v].push_back(i);
}

// Edges List + Structure + Net Flows (Oriented)
// Edges List + Structure + Net Flows (Oriented)

```

```

struct edge {
    int to, cap, flow, weight;
    edge(int to, int cap, int flow = 0, int weight = 0)
        : to(to), cap(cap), flow(flow), weight(weight) {}
    int res() {
        return cap - flow;
    }
};

int sz;

vector<edge> edges;
vector<vector<int>> fgraph;

fgraph.assign(n, {});

void add_edge(int u, int v, int limit, int flow = 0, int
    weight = 0) {
    fgraph[u].push_back(edges.size());
    edges.push_back({ v, limit, flow, weight });
    fgraph[v].push_back(edges.size());
    edges.push_back({ u, 0, 0, -weight });
}

// Adjacency Matrix
vector<vector<int>> graph;

for (int i = 0; i < n; i++)
    for (int j = 0; j < n; j++)
        cin >> graph[i][j];

```

7.2 Graph Traversing

```

// Theme: Graph Traversing

vector<vector<int>> graph;
vector<int> used;

// Algorithm: Depth-First Search (Adjacency List)
// Complexity: O(N + M)

void dfs(int cur, int p = -1) {
    used[cur] = 1;

    for (auto &to : graph[cur]) {
        if (to == p || used[to]) continue;
        dfs(to, cur);
    }
}

// Algorithm: Breadth-First Search (Adjacency List)
// Complexity: O(N + M)

void bfs(int u) {
    queue<int> q; q.push(u);

    while (q.size()) {
        int cur = q.front(); q.pop();

        for (auto &to : graph[cur]) {
            if (used[to]) continue;
            q.push(to);
        }
    }
}

```

7.3 Topological Sort

```

// Theme: Topological Sort

vector<vector<int>> graph;
vector<int> used;

// Algorithm: Topological Sort
// Complexity: O(N + M)

vector<int> topsort;

void dfs_topsort(int cur, int p = -1) {

```

```

    used[cur] = 1;

    for (auto &to : graph[cur]) {
        if (to == p || used[to]) continue;
        dfs(to, cur);
    }

    topsort.push_back(cur);
}

for (int u = 0; u < n; u++)
    if (!used[u])
        dfs_topsort(u);

reverse(all(topsort));

```

7.4 Connected Components

```

// Theme: Connectivity Components

vector<vector<int>> graph;
vector<int> used;

// Algorithm: Connected Components
// Complexity: O(N + M)

vector<vector<int>> cc;

void dfs_cc(int cur, int p = -1) {
    used[cur] = 1;
    cc.back().push_back(cur);

    for (auto &to : graph[cur]) {
        if (to == p || used[to]) continue;
        dfs_cc(to, cur);
    }
}

for (int u = 0; u < n; u++)
    if (!used[u])
        dfs_cc(u);

// Algorithm: Strongly Connected Components
// Complexity: O(N + M)

vector<vector<int>> rgraph;

vector<vector<int>> topsort;

vector<vector<int>> scc;

void dfs_scc(int cur, int p = -1) {
    used[cur] = 1;
    scc.back().push_back(cur);

    for (auto &to : rgraph[cur]) {
        if (to == p || used[to]) continue;
        dfs_scc(to, cur);
    }
}

for (auto &u: topsort)
    if (!used[u])
        dfs_scc(u);

```

7.5 2 Sat

```

// Theme: 2-SAT

// Algorithm: Adding Edges To 2-SAT

vector<vector<int>> ts_graph;
vector<vector<int>> ts_rgraph;

// Vertex By Var Number
int to_vert(int x) {
    if (x < 0) {
        return ((abs(x) - 1) << 1) ^ 1;
    }
    else {
        return (x - 1) << 1;
    }
}

```



```

}

// Adding Implication
void add_impl(int a, int b) {
    ts_graph[a].insert(b);
    ts_rgraph[b].insert(a);
}

// Adding Disjunction
void add_or(int a, int b) {
    add_impl(a ^ 1, b);
    add_impl(b ^ 1, a);
}

```

7.6 Bridges

// Theme: Bridges And ECC

```

vector<pair<int, int>> edges;
vector<vector<int>> graph;
vector<int> used;

```

```

vector<int> height;
vector<int> up;

```

// Algorithm: Bridges
// Complexity: $O(N + M)$

```

vector<int> bridges;

```

```

void dfs_bridges(int cur, int p = -1) {
    used[cur] = 1;
    up[cur] = height[cur];
    for (auto &ind : g[cur]) {
        int to = cur ^ edges[ind].ff ^ edges[ind].ss;
        if (to == p) continue;
        if (used[to]) {
            up[cur] = min(up[cur], height[to]);
        }
        else {
            height[to] = height[cur] + 1;
            dfs_bridges(to, cur);
            up[cur] = min(up[cur], up[to]);
            if (up[to] > height[cur])
                bridges.push_back(ind);
        }
    }
}

```

// Algorithm: ECC
// Complexity: $O(N + M)$

```

vector<int> st;

```

```

vector<int> add_comp(vector<int> &st, int sz) {
    vector<int> comp;

```

```

    while (st.size() > sz) {
        comp.push_back(st.back());
        st.pop_back();
    }

```

```

    return comp;
}

```

```

vector<vector<int>> ecc;

```

```

void dfs_bridges_comps(int cur, int p = -1) {
    used[cur] = 1;
    up[cur] = height[cur];

    for (auto &ind : g[cur]) {
        int to = cur ^ edges[ind].ff ^ edges[ind].ss;
        if (to == p) continue;
        if (used[to]) {
            up[cur] = min(up[cur], height[to]);
        }
        else {
            int sz = st.size();
            st.push_back(to);
            height[to] = height[cur] + 1;
            dfs_bridges_comps(to, cur);
            up[cur] = min(up[cur], up[to]);
            if (up[to] > height[cur])
                ecc.push_back(add_comp(st, sz));
        }
    }
}

```

```

    }
}

```

7.7 Articulation Points

// Theme: Articulation Points And VCC

```

vector<pair<int, int>> edges;
vector<vector<int>> graph;
vector<int> used;

```

```

vector<int> height;
vector<int> up;

```

// Algorithm: Articulation Points
// Complexity: $O(N + M)$

```

set<int> art_points;

```

```

void dfs_artics(int cur, int p = -1) {
    used[cur] = 1;
    up[cur] = height[cur];

    int desc_count = 0;

    for (auto &ind : g[cur]) {
        int to = cur ^ edges[ind].ff ^ edges[ind].ss;
        if (to == p) continue;
        if (used[to]) {
            up[cur] = min(up[cur], height[to]);
        }
        else {
            desc_count++;
            height[to] = height[cur] + 1;
            dfs_artics(to, cur);
            up[cur] = min(up[cur], up[to]);
            if (up[to] >= height[cur] && p != -1)
                art_points.insert(cur);
        }
    }

    if (p == -1 && desc_count > 1) {
        art_points.insert(cur);
    }
}

```

// Algorithm: VCC
// Complexity: $O(N + M)$

```

vector<vector<int>> vcc;

```

```

void dfs_artics_comps(int cur, int p = -1) {
    used[cur] = 1;
    up[cur] = height[cur];

    for (auto &ind : g[cur]) {
        int to = cur ^ edges[ind].ff ^ edges[ind].ss;
        if (to == p) continue;
        if (used[to]) {
            up[cur] = min(up[cur], height[to]);
            if (height[to] < height[cur]) st.push_back(ind);
        }
        else {
            int sz = st.size();
            st.push_back(ind);
            height[to] = height[cur] + 1;
            dfs_artics_comps(to, cur);
            up[cur] = min(up[cur], up[to]);
            if (up[to] >= height[cur])
                vcc.push_back(add_comp(st, sz));
        }
    }
}

```

7.8 Kuhn

// Maximum Matching

// Algorithm: Kuhn Algorithm
// Complexity: $O(|\text{Left Part}| \cdot \Delta)$

```

vector<vector<int>> bigraph;
vector<int> used;

vector<int> mt;

bool kuhn(int u) {
    if (used[u]) return false;

    used[u] = 1;

    for (auto &v : bigraph[u]) {
        if (mt[v] == -1 || kuhn(mt[v])) {
            mt[v] = u;
            return true;
        }
    }

    return false;
}

```

7.9 Kruskal

```

// Theme: Minimum Spanning Tree

int sz;

vector<edge> edges;
vector<vector<int>> graph;

// Algorithm: Kruskal Algorithm
// Complexity: O(M)

vector<edge> mst;

void kruskal() {
    dsu d(sz);

    auto tedges = edges;
    sort(all(tedges), [](edge &e1, edge &e2) { return e1.w
        < e2.w; });

    for (auto &e : tedges) {
        if (d.get(e.u) != d.get(e.v)) {
            mst.push_back(e);
            d.unite(e.u, e.v);
        }
    }
}

```

7.10 Lowest Common Ancestor

```

// Theme: Lowest Common Ancestor

// Algorithm: Binary Lifting
// Complexity: O(N * log(N) * log(N))

vector<vector<int>> graph;
vector<vector<int>> up;
vector<int> tin, tout;

int timer;

// l == log(N) (~20)
int l;

void dfs(int cur, int p = -1) {
    tin[cur] = timer++;

    up[cur][0] = p;
    for (int i = 1; i <= l; i++)
        up[cur][i] = up[up[cur][i - 1]][i - 1];

    for (auto &to : graph[cur]) {
        if (to == p) continue;
        dfs(to, cur);
    }

    tout[cur] = timer++;
}

void preprocess(int u) {
    l = (int) ceil(log2(sz));
}

```

```

up.assign(sz, vector<int>(l + 1));
timer = 0;
dfs(u, u);
}

bool is_anc(int u, int v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}

int lca(int u, int v) {
    if (is_anc(u, v))
        return v;
    if (is_anc(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!is_anc(up[v][i], u))
            v = up[v][i];
    }
    return up[v][0];
}

```

7.11 Shortest Paths

```

// Theme: Shortest Paths

int sz;

vector<edge> edges;
vector<vector<int>> graph;

// Algorithm: Dijkstra Algorithm
// Complexity: O(M*log(N))

vector<int> d;
vector<int> p;

void dijkstra(int u) {
    d.assign(sz, INF); d[u] = 0;
    p.assign(sz, -1);

    priority_queue<pair<int, int>> q;
    q.push({ 0, u });

    while (q.size()) {
        int dist = -q.top().ff, v = q.top().ss; q.pop();

        if (dist > d[v]) continue;

        for (auto &ind : graph[v]) {
            int to = v ^ edges[ind].u ^ edges[ind].v,
                w = edges[ind].w;
            if (d[v] + w < d[to]) {
                d[to] = d[v] + w;
                p[to] = v;
                q.push({ -d[to], -to });
            }
        }
    }
}

// Algorithm: Shortest Path Faster Algorithm
// Complexity: ...

vector<int> d;

void bfs_spfa(int u) {
    d.assign(sz, INF); d[u] = 0;

    queue<int> q; q.push(u);
    vector<int> in_q(sz, 0); in_q[u] = 1;

    while (q.size()) {
        auto [v, f] = q.front(); q.pop();

        in_q[v] = 0;

        for (auto &ind : graph[v]) {
            int to = v ^ edges[ind].u ^ edges[ind].v,
                w = edges[ind].w;
            if (d[v] + w < d[to]) {
                d[to] = d[v] + w;
                if (!in_q[to]) {
                    in_q[to] = 1;
                    q.push(to);
                }
            }
        }
    }
}

```

```

    }
  }
}

// Algorithm: Belman-Ford Algorithm
// Complexity: (N*M)

vector<int> d;

void bfa(int u) {
  d.assign(sz, INF); d[u] = 0;

  for (;;) {
    bool any = false;

    for (auto &e : edges) {
      if (d[e.u] != INF && d[e.u] + e.w < d[e.v]) {
        d[e.v] = d[e.u] + e.w;
        any = true;
      }
      if (d[e.v] != INF && d[e.v] + e.w < d[e.u]) {
        d[e.u] = d[e.v] + e.w;
        any = true;
      }
    }

    if (!any) break;
  }
}

// Algorithm: Floyd-Warshall Algorithm
// Complexity: O(N^3)

vector<vector<int>>> d;

void fwa() {
  d.assign(sz, vector<int>(sz, INF));

  for (int i = 0; i < sz; i++)
    for (int j = 0; j < sz; j++)
      for (int k = 0; k < sz; k++)
        if (d[i][k] != INF && d[k][j] != INF)
          d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
}

```

7.12 Maximum Flow

```

// Theme: Maximum Flow

int s, t, sz;

vector<edge> edges;
vector<vector<int>>> fgraph;

vector<int> used;

// Algorithm: Ford-Fulkerson Algorithm
// Complexity: O(MF)

int dfs_fordfulk(int u, int bound, int flow = INF) {
  if (used[u]) return 0;
  if (u == t) return flow;

  used[u] = 1;

  for (auto &ind : fgraph[u]) {
    auto &e = edges[ind];
    &_e = edges[ind ^ 1];
    int to = e.to, res = e.res();

    if (res < bound) continue;

    int pushed = dfs_fordfulk(to, bound, min(res, flow));
    ;

    if (pushed) {
      e.flow += pushed;
      _e.flow -= pushed;
      return pushed;
    }
  }

  return 0;
}

```

```

// Algorithm: Edmonds-Karp Algorithm
// Complexity: O(N(M^2))

vector<int> p;
vector<int> pe;

void augment(int pushed) {
  int cur = t;
  while (cur != s) {
    auto &e = edges[pe[cur]];
    &_e = edges[pe[cur] ^ 1];
    e.flow += pushed;
    _e.flow -= pushed;
    cur = p[cur];
  }
}

int bfs_edmskarp(int u, int bound) {
  p.assign(sz, -1);
  pe.assign(sz, -1);

  int pushed = 0;

  queue<pair<int, int>> q;
  q.push({ u, INF });

  used[u] = 1;

  while (q.size()) {
    auto [v, f] = q.front(); q.pop();

    for (auto &ind : fgraph[v]) {
      auto &e = edges[ind];
      int to = e.to, res = e.res();

      if (used[to] || res < bound) continue;

      p[to] = v;
      pe[to] = ind;
      used[to] = 1;

      if (to == t) {
        pushed = min(f, res);
        break;
      }

      q.push({ to, min(f, res) });
    }
  }

  if (pushed)
    augment(pushed);

  return pushed;
}

// Algorithm: Dinic Algorithm
// Complexity: O((N^2)M)

vector<int> d;

bool bfs_dinic(int u, int bound) {
  d.assign(sz, INF); d[u] = 0;

  queue<int> q; q.push(u);

  while (q.size()) {
    int v = q.front(); q.pop();

    for (auto &ind : fgraph[v]) {
      auto &e = edges[ind];
      int to = e.to, res = e.res();

      if (d[v] + 1 >= d[to] || res < bound) continue;

      d[to] = d[v] + 1;
      q.push(to);
    }
  }

  return d[t] != INF;
}

vector<int> lst;

int dfs_dinic(int u, int mx = INF) {
  if (u == t) return mx;
}

```

```

int smf = 0;

for (int i = lst[u]; i < fgraph[u].size(); i++) {
    int ind = fgraph[u][i];

    auto &e = edges[ind],
        &_e = edges[ind ^ 1];
    int to = e.to, res = e.res();

    if (d[to] == d[u] + 1 && res) {
        int pushed = dfs_dinic(to, min(res, mx - smf));

        if (pushed) {
            smf += pushed;
            e.flow += pushed;
            _e.flow -= pushed;
        }
    }

    lst[u]++;

    if (smf == mx)
        return smf;
}

return smf;
}

int dinic(int u) {
    int pushed = 0;

    for (int bound = 1; bound <= 30; bound << 1) {
        while (true) {
            bool bfs_ok = bfs_dinic(u, bound);
            if (!bfs_ok) break;

            lst.assign(sz, 0);

            while (true) {
                int dfs_pushed = dfs_dinic(u);
                if (!dfs_pushed) break;

                pushed += dfs_pushed;
            }
        }
    }

    return pushed;
}

// Algorithm: Maximum Flow Of Minimum Cost (SPFA)
// Complexity: ...

vector<int> d;
vector<int> p;
vector<int> pe;

void augment(int pushed) {
    int cur = t;
    while (cur != s) {
        auto &e = edges[pe[cur]],
            &_e = edges[pe[cur] ^ 1];
        e.flow += pushed;
        _e.flow -= pushed;
        cur = p[cur];
    }
}

int bfs_spfa(int u, int flow = INF) {
    d.assign(sz, INF); d[u] = 0;
    p.assign(sz, -1);
    pe.assign(sz, -1);

    queue<pair<int, int>> q; q.push({ u, flow });
    vector<int> in_q(sz, 0); in_q[u] = 1;

    int pushed = 0;

    while (q.size()) {
        auto [v, f] = q.front(); q.pop();

        in_q[v] = 0;

        if (v == t) {
            pushed = f;
            break;
        }
    }
}

```

```

}

for (auto &ind : fgraph[v]) {
    auto &e = edges[ind];
    int to = e.to, res = e.res(),
        w = e.weight;

    if (d[v] + w >= d[to] || !res) continue;

    d[to] = d[v] + w;
    p[to] = v;
    pe[to] = ind;

    if (!in_q[to]) {
        in_q[to] = 1;
        q.push({ to, min(f, res) });
    }
}

if (pushed)
    augment(pushed);

return pushed;
}

```

7.13 Eulerian Path

```

// Theme: Eulerian Path

int sz;

vector<vector<int>> graph;

// Algorithm: Eulerian Path
// Complexity: O(M)

vector<int> eul;

// 0 - path not exist
// 1 - cycle exists
// 2 - path exists
int euler_path() {
    vector<int> deg(sz);

    for (int i = 0; i < sz; i++)
        for (int j = 0; j < sz; j++)
            deg[i] += g[i][j];

    int v1 = -1, v2 = -1;
    for (int i = 0; i < sz; i++)
        if (deg[i] & 1)
            if (v1 == -1) v1 = i;
            else if (v2 == -1) v2 = i;
            else return 0;

    if (v1 != -1) {
        if (v2 == -1)
            return 0;
        graph[v1][v2]++;
        graph[v2][v1]++;
    }

    stack<int> st;

    for (int i = 0; i < sz; i++) {
        if (deg[i]) {
            st.push(i);
            break;
        }
    }

    while (st.size()) {
        int u = st.top();

        int ind = -1;

        for (int i = 0; i < sz; i++)
            if (graph[u][i]) {
                ind = i;
                break;
            }

        if (ind == -1) {
            eul.push_back(u);
        }
    }
}

```

```

        st.pop();
    }
    else {
        graph[u][ind]--;
        graph[ind][u]--;
        st.push(ind);
    }
}

int res = 2;
if (v1 != -1) {
    res = 1;

    for (int i = 0; i < eul.size() - 1; i++)
        if (eul[i] == v1 && eul[i + 1] == v2 ||
            eul[i] == v2 && eul[i + 1] == v1) {
            vector<int> teul;
            for (int j = i + 1; j < eul.size(); j++)
                teul.push_back(eul[j]);
            for (int j = 0; j <= i; j++)
                teul.push_back(eul[j]);
            eul = teul;
            break;
        }

    for (int i = 0; i < sz; i++)
        for (int j = 0; j < sz; j++)
            if (graph[i][j])
                return 0;

    return res;
}

```

```

    else
        l = m1;
}

return min(f(l), min(f(l + 1), f(r)));
}

```

8 Miscellaneous

8.1 Ternary Search

// Theme: Ternary Search

// Algorithm: Continuous Search With Golden Ratio
 // Complexity: $O(\log(N))$

// Golden Ratio
 // Phi = 1.618...

double phi = (1 + sqrt(5)) / 2;

```

double cont_tern_srch(double l, double r) {
    double m1 = l + (r - l) / (1 + phi);
    double m2 = r - (r - l) / (1 + phi);

```

```

    double f1 = f(m1), f2 = f(m2);

```

```

    int count = 200;
    while (count-- > 0) {
        if (f1 < f2) {
            r = m2;
            m2 = m1;
            f2 = f1;
            m1 = l + (r - l) / (1 + phi);
            f1 = f(m1);
        }
        else {
            l = m1;
            m1 = m2;
            f1 = f2;
            m2 = r - (r - l) / (1 + phi);
            f2 = f(m2);
        }
    }

```

```

    return f((l + r) / 2);
}

```

// Algorithm: Discrete Search
 // Complexity: $O(\log(N))$

```

double discr_tern_srch(int l, int r) {
    while (r - l > 2) {
        int m1 = l + (r - l) / 3;
        int m2 = r - (r - l) / 3;
        if (f(m1) < f(m2))
            r = m2;
    }
}

```