A Practical Use of Principal Component Analysis

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Dimensionality Reduction: Why?

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Team	W	D	L	G	GA	GD
Liverpool	32	3	3	85	33	52
Manchester City	26	3	9	102	35	67
Manchester United	18	12	8	66	36	30
Chelsea	20	6	12	69	54	15
Leicester City	18	8	12	67	41	26
Tottenham Hotspur	16	11	11	61	47	14
Wolverhampton	15	14	9	51	40	11
Arsenal	14	14	10	56	48	8
Sheffield United	14	12	12	39	39	0
Burnley	15	9	14	43	50	-7

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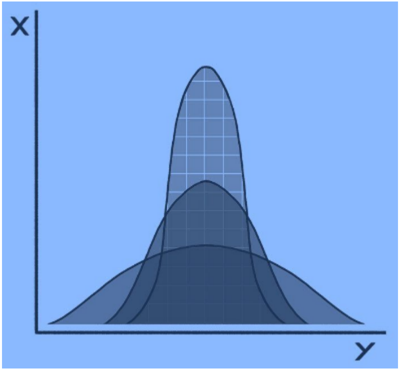
Variance

The measure of the spread of data within a data set.



Covariance Matrix

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$$C = \begin{bmatrix} cov(x_1, x_1) & \dots & cov(x_1, x_n) \\ \vdots & \ddots & \vdots \\ cov(x_n, x_1) & \dots & cov(x_n, x_n) \end{bmatrix}$$

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Note: The values are listed such that $\lambda_1 \geq ... \geq \lambda_n$

Interpreting the Principal Components

Principal Component 1 to n

The first principal component (corresponding to λ_1) explains the *highest* proportion of the variability in the data set. The nth component accounts for the *lowest* proportion of variability.

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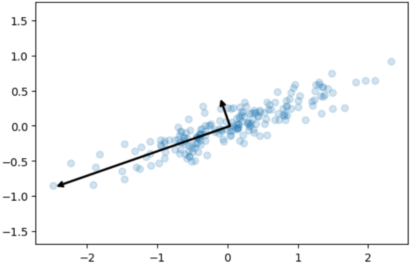

Figure: Eigenvectors (Principal Components) point in direction of highest variance

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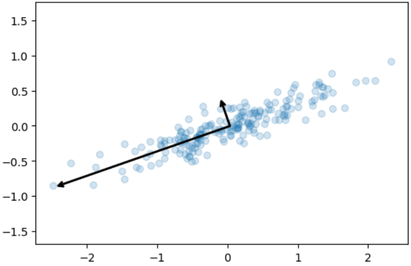

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We can choose to keep some p (with $p \leq n$) number of PC's to reduce our data's dimension!

A Quick Demonstration

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Figure: 2019-2020 Premier League data

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$$D = \text{diag}(1300,\ 71.9,\ 8.05,\ 4.62,\ -2.65e-14,\ -3.73e-14)$$

- Two of our eigenvalues come out very close to zero!

- We can see the proportion of variability explained by each eigenvalue if we take

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$$P = (0.939,\ 0.052,\ 0.00583,\ 0.00334,\ -192e-17,\ -2.7e-17)$$

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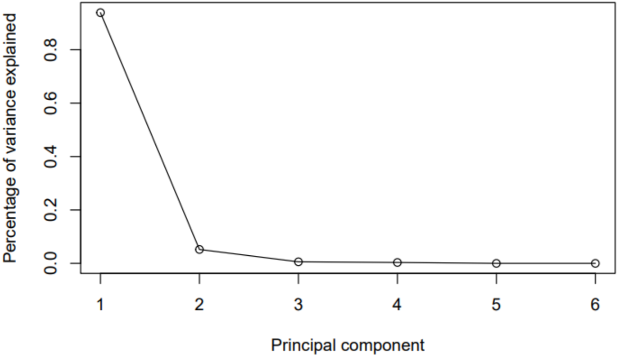

Figure: Scree Plot of our PCs

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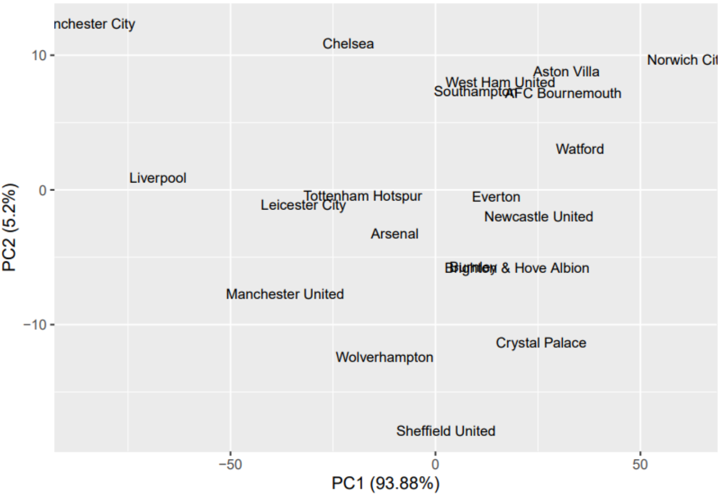

Figure: Visualization of differences between teams

Thank you for listening!