A MATLAB Script for Creating Pork Chop Plots of Ballistic Earth-to-Mars Trajectories

This document describes a MATLAB script called `porkchop` that can be used to create and plot interplanetary "pork chop" plots for Type I and Type II Earth-to-Mars trajectories. These plots illustrate the behavior of launch energy (C3L), right ascension (RLA) and declination (DLA) of the departure hyperbola, time-of-flight, and arrival v-infinity, DLA and RLA, and total heliocentric scalar delta-v for a range of user-defined launch and arrival calendar dates.

The data required for these contour plots is created by solving the heliocentric, two-body "patched-conic" Lambert problem. A patched-conic trajectory ignores the gravitational effect of both the launch and arrivals planets on the heliocentric trajectory. Type I trajectories are characterized by heliocentric Earth-to-Mars transfer angles which are less than 180 degrees while Type II trajectories have transfer angles greater than 180 degrees. Pork chop plots are typically used for preliminary mission analysis.

The angular coordinates of the launch hyperbola (DLA and RLA) are provided in the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The angular coordinates of the arrival hyperbola (DLA and RLA) are provided in the Mars mean equator and IAU node of epoch coordinate system. The total delta-v is the sum of the heliocentric departure and arrival scalar delta-v values. The `porkchop` script can be easily modified for other departure and arrival planets.

Typical user interaction

The following is typical user interaction with this MATLAB application. This example creates typical data and plots for the Phoenix 2007 mission. In the following discussion the user inputs are in bold *courier* font and all explanations are in *times italic* font.

```
program porkchop

< interplanetary pork chop plots >
```

The first user input is the nominal launch calendar date in the order month, day, and year. Please be sure to include all four digits of the calendar year.

```
nominal launch date

please input the calendar date
(1 <= month <= 12, 1 <= day <= 31, year = all digits!)
? 9,25,2007
```

The next user input is the calendar date of the nominal arrival at Mars.

```
nominal arrival date

please input the calendar date
(1 <= month <= 12, 1 <= day <= 31, year = all digits!)
? 8,15,2008
```

The next input is the analysis "span" centered about the nominal launch date. This number should be input in days. The software will create data over the range (nominal – span) and (nominal + span).

```
please input the launch date span in days
```

```
? 60
```

The next input is the time span centered about the nominal arrival date. This number should also be input in days.

```
please input the arrival date span in days
? 180
```

The next input is the step size for the parametric scan. This number should be input in days and need not be an integer. A value between 2 and 5 days is recommended.

```
please input the step size in days
? 1
```

The next six inputs are MATLAB vectors that define the contour levels for each flight parameter. To accept default values for the contour levels, the user should input []. Otherwise, the input should be in the format [first level, second level, third level, …, last level].

```
please input the launch energy contour levels in km^2/sec^2 ([] for defaults)
? []
```

The default launch energy contour levels are

[6,7,8,9,10,11,12,13,14,15,20,25,30,35,40,45,50]

```
please input the arrival v-infinity contour levels in km/sec ([] for defaults)
? []
```

The default arrival v-infinity contour levels are

[1.5,1.8,2.0,2.1,2.2,2.3,2.4,2.5,2.6,2.7,2.8,2.9,3.0,3.5,4.0,5.0,6.0,7.0,8.0]

```
please input the launch and arrival declination contour levels in degrees ([] for
defaults)
? []
```

The default launch and arrival declination contour levels are

[-30, -25, -20, -15, -10, -5, 0, 5, 10, 15, 20, 25, 30]

```
please input the launch and arrival right ascension contour levels in degrees ([]
for defaults)
? []
```

The default launch and arrival right ascension contour levels are

[0, 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, 195, 210, 225, 240, 255, 270, 285, 300, 315, 330]

```
please input the time-of-flight contour levels in days ([] for defaults)
? []
```

The default time-of-flight contour levels are

[100, 150, 200, 250, 300, 350, 400]

```
please input the total delta-v contour levels in kilometers/second ([] for
defaults)
? []
```

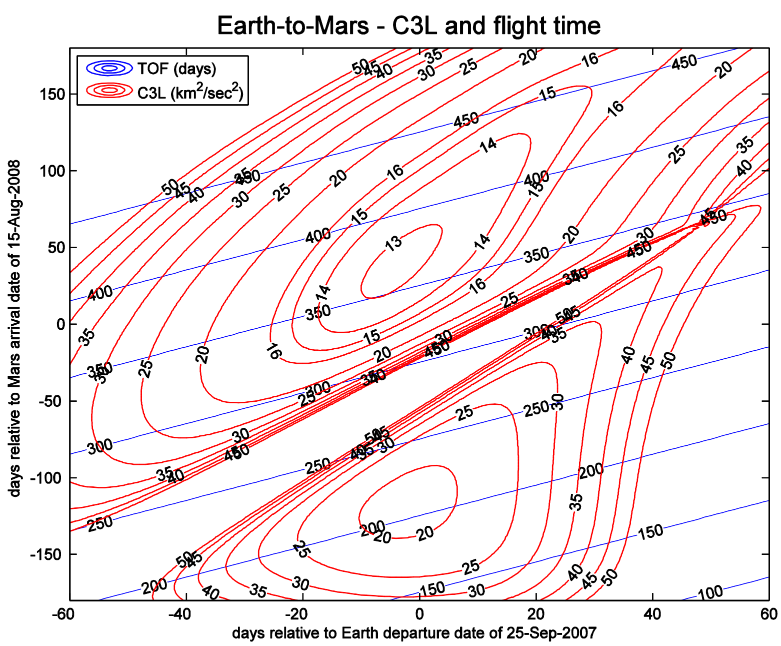
The default total delta-v contour levels are

[5, 5.5, 6, 6.5, 7, 7.5, 8, 8.5, 9, 9.5, 10, 10.5, 11, 11.5, 12.0, 13.0, 14.0, 15.0]

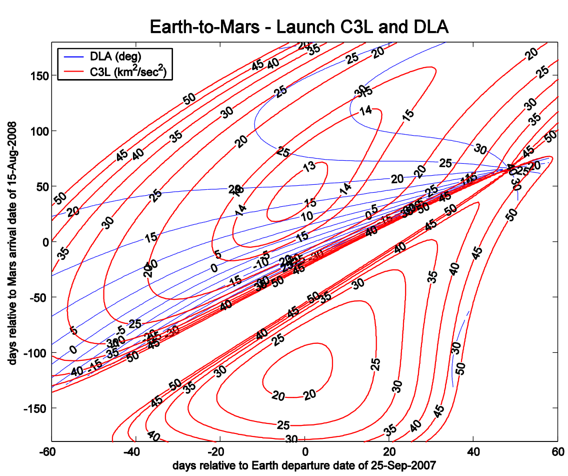
Of course, the user can directly edit the MATLAB source code and change any default values.

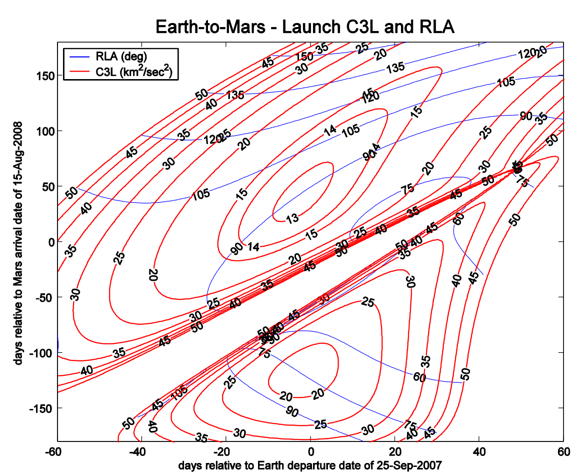
Pork chop graphics

This section contains pork chop plots for this example. The first contour plot summarizes the behavior of the launch energy (C3L) and the flight time in days.

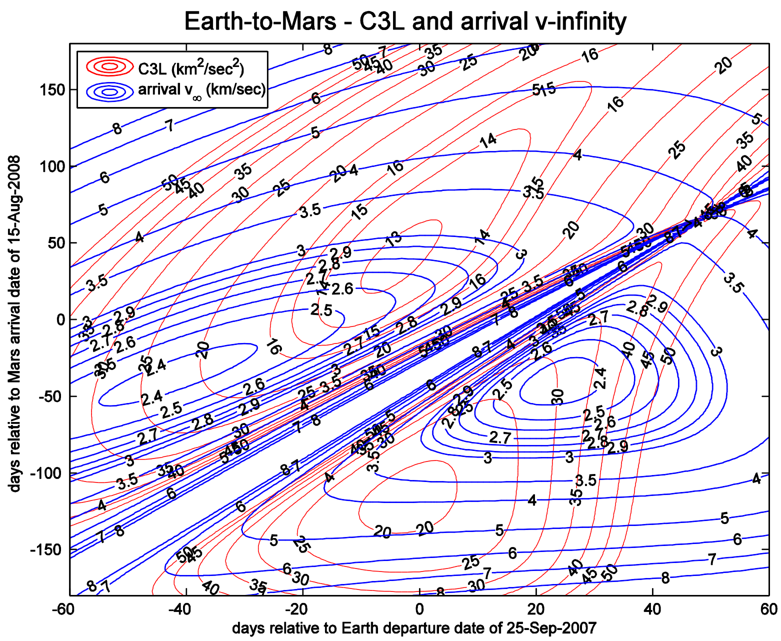


The next two plots characterize the declination and right ascension of the launch hyperbola.

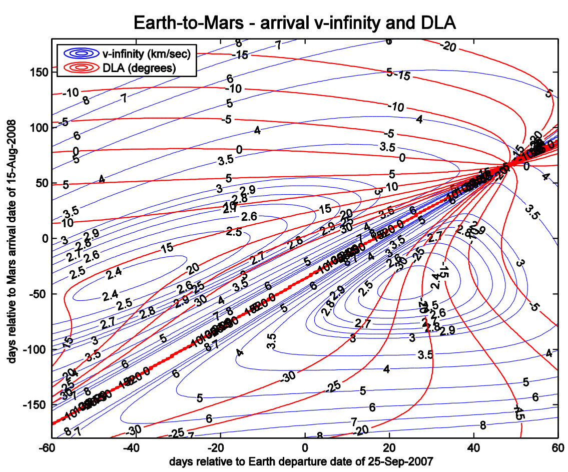


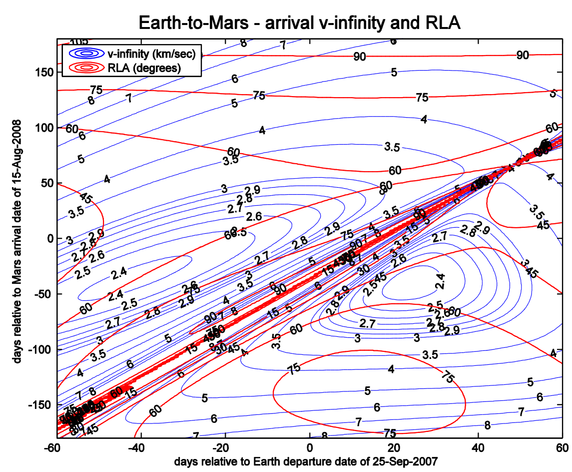


This plot summarizes the arrival v-infinity at Mars as a function of the launch and arrival calendar dates.



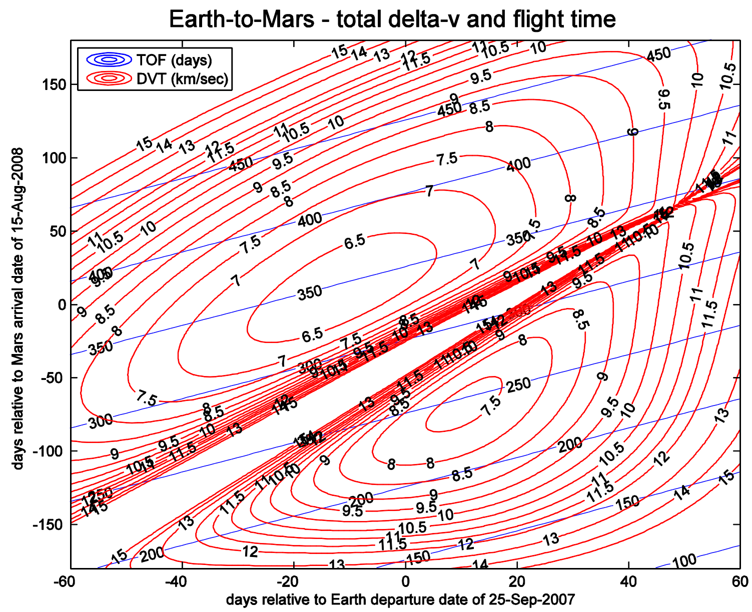
The next two plots illustrate the behavior of the orientation (DLA and RLA) of the incoming hyperbola at Mars.





Please note that the declination and right ascension of the incoming hyperbolic trajectory are displayed in the Mars mean equator and IAU node of epoch coordinate system. The coordinate transformation used to compute these flight characteristics is described in Appendix A.

The final plot illustrates the total heliocentric scalar delta-v required.



Technical Discussion

A solution for the launch and arrival impulsive delta-v vectors can be determined from the solution of the Lambert two-point boundary-value problem (TPBVP). Lambert's Theorem states that the time to traverse a trajectory depends only upon the length of the semimajor axis a of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length c of the chord joining these two positions.

Lambert's Problem

Lambert's problem is concerned with the determination of an orbit that passes between two positions within a specified time-of-flight. This classic astrodynamic problem is also known as the orbital two-point boundary value problem (TPBVP).

The time to traverse a trajectory depends only upon the length of the semimajor axis a of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length c of the chord joining these two positions. This relationship can be stated as follows:

$$tof = tof\left(r_i + r_f, c, a\right)$$

From the following form of Kepler's equation

$$t - t_0 = \sqrt{\frac{a^3}{\mu}}\left(E - e\sin E\right)$$

we can write

$$t = \sqrt{\frac{a^3}{\mu}}\left[E - E_0 - e\left(\sin E - \sin E_0\right)\right]$$

where E is the eccentric anomaly associated with radius r, E_0 is the eccentric anomaly at r_0, and $t = 0$ when $r = r_0$.

At this point we need to introduce the following trigonometric sun and difference identities:

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{a+\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha-\beta}{2}\sin\frac{a+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha-\beta}{2}\cos\frac{a+\beta}{2}$$

If we let $E = \alpha$ and $E_0 = \beta$ and substitute the first trig identity into the second equation above, we have the following equation:

$$t = \sqrt{\frac{a^3}{\mu}}\left\{E - E_0 - 2\sin\frac{E-E_0}{2}\left(e\cos\frac{E+E_0}{2}\right)\right\}$$

With the two substitutions given by

$$e\cos\frac{E+E_0}{2} = \cos\frac{\alpha+\beta}{2}$$

$$\sin\frac{E-E_0}{2} = \sin\frac{\alpha-\beta}{2}$$

the time equation becomes

$$t = \sqrt{\frac{a^3}{\mu}\left\{(\alpha-\beta) - 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}\right\}}$$

From the elliptic relationships given by

$$r = a(1 - e\cos E)$$

$$x = a(\cos E - e)$$

$$y = a\sin E\sqrt{1-e^2}$$

and some more manipulation, we have the following two equations:

$$\cos\alpha = \left(1 - \frac{r+r_0}{2a}\right) - \frac{c}{2a} = 1 - \frac{r+r_0+c}{2a} = 1 - \frac{s}{a}$$

$$\sin\beta = \left(1 - \frac{r+r_0}{2a}\right) + \frac{c}{2a} = 1 - \frac{r+r_0-c}{2a} = 1 - \frac{s-c}{a}$$

This part of the derivation makes use of the following three relationships:

$$\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}=1-\frac{r+r_0}{2}$$

$$\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}=\sin\frac{E-E_0}{2}\sqrt{1-\left(e\cos\frac{E+E_0}{2}\right)^2}$$

$$\left(\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}\right)^2=\left(\frac{x-x_0}{2a}\right)^2+\left(\frac{y-y_0}{2a}\right)^2=\left(\frac{c}{2a}\right)^2$$

With the use of the half angle formulas given by

$$\sin\frac{\alpha}{2}=\sqrt{\frac{s}{2a}} \qquad \sin\frac{\beta}{2}=\sqrt{\frac{s-c}{2a}}$$

and several additional substitutions, we have the time-of-flight form of Lambert's theorem

$$t=\sqrt{\frac{a^3}{\mu}}\left[(\alpha-\beta)-(\sin\alpha-\sin\beta)\right]$$

A discussion about the angles α and β can be found in "Geometrical Interpretation of the Angles α and β in Lambert's Problem" by J. E. Prussing, AIAA *Journal of Guidance and Control*, Volume 2, Number 5, Sept.-Oct. 1979, pages 442-443.

The algorithm used in this MATLAB script is based on the method described in "A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem" by R. H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48:** 145-165, 1990. This iterative solution is valid for elliptic, parabolic and hyperbolic transfer orbits which may be either posigrade or retrograde, and involve one or more revolutions about the central body.

The heliocentric $\Delta V's$ required at launch and arrival are simply the differences between the velocity on the transfer trajectory determined by the solution of Lambert's problem and the heliocentric velocities of the two planets. If we treat each planet as a point mass and assume *impulsive* maneuvers, the *planet-centered* magnitude and direction of the required maneuvers are given by the two vector equations:

$$\Delta\mathbf{V}_L=\mathbf{V}_{T_L}-\mathbf{V}_{P_L}$$

$$\Delta\mathbf{V}_A=\mathbf{V}_{P_A}-\mathbf{V}_{T_A}$$

where

$\mathbf{V}_{T_L}=$ heliocentric velocity vector of the transfer trajectory at launch

$\mathbf{V}_{T_A}=$ heliocentric velocity vector of the transfer trajectory at arrival

$\mathbf{V}_{P_L}=$ heliocentric velocity vector of the launch planet

$\mathbf{V}_{P_A}=$ heliocentric velocity vector of the arrival planet

The scalar magnitude of each maneuver is also called the "hyperbolic excess velocity" or v_∞ at launch and arrival. The hyperbolic excess velocity is the speed of the spacecraft relative to each planet at an *infinite* distance from the planet. Furthermore, the *energy* or C_3 at launch or arrival is equal to v_∞^2 for the respective maneuver. C_3 is also equal to <u>twice</u> the orbital energy per unit mass (the specific orbital energy). The orientation of the departure and arrival hyperbolas is specified in terms of the right ascension and declination of the asymptote. These coordinates can be calculated using the components of the v-infinity velocity vector.

The right ascension of the asymptote is determined from $\alpha = \tan^{-1}\left(v_{\infty_y}, v_{\infty_x}\right)$ and the geocentric declination of the asymptote is given by $\delta = 90^0 - \cos^{-1}\left(\hat{v}_{\infty_z}\right)$ where $\hat{v}_{\infty_z}$ is z-component of the unit v-infinity vector.

This MATLAB script models the planetary coordinates using the DE421 binary ephemeris from the Jet Propulsion Laboratory (JPL).

The DE421 ephemeris and the declination and right ascension of the departure v-infinity vector are computed with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this Earth-centered-inertial (ECI) inertial coordinate system is the geocenter and the fundamental plane is the Earth's mean equator. The z-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time (TT).

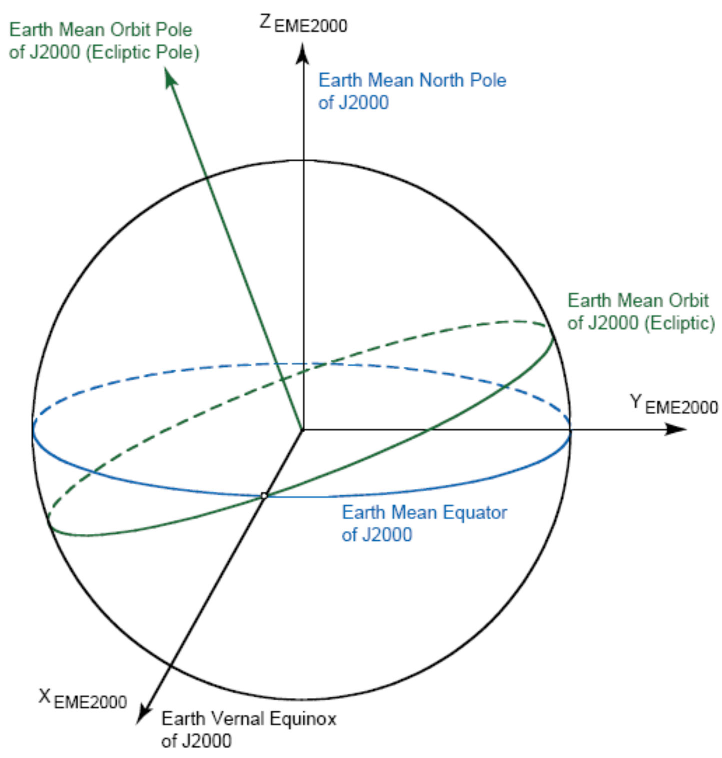


Figure 1. Earth mean equator and equinox of J2000 coordinate system

Algorithm Resources

"Modern Astrodynamics", Victor R. Bond and Mark C. Allman, Princeton University Press, 1996.

A. B. Sergeyevsky, G. C. Synder, and R. A. Cunniff, "Interplanetary Mission Design Handbook, Volume 1, Part 2", JPL Publication 82-43, September 15, 1983.

Explanatory Supplement to the Astronomical Almanac, Edited by P. K. Seidelmann, University Science Books, 1992.

"Update to Mars Coordinate Frame Definitions", R. A. Mase, JPL IOM 312.B/015-99, 15 July 1999.

R. H. Battin, *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA, 1987.

"The Planetary and Lunar Ephemeris DE 421", W. M. Folkner, J. G. Williams, D. H. Boggs, JPL IOM 343R-08-003, 31-March-2008.

"Report of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites: 2009", *Celestial Mechanics and Dynamical Astronomy*, **109**: 101-135, 2011.

"IERS Conventions (2003)", IERS Technical Note 32, November 2003.

"Planetary Constants and Models", R. Vaughan, JPL D-12947, December 1995.

Ryan C. Woolley and Charles W. Whetsel, "On the Nature of Earth-Mars Porkchop Plots", AAS 13-226, 23rd AAS/AIAA Space Flight Mechanics Meeting, Kauai, Hawaii, February 10-14, 2013.

S. Matousek and A. B. Sergeyevsky, "To Mars and Back: 2002-2020 Ballistic Trajectory Data for the Mission Architect", AIAA 98-4396, AIAA/AAS Astrodynamics Specialist Conference, Boston, MA, August 10-12, 1998.

L. E. George and L. D. Koss, "Interplanetary Mission Design Handbook: Earth-to-Mars Mission Opportunities and Mars-to-Earth Return Opportunities 2009-2024", NASA TM-1998-208533, 1998.

APPENDIX A

EME2000-to-Areocentric Coordinate Transformation

This appendix describes the transformation of coordinates between the Earth mean equator and equinox of J2000 (EME2000) and the areocentric (Mars-centered) mean equator and IAU node of epoch coordinate systems. This transformation is used to compute the right ascension and declination of the incoming asymptote at the time of Mars arrival.

A unit vector in the direction of the pole of Mars can be determined from

$$\hat{\mathbf{p}}_{Mars} = \begin{bmatrix} \cos\alpha_p \cos\delta_p \\ \sin\alpha_p \cos\delta_p \\ \sin\delta_p \end{bmatrix}$$

The IAU 2000 right ascension and declination of the pole of Mars in the Earth mean equator and equinox of J2000 (EME2000) coordinate system are given by the following expressions

$$\alpha_p = 317.68143 - 0.1061T$$

$$\delta_p = 52.88650 - 0.0609T$$

where T is the time in Julian centuries given by $T = \left(JD - 2451545.0\right)/36525$ and JD is the TDB Julian Date.

The unit vector in the direction of the *IAU-defined* x-axis is computed from

$$\hat{\mathbf{x}} = \hat{\mathbf{p}}_{J2000} \times \hat{\mathbf{p}}_{Mars}$$

where $\hat{\mathbf{p}}_{J2000} = \left[0\ 0\ 1\right]^T$ is unit vector in the direction of the pole of the J2000 coordinate system.

The unit vector in the y-axis direction of this coordinate system is

$$\hat{\mathbf{y}} = \hat{\mathbf{p}}_{Mars} \times \hat{\mathbf{x}}$$

Finally, the components of the matrix that transforms coordinates from the EME2000 system to the Mars-centered mean equator and IAU node of epoch system are as follows:

$$\mathbf{M} = \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{p}}_{Mars} \end{bmatrix}$$