Bayes Decision Theory

Assume $P(\underline{x}|s_i)$ is known $\forall i$

← can be estimated by density estimation

Assume $P(s_i)$ is known $\forall i$

← can be estimated by $\hat{P}(s_i) = \frac{N_i}{N}$

Minimum Error Classifier

$C=2$ $\quad P(\underline{x}|s_1)P(s_1) \underset{s_2}{\overset{s_1}{\gtrless}} P(\underline{x}|s_2)P(s_2)$

or $P(s_1|\underline{x}) \underset{s_2}{\overset{s_1}{\gtrless}} P(s_2|\underline{x})$ ← in terms of posterior prob.

$C>2$ $\quad P(\underline{x}|s_k)P(s_k) > P(\underline{x}|s_j)P(s_j) \quad \forall j \neq k$

then $\underline{x} \in s_k$

Minimum Risk

$$\sum_{j=1}^{C} L_{ji}\, p(s_j|\underline{x}) < \sum_{j=1}^{C} L_{jk}\, p(s_j|\underline{x}) \quad \forall k \neq i \quad \Rightarrow \underline{x} \in s_i$$

where $\underline{\underline{L}} = \begin{bmatrix} L_{11} & L_{12} & & \\ L_{21} & L_{22} & & \\ & & \ddots & \\ & & & L_{cc} \end{bmatrix}$, $L_{ii} = 0 \quad \forall i$

where L_{ij} = loss of assigning $\underline{x}$ to s_j when it actually belongs to i.

Ex: if $c=2$, then if $L_{11}P(s_1|\underline{x}) + L_{21}P(s_2|\underline{x}) < L_{12}P(s_1|\underline{x}) + L_{22}P(s_2|\underline{x})$

$\Rightarrow L_{21}P(s_2|\underline{x}) < L_{12}P(s_1|\underline{x}) \xrightarrow{\text{let } L_{21}=L_{12}} P(s_2|\underline{x}) < P(s_1|\underline{x})$

then $\underline{x} \in s_1$