K-nearest neighbors

Wrong

1. Randomly generate $\underbrace{k \text{ points}}$. $\in S_m$ ← middle points.
2. calculate distances between each point $\underline{x}_i$ and each point in S_m.
3. For each point $\underline{x}_i$, classify it to class p if $D_{ip} < D_{iq}$ $\forall p \neq q$

   D_{ip}, D_{iq}: distance between point $\underline{x}_i$ and point p or q in S_m.
4. For each class, recalculate new midde point and replace corresponding middle points in S_m.

KNN classification [discriminative Approch]

1. choose proper k
2. For $\underline{x}$

   Find k nearest points and count # of points in each class.

   e.g. $K=6$

   $\# x = 3$ $\quad S_i$

   $\# o = 2$ $\quad S_j$

   $\# \Delta = 1$ $\quad S_l$

   $\therefore \underline{x} \in S_i$

   if $\# \text{ of } p^{(i)} > \# \text{ of } p^{(j)}$ $\forall i \neq j$

   then $\underline{x} \in S_i$

Generative Approach

use $P(\underline{x}) = \dfrac{\frac{K}{N}}{V}$ for each class

then Bayes classifier

$$P(\underline{x}|s_i)P(s_i) \underset{s_j}{\overset{s_i}{\gtrless}} P(\underline{x}|s_j)P(s_j)$$

KNN regression [discriminative Approach]

e.g. $K=6$

① estimate $\hat{y}(\underline{x}) = \frac{1}{K}\sum_{n=1}^{K} y_n$ ← y_n is the value for points in column

② or, estimate $\hat{y}(\underline{x}) = \frac{1}{\sum_{n=1}^{K} w_n}\sum_{n=1}^{K} w_n y_n$ ← w_n is the weight of points.

e.g. $w_n = 1 - \frac{d(\underline{x}, \underline{x}_n)}{(1+\varepsilon)\, d_{max}}$

d_{max} : maximum distance between $\underline{x}$ and x_n in column.

$d(\underline{x}, \underline{x}_n)$: distance between $\underline{x}$ and $\underline{x}_n$ in column.

$\varepsilon > 0$ in case for $\underline{x}_i$: $d(\underline{x}, \underline{x}_i) = d_{max}$, then $w_n = 0$

e.g. $w_n = \frac{1}{\varepsilon + d(\underline{x}, \underline{x}_n)}$

$\varepsilon > 0$ in case for $\underline{x}_i$: $d(\underline{x}, \underline{x}_i) \to 0$, then $w_n \to \infty$

Note: $d(\underline{x}, \underline{x}_n)$ could be
- Euclidean $\|\underline{x}_n - \underline{x}\|_2$
- Mahalanobis $\left[(\underline{x}-\underline{x}_n)^T \underline{\underline{\Sigma}}_{\underline{x}}^{-1} (\underline{x}-\underline{x}_n)\right]^{\frac{1}{2}}$ ← estimated from data

option 1: $\underline{\underline{\Sigma}}_{\underline{x}} = \mathrm{diag}\{\sigma_1^2, \cdots, \sigma_D^2\}$

$\hat{\sigma}_i^2 = \frac{1}{N_{Tr}-1}\sum_{n=1}^{N_{Tr}} (x_{ni} - \bar{x}_i)^2$

$\bar{x}_i = \frac{1}{N_{Tr}}\sum_{n=1}^{N_{Tr}} x_{ni}$

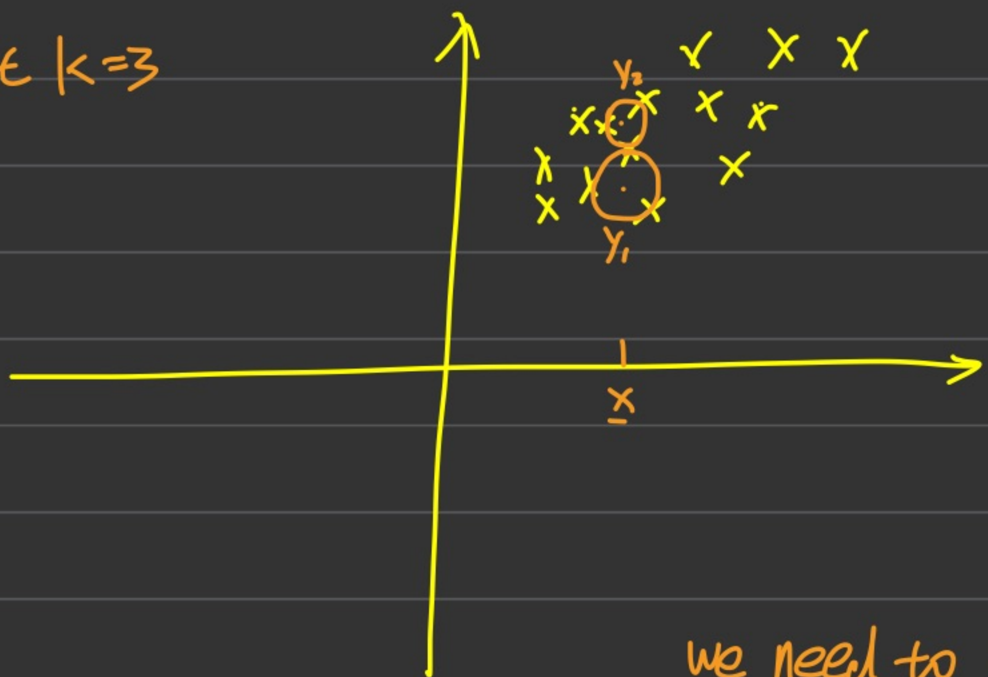
option 2: $\underline{\underline{\Sigma}}_{\underline{x}} = \frac{1}{N_{Tr}-1}\sum_{n=1}^{N_{Tr}} (\underline{x}_n - \bar{\underline{x}})(\underline{x}_n - \bar{\underline{x}})^T$

$\bar{\underline{x}} = \frac{1}{N_{Tr}}\sum_{n=1}^{N_{Tr}} \underline{x}_n$

KNN Regression [Generative Approach] ← not mentioned in lecture (just my thoughts)

use $P(y|\underline{x}) = \frac{k}{V}$ for different y on $\underline{x}$

let $k=3$



$P(y_1|\underline{x}) = \frac{3}{31}$

$P(y_2|\underline{x}) = \frac{3}{22}$

$\therefore$ y_2 is more likely than y_1

we need to calculate all y_n for $\underline{x}$ and take the one with largest probability.