Mixed-Integer Linear Programming (MILP)

MILP extends LP by allowing some decision variables to take only integer values. This is useful for problems where decisions must be whole numbers (e.g., number of machines, assignment of workers, on/off binary decisions). MILP problems are formulated as:

$$\text{Maximize (or Minimize)} \quad c^T x$$

$$\text{Subject to} \quad Ax \leq b, \quad x_i \in \mathbb{Z} \text{ (for some } i)$$

Key Differences Between LP and MILP:

- **Complexity:** MILP is NP-hard, meaning it is computationally more challenging than LP.
- **Solving Methods:** MILP is solved using **Branch and Bound, Cutting Plane, and Branch and Cut** methods instead of the simplex method alone.
- **Discrete vs. Continuous Variables:** LP has continuous variables, while MILP has a mix of continuous and integer variables.

MILP is widely used in scheduling, logistics, network design, and energy system optimization where discrete decisions are required.

Solving Methods for MILP vs. Simple LP

Linear Programming (LP) problems can be solved efficiently using methods like:

1. **Simplex Method:** Iteratively moves from one extreme point to another until the optimal solution is reached.
2. **Interior-Point Methods:** Uses matrix computations to find the optimal solution in polynomial time.
3. **Graphical Method:** Only applicable for two-variable problems by visually identifying the feasible region.

However, **Mixed-Integer Linear Programming (MILP)** problems are more complex due to the presence of integer constraints, requiring specialized techniques:

Solving Methods for MILP

1. **Branch and Bound (B&B):**
   - Divides the problem into smaller subproblems by relaxing integer constraints (solving them as LPs).
   - If a non-integer solution is found, it branches into new subproblems with additional constraints.
   - The best feasible integer solution is selected.
2. **Cutting Plane Method:**
   - Starts by solving the relaxed LP (ignoring integer constraints).
   - Adds linear inequalities (cuts) to eliminate non-integer solutions while preserving feasible integer solutions.
   - Repeats until an integer solution is reached.
3. **Branch and Cut:**
   - A combination of **Branch and Bound** and **Cutting Plane** methods.
   - Solves subproblems via branching while adding cuts dynamically to improve efficiency.
4. **Heuristics & Metaheuristics (for large MILPs):**
   - Includes Genetic Algorithms, Simulated Annealing, and Tabu Search.
   - Provides near-optimal solutions quickly when exact methods are too slow.

Differences Between LP and MILP Solving Methods

Aspect	LP (Linear Programming)	MILP (Mixed-Integer Linear Programming)
Solution Complexity	Solvable in polynomial time (P)	NP-hard (exponential complexity)
Variables	Continuous (real numbers)	Some variables must be integers
Solving Methods	Simplex, Interior-Point	Branch & Bound, Branch & Cut, Cutting Plane
Computational Cost	Relatively low	High, especially for large problems
Guaranteed Optimality	Always	Sometimes requires heuristics