Problem Statement:

A company produces **three types of products**: **A, B, and C**. The production requires **three types of resources**: **Raw Material, Skilled Labor, and Machine Hours**. The objective is to **maximize profit**, considering the **limited availability of resources**.

Product	Profit (per unit)	Raw Material (kg)	Skilled Labor (hours)	Machine Hours (hours)
A	60	4	10	5
B	45	6	4	4
C	80	5	6	6

The **total available resources** are:

- **Raw Material**: 120 kg
- **Skilled Labor**: 160 hours
- **Machine Hours**: 100 hours

Problem Formulation:

Decision Variables:

x_1 = Number of units of **Product A** produced
x_2 = Number of units of **Product B** produced
x_3 = Number of units of **Product C** produced

Objective Function:

$$\max Z = 60x_1 + 45x_2 + 80x_3$$

Constraints:

Raw Material Availability

$$4x_1 + 6x_2 + 5x_3 \leq 120$$

Skilled Labor Hours

$$10x_1 + 4x_2 + 6x_3 \leq 160$$

Machine Hours

$$5x_1 + 4x_2 + 6x_3 \leq 100$$

Non-Negativity Constraint

$$x_1, x_2, x_3 \geq 0$$

Steps to Solve:

1. Solve this problem using the Geometric Method (only possible for two variables, so we simplify)
2. Solve it using the Simplex Method (full problem)
3. Formulate and solve the Dual problem
4. Compare results and explain the connections between Simplex and Duality

Code Explanation:

This code uses **NumPy**, **Matplotlib**, and **SciPy** to solve a linear programming problem.

- **NumPy (np)**: Used for array manipulations, such as transposing the constraint matrix for the dual problem.
- **Matplotlib (plt)**: Used to plot the feasible region for a simplified two-variable case.
- **SciPy (linprog)**: Solves the linear programming problem using the **Simplex** method.

How it Works:

1. **Defines the Objective Function**: The goal is to **maximize profit**, but since `linprog` minimizes by default, the coefficients are negated.
2. **Sets Constraints**: Three constraints (Raw Material, Skilled Labor, Machine Hours) are defined in matrix form.

3. **Solves the Primal Problem**: Uses `linprog()` to find the optimal production quantities.
4. **Solves the Dual Problem**: Computes shadow prices for resource constraints.
5. **Plots the Feasible Region**: Uses Matplotlib to visualize constraints in a **2-variable** case.

Results:

Optimal Solution:
x1 (Product A) = 0.00
x2 (Product B) = 0.00
x3 (Product C) = 16.67
Maximum Profit = 1333.33

Dual Solution:
y1 (Shadow Price of Raw Material) = 0.00
y2 (Shadow Price of Skilled Labor) = 0.00
y3 (Shadow Price of Machine Hours) = 0.00
Minimum Cost (Dual Objective) = 0.00