Importance of Convexity & Convex Hulls

Convexity plays a crucial role in optimization, machine learning, and computational geometry. Convex functions guarantee a global minimum, making optimization easier and more efficient. Convex hulls help in clustering, pattern recognition, and computational geometry by identifying the smallest convex shape enclosing a set of points.

Brief Code Explanation

The code is divided into three parts:

1. **Convex Function Check**

   - Uses `sympy` for symbolic differentiation.

   - Computes the second derivative to check if a function is convex.

2. **Convex Hull Computation**

   - Uses `scipy.spatial.ConvexHull` to compute the convex hull of a set of points.

   - `matplotlib` is used to visualize the hull, showing the outer boundary.

3. **Visualization of Convex vs. Non-Convex Functions**

   - Uses `matplotlib` to plot x^2 (convex) and $\sin(x)$ (non-convex).

   - Demonstrates how convex functions differ in shape and behavior.

Results:

1st Figure: Convex Hull of Random Points

• **What it shows:** A set of randomly generated points in 2D space with their convex hull outlined in red.

• **Meaning:** The convex hull represents the smallest convex shape enclosing all points, useful in computational geometry, clustering, and pattern recognition.
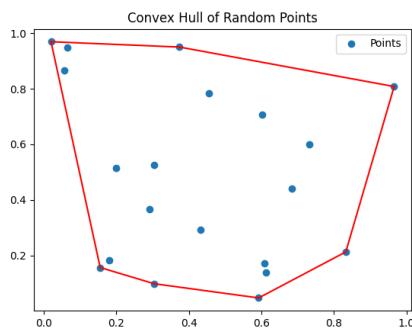
2nd Figure: Convex vs. Non-Convex Functions

• **Left Plot (Convex Function: x^2)**

   - The curve is always **curving upwards** (positive second derivative).

   - Any line segment between two points on the curve lies **above or on** the curve.

   - **Key insight:** Convex functions are easier to optimize since they have a single global minimum.

• **Right Plot (Non-Convex Function: $\sin(x)$)**

   - The curve **oscillates** up and down, meaning it has **multiple local minima and maxima**.

   - Some line segments between points fall **below the curve**, violating convexity.

   - **Key insight:** Non-convex functions make optimization harder due to multiple local minima.

Convex Function

Non-Convex Function

Convex: x^2

Non-Convex: sin(x)