Differences Between the Geometry Method and the Simplex Method in Linear Programming (LP)

1. **Geometry Method (Graphical Method)**

- The geometric or graphical method is used for solving **Linear Programming (LP) problems with two variables** because they can be represented on a 2D graph.
- The feasible region, defined by constraints, is plotted as a polygon or a bounded region.
- The objective function is represented as a line, which is moved parallel to itself until the optimal solution is reached at one of the feasible region's vertices.
- This method provides an intuitive, visual understanding of LP problems but is **limited to two-variable cases** and cannot handle higher dimensions.

2. **Simplex Method**

- The simplex method is an **iterative algebraic approach** used for solving LP problems, even with **many variables**.
- It works by moving from one **vertex (corner point) of the feasible region** to another, ensuring that each move improves (or maintains) the objective function value.
- It stops when no further improvement is possible, reaching the optimal solution.
- Unlike the graphical method, the simplex method can efficiently handle LP problems with **hundreds or thousands of variables**.

Meaning of Duality in Linear Programming

Duality in LP refers to the **relationship between two LP problems**, where every LP problem (called the **primal**) has a corresponding **dual** problem. The key insights of duality include:

- The **objective function of the dual** gives a bound on the **optimal value of the primal**.
- The **feasible solutions of one problem correspond to constraints of the other**.
- The **strong duality theorem** states that if an optimal solution exists for the primal problem, then the optimal solution for the dual problem has the same objective function value.

Example of Duality:

For a primal **maximization** LP problem:
$$\text{Maximize } Z = c_1x_1 + c_2x_2$$

subject to:
$$a_{11}x_1 + a_{12}x_2 \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 \leq b_2$$
$$x_1, x_2 \geq 0$$

The corresponding **dual** problem is a **minimization** LP problem:
$$\text{Minimize } W = b_1y_1 + b_2y_2$$

subject to:
$$a_{11}y_1 + a_{21}y_2 \geq c_1$$
$$a_{12}y_1 + a_{22}y_2 \geq c_2$$
$$y_1, y_2 \geq 0$$

This dual formulation is useful in **economic interpretations**, **sensitivity analysis**, and **algorithmic efficiency**.

[https://youtu.be/E72DWgKP_1Y?si=L7v1JDdc7zy5sLoq]