1 Definitions

$$\hat{e}_t = y_t - \hat{y}_t$$

$$\hat{\Sigma}_\ell^r = \frac{1}{r-\ell} \sum_{t=\ell}^{r} \hat{e}_t \hat{e}_t^\top$$

$$A_\ell^r(h) = \sum_{t=\ell}^{h} \hat{e}_t^\top \left(\hat{\Sigma}_\ell^r\right)^{-1} \hat{e}_t$$

$$C_\ell^r(h) = \frac{h}{\sqrt{2k(r-\ell+1)}} \left(\frac{A_\ell^r(h)}{h} - \frac{A_\ell^r(r-\ell+1)}{r-\ell+1} \right)$$

$$\Gamma_\ell^r = \max_{h \in \{\ell,\dots,r\}} |C_\ell^r(h)|$$

$$\bar{h}_\ell^r = \operatorname*{argmax}_{h \in \{\ell,\dots,r\}} |C_\ell^r(h)|$$

$$d = k(p+q+1) + \frac{k(k+1)}{2} + 1$$

Given a significance level of α, define C_α such that

$$\Pr\left(\sup_{0 \le v \le 1} |M_v^0| \le C_\alpha \right) = 1 + 2 \sum_{i=1}^{\infty} (-1)^i \exp(-2i^2 a^2) = 1 - \alpha,$$

where M_v^0 is the random variable of a standard Brownian bridge at time v.

2 Cusum algorithm

Algorithm 1: Cusum algorithm by Galeano and Peña.

Step 1 fit VARIMA model to y_t;
compute residuals $\hat{e}_t$;
$\mathit{candidates} \leftarrow \{1, T\}$;
$h_{\text{first}} \leftarrow d + 1$; $h_{\text{last}} \leftarrow T - d$;
while *True* **do**
Step 2   **if** $\Gamma_{h_{\text{first}}}^{h_{\text{last}}} < C_\alpha$ **then**
        break;
    else
        $\Gamma_{\text{old}} \leftarrow \Gamma_{h_{\text{first}}}^{h_{\text{last}}}$; $\bar{h}_{\text{old}} \leftarrow \bar{h}_{h_{\text{first}}}^{h_{\text{last}}}$;
        $\Gamma \leftarrow \Gamma_{\text{old}}$; $\bar{h} \leftarrow \bar{h}_{\text{old}}$;
Step 3a   **while** $\Gamma > C_\alpha$ **do**
            $t_2 \leftarrow \bar{h} - 1$;
            $\Gamma = \Gamma_{h_{\text{first}}}^{t_2}$;
        $h_{\text{first}} \leftarrow t_2$;
        $\Gamma \leftarrow \Gamma_{\text{old}}$; $\bar{h} \leftarrow \bar{h}_{\text{old}}$;
Step 3b   **while** $\Gamma > C_\alpha$ **do**
            $t_1 \leftarrow \bar{h} + 1$;
            $\Gamma = \Gamma_{t_1}^{h_{\text{last}}}$;
        $h_{\text{last}} \leftarrow t_1$;
Step 3c   **if** $|h_{\text{last}} - h_{\text{first}}| > d$ **then**
            append $h_{\text{first}}, h_{\text{last}}$ to *candidates*;
            $h_{\text{first}} = h_{\text{first}} + d$; $h_{\text{last}} = h_{\text{last}} - d$;
        else
            append $\bar{h}_{\text{old}}$ to *candidates*;
            break;
sort *candidates*;
$\{x_1, \ldots, x_s\} \leftarrow \mathit{candidates}$;
Step 4 **repeat**
    for $i \in \{1, \ldots, s-2\}$ **do**
        if $\Gamma_{x_i+1}^{x_{i+2}-1} < C_\alpha$ **then**
            remove x_{i+1} from *candidates*;
until *convergence* ;
remove $1, T$ from *candidates*;
$\mathit{changepoints} \leftarrow \{x + 1 : x \in \mathit{candidates}\}$;