SYNCOPE: Automatic Enforcement of Distributed Consistency Guarantees

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Abstract. Designing reliable and highly available distributed applications typically requires data to be replicated over geo-distributed stores. But, such architectures force application developers to make an undesirable tradeoff between ease of reasoning, possible when replicated data is required to be strongly consistent, and performance, possible only when such guarantees are weakened. Unfortunately, undesirable behaviors may arise under weak consistency that can violate application correctness, forcing designers to either implement ad-hoc mechanisms to avoid these anomalies, or choose to run applications using stronger levels of consistency than necessary. The former approach introduces unwanted complexity, while the latter sacrifices performance.
In this paper, we describe a lightweight runtime verification system that relieves developers from having to make such tradeoffs. Instead, our approach leverages declarative axiomatic specifications that reflect the necessary constraints any correct implementation must satisfy to guide a runtime consistency enforcement mechanism. This mechanism guarantees a *provably optimal* strategy that imposes no additional communication or blocking overhead beyond what is required to satisfy the specification, allowing distributed operations to run in a *provably safe* environment. Experimental results show that the performance of our automatically derived mechanisms is better than both specialized hand-written protocols and common store-offered consistency guarantees, providing strong evidence of its practical utility.

Keywords: runtime safety enforcement, weak consistency, distributed systems, Haskell

1 Introduction

Historically, the *de facto* system abstraction for developing distributed programs has always included the ACID[1] properties. These properties, guarantee replication transparency (i.e. requiring distributed systems to *appear* as a single compute and storage server to users), and have resulted in development of standardized implementation and reasoning techniques around *strongly consistent* (SC) distributed stores. Although strong notions of consistency, are ideal for developement and reasoning about distributed applications, they require extensive

[1] Atomicity, Consistency, Isolation and Durability

synchronization overhead which is unacceptable for web-scale applications that wish to be "always-on" despite network partitioning. Applications are therefore usually designed to tolerate certain *inconsistencies*, in exchange for availability and low-latency. An extreme example is *eventual consistency* (EC), where the local state of each node at all time, only represents an *unspecified order* of an *unspecified subset* of the set of all updates submitted to the system globally. Applications that cannot tolerate the anomalous behaviors allowed under EC, may choose to use various stronger instantiations, that are collectively refered to as *weak consistency* guarantees. Unfotunately, weak notions of consistency, are closely tied to the specific data-store implementations, and in very few cases, such as *causal consistency* (CC), for which there exists relatively standard definitions and known implementation techniques, users are usually offered with unnecessary levels of consistency and potental performance loss[2]. In order to face this problem, developers are forced to inject their code with *ad-hoc* anomaly tolerance mechanisms that are closely tied to the application logic and conflates it with concerns orthogonal to its semantics. To illustrate this problem, we will present an example in section 3, where we introduce a simple distributed application developed on top of an off-the-shelf eventually consistent data-store (ECDS), and explain how it must be re-engineered from the scratch in order to preempt certain undesired behaviors (i.e. enforce fine-grained weak consistency requirements). As we will see, the ad-hoc nature of such mechanisms confounds standardization, and complicates reasoning, maintainability and reusability of the applications.

In this paper, we propose an alternative to the aforementioned approaches that overcomes their weaknesses. SYNCOPE is a lightweight runtime system for Haskell that allows application developers to take advantage of weak consistency without having to re-engineer their code to accommodate anomaly preemption mechanisms. The key insight that drives SYNCOPE's design is that the hardness of reasoning about the integrity of a distributed application stems from conflating application logic with the consistency enforcement logic, and reasoning about both *operationally*. By separating application semantics from consistency enforcement semantics, admitting operational reasoning for the former, and declarative reasoning for the latter programmers are liberated from having to worry about implementation details of preemption mechanisms, and instead focus on reasoning about application semantics, under the assumption that specified consistency requirements are automatically enforced by the data store at runtime. Our approach admits declarative reasoning for consistency enforcement via a specification language that allows programmers formally specify the consistency requirements of their application. The design of our specification language is based on the observation that all anomalous behaviors allowed under EC, occur as the result of nodes executing operations, before a certain set of *dependencies* arrive at that node. Users in SYNCOPE, can specify arbitary dependency relations between updates, and the runtime monitoring system working on top

[2] In fact, CC is the strongest consistency guarantee that remains available under network partitioning

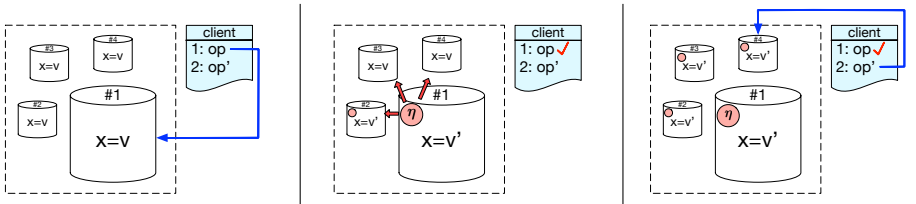
of each ECDS replica, guarantees that an operation will only proceed if it can witness all of its dependencies. For example, *lost-updates*, which is a very well known anomaly under EC, occures when an operation from a session is routed to a replica different than the replica that served the earlier operaitons of the same session (because of transient system properties, such as load balancing or network partitions) and is successfully executed, without witnessing the update from those earlier operations. In this case, the dependency of the operations can be defined as the updates from *all previous operations from the same sessions*, and SYNCOPE is guaranteed to temporarily block operations untill all such dependencies become available at a replica.

To summarize the contributions of this paper: (i) We propose a specification language to express the fine-grained consistency requirements of applications in terms of the dependencies between operations. (ii) We describe a generic consistency enforcement runtime that analyzes each operation's consistency specification, and ensures that its dependencies are satisfied before it is executed. We formalize the operational semantics of the runtime, and prove its correctness and optimality (including *minimum blocking* and *minimum staleness*) guarantees. (iii) We describe an implementation of our specification language and consistency enforcement runtime in a tool called SYNCOPE, which works on top of an off-the-shelf EC data store. We evaluate SYNCOPE over realistic applications and microbenchmarks, and present results demonstrating the performance benefits of making fine-grained distinctions between consistency guarantees, and the ease of doing so via our specification language.

The remainder of the paper is organized as follows. A system model that describes the key notions of consistency and replication is presented in Sec. 2. In Sec. 3 we provide a detailed example to further motivate the problem. In Sec. 4 and Sec. 5, we formally present our specification language and the high level operational semantics of the runtime system, with correctness and optimality theorems. Sec. 6 elaborates on the algorithmic aspects of our runtime that is key to its efficient realization. Sec. 7 describes implementation of SYNCOPE, and evaluates its applicability and practical utility. Related works and conclusion are presented in Sec. 8 and Sec. 9

2 System Model

A data store in our system model is a collection of *replicas* (#1,#2,...), each of which maintains a copy of a set of replicated *data object* (x,y,...). Each data object includes maintains a *state value* (v,v',...) and is equipped with a set of *operations* (op,op',...). Operations may read the state of an object residing in a replica, and modify it by generating *update effects* (η,η',...). Update effects or simply effects are asynchronously sent to all other replicas, where, by using a user-defined function, are *applied* to the state of the object instance in the recipient replica. Fig. 1a and 1b illustrate this process, where the example shows how effects are locally created and remotely applied.



(a) A client submits an operation `op` to the store, which is routed to the replica `#1`.

(b) The state of the replica `#1` is updated, an effect is created and is being propagated

(c) Second operation `op'` is submitted to the store, which is routed to the replica `#4`

Fig. 1: system model of SYNCOPE

Observe that in our system model, there is no direct synchronization between replicas when an operation is executed, which means concurrent and possibly conflicting updates can be generated at different replicas. We require the user-defined *apply* function to implement a conflict resolution strategy for replicas to eventually converge. This model admits all inconsistencies and anomalies associated with eventual consistency [1, 2], and our goal is equip applications and implementations with mechanisms to specify and prevent such inconsistencies.

Clients in our model, interact with the store by invoking operations on objects. A *session* is a sequence of operations invoked by a particular client. Consequently, operations (and effects) can be uniquely identified by the *session id* that invoked them, and their *sequence number* in that particular session, which is used by replicas, to record the set of all updates locally applied. Since, the data store is concurrently accessed by a typically large number of clients, and as a result of the load balancing regulations, operations (even from the same session) might be routed to different replicas (Fig. 1a and 1c).

Lastly, we define two relations over effects created in the store. *Session order* (so) is an irreflexive, transitive relation that relates an effect to all subsequent effects from the same session. Moreover, we define *visibility* (vis) as an irreflexive and assymetric relation that relates an effects to all others that are influenced by it (witnessed its update) at the time of their generation. For example, in Fig. 1c $\texttt{vis}(\eta, \eta')$ holds, since η (the effect of `op`) has already been delivered and applied to the replica `#4`, when `op` is executed and thus has influenced generation of η'.

3 Motivation

3.1 Replicated Data Types in ECDS

To provide further motivation, consider a highly available (low latency) application for managing comments on posts in a photo sharing web site. Fig. 2a presents a simple Haskell implementation of such an application cognizant of our system model.

In the implementation, `Effect` and `State` strings are respectively defined as the text of a single comment, and the concatentation of all visible comments associated with a post. A new `Effect` is generated every time a user wants to

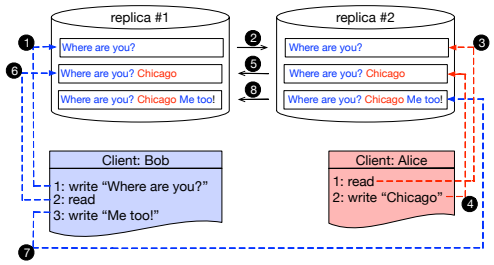
```
type Effect = String
type State =  String

read :: State -> (String,Maybe Effect)
read s = (s,Nothing)

write :: String -> ((),Maybe Effect)
write comment = ((),Just comment)

apply :: State -> Effect -> State
apply s comment = s ++ comment
```

(a) A simple implementation



(b) Example execution

Fig. 2: A distributed application for comment section management

comment on a post by calling the `write` function, and a `read` call simply returns the `State` of the object at the serving replica. The `apply` function, simply returns the updated state of the replica, which is a concatination of the old state and a given effect. For perspicuity, we omit any conflict resolution strategy in the code; however, developers (using roll-backs, etc) can design the `apply` function to resolve conflicting concurrent updates as they desire.

An example of how users interact with this application is presented in Fig. 2b, where Alice and Bob are invoking operations on an object (here, a photo of Alice in Chicago), and the chronological order of events is given in black circles. At time ❶, Bob writes a comment, which is routed to replica #1, whose effect is then propagated and delivered to replica #2 at ❷; where Alice's first read operation is routed to at ❸. Alice and Bob then keep talking through more read and write events, while updates are propagated between the two replica.

As mentioned before, lost-updates, is a well known undesirable behavior admitted by ECDS. An example of sych anomaly can occure here if at time time ❻, Bob is temporarily disconnected from both replicas in the figure, and his read operation is routed to another replica that has not yet received any updates from #1 or #2. Consequently, Bob cannot see his first comment and would retry submitting it, assuming the first time it was failed.

3.2 Ad-hoc Anomaly Prevention

A known technique to prevent the above anomaly, is to tag each effect using a unique identifier as mentioned in Sec. 2. Using these tags, replicas will be able to track all locally available effects, and temporarily *block* operations, until all the preceding effects from the same session arrive at the replica. For example, the replica #3 that receives Bob's read in the example scenario, can simply postpone its execution until all dependencies arrive.

In order to reduce the overhead of tracking dependencies per operation, the above idea is further realized using another technique called *filteration*, which is based on separating the locally available effects at each replica that have not been applied to the state yet and those who have. By this separation, in the

above example each replica can maintain a *safe environment* for operations (e.g. using a soft-state cache), that contains an effect only if it also contains all the previous effects from the same session. This way, an operation can proceed, when the effect of the very exact previous operation from the same session is already applied to the state (which transitively yields the presence of all dependencies).

We present a new version of our running example in appendix A, which is modified to tolerate the lost-update anomaly by implementing the blocking mechanism in the `read` function and the filteration in the `apply` function as explained above. Unfortunately, these modifications require fundamental and pervasive changes to the original code including almost all type and function definitions. Additionally, the changes are heavily tangled with application logic, complicating reasoning and hampering correctness arguments.

A major drawback of this approach in stores that do not admit metadata queries (e.g. Cassandra), is the *lost histories*[3] problem. To face this problem, for each new session joining, the replicas must perforom a table alteration at the data store level, to accomodate the data on the newly joined session. This requires strong synchronization of replicas, degrading application performance and availability. Moreover, to make the matter worse, new anomalies are constantly found in the systems after the design phase, which require non-trivial, further polluting, ad-hoc solutions that leave the old implementation obsolete.

3.3 An Alternative

We now present our generic consistency management tool. SYNCOPE allows developers to define a consistency level for each operation *a priory*, and rely on the runtime system for its satisfaction. Our approach is consisted of generalized blocking and filteration mechanisms, which admits arbitrary user-defined dependence relations for each operation and maintaines a multi-consistent *shim layer* on top of each ECDS replica.

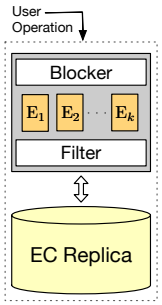


Fig. 3: SYNCOPE

The SYNCOPE shim layer maintains multiple safe environments $(E_1, E_2, ...)$ by periodic (or on-demand) reads from the underlying ECDS database, and adding effects to each environment, only if its dependencies have already been added (Fig. 3). SYNCOPE realizes this idea efficiently, using a simple tagging mechanism that represents effects in an environment by giving them a tag associated with that environment. Each operation in SYNCOPE only witnesses its associated environement, and is blocked by the runtime system, if the necessary effects are not in there yet.

Users in our tool can specify arbitrary consistency guarantees in a language that is seeded with `so` and `vis` relations and allows them to define constraints on read operations, that can be used to synthesize appropriate filteration and blocking mechanisms. For example, the following *contract*, eliminates the possibilty of lost-update anomaly, by establishing the appropriate condition under which an effect may become visible to $\hat{\eta}$, the effect of the current operation:

$$\psi_1 : \forall a.\ \xrightarrow{\mathtt{so}} \hat{\eta} \Rightarrow a \xrightarrow{\mathtt{vis}} \hat{\eta}$$

$$
\begin{array}{lll}
\texttt{r} \in \texttt{rel.seed} & \coloneqq & \mathsf{vis} \mid \mathsf{so} \mid \texttt{r} \cup \texttt{r} \\
\mathsf{R} \in \texttt{relation} & \coloneqq & \texttt{r} \mid \mathsf{R};\texttt{r} \mid \texttt{null} \\
\pi \in \texttt{prop} & \coloneqq & \forall a.\ a \xrightarrow{R} \hat{\eta} \ \Rightarrow\ a \xrightarrow{\mathsf{vis}} \hat{\eta} \\
\psi \in \texttt{spec} & \coloneqq & \pi \mid \pi \wedge \pi
\end{array}
$$

(a) syntax

Guarantee	Contract
RMW	$\forall a.a \xrightarrow{\mathsf{so}} \hat{\eta} \Rightarrow a \xrightarrow{\mathsf{vis}} \hat{\eta}$
MW	$\forall a.a \xrightarrow{\mathsf{so};\mathsf{vis}} \hat{\eta} \Rightarrow a \xrightarrow{\mathsf{vis}} \hat{\eta}$
MR	$\forall a.a \xrightarrow{\mathsf{vis};\mathsf{so}} \hat{\eta} \Rightarrow a \xrightarrow{\mathsf{vis}} \hat{\eta}$

(b) examples

Fig. 4: SYNCOPE Specification Language

4 Specification Language

The formal syntax of our specification language is presented in Fig. 4a, which allows definition of propositions (`prop`), FOL formulae that establish dependence relations between effects in order to determine the effects an operation should witness. The language is seeded with so and vis, respectively representing session order and visibility over effects, and defines a `relation` as a sequence[3] of relation seeds, representing dependencies between effects, which should be desugared as follows:

$$a \xrightarrow{\mathtt{r}_1;\ldots;\mathtt{r}_k} b \iff \exists c.(a \xrightarrow{\mathtt{r}_1;\ldots;\mathtt{r}_{k-1}} c \wedge c \xrightarrow{\mathtt{r}_k} b) \tag{1}$$

Additionally, the language allows definition of `spec`, that is a conjunction of propositions, and is used to define safe environments that are free of *multiple* inconsistencies. Our language is crafted to capture fine-grained weak consistency requirements, including the famous session guarantees [2], presented in Fig 4b.

We finish this section by syntactically classifying contracts, and explaining how each of them requires different enforcement techniques.

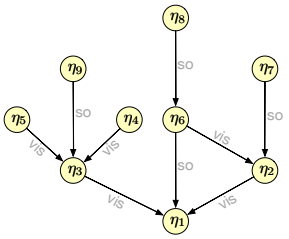
- LB: If the defined dependency relation for a contract ends with an so, i.e. is of the following form ($\forall a.a \xrightarrow{r_1;r_2;\ldots;\mathsf{so}} \hat{\eta} \Rightarrow a \xrightarrow{\mathsf{vis}} \hat{\eta}$), we call it a *lower bound* (LB) contract, since it specificly defines the smallest set of effects that any operation should witness to maintain consistency, e.g. RMW and MR in Fig. 4b.
- UB: Similarly, we define the *upper bound* (UB) contracts, as the ones with dependency relations ending with a vis. These contracts define constraints on the set of effects made visible to each operation, by enforcing that if an effect is being witnessed certain set of dependencies must also be witnessed.

Our consistency enforcement approach is based on blocking operations with LB contracts to make sure that they witness *all effects that they are supposed to*, and filteration for UB contracts to make sure that they would not witness *effects that they are not supposed to*. A combination of both approaches is also taken for contracts that are neither LB nor UB, i.e hybrid contracts.

5 Semantics

In this section, we present the consistency enforcement mechanism of SYNCOPE, abstracted as a formal operational semantics. Our approach is complete for the

[3] SYNCOPE also allows definition of the closure of relations, however we omitted them here for simplicity reasons



(a) An execution state E

$$
\begin{aligned}
\mathsf{E.A} &= \{\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \\
&\quad \eta_6, \eta_7, \eta_8, \eta_9\} \\
\mathsf{E.vis} &= \{(\eta_5, \eta_3), (\eta_4, \eta_3), \\
&\quad (\eta_3, \eta_1), (\eta_2, \eta_1), \\
&\quad (\eta_6, \eta_2)\} \\
\mathsf{E.so} &= \{(\eta_9, \eta_3), (\eta_8, \eta_6), \\
&\quad (\eta_6, \eta_1), (\eta_8, \eta_1), \\
&\quad (\eta_7, \eta_2)\}
\end{aligned}
$$

(b) Effect soup and primitive relations

$$
\begin{aligned}
\mathsf{vis}^{-1}(\eta_1) &= \{\eta_2, \eta_3\} \\
\mathsf{so}^{-1}(\eta_1) &= \{\eta_6, \eta_8\} \\
(\mathsf{so} \cup \mathsf{vis})^{-1}(\eta_1) &= \{\eta_2, \eta_3, \eta_6, \eta_8\} \\
(\mathsf{vis}; \mathsf{vis})^{-1}(\eta_1) &= \{\eta_2, \eta_3, \eta_4, \eta_5, \eta_6\} \\
(\mathsf{so}; \mathsf{vis})^{-1}(\eta_1) &= \{\eta_7, \eta_9\}
\end{aligned}
$$

(c) Relation inverse examples

Fig. 5: A simple execution state

specification language defined in Sec. 4, however for better comprehensibility, here we present the semantics and the theorems paramterized over a non-hybrid contract consisting of one proposition. Therefore, in the rest of this section, we will assume a given contracat ψ of the following form:

$$\psi = \forall a.a \xrightarrow{\mathtt{r}_1;\mathtt{r}_2;\ldots;\mathtt{r}_k} \hat{\eta} \Rightarrow a \xrightarrow{\mathsf{vis}} \hat{\eta} \qquad \mathtt{r}_i \in \{\mathsf{vis}; \mathsf{so}\}$$

The operational semantics is defined via a small-step relation over *execution states*, which are tuples of the form $\mathsf{E} = (\mathsf{A}, \mathsf{vis}, \mathsf{so})$. The *effect soup* A, stands for the set of all effects produced in the system, and *primitive relations* vis, $\mathsf{so} \subseteq \mathsf{A} \times \mathsf{A}$, respectively represent the visibility and session order among such effects. Figures 5a and 5b represent a simple execution state consisting of 9 effects with associated primitive relations[4]. We denote the subset of A consisting of effects that satisfy a certain condition as $\mathsf{A}_{(\texttt{condition})}$.

Note that SYNCOPE's contracs are in fact constraints over execution states, where the domain of quantification is fixed to the effect soup A, and interpretation for so and vis relations (which occur free in the contract formulae) are also provided. Thus, execution states are potential models for any first-order formula expressable in the specification language. If an execution state E is in fact a valid model for a contract ψ, we say that E satisfies ψ, written as $\mathsf{E} \models \psi$.

The reduction relation in the semantics is of the form $(\mathsf{E}, \mathsf{op}_{<s,i>}) \xrightarrow{\mathsf{V}} (\mathsf{E}', \eta)$, which can be interpreted as the reduction of the initial execution state E, performed by a replica with a local set of effects V when it executes op, which is the i^{th} operation from the session s. During this reduction step a new effect η is produced and added to the system, resulting in a new execution state E' with updated effect soup and primitive relations.

5.1 Preliminaries

Before introducing the operational semantics, we will first formally present the required definitions in the next section. We start by formally defining the inverse of a seed relation ($\mathtt{r} \in \mathtt{rel.seed}$) given an execution state E:

$$\mathtt{r}^{-1}(\mathsf{S}) = \begin{cases} \bigcup_{b \in \mathsf{S}} \{a | (a, b) \in \mathsf{E}.\mathtt{r}\} & \mathtt{if\ r} \in \{\mathsf{so}, \mathsf{vis}\} \\ \mathtt{r}_1^{-1}(\mathsf{S}) \cup \mathtt{r}_2^{-1}(\mathsf{S}) & \mathtt{if\ r} = \mathtt{r}_1 \cup \mathtt{r}_2 \end{cases} \tag{2}$$

[4] we ommit drawing transitive so edges (e.g. between η_8 and η_1) for better readability

Note that when the input of an inversed relation is a singleton $\{\eta\}$, we drop the brackets and simply write it as $\mathtt{r}^{-1}(\eta)$. We now present the definition of the inverse of sequences (of size larger than 1) of `rel.seed` as follows:

$$b \in (R'; r)^{-1}(a) \iff \exists c.c \in r^{-1}(a) \wedge b \in (R')^{-1}(c) \tag{3}$$

Inverse of sequences of length 1 is also implicitely defined as the inverse of the enclosed `rel.seed`.

Following definitions (2) and (3), since `relation` in our specification language is defined as either a `rel.seed`, or a sequence of them, we are now ready to formally define the inverse of any given relation $R \in$ `relation`. However, note that the definition (3) fails to capture the reality of distributed systems, where all computations are done locally by replicas, which might have access to only a *subset of all effects* at any given moment. For example, consider $(\mathsf{so}; \mathsf{vis})^{-1}(\eta_1)$ of the execution state in figure 5. In order to compute this set, based on the recursive defintion of (3) we have:

$$b \in (\mathsf{so}; \mathsf{vis})^{-1}(\eta_1) \iff \exists c.c \in \mathsf{vis}^{-1}(\eta_1) \wedge b \in (\mathsf{so})^{-1}(c)$$

Since there exist *mid-level* effects η_2 and η_3, such that satisfy the above definition respectively for $b = \eta_7$ and $b = \eta_9$, we have: $(\mathsf{so}; \mathsf{vis})^{-1}(\eta_1) = \{\eta_7, \eta_9\}$. Now assume a replica only contains $\{\eta_1, \eta_6, \eta_7, \eta_8, \eta_9\}$ and wants to check if the dependencies of η_1 are locally present or not. Even though based on the above definitions the answer is yes (since the replica does contain $\{\eta_7, \eta_9\}$), but in reality the replica would not be able to verify that, since the mid-level effects η_2 and η_3 are not present at the replica yet.

To capture the above property, we now present partial definition of the inverse of a given relation $R \in$ `relation` *according to a set of available effects* V. We define the inverse, only if all the required mid-level effects are present in V using definition (2) and a slightly different version of the definition (3).

$$b \in R_V^{-1}(a) \iff \begin{cases} \bot & \texttt{if } R = \texttt{null} \\ b \in \mathtt{r}^{-1}(a) & \texttt{if } R = \mathtt{r} \\ \exists c.c \in \mathtt{r}^{-1}(a) \wedge b \in (R')^{-1}(c) \wedge \mathtt{r}^{-1}(a) \subseteq V & \texttt{if } R = R'; \mathtt{r} \end{cases} \tag{4}$$

Note that the only difference between the third case in above definition and the definition (3), is the last conjunct which is added to ensure the presence of mid-level effects before performing the next recursive call.

Now, we define $\mathsf{trunc}()$ as a function that given $R \in$ `relation`, removes the last element from the sequence (if there is any) in R, i.e.

$$\mathsf{trunc}(R) = \begin{cases} \texttt{null if } R = \mathtt{r} \quad \texttt{or} \quad R = \texttt{null} \\ R' \quad \texttt{if } R = R'; \mathtt{r} \end{cases} \tag{5}$$

Finally, we define *closed subsets* of a given set of effects V under the contract ψ, which the maxiamal element among such subsets is also defined next[5]:

$$\begin{aligned} \texttt{closed subsets} : V' \in \lfloor V \rfloor \quad &\iff V' \subseteq V \quad \wedge (\mathsf{trunc}(R))_V^{-1}(V') \subseteq V' \\ \texttt{maximally closed subset} : V' = \lfloor V \rfloor_{\max} \quad &\iff V' \in \lfloor V \rfloor \wedge \nexists V'' \in \lfloor V \rfloor . |V''| > |V'| \end{aligned} \tag{6}$$

[5] We abuse the previously defined notation slightly and use a *set* of effects as the input to the inverse of $R \in$`relation`, which simply means the union of the results of apply the function for all the effects in the input set

Auxiliary Definitions

$$\begin{array}{llll} \mathsf{op} & \in \texttt{Operation Name} & F_{\mathsf{op}} & \in \texttt{Op. Def.} \quad := \mathcal{P}(\eta) \mapsto v \\ v & \in \texttt{Return Value} & \mathsf{A} & \in \texttt{Eff Soup} \quad := \mathcal{P}(\eta) \\ s & \in \texttt{Session ID} & \mathsf{vis}, \mathsf{so} & \in \texttt{Relations} \quad := \mathcal{P}((\eta, \eta)) \\ \eta & \in \texttt{Effect} := (s, op, v) & \mathsf{E} & \in \texttt{Exec State} := (\mathsf{A}, \mathsf{vis}, \mathsf{so}) \end{array}$$

Auxiliary Reduction

$$\boxed{S \vdash (\mathsf{E}, op_{<s,i>}) \hookrightarrow (\mathsf{E}', \eta)}$$

[Oper]

$$\frac{S \subseteq \mathsf{A} \qquad F_{op}(S) = v \qquad \eta \notin S \qquad \eta = (s, op, v) \qquad \mathsf{A}' = \mathsf{A} \cup \{\eta\} \qquad \mathsf{vis}' = \mathsf{vis} \cup S \times \{\eta\} \qquad \mathsf{so}' = \mathsf{so} \cup \{(\eta', \eta) \mid \eta' \in \mathsf{A}_{(\mathsf{SessID}=s)}\}}{S \vdash ((\mathsf{A}, \mathsf{vis}, \mathsf{so}), op_{<s,i>})) \hookrightarrow ((\mathsf{A}', \mathsf{vis}', \mathsf{so}'), \eta)}$$

Operational Semantics

$$\boxed{(\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}', \eta)}$$

[UB Exec]

$$\frac{r_k = \mathsf{vis} \quad V \subseteq E.A \quad V' = \lfloor V \rfloor_V \quad V' \vdash (E, op_{<s,i>})) \hookrightarrow (E', \eta)}{(\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}', \eta)}$$

[LB Exec]

$$\frac{r_k = \mathsf{so} \quad V \subseteq E.A \quad R_V^{-1}(\eta) = R_{E'.A}^{-1}(\eta) \quad R_V^{-1}(\eta) \subseteq V \qquad V \vdash (E, op_{<s,i>})) \hookrightarrow (E', \eta)}{(\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}', \eta)}$$

Fig. 6: Core Operational semantics of a replicated data store.

5.2 Core Operational Semantics

In this part we present the reduction rules, representing our consistency preservation approach. Figure 6 presents the set of rules defining the auxiliary relation ($\hookrightarrow$) and small-step reduction relation ($\rightarrow$) over executions. The latter relation is parametrized over a set V, that represents the set of effects that are available at the replica taking the step. Obviously V must be a subset of the effect soup of the initial execution, however, there is no other restrictions on V, since we only assume eventual consistency at the underlying store.

The rule [oper] represtns the procedure of producing a new effect η, by witnessing a set of effects S. An effect is formally defined as a tuple $\eta = (s, op, v)$, representing the session and the operation name whose execution created η, and the value that the replica returns as the response to that operation. The rule explains how the execution state changes after producing an effect at a replica. Specifially, in the new state, the effect soup A' contains the newly created effect η, and the relations vis' and so' capture the fact that all effects in the set S were made visible to η, and all effects from the same session that were already presenet in the intial execution state, should be in session order with η in the final execution state.

Now we explain the rules for reduction relation ($\xrightarrow{V}$), starting with [ub exec], which represents the execution of operations in a replica that updates the global state and produces a new effect under a UB contract. The rule requires operations witnessing only the maximally consistent subset V' of the local set of available

effects V. In other words, the rule filters out the effects that may result anomalies and shows the safe environment to the operation.

The next rule, [LB EXEC], represents the step taken when an operation is performed under an LB contract. The precondition $R_V^{-1}(\eta) \subseteq V$ in the rule, ensures that the reduction happens only if the effects necessary to avoid the specified anomaly are present in V. The operations performing under an LB contract must be blocked, untill all the necessary effects (and possibly required mid-level effects) become available in the locally available set of effects V. Note that in this case effects are not filtered out, and the operation witnesses all effects in set V.

5.3 Soundness

In order to prove a meta-theoretic correctness property for our semantics, we first define a ψ-consistent set of effects S given a execution state E as follows:

$$S \texttt{ is } \psi\texttt{−consistent} \iff \forall(\eta \in S).\forall(a \in \mathsf{E.A}).R(a,\eta) \Rightarrow a \in S \tag{7}$$

Theorem 1. *For all reduction steps* $(\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}', \eta)$,

(i) $\texttt{if } V \texttt{ is } \psi\texttt{−consistent under } \mathsf{E}, \texttt{ then } V \cup \{\eta\} \texttt{ is } \psi\texttt{−consistent under } E'$
(ii) $E' \models \psi[\eta/\hat{\eta}]$

Proof. Appendix B.1

5.4 Optimality

Now we will present theorems 2 and 3, the former showing that the set of effects made visible during each operation execution is the largest one possible, and the later presenting the liveness property of the semantics, which states that the store will take a step, if the required dependencies are locally present. This guarantees that the store would never get stuck, since the eventual delivery of all updates to all replicas is guaranteed by the underlying ECDS.

Theorem 2. *For all operation executions* $(\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}', \eta)$*, the set of effects made visible to* η *is maximal. i.e. for all* $a \in V$ *that* $(\mathsf{trunc}(R)_V^{-1}(a) = \mathsf{trunc}(R)_{E.A}^{-1}(a))$ *the following holds:*

$$(a,\eta) \notin \mathsf{E}'.\mathsf{vis} \Rightarrow (\mathsf{E}'.A, \mathsf{E}'.\mathsf{vis} \cup \{a,\eta\}, \mathsf{E}'.so) \not\models \psi[\eta/\hat{\eta}]$$

Proof. Appendix B.2.

Theorem 3. *For all execution states* E*, set of effects* $(S \subseteq E.A)$*, if:*

$$S \vdash (\mathsf{E}, op_{<s,i>}) \hookrightarrow (\mathsf{E}', \eta) \quad \wedge \quad (\mathsf{S} \cup \{\eta\} \texttt{ is } \psi\texttt{−consistent under } E')$$

then there exist E''*,* η' *and* $\mathsf{V} \subseteq \mathsf{E.A}$ *such that:* $((\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}'', \eta'))$

Proof. Appendix B.3

6 Algorithm

In this section, we present a detailed and practical implementation strategy of the operational semantics presented in section 5, where we introduced an abstract outline of our consistency preservation technique. Here we realize our ideas by equipping each replica with a *cache*, that is guaranteed to preserve a specified consistency level. Here we assume a contract of the form $\psi = \forall(a, b).a \xrightarrow{R} b \Rightarrow a \xrightarrow{\mathsf{vis}} b$ and a replica containg a local set V and explain SYNCOPE's behavior in more detail.

Let's first define a *truncated relation* as the relation derived by removing the last element from a given relation:

$$\mathsf{trunc}(r_1; r_2; ...; r_k) = r_1; r_2; ...; r_{k-1}$$

We now extend the above definition to the given contract ψ, by replacing R with $\mathsf{trunc}(R)$ and argue that an effect η can only enter the cache, if its presence would not violate the truncated contract in the replica, i.e. η's dependency set $(\mathsf{trunc}(R))_V^{-1}(\eta)$ is already present in the cache (replica) for a UB (LB) contract. Now we consider two possible types of contracs and explain the replicas' behavior when an operation is submitted:

1. LB contracts: In this case, the replica makes sure that the operation is blocked until effects of earlier operations from the same session, are already present in the cache. This guarantees the presence of all the dependencies of the current operation, accoring to the *original* contract. Note that in this case, the operation can witness *all* effects at the replica.
2. UB contracts: In this case, operations are not blocked, however, they should only witness the effects that are already present in the cache. This guarantees the preservation of the original contract, which puts a maximal bound on the set of effects to be made visible to an operation.
3. Hybrid Contracts: Here, the oeprations should both be blocked similar to the LB case, and also witness only the effects in the cache.

6.1 Degree of Dependency Presence

In the above description of our consistency management tool, the notion of *the presence of the depency set* is treated as a true/false property, that is checked before allowing an effect enter the cache. However, as an astute reader might have noticed, a naive implementation of this idea, could result in poor performance. That is because contracts in our specification language can be arbitrarily large and might contain closures of relations, computing the inverse of which can become very large. A naive implementation that drops all the computations done for a failed dependency check of an effect, results in redunencies the next time the same property is being validated, which is not practically reasonable.

To address the mentioned difficulties, we introduce the *Degree of Dependency Presence*, DDP, that extends the above binary property, by marking the

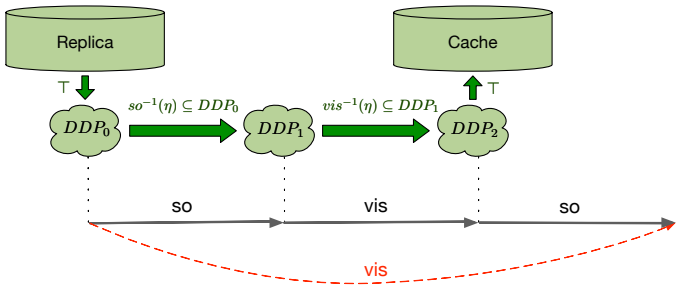


Fig. 7: Representation of the stepwise process where effects become closer to the cache, before actually entering it

effects with a number, that represents how far the presence of their dependencies have been checked so far. The DDP of an effect according to a relation $R = r_1; r_2; ...; r_k$ and a given set of effects V is defined as the length of the longest prefix of R, under which $V \cup \{\eta\}$ is consistent, that is,

$$DDP_V(\eta) = m \iff (r_1; r_2; ...; r_m)^{-1}(\eta) \subseteq V$$

For example, $DDP_V(\eta) = 0$ means that η has just arrived to the replica and no degree of its dependencies are checked yet, and $DDP_V(\eta) = k - 1$ means that all of η's dependencies according to the truncated relation are present and it is safe now to add it to the cache. We use the notation DDP^i to refer to the set of all effects whose DDP value is equal to i.

This way, by periodically refreshing the DDP of effects, the porcess of moving effects from the replica to the cache is recorded while the the dependencies arrive, and as we will explain shortly, we can totally avoid computing closures of relations and redundant computations at replicas by a simple memoization technique. Finally, note that (following the discussion in the previous section) we require the dependencies to be looked for, in the replica and the cache respectively, for the LB and non LB contracts. i.e. in the case of LB contracts $DDP_{replica}$ should be computed and DDP_{cache} for UB and hybrid contracts.

6.2 Example

In this part, we will explain the behavioral outline of our algorithm using an example. The formal operational semantics of this approach can be found in appendix C.

Let's assume we are given a contract $\psi = \forall(a, b).a \xrightarrow{\text{so;vis;so}} b \Rightarrow a \xrightarrow{\text{vis}} b$, and we want replicas to maintain consistent caches according to this contract. We explain our approach by explaining a replicas' behavior when certain events occur:

– **Remote Effect Arrival:** When a new remote effect arrives to the replica, it is simply added to the set of local effects and its DDP is initially set to 0. Since the length of the given contract is 3, as we will see shortly the effect requires two steps of DDP refreshes, before it can enter the cache.

- **Operation Submission:** Since the given contract is an LB type, the replica must now make sure that all effects from ealier operations of the same session, are present in the *cache* and if not, block the operation temporarily. The operation can proceed and wtiness *all* effects at the replica, after the mentioned effects enter the cache.
- **Cache Refresh:** The replica, must periodically perform cache refreshes and move effects from DDP_2 to the cache. As explained we know that the complete set of dependencies for these effects are already present.
- **DDP Refresh:** At this periodic step, DDP of effects are updated by checking if they can be given a larger one. At each step the DDP of an effect η is increased from i to $i+1$ if all effects in $r_{i+1}^{-1}(\eta)$ already have DDP value at least equal to i. In this example, an effect η that has the initial DDP value 0, can get the value 1, only if all effects in $\mathsf{so}^{-1}(\eta)$ are already present at the replica which means they have DDP value of at least 0. Similarly, effects that have the DDP value of 1 can get the value 2, if all effects in their vis^{-1} set, have the DDP value of minimum 1. At this point, effects have reached the value 2, which means they can be now moved to the cache at the next cache refresh (Figure 7).

Note that, in case one of the elements of the given relation is a closure, for example assume the given relation is $\xrightarrow{so;vis^*;so}$, we can avoid all recursive computations, by allowing an effect η to go from DDP_1 to DDP_2, only if $vis^{-1}(\eta)$ is present in DDP_1 **and** in DDP_2. This way, we are sure that the same condition was also checked for all effects in $vis^{-1}(\eta)$ before they entered DDP_2, which brings us the desired behavior, without any computations involving closures.

Benchmark	LoC	Consistency Reqs.	Description
Counter	65	MR	Monotonicly increasing counter, e.g. YouTubes' watch count
DynamoDB	126	RMW	An integer register that allos different types of (conditional) puts and gets
Online Store	236	RMW	Online store with shopping carts and modifiable item prices
Bankaccount	85	2VIS ∧ RMW	A bankaccount application with deposit, withdraw and get balance operatio
Shopping List	140	MW ∧ RMW	A shopping list with concurrent adds and deletes functionality
Microblog	395	MW, RMW	A Twitter-like application modeled after Twissandra, with posting and reply to tweets, following, unfollowing and blocking users, etc.
Rubis	417	RMW, RMW∧2VIS	eBay-like application with browsing, bidding and making payments from a v

Fig. 8: Usage of weak consistency requirements in benchmark applications

7 Evaluation

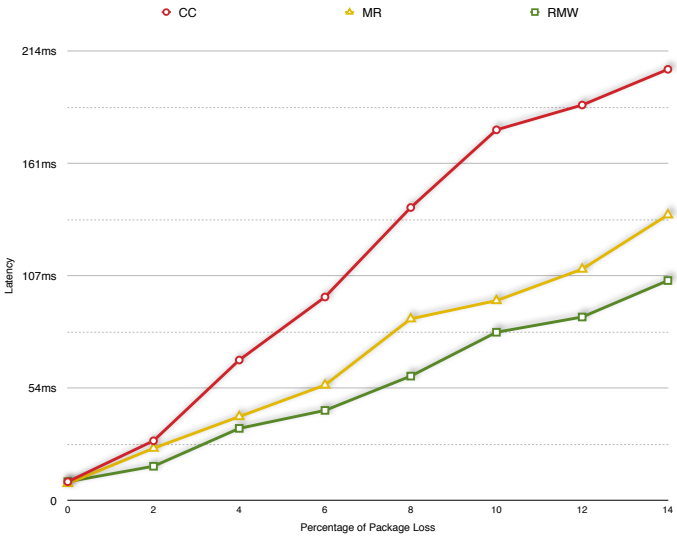
In this section, we present our evaluation study of SYNCOPE. The results are presented in three parts, where we first present the distribution of weak consistency requirements on benchmark programs. Second, we presents our studies on the performance of programs running on various consistency levels and finally, we present the complexity and perforamnce results from our study of implementing a well-understood ad-hoc prevention mechanism for lost-updates anomaly, compared to writing the same program in SYNCOPE.

7.1 Weak Consistency in Benchmark Programs

In this section, we present seven different benchmark applications we collected, in which various types of anomalous behavior under eventual consistency have been detected. We present these programs and their detected consistency requirements, in figure 8. For example, two following anomalies have been detected for operations of the microblog application:

1. When Alice unfollows Donald, but later sees more tweets from him. This is because the `getFolloweeList` operation did not witness the effect of the `unfollow` operation; a clear example of lost-updates anomaly which can be prevented by RMW guarantee.
2. When Donald posts a series of tweets, but after Alice refreshes her timline, only sees the fifth tweet. This can be prevented by requiring `getTweet` operation to return only tweets, whose prior tweets are also visible; which is exacly what is provided by enforcing MW contract.

In addition to having a large number of operations each of which might require a different level of consistency, above examples also show how in practice, some programs might inlcude operations that are involved in *multiple* types of anomalies. For example the `getBalance` operation of the bank account application above, shows two different types of anomalies, whose prevention requires 2VIS *and* RMW. The possiblity of showing *combinations* of anomalies, considering the large number of known anomalies, shows the inefficiency of any consistency enforcement technique specific to a certain type of anomaly.



(a) Example execution.

Fig. 9: A distributed application for comment section management

7.2 Latency and Staleness Comparison

7.3 Ad-hoc vs SYNCOPE

8 Related Works

9 Conclusion

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A Modified Haskell Program

```
data Sess = Bob | Alice
type ID = (Sess,Int)
type Effect= (ID,String)
type State = (String,Int,Int)

read :: ID -> State -> String
read (sess,seq) (st,sq1,sq2) =
 case sess of
  Bob ->   if (seq==sq1+1) then st
           else read (sess,seq)(st,sq1,sq2)
  Alice -> if (seq==sq2+1) then st
           else read (sess,seq)(st,sq1,sq2)
```

```
apply :: State -> Effect -> State
apply (st,sq1,sq2) ((sess,seq),cm) =
  case sess of
    Bob ->   if (sq1==seq-1)
             then (st++cm,sq1+1,sq2)
             else (st,sq1,sq2)
    Alice -> if (sq2==seq-1)
             then (st++cm,sq1,sq2+1)
             else (st,sq1,sq2)
```

Fig. 10: Guarded Application to Prevent Lost-updates Anomaly When Serving Bob and Alice

B Proofs

Here, we present the detailed proofs of the theorems of the paper. Let's first present a useful lemma:

Lemma 1. *For all relations R and execution steps:*

$$(\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}', \eta)$$

interpretatin of R under E and E' only differs considering η, i.e. $a, b \neq \eta \Rightarrow (R'(a,b) \Leftrightarrow R(a,b))$

Proof. We only prove $\Rightarrow$, the other part can be done similarly. We have the following goal and hypotheses:

$$\begin{array}{l} H_0 : (\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}', \eta) \\ H_1 : a, b \neq \eta \\ H_2 : R'(a,b) \\ G_0 : R(a,b) \end{array}$$

Now by destructing R we have the followings from new hypothesis and goal:

$$\begin{array}{l} H_3 : (\mathsf{trunc}(R); r)'(a,b) \\ G_1 : (\mathsf{trunc}(R); r)(a,b) \end{array}$$

which can be rewritten by the definition to get that y exists s.t.

$$\begin{array}{l} H_4 : (\mathsf{trunc}(R))'(a,y) \\ H_5 : r'(y,b) \\ G_1 : \exists x.(\mathsf{trunc}(R))(a,x) \wedge (r)(x,b) \end{array}$$

Now we instantiate the goal with y itself and by using induction on the length of R, the first conjunct is proved, and we are left with the following:

$$\begin{array}{l} H_6 : r'(y,b) \\ G_1 : r(y,b) \end{array}$$

Now by inversion on H_0 we get two cases, at both of which the following can be derived. (In one case V should be replaced by V' but has no effect on the proof):

$$\begin{array}{l} H_7 : \mathsf{vis}' = \mathsf{vis} \cup V \times \{\eta\} \\ H_8 : \mathsf{so}' = \mathsf{so}' = \mathsf{so} \cup \{(\eta', \eta) \mid \eta' \in \mathsf{A}_{(\mathsf{SessID}=s)}\} \end{array}$$

Now, because of H_1 (and the fact that $y \neq \eta$) it is easy to get the following from H_7 and H_8:

$$\begin{array}{l} H_9 : \ \mathsf{vis}(y,b) \Rightarrow \mathsf{vis}(y,b) = \\ H_{10} : \mathsf{so}(y,b) \Rightarrow \mathsf{so}(y,b) = \end{array}$$

Which directly prove the goal, after destructing r.

B.1 Proof of Theorem 1

(Part i) We have the following two hypotheses and the goal:

$$\frac{\begin{array}{l}H_0 : (\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}', \eta) \\ H_1 : V \texttt{ is } \psi\texttt{−consistent under } \mathsf{E}\end{array}}{G_0 : V \cup \{\eta\} \texttt{ is } \psi\texttt{−consistent under } \mathsf{E}'}$$

Rewriting the definition in G_0 results in the following. We denote the interpretation of R under E' as R':

$$G_1 : \forall(b \in V \cup \{\eta\}).\forall(a \in E'.A).R'(a, b) \Rightarrow a \in V \cup \{\eta\}$$

By intros we have:

$$\frac{\begin{array}{l}H_2 : b \in V \cup \{\eta\} \\ H_3 : a \in E'.A \\ H_4 : R'(a, b)\end{array}}{G_2 : a \in V \cup \{\eta\}}$$

by inversion on H_0, there is two cases, in case one (UB reduction) we have the following:

$$T_1 : V' \vdash (\mathsf{E}, op_{<s,i>}) \hookrightarrow (\mathsf{E}', \eta)$$

by inversion on T_1 we will have the following:

$$T_2 : E'.A = E.A \cup \{\eta\}$$

Since the other case (LB reduction) also includes similar premises which yields T_2, we can add it to the hypothesis:

$$H_5 : E'.A = E.A \cup \{\eta\}$$

by rewriting H_5 in H_3 and by inversion, we get two cases: $a = \eta$ and $a \in E.A$. The first case immediatly proves G_2, so we only consider the second case where we have:

$$H_6 : a \in E.A$$

Now, by inversion on H_2, we have two cases:

- **Case 1:**

  $$b \in V$$

  by inversion in H_1 we have:

  $$H_7 : \forall(x \in V).\forall(y \in E.A).R(y, x) \Rightarrow y \in V$$

  by instantiation with a and b:

  $$H_8 : R(a, b) \Rightarrow a \in V$$

  Now by applying the lemma 1 on H_4 we get that $R(a, b)$ holds (since $a, b \neq \eta$), which can be applied on H_8 to get $a \in V$ which proves the goal G_2.
- **Case 2:**

  $$\begin{array}{ll} & H_9 : \ b = \eta \\ (\texttt{by rewriting } H_9 \texttt{ in } H_4) & H_{10} : R'(a, \eta)\end{array}$$

  Now we use inversion on H_0 and get two cases: (LB exec) and (UB exec)
  - **SCase (LB exec):** we have H_{11} and H_{12} from the reduction rule premises:

    $$\begin{array}{l}H_{11} : R_V^{-1}(\eta) = R_{E'.A}^{-1}(\eta) \\ H_{12} : R_V^{-1}(\eta) \subseteq V\end{array}$$

now from H_{10} we have H_{13} which can be rewritten by H_{11} to get H_{H14}:

$$H_{13} : a \in R^{-1}_{E'.A}(\eta)$$
$$H_{14} : a \in R^{-1}_{V}(\eta)$$

The goal G_2 is now proved from H_{12} and H_{14}.
– **SCase (UB exec):** We have the following from the premises:

$$H_{15} : V' = \lfloor V \rfloor_{\mathsf{max}}$$
$$H_{16} : V' \subseteq V$$

now destruct R, the only non-trivial cases are $(R = \mathsf{trunc}(R); \mathsf{vis})$ and $(R = \mathsf{vis})$:
SSCase $(R = \mathsf{trunc}(R); \mathsf{vis})$:
From H_{10} we get H_{17} which based on the definition, yields that there exists c such that H_{18}, H_{19} and H_{20} hold:

$$H_{17} : a \in (\mathsf{trunc}(R)'; \mathsf{vis}')^{-1}_{E'.A}(\eta)$$
$$H_{18} : c \in \mathsf{vis}'^{-1}(\eta)$$
$$H_{19} : a \in \mathsf{trunc}(R)'^{-1}(c)$$
$$H_{20} : \mathsf{vis}'^{-1}(\eta) \subseteq E'.A$$

from H_{15} we have:

$$H_{21} : (\mathsf{trunc}(R))^{-1}_{V}(V') \subseteq V'$$

Now from H_{18} is straightforward to get:

$$H_{22} : c \in V'$$

which after appying the lemma 1 on H_{19}, and by H_{21} yields the following, which proves the goal G_2:

$$H_{23} : a \in V'$$

SSCase $(R = \mathsf{vis})$: From H_{10} we get that $\mathsf{vis}'(a, \eta)$, which -with a similar argument to the previous subcase- yields the following and the goal is proved:

$$H_{24} : a \in V'$$

QED.

(Part ii)
For this part we have the following hypothesis and the goal:

$$H_0 : (\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}', \eta)$$
$$G_0 : E' \models [\eta/\hat{\eta}]$$

By inversion on H_0, we have two cases:
Case1 (UB exec):

$$H_1 : r_k = \mathsf{vis}$$
$$H_2 : V \subseteq E.A$$
$$H_3 : V' = \lfloor V \rfloor_{\mathsf{max}}$$
$$H_4 : V' \vdash (\mathsf{E}, op_{<s,i>}) \hookrightarrow (\mathsf{E}', \eta)$$

The goal G_0 can be rewritten as:

$$G_1 : E' \models \forall a.a \xrightarrow{R} \eta \Rightarrow a \xrightarrow{vis} \eta$$

Since the $E'.A$ gives the interpretation for the universe of quantification:

$$G_2 : \forall(a \in E'.A).E' \models a \xrightarrow{R} \eta \Rightarrow a \xrightarrow{vis} \eta$$

by intros:

$$\begin{array}{l} H_5 : a \in E'.A \\ G_3 : E' \models a \xrightarrow{R} \eta \Rightarrow a \xrightarrow{vis} \eta \end{array}$$

Now since $((\mathcal{M} \models A \Rightarrow B) \Leftrightarrow (\mathcal{M} \models A \Rightarrow \mathcal{M} \models B))$ we can rewrite G_3 as:

$$G_4 : (E' \models a \xrightarrow{R} \eta) \Rightarrow (E' \models a \xrightarrow{vis} \eta)$$

intros:

$$\begin{array}{l} H_6 : E' \models a \xrightarrow{R} \eta \\ G_5 : E' \models a \xrightarrow{vis} \eta \end{array}$$

Now we use the interpretation given by E', to rewrite the relations as follows. Note that we denote the interpretation of R under E' as R' and $E.\mathsf{vis}$ as vis'.

$$\begin{array}{l} H_7 : R'(a, \eta) \\ G_6 : \mathsf{vis}'(a, \eta) \end{array}$$

by inversion on H_4:

$$H_8 : \mathsf{vis}' = \mathsf{vis} \cup V' \times \{\eta\}$$

Now since η is a fresh effect, we get that $a \in V' \Rightarrow \mathsf{vis}'(a, \eta)$ which can be applied to G_6 to get the following:

$$G_7 : a \in V'$$

Now, destructing R yileds multiple cases, only one of which is non-trivial: $R = \mathsf{trunc}(R); \mathsf{vis}$, which can be rewritten in H_7 to get:

$$H_9 : (\mathsf{trunc}(R); \mathsf{vis})'(a, \eta)$$

Now we can rewrite the definition in H_9, and derive that there exists b such that:

$$\begin{array}{l} H_{10} : \mathsf{trunc}(R)'(a, b) \\ H_{11} : \mathsf{vis}'(b, \eta) \end{array}$$

Now using a similar argument, from H_8 and H_{11} we get:

$$H_{12} : b \in V'$$

Now by applying the lemma 1 on H_{10} we get:

$$H_{13} : \mathsf{trunc}(R)(a, b)$$

since we have $V' \in \lfloor V \rfloor$, we get the following:

$$H_{14} : \forall(x \in V').(\mathsf{trunc}(R))^{-1}_{E.A}(V') \Rightarrow x \in V'$$

which yields the following from H_{12} and H_{13}:

$$H_{15} : a \in V'$$

which proves the goal G_7.

Case2 (LB exec):

We prove this case by induction on the length of the given relation R. We have the followings, from the premises of the reduction rule:

$$\begin{array}{l} H_1 : r_k = \mathsf{so} \\ H_2 : V \subseteq E.A \\ H_3 : R_V^{-1}(\eta) = R_{E.A}^{-1}(\eta) \\ H_4 : R_V^{-1}(\eta) \subseteq V \\ H_5 : V \vdash (\mathsf{E}, op_{<s,i>}) \hookrightarrow (\mathsf{E}', \eta) \end{array}$$

Using the same argument as the previous section, we get the following new goal and hypotheses:

$$\begin{array}{l} H_6 : a \in E'.A \\ H_7 : R'(a, \eta) \\ G_1 : \mathsf{vis}'(a, \eta) \end{array}$$

We now destruct R to get H_8 from H_7, and rewrite the definition in it to get the next two hypotheses. Note that by destructing R, there are only two non-trivial cases $R = \mathsf{trunc}(R); \mathsf{so}$ and $R = \mathsf{so}$, which we are only consideing the former, since the latter can be proved similarly.

$$\begin{array}{ll} H_8 : & (\mathsf{trunc}(R); \mathsf{so})'(a, \eta) \\ H_9 : & \mathsf{trunc}(R)'(a, b) \\ H_{10} : & \mathsf{so}'(b, \eta) \end{array}$$

Now, from the previous section we know that $(so')^{-1}(\eta) \subseteq V$ which yields the following from H_{10}:

$$H_{11} : b \in V$$

The goal is proved by the induction hypothesis, H_9 and H_{11}.
QED.

B.2 Proof of Theorem 2

We prove the theorem by contradiction:

$$\begin{array}{l} H_0 : (\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}', \eta) \\ H_1 : a \in V \\ H_2 : (a, \eta) \notin E'.\mathsf{vis} \\ H_3 : (E'.A, E'.\mathsf{vis} \cup \{(a, \eta)\}, E'.\mathsf{so}) \models \psi[\eta/\hat{\eta}] \\ H_4 : (\mathsf{trunc}(R)_V^{-1}(a) = \mathsf{trunc}(R)_{E.A}^{-1}(a)) \\ G_0 : \bot \end{array}$$

Now we call $(E'.A, E'.\mathsf{vis} \cup \{(a, \eta)\}, E'.\mathsf{so})$ as E'' and derive the following from H_3:

$$H_5 : E'' \models \forall x.x \xrightarrow{R} \eta \Rightarrow x \xrightarrow{\mathbf{vis}} \eta$$

because E'' defines the universe of quantification (and since $E''.A = E'.A$), we get the following:

$$H_6 : \forall (x \in E'.A).E'' \models x \xrightarrow{R} \eta \Rightarrow x \xrightarrow{\mathbf{vis}} \eta$$

and is rewritten as the following:

$$H_7 : \forall (x \in E'.A).(E'' \models x \xrightarrow{R} \eta) \Rightarrow (E'' \models x \xrightarrow{\mathbf{vis}} \eta)$$

Now by inversion on H_0 we get two cases, one of which is trivial. We skip the formal proof for it but it is easy to see that in [LB exec] case, ALL effects in V are made

visible to η, so the set is trivially maximal, i.e. H_1 and H_2 yield $\bot$. For the other case (UB exec), we get the following:

$$H_8 : V' = \lfloor V \rfloor_{\texttt{max}}$$
$$H_9 : V' \vdash (\mathsf{E}, op_{<s,i>}) \hookrightarrow (\mathsf{E}', \eta)$$

by inversion on H_9 we get H_{10} and from that and from H_2, following a similar argument from the proof of theorem 1, we get H_{11}:

$$H_{10} : \mathsf{vis}' = \mathsf{vis} \cup V' \times \{\eta\}$$
$$H_{11} : a \notin V'$$

Now by denoting the interpretation of R under E'' as R'', H_7 can be rewritten as follows:

$$H_{12} : \forall (x \in E'.A).R''(x, \eta) \Rightarrow \mathsf{vis}''(x, \eta)$$

Now by inversion on H_8, we get the following:

$$H_{13} : V' \in \lfloor V \rfloor$$
$$H_{14} : \nexists V'' \in \lfloor V \rfloor .|V''| > |V'|$$
$$(\texttt{from H}_{13})\ H_{15} : V' \subseteq V \ \wedge\ (\mathsf{trunc}(R))_V^{-1}(V') \subseteq V' \wedge (\mathsf{trunc}(R))_V^{-1}(V') = (\mathsf{trunc}(R))_{E.A}^{-1}(V')$$

Now we can destruct R, where we get multiple cases, only two of which are non-trivial, $(R = \mathsf{vis})$ and $(R = \mathsf{trunc}(R); \mathsf{vis})$

- **Case1**$(R = \mathsf{vis})$:
  $\mathsf{trunc}(R) = \texttt{null}$, thus V itself satisfies the requirements in H_{15} and we get that $(V = \lfloor V \rfloor_{\texttt{max}})$ and the following holds:

  $$H_{16} : V = V'$$

  which results in contradiction from H_1 and H_{11}.

- **Case2**$(R = \mathsf{trunc}(R); \mathsf{vis})$:
  Since $|V' \cup \{a\}| > |V'|$ we have the following:

  $$H_{17} : (V' \cup \{a\}) \notin \lfloor V \rfloor$$

  which based on the definition yields that the conditiones for holding the above relation are not true, i.e.

  $$H_{18} : \neg((V' \cup \{a\}) \subseteq V \ \wedge\ (\mathsf{trunc}(R))_V^{-1}(V' \cup \{a\}) \subseteq (V' \cup \{a\}) \wedge (\mathsf{trunc}(R))_V^{-1}(V' \cup \{a\}) = (\mathsf{trunc}(R))_{E.A}^{-1}(V' \cup \{a\}))$$

  or equally:

  $$H_{19} : (V' \cup \{a\}) \not\subseteq V \vee (\mathsf{trunc}(R))_V^{-1}(V' \cup \{a\}) \not\subseteq (V' \cup \{a\}) \vee (\mathsf{trunc}(R))_V^{-1}(V' \cup \{a\}) \neq (\mathsf{trunc}(R))_{E.A}^{-1}(V' \cup \{a\})$$

  By inversion on the above, we get three cases, two of which are trivial. The last conjunct can't hold because of H_4 and the first one also contradics with H_1 and H_{15}. Thus, we are left with only one case:

  $$H_{20} : (\mathsf{trunc}(R))_V^{-1}(V' \cup \{a\}) \not\subseteq (V' \cup \{a\})$$

  Now, from the second conjunct in H_{15} we know that it should be the case that:

  $$(\texttt{from H}_{15} : (\mathtt{trunc(R)})_{\mathtt{V}}^{-1}(\mathtt{V}') \subseteq \mathtt{V}') \qquad H_{21} : ((\mathsf{trunc}(R))_V^{-1}(a) \not\subseteq (V' \cup \{a\}))$$

The above hypothesis yields the existance of $c \neq a$ such that:

$$H_{22} : c \in (\mathsf{trunc}(R))_V^{-1}(a)$$
$$H_{23} : c \notin V'$$

Now, by rewriting $(R = \mathsf{trunc}(R); \mathsf{vis})$ in H_{12} we get H_{24}, which can be rewritten again into H_{25} from the definition:

$$H_{24} : \forall (x \in E'.A).((\mathsf{trunc}(R); \mathsf{vis})''(x, \eta) \Rightarrow \mathsf{vis}''(x, \eta))$$
$$H_{25} : \forall (x \in E'.A).(\exists b.\mathsf{trunc}(R)''(x, b) \wedge \mathsf{vis}''(b, \eta) \Rightarrow \mathsf{vis}''(x, \eta))$$

Now, we instantiate H_{25} with $x = c$:

$$H_{26} : \exists b.\mathsf{trunc}(R)''(c, b) \ \wedge \ \mathsf{vis}''(b, \eta) \Rightarrow \mathsf{vis}''(c, \eta)$$

we can replace $\mathsf{trunc}(R)''$ with $\mathsf{trunc}(R)'$ in above definition, since from H_3, the only difference in interpretation under E' and E'' is the extra element (a, η) in $E''.\mathsf{vis}$ which does not effect $\mathsf{trunc}(R)''(c, b)$:

$$H_{27} : \exists b.\mathsf{trunc}(R)'(c, b) \ \wedge \ \mathsf{vis}''(b, \eta) \Rightarrow \mathsf{vis}''(c, \eta)$$

Moreover, since $c \neq a$, we can replace $\mathsf{vis}''(c, \eta)$ with $\mathsf{vis}'(c, \eta)$:

$$H_{28} : \exists b.\mathsf{trunc}(R)'(c, b) \ \wedge \ \mathsf{vis}''(b, \eta) \Rightarrow \mathsf{vis}'(c, \eta)$$

From H_{15} and H_{22} we get H_{29}, and H_{30} also holds trivially from H_3:

$$H_{29} : \mathsf{trunc}(R)'(c, a)$$
$$H_{30} : \mathsf{vis}''(a, \eta)$$

which can be used in instantiation of H_{28} with $b = a$ and derive the following:

$$H_{31} : \mathsf{vis}'(c, \eta)$$

However, we know -from the previously explained argument- that H_{31} results in H_{32}, which results in contradiction with H_{23}.

$$H_{32} : c \in V'$$

QED.

B.3 Proof of Theorem 3

Before proving the theorem, we first present and prove a useful lemma and then we will present a new definition, regarding sets of effects.

Lemma 2. *Under an execution state E and for a given set $S \subseteq E.A$, if S is ψ-consistent under E, then $\forall (x \in S).R_S^{-1}(x) \subseteq S$ under E.*

Proof.

$$H_0 : Sis\psi\text{-}\mathtt{consistent}$$
$$G_0 : \forall (x \in S).R_S^{-1}(x) \subseteq S$$

after intros:

$$H_1 : x \in S$$
$$G_1 : R_S^{-1}(x) \subseteq S$$

inversion on H_0 gives the following:

$$H_2 : \forall(\eta \in S).\forall(a \in E.A).R(a,\eta) \Rightarrow a \in S$$

which can be rewritten to:

$$H_3 : \forall(\eta \in S).R^{-1}(\eta) \subseteq S$$

however, since $S \subseteq E.A$ then[6]:

$$H_4 : \forall(a \in E.A).R_S^{-1}(a) \subseteq R^{-1}(a)$$

Now we can instantiate H_3 and H_4 into:

$$H_5 : R^{-1}(x) \subseteq S$$
$$H_6 : R_S^{-1}(x) \subseteq R^{-1}(x)$$

which trivially yields G_1 and the proof is completed.
QED.

Definition 1. *We define the completment of a given set of effects S (under an execution state E) as the super set of S, containing ALL the mid-level effects required to determine ALL the dependencies of the effects in S, i.e.*

$$S' \in \lceil S \rceil \iff R_{S'}^{-1}(S) = R_{E.A}^{-1}(S)$$

Now, using the above theorem and lemma, we present the proof of the theorem 3, which starts by listing the following hypotheses and the goal:

$$H_0 : S \vdash (\mathsf{E}, op_{<s,i>}) \hookrightarrow (\mathsf{E}', \eta)$$
$$H_1 : S \cup \{\eta\} \; is \; \psi\text{-}\texttt{consistent}$$
$$G_0 : \exists E''.\exists \eta'.\exists V.((\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}'', \eta'))$$

Now, by destructing R we get two non-trivial cases:

- **Case1**$(R = \mathsf{trunc}(R); \mathsf{vis})$:
  In this case[7], we generate the premises of the [UB EXEC] to achieve the goal as follows. Firstly, we define S' and present η':

  $$H_3 : S' = \lfloor S \rfloor_{\texttt{max}}$$
  $$H_4 : \eta' = (s, op, F_{op}(S'))$$

  Moreover, we will define the followings, which will be used when presenting E'':

  $$H_5 : \mathsf{so}'' = \mathsf{so} \cup A_{(\texttt{SessID=s})} \times \{\eta'\}$$
  $$H_6 : \mathsf{vis}'' = \mathsf{vis} \cup S' \times \{\eta'\}$$
  $$H_7 : A'' = E.A \cup \{\eta'\}$$

  Now we present V and E'' as follows and rewrite the goal:

  $$H_8 : V = S$$
  $$H_9 : E'' = (A'', \mathsf{so}'', \mathsf{vis}'')$$
  $$G_1 : (\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}'', \eta')$$

[6] we skip the formal proof of this claim, however, since the only difference in the definitions of R^{-1} and R_S^{-1} is the extra requirement about mid-level effects in the latter, it should be a subset of the former.

[7] Note that in this case the goal G_0, trivially holds. That is because the contract in this case is [UB], which represents executions whitout blocking or waiting, that can always make progress by showing *some* set of effects to the operations

by applying [UB EXEC] on G_1 we get the following new goals (after rewriting H_9 and H_3):

$$\begin{array}{l} G_2 : r_k = \mathsf{vis} \\ G_3 : S \subseteq E.A \\ G_4 : S' = \lfloor S \rfloor \\ G_5 : S' \vdash (\mathsf{E}, op_{<s,i>}) \hookrightarrow (\mathsf{E}'', \eta') \end{array}$$

first three goals are proved via the assumptions, and the last one can be easily shown to hold by applying [OPER] and deriving the following new goals:

$$\begin{array}{ll} G_6 : & S' \subseteq E.A \\ G_7 : & F_{op}(S') = v \\ G_8 : & \eta' = (s, op, v) \\ G_9 : & \eta \notin S' \\ G_{10} : & E''.A = E.A \cup \{\eta'\} \\ G_{11} : & E''.\mathsf{vis} = E.\mathsf{vis} \cup S' \times \{\eta\} \\ G_{12} : & E''.\mathsf{so} = E.\mathsf{so} \cup (A_{(\mathtt{SessID=s})}) \times \{\eta\} \end{array}$$

all the above goals have already been shown in the assumptions and the case is proved.

– **Case2**($R = \mathsf{trunc}(R); \mathsf{so}$):
Similarly in this case we define the following:

$$H_{13} : V = \lceil S \cup \{\eta\} \rceil$$

which yields:

$$H_{14} : \forall (x \in S \cup \{\eta\}).R_V^{-1}(x) = R_{E'.A}^{-1}(x)$$

and also:

$$H_{15} : R_V^{-1}(\eta) = R_{E'.A}^{-1}(\eta)$$

Similar to the previous case, we now define the followings:

$$\begin{array}{l} H_{16} : \eta' = (s, op, F_{op}(V)) \\ H_{17} : \mathsf{so}'' = \mathsf{so} \cup A_{(\mathtt{SessID=s})} \times \{\eta'\} \\ H_{18} : \mathsf{vis}'' = \mathsf{vis} \cup V \times \{\eta'\} \end{array}$$

Now we presentE'' as follows and rewrite the goal:

$$\begin{array}{ll} H_{19} : E'' = (A'', \mathsf{so}'', \mathsf{vis}'') \\ G_1 : & (\mathsf{E}, op_{<s,i>}) \xrightarrow{V} (\mathsf{E}'', \eta') \end{array}$$

by applying [LB EXEC] on G_1 we get the following new goals

$$\begin{array}{l} G_2 : r_k = \mathsf{so} \\ G_3 : V \subseteq E.A \\ G_4 : R_V^{-1}(\eta') = R_{E''.A}^{-1}(\eta') \\ G_5 : R_V^{-1}(\eta') \subseteq V \\ G_6 : V \vdash (\mathsf{E}, op_{<s,i>}) \hookrightarrow (\mathsf{E}'', \eta') \end{array}$$

Now, G_2 and G_3 are trivially proved from the assumptions, and G_6 also can be easily proved following the argument from the previous case. We prove G_4 and G_5, by a new claim that $R_{E'.A}^{-1}(\eta) = R_{E''.A}^{-1}(\eta')$ which will be proved separately. Thus, we can rewrite the goals and add the new claim:

$$\begin{array}{l} G_7 : R^{-1}(\eta) = R_{E'.A}^{-1}(\eta) \\ G_8 : R^{-1}(\eta) \subseteq V \\ G_9 : R_{E'.A}^{-1}(\eta) = R_{E''.A}^{-1}(\eta') \end{array}$$

Now G_7 is equal to the assumption H_{15}, and G_8 is the direct result of applying the lemma 2 on H_1. Now by rewriting $R = \mathsf{trunc}(R); \mathsf{so}$ in G_9 we have the following:

$$G_{10} : (\mathsf{trunc}(R); \mathsf{so})^{-1}_{E'.A}(\eta) = (\mathsf{trunc}(R); \mathsf{so})^{-1}_{E''.A}(\eta')$$

Now, note that the only difference in E' and E'' is in how the update the vis relation from E, the former makes the set S visible to the operation and the latter the set $\lceil S \cup \{\eta\} \rceil$. Now since the given relation R ends with an so relation, it is straigtforward to show that G_10 holds and thus the case (and the theorem) is proved.

QED.

C Operational Semantics of the Augmented algorithm

Here, we explain our detailed operational semantics, to maintain multi-consistent replicated stores. We assume a given function from operation names, to consistency contracts: $\Psi : op \mapsto \psi$ and for simplicity reasons (again, it can be easily generalized) we consider contracts made by a single prop:

$$\Psi(op) = \forall(a, b).a \xrightarrow{R_{op}} \Rightarrow a \xrightarrow{vis} b.$$

For a given realtion R we also define $R[m]$ to refer to the m'th relation seed in R:

$$(r_1; r_2; ...; r_m; ...; r_k)[m] = r_m$$

Each replica in this semantics, maintains a pool of available effects, and a cache of filtered effects for each operation, each of which is a subset of pool that is closed under its associated contract, i.e. $\forall \eta \in \mathsf{cache}(op).(\mathsf{trunc}(R_{op}))^{-1}_{\mathsf{pool}}(\eta) \subseteq \mathsf{cache}(op)$ We also define DDP of effects which is maintained according to section 6. Following is the formal definitions and the operation semantics.

$$\begin{array}{llll}
\multicolumn{4}{l}{\delta \in \texttt{Replicated Data Type} \qquad v \in \texttt{Value} \qquad op \in \texttt{Operation Name}} \\
\multicolumn{4}{l}{\qquad s \in \texttt{Session Id} \qquad i \in \texttt{Effect Id} \qquad \rho \in \texttt{Replica Id}} \\
\eta & \in \texttt{Effect} & \coloneqq & (s, i, op, v) \\
\mathsf{pool} & \in \texttt{Pool} & \coloneqq & (v, \mathcal{P}(\eta)) \\
\mathsf{cache} & \in \texttt{Cache} & \coloneqq & op \mapsto (v, \mathcal{P}(\eta)) \\
\mathsf{DDP} & \in \texttt{Deps.Presence} & \coloneqq & op \mapsto (\eta \mapsto \{0, 1, ..., k-1\}) \\
F_{op} & \in \texttt{Op. Def.} & \coloneqq & v \to \eta \\
\mathsf{A} & \in \texttt{Eff Soup} & \coloneqq & \mathcal{P}(\eta) \\
\mathsf{vis}, \mathsf{so} & \in \texttt{Relations} & \coloneqq & \mathcal{P}((\eta, \eta)) \\
\mathsf{E} & \in \texttt{Exec State} & \coloneqq & (\mathsf{A}, \mathsf{vis}, \mathsf{so}) \\
\Theta & \in \texttt{Store} & \coloneqq & \rho \mapsto (\mathsf{pool}, \mathsf{cache}, \mathsf{DDP}) \\
\sigma & \in \texttt{Session} & \coloneqq & \cdot \mid op :: \sigma \\
\Sigma & \in \texttt{Session Soup} & \coloneqq & \langle s, i, \sigma \rangle \parallel \Sigma \mid \emptyset
\end{array}$$

$$\mathsf{ssn}(s, \_, \_, \_) = s \qquad \mathsf{id}(\_, j, \_, \_) = j \qquad \mathsf{oper}(\_, \_, op, \_) = op \qquad \mathsf{rval}(\_, \_, \_, n) = n$$

Auxiliary Reduction $\boxed{v \vdash (\mathsf{E}, \langle s, i, op \rangle) \hookrightarrow (\mathsf{E}', \eta)}$

[Oper]

$$\frac{F_{op}(v) = \eta \qquad \mathsf{ssn}(\eta) = s \qquad \mathsf{id}(\eta) = i \qquad \mathsf{A}' = \mathsf{A} \cup \{\eta\} \qquad \mathsf{vis}' = \mathsf{vis} \cup S \times \{\eta\} \qquad \mathsf{so}' = \mathsf{so} \cup \{(\eta', \eta) \mid \eta' \in \mathsf{A} \ \wedge \ \mathsf{ssn}(\eta') = s \ \wedge \ \mathsf{id}(\eta') < i\}}{v \vdash ((\mathsf{A}, \mathsf{vis}, \mathsf{so}), \langle s, i, op \rangle \hookrightarrow ((\mathsf{A}', \mathsf{vis}', \mathsf{so}'), \eta)}$$

Operational Semantics $\boxed{(\mathsf{E}, \Theta, \Sigma) \xrightarrow{\eta} (\mathsf{E}', \Theta', \Sigma')}$

[Pool Refresh]

$$\frac{\eta \in \mathsf{E.A} \quad \Theta(\rho) = (\mathsf{pool}, \mathsf{cache}, \mathsf{DDP}) \quad \eta \notin \mathsf{pool}_e \quad \mathsf{pool}' = (apply \ \eta \ \mathsf{pool}_v, \mathsf{pool}_e \cup \{\eta\}) \quad \Theta' = \Theta[\rho \mapsto (\mathsf{pool}', \mathsf{cache}, \mathsf{DDP})]}{(\mathsf{E}, \Theta, \Sigma) \xrightarrow{\eta} (\mathsf{E}, \Theta', \Sigma)}$$

[DDP Refresh]

$$\frac{\Theta(\rho) = (\mathsf{pool}, \mathsf{cache}, \mathsf{DDP}) \quad \eta \in \mathsf{pool}_e \quad \mathsf{oper}(\eta) = op \quad \mathsf{DDP}(op)(\eta) = i \quad i < k \quad \mathsf{DDP}'(op) = \mathsf{DDP}(op)[\eta \mapsto i+1] \quad \mathsf{DDP}(op)((R_{op}[i+1])^{-1}(\eta)) \subseteq \mathsf{DDP}^i \quad \Theta' = \Theta[\rho \mapsto (\mathsf{pool}, \mathsf{cache}, \mathsf{DDP}[op \mapsto \mathsf{DDP}'(op)])]}{(\mathsf{E}, \Theta, \Sigma) \xrightarrow{\eta} (\mathsf{E}, \Theta', \Sigma)}$$

[Cache Refresh]

$$\frac{\Theta(\rho) = (\mathsf{pool}, \mathsf{cache}, \mathsf{DDP}) \quad \eta \in \mathsf{pool}_e \quad \mathsf{oper}(\eta) = op \quad \eta \notin \mathsf{cache}(op)_e \quad \mathsf{cache}' = (apply \ \eta \ \mathsf{cache}(op)_v, \mathsf{cache}(op)_e \cup \{\eta\}) \quad \mathsf{DDP}(op)(\eta) = k-1 \quad \Theta' = \Theta[\rho \mapsto (\mathsf{pool}, \mathsf{cache}', \mathsf{DDP})]}{(\mathsf{E}, \Theta, \Sigma) \xrightarrow{\eta} (\mathsf{E}, \Theta', \Sigma)}$$

[LB Exec] [UB Exec]

$$\frac{\Theta(\rho) \vdash (\mathsf{E}, \langle s, i, op \rangle) \hookrightarrow (\mathsf{E}', \eta) \qquad \Theta(\rho) = (\mathsf{pool}, \mathsf{cache}, \_) \qquad \Theta(\rho) = (\mathsf{pool}, \mathsf{cache}, \_) \quad \mathsf{so}^{-1}(\eta) \subseteq \mathsf{cache}(op)_e \quad \mathsf{cache}(\mathsf{op}) \vdash (\mathsf{E}, \langle s, i, op \rangle) \hookrightarrow (\mathsf{E}', \eta)}{(\mathsf{E}, \Theta, \langle s, i, op :: \sigma \rangle \parallel \Sigma) \xrightarrow{\eta} (\mathsf{E}', \Theta, \langle s, i+1, \sigma \rangle \parallel \Sigma) \qquad (\mathsf{E}, \Theta, \langle s, i, op :: \sigma \rangle \parallel \Sigma) \xrightarrow{\eta} (\mathsf{E}', \Theta, \langle s, i+1, \sigma \rangle \parallel \Sigma)}$$

Fig. 11: Operational semantics of a replicated data store.