# hw4 4

### February 23, 2023

```
[99]: import numpy as np
      import random as rm
      import pandas as pd
      import csv
      import sys
      from sklearn import preprocessing
      # from sklearn.preprocessing import StandardScaler
      from sklearn.metrics import confusion matrix, ConfusionMatrixDisplay
      import matplotlib.pyplot as plt
[21]: def getData(fname, dimension):
          # create a new array to store the data
          data = np.empty([0,dimension])
          label = []
          with open(fname, mode = 'r') as file:
              # reading the CSV file
              csvFile = csv.reader(file)
              # displaying the contents of the CSV file
              for lines in csvFile:
                  data = np.row_stack((data,[float(lines[0]), float(lines[1])]))
                  label.append(float(lines[2]))
          label = np.array(label)
          return (data, label)
[24]: # Obtain data
      xdata1_train, ydata1_train = getData('dataset1_train.csv', 2)
      xdata1_test, ydata1_test = getData('dataset1_test.csv', 2)
      xdata2_train, ydata2_train = getData('dataset2_train.csv', 2)
      xdata2_test, ydata2_test = getData('dataset2_test.csv', 2)
      xdata3_train, ydata3_train = getData('dataset3_train.csv',2 )
      xdata3_test, ydata3_test = getData('dataset3_test.csv', 2)
      ## preprocessing
      # Standarlize
```

```
scaler1_train = preprocessing.StandardScaler().fit(xdata1_train)
scaler2_train = preprocessing.StandardScaler().fit(xdata2_train)
scaler3_train = preprocessing.StandardScaler().fit(xdata3_train)

xdata1_train_scaled = scaler1_train.transform(xdata1_train)
xdata1_test_scaled = scaler1_train.transform(xdata1_test)

xdata2_train_scaled = scaler2_train.transform(xdata2_train)
xdata2_test_scaled = scaler2_train.transform(xdata2_test)

xdata3_train_scaled = scaler3_train.transform(xdata3_train)
xdata3_test_scaled = scaler3_train.transform(xdata3_test)
```

```
## EE559 HW4, Prof. Chuqq
      def plotDecBoundaries_Nonlinear(feature, labels, weight, non_linear_trans,_
       →predictor, fsize=(6,4),legend_on = False):
          111
         Plot the decision boundaries and data points for any binary classifiers
         feature: original2D feautre, N x 2 array:
             N: number of data points
             2: number of features
          labels: class lables correspond to feature, N x 1 array: [0,0,1,1,0,0,...]
             N: number of data points
          legend\_on: add the legend in the plot. potentially slower for datasets with _{\! \sqcup}
       ⇔large number of clases and data points
          You need to write the following two functions
         non_linear_trans: your custom non-linear transforation function.
             <feature_nonlinear> = non_linear_trans(<feature_original>),
                 Input: <feature_original>, Nx2 array,
                 Output: <feature_nonlinear>: Nx? array.
             if no nonlinear transformation performs, then,
             let non\_linear\_trans = lambda \ x:x, which just output your original \sqcup
       \hookrightarrow feature
         predictor: your custom predictor.
             <predictions> = predictor(<feature>)
                 Input: <feature> Nx? array.
                 Output: constant
```

```
If you don't want write custom functions, you can modify this plot function
 ⇔based on your need,
    do non-linear transformation and class prediction inside this plot function.
    IIII
   labels = labels.astype(int)
   # Set the feature range for ploting
   max_x = np.ceil(max(feature[:, 0])) + 1
   min_x = np.floor(min(feature[:, 0])) - 1
   max_y = np.ceil(max(feature[:, 1])) + 1
   min_y = np.floor(min(feature[:, 1])) - 1
   xrange = (min_x, max_x)
   yrange = (min_y, max_y)
   # step size for how finely you want to visualize the decision boundary.
   inc = 0.05
   # generate grid coordinates. this will be the basis of the decision
    # boundary visualization.
    (x, y) = np.meshgrid(np.arange(xrange[0], xrange[1]+inc/100, inc), np.
 →arange(yrange[0], yrange[1]+inc/100, inc))
    # size of the (x, y) image, which will also be the size of the
    # decision boundary image that is used as the plot background.
   image_size = x.shape
   xy = np.hstack( (x.reshape(x.shape[0]*x.shape[1], 1, order='F'), y.
 \negreshape(y.shape[0]*y.shape[1], 1, order='F'))) # make (x,y) pairs as a_{\sqcup}
 ⇒bunch of row vectors.
    111
    You should write the custom functions, non linear trans and predictor
    # apply non-linear transformation to all points in the map (not only data_
 ⇔points)
   xy = non_linear_trans(xy)
   # predict the class of all points in the map
    pred_label = predictor(xy)
#
   pred_label = predictor(xy, weight)
    # reshape the idx (which contains the class label) into an image.
   decisionmap = pred_label.reshape(image_size, order='F')
```

```
# documemtation: https://matplotlib.org/stable/api/_as_gen/matplotlib.
⇒pyplot.plot.html
  symbols_ar = np.array(['rx', 'bo', 'ms',_
#show the image, give each coordinate a color according to its class label
  plt.figure(figsize=fsize)
  plt.imshow(decisionmap, extent=[xrange[0], xrange[1], yrange[0],__
# plot the class data.
  plot index = 0
  class_list = []
  class_list_name = [] #for legend
  for cur_label in np.unique(labels):
      # print(cur_label,plot_index,np.sum(label_train == cur_label))
      d1, = plt.plot(feature[labels == cur_label, 0],feature[labels ==_

cur_label, 1], symbols_ar[plot_index])
      if legend on:
         class list.append(d1)
          class_list_name.append('Class '+str(plot_index))
         1 = plt.legend(class_list,class_list_name, loc=2)
         plt.gca().add_artist(1)
      plot_index = plot_index + 1
  plt.show()
```

```
# Set the feature range for ploting
  max_x = np.ceil(max(training[:, 0])) + 1
  min_x = np.floor(min(training[:, 0])) - 1
  max_y = np.ceil(max(training[:, 1])) + 1
  min_y = np.floor(min(training[:, 1])) - 1
  xrange = (min_x, max_x)
  yrange = (min_y, max_y)
  # step size for how finely you want to visualize the decision boundary.
  inc = 0.01
  # generate grid coordinates. this will be the basis of the decision
  # boundary visualization.
  (x, y) = np.meshgrid(np.arange(xrange[0], xrange[1] + inc / 100, inc),
                       np.arange(yrange[0], yrange[1] + inc / 100, inc))
  # size of the (x, y) image, which will also be the size of the
  # decision boundary image that is used as the plot background.
  image_size = x.shape
  xy = np.hstack((x.reshape(x.shape[0] * x.shape[1], 1, order='F'),
                  y.reshape(y.shape[0] * y.shape[1], 1, order='F'))) # make_
\hookrightarrow (x,y) pairs as a bunch of row vectors.
  # distance measure evaluations for each (x,y) pair.
  aug = np.zeros(np.shape(xy)[0]) + 1
  xy_aug = np.concatenate((aug[:, None], xy), axis=1)
  pred_label = np.zeros(np.shape(xy)[0])
  for i in range(np.shape(xy)[0]):
      if w.T @ xy_aug[i] > 0:
          pred_label[i] = 1
      else:
          pred_label[i] = 2
  decisionmap = pred_label.reshape(image_size, order='F')
  plt.imshow(decisionmap, extent=[xrange[0], xrange[1], yrange[0],
# plot the class training data.
  plt.plot(training[label_train == 1, 0], training[label_train == 1, 1], 'rx')
  plt.plot(training[label_train == 2, 0], training[label_train == 2, 1], 'go')
  1 = plt.legend(('Class 1', 'Class 2'), loc=2)
  plt.gca().add_artist(1)
```

```
# plot the class mean vector.
          plt.show()
[50]: def shuffle(xdata, ydata):
          newX = np.copy(xdata)
          newY = np.copy(ydata)
          N = len(newX)
          shuff = np.random.permutation(N)
          for i in range(N):
              newX[i] = xdata[shuff[i]]
              newY[i] = ydata[shuff[i]]
          return (newX, newY)
[41]: def perceptronLearning(data, label, w0, eta = 1, maxEpochs = 100):
          data: (N, D) data array, non-augmented format with labels(1.0, 2.0)
          eta: learning rate (constant)
          maxEpochs: max number of passes through the data. Halts if reach the max_{\sqcup}
       \hookrightarrow epoch
          11 11 11
          N, D = data.shape
          wHat = np.copy(w0)
          minJ = sys.float_info.max
          finalWHat = np.copy(w0)
          i1 = False
          for m in range(1, maxEpochs + 1):
              # 1. shuffle
              shuffledData, shuffledLabel = shuffle(data,label)
              # 2. Augment and reflected
              z = (-1.0) ** (shuffledLabel + 1)
                z = shuffledLabel
              dataAug = np.ones((N, D + 1))
              dataAug[:, 1:] = shuffledData
              zData = (dataAug.T * z).T
              for n in range(0, N):
                  condition = np.dot(wHat ,zData[n])
                  index = (m - 1) * N + n
                   # compute new J(w)
```

gx\_matrix = np.dot(wHat, zData.T)

```
gx_matrix = gx_matrix * -1
        loss = np.sum(gx_matrix > 0)
        if( loss < minJ ):</pre>
            minJ = loss
            finalWHat = np.copy(wHat)
        if(condition <= 0):</pre>
            wHat = wHat + eta * zData[n]
    if loss == 0:
        print("i1 reach. Data is linearly separable")
        i1 = True
        break
if(not i1):
    print("i2 reach.")
print("Weight matrix is:" , finalWHat)
print("Min J is:" , minJ)
return finalWHat
```

```
[42]: def linear_classification():
    weight = np.ones(3)

    weight1 = perceptronLearning(xdata1_train_scaled, ydata1_train, weight)

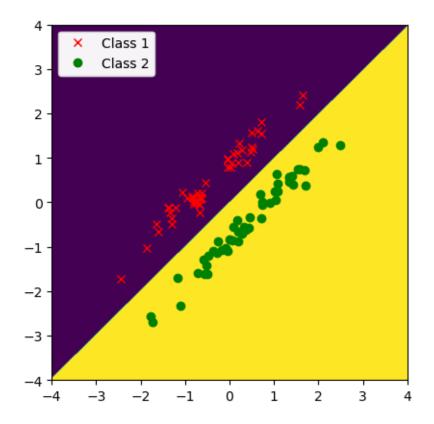
    plotDecBoundaries(xdata1_train_scaled, ydata1_train, weight1)
    acc_train1 = accuracy(xdata1_train_scaled, ydata1_train, weight1)
    print("Accuracy for training data 1 is: ", acc_train1, "%")

    plotDecBoundaries(xdata1_test_scaled, ydata1_test, weight1)
```

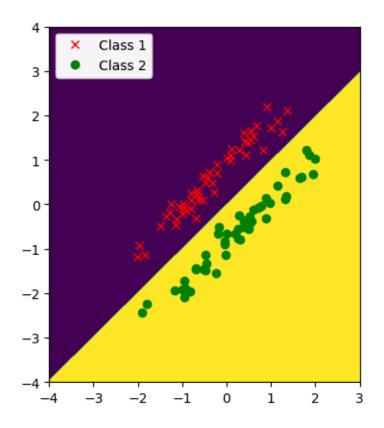
```
acc_test1 = accuracy(xdata1_test_scaled, ydata1_test, weight1)
print("Accuracy for testing data 1 is: ", acc_test1, "%")
weight2 = perceptronLearning(xdata2_train_scaled, ydata2_train, weight)
plotDecBoundaries(xdata2_train_scaled, ydata2_train, weight2)
acc_train2 = accuracy(xdata2_train_scaled, ydata2_train, weight2)
print("Accuracy for training data 2 is: ", acc_train2, "%")
plotDecBoundaries(xdata2_test_scaled, ydata2_test, weight2)
acc_test2 = accuracy(xdata2_test_scaled, ydata2_test, weight2)
print("Accuracy for testing data 2 is: ", acc_test2, "%")
weight3 = perceptronLearning(xdata3_train_scaled, ydata3_train, weight)
plotDecBoundaries(xdata3 train scaled, ydata3 train, weight3)
acc_train3 = accuracy(xdata3_train_scaled, ydata3_train, weight3)
print("Accuracy for training data 3 is: ", acc_train3, "%")
plotDecBoundaries(xdata3_test_scaled, ydata3_test, weight3)
acc_test3 = accuracy(xdata3_test_scaled, ydata3_test, weight3)
print("Accuracy for testing data 3 is: ", acc_test3, "%")
```

## [49]: linear\_classification()

```
i1 reach. Data is linearly separable
Weight matrix is: [ 0. -2.09552505 2.11386957]
Min J is: 0
```



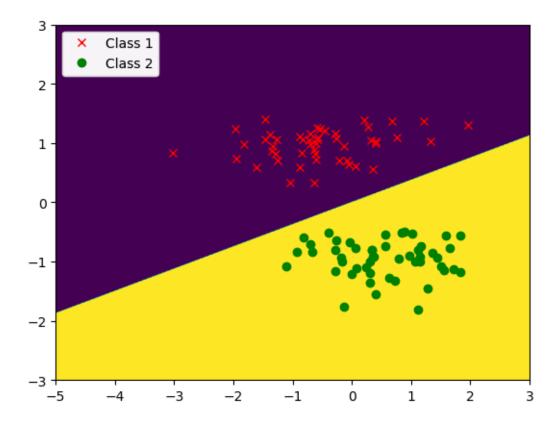
Accuracy for training data 1 is: 100.0 %



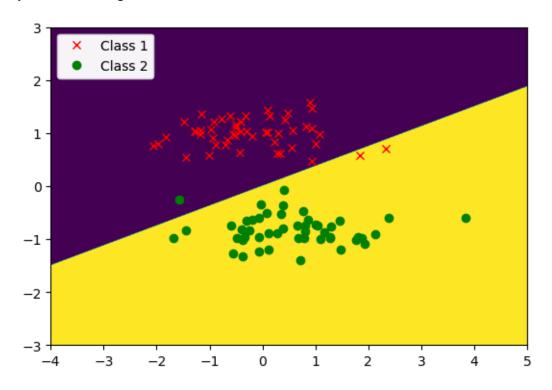
Accuracy for testing data 1 is: 100.0 % i1 reach. Data is linearly separable

Weight matrix is: [ 0. -0.58388862 1.55803305]

Min J is: 0



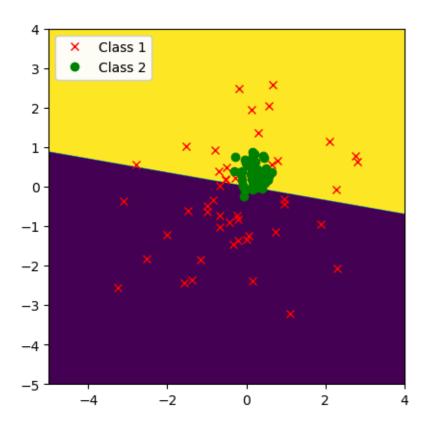
Accuracy for training data 2 is: 100.0 %



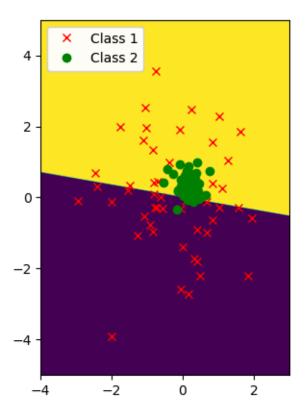
i2 reach.

Weight matrix is: [ 0. -0.00949884 -0.05430716]

Min J is: 1.0980766158415043



Accuracy for training data 3 is: 74.74747474747475 %



Accuracy for testing data 3 is: 75.75757575757575 %

```
def run_10_linear():
    weight = np.ones(3)

    accuracy_train1 = np.zeros(10)
    accuracy_test1 = np.zeros(10)

    accuracy_train2 = np.zeros(10)
    accuracy_test2 = np.zeros(10)

    accuracy_test3 = np.zeros(10)

    for i in range(10):

        weight1 = perceptronLearning(xdata1_train_scaled, ydata1_train, weight)
        accuracy_train1[i] = accuracy(xdata1_train_scaled, ydata1_train, usight)
        accuracy_test1[i] = accuracy(xdata1_test_scaled, ydata1_test, weight1)

        weight2 = perceptronLearning(xdata2_train_scaled, ydata2_train, weight)
```

```
accuracy_train2[i] = accuracy(xdata2_train_scaled, ydata2_train,_
⇒weight2)
      accuracy test2[i] = accuracy(xdata2 test scaled, ydata2 test, weight2)
      weight3 = perceptronLearning(xdata3 train scaled, ydata3 train, weight)
      accuracy_train3[i] = accuracy(xdata3_train_scaled, ydata3_train,__
⇒weight3)
      accuracy_test3[i] = accuracy(xdata3_test_scaled, ydata3_test, weight3)
  print("The mean of accuracy for the training data 1 is : ", np.
→mean(accuracy_train1), "%")
  print("The mean of accuracy for the testing data 1 is :", np.
→mean(accuracy_test1), "%")
  print("The std of accuracy for the training data 1 is :", np.
⇔std(accuracy_train1))
  print("The std of accuracy for the testing data 1 is :", np.
⇔std(accuracy_test1))
  print("\n")
  print("The mean of accuracy for the training data 2 is :", np.
→mean(accuracy train2), "%")
  print("The mean of accuracy for the testing data 2 is :", np.
→mean(accuracy_test2), "%")
  print("The std of accuracy for the training data 2 is :", np.
⇒std(accuracy_train2))
  print("The std of accuracy for the testing data 2 is :", np.
⇔std(accuracy_test2))
  print("\n")
  print("The mean of accuracy for the training data 3 is :", np.
→mean(accuracy_train3), "%")
  print("The mean of accuracy for the testing data 3 is :", np.
→mean(accuracy_test3), "%")
  print("The std of accuracy for the training data 3 is :", np.
⇒std(accuracy_train3))
  print("The std of accuracy for the testing data 3 is :", np.
⇔std(accuracy_test3))
```

#### [47]: run\_10\_linear()

```
i1 reach. Data is linearly separable
Weight matrix is: [ 0.
                           -3.52481552 3.50225845]
Min J is: 0
Accuracy: 100.0 %
Accuracy: 100.0 %
i1 reach. Data is linearly separable
```

```
Weight matrix is: [ 0. -0.153983 1.91133793]
Min J is: 0
Accuracy: 100.0 %
Accuracy: 100.0 %
i2 reach.
Weight matrix is: [ 0.
                       -0.03543798 0.02314264]
Min J is: 1.225349059469708
Accuracy: 53.5353535353536 %
Accuracy: 46.4646464646464 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0. -0.70940556 0.61468782]
Min J is: 0
Accuracy: 100.0 %
Accuracy: 100.0 %
i1 reach. Data is linearly separable
Weight matrix is: [ 1. -0.00874213 2.01612548]
Min J is: 0
Accuracy: 100.0 %
Accuracy: 94.949494949495 %
i2 reach.
Weight matrix is: [ 0.
                             0.01542985 -0.04514293]
Min J is: 0.9244808153892415
Accuracy: 67.676767676768 %
Accuracy: 67.67676767676768 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.
                       -1.82931335 2.21360549]
Min J is: 0
Accuracy: 100.0 %
Accuracy: 100.0 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0. -0.0247717 1.52412521]
Min J is: 0
Accuracy: 100.0 %
Accuracy: 100.0 %
i2 reach.
Weight matrix is: [ 0. -0.00547731 -0.03518379]
Min J is: 0.7079176600142785
Accuracy: 74.74747474747475 %
Accuracy: 76.767676767676 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.
                       -2.62304313 2.08564099]
Min J is: 0
Accuracy: 100.0 %
Accuracy: 98.98989898989899 %
i1 reach. Data is linearly separable
                     0.26978371 2.32760393]
Weight matrix is: [0.
Min J is: 0
Accuracy: 100.0 %
```

Accuracy: 100.0 % i2 reach. Weight matrix is: [ 0. -0.02013134 -0.08456868] Min J is: 1.7473778985840835 Accuracy: 74.747474747475 % Accuracy: 75.75757575757575 % i1 reach. Data is linearly separable -3.15542044 3.06719648] Weight matrix is: [ 0. Min J is: 0 Accuracy: 100.0 % Accuracy: 100.0 % i1 reach. Data is linearly separable Weight matrix is: [ 0. -0.11943942 2.8135676 ] Min J is: 0 Accuracy: 100.0 % Accuracy: 100.0 % i2 reach. Weight matrix is: [0. 0.00075788 0.00378839] Min J is: 0.21160802058431658 Accuracy: 25.252525252525253 % Accuracy: 25.252525252525253 % i1 reach. Data is linearly separable Weight matrix is: [-1. -3.04622363 2.69569424] Min J is: 0 Accuracy: 100.0 % Accuracy: 97.979797979798 % i1 reach. Data is linearly separable Weight matrix is: [ 0. -0.16897988 1.73812694] Min J is: 0 Accuracy: 100.0 % Accuracy: 100.0 % i2 reach. Weight matrix is: [ 0. -0.00152637 -0.03085155] Min J is: 0.6063426545733005 Accuracy: 72.727272727273 % Accuracy: 72.727272727273 % i1 reach. Data is linearly separable Weight matrix is: [ 0. -0.8561001 1.17339924] Min J is: 0 Accuracy: 100.0 % Accuracy: 100.0 % i1 reach. Data is linearly separable Weight matrix is: [ 0. -0.71908314 2.12224397] Min J is: 0 Accuracy: 100.0 % Accuracy: 96.96969696969697 % i2 reach.

Weight matrix is: [ 0. -0.06606927 0.04531855]

Min J is: 2.3581858834547638 Accuracy: 51.5151515151516 % il reach. Data is linearly separable Weight matrix is: [ 0. -2.9573079 2.78736034] Min J is: 0 Accuracy: 100.0 % Accuracy: 100.0 % i1 reach. Data is linearly separable Weight matrix is: [ 0. -0.8240739 3.03591588] Min J is: 0 Accuracy: 100.0 % Accuracy: 98.98989898989899 % i2 reach. Weight matrix is: [0. 0.0035151 0.02225743] Min J is: 1.2207465410559901 Accuracy: 25.252525252525253 % Accuracy: 23.232323232323232 % i1 reach. Data is linearly separable Weight matrix is: [ 0. -2.55001648 1.93274305] Min J is: 0 Accuracy: 100.0 % Accuracy: 96.96969696969697 % i1 reach. Data is linearly separable Weight matrix is: [ 0. -0.55098496 2.14489367] Min J is: 0 Accuracy: 100.0 % Accuracy: 98.989898989899 % i2 reach. Weight matrix is: [0. 0.00923827 0.00011089] Min J is: 0.3852259545630488 Accuracy: 31.3131313131315 % Accuracy: 34.343434343434 % il reach. Data is linearly separable Weight matrix is: [ 0. -3.53397421 3.33131186] Min J is: 0 Accuracy: 100.0 % Accuracy: 100.0 % i1 reach. Data is linearly separable Weight matrix is: [ 0. -0.8279724 2.17683291] Min J is: 0 Accuracy: 100.0 % Accuracy: 96.96969696969697 % i2 reach. Weight matrix is: [ 0. -0.05487033 -0.06343353] Min J is: 1.8042490928978543

Accuracy: 77.777777777779 % Accuracy: 76.76767676767676 %

```
The mean of accuracy for the training data 1 is : 100.0 \%
    The std of accuracy for the training data 1 is: 0.0
    The std of accuracy for the testing data 1 is : 1.0301049522409669
    The mean of accuracy for the training data 2 is : 100.0 \%
    The mean of accuracy for the testing data 2 is: 98.686868686868888 %
    The std of accuracy for the training data 2 is: 0.0
    The std of accuracy for the testing data 2 is: 1.693237839822244
    The mean of accuracy for the training data 3 is : 55.4545454545454546 %
    The std of accuracy for the training data 3 is : 20.28996011281075
    The std of accuracy for the testing data 3 is: 20.866838683353016
[46]: def nonlinear_quadratic_mapping(xdata):
         11 11 11
         xdata : nonagument data N * D
         N, D = xdata.shape
         D_prime = 1/2 * (D ** 2 + 3 * D)
         xdata_mapping = []
         # Augment xdata
         xdata_aug = np.ones((N, D + 1))
         xdata_aug[:,1:] = xdata
         for i in range(D + 1):
            for j in range(i, D + 1):
                xdata_mapping.append(xdata_aug[:,i] * xdata_aug[:,j])
         xdata_mapping = np.array(xdata_mapping).T.reshape(N, int(D_prime) + 1)
         return xdata_mapping
[53]: def predictor(xdata, weight):
         xdata: augment data N * D_primes + 1 matrix
         weight : augmnet weight D_primes + 1 vector
         gx = np.dot(xdata, weight)
         predict_label = np.ones(len(gx))
         for i in range(len(gx)):
             if gx[i] < 0:
                predict label[i] += 1
```

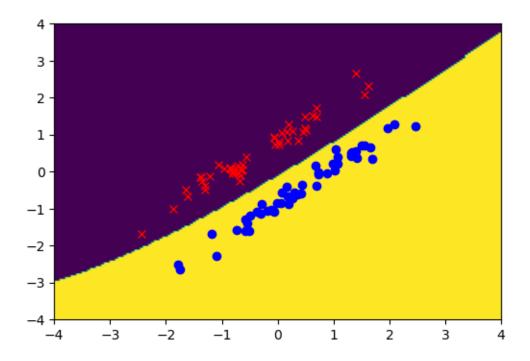
### return predict\_label

```
[115]: # Preprocessing: create an initial weight and get the transformed data
      N, D = xdata1_train_scaled.shape
      D_{prime} = 1/2 * (D ** 2 + 3 * D)
      weight_prime = np.ones(int(D_prime) + 1)
      xdata1_train_mapping = nonlinear_quadratic_mapping(xdata1_train_scaled)
      xdata1_test_mapping = nonlinear_quadratic_mapping(xdata1_test_scaled)
      xdata2_train_mapping = nonlinear_quadratic_mapping(xdata2_train_scaled)
      xdata2_test_mapping = nonlinear_quadratic_mapping(xdata2_test_scaled)
      xdata3_train_mapping = nonlinear_quadratic_mapping(xdata3_train_scaled)
      xdata3_test_mapping = nonlinear_quadratic_mapping(xdata3_test_scaled)
[117]: def quadratic_classfication():
          weight_prime_1 = perceptronLearning(xdata1_train_mapping[:,1:],__
        ⇔ydata1_train, weight_prime)
          plotDecBoundaries_Nonlinear(xdata1_train_scaled, ydata1_train,__
        ⇔weight_prime_1, nonlinear_quadratic_mapping,predictor)
          print("Accuracy rate for training data 1 :",accuracy(xdata1_train_mapping[:
        →,1:], ydata1_train, weight_prime_1), "%")
          plotDecBoundaries_Nonlinear(xdata1_test_scaled, ydata1_test,__
        Gweight_prime_1, nonlinear_quadratic_mapping,predictor)
          print("Accuracy rate for testing data 1 :",accuracy(xdata1_test_mapping[:,1:
        →], ydata1_test, weight_prime_1), "%")
          print("\n")
          weight_prime_2 = perceptronLearning(xdata2_train_mapping[:,1:],_
        plotDecBoundaries_Nonlinear(xdata2_train_scaled, ydata2_train,_
        ⇔weight_prime_2, nonlinear_quadratic_mapping,predictor)
          print("Accuracy rate for training data 2 : ",accuracy(xdata2_train_mapping[:
        →,1:], ydata2_train, weight_prime_2), "%")
          plotDecBoundaries_Nonlinear(xdata2_test_scaled, ydata2_test,_
        weight_prime_2, nonlinear_quadratic_mapping,predictor)
          print("Accuracy rate for testing data 2 :",accuracy(xdata2_test_mapping[:,1:

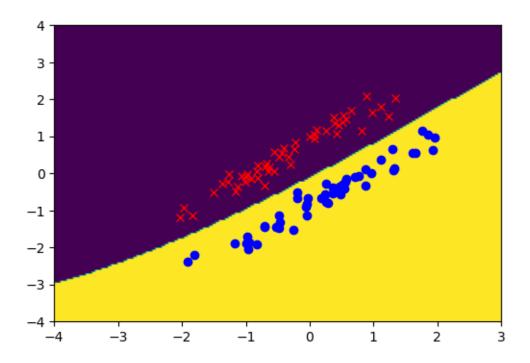
→], ydata2_test, weight_prime_2), "%")
          print("\n")
          weight_prime_3 = perceptronLearning(xdata3_train_mapping[:,1:],__

→ydata3_train, weight_prime)
```

i1 reach. Data is linearly separable
Weight matrix is: [ 1. -7.66323998 8.7350138 -0.6293219 0.07998281 0.39826552]
Min J is: 0



Accuracy rate for training data 1 : 100.0 %



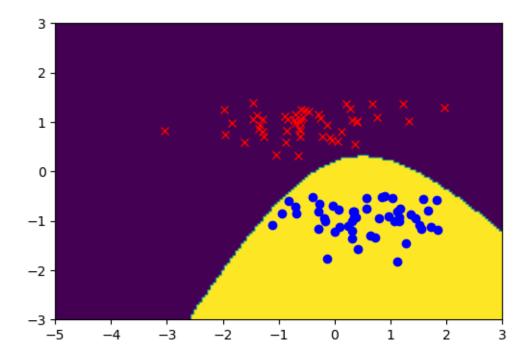
Accuracy rate for testing data 1 : 100.0 %

i1 reach. Data is linearly separable

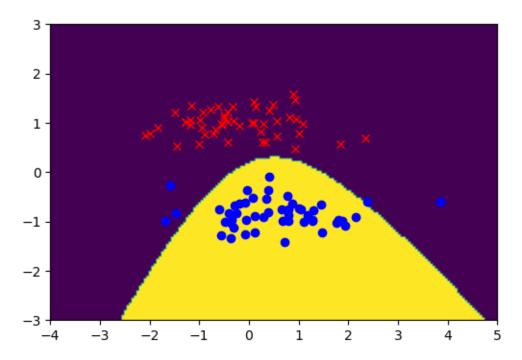
Weight matrix is: [-1. -2.23120865 5.25456869 1.97562164 0.68693003

-0.86031339]

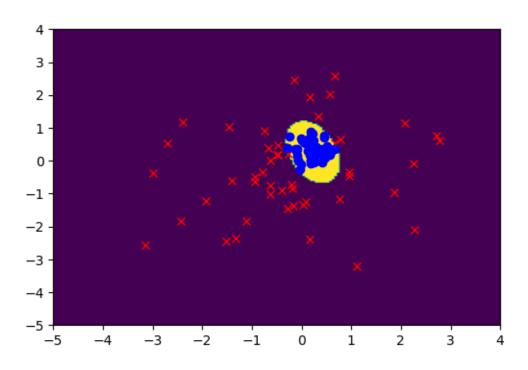
Min J is: 0



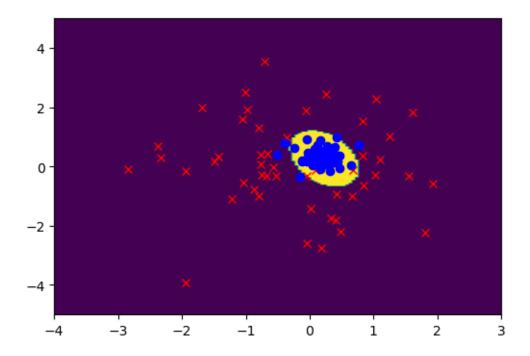
Accuracy rate for training data 2 : 100.0 %



Accuracy rate for testing data 2 : 95.0 %



Accuracy rate for training data 3 : 98.0 %



Accuracy rate for testing data 2 : 93.0 %

```
[151]: def run_10_quadratic():
           accuracy_train1 = np.zeros(10)
           accuracy_test1 = np.zeros(10)
           accuracy_train2 = np.zeros(10)
           accuracy_test2 = np.zeros(10)
           accuracy_train3 = np.zeros(10)
           accuracy_test3 = np.zeros(10)
           for i in range(10):
               weight_prime_1 = perceptronLearning(xdata1_train_mapping[:,1:],__

→ydata1_train, weight_prime)
               weight_prime_2 = perceptronLearning(xdata2_train_mapping[:,1:],__

→ydata2_train, weight_prime)
               weight_prime_3 = perceptronLearning(xdata3_train_mapping[:,1:],__

→ydata3_train, weight_prime)
               accuracy_train1[i] = accuracy(xdata1_train_mapping[:,1:], ydata1_train,__
        →weight_prime_1)
```

```
accuracy_test1[i] = accuracy(xdata1_test_mapping[:,1:], ydata1_test,__
 ⇔weight_prime_1)
        accuracy train2[i] = accuracy(xdata2 train mapping[:,1:], ydata2 train,
 →weight_prime_2)
        accuracy_test2[i] = accuracy(xdata2_test_mapping[:,1:], ydata2_test,__
 →weight_prime_2)
        accuracy_train3[i] = accuracy(xdata3_train_mapping[:,1:], ydata3_train,_u
 →weight_prime_3)
        accuracy_test3[i] = accuracy(xdata3_test_mapping[:,1:], ydata3_test,__
 ⇔weight_prime_3)
   print("The mean of accuracy for the training data 1 is :", np.
 →mean(accuracy_train1), "%")
   print("The mean of accuracy for the testing data 1 is :", np.
 →mean(accuracy_test1), "%")
   print("The std of accuracy for the training data 1 is :", np.
 ⇒std(accuracy train1))
   print("The std of accuracy for the testing data 1 is :", np.
 ⇔std(accuracy_test1))
   print("\n")
   print("The mean of accuracy for the training data 2 is :", np.
 →mean(accuracy_train2), "%")
   print("The mean of accuracy for the testing data 2 is :", np.
 →mean(accuracy_test2), "%")
   print("The std of accuracy for the training data 2 is :", np.
 ⇒std(accuracy train2))
   print("The std of accuracy for the testing data 2 is :", np.
 ⇒std(accuracy_test2))
   print("\n")
   print("The mean of accuracy for the training data 3 is : ", np.
 →mean(accuracy_train3), "%")
    print("The mean of accuracy for the testing data 3 is :", np.
 →mean(accuracy_test3), "%")
   print("The std of accuracy for the training data 3 is :", np.
 ⇔std(accuracy_train3))
   print("The std of accuracy for the testing data 3 is :", np.

std(accuracy_test3))
run_10_quadratic()
```

```
i1 reach. Data is linearly separable
Weight matrix is: [ -2. -14.37311012 13.15115425 1.63934042
```

```
1.76400355
 -1.6091564 ]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-1. -1.12933732 8.36800763 -0.61211116 1.35499821
0.736362621
Min J is: 0
i2 reach.
Weight matrix is: [-1. -3.4761001 -1.45441152 6.59679023 2.31566287
2.158222521
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ 2. -14.15847923 14.66324413 1.22354825
-0.046896
  0.4798287 ]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 0.
                      -1.61757337 5.627948 1.48937733 0.60085165
-2.34590643
Min J is: 0
i2 reach.
Weight matrix is: [-1. -4.19888817 -2.37571506 7.47137138 3.37030289
2.472736431
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ -1. -10.17791242 9.71769867 -1.75130274
0.66391675
  1.3751295 ]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-2. -5.60474477 9.60734888 1.6357955 -1.22020546
-0.36435115]
Min J is: 0
i2 reach.
Weight matrix is: [-1. -3.78220458 -1.58370942 6.55332203 3.43743455
2.242465611
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ 0. -12.84893434 13.18091731 -0.78387109
0.33967744
 -0.83122035]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-1.
                       -0.03200588 3.47644623 0.29721 0.50448169
0.12918827]
Min J is: 0
i2 reach.
Weight matrix is: [-1.
                          -4.14520293 -1.8375234 6.85592478 3.101276
```

```
2.24553155]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ -3. -11.92021152 16.67762127 1.02970221
0.76314312
  -1.202867021
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-1.
                       -3.45807899 9.29396772 2.60828993 1.5040015
0.0470346 1
Min J is: 0
i2 reach.
Weight matrix is: [-1. -3.91056102 -1.95672937 6.96921311 3.15700443
2.673841047
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ 0. -9.27378006 10.14364156 0.45487066 -0.79106118
-1.13223094]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 0. -1.23995226 3.58775562 0.31774209 1.05814183
-0.70960256]
Min J is: 0
i2 reach.
Weight matrix is: [-1. -3.83404934 -1.88671455 7.14938155 2.85661554
2.82642925]
Min J is: 2
i1 reach. Data is linearly separable
                       -8.54532424 9.82140665 -0.74031233 -0.12017084
Weight matrix is: [ 2.
1.15082343]
Min J is: 0
i1 reach. Data is linearly separable
                       -4.79736521 9.02599743 2.88052234 -0.60488379
Weight matrix is: [-3.
-0.153348787
Min J is: 0
i2 reach.
Weight matrix is: [-1. -4.24271236 -2.27383013 7.28186138 2.8860311
2.930269981
Min J is: 2
il reach. Data is linearly separable
Weight matrix is: [ -1. -18.69572391 15.94743714 -2.34073218
1.37407603
  0.86961898]
Min J is: 0
i1 reach. Data is linearly separable
                       -3.81391111 7.72988259 3.82916627 1.41079492
Weight matrix is: [-3.
-1.3838321 ]
Min J is: 0
```

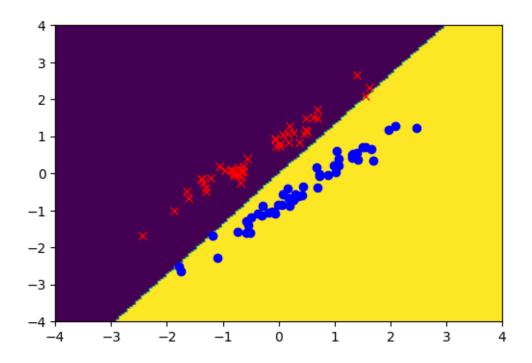
```
i2 reach.
Weight matrix is: [-1. -4.0317986 -2.25217464 7.39099968 3.3326658
2.700045091
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ -2. -10.73828749 8.91720104 -0.55826933
1.11305686
  -0.201598287
Min J is: 0
il reach. Data is linearly separable
                       -2.31644654 6.58540398 -0.0937659 -0.3247009
Weight matrix is: [ 0.
1.07754565]
Min J is: 0
i2 reach.
Weight matrix is: [-1. -3.98268869 -1.94221411 7.27036853 3.09007418
2.79148462]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ -1. -19.69255506 19.54385602 0.29374475
1.51574983
  -2.10647268]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-1. -5.24444232 11.06485385 2.12370647 0.44534914
-2.223766361
Min J is: 0
i2 reach.
Weight matrix is: [-1. -4.4563076 -2.49453314 8.5177586 2.90341355
2.48982312]
Min J is: 2
The mean of accuracy for the training data 1 is : 100.0 \%
The mean of accuracy for the testing data 1 is : 99.6 \%
The std of accuracy for the training data 1 is: 0.0
The std of accuracy for the testing data 1 is: 0.8
The mean of accuracy for the training data 2 is : 100.0 %
The mean of accuracy for the testing data 2 is : 96.7 \%
The std of accuracy for the training data 2 is: 0.0
The std of accuracy for the testing data 2 is : 1.6155494421403513
The mean of accuracy for the training data 3 is : 98.0 \%
The mean of accuracy for the testing data 3 is : 92.0 %
The std of accuracy for the training data 3 is : 0.0
The std of accuracy for the testing data 3 is: 0.6324555320336759
```

```
[140]: def nonlinear_cubic_mapping(xdata):
           xdata : nonagument data N * D
           N, D = xdata.shape
           D_prime = 9
           xdata_mapping = []
           # Augment xdata
           xdata_aug = np.ones((N, D + 1))
           xdata aug[:,1:] = xdata
           for i in range(D + 1):
               for j in range(i, D + 1):
                   for k in range(j, D + 1):
                       xdata_mapping.append(xdata_aug[:,i] * xdata_aug[:,j] *__
        →xdata_aug[:,k])
           xdata_mapping = np.array(xdata_mapping).T.reshape(N, D_prime + 1)
           return xdata_mapping
[145]: xdata1_train_mapping_cubic = nonlinear_cubic_mapping(xdata1_train_scaled)
       xdata1_test_mapping_cubic = nonlinear_cubic_mapping(xdata1_test_scaled)
       xdata2_train_mapping_cubic = nonlinear_cubic_mapping(xdata2_train_scaled)
       xdata2_test_mapping_cubic = nonlinear_cubic_mapping(xdata2_test_scaled)
       xdata3_train_mapping_cubic = nonlinear_cubic_mapping(xdata3_train_scaled)
       xdata3_test_mapping_cubic = nonlinear_cubic_mapping(xdata3_test_scaled)
       weight_prime_cubic = np.ones(10)
[147]: def cubic_classfication():
           weight_prime_1 = perceptronLearning(xdata1_train_mapping_cubic[:,1:],__
        →ydata1_train, weight_prime_cubic)
           plotDecBoundaries_Nonlinear(xdata1_train_scaled, ydata1_train,_
        →weight_prime_1, nonlinear_cubic_mapping,predictor)
           print("Accuracy rate for training data 1 :
        -",accuracy(xdata1_train_mapping_cubic[:,1:], ydata1_train, weight_prime_1),__
        "%")
           plotDecBoundaries_Nonlinear(xdata1_test_scaled, ydata1_test,_
        ⇔weight_prime_1, nonlinear_cubic_mapping,predictor)
           print("Accuracy rate for testing data 1 :
        →",accuracy(xdata1_test_mapping_cubic[:,1:], ydata1_test, weight_prime_1),
        "%")
```

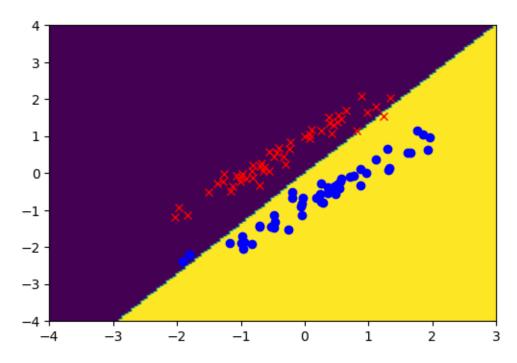
```
print("\n")
    weight_prime_2 = perceptronLearning(xdata2_train_mapping_cubic[:,1:],__

→ydata2_train, weight_prime_cubic)
    plotDecBoundaries_Nonlinear(xdata2_train_scaled, ydata2_train,_
  →weight_prime_2, nonlinear_cubic_mapping,predictor)
    print("Accuracy rate for training data 2 :
  →",accuracy(xdata2_train_mapping_cubic[:,1:], ydata2_train, weight_prime_2),
  "%")
    plotDecBoundaries_Nonlinear(xdata2_test_scaled, ydata2_test,__
  →weight_prime_2, nonlinear_cubic_mapping,predictor)
    print("Accuracy rate for testing data 2 :
  →",accuracy(xdata2_test_mapping_cubic[:,1:], ydata2_test, weight_prime_2),__
  "%")
    print("\n")
    weight_prime_3 = perceptronLearning(xdata3_train_mapping_cubic[:,1:],__

→ydata3_train, weight_prime_cubic)
    plotDecBoundaries_Nonlinear(xdata3_train_scaled, ydata3_train,_
  weight_prime_3, nonlinear_cubic_mapping,predictor)
    print("Accuracy rate for training data 3 :", ...
  →accuracy(xdata3_train_mapping_cubic[:,1:], ydata3_train, weight_prime_3), ___
  "%")
    plotDecBoundaries_Nonlinear(xdata3_test_scaled, ydata3_test,_
  →weight_prime_3, nonlinear_cubic_mapping,predictor)
    print("Accuracy rate for testing data 2 :
  →",accuracy(xdata3_test_mapping_cubic[:,1:], ydata3_test, weight_prime_3),
  "%")
cubic_classfication()
i1 reach. Data is linearly separable
                            -1.61543874 1.24146949 -0.28421054 0.65143742
Weight matrix is: [ 0.
-0.0449748
 -5.97039289 -0.67180192 0.92055614 2.10049763]
Min J is: 0
```

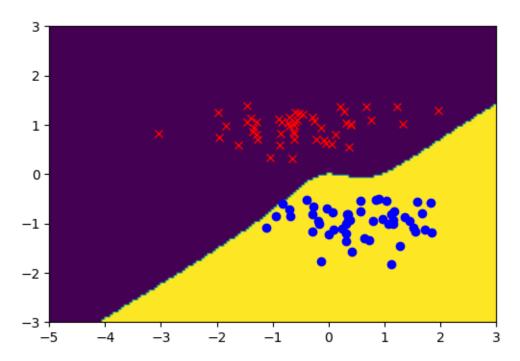


Accuracy rate for training data 1 : 100.0 %

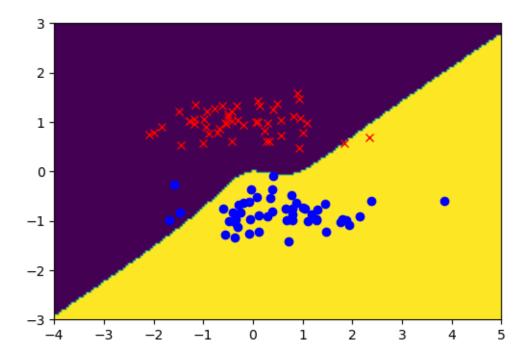


Accuracy rate for testing data 1 : 97.0 %

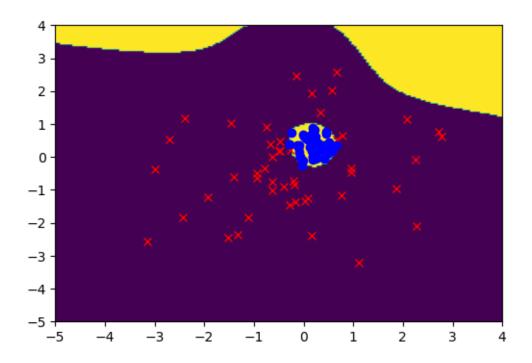
i1 reach. Data is linearly separable
Weight matrix is: [ 0. -0.21279609 3.29674302 3.02243026 -0.14617657 -0.16473553 -2.91061976 2.98469536 0.49709636 2.56366838]
Min J is: 0



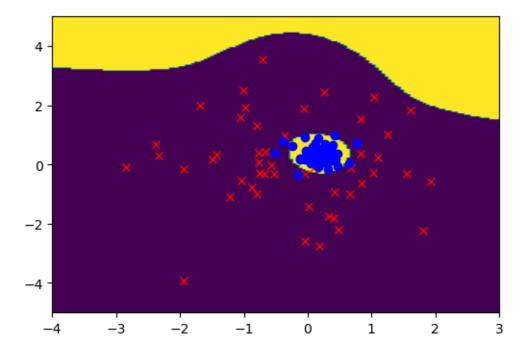
Accuracy rate for training data 2 : 100.0 %



Accuracy rate for testing data 2 : 95.0 %



Accuracy rate for training data 3 : 97.0 %



Accuracy rate for testing data 2 : 92.0 %

```
[152]: def run_10_cubic():
          accuracy_train1 = np.zeros(10)
          accuracy_test1 = np.zeros(10)
          accuracy_train2 = np.zeros(10)
          accuracy_test2 = np.zeros(10)
          accuracy train3 = np.zeros(10)
          accuracy_test3 = np.zeros(10)
          for i in range(10):
              weight_prime_1 = perceptronLearning(xdata1_train_mapping_cubic[:,1:],__

→ydata1_train, weight_prime_cubic)
              weight_prime_2 = perceptronLearning(xdata2_train_mapping_cubic[:,1:],_

→ydata2_train, weight_prime_cubic)
              weight_prime_3 = perceptronLearning(xdata3_train_mapping_cubic[:,1:],__
       accuracy_train1[i] = accuracy(xdata1_train_mapping_cubic[:,1:],__
       →ydata1_train, weight_prime_1)
              accuracy test1[i] = accuracy(xdata1 test mapping cubic[:,1:],...
       accuracy_train2[i] = accuracy(xdata2_train_mapping_cubic[:,1:],__
       →ydata2_train, weight_prime_2)
              accuracy test2[i] = accuracy(xdata2 test mapping cubic[:,1:],,,
       →ydata2_test, weight_prime_2)
              accuracy_train3[i] = accuracy(xdata3_train_mapping_cubic[:,1:],__
       →ydata3_train, weight_prime_3)
              accuracy test3[i] = accuracy(xdata3 test mapping cubic[:,1:],...
       print("The mean of accuracy for the training data 1 is :", np.
       →mean(accuracy_train1), "%")
          print("The mean of accuracy for the testing data 1 is :", np.
       →mean(accuracy_test1), "%")
          print("The std of accuracy for the training data 1 is :", np.
       ⇔std(accuracy_train1))
          print("The std of accuracy for the testing data 1 is :", np.
       ⇒std(accuracy test1))
          print("\n")
```

```
print("The mean of accuracy for the training data 2 is :", np.
  →mean(accuracy_train2), "%")
    print("The mean of accuracy for the testing data 2 is :", np.
  →mean(accuracy test2), "%")
    print("The std of accuracy for the training data 2 is :", np.
  ⇒std(accuracy_train2))
    print("The std of accuracy for the testing data 2 is :", np.
  ⇔std(accuracy_test2))
    print("\n")
    print("The mean of accuracy for the training data 3 is :", np.
  →mean(accuracy_train3), "%")
    print("The mean of accuracy for the testing data 3 is :", np.
 →mean(accuracy_test3), "%")
    print("The std of accuracy for the training data 3 is :", np.
  ⇔std(accuracy_train3))
    print("The std of accuracy for the testing data 3 is :", np.
 ⇔std(accuracy test3))
run_10_cubic()
i1 reach. Data is linearly separable
Weight matrix is: [-1.
                             -3.19325329 2.39589446 -1.20055382 3.36119949
4.26498433
 -7.05501674 2.54994615 6.39661089 10.34622777]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ -1.
                                 3.6313337
                                              8.19029064 12.32073
1.19744872
   2.39701627 -16.04666591 18.19329743 5.41832681
                                                     8.56590471]
Min J is: 0
i2 reach.
Weight matrix is: [-3. -8.46960358 -7.13806135 20.50116809 0.26687748
13.19159749
  0.60849901 -4.36401562 7.03271722 -2.80559863]
Min J is: 3
i1 reach. Data is linearly separable
Weight matrix is: [ 2.
                                -5.64921969
                                              0.56262001
                                                          5.31905921
0.95184137
  -2.21272912 -19.66778651 -5.53423642 3.67571804 11.93683492
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 0.
                        -0.55750747 2.1550173 -1.42582952 2.79894808
-0.33406497
 -2.77824761 3.80187508 -1.07781616 2.54086813]
Min J is: 0
i2 reach.
Weight matrix is: [ -4. -15.91808354 -9.14466603 38.79119612
```

```
12.57828645
  15.34553412 -3.74832756 -22.10840048 3.24412398 -3.68140448]
Min J is: 2
il reach. Data is linearly separable
Weight matrix is: [ 0. -3.02101957 1.98710986 -0.36972413 0.70606103
1.03560909
 -1.79173075 0.68323635 0.72181011 1.59789735]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 2. -0.34292846 5.02008043 4.51384146 1.72968692
1.81374949
-3.85260307 12.30376063 3.89228773 5.08513791]
Min J is: 0
i2 reach.
Weight matrix is: [-5. -2.83157781 -6.22558909 17.38118561 0.20704133
11.78074358
 -4.50066121 0.60973929 2.4032791 -0.55878507]
Min J is: 3
il reach. Data is linearly separable
Weight matrix is: [ 1. 0.64005397 3.76834699 -2.18061239 2.34963432
5.90976011
 -9.83182478 -0.60097797 5.65682775 11.67090527]
il reach. Data is linearly separable
Weight matrix is: [ 0. -0.07085133 3.22098428 2.03242838 -0.85390597
-0.14132025
-2.01863083 4.08316105 1.60179476 2.65325768]
Min J is: 0
i2 reach.
Weight matrix is: [ -4. -15.86973792 -10.41818294 31.58953964
-0.25160302
  12.48664067 0.7172856 0.80188143 14.87066134 -1.07772038]
Min J is: 2
il reach. Data is linearly separable
Weight matrix is: [ 0. -1.13182876 1.34379215 2.25677305 0.64389327
-1.46714345
-4.10649497 -0.92888145 1.22509077 4.5849209 ]
Min J is: 0
il reach. Data is linearly separable
Weight matrix is: [ -1. 3.90600593 8.1814851 12.84652803
0.96545373
  0.80928335 -15.22569425 17.92073856 4.19666927 8.34845111]
Min J is: 0
i2 reach.
Weight matrix is: [ -5. -18.70172676 -11.56800037 46.49445244
15.28163393
  17.41366757 -5.21697907 -19.50062882 -3.11077471 -2.02951056
Min J is: 2
```

```
i1 reach. Data is linearly separable
Weight matrix is: [ -2.
                              -3.94246901 3.21546696 -6.44414543
-5.02819102
  -6.67244774 -18.42215445 -2.61765968 6.93375816 15.96591516]
Min J is: 0
i1 reach. Data is linearly separable
                           0.02730717 1.91326997 0.05386866 1.88833116
Weight matrix is: [0.
0.16593795
0.07970483 1.86407335 0.18871383 1.76172382]
Min J is: 0
i2 reach.
Weight matrix is: [ -5. -19.40318163 -11.02912748 40.6493568
13.48407669
  20.67300565
               0.46015311 -20.6934664 4.97114348 -3.64727937]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ -3.
                              -5.51054782 3.58056218 -1.10902785
-0.30612346
  -3.31618327 -21.39091947 -3.80340039 9.20881948 23.61182548
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-1. -0.06357705 5.39249727 5.12317834 2.66272196
1.82546018
 -2.471839
            10.92586178 2.95474769 4.4041211 ]
Min J is: 0
i2 reach.
Weight matrix is: [ -4.
                            -17.32007788 -9.83425553 31.02755758
6.80218615
  11.46717791 7.86077598 -7.86158872 5.89929833
                                                   2.187822 ]
Min J is: 2
i1 reach. Data is linearly separable
                      -4.98174567 1.63251709
Weight matrix is: [ 0.
                                                        0.16742558
-3.38202535
 -6.80159942 -16.31389932 -1.67852194 6.60528878 14.8436942 ]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 0. 2.70256443 8.43848916 12.33838474
1.32326339
  2.19765038 -15.95922298 23.91317758 7.40657274 8.43858545]
Min J is: 0
i2 reach.
Weight matrix is: [ -6. -9.56823438 -14.26393998 27.00183139
0.73618758
  24.69876161 7.07136178 -4.8830188
                                       5.2496867 -0.38183264]
Min J is: 3
i1 reach. Data is linearly separable
Weight matrix is: [ 0.
                            -1.65413538 1.62578599 0.4959464
                                                               0.71093139
0.3932999
```

```
-3.43483005 -0.44417127 0.97920219 2.52965019]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 1.
                               0.50646539 4.2589735
                                                       4.07942587 3.42566164
2.04938897
  0.37613022 10.13571814 3.01585047 4.06299535]
Min J is: 0
i2 reach.
Weight matrix is: [ -4.
                             -14.50791123 -8.68169626 27.05734734
5.20866973
               1.94435882 -1.42184934
                                        4.44093119 -2.389114 ]
  14.17115238
Min J is: 2
i1 reach. Data is linearly separable
                         -1.45829274 1.09762932 -0.21175947 0.58395041
Weight matrix is: [ 0.
0.36867919
 -1.80206718 0.20489922 0.53929096 1.11440734]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 0.
                               0.7694296
                                           3.86254852 -0.65941393 4.37429351
0.22175095
 -2.7856921
             6.87811528 -0.06704011 4.02656855]
Min J is: 0
i2 reach.
                             -16.52750914 -11.21776605 31.56551197
Weight matrix is: [ -5.
7.72424132
  19.10848872
               7.32112326 -12.05266387
                                         4.33235968 -1.33057282]
Min J is: 3
The mean of accuracy for the training data 1 is : 100.0 \%
The mean of accuracy for the testing data 1 is: 99.3 %
The std of accuracy for the training data 1 is: 0.0
The std of accuracy for the testing data 1 is: 1.1874342087037917
The mean of accuracy for the training data 2 is : 100.0 \%
The mean of accuracy for the testing data 2 is: 96.8 %
The std of accuracy for the training data 2 is: 0.0
The std of accuracy for the testing data 2 is: 2.2271057451320084
The mean of accuracy for the training data 3 is : 97.6 %
The mean of accuracy for the testing data 3 is : 91.3 \%
The std of accuracy for the training data 3 is: 0.4898979485566356
The std of accuracy for the testing data 3 is: 1.676305461424021
```

[]: