hw4_1

February 22, 2023

[102]:
```
import numpy as np
import random as rm
import pandas as pd
from sklearn import preprocessing
# from sklearn.preprocessing import StandardScaler
from sklearn.metrics import confusion_matrix, ConfusionMatrixDisplay
import matplotlib.pyplot as plt
```

[4]:
```
def getdata(fname):
    data = pd.read_csv(fname)
    xdata = data.drop("Class", axis=1)
    ydata = data['Class']
    return xdata, ydata
```

[149]:
```
xdata_train, ydata_train = getdata('Dry_Bean_train.csv')
xdata_test, ydata_test = getdata('Dry_Bean_test.csv')
# print(xdata_train)

## preprocessing
# Convert Class String labels into Integers
lab_enc = preprocessing.LabelEncoder()
ydata_train = lab_enc.fit_transform(ydata_train)
ydata_test = lab_enc.transform(ydata_test)

# Standarlize
scaler_train = preprocessing.StandardScaler().fit(xdata_train)
# scaler_test = preprocessing.StandardScaler().fit(xdata_test)

xdata_train_scaled = scaler_train.transform(xdata_train)
xdata_test_scaled = scaler_train.transform(xdata_test)
```

[150]:
```
def shuffle(xdata, ydata):
    newX = np.copy(xdata)
    newY = np.copy(ydata)
    N = len(newX)
    shuff = np.random.permutation(N)
    for i in range(N):
```

```
        newX[i] = xdata[shuff[i]]
        newY[i] = ydata[shuff[i]]
    return (newX, newY)
```

[128]:
```
def calculate_classify_accuracy(xdata, ydata, weights):
    gx = np.dot(xdata,weights.T)
    accuracy = np.sum(np.argmax(gx, axis = 1) == ydata) / len(xdata)
    return accuracy
```

[129]:
```
def calculate_J(xdata, ydata, weights):
    ans = 0
    N = len(xdata)
    for i in range(N):
        target = ydata[i]
        gx = weights @ xdata[i]
        predict = np.argmax(gx)

        if(target != predict):
            ans += np.dot(xdata[i],weights[target].T) - np.
 ↪dot(xdata[i],weights[predict].T)
    return ans * -1
```

[151]:
```
def multiclass_perceptron_learning(xdata, ydata, maxEpochs):
    """
    xdata: (N, D) data array, non-augmented format
    ydata: (N, ) labels(1.0, 2.0)
    maxEpochs: max number of passes through the data.  Halts sooner if no␣
 ↪classififcation errors
    """
    N, D = xdata.shape
    C = np.argmax(np.unique(ydata)) + 1
#     print(C, N, D)
    eta = 1

    weights = np.ones((C, D + 1))
    xdata_aug = np.ones((N, D + 1))
    xdata_aug[:, 1:] = xdata
    acc = 0

    min_J = 99999999
    final_weights = np.copy(weights)

    for e in range(maxEpochs):
        # 1.shuffle
        xdata_aug, ydata = shuffle(xdata_aug, ydata)

        # 2.For each data point x, update w
```

```python
        for i in range(N):
            target = ydata[i]
            gx = weights @ xdata_aug[i]
            predict = np.argmax(gx)

            if(target != predict):
                weights[target] = weights[target] + eta * xdata_aug[i]
                weights[predict] = weights[predict] - eta * xdata_aug[i]
            if( e == maxEpochs - 1 and N - i <= 100):
                J = calculate_J(xdata_aug, ydata, weights)
                if( J <= min_J ):
                    min_J = J
                    final_weights = np.copy(weights)

    return final_weights
```

[142]:
```python
def predict_label(xdata, weights):
    weights = np.asarray(weights)
    gx = np.dot(xdata,weights.T)
    return np.argmax(gx, axis = 1)
```

[176]:
```python
def plot_confusion_matrix(target_label, predict_label):
    cm = confusion_matrix(target_label, predict_label)
    disp = ConfusionMatrixDisplay(confusion_matrix=cm)
    disp.plot()
    plt.figure()
    plt.show()
```

[156]:
```python
final_weights = multiclass_perceptron_learning(xdata_train_scaled, ydata_train,␣
 ↪100)
```

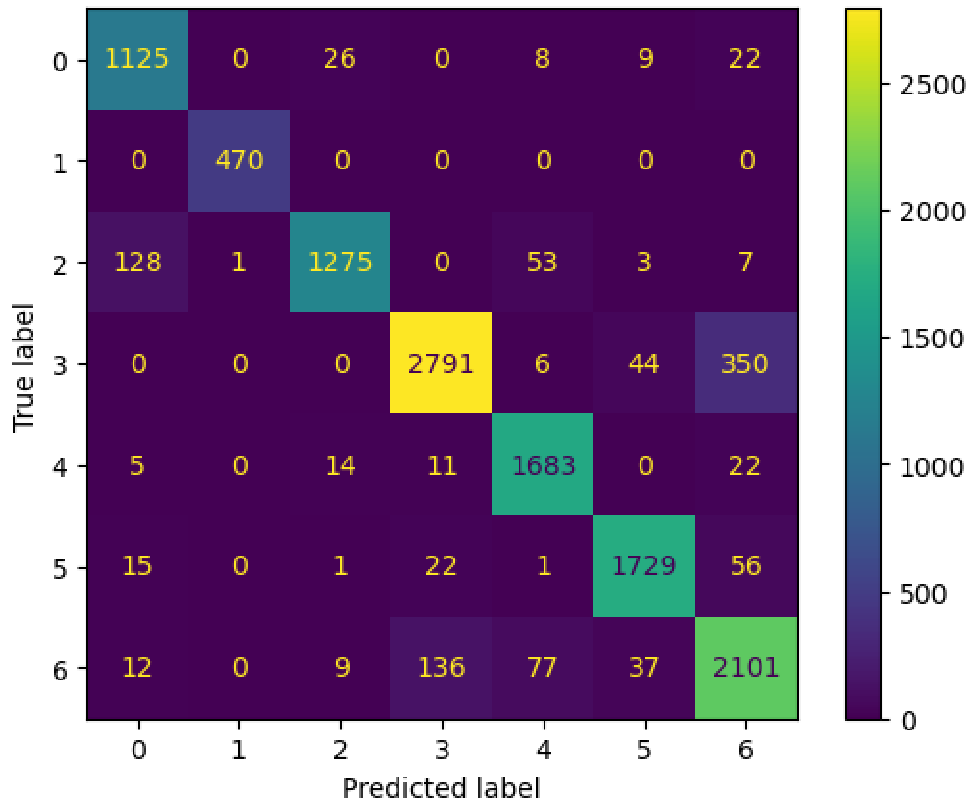
[161]:
```python
print("Final weights is :", final_weights)
print("Magtitude is :", np.linalg.norm(final_weights, axis = 1))
```

```
Final weights is : [[ 5.00000000e+00  2.12796720e+00  7.60585752e+00
1.34295771e+00
   1.19911622e+01 -6.75631691e+00 -1.00905513e-01  2.29432636e+00
   6.88080450e+00 -1.67181508e+00 -4.51363017e+00 -6.92410801e+00
   6.26528919e+00 -1.32294487e+01 -8.50954425e+00  5.28559780e+00
   1.25934303e+01]
 [-2.10000000e+01  2.28191891e+01  8.81236711e+00  8.42526643e+00
   1.09445976e+01 -2.58056370e+00 -4.59697369e+00  2.29915094e+01
   9.82708714e+00  1.87271995e+00 -1.82682445e+00  9.28829013e+00
   5.56959815e+00  1.04208363e+01  1.12187898e+01  5.86054176e+00
   1.94915034e+00]
 [-3.00000000e+00 -1.32628798e+00  5.43166932e+00  9.50334710e+00
   4.57098025e+00  2.58745515e+00  4.59194022e+00 -1.40322074e+00
```

```
  7.59502709e+00  2.37526651e+00  2.08404690e+00  9.99850477e+00
 -2.50000669e+00 -1.73368610e+01 -5.91919045e+00 -2.79498942e+00
 -7.72103551e+00]
[ 4.00000000e+00 -5.89423748e+00 -8.55906568e+00 -8.07684600e+00
 -9.08011520e+00 -1.99306440e+00  9.20299976e+00 -5.93997408e+00
 -8.66012583e+00  1.45232818e+00 -1.07276886e+00  8.13324313e+00
  6.11932068e-01  1.72754049e+01  6.33213989e+00 -7.84860728e-01
  1.54121928e-01]
[-8.00000000e+00 -2.90267097e+00 -7.10516327e-01  2.09963204e+00
 -6.88504690e+00  1.81612589e+01  2.65651853e+00 -3.01660356e+00
 -3.07532296e+00  3.22282938e+00  7.69690431e+00 -8.27527311e+00
 -9.02104293e+00  1.03250661e+01  2.73124270e-02 -6.77495571e+00
 -1.44022702e+01]
[ 7.00000000e+00 -7.32537542e-01  1.43799397e+00  4.48120022e-02
  2.12533879e-01 -3.60505421e-01 -1.06760657e+01 -7.61718923e-01
 -6.79343389e-03 -1.73964978e+00  4.19153617e+00 -6.36565341e+00
  5.62315941e+00 -2.54728797e+00  7.05420167e+00  6.86544364e+00
  1.71340154e+01]
[ 2.30000000e+01 -7.09142231e+00 -7.01830591e+00 -6.33916928e+00
 -4.75411188e+00 -2.05826360e+00  5.92248642e+00 -7.16431843e+00
 -5.56067649e+00  1.48832083e+00  4.40736103e-01  1.14499651e+00
  4.51070807e-01  2.09229039e+00 -3.20370907e+00 -6.56777343e-01
 -2.70741226e+00]]
Magtitude is : [29.58992822 47.74780091 27.53432469 29.65791159 32.5516521
25.61320525
 28.96927871]
```

[177]:
```python
# Augment scaled data
N_train, D_train = xdata_train_scaled.shape
xdata_train_aug = np.ones((N_train, D_train + 1))
xdata_train_aug[:, 1:] = xdata_train_scaled

# Calculate accuracy and plot the confusion matrix
train_accuracy = calculate_classify_accuracy(xdata_train_aug, ydata_train,␣
 ↪final_weights)
print("The accuracy on the training set is :", train_accuracy * 100 , " %")

predict_train_label = predict_label(xdata_train_aug,final_weights )
plot_confusion_matrix(ydata_train, predict_train_label)
```

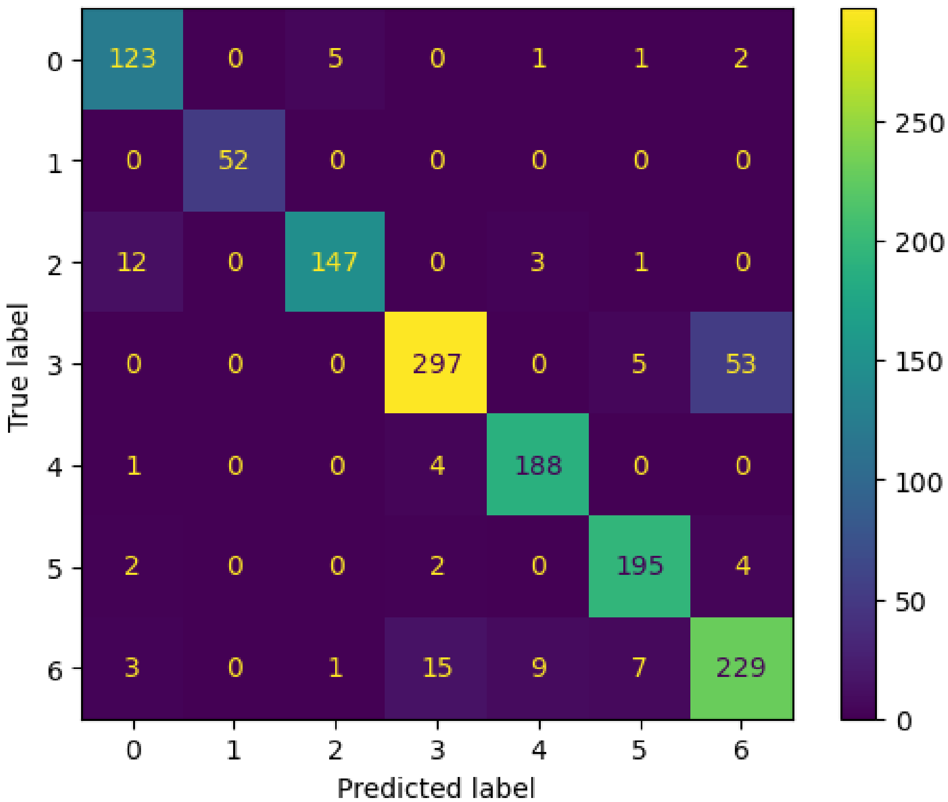
```
The accuracy on the training set is : 91.22377336925463  %
```




```
<Figure size 640x480 with 0 Axes>
```

[178]:
```python
# Augment scaled data
N_test, D_test = xdata_test_scaled.shape
xdata_test_aug = np.ones((N_test, D_test + 1))
xdata_test_aug[:, 1:] = xdata_test_scaled

# Calculate accuracy and plot the confusion matrix
test_accuracy = calculate_classify_accuracy(xdata_test_aug, ydata_test,␣
    ↪final_weights)
print("The accuracy on the training set is :", test_accuracy * 100 , " %")

predict_test_label = predict_label(xdata_test_aug,final_weights )
plot_confusion_matrix(ydata_test, predict_test_label)
```

```
The accuracy on the training set is : 90.38179148311308  %
```




```
<Figure size 640x480 with 0 Axes>
```

[198]:

```
# Repeat 10 times
C, D = final_weights.shape
weight_10_mag = np.zeros((10, C))
train_accuracy_10 = np.zeros(10)
test_accuracy_10 = np.zeros(10)
# predict_label_train_10 = np.zeros((10, N_train))
# predict_label_test_10 = np.zeros((10, N_test))
cm_train = []
cm_test = []

for i in range(10):
    weight_10 = multiclass_perceptron_learning(xdata_train_scaled, ydata_train,␣
 ↪100)
    train_accuracy_10[i] = calculate_classify_accuracy(xdata_train_aug,␣
 ↪ydata_train, weight_10)
    test_accuracy_10[i] = calculate_classify_accuracy(xdata_test_aug,␣
 ↪ydata_test, weight_10)
    weight_10_mag[i] = np.linalg.norm(weight_10, axis = 1)
    predict_label_train_10 = predict_label(xdata_train_aug,weight_10 )
```

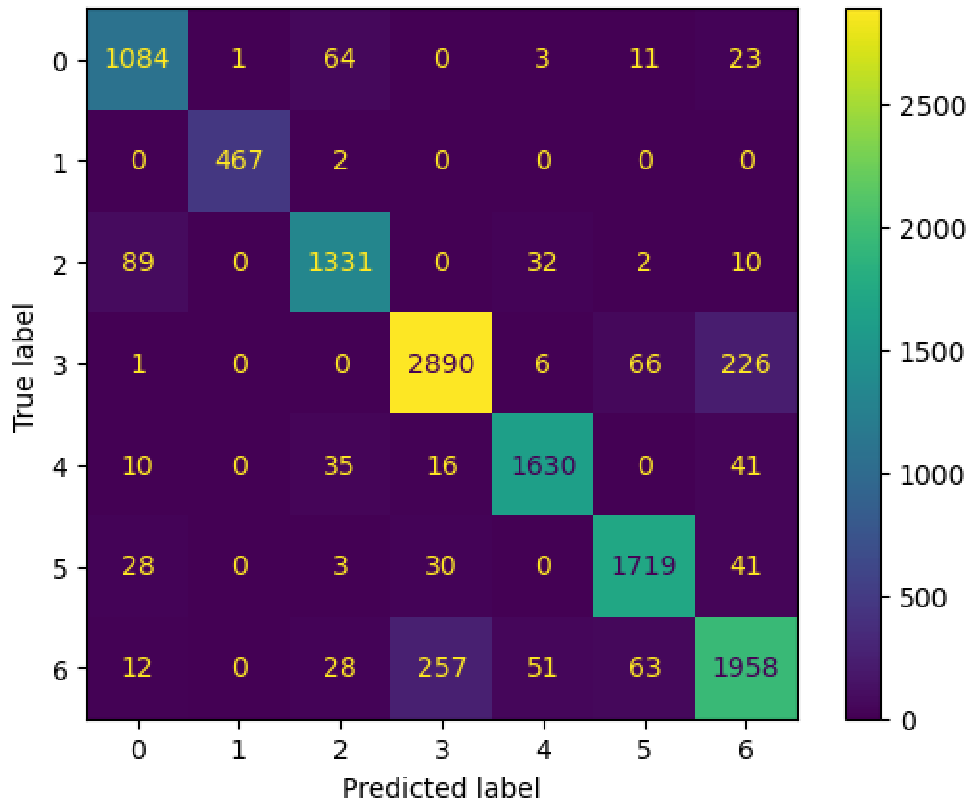
```
    predict_label_test_10 = predict_label(xdata_test_aug,weight_10 )
    cm_train.append(confusion_matrix(ydata_train, predict_label_train_10))
    cm_test.append(confusion_matrix(ydata_test, predict_label_test_10))
```

[257]:

```
print("The mean for the trainning accuracy is:", np.mean(train_accuracy_10))
print("The mean for the testing accuracy is:", np.mean(test_accuracy_10))
print("\n")
print("The std for the trainning accuracy is:", np.std(train_accuracy_10))
print("The std for the testing accuracy is:", np.std(test_accuracy_10))
print("\n")
print("The mean for the magnitude is:", np.mean(weight_10_mag, axis = 0))
print("The std for the magnitude is:", np.std(weight_10_mag, axis = 0))
print("\n")

cm_train_array = np.asarray(cm_train)
cm_test_array = np.asarray(cm_test)

cm_mean_train = np.copy(cm_train_array[0])
cm_std_train = np.copy(cm_train_array[0])

cm_mean_test = np.copy(cm_test_array[0])
cm_std_test = np.copy(cm_test_array[0])


temp_train = np.zeros(10)
temp_test = np.zeros(10)
# idx = 0
for i in range(7):
    for j in range(7):
        for n in range(10):
            temp_train[n] = cm_train_array[n][i][j]
            temp_test[n] = cm_test_array[n][i][j]

        cm_mean_train[i][j] = np.mean(temp_train)
        cm_std_train[i][j] = np.std(temp_train)

        cm_mean_test[i][j] = np.mean(temp_test)
        cm_std_test[i][j] = np.std(temp_test)


print("The confusion matrix for the mean of training set is:")
disp = ConfusionMatrixDisplay(confusion_matrix=cm_mean_train)
disp.plot()
plt.figure()
plt.show()
```

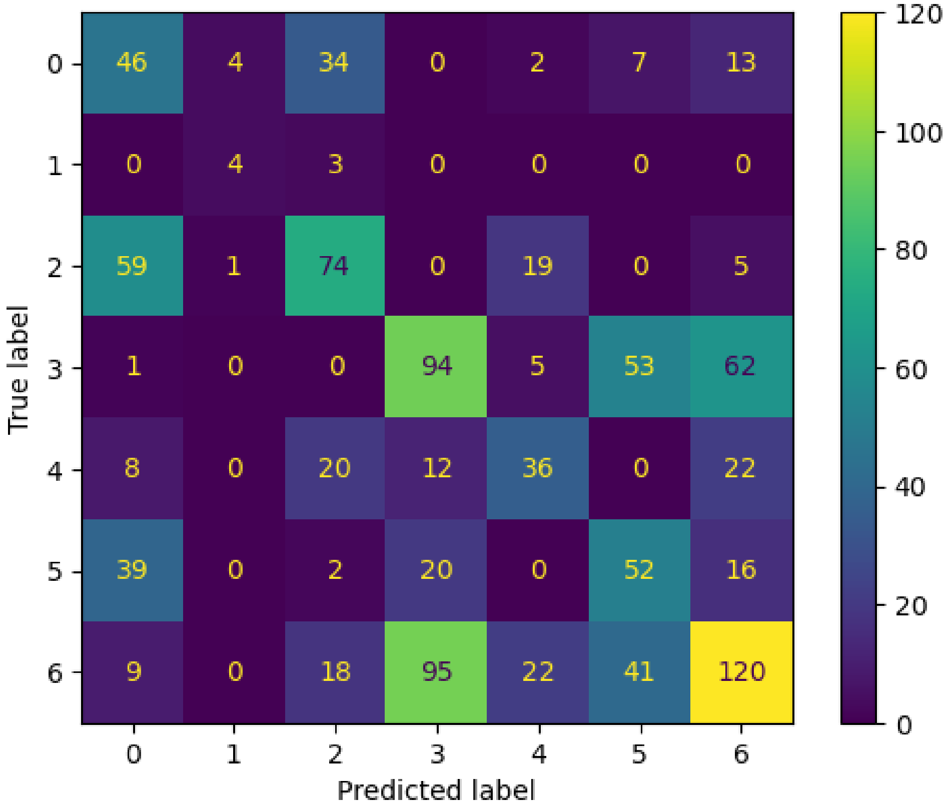
```python
print("The confusion matrix for the std of training set is:")
disp = ConfusionMatrixDisplay(confusion_matrix=cm_std_train)
disp.plot()
plt.figure()
plt.show()

print("The confusion matrix for the mean of testing set is:")
disp = ConfusionMatrixDisplay(confusion_matrix=cm_mean_test)
disp.plot()
plt.figure()
plt.show()

print("The confusion matrix for the std of testing set is:")
disp = ConfusionMatrixDisplay(confusion_matrix=cm_std_test)
disp.plot()
plt.figure()
plt.show()
```

```
The mean for the trainning accuracy is: 0.9047432443464771
The mean for the testing accuracy is: 0.8960352422907489


The std for the trainning accuracy is: 0.0074947205649701755
The std for the testing accuracy is: 0.008137533628485993


The mean for the magnitude is: [29.56702155 45.90045369 30.87991092 29.25864678
32.27456786 25.35509257
 28.82129615]
The std for the magnitude is: [2.32968867 1.71676904 1.60612051 1.78060799
2.88614301 1.19451596
 1.45609997]


The confusion matrix for the mean of training set is:
```



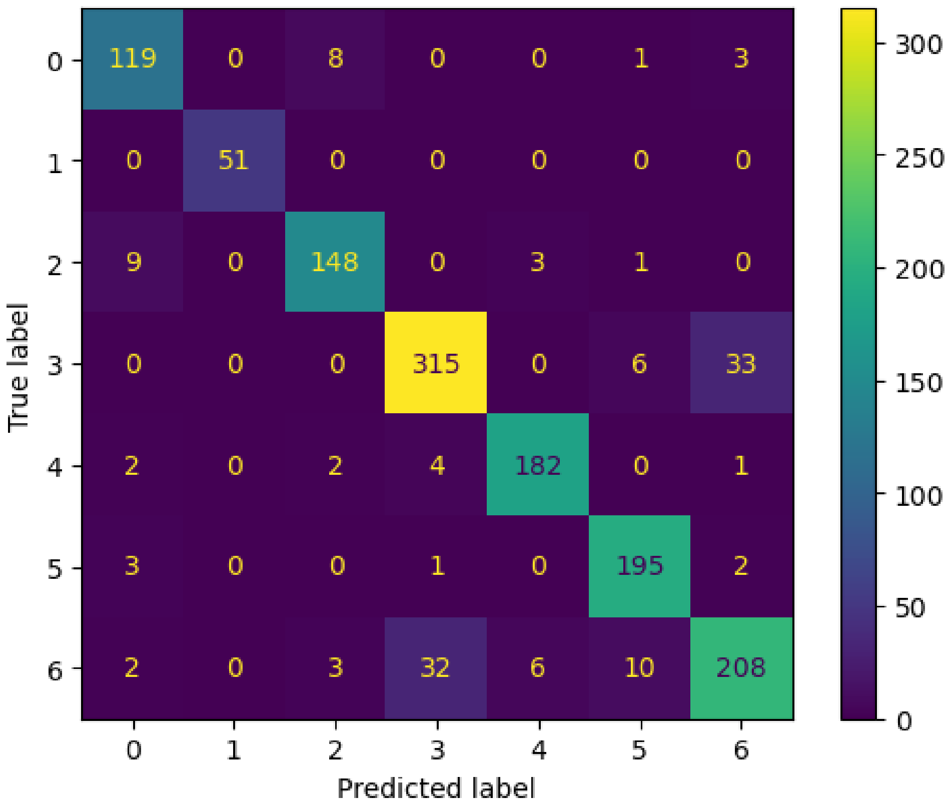
<Figure size 640x480 with 0 Axes>

The confusion matrix for the std of training set is:



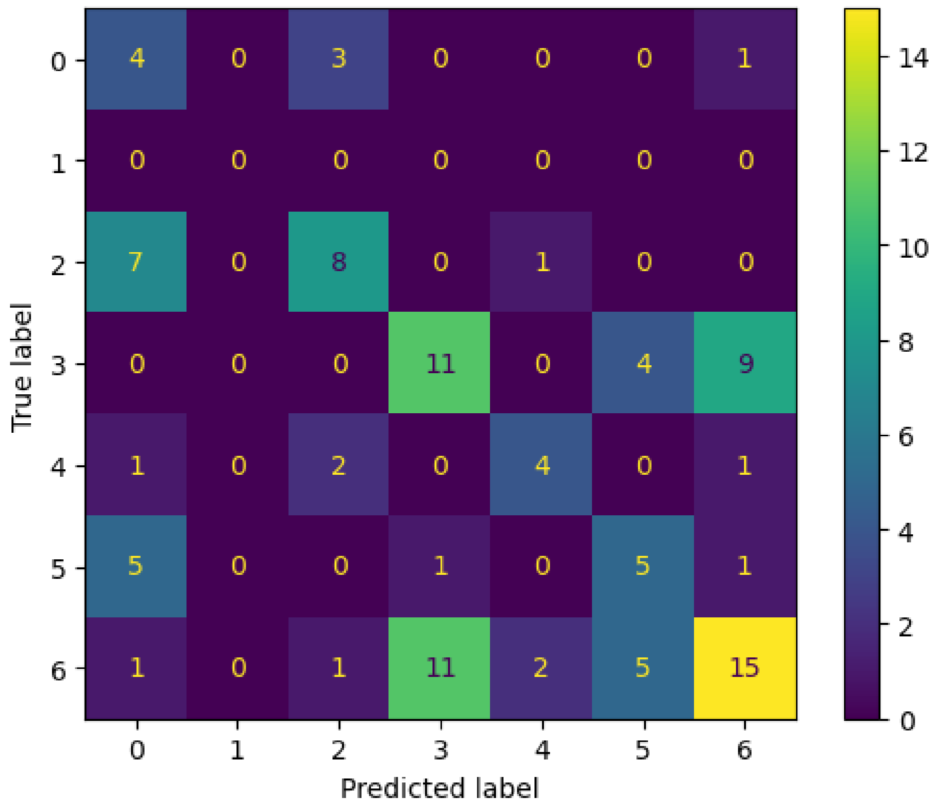
<Figure size 640x480 with 0 Axes>

The confusion matrix for the mean of testing set is:



<Figure size 640x480 with 0 Axes>

The confusion matrix for the std of testing set is:



<Figure size 640x480 with 0 Axes>

[ ]:

hw4_2

February 23, 2023

[1]:
```
import numpy as np
import random as rm
import pandas as pd
import sys
from sklearn import preprocessing
# from sklearn.preprocessing import StandardScaler
from sklearn.metrics import confusion_matrix, ConfusionMatrixDisplay
import matplotlib.pyplot as plt
```

[2]:
```
def getdata(fname):
    data = pd.read_csv(fname)
    xdata = data.drop("Class", axis=1)
    ydata = data['Class']
    return xdata, ydata
```

[3]:
```
def shuffle(xdata, ydata):
    newX = np.copy(xdata)
    newY = np.copy(ydata)
    N = len(newX)
    shuff = np.random.permutation(N)
    for i in range(N):
        newX[i] = xdata[shuff[i]]
        newY[i] = ydata[shuff[i]]
    return (newX, newY)
```

[4]:
```
xdata_train, ydata_train = getdata('Dry_Bean_train.csv')
xdata_test, ydata_test = getdata('Dry_Bean_test.csv')
# print(xdata_train)

## preprocessing
# Convert Class String labels into Integers
lab_enc = preprocessing.LabelEncoder()
ydata_train = lab_enc.fit_transform(ydata_train)
ydata_test = lab_enc.transform(ydata_test)

# Standarlize
scaler_train = preprocessing.StandardScaler().fit(xdata_train)
```

```
# scaler_test = preprocessing.StandardScaler().fit(xdata_test)

xdata_train_scaled = scaler_train.transform(xdata_train)
xdata_test_scaled = scaler_train.transform(xdata_test)
```

[109]:
```
def perceptronLearning(data, label, w0, eta = 1, maxEpochs = 100):
    """
    data: (N, D + 1) data array, non-augmented format with labels(1.0, 2.0)
    eta: learning rate (constant)
    w0: 1 *
    maxEpochs: max number of passes through the data.  Halts if reach the max␣
 ↪epoch
    """

    N, D = data.shape
    wHat = np.copy(w0) # D + 1 * 1
#     print(wHat.shape)
#     print(zData)
#     wHats = np.zeros((maxIter + 1, D + 1))


    minJ = sys.float_info.max
    finalWHat = np.copy(w0)
    i1 = False


    for m in range(1, maxEpochs + 1):
        # 1. shuffle and preprocessing
        shuffledData, shuffledLabel = shuffle(data,label)
#         print(shuffledLabel)
#         break

        # 2. Augment and reflected
#         z = (-1.0) ** (shuffledLabel + 1)
        z = shuffledLabel
        dataAug = np.ones((N, D + 1))
        dataAug[:, 1:] = shuffledData
        zData = (dataAug.T * z).T
        J_iter = 0
        correctClass = 0

#         for n in range(1, N + 1):
        for n in range(0, N):
            condition = np.dot(wHat ,zData[n])
#             print("condition", condition)
            index = (m - 1) * N + n
```

```
            # compute new J(w) and misclassfication
            J_iter = 0
            correctClass = 0
#             for i in range(0, N):
#                 gx = np.dot(wHat ,zData[i])
#                 if gx <= 0:
#                     J_iter -= gx
#                 else:
#                     correctClass += 1
            gx_matrix = np.dot(wHat, zData.T)
#             print(J_iter.shape)
            gx_matrix = gx_matrix * -1
            loss = np.sum(gx_matrix > 0)
#             print("loss",loss)




            if(m == maxEpochs and N - n <= 100 and loss < minJ ):
                minJ = loss
                finalWHat = np.copy(wHat)

            if(condition <= 0):
                wHat = wHat + eta * zData[n]

        if minJ == 0:
            print("i1 reach. Data is linearly separable")
            i1 = True
            break
    if(not i1):
        print("i2 reach.")
    print("Weight matrix is:" , finalWHat)
    print("Min J is:" , minJ)
#     print("Misclassification rate is :", misEpoch[-1])

    return finalWHat
```

[36]:
```
def change_label(ydata, c_num):
    N = len(ydata)
    changed_label = np.copy(ydata)
    for i in range(N):
        if(ydata[i] == c_num):
            changed_label[i] = 1
        else:
            changed_label[i] = -1
    return changed_label
```

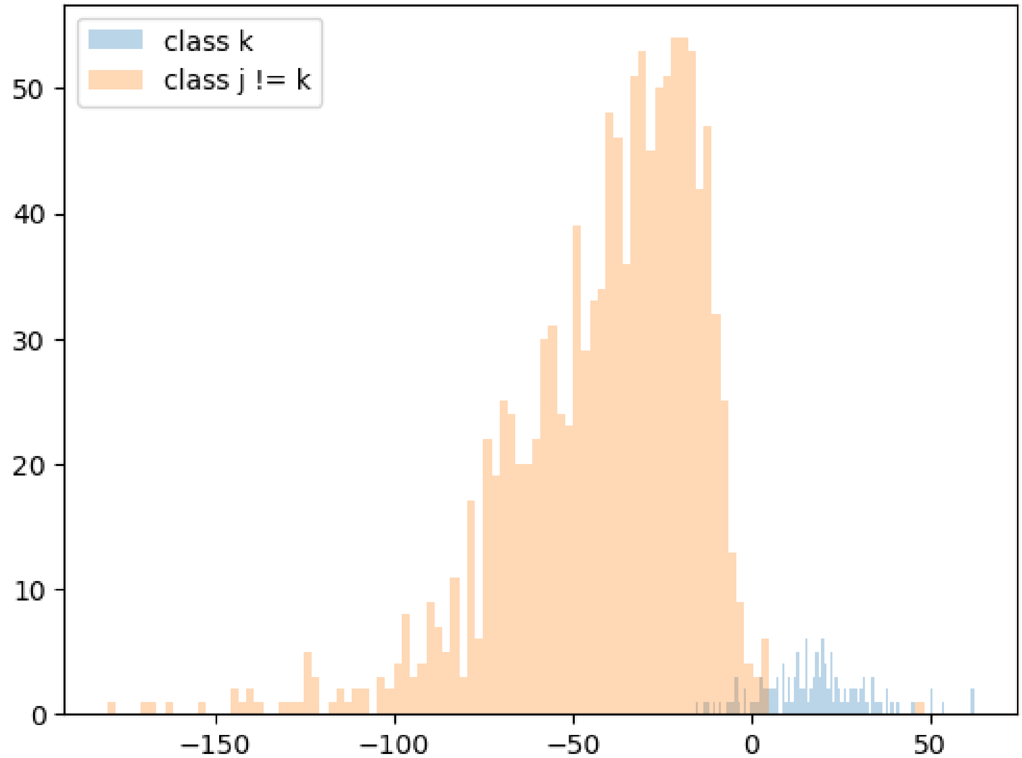
[101]:
```
def plot_hist(data, label, weight):
    """
    data : N * D non augment
    label : N * 1 label vector
    weight : 1 * D + 1 augment weight
    """
    N, D = data.shape
    data_aug = np.ones((N, D + 1))
    data_aug[:, 1:] = data
    class1 = []
    class2 = []
    for i in range(N):
        gx = weight @ data_aug[i]
#         print(gx)
        if(label[i] > 0):
            class1.append(gx)
        else:
#             print(gx)
            class2.append(gx)

    plt.hist(class1,bins = 100, alpha=0.3)
    plt.hist(class2,bins = 100, alpha = 0.3)
    plt.legend(('class k', 'class j != k'), loc=2)
    plt.show()
```

[127]:
```
def main(xdata_train, ydata_train, xdata_test, ydata_test ):
    N, D = xdata_train.shape
    weights = np.zeros((7, D + 1))
    weight = np.ones(D + 1)
    for c in range(7):
        label_train = change_label(ydata_train, c)
        label_test = change_label(ydata_test, c)

        weights[c] = np.copy(perceptronLearning(xdata_train, label_train␣
 ↪,weight))

        print("Accuracy for training data is:", test(xdata_train, label_train,␣
 ↪weights[c]), "%")
        print("Accuracy for testing data is:", test(xdata_test, label_test,␣
 ↪weights[c]), "%")

        plot_hist(xdata_test, label_test, weights[c] )
        print("\n")
    return weights
```

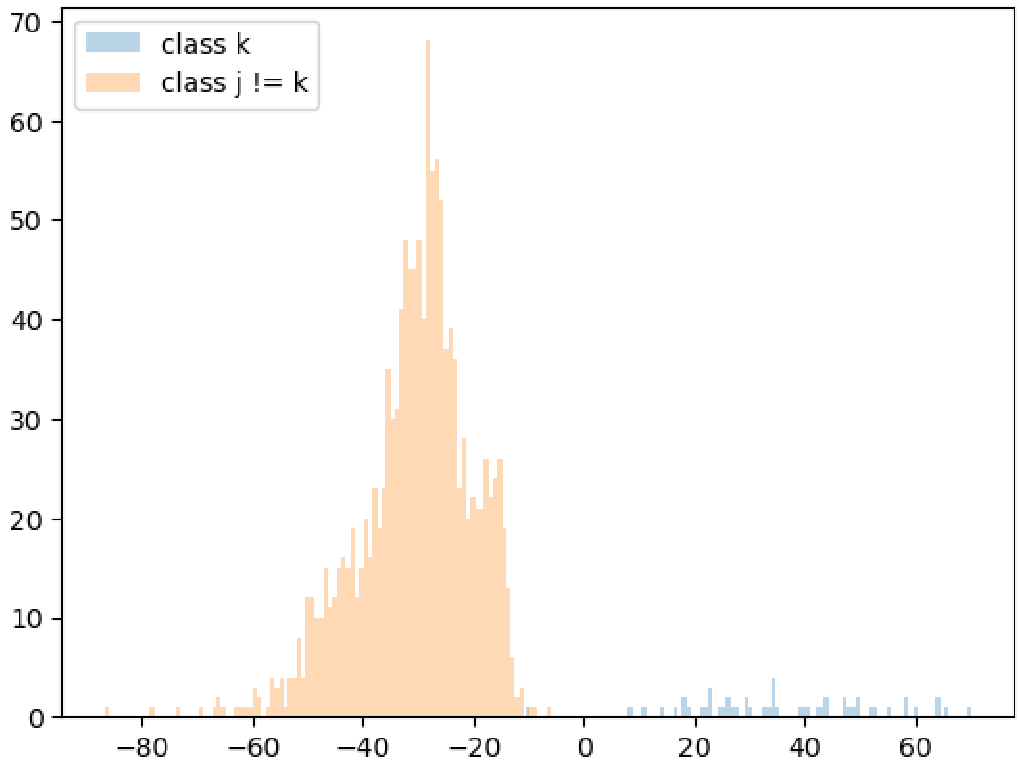
[117]:
```
def test(data, label, wHat):
    '''
    data: data is a matrix with dimension: num of data points * num features
    '''
    N, D = data.shape
    z = label
    wHat = np.copy(wHat)
    dataAug = np.ones((N, D + 1))
    dataAug[:, 1:] = data
    zData = (dataAug.T * z).T

    count = 0
    for i in range(N):
        if np.dot(wHat ,zData[i]) > 0:
            count += 1
#     print("Accuracy rate: ", (count) / N * 100 , "%")
    return (count) / N * 100
```

[128]:
```
weights = main(xdata_train_scaled, ydata_train, xdata_test_scaled, ydata_test)
```

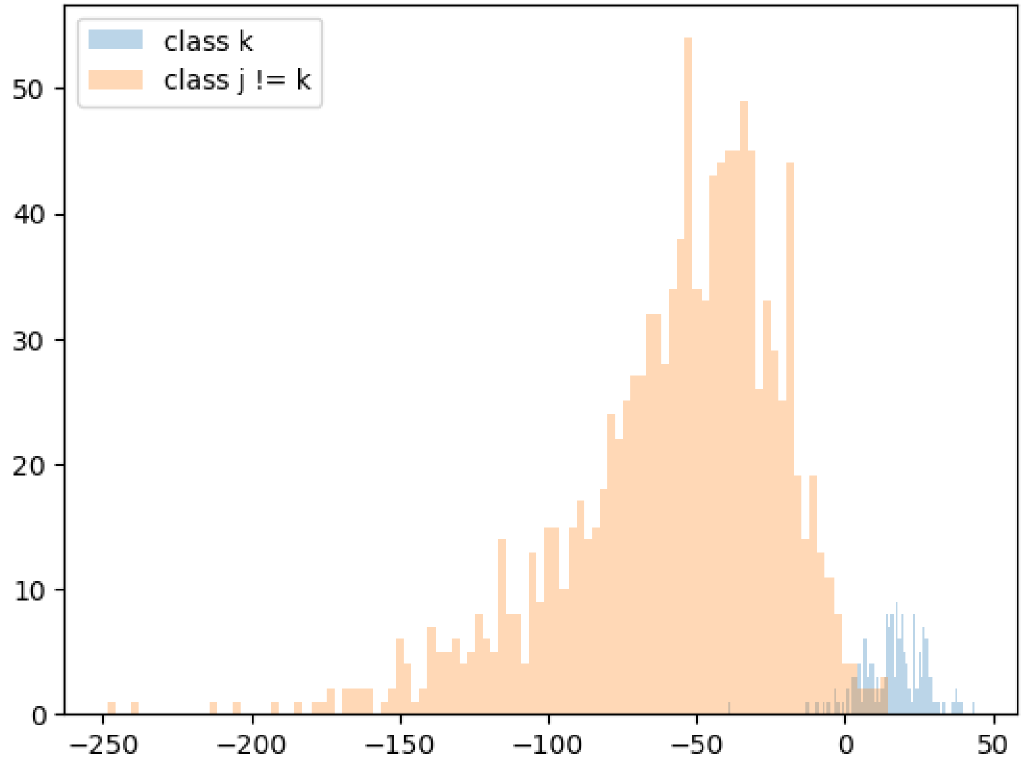
```
i2 reach.
Weight matrix is: [-36.         -28.60087316  54.73154488 -28.1266936
-14.98698367
 -49.62951441 -21.90425284 -15.73967196 -21.68719729   2.1055915
  -4.79083549  -1.15495649  -8.08200612 -21.48593591 -72.43551164
 -22.73420473  13.20265585]
Min J is: 186
Accuracy for training data is: 98.48150869458732 %
Accuracy for testing data is: 98.09104258443465 %
```



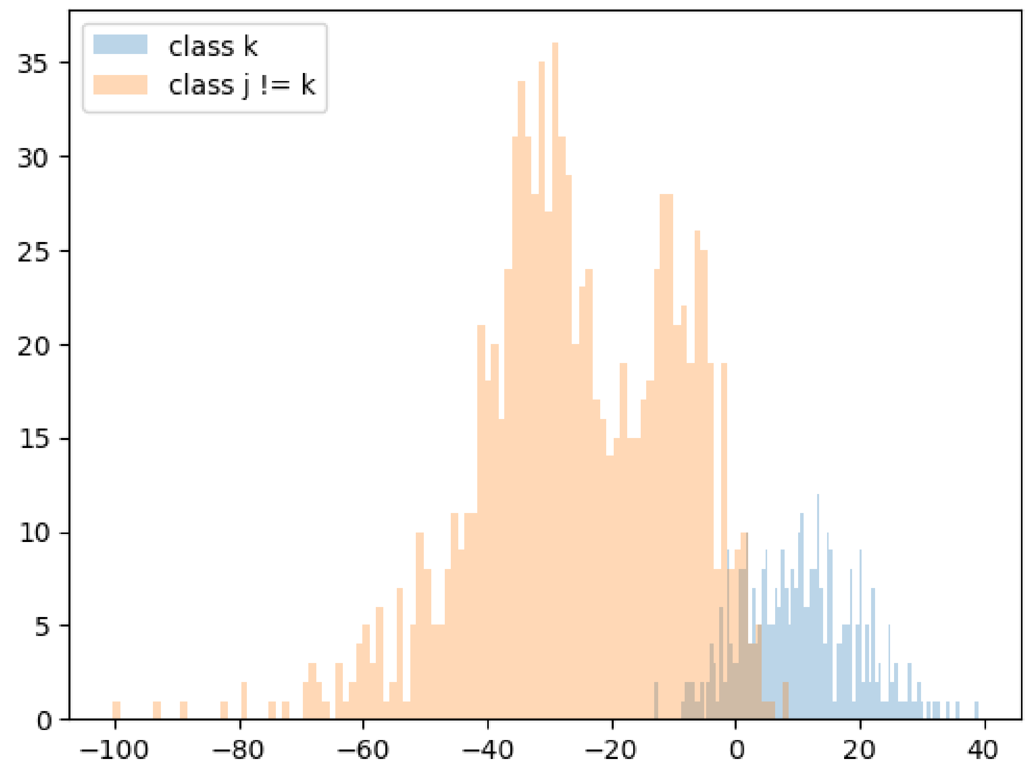

```
i1 reach. Data is linearly separable
Weight matrix is: [-28.          8.44727203   1.66088407   0.81776488
3.3682316
  -2.63667497   2.79285994   8.45797429   2.22162419   1.74867361
   2.05566716   4.02851812   2.87062772   8.63314079   4.27164211
   2.15745894   2.73616084]
Min J is: 0
Accuracy for training data is: 100.0 %
Accuracy for testing data is: 99.92657856093979 %
```



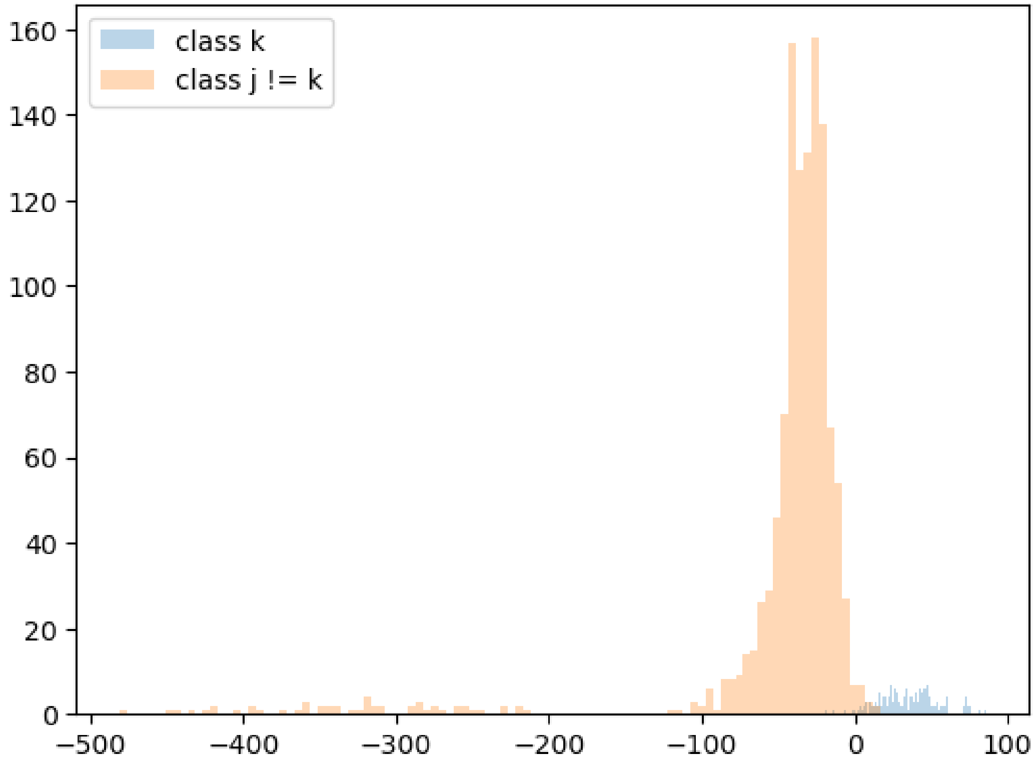

```
i2 reach.
Weight matrix is: [ -49.          -30.65660599  -31.85917368   69.815186
-77.37273476
  -67.3783354    28.80888287   -9.09333362   -4.83854503    0.81856057
   -3.2413633    13.70298145  -35.74025225 -140.06376002   68.47276459
  -65.31643919   -3.88222522]
Min J is: 285
Accuracy for training data is: 97.67327945138379 %
Accuracy for testing data is: 98.16446402349486 %
```



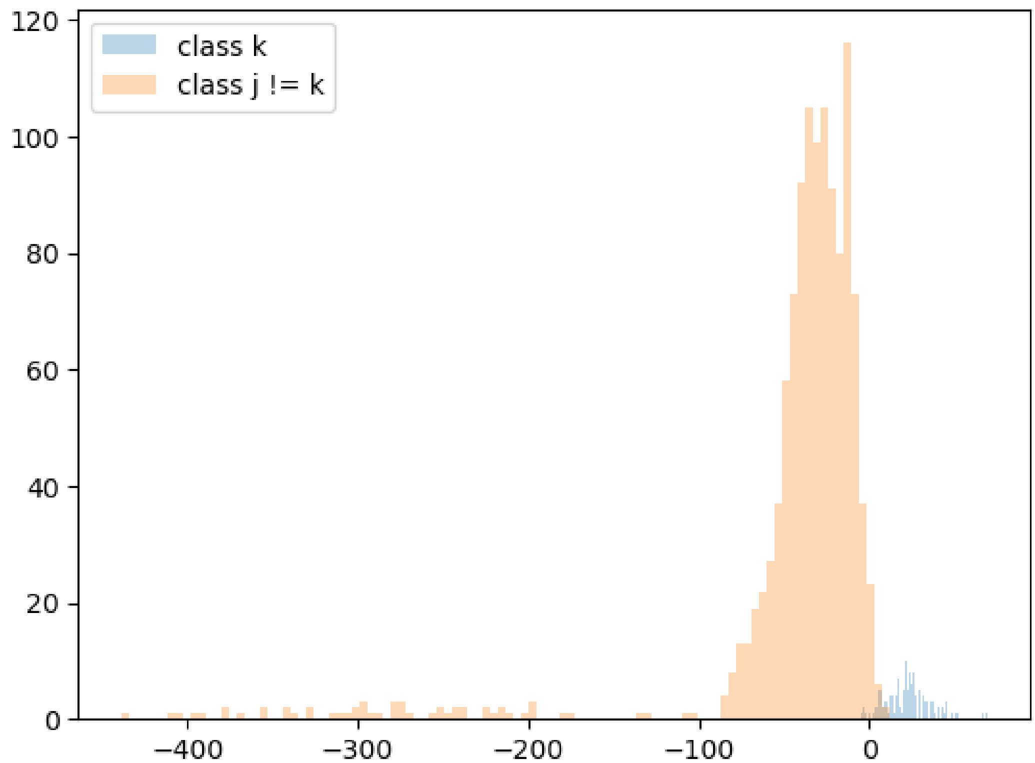

```
i2 reach.
Weight matrix is: [-1.60000000e+01 -4.95161341e+00  1.92317028e+01
-7.08014034e+00
  1.12832912e+01  2.45856703e-02  5.87694159e+01 -1.30612487e+01
  3.73229927e+00 -2.81903386e+00  1.86703210e+00  6.00631650e+00
  2.25374026e+01  1.18678333e+01  2.64575279e+01  1.10847086e+01
  1.31197165e+00]
Min J is: 592
Accuracy for training data is: 95.16695240427791 %
Accuracy for testing data is: 94.27312775330397 %
```



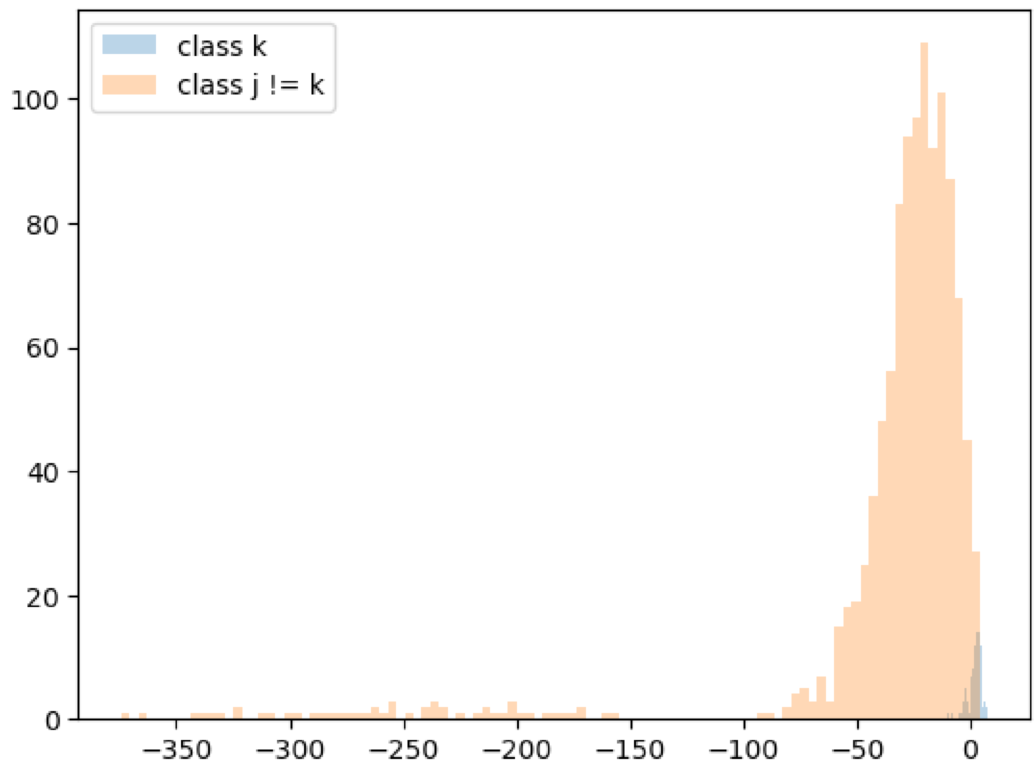

```
i2 reach.
Weight matrix is: [-36.         -45.50515993   9.49695167   1.83255746
-10.75525762
  69.73096833   9.81141473 -47.66518436 -12.75444641   0.64219741
   3.69507221  -3.18176461  11.31827127 -48.10524125   0.85186481
  27.54056961  -6.12319121]
Min J is: 220
Accuracy for training data is: 98.20393501510327 %
Accuracy for testing data is: 98.38472834067548 %
```




```
i2 reach.
Weight matrix is: [ -34.          -29.33074247  -26.20480651   16.98748079
-49.9632721
   -6.51176292  -22.29610256  -35.63303252   -5.39689601   -2.64479815
   -0.52265602    1.0786598   -26.26811941 -103.93958625   26.43716586
  -27.07839638   -0.18875012]
Min J is: 265
Accuracy for training data is: 97.83655808637441 %
Accuracy for testing data is: 97.79735682819384 %
```




```
i2 reach.
Weight matrix is: [ -28.          11.56025143 -180.07677194   13.84223432
-4.35203167
  -39.61881869  -14.80187212  -15.87783617   27.98203347    1.05487084
    0.46028178  -25.72585778  -23.09950017  -87.93645621  -11.45523474
  -58.81522415    0.2653616 ]
Min J is: 752
Accuracy for training data is: 93.86072332435302 %
Accuracy for testing data is: 93.3186490455213 %
```



[119]:
```
def decision_rule_1(xdata, ydata, weights):
    """
    weights : C * D + 1 augment weights matrix
    xdata : N * D non-augment data matrix
    ydata : N * 1 label vector
    """
    N , D = xdata.shape
    C = 7
    mis = 0
    unclassify = 0
    correct = 0

    xdata_aug = np.ones((N, D + 1))
    xdata_aug[:, 1:] = xdata

    for i in range(N):
        gx = weights @ xdata_aug[i]
#         print(gx)
        target = ydata[i]
```

```
#         print(target)

        filter_bool = gx > 0
        filter_gx = gx[filter_bool]
        l_fil = len(filter_gx)
        if(l_fil > 1 or l_fil == 0):
            unclassify += 1
        else:
            if(target != np.where(gx == filter_gx[0])[0][0]):
                mis += 1
            else:
                correct += 1

    print("The accuracy rate is :", correct / N * 100, "%")
    print("The error rate is :", mis / N * 100, "%")
    print("The unclassified rate is :", unclassify / N * 100, "%")
```

[129]:
```
print("Classify the training data using decision rule 1:")
decision_rule_1(xdata_train_scaled, ydata_train, weights)
print("\n")
print("Classify the testing data using decision rule 1:")
decision_rule_1(xdata_test_scaled, ydata_test, weights)
```

```
Classify the training data using decision rule 1:
The accuracy rate is : 86.71728304351376 %
The error rate is : 5.208588456200506 %
The unclassified rate is : 8.074128500285738 %


Classify the testing data using decision rule 1:
The accuracy rate is : 86.04992657856094 %
The error rate is : 6.093979441997063 %
The unclassified rate is : 7.856093979441997 %
```

[121]:
```
def decision_rule_2(xdata, ydata, weights):
    """
    weights : C * D + 1 augment weights matrix
    xdata : N * D non-augment data matrix
    ydata : N * 1 label vector
    """
    N , D = xdata.shape
    C = 7
    mis = 0
    unclassify = 0
    correct = 0

    xdata_aug = np.ones((N, D + 1))
```

```
    xdata_aug[:, 1:] = xdata

    for i in range(N):
        gx = weights @ xdata_aug[i]
#         print(gx)
        target = ydata[i]
#         print(target)

        if( np.argmax(gx) != target ):
            mis += 1
        else:
            correct += 1

    print("The accuracy rate is :", correct / N * 100, "%")
    print("The error rate is :", mis / N * 100, "%")
    print("The unclassified rate is :", unclassify / N * 100, "%")
```

[130]:
```
print("Classify the training data using decision rule 2:")
decision_rule_2(xdata_train_scaled, ydata_train, weights)
print("\n")
print("Classify the testing data using decision rule 2:")
decision_rule_2(xdata_test_scaled, ydata_test, weights)
```

```
Classify the training data using decision rule 2:
The accuracy rate is : 91.26459302800228 %
The error rate is : 8.735406971997714 %
The unclassified rate is : 0.0 %


Classify the testing data using decision rule 2:
The accuracy rate is : 90.30837004405286 %
The error rate is : 9.691629955947137 %
The unclassified rate is : 0.0 %
```

[123]:
```
def decision_rule_3(xdata, ydata, weights):
    """
    weights : C * D + 1 augment weights matrix
    xdata : N * D non-augment data matrix
    ydata : N * 1 label vector
    """
    N , D = xdata.shape
    C = 7
    mis = 0
    unclassify = 0
    correct = 0
```

```
    xdata_aug = np.ones((N, D + 1))
    xdata_aug[:, 1:] = xdata

    for i in range(N):
        gx = weights @ xdata_aug[i] # C * 1 vector

        weights_nonaug = weights[:,1:] # C * D
        weight_norm = np.linalg.norm(weights_nonaug, axis = 1)

        gkx = gx / weight_norm

        target = ydata[i]
#         print(target)

        if( np.argmax(gkx) != target ):
            mis += 1
        else:
            correct += 1

    print("The accuracy rate is :", correct / N * 100, "%")
    print("The error rate is :", mis / N * 100, "%")
    print("The unclassified rate is :", unclassify / N * 100, "%")
```

[131]:

```
print("Classify the training data using decision rule 3:")
decision_rule_3(xdata_train_scaled, ydata_train, weights)
print("\n")
print("Classify the testing data using decision rule 3:")
decision_rule_3(xdata_test_scaled, ydata_test, weights)
```

```
Classify the training data using decision rule 3:
The accuracy rate is : 91.11764225651073 %
The error rate is : 8.882357743489266 %
The unclassified rate is : 0.0 %


Classify the testing data using decision rule 3:
The accuracy rate is : 90.08810572687224 %
The error rate is : 9.911894273127754 %
The unclassified rate is : 0.0 %
```

[ ]:

hw4_4

February 23, 2023

[99]:
```
import numpy as np
import random as rm
import pandas as pd
import csv
import sys
from sklearn import preprocessing
# from sklearn.preprocessing import StandardScaler
from sklearn.metrics import confusion_matrix, ConfusionMatrixDisplay
import matplotlib.pyplot as plt
```

[21]:
```
def getData(fname, dimension):
    # create a new array to store the data
    data = np.empty([0,dimension])
    label = []
    with open(fname, mode ='r')as file:
        # reading the CSV file
        csvFile = csv.reader(file)

        # displaying the contents of the CSV file
        for lines in csvFile:
            data = np.row_stack((data,[float(lines[0]), float(lines[1])]))
            label.append(float(lines[2]))
    label = np.array(label)
    return (data, label)
```

[24]:
```
# Obtain data
xdata1_train, ydata1_train = getData('dataset1_train.csv', 2)
xdata1_test, ydata1_test = getData('dataset1_test.csv', 2)

xdata2_train, ydata2_train = getData('dataset2_train.csv', 2)
xdata2_test, ydata2_test = getData('dataset2_test.csv', 2)

xdata3_train, ydata3_train = getData('dataset3_train.csv',2 )
xdata3_test, ydata3_test = getData('dataset3_test.csv', 2)

## preprocessing
# Standarlize
```

```
scaler1_train = preprocessing.StandardScaler().fit(xdata1_train)
scaler2_train = preprocessing.StandardScaler().fit(xdata2_train)
scaler3_train = preprocessing.StandardScaler().fit(xdata3_train)

xdata1_train_scaled = scaler1_train.transform(xdata1_train)
xdata1_test_scaled = scaler1_train.transform(xdata1_test)

xdata2_train_scaled = scaler2_train.transform(xdata2_train)
xdata2_test_scaled = scaler2_train.transform(xdata2_test)

xdata3_train_scaled = scaler3_train.transform(xdata3_train)
xdata3_test_scaled = scaler3_train.transform(xdata3_test)
```

[101]:
```
################################################
## EE559 HW4, Prof. Chugg
################################################

def plotDecBoundaries_Nonlinear(feature, labels, weight, non_linear_trans,␣
 ↪predictor, fsize=(6,4),legend_on = False):

    '''
    Plot the decision boundaries and data points for any binary classifiers

    feature: origianl2D feautre, N x 2 array:
        N: number of data points
        2: number of features
    labels: class lables correspond to feature, N x 1 array: [0,0,1,1,0,0,...]
        N: number of data points
    legend_on: add the legend in the plot. potentially slower for datasets with␣
 ↪large number of clases and data points
    ----------------------------
    You need to write the following two functions

    non_linear_trans: your custom non-linear transforation function.
        <feature_nonlinear> = non_linear_trans(<feature_original>),
            Input: <feature_original>, Nx2 array,
            Output: <feature_nonlinear>: Nx? array.
        if no nonlinear transformation performs, then,
        let non_linear_trans = lambda x:x, which just output your original␣
 ↪feature

    predictor: your custom predictor.
        <predictions> = predictor(<feature>)
            Input: <feature> Nx? array.
            Output: <predictions> binary labels, i.e., array ([0,1,0,0,1...])
```

```
    If you don't want write custom functions, you can modify this plot function␣
↪based on your need,
    do non-linear transformation and class prediction inside this plot function.
    ----------------------------
    '''

    labels = labels.astype(int)

    # Set the feature range for ploting
    max_x = np.ceil(max(feature[:, 0])) + 1
    min_x = np.floor(min(feature[:, 0])) - 1
    max_y = np.ceil(max(feature[:, 1])) + 1
    min_y = np.floor(min(feature[:, 1])) - 1

    xrange = (min_x, max_x)
    yrange = (min_y, max_y)

    # step size for how finely you want to visualize the decision boundary.
    inc = 0.05

    # generate grid coordinates. this will be the basis of the decision
    # boundary visualization.
    (x, y) = np.meshgrid(np.arange(xrange[0], xrange[1]+inc/100, inc), np.
↪arange(yrange[0], yrange[1]+inc/100, inc))

    # size of the (x, y) image, which will also be the size of the
    # decision boundary image that is used as the plot background.
    image_size = x.shape
    xy = np.hstack( (x.reshape(x.shape[0]*x.shape[1], 1, order='F'), y.
↪reshape(y.shape[0]*y.shape[1], 1, order='F')) ) # make (x,y) pairs as a␣
↪bunch of row vectors.

    '''
    You should write the custom functions, non_linear_trans and predictor
    '''
    # apply non-linear transformation to all points in the map (not only data␣
↪points)
    xy = non_linear_trans(xy)
    # predict the class of all points in the map
#     pred_label = predictor(xy)

    pred_label = predictor(xy, weight)

    # reshape the idx (which contains the class label) into an image.
    decisionmap = pred_label.reshape(image_size, order='F')
```

```
    # documemtation: https://matplotlib.org/stable/api/_as_gen/matplotlib.
 ↪pyplot.plot.html
    symbols_ar = np.array(['rx', 'bo', 'ms',␣
 ↪'cd','gp','y*','kx','gP','r+','bh'])
    #show the image, give each coordinate a color according to its class label
    plt.figure(figsize=fsize)

    plt.imshow(decisionmap, extent=[xrange[0], xrange[1], yrange[0],␣
 ↪yrange[1]], origin='lower', aspect='auto')

    # plot the class data.
    plot_index = 0
    class_list = []
    class_list_name = [] #for legend
    for cur_label in np.unique(labels):
        # print(cur_label,plot_index,np.sum(label_train == cur_label))
        d1, = plt.plot(feature[labels == cur_label, 0],feature[labels ==␣
 ↪cur_label, 1], symbols_ar[plot_index])

        if legend_on:
            class_list.append(d1)
            class_list_name.append('Class '+str(plot_index))
            l = plt.legend(class_list,class_list_name, loc=2)
            plt.gca().add_artist(l)

        plot_index = plot_index + 1

    plt.show()
```

[20]:

```
################################################
## EE559 HW1, Prof. Jenkins
## Created by Arindam Jati
## Tested in Python 3.6.3, OSX El Capitan, and subsequent versions
################################################

import numpy as np
import matplotlib.pyplot as plt
from scipy.spatial.distance import cdist


def plotDecBoundaries(training, label_train, w):
    # Plot the decision boundaries and data points for perceptron learning␣
 ↪classification result
    # training: traning data
    # label_train: class lables correspond to training data
    # w: weight vector
    nclass = max(np.unique(label_train))
```

```
# Set the feature range for ploting
max_x = np.ceil(max(training[:, 0])) + 1
min_x = np.floor(min(training[:, 0])) - 1
max_y = np.ceil(max(training[:, 1])) + 1
min_y = np.floor(min(training[:, 1])) - 1

xrange = (min_x, max_x)
yrange = (min_y, max_y)

# step size for how finely you want to visualize the decision boundary.
inc = 0.01

# generate grid coordinates. this will be the basis of the decision
# boundary visualization.
(x, y) = np.meshgrid(np.arange(xrange[0], xrange[1] + inc / 100, inc),
                     np.arange(yrange[0], yrange[1] + inc / 100, inc))

# size of the (x, y) image, which will also be the size of the
# decision boundary image that is used as the plot background.
image_size = x.shape
xy = np.hstack((x.reshape(x.shape[0] * x.shape[1], 1, order='F'),
                y.reshape(y.shape[0] * y.shape[1], 1, order='F')))  # make
↪(x,y) pairs as a bunch of row vectors.

# distance measure evaluations for each (x,y) pair.
aug = np.zeros(np.shape(xy)[0]) + 1
xy_aug = np.concatenate((aug[:, None], xy), axis=1)
pred_label = np.zeros(np.shape(xy)[0])

for i in range(np.shape(xy)[0]):
    if w.T @ xy_aug[i] > 0:
        pred_label[i] = 1
    else:
        pred_label[i] = 2

decisionmap = pred_label.reshape(image_size, order='F')

plt.imshow(decisionmap, extent=[xrange[0], xrange[1], yrange[0],
↪yrange[1]], origin='lower')

# plot the class training data.
plt.plot(training[label_train == 1, 0], training[label_train == 1, 1], 'rx')
plt.plot(training[label_train == 2, 0], training[label_train == 2, 1], 'go')

l = plt.legend(('Class 1', 'Class 2'), loc=2)
plt.gca().add_artist(l)
```

```
    # plot the class mean vector.
    plt.show()
```

[50]:
```python
def shuffle(xdata, ydata):
    newX = np.copy(xdata)
    newY = np.copy(ydata)
    N = len(newX)
    shuff = np.random.permutation(N)
    for i in range(N):
        newX[i] = xdata[shuff[i]]
        newY[i] = ydata[shuff[i]]
    return (newX, newY)
```

[41]:
```python
def perceptronLearning(data, label, w0, eta = 1, maxEpochs = 100):
    """
    data: (N, D) data array, non-augmented format with labels(1.0, 2.0)
    eta: learning rate (constant)
    maxEpochs: max number of passes through the data.  Halts if reach the max␣
 ↪epoch
    """

    N, D = data.shape
    wHat = np.copy(w0)

    minJ = sys.float_info.max
    finalWHat = np.copy(w0)
    i1 = False


    for m in range(1, maxEpochs + 1):
        # 1. shuffle
        shuffledData, shuffledLabel = shuffle(data,label)

        # 2. Augment and reflected
        z = (-1.0) ** (shuffledLabel + 1)
#          z = shuffledLabel
        dataAug = np.ones((N, D + 1))
        dataAug[:, 1:] = shuffledData
        zData = (dataAug.T * z).T

        for n in range(0, N):
            condition = np.dot(wHat ,zData[n])
            index = (m - 1) * N + n

            # compute new J(w)
            gx_matrix = np.dot(wHat, zData.T)
```

```
            gx_matrix = gx_matrix * -1
            loss = np.sum(gx_matrix > 0)
            if( loss < minJ ):
                minJ = loss
                finalWHat = np.copy(wHat)

            if(condition <= 0):
                wHat = wHat + eta * zData[n]

        if loss == 0:
            print("i1 reach. Data is linearly separable")
            i1 = True
            break
    if(not i1):
        print("i2 reach.")
    print("Weight matrix is:" , finalWHat)
    print("Min J is:" , minJ)

    return finalWHat
```

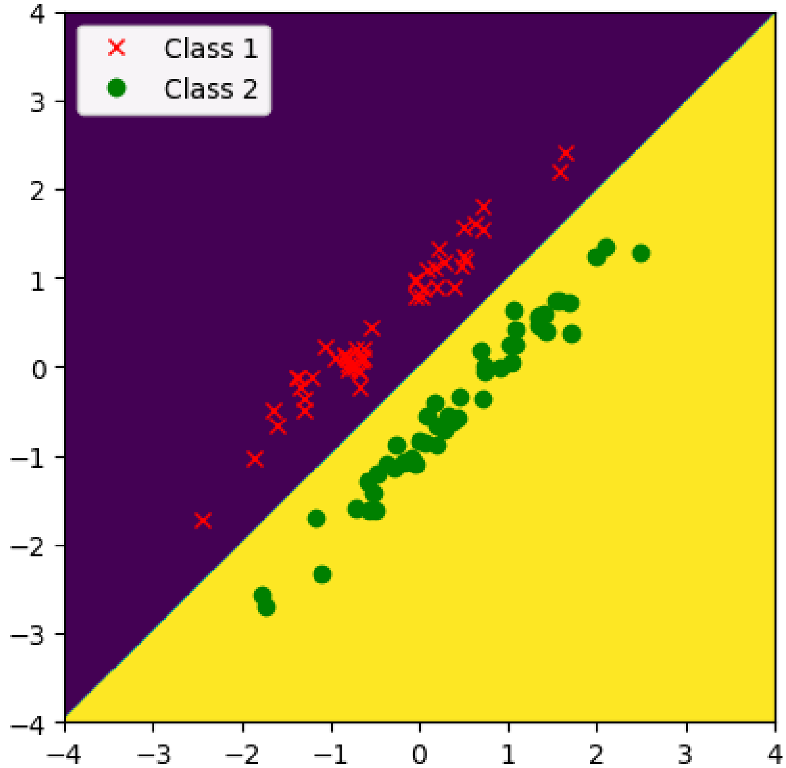
[134]:
```
def accuracy(data, label, wHat):
    '''
    data: non augment data
    '''
    N, D = data.shape
    z = (-1.0) ** (label + 1)
    wHat = np.copy(wHat)
    dataAug = np.ones((N, D + 1))
    dataAug[:, 1:] = data
    zData = (dataAug.T * z).T

    count = 0
    for i in range(N):
        if np.dot(wHat ,zData[i]) > 0:
            count += 1
    return (count) / N * 100
```

[42]:
```
def linear_classification():
    weight = np.ones(3)

    weight1 = perceptronLearning(xdata1_train_scaled, ydata1_train, weight)

    plotDecBoundaries(xdata1_train_scaled, ydata1_train, weight1)
    acc_train1 = accuracy(xdata1_train_scaled, ydata1_train, weight1)
    print("Accuracy for training data 1 is: ", acc_train1, "%")

    plotDecBoundaries(xdata1_test_scaled, ydata1_test, weight1)
```

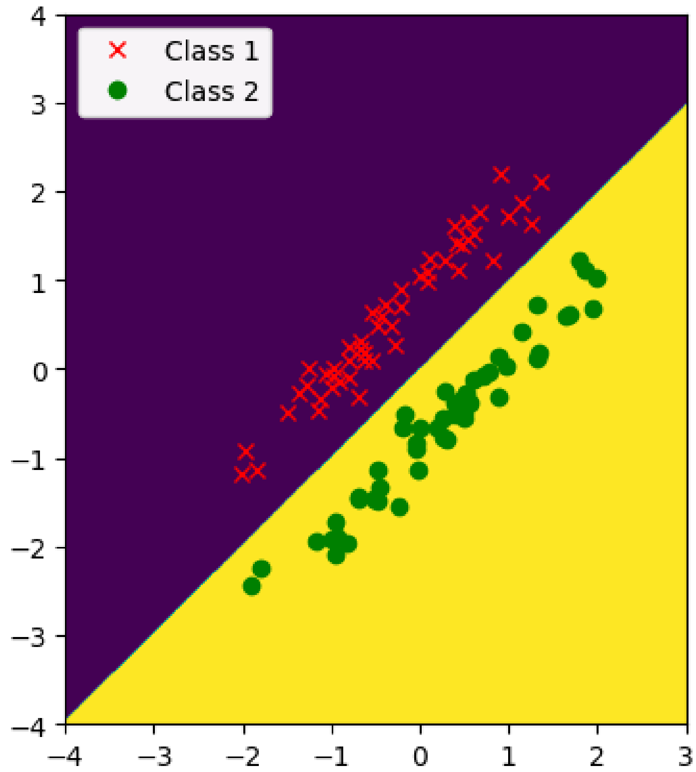
```
    acc_test1 = accuracy(xdata1_test_scaled, ydata1_test, weight1)
    print("Accuracy for testing data 1 is: ", acc_test1, "%")


    weight2 = perceptronLearning(xdata2_train_scaled, ydata2_train, weight)

    plotDecBoundaries(xdata2_train_scaled, ydata2_train, weight2)
    acc_train2 = accuracy(xdata2_train_scaled, ydata2_train, weight2)
    print("Accuracy for training data 2 is: ", acc_train2, "%")

    plotDecBoundaries(xdata2_test_scaled, ydata2_test, weight2)
    acc_test2 = accuracy(xdata2_test_scaled, ydata2_test, weight2)
    print("Accuracy for testing data 2 is: ", acc_test2, "%")

    weight3 = perceptronLearning(xdata3_train_scaled, ydata3_train, weight)

    plotDecBoundaries(xdata3_train_scaled, ydata3_train, weight3)
    acc_train3 = accuracy(xdata3_train_scaled, ydata3_train, weight3)
    print("Accuracy for training data 3 is: ", acc_train3, "%")

    plotDecBoundaries(xdata3_test_scaled, ydata3_test, weight3)
    acc_test3 = accuracy(xdata3_test_scaled, ydata3_test, weight3)
    print("Accuracy for testing data 3 is: ", acc_test3, "%")
```

[49]:
```
linear_classification()
```

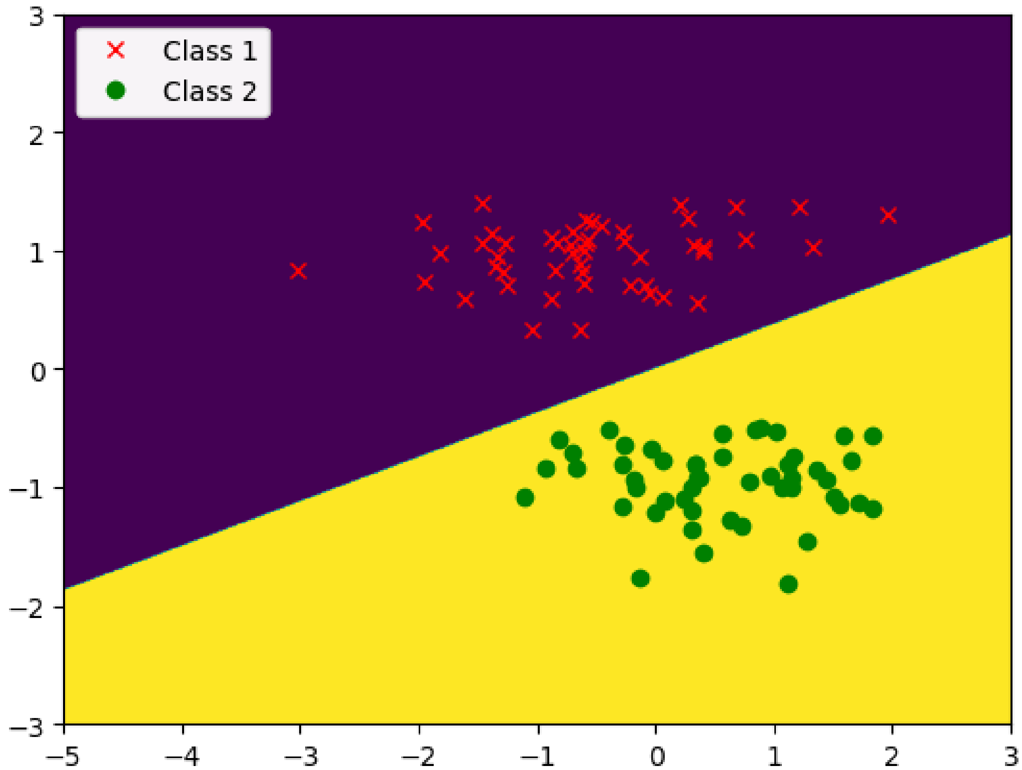
```
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -2.09552505  2.11386957]
Min J is: 0
```



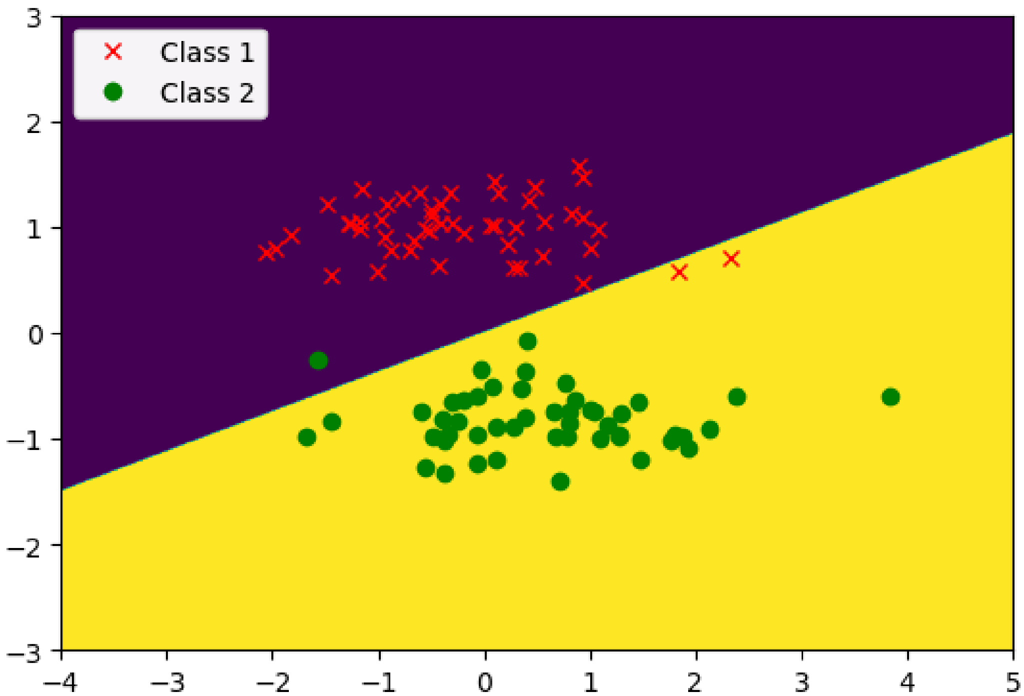

```
Accuracy for training data 1 is:  100.0 %
```



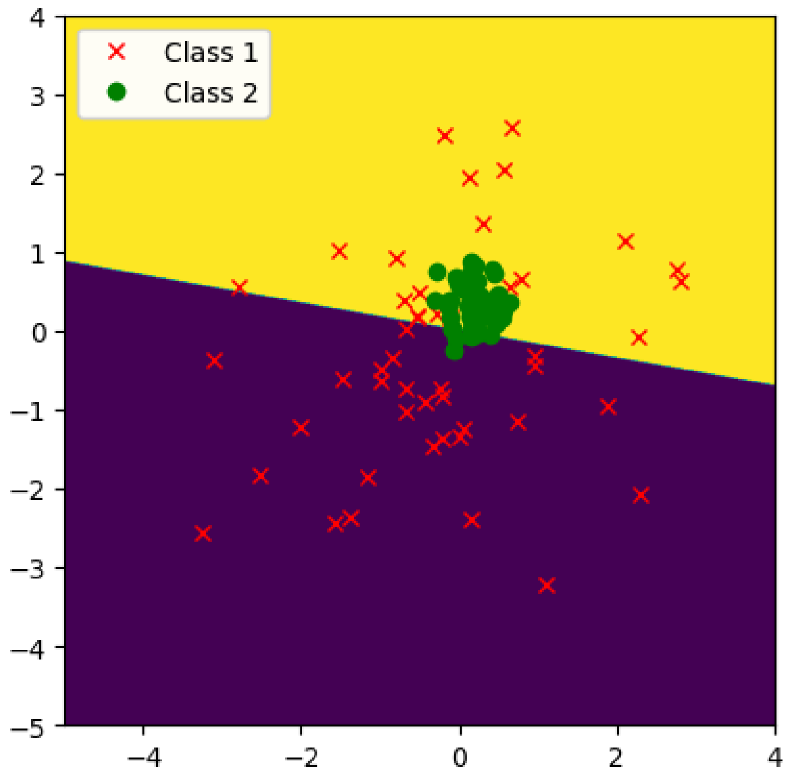

```
Accuracy for testing data 1 is:  100.0 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.          -0.58388862  1.55803305]
Min J is: 0
```



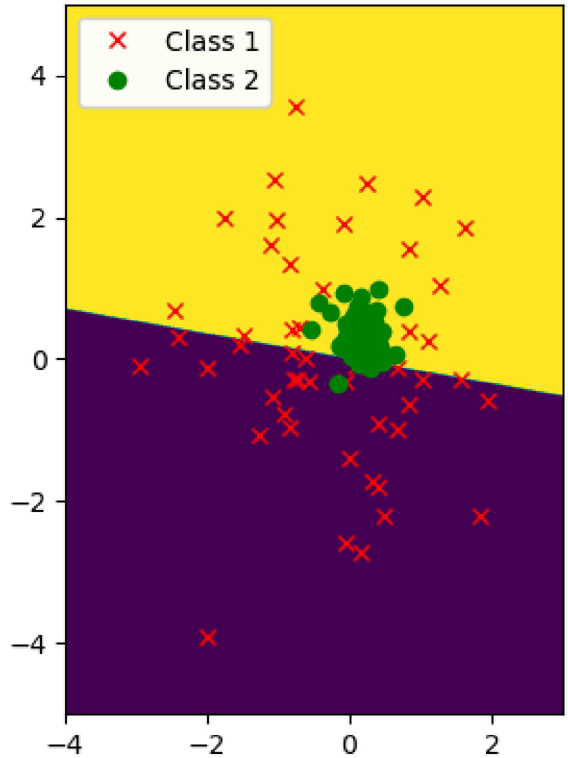

```
Accuracy for training data 2 is:  100.0 %
```



```
Accuracy for testing data 2 is:  96.96969696969697 %
i2 reach.
Weight matrix is: [ 0.          -0.00949884 -0.05430716]
Min J is: 1.0980766158415043
```




```
Accuracy for training data 3 is:  74.74747474747475 %
```




```
Accuracy for testing data 3 is:  75.75757575757575 %
```

[46]:
```python
def run_10_linear():
    weight = np.ones(3)

    accuracy_train1 = np.zeros(10)
    accuracy_test1 = np.zeros(10)

    accuracy_train2 = np.zeros(10)
    accuracy_test2 = np.zeros(10)

    accuracy_train3 = np.zeros(10)
    accuracy_test3 = np.zeros(10)

    for i in range(10):

        weight1 = perceptronLearning(xdata1_train_scaled, ydata1_train, weight)
        accuracy_train1[i] = accuracy(xdata1_train_scaled, ydata1_train,␣
↪weight1)
        accuracy_test1[i] = accuracy(xdata1_test_scaled, ydata1_test, weight1)

        weight2 = perceptronLearning(xdata2_train_scaled, ydata2_train, weight)
```

```
        accuracy_train2[i] = accuracy(xdata2_train_scaled, ydata2_train,␣
↪weight2)
        accuracy_test2[i] = accuracy(xdata2_test_scaled, ydata2_test, weight2)

        weight3 = perceptronLearning(xdata3_train_scaled, ydata3_train, weight)

        accuracy_train3[i] = accuracy(xdata3_train_scaled, ydata3_train,␣
↪weight3)
        accuracy_test3[i] = accuracy(xdata3_test_scaled, ydata3_test, weight3)

    print("The mean of accuracy for the training data 1 is :", np.
↪mean(accuracy_train1), "%")
    print("The mean of accuracy for the testing data 1 is :", np.
↪mean(accuracy_test1), "%")
    print("The std of accuracy for the training data 1 is :", np.
↪std(accuracy_train1))
    print("The std of accuracy for the testing data 1 is :", np.
↪std(accuracy_test1))
    print("\n")
    print("The mean of accuracy for the training data 2 is :", np.
↪mean(accuracy_train2), "%")
    print("The mean of accuracy for the testing data 2 is :", np.
↪mean(accuracy_test2), "%")
    print("The std of accuracy for the training data 2 is :", np.
↪std(accuracy_train2))
    print("The std of accuracy for the testing data 2 is :", np.
↪std(accuracy_test2))
    print("\n")
    print("The mean of accuracy for the training data 3 is :", np.
↪mean(accuracy_train3), "%")
    print("The mean of accuracy for the testing data 3 is :", np.
↪mean(accuracy_test3), "%")
    print("The std of accuracy for the training data 3 is :", np.
↪std(accuracy_train3))
    print("The std of accuracy for the testing data 3 is :", np.
↪std(accuracy_test3))
```

[47]:
```
run_10_linear()
```

```
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -3.52481552  3.50225845]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  100.0 %
i1 reach. Data is linearly separable
```

```
Weight matrix is: [ 0.         -0.153983    1.91133793]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  100.0 %
i2 reach.
Weight matrix is: [ 0.         -0.03543798  0.02314264]
Min J is: 1.225349059469708
Accuracy:  53.535353535353536 %
Accuracy:  46.464646464646464 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.70940556  0.61468782]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  100.0 %
i1 reach. Data is linearly separable
Weight matrix is: [ 1.         -0.00874213  2.01612548]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  94.94949494949495 %
i2 reach.
Weight matrix is: [ 0.          0.01542985 -0.04514293]
Min J is: 0.9244808153892415
Accuracy:  67.67676767676768 %
Accuracy:  67.67676767676768 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -1.82931335  2.21360549]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  100.0 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.0247717   1.52412521]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  100.0 %
i2 reach.
Weight matrix is: [ 0.         -0.00547731 -0.03518379]
Min J is: 0.7079176600142785
Accuracy:  74.74747474747475 %
Accuracy:  76.76767676767676 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -2.62304313  2.08564099]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  98.98989898989899 %
i1 reach. Data is linearly separable
Weight matrix is: [0.         0.26978371 2.32760393]
Min J is: 0
Accuracy:  100.0 %
```

```
Accuracy:  100.0 %
i2 reach.
Weight matrix is: [ 0.         -0.02013134 -0.08456868]
Min J is: 1.7473778985840835
Accuracy:  74.74747474747475 %
Accuracy:  75.75757575757575 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -3.15542044  3.06719648]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  100.0 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.11943942  2.8135676 ]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  100.0 %
i2 reach.
Weight matrix is: [0.         0.00075788 0.00378839]
Min J is: 0.21160802058431658
Accuracy:  25.252525252525253 %
Accuracy:  25.252525252525253 %
i1 reach. Data is linearly separable
Weight matrix is: [-1.         -3.04622363  2.69569424]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  97.97979797979798 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.16897988  1.73812694]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  100.0 %
i2 reach.
Weight matrix is: [ 0.         -0.00152637 -0.03085155]
Min J is: 0.6063426545733005
Accuracy:  72.72727272727273 %
Accuracy:  72.72727272727273 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.8561001   1.17339924]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  100.0 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.71908314  2.12224397]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  96.96969696969697 %
i2 reach.
Weight matrix is: [ 0.         -0.06606927  0.04531855]
```

```
Min J is: 2.3581858834547638
Accuracy:  51.515151515151516 %
Accuracy:  44.44444444444444 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -2.9573079   2.78736034]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  100.0 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.8240739   3.03591588]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  98.98989898989899 %
i2 reach.
Weight matrix is: [0.         0.0035151  0.02225743]
Min J is: 1.2207465410559901
Accuracy:  25.252525252525253 %
Accuracy:  23.232323232323232 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -2.55001648  1.93274305]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  96.96969696969697 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.55098496  2.14489367]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  98.98989898989899 %
i2 reach.
Weight matrix is: [0.         0.00923827 0.00011089]
Min J is: 0.3852259545630488
Accuracy:  31.313131313131315 %
Accuracy:  34.34343434343434 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -3.53397421  3.33131186]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  100.0 %
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.8279724   2.17683291]
Min J is: 0
Accuracy:  100.0 %
Accuracy:  96.96969696969697 %
i2 reach.
Weight matrix is: [ 0.         -0.05487033 -0.06343353]
Min J is: 1.8042490928978543
Accuracy:  77.77777777777779 %
Accuracy:  76.76767676767676 %
```

```
The mean of accuracy for the training data 1 is : 100.0 %
The mean of accuracy for the testing data 1 is : 99.3939393939394 %
The std of accuracy for the training data 1 is : 0.0
The std of accuracy for the testing data 1 is : 1.0301049522409669


The mean of accuracy for the training data 2 is : 100.0 %
The mean of accuracy for the testing data 2 is : 98.68686868686868 %
The std of accuracy for the training data 2 is : 0.0
The std of accuracy for the testing data 2 is : 1.693237839822244


The mean of accuracy for the training data 3 is : 55.45454545454546 %
The mean of accuracy for the testing data 3 is : 54.34343434343434 %
The std of accuracy for the training data 3 is : 20.28996011281075
The std of accuracy for the testing data 3 is : 20.866838683353016
```

[46]:
```
def nonlinear_quadratic_mapping(xdata):
    """
    xdata : nonagument data N * D
    """
    N, D = xdata.shape
    D_prime = 1/2 * (D ** 2 + 3 * D)
    xdata_mapping = []

    # Augment xdata
    xdata_aug = np.ones((N, D + 1))
    xdata_aug[:,1:] = xdata

    for i in range(D + 1):
        for j in range(i, D + 1):
            xdata_mapping.append(xdata_aug[:,i] * xdata_aug[:,j])

    xdata_mapping = np.array(xdata_mapping).T.reshape(N, int(D_prime) + 1)

    return xdata_mapping
```

[53]:
```
def predictor(xdata, weight):
    """
    xdata : augment data N * D_primes + 1 matrix
    weight : augmnet weight D_primes + 1 vector
    """
    gx = np.dot(xdata, weight)
    predict_label = np.ones(len(gx))
    for i in range(len(gx)):
        if gx[i] < 0:
            predict_label[i] += 1
```

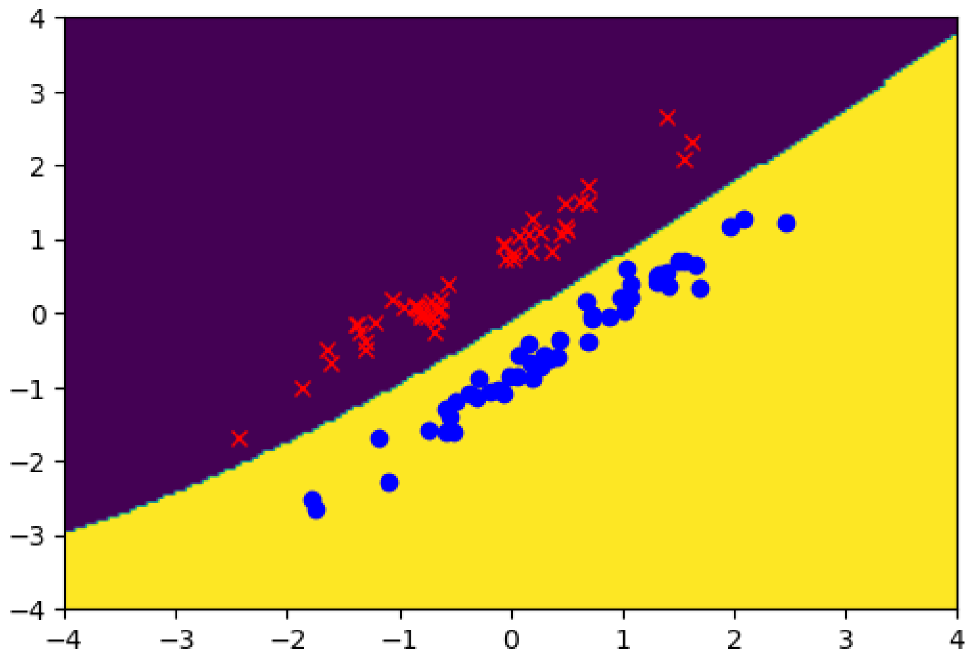
```
    return predict_label
```

[115]:
```
# Preprocessing: create an initial weight and get the transformed data
N, D = xdata1_train_scaled.shape
D_prime = 1/2 * (D ** 2 + 3 * D)
weight_prime = np.ones(int(D_prime) + 1)

xdata1_train_mapping = nonlinear_quadratic_mapping(xdata1_train_scaled)
xdata1_test_mapping = nonlinear_quadratic_mapping(xdata1_test_scaled)

xdata2_train_mapping = nonlinear_quadratic_mapping(xdata2_train_scaled)
xdata2_test_mapping = nonlinear_quadratic_mapping(xdata2_test_scaled)

xdata3_train_mapping = nonlinear_quadratic_mapping(xdata3_train_scaled)
xdata3_test_mapping = nonlinear_quadratic_mapping(xdata3_test_scaled)
```

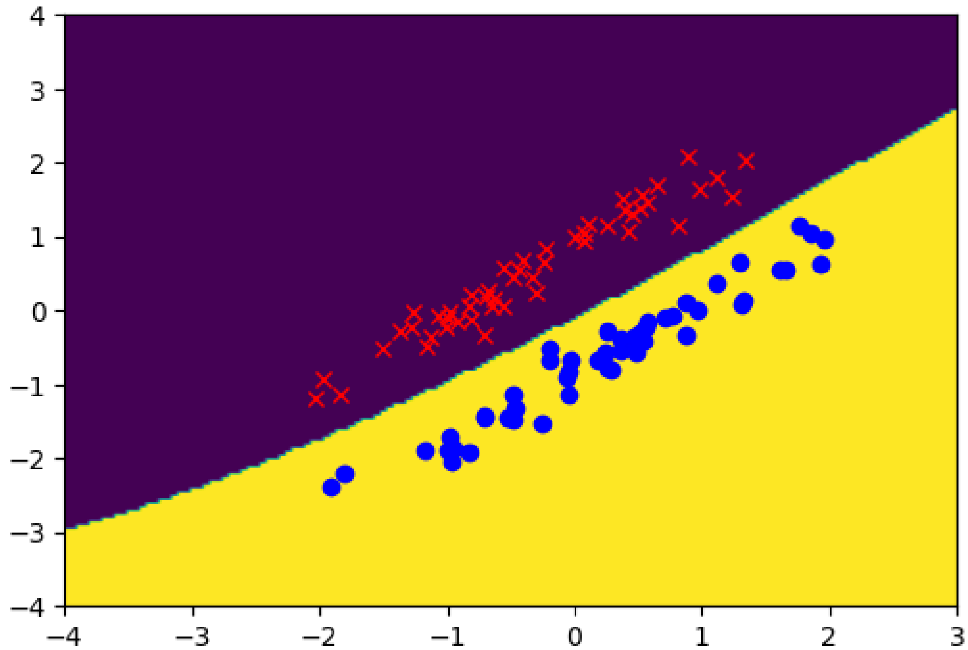
[117]:
```
def quadratic_classfication():
    weight_prime_1 = perceptronLearning(xdata1_train_mapping[:,1:],␣
 ↪ydata1_train, weight_prime)
    plotDecBoundaries_Nonlinear(xdata1_train_scaled, ydata1_train,␣
 ↪weight_prime_1, nonlinear_quadratic_mapping,predictor)
    print("Accuracy rate for training data 1 :",accuracy(xdata1_train_mapping[:
 ↪,1:], ydata1_train, weight_prime_1), "%")

    plotDecBoundaries_Nonlinear(xdata1_test_scaled, ydata1_test,␣
 ↪weight_prime_1, nonlinear_quadratic_mapping,predictor)
    print("Accuracy rate for testing data 1 :",accuracy(xdata1_test_mapping[:,1:
 ↪], ydata1_test, weight_prime_1), "%")
    print("\n")

    weight_prime_2 = perceptronLearning(xdata2_train_mapping[:,1:],␣
 ↪ydata2_train, weight_prime)
    plotDecBoundaries_Nonlinear(xdata2_train_scaled, ydata2_train,␣
 ↪weight_prime_2, nonlinear_quadratic_mapping,predictor)
    print("Accuracy rate for training data 2 :",accuracy(xdata2_train_mapping[:
 ↪,1:], ydata2_train, weight_prime_2), "%")

    plotDecBoundaries_Nonlinear(xdata2_test_scaled, ydata2_test,␣
 ↪weight_prime_2, nonlinear_quadratic_mapping,predictor)
    print("Accuracy rate for testing data 2 :",accuracy(xdata2_test_mapping[:,1:
 ↪], ydata2_test, weight_prime_2), "%")
    print("\n")

    weight_prime_3 = perceptronLearning(xdata3_train_mapping[:,1:],␣
 ↪ydata3_train, weight_prime)
```

```
    plotDecBoundaries_Nonlinear(xdata3_train_scaled, ydata3_train,␣
↪weight_prime_3, nonlinear_quadratic_mapping,predictor)
    print("Accuracy rate for training data 3 :", accuracy(xdata3_train_mapping[:
↪,1:], ydata3_train, weight_prime_3), "%")

    plotDecBoundaries_Nonlinear(xdata3_test_scaled, ydata3_test,␣
↪weight_prime_3, nonlinear_quadratic_mapping,predictor)
    print("Accuracy rate for testing data 2 :",accuracy(xdata3_test_mapping[:,1:
↪], ydata3_test, weight_prime_3), "%")

quadratic_classfication()
```

```
i1 reach. Data is linearly separable
Weight matrix is: [ 1.          -7.66323998  8.7350138  -0.6293219   0.07998281
0.39826552]
Min J is: 0
```

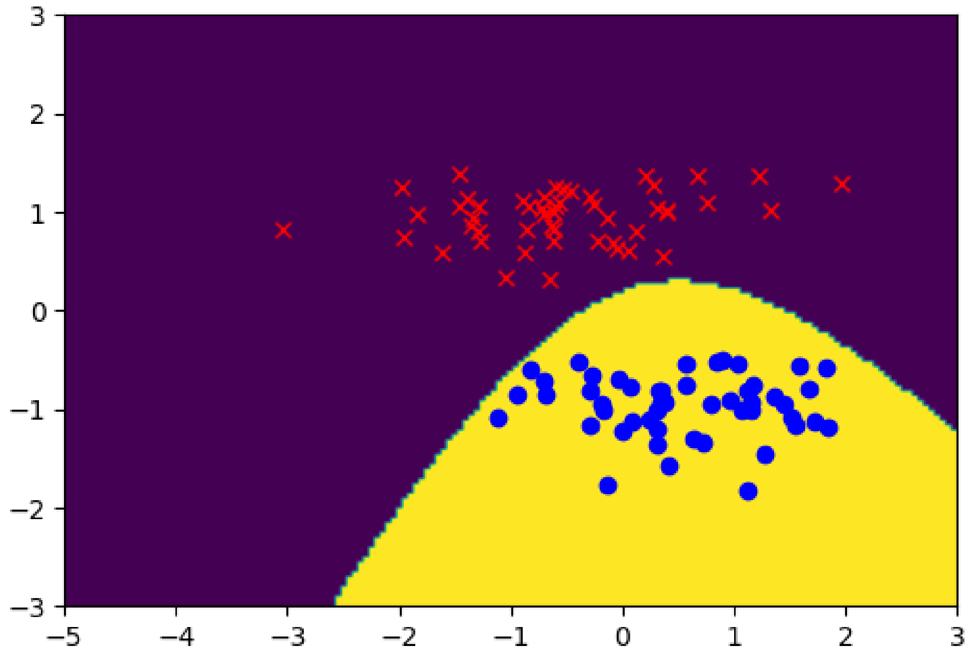


```
Accuracy rate for training data 1 : 100.0 %
```

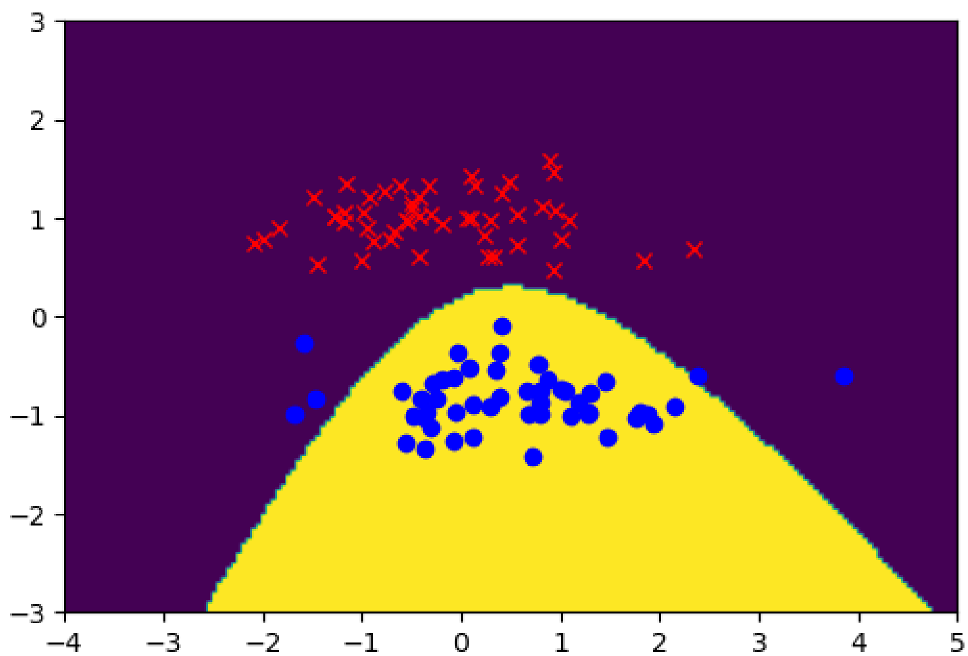



```
Accuracy rate for testing data 1 : 100.0 %
```

```
i1 reach. Data is linearly separable
Weight matrix is: [-1.         -2.23120865  5.25456869  1.97562164  0.68693003
-0.86031339]
Min J is: 0
```

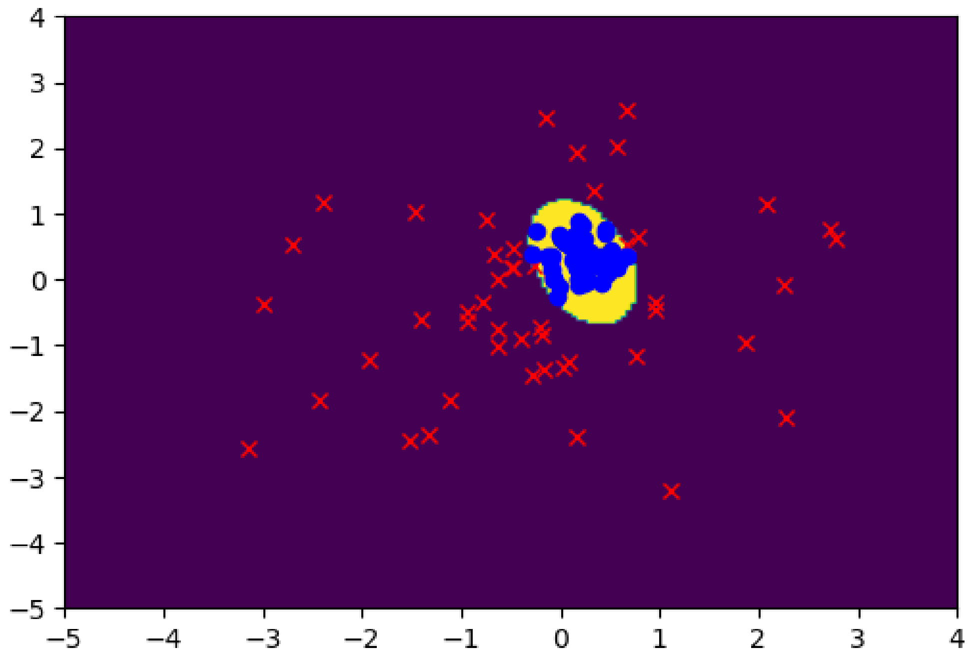



```
Accuracy rate for training data 2 : 100.0 %
```

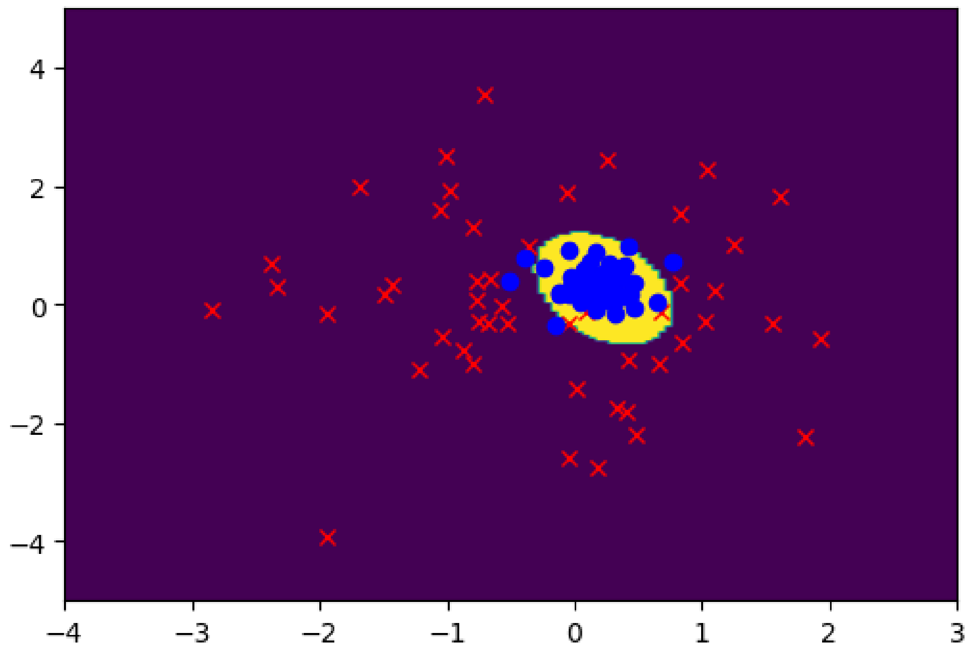



```
Accuracy rate for testing data 2 : 95.0 %
```

```
i2 reach.
Weight matrix is: [-1.         -3.86819942 -1.75654489  6.64888088  2.73306438
2.14209377]
Min J is: 2
```




```
Accuracy rate for training data 3 : 98.0 %
```



Accuracy rate for testing data 2 : 93.0 %

[151]:
```
def run_10_quadratic():

    accuracy_train1 = np.zeros(10)
    accuracy_test1 = np.zeros(10)

    accuracy_train2 = np.zeros(10)
    accuracy_test2 = np.zeros(10)

    accuracy_train3 = np.zeros(10)
    accuracy_test3 = np.zeros(10)

    for i in range(10):
        weight_prime_1 = perceptronLearning(xdata1_train_mapping[:,1:],␣
↪ydata1_train, weight_prime)
        weight_prime_2 = perceptronLearning(xdata2_train_mapping[:,1:],␣
↪ydata2_train, weight_prime)
        weight_prime_3 = perceptronLearning(xdata3_train_mapping[:,1:],␣
↪ydata3_train, weight_prime)


        accuracy_train1[i] = accuracy(xdata1_train_mapping[:,1:], ydata1_train,␣
↪weight_prime_1)
```

```
        accuracy_test1[i] = accuracy(xdata1_test_mapping[:,1:], ydata1_test,␣
↪weight_prime_1)

        accuracy_train2[i] = accuracy(xdata2_train_mapping[:,1:], ydata2_train,␣
↪weight_prime_2)
        accuracy_test2[i] = accuracy(xdata2_test_mapping[:,1:], ydata2_test,␣
↪weight_prime_2)


        accuracy_train3[i] = accuracy(xdata3_train_mapping[:,1:], ydata3_train,␣
↪weight_prime_3)
        accuracy_test3[i] = accuracy(xdata3_test_mapping[:,1:], ydata3_test,␣
↪weight_prime_3)


    print("The mean of accuracy for the training data 1 is :", np.
↪mean(accuracy_train1), "%")
    print("The mean of accuracy for the testing data 1 is :", np.
↪mean(accuracy_test1), "%")
    print("The std of accuracy for the training data 1 is :", np.
↪std(accuracy_train1))
    print("The std of accuracy for the testing data 1 is :", np.
↪std(accuracy_test1))
    print("\n")
    print("The mean of accuracy for the training data 2 is :", np.
↪mean(accuracy_train2), "%")
    print("The mean of accuracy for the testing data 2 is :", np.
↪mean(accuracy_test2), "%")
    print("The std of accuracy for the training data 2 is :", np.
↪std(accuracy_train2))
    print("The std of accuracy for the testing data 2 is :", np.
↪std(accuracy_test2))
    print("\n")
    print("The mean of accuracy for the training data 3 is :", np.
↪mean(accuracy_train3), "%")
    print("The mean of accuracy for the testing data 3 is :", np.
↪mean(accuracy_test3), "%")
    print("The std of accuracy for the training data 3 is :", np.
↪std(accuracy_train3))
    print("The std of accuracy for the testing data 3 is :", np.
↪std(accuracy_test3))

run_10_quadratic()
```

```
i1 reach. Data is linearly separable
Weight matrix is: [ -2.         -14.37311012  13.15115425   1.63934042
```

```
1.76400355
  -1.6091564 ]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-1.         -1.12933732  8.36800763 -0.61211116  1.35499821
0.73636262]
Min J is: 0
i2 reach.
Weight matrix is: [-1.         -3.4761001  -1.45441152  6.59679023  2.31566287
2.15822252]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [  2.         -14.15847923  14.66324413   1.22354825
-0.046896
   0.4798287 ]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -1.61757337  5.627948    1.48937733  0.60085165
-2.34590643]
Min J is: 0
i2 reach.
Weight matrix is: [-1.         -4.19888817 -2.37571506  7.47137138  3.37030289
2.47273643]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ -1.         -10.17791242   9.71769867  -1.75130274
0.66391675
   1.3751295 ]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-2.         -5.60474477  9.60734888  1.6357955  -1.22020546
-0.36435115]
Min J is: 0
i2 reach.
Weight matrix is: [-1.         -3.78220458 -1.58370942  6.55332203  3.43743455
2.24246561]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [  0.         -12.84893434  13.18091731  -0.78387109
0.33967744
  -0.83122035]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-1.         -0.03200588  3.47644623  0.29721     0.50448169
0.12918827]
Min J is: 0
i2 reach.
Weight matrix is: [-1.         -4.14520293 -1.8375234   6.85592478  3.101276
```

```
2.24553155]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ -3.         -11.92021152  16.67762127   1.02970221
0.76314312
  -1.20286702]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-1.         -3.45807899  9.29396772  2.60828993  1.5040015
0.0470346 ]
Min J is: 0
i2 reach.
Weight matrix is: [-1.         -3.91056102 -1.95672937  6.96921311  3.15700443
2.67384104]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -9.27378006 10.14364156  0.45487066 -0.79106118
-1.13223094]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -1.23995226  3.58775562  0.31774209  1.05814183
-0.70960256]
Min J is: 0
i2 reach.
Weight matrix is: [-1.         -3.83404934 -1.88671455  7.14938155  2.85661554
2.82642925]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ 2.         -8.54532424  9.82140665 -0.74031233 -0.12017084
1.15082343]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-3.         -4.79736521  9.02599743  2.88052234 -0.60488379
-0.15334878]
Min J is: 0
i2 reach.
Weight matrix is: [-1.         -4.24271236 -2.27383013  7.28186138  2.8860311
2.93026998]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ -1.         -18.69572391  15.94743714  -2.34073218
1.37407603
   0.86961898]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-3.         -3.81391111  7.72988259  3.82916627  1.41079492
-1.3838321 ]
Min J is: 0
```

```
i2 reach.
Weight matrix is: [-1.         -4.0317986  -2.25217464  7.39099968  3.3326658
2.70004509]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ -2.         -10.73828749   8.91720104  -0.55826933
1.11305686
  -0.20159828]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -2.31644654  6.58540398 -0.0937659  -0.3247009
1.07754565]
Min J is: 0
i2 reach.
Weight matrix is: [-1.         -3.98268869 -1.94221411  7.27036853  3.09007418
2.79148462]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ -1.         -19.69255506  19.54385602   0.29374475
1.51574983
  -2.10647268]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-1.         -5.24444232 11.06485385  2.12370647  0.44534914
-2.22376636]
Min J is: 0
i2 reach.
Weight matrix is: [-1.         -4.4563076  -2.49453314  8.5177586   2.90341355
2.48982312]
Min J is: 2
The mean of accuracy for the training data 1 is : 100.0 %
The mean of accuracy for the testing data 1 is : 99.6 %
The std of accuracy for the training data 1 is : 0.0
The std of accuracy for the testing data 1 is : 0.8


The mean of accuracy for the training data 2 is : 100.0 %
The mean of accuracy for the testing data 2 is : 96.7 %
The std of accuracy for the training data 2 is : 0.0
The std of accuracy for the testing data 2 is : 1.6155494421403513


The mean of accuracy for the training data 3 is : 98.0 %
The mean of accuracy for the testing data 3 is : 92.0 %
The std of accuracy for the training data 3 is : 0.0
The std of accuracy for the testing data 3 is : 0.6324555320336759
```

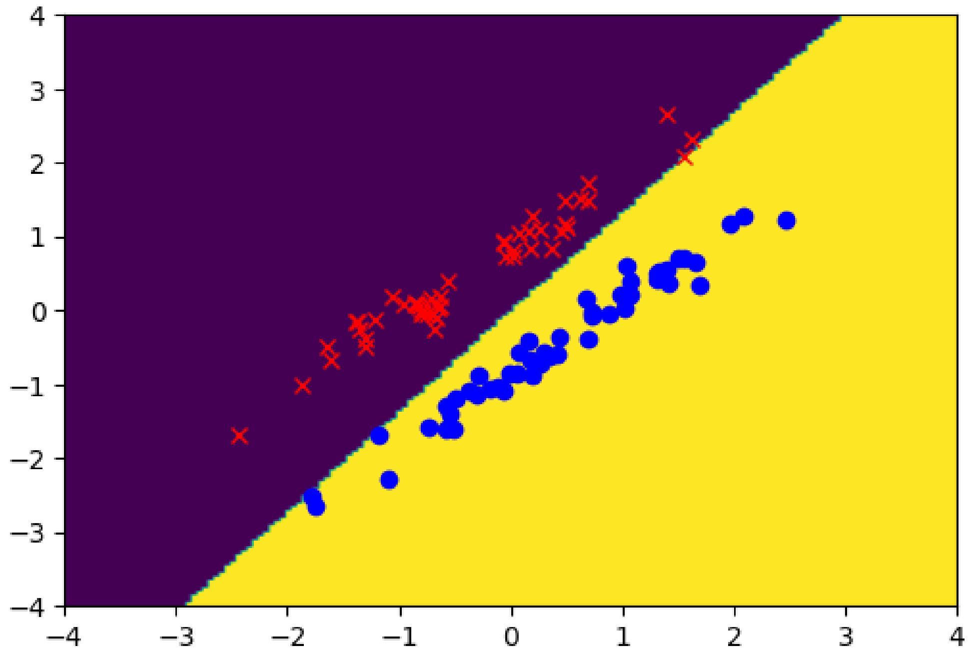
[140]:
```
def nonlinear_cubic_mapping(xdata):
    """
    xdata : nonagument data N * D
    """
    N, D = xdata.shape
    D_prime = 9
    xdata_mapping = []

    # Augment xdata
    xdata_aug = np.ones((N, D + 1))
    xdata_aug[:,1:] = xdata

    for i in range(D + 1):
        for j in range(i, D + 1):
            for k in range(j, D + 1):
                xdata_mapping.append(xdata_aug[:,i] * xdata_aug[:,j] *␣
↪xdata_aug[:,k])

    xdata_mapping = np.array(xdata_mapping).T.reshape(N, D_prime + 1)
    return xdata_mapping
```

[145]:
```
xdata1_train_mapping_cubic = nonlinear_cubic_mapping(xdata1_train_scaled)
xdata1_test_mapping_cubic = nonlinear_cubic_mapping(xdata1_test_scaled)

xdata2_train_mapping_cubic = nonlinear_cubic_mapping(xdata2_train_scaled)
xdata2_test_mapping_cubic = nonlinear_cubic_mapping(xdata2_test_scaled)

xdata3_train_mapping_cubic = nonlinear_cubic_mapping(xdata3_train_scaled)
xdata3_test_mapping_cubic = nonlinear_cubic_mapping(xdata3_test_scaled)

weight_prime_cubic = np.ones(10)
```

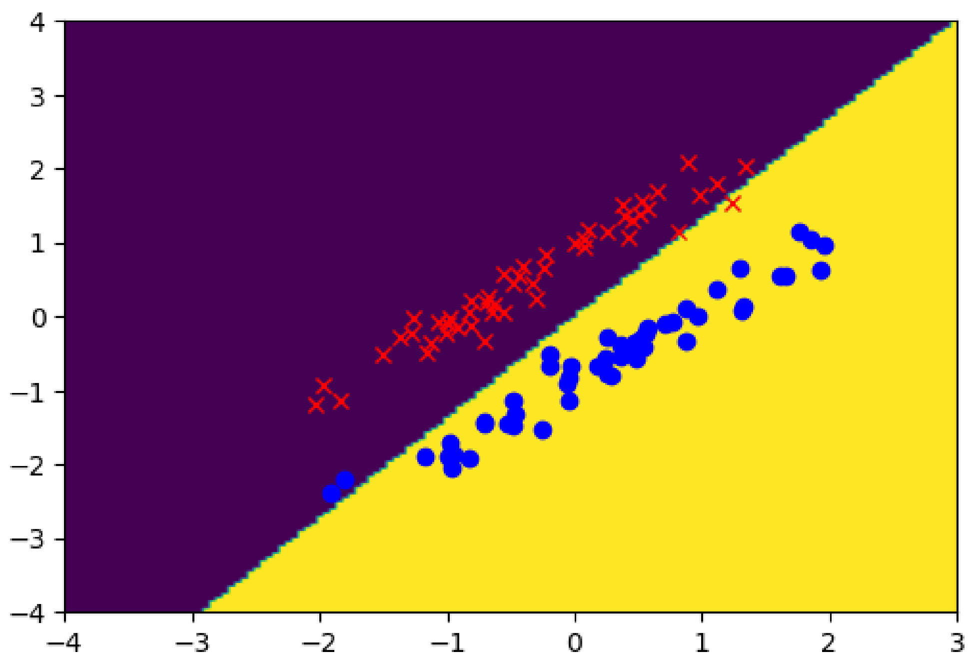
[147]:
```
def cubic_classfication():
    weight_prime_1 = perceptronLearning(xdata1_train_mapping_cubic[:,1:],␣
↪ydata1_train, weight_prime_cubic)
    plotDecBoundaries_Nonlinear(xdata1_train_scaled, ydata1_train,␣
↪weight_prime_1, nonlinear_cubic_mapping,predictor)
    print("Accuracy rate for training data 1 :
↪",accuracy(xdata1_train_mapping_cubic[:,1:], ydata1_train, weight_prime_1),␣
↪"%")

    plotDecBoundaries_Nonlinear(xdata1_test_scaled, ydata1_test,␣
↪weight_prime_1, nonlinear_cubic_mapping,predictor)
    print("Accuracy rate for testing data 1 :
↪",accuracy(xdata1_test_mapping_cubic[:,1:], ydata1_test, weight_prime_1),␣
↪"%")
```

```
    print("\n")

    weight_prime_2 = perceptronLearning(xdata2_train_mapping_cubic[:,1:],␣
↪ydata2_train, weight_prime_cubic)
    plotDecBoundaries_Nonlinear(xdata2_train_scaled, ydata2_train,␣
↪weight_prime_2, nonlinear_cubic_mapping,predictor)
    print("Accuracy rate for training data 2 :
↪",accuracy(xdata2_train_mapping_cubic[:,1:], ydata2_train, weight_prime_2),␣
↪"%")

    plotDecBoundaries_Nonlinear(xdata2_test_scaled, ydata2_test,␣
↪weight_prime_2, nonlinear_cubic_mapping,predictor)
    print("Accuracy rate for testing data 2 :
↪",accuracy(xdata2_test_mapping_cubic[:,1:], ydata2_test, weight_prime_2),␣
↪"%")
    print("\n")

    weight_prime_3 = perceptronLearning(xdata3_train_mapping_cubic[:,1:],␣
↪ydata3_train, weight_prime_cubic)
    plotDecBoundaries_Nonlinear(xdata3_train_scaled, ydata3_train,␣
↪weight_prime_3, nonlinear_cubic_mapping,predictor)
    print("Accuracy rate for training data 3 :",␣
↪accuracy(xdata3_train_mapping_cubic[:,1:], ydata3_train, weight_prime_3),␣
↪"%")

    plotDecBoundaries_Nonlinear(xdata3_test_scaled, ydata3_test,␣
↪weight_prime_3, nonlinear_cubic_mapping,predictor)
    print("Accuracy rate for testing data 2 :
↪",accuracy(xdata3_test_mapping_cubic[:,1:], ydata3_test, weight_prime_3),␣
↪"%")

cubic_classfication()
```

```
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -1.61543874  1.24146949 -0.28421054  0.65143742
-0.0449748
 -5.97039289 -0.67180192  0.92055614  2.10049763]
Min J is: 0
```

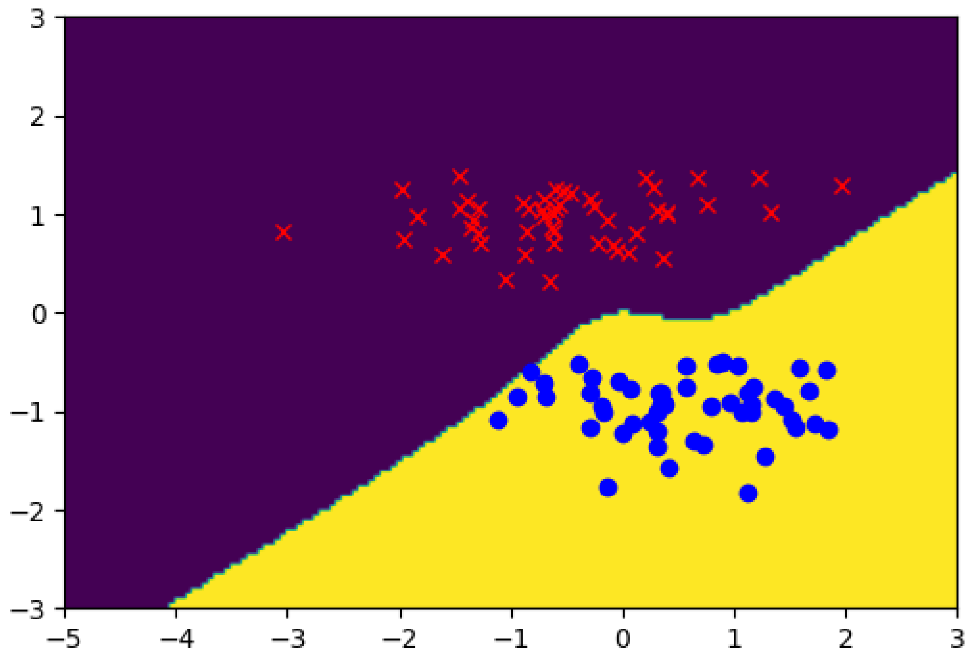


Accuracy rate for training data 1 : 100.0 %

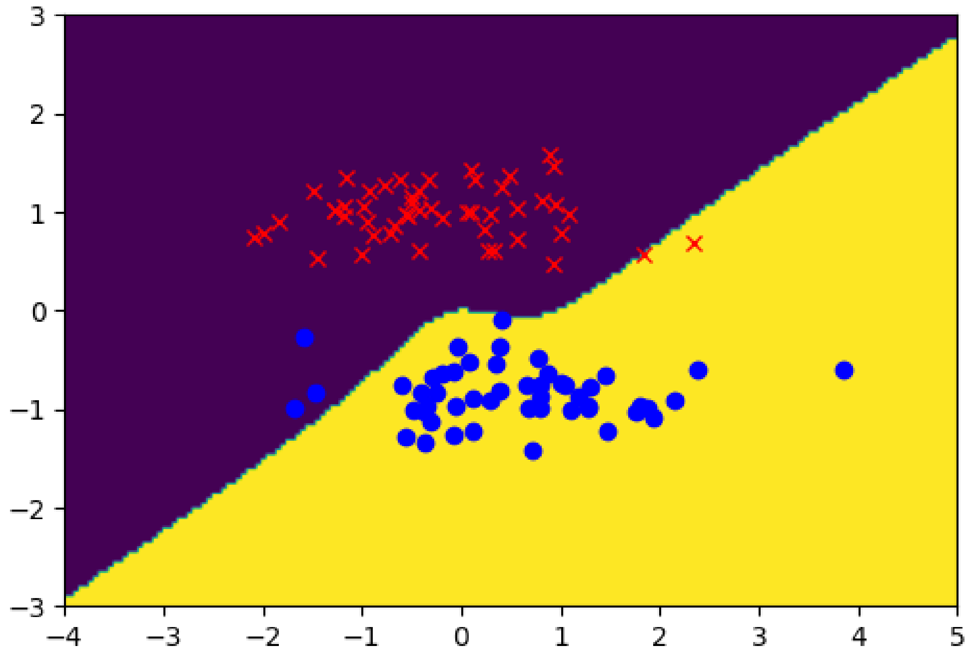


Accuracy rate for testing data 1 : 97.0 %

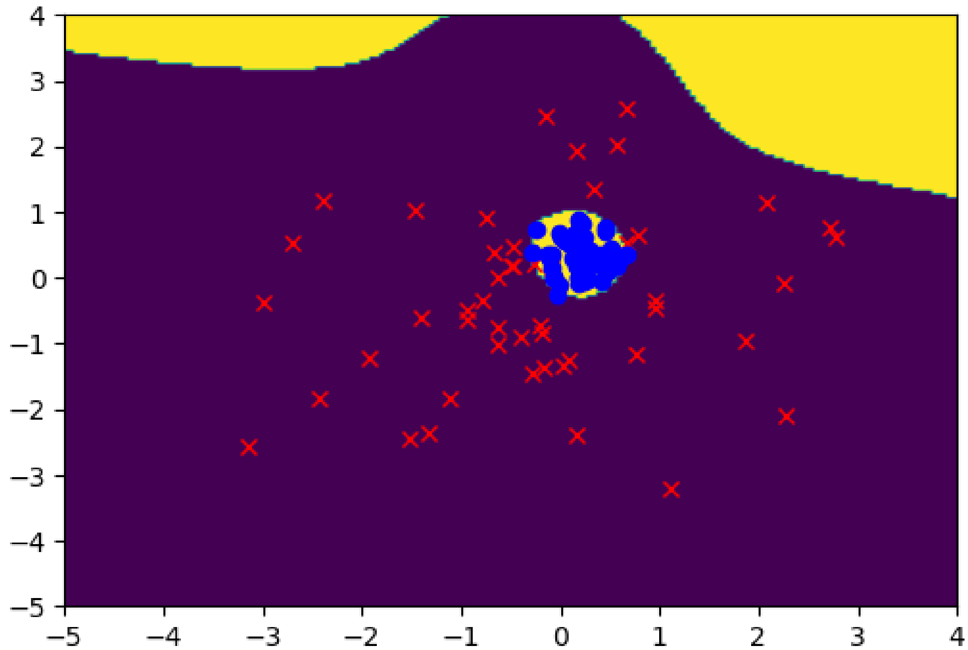
```
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.21279609  3.29674302  3.02243026 -0.14617657
-0.16473553
 -2.91061976  2.98469536  0.49709636  2.56366838]
Min J is: 0
```



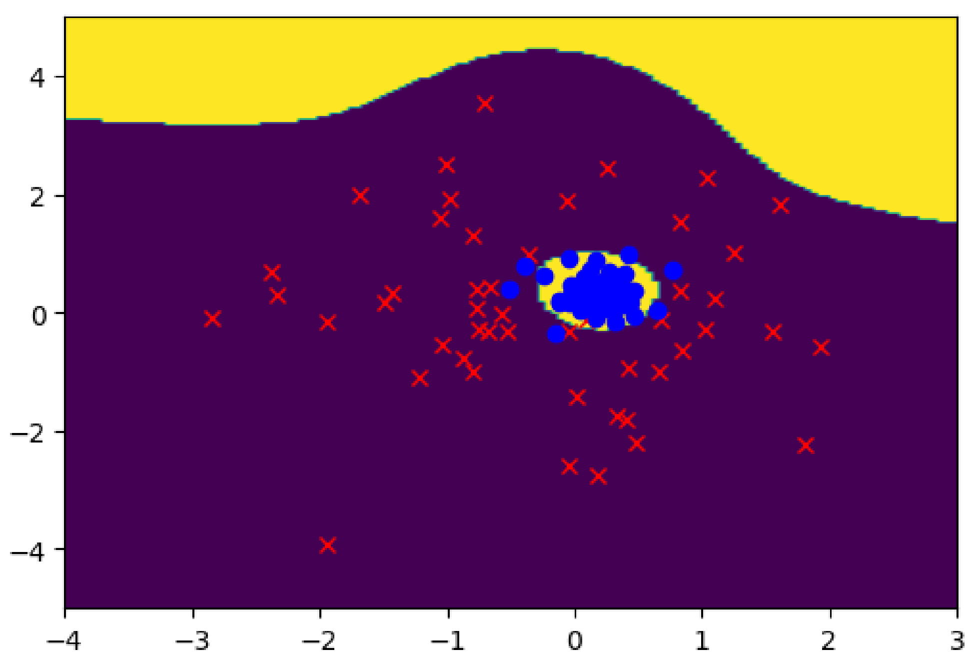

```
Accuracy rate for training data 2 : 100.0 %
```




```
Accuracy rate for testing data 2 : 95.0 %


i2 reach.
Weight matrix is: [ -4.          -15.39150474 -13.2885659   40.41485695
13.15847859
  21.44429684  -4.17019256 -18.59555545  -3.22875261  -4.12572051]
Min J is: 3
```



Accuracy rate for training data 3 : 97.0 %



Accuracy rate for testing data 2 : 92.0 %

```
[152]: def run_10_cubic():

    accuracy_train1 = np.zeros(10)
    accuracy_test1 = np.zeros(10)

    accuracy_train2 = np.zeros(10)
    accuracy_test2 = np.zeros(10)

    accuracy_train3 = np.zeros(10)
    accuracy_test3 = np.zeros(10)

    for i in range(10):
        weight_prime_1 = perceptronLearning(xdata1_train_mapping_cubic[:,1:],␣
↪ydata1_train, weight_prime_cubic)
        weight_prime_2 = perceptronLearning(xdata2_train_mapping_cubic[:,1:],␣
↪ydata2_train, weight_prime_cubic)
        weight_prime_3 = perceptronLearning(xdata3_train_mapping_cubic[:,1:],␣
↪ydata3_train, weight_prime_cubic)


        accuracy_train1[i] = accuracy(xdata1_train_mapping_cubic[:,1:],␣
↪ydata1_train, weight_prime_1)
        accuracy_test1[i] = accuracy(xdata1_test_mapping_cubic[:,1:],␣
↪ydata1_test, weight_prime_1)

        accuracy_train2[i] = accuracy(xdata2_train_mapping_cubic[:,1:],␣
↪ydata2_train, weight_prime_2)
        accuracy_test2[i] = accuracy(xdata2_test_mapping_cubic[:,1:],␣
↪ydata2_test, weight_prime_2)


        accuracy_train3[i] = accuracy(xdata3_train_mapping_cubic[:,1:],␣
↪ydata3_train, weight_prime_3)
        accuracy_test3[i] = accuracy(xdata3_test_mapping_cubic[:,1:],␣
↪ydata3_test, weight_prime_3)


    print("The mean of accuracy for the training data 1 is :", np.
↪mean(accuracy_train1), "%")
    print("The mean of accuracy for the testing data 1 is :", np.
↪mean(accuracy_test1), "%")
    print("The std of accuracy for the training data 1 is :", np.
↪std(accuracy_train1))
    print("The std of accuracy for the testing data 1 is :", np.
↪std(accuracy_test1))
    print("\n")
```

```
    print("The mean of accuracy for the training data 2 is :", np.
 ↪mean(accuracy_train2), "%")
    print("The mean of accuracy for the testing data 2 is :", np.
 ↪mean(accuracy_test2), "%")
    print("The std of accuracy for the training data 2 is :", np.
 ↪std(accuracy_train2))
    print("The std of accuracy for the testing data 2 is :", np.
 ↪std(accuracy_test2))
    print("\n")
    print("The mean of accuracy for the training data 3 is :", np.
 ↪mean(accuracy_train3), "%")
    print("The mean of accuracy for the testing data 3 is :", np.
 ↪mean(accuracy_test3), "%")
    print("The std of accuracy for the training data 3 is :", np.
 ↪std(accuracy_train3))
    print("The std of accuracy for the testing data 3 is :", np.
 ↪std(accuracy_test3))

run_10_cubic()
```

```
i1 reach. Data is linearly separable
Weight matrix is: [-1.         -3.19325329  2.39589446 -1.20055382  3.36119949
4.26498433
 -7.05501674  2.54994615  6.39661089 10.34622777]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ -1.           3.6313337    8.19029064  12.32073
1.19744872
   2.39701627 -16.04666591  18.19329743   5.41832681   8.56590471]
Min J is: 0
i2 reach.
Weight matrix is: [-3.         -8.46960358 -7.13806135 20.50116809  0.26687748
13.19159749
  0.60849901 -4.36401562  7.03271722 -2.80559863]
Min J is: 3
i1 reach. Data is linearly separable
Weight matrix is: [  2.          -5.64921969   0.56262001   5.31905921
0.95184137
  -2.21272912 -19.66778651  -5.53423642   3.67571804  11.93683492]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.55750747  2.1550173  -1.42582952  2.79894808
-0.33406497
 -2.77824761  3.80187508 -1.07781616  2.54086813]
Min J is: 0
i2 reach.
Weight matrix is: [ -4.         -15.91808354  -9.14466603  38.79119612
```

```
12.57828645
  15.34553412  -3.74832756 -22.10840048   3.24412398  -3.68140448]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -3.02101957  1.98710986 -0.36972413  0.70606103
1.03560909
 -1.79173075  0.68323635  0.72181011  1.59789735]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 2.         -0.34292846  5.02008043  4.51384146  1.72968692
1.81374949
 -3.85260307 12.30376063  3.89228773  5.08513791]
Min J is: 0
i2 reach.
Weight matrix is: [-5.         -2.83157781 -6.22558909 17.38118561  0.20704133
11.78074358
 -4.50066121  0.60973929  2.4032791  -0.55878507]
Min J is: 3
i1 reach. Data is linearly separable
Weight matrix is: [ 1.          0.64005397  3.76834699 -2.18061239  2.34963432
5.90976011
 -9.83182478 -0.60097797  5.65682775 11.67090527]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -0.07085133  3.22098428  2.03242838 -0.85390597
-0.14132025
 -2.01863083  4.08316105  1.60179476  2.65325768]
Min J is: 0
i2 reach.
Weight matrix is: [ -4.         -15.86973792 -10.41818294  31.58953964
-0.25160302
  12.48664067   0.7172856    0.80188143  14.87066134  -1.07772038]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -1.13182876  1.34379215  2.25677305  0.64389327
-1.46714345
 -4.10649497 -0.92888145  1.22509077  4.5849209 ]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ -1.           3.90600593   8.1814851   12.84652803
0.96545373
   0.80928335 -15.22569425  17.92073856   4.19666927   8.34845111]
Min J is: 0
i2 reach.
Weight matrix is: [ -5.         -18.70172676 -11.56800037  46.49445244
15.28163393
  17.41366757  -5.21697907 -19.50062882  -3.11077471  -2.02951056]
Min J is: 2
```

```
i1 reach. Data is linearly separable
Weight matrix is: [ -2.          -3.94246901   3.21546696  -6.44414543
-5.02819102
  -6.67244774 -18.42215445  -2.61765968   6.93375816  15.96591516]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [0.         0.02730717 1.91326997 0.05386866 1.88833116
0.16593795
 0.07970483 1.86407335 0.18871383 1.76172382]
Min J is: 0
i2 reach.
Weight matrix is: [ -5.         -19.40318163 -11.02912748  40.6493568
13.48407669
  20.67300565   0.46015311 -20.6934664    4.97114348  -3.64727937]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ -3.          -5.51054782   3.58056218  -1.10902785
-0.30612346
  -3.31618327 -21.39091947  -3.80340039   9.20881948  23.61182548]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [-1.         -0.06357705  5.39249727  5.12317834  2.66272196
1.82546018
 -2.471839   10.92586178  2.95474769  4.4041211 ]
Min J is: 0
i2 reach.
Weight matrix is: [ -4.         -17.32007788  -9.83425553  31.02755758
6.80218615
  11.46717791   7.86077598  -7.86158872   5.89929833   2.187822  ]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [  0.          -4.98174567   1.63251709   0.16742558
-3.38202535
  -6.80159942 -16.31389932  -1.67852194   6.60528878  14.8436942 ]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [  0.           2.70256443   8.43848916  12.33838474
1.32326339
   2.19765038 -15.95922298  23.91317758   7.40657274   8.43858545]
Min J is: 0
i2 reach.
Weight matrix is: [ -6.          -9.56823438 -14.26393998  27.00183139
0.73618758
  24.69876161   7.07136178  -4.8830188    5.2496867   -0.38183264]
Min J is: 3
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -1.65413538  1.62578599  0.4959464   0.71093139
0.3932999
```

```
 -3.43483005 -0.44417127  0.97920219  2.52965019]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 1.          0.50646539  4.2589735   4.07942587  3.42566164
2.04938897
  0.37613022 10.13571814  3.01585047  4.06299535]
Min J is: 0
i2 reach.
Weight matrix is: [ -4.         -14.50791123  -8.68169626  27.05734734
5.20866973
  14.17115238   1.94435882  -1.42184934   4.44093119  -2.389114  ]
Min J is: 2
i1 reach. Data is linearly separable
Weight matrix is: [ 0.         -1.45829274  1.09762932 -0.21175947  0.58395041
0.36867919
 -1.80206718  0.20489922  0.53929096  1.11440734]
Min J is: 0
i1 reach. Data is linearly separable
Weight matrix is: [ 0.          0.7694296   3.86254852 -0.65941393  4.37429351
0.22175095
 -2.7856921   6.87811528 -0.06704011  4.02656855]
Min J is: 0
i2 reach.
Weight matrix is: [ -5.         -16.52750914 -11.21776605  31.56551197
7.72424132
  19.10848872   7.32112326 -12.05266387   4.33235968  -1.33057282]
Min J is: 3
The mean of accuracy for the training data 1 is : 100.0 %
The mean of accuracy for the testing data 1 is : 99.3 %
The std of accuracy for the training data 1 is : 0.0
The std of accuracy for the testing data 1 is : 1.1874342087037917


The mean of accuracy for the training data 2 is : 100.0 %
The mean of accuracy for the testing data 2 is : 96.8 %
The std of accuracy for the training data 2 is : 0.0
The std of accuracy for the testing data 2 is : 2.2271057451320084


The mean of accuracy for the training data 3 is : 97.6 %
The mean of accuracy for the testing data 3 is : 91.3 %
The std of accuracy for the training data 3 is : 0.4898979485566356
The std of accuracy for the testing data 3 is : 1.676305461424021
```

[ ]: