Problem 1: Cantilever Beam under Uniform Load

Governing Equation

The Euler-Bernoulli beam equation for deflection $u(x)$ is given by:

$$\frac{d^4u}{dx^4} = -\frac{q}{EI}$$

where $E = 200 \times 10^9\,\text{Pa}$, $I = 10^{-6}\,\text{m}^4$, $q = 10000\,\text{N/m}$, and $L = 1\,\text{m}$.

Boundary Conditions

- **Fixed end** ($x = 0$):

$$u(0) = 0, \quad \frac{du}{dx}\Big|_{x=0} = 0$$

- **Free end** ($x = L$):

$$\frac{d^2u}{dx^2}\Big|_{x=L} = 0, \quad \frac{d^3u}{dx^3}\Big|_{x=L} = 0$$

Analytical Solution

$$u_{\text{exact}}(x) = -\frac{q}{EI}\left(\frac{x^4}{24} - \frac{x^3}{6} + \frac{x^2}{4}\right)$$

PINN Implementation

- **Architecture**: FNN with 4 layers (1-30-30-30-1)
- **Activation**: tanh
- **Optimizer**: Adam (lr = 0.001)
- **Loss**: PDE residual + 4 boundary operators
- **Training**: 4000 iterations

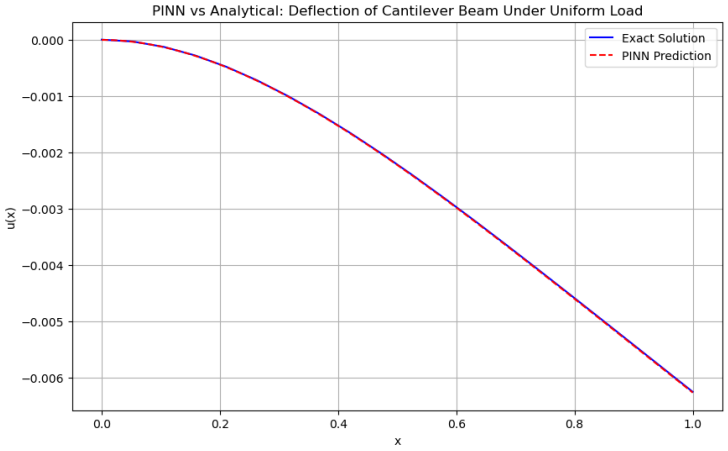

Figure 1: PINN prediction vs analytical solution for cantilever beam deflection