

## Problem 2: Fully Restrained Beam under Uniform Load

### Governing Equation

The beam equation is identical to Problem 1:

$$\frac{d^4 w}{dx^4} = -\frac{q}{EI}$$

with identical parameters  $E$ ,  $I$ ,  $q$ , and  $L$ .

### Boundary Conditions

- **Both ends** ( $x = 0$  and  $x = L$ ):

$$w(0) = w(L) = 0, \quad \left. \frac{dw}{dx} \right|_{x=0} = \left. \frac{dw}{dx} \right|_{x=L} = 0$$

### Analytical Solution

$$w_{\text{exact}}(x) = -\frac{q}{24EI} (x^4 - 2Lx^3 + L^2x^2)$$

### PINN Implementation

- **Architecture:** FNN with 4 layers (1-50-50-50-1)
- **Activation:** Swish
- **Optimizer:** Adam ( $\text{lr} = 5 \times 10^{-5}$ ) + L-BFGS
- **Loss:** PDE residual + 4 boundary conditions
- **Training:** 15,000 Adam + L-BFGS iterations

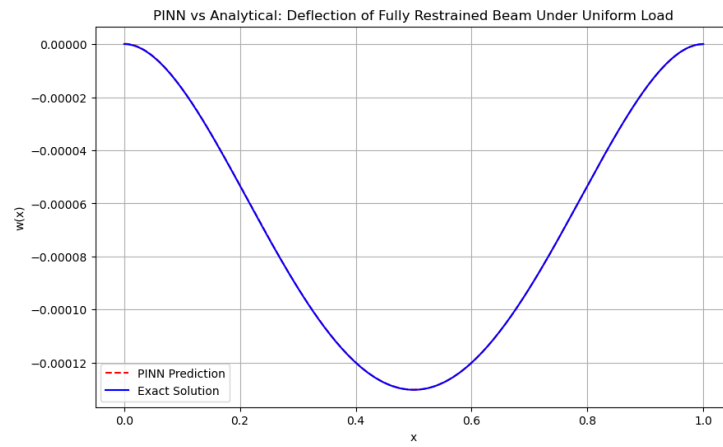


Figure 1: PINN prediction vs analytical solution for fully restrained beam deflection