Quantitative Review
Based off the “Official ETS Math Review”

Part 1: Arithmetic

Integers

- Whole numbers (no decimals/fractions)
- (+)(+) = (+)
- (-)(-) = (+)
- (+)(-) = (-)
- Factor: numbers or that can be multiplied together to get the original number: 6 = (3)(2), so 3 and 2 are factors of 6.
- Divisor: a number that divides an integer exactly: 12 ÷ 3 = 4, so 3 is the divisor.
- Least Common Multiple: the smallest number that both numbers go into: LCM of 3 and 5 is 15.
- Greatest Common Divisor: largest number that both numbers can be divided by: GCD of 30 and 75 is 15.
- Even (e) and Odd (o):
- (e) + (e) = (e)
- (o) + (o) = (e)
- (e) + (o) = (o)
- (e)(e) = (e)
- (o)(o) = (o)
- (e)(o) = (e)
- Prime Numbers: numbers greater than 1 that cannot be formed by multiplying two smaller natural numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc…
- Prime Factorization: the process of finding which prime numbers multiply together to make the original number: 81 = (3)^4
- Composite Numbers: Non-prime numbers: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, etc…

Fractions

- Rational Number: any number that can be expressed as a fraction of decimal.
- Adding/Subtracting Fractions:
- To add two fractions with the same denominator, you add the numerators and keep the same denominator.
- To add two fractions with the different denominators, you must find the common denominator which is merely the common multiple of both denominators (make both denominators equal).
- Multiply/Dividing Fractions:
- To multiply two fractions, just multiply the two numerators together and multiply the two denominators together.
- To divide one fraction by another, first flip the second fraction (this is called the reciprocal), then multiply the first fraction by the second (flipped) fraction or reciprocal.
- Mixed Numbers: a number consisting of an integer and a fraction: 4 3/8 = 27/8 (multiply the denominator by the integer and add the remaining value in the numerator to find convert to fraction form.

- Fractional Expressions: numbers where the numerator or denominator are not in integer form; however, the same rules for basic fractions apply to fractional expressions: pi/3 + pi/2 = 3pi/6 + 2pi/6 = 5pi/6.

Exponents

- The base number is multiplied by itself x amount of times, where x represents an exponent: 4^3 = (4)(4)(4) as 4 is the base and 3 is the exponent.
- Squaring a number represents an exponent of 2 and cubing a number represents an exponent of 3.
- A negative number raised to an even power is always positive, and a negative number raised to an odd power is always negative.
- Anything [excluding 0 as 0^anything = 0) raised to the 0th power equals 1: 5^0 = 1 or 157^0 = 1 (this always applies).
- Anything raised to a negative power takes the value of the expression and puts it in the denominator while the numerator equals 1: 4^-3 = 1/64 or 5^-2 = 1/25 (this always applies).
- Important Note: (−3)^2 = 9 [(−3)(−3)] however, −3^2 = −9 [if there are no parentheses, only the 3 is raised to the exponent, not the negative value which would then be multiplied in afterwards]

Roots

- Square Root: a number which produces a specified quantity when multiplied by itself (values inside the root can't be negative).
- Any square root raised to the 2nd power gets rid of the square root: (√5)^2 = 5
- Anything value within the square root raised to the 2nd power comes out of the square root: √5^2 = 5
- A square root multiplied by a square root combines the terms inside of the roots all under one root: (√5)(√3) = √15
- A square root divided by a square root combines the expression all under one root: (√5)/(√3) = √5/3
- These same rules apply to every situation and only change if the root changes. For instance, if the root is a cube root (3), then the expressions must be cubed to get rid of the root, or the values inside the parentheses must be cubed.
- For odd order roots, there is exactly one root for every number n, even when n is negative. For even order roots, there are exactly two roots for every positive number n and no roots for any negative number n: 8 has exactly one cube root, 3√8 = 2, but 8 has two fourth roots, 4√8.

Decimals

- Every fraction with integers in the numerator and denominator is equivalent to a decimal that either terminates or repeats. That is, every rational number can be expressed as a terminating or repeating decimal. The converse is also true; that is, every terminating or repeating decimal represents a rational number.
- Irrational Numbers: never end as they go on forever (does not include repeating decimals): pi.
- Terminating decimals: decimals that end: 10/2 = 2.0.
- Repeating decimals: represented by a line above the sequence that repeats.

Real Numbers
- Consists of all rational numbers and all irrational numbers. The real numbers include all integers, fractions, and decimals.
- Interval: a set of all real numbers that are between given points. These can be respected as less than, greater than, less than or equal to, or greater than or equal to.

Absolute Value
- The distance between a number (x) and 0 which is always positive: | -5 | = 5 (5 away from 0)

Properties of Real Numbers (let x, s, and t equal real numbers)
- (x) + (s) = (s) + (x) and (xs) = (sx)
- (x + s) + t = x + (s + t) and t(xs) = x(st)
- x(s + t) = xs + xt
- x + 0 = x, (x)(0) = 0, (x)(1) = x
- If (xs) = 0, then x, s, or both values equal 0.
- Division by 0 is undefined.
- If two values are positive, then the sum and product of the two values are positive.
- If two values are negative, then the sum is negative and the product is positive.
- If a value is positive and the other is negative, the product is negative.
- Triangle inequality states that the sum of the lengths of any two sides must be greater than or equal to the length of the remaining side.
- |r||s| = |rs|
- r > 1, then r^2 > r. If 0 < s < 1, then s^2 < s.

Ratios
- The ratio of one quantity to another is a way to express their relative sizes, often in the form of a fraction, where the first quantity is the numerator and the second quantity is the denominator.
- Proportion: an equation relating two ratios; for example, 9/12 = ¾. To solve a problem involving ratios, you can often write a proportion and solve it by cross multiplication.

Percents

- The term percent means per hundred, or hundredths. Percents are ratios that are often used to represent parts of a whole, where the whole is considered as having 100 parts (out of 100, so 5% = 5/100).
- To compute a percent, given the part and the whole, first divide the part by the whole to get the decimal equivalent, then multiply the result by 100. The percent is that number followed by the word “percent” or the % symbol: 13/20 = (0.65)(100) = 65%
- To find the part that is a certain percent of a whole, you can either multiply the whole by the decimal equivalent of the percent or set up a proportion to find the part: (30% of 350 would be 0.3 x 350).
- Given the percent and the part, you can calculate the whole. To do this, either you can use the decimal equivalent of the percent or you can set up a proportion and solve it: (15 is 60% of what number? 0.6z = 15, solve for z).
- Percentages above 100 could be represented as values above 1 (250% = 2.50, etc...)

Percent Increase, Percent Decrease, and Percent Change

- Percent increase and decrease is found by dividing the amount of change by the original amount: 750-600 / 600 = 25%.

[**pages 29-36 provides ETS questions and answers]**

Part 2: Algebra

Algebraic Expressions

- Like Terms: contain the same variables and corresponding variables have the same exponents.
- Constant: term with no variable.
- Coefficient: term with a variable.
- Polynomial: is the sum of a finite number of terms in which each term is either a constant term or a product of a coefficient and one or more variables with positive integer exponents.
- Degree of each term is the sum of the exponents of the variables in the term (variable w/o an exponent has a degree of 1)
- The overall degree of the polynomial is the greatest degree of its terms.
- ca + cb = c(a + b) and ca − cb = c(a − b)
- (a+b)^2 =a^2 +2ab+b^2 and (a−b)^2 =a^2 −2ab+b^2
- a^2 - b^2 = (a+b)(a-b)
- (a+b)^3=a^3+ 3a^2b +3ab^2+b^3
- (a-b)^3=a^3-3a^2b+3ab^2+ b^3

Rules of Exponents

- x^-a = 1/x^a
- (x^a)(x^b) = x^a+b
- x^a /x^b = x^a-b
- x^0 = 1
- Remember: when solving problems, always makes the bases (x) like, so you can then solve.

Solving Linear Equations

- When solving equations, anything done to one side must be done to the other (multiplication, division, simplifying by substituting terms, etc…) which preserves the equation, yet makes it solvable.
- Linear Equations only involve a variable raised to the 1st degree.

Linear Equations in One Variable

- To solve a linear equation in one variable, find successively simpler equivalent equations by combining like terms and applying the rules for producing simpler equivalent equations until the solution is obvious.
- You can always check your solution by plugging in your answer for the variable.

Linear Equations in Two Variables

- Equations with 2 variables have answers in the form of ordered pairs (x,y).
- Substitution: isolate one of the variables in the equations, then plug it into the other to solve.
- Elimination: find the LCM of the first terms to make them equivalent and then subtract, thus cancelling out the first variable and solving for the other.

Solving Quadratics

- Equations that have an exponent to the second degree and can have either 2, 1, or 0 solutions.
- Solve methods include quadratic formula and factoring.

Solving Linear Inequalities

- Solving these types of problems means to find the set of all values of the variable that make the inequality true. This set of values is also known as the solution set of an inequality. Two inequalities that have the same solution set are called equivalent inequalities.
- When solving equations, anything done to one side must be done to the other (multiplication, division, simplifying by substituting terms, etc…) which preserves the equation, yet makes it solvable; HOWEVER, when you divide by a negative, the direction of the inequality is reversed.

Functions

- f (x) is called the value of f at x and is obtained by substituting the value of x in the expression above. It is helpful to think of an analogy similar to that of a machine, where x is in input and f(x) or y, is the output: if x = 1 is substituted in the expression above, the result is f (1) = 3(1) 5+ 8.
- Domain: the set of all permissible inputs, that is, all permissible values of the variable x. The domain is the set of all real numbers. Sometimes the domain of the function is given explicitly and is restricted to a specific set of values of x, yet when the domain is not stated, we know that 1.) we can never have 0 in the denominator 2.) a negative value can't can't in a square root 3.) a square root value will always be positive.

Applications

- Translating verbal descriptions into algebraic expressions is an essential initial step in solving word problems (improved through practice).
- Important examples are below...

Solution: Let x be the time, in minutes, that it took Dennis to drive the course. The distance d, in miles, is equal to the product of the rate r, in miles per hour, and the time t, in hours; that is,

$$d = rt$$

Note that since the rates are given in miles per *hour*, it is necessary to express the times in hours; for example, 40 minutes equals $\frac{40}{60}$ of an hour. Thus, the distance traveled by Jeff is the product of his speed and his time, $(51)\left(\frac{40}{60}\right)$ miles, and the distance traveled by Dennis is similarly represented by $(54)\left(\frac{x}{60}\right)$ miles.

Since the distances are equal, it follows that $(51)\left(\frac{40}{60}\right) = (54)\left(\frac{x}{60}\right)$.

From this equation it follows that $(51)(40) = 54x$ and $x = \frac{(51)(40)}{54} \approx 37.8$.

Thus, it took Dennis approximately 37.8 minutes to drive the course.

Example 2.7.7: A batch of computer parts consists of n identical parts, where n is a multiple of 60. Working alone at its constant rate, machine A takes 3 hours to produce a batch of computer parts. Working alone at its constant rate, machine B takes 2 hours to produce a batch of computer parts. How long will it take the two machines, working simultaneously at their respective constant rates, to produce a batch of computer parts?

Solution: Since machine A takes 3 hours to produce a batch, machine A can produce $\frac{1}{3}$ of the batch in 1 hour. Similarly, machine B can produce $\frac{1}{2}$ of the batch in 1 hour. If we let x represent the number of hours it takes both machines, working simultaneously, to produce the batch, then the two machines will produce $\frac{1}{x}$ of the batch in 1 hour. When

GRE Math Review 56

the two machines work together, adding their individual production rates, $\frac{1}{3}$ and $\frac{1}{2}$, gives their combined production rate $\frac{1}{x}$. Therefore, it follows that $\frac{1}{3} + \frac{1}{2} = \frac{1}{x}$.

This equation is equivalent to $\frac{2}{6} + \frac{3}{6} = \frac{1}{x}$. So $\frac{5}{6} = \frac{1}{x}$ and $\frac{6}{5} = x$.

Thus, working together, the machines will take $\frac{6}{5}$ hours, or 1 hour 12 minutes, to produce a batch of computer parts.

Example 2.7.8: At a fruit stand, apples can be purchased for $0.15 each and pears for $0.20 each. At these rates, a bag of apples and pears was purchased for $3.80. If the bag contained 21 pieces of fruit, how many of the pieces were pears?

Solution: If a represents the number of apples purchased and p represents the number of pears purchased, then the total cost of the fruit can be represented by the equation $0.15a + 0.20p = 3.80$, and the total number of pieces of fruit can be represented by the equation $a + p = 21$. Thus to answer the question, you need to solve the following system of equations.

$$\begin{aligned} 0.15a + 0.20p &= 3.80 \\ a + p &= 21 \end{aligned}$$

From the equation for the total number of fruit, $a = 21 - p$.

Substituting $21 - p$ for a in the equation for the total cost gives the equation

$$0.15(21 - p) + 0.20p = 3.80$$

So, $(0.15)(21) - 0.15p + 0.20p = 3.80$, which is equivalent to

$$3.15 - 0.15p + 0.20p = 3.80$$

Therefore $0.05p = 0.65$, and $p = 13$.

GRE Math Review 57

Thus, of the 21 pieces of fruit, 13 were pears.

Example 2.7.9: To produce a particular radio model, it costs a manufacturer $30 per radio, and it is assumed that if 500 radios are produced, all of them will be sold. What must be the selling price per radio to ensure that the profit (revenue from the sales minus the total production cost) on the 500 radios is greater than $8,200 ?

Solution: If the selling price per radio is y dollars, then the profit is $500(y - 30)$ dollars.

Therefore, $500(y - 30) > 8,200$.

Simplifying further gives $500y - 15,000 > 8,200$, which simplifies to $500y > 23,200$ and then to $y > 46.4$.

Thus, the selling price must be greater than $46.40 to ensure that the profit is greater than $8,200.

Interest

- Simple interest is based only on the initial deposit, which serves as the amount on which interest is computed, called the principal, for the entire time period. If the amount P is invested at a simple annual interest rate of r percent, then the value V of the investment at the end of t years is given by the formula: V = P(1+rt/100)
- Compound interest: interest is added to the principal at regular time intervals, such as annually, quarterly, and monthly. Each time interest is added to the principal, the interest is said to be compounded. After each compounding, interest is earned on the new principal, which is the sum of the preceding principal and the interest just added. If the amount P is invested at an annual interest rate of r percent, compounded annually, then

the value V of the investment at the end of t years is given by the formula: V = P(1 + r/100)^t

- If the amount P is invested at an annual interest rate of r percent, compounded n times per year, then the value V of the investment at the end of t years is given by the formula: V = P(1+r/100n)^nt

Coordinate Geometry

- Two real number lines that are perpendicular to each other and that intersect at their respective zero points define a rectangular coordinate system, often called the xy-coordinate system or xy-plane. The horizontal number line is called the x-axis and the vertical number line is called the y-axis. The point where the two axes intersect is called the origin, denoted by O. The positive half of the x-axis is to the right of the origin, and the positive half of the y-axis is above the origin. The two axes divide the plane into four regions called quadrants. The four quadrants are labeled I, II, III, and IV.
- Each point on the coordinate geometric plane represents an ordered pair which can be reflected, translated, etc…

Distance Between Points

- The distance between two points in the xy-plane can be found by using the Pythagorean theorem as you use the coordinate points to see how long 2 of the sides are as you can then solve for the 3rd side.

Graphing Linear Expressions/Inequalities

- y=mx+b (m= slope, y2-y1/x2-x1 which is also known as rise/run) (b=y-intercept)
- x intercept can be found by plugging 0 in for y
- y intercept can be found by plugging 0 in for x
- If slope is equal, then the lines are parallel
- If slope is the reciprocal (flipped) and negative, then the lines are perpendicular to each other

Graphing Quadratics

- y=ax^2+bx+c
- If a is negative, parabola opens downward (vertex is the highest point).
- If a is positive, parabola opens upwards (vertex is lowest point).
- Line of Symmetry: the area between the x-intercepts
- To find vertex, use (-b/2a) which finds x, then plug that value into equation to find y: this yields the x,y coordinate of the vertex.

Graphing Circles

- The graph of an equation of the form (x − a)^2 + (y − b)^2 = r^2 is a circle with its center at the point (a, b) and with radius r > 0.

Graphs of Functions

- The coordinate plane can be used for graphing functions. To graph a function in the xy-plane, you represent each input x and its corresponding output f (x) as a point (x, y), where y = f (x). In other words, you use the x-axis for the input and the y-axis for the output.

- You can plus variables from other lines into the equation of another line to find the points of intersection.
- The graph of h(x) + c is the graph of h(x) shifted upward by c units.
- The graph of h(x) − c is the graph of h(x) shifted downward by c units.
- The graph of h(x + c) is the graph of h(x) shifted to the left by c units.
- The graph of h(x − c) is the graph of h(x) shifted to the right by c units.
- The graph of ch(x) is the graph of h(x) stretched vertically by a factor of c if
- c > 1.
- The graph of ch(x) is the graph of h(x) shrunk vertically by a factor of c if
- 0 < c < 1.

[pages 81-92 provide ETS questions and answers]

Part 3: Geometry

Lines and Angles

- When 2 lines intersect at a point they form 4 angles. Angles across from each other are called opposite angles or vertical angles and they have equal measures. Also, the sum of all 4 angles is 360 degrees.
- Two lines that intersect to form four congruent angles are called perpendicular lines. Each of the four angles has a measure of 90. An angle with a measure of 90 is called a right angle.
- Obtuse angle > 90 degrees
- Acute angle < 90 degrees
- Two lines in the same plane that won't ever intersect are called parallel lines. Any 2 sides add up to 180 because of their parallel nature.

Polygons

- A closed figure formed by three or more line segments all of which are in the same plane. The line segments are called the <u>sides</u> of the polygon. Each side is joined to two other sides at its endpoints, and the endpoints are called <u>vertices</u>.
- If a polygon has n sides, it can be divided into n-2 triangles. Since the sum of the measures of the interior angles of a triangle is 180, it follows that the sum of the measures of the interior angles of an n-sided polygon is (n-2)(180). For example, since a quadrilateral has 4 sides, the sum of the measures of the interior angles of a quadrilateral is 4-2(180)=360 , and since a hexagon has 6 sides, the sum of the measures of the interior angles of a hexagon is 6-2(180)=720.
- You can find the degree of each angle in a regular polygon with congruent sides by taking the total degrees (which can be found as shown above) and dividing it by the amount of sides or in other words n-2(180) / n.
- Perimeter = sum of side lengths.
- Area = area enclosed by shape.

Triangles

- Every triangle has three sides and three interior angles. The measures of the interior angles add up to 180°. The length of each side must be less than the sum of the lengths of the other two sides. For example, the sides of a triangle could not have the lengths 4, 7, and 12 because 12 is greater than 4 + 7.
- <u>Equilateral Triangle</u>: three congruent sides with equal measures of the three interior angles: (60)(60)(60).
- <u>Isosceles Triangle</u>: at least two congruent sides as the angles opposite of the two congruent sides are congruent: sides are (x)(x)(180-2x)
- <u>Right Triangle</u>: has interior right angle and the side opposite the right angle is called the hypotenuse; the other two sides are called legs: (90)(x)(y)

Pythagorean Theorem

- In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs (a^2+b^2=c^2)
- 30, 60, 90 (angles) =1, √3, 1 (sides opposite of angles)
- 45, 45, 90 (angles) = 1, 1, √2 (sides opposite of angles)

Area of Triangle

- A = (½)(base)(height)

Congruent/Similar Triangles

- Two triangles that have the same shape and size are called congruent triangles. More precisely, two triangles are congruent if their vertices can be matched up so that the corresponding angles and the corresponding sides are congruent.
- Can determine if 2 triangle are congruent by 3 steps… 1.) If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent. This proposition is called Side- Side-Side, or SSS, congruence. 2.) If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. This proposition is called Side-Angle-Side, or SAS, congruence. 3.) If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. This proposition is called Angle-Side-Angle, or ASA, congruence. Note that if two angles of one triangle are congruent to two angles of another triangle, then the remaining angles are also congruent to each other, since the sum of the angle measures in any triangle is 180 degrees. Therefore, a similar proposition, called Angle-Angle-Side, or AAS, congruence, follows from ASA congruence.
- Similar Triangles have the same shape but are different sizes: two triangles are similar if their vertices can be matched up so that the corresponding angles are congruent or, equivalently, the lengths of the corresponding sides have the same ratio, called the scale factor of similarity (same angles but different side lengths, vise versa, or side lengths set to a different ratio such as 3:4:5 to 6:8:10).

Quadrilaterals

- Every quadrilateral has four sides and four interior angles. The measures of the interior angles add up to 360°.
- Rectangle: four right angles and opposite sides of a rectangle are parallel and congruent, and the two diagonals are also congruent.
- Square: a rectangle with 4 congruent (equal) sides.
- Parallelogram: quadrilateral in which both pairs of opposite sides are parallel as the opposite sides are congruent and opposite angles are congruent (all rectangles are parallelograms).
- Trapezoid: quadrilateral with only one pair of parallel sides.

Areas of Special Types of Quadrilaterals

- A = (base)(height)
- Trapezoid: A = (½)(base1+base2)(h)

Circles

- Radius: length from center to edge
- Diameter: length across entirely of circle (double the radius)
- Congruent Circles: 2 circles with equal radii
- Circumference: distance around the circle (C=2pi(r))
- Arc: the part of the circle containing the two points and all the points between them. Two points on a circle are always the endpoints of two arcs. An arc is frequently identified by three points to avoid ambiguity (2 on edge, 1 in center).
- A central angle of a circle is an angle with its vertex at the center of the circle. The measure of an arc is the measure of its central angle, which is the angle formed by two radii that connect the center of the circle to the two endpoints of the arc. An entire circle is considered to be an arc with measure 360: SO…. arc length = (central angle)(circumference)/(360)
- Area of circle = pi(r)^2
- Sector of a circle is a region bounded by an arc of the circle and two radii. To find the area of a sector, note that the ratio of the area of a sector of a circle to the area of the entire circle is equal to the ratio of the degree measure of its arc to 360: follows the arc length equation.
- A tangent to a circle is a line that lies in the same plane as the circle and intersects the circle at exactly one point, called the point of tangency. If a line is tangent to a circle, then a radius drawn to the point of tangency is perpendicular to the tangent line. The converse is also true; that is, if a radius and a line intersect at a point on the circle and the line is perpendicular to the radius, then the line is a tangent to the circle at the point of intersection.
- A polygon is inscribed in a circle if all its vertices lie on the circle, or equivalently, the circle is circumscribed about the polygon.
- If one of the lines in the inscribed shape goes through the center, that line is the diameter and if one of the sides of the inscribed shape is the diameter, then it makes a right triangle.
- Two or more circles with the same center are called concentric circles.
- (Review inscribed shapes, angles, tangency).

3D-Figures

- Volume of rectangular solid: V = (length)(width)(height)
- Surface area of rectangular solid: A = 2(lw+lh+wh)
- Volume of right circular cylinder: $V = \pi(r^2)h$
- Surface area of right circular cylinder: $A = 2(\pi r^2) + 2\pi rh$

[pages 116-125 provide ETS questions and answers]

Part 4: Data Analysis

Methods for Presenting Data

- Data can be quantitative (numerical) or categorical (non numerical).

- The frequency, or count, of a particular category or numerical value is the number of times that the category or numerical value appears in the data. A frequency distribution is a table or graph that presents the categories or numerical values along with their corresponding frequencies. The relative frequency of a category or a numerical value is the corresponding frequency divided by the total number of data. Relative frequencies may be expressed in terms of percents, fractions, or decimals. A relative frequency distribution is a table or graph that presents the relative frequencies of the categories or numerical values.

Tables

- Tables are used to present a wide variety of data, including frequency distributions and relative frequency distributions. The rows and columns provide clear associations between categories and data. A frequency distribution is often presented as a 2-column table in which the categories or numerical values of the data are listed in the first column and the corresponding frequencies are listed in the second column. A relative frequency distribution table has the same layout but with relative frequencies instead of frequencies. When data include a large number of categories or numerical values, the categories or values are often grouped together in a smaller number of groups and the corresponding frequencies are given.

Bar Graphs

- A frequency distribution or relative frequency distribution of data collected from a population by observing one or more variables can be presented using a bar graph, or bar chart. In a bar graph, each of the data categories or numerical values is represented by a rectangular bar, and the height of each bar is proportional to the corresponding frequency or relative frequency. All of the bars are drawn with the same width, and the bars can be presented either vertically or horizontally. When data include a large number of different categories of numerical values, the categories or values are often grouped together in several groups and the corresponding frequencies or relative frequencies are given. Bar graphs enable comparisons across several categories more easily than tables do. For example, in a bar graph it is easy to identify the category with the greatest frequency by looking for the bar with the greatest height.

Segmented Bar Graphs

- A segmented bar graph, or stacked bar graph, is similar to a regular bar graph except that in a segmented bar graph, each rectangular bar is divided, or segmented, into smaller rectangles that show how the variable is "separated" into other related variables. For example, rectangular bars representing enrollment can be divided into two smaller rectangles, one representing full-time enrollment and the other representing part-time enrollment, as shown in the following example.

Histograms

- When a list of data is large and contains many different values of a numerical variable, it is useful to organize the data by grouping the values into intervals, often called classes.

To do this, divide the entire interval of values into smaller intervals of equal length and then count the values that fall into each interval. In this way, each interval has a frequency and a relative frequency. The intervals and their frequencies (or relative frequencies) are often displayed in a histogram. Histograms are graphs of frequency distributions that are similar to bar graphs, but they must have a number line for the horizontal axis, which represents the numerical variable. Also, in a histogram, there are no regular spaces between the bars. Any spaces between bars in a histogram indicate that there are no data in the intervals represented by the spaces.

- Histograms are useful for identifying the general shape of a distribution of data. Also evident are the “center” and degree of “spread” of the distribution, as well as high-frequency and low-frequency intervals.

Circle Graphs

- In other words, pie charts are used to represent data that have been separated into a small number of categories. They illustrate how a whole is separated into parts. The data are presented in a circle such that the area of the circle representing each category is proportional to the part of the whole that the category represents.

Scatterplots

- A type of graph that is useful for showing the relationship between two numerical variables whose values can be observed in a single population of individuals or objects. In a scatterplot, the values of one variable appear on the horizontal axis of a rectangular coordinate system and the values of the other variable appear on the vertical axis. For each individual or object in the data, an ordered pair of numbers is collected, one number for each variable, and the pair is represented by a point in the coordinate system.
- Scatterplots makes it possible to observe an overall pattern, or trend, in the relationship between the two variables. Also, the strength of the trend as well as striking deviations from the trend are evident. In many cases, a line or a curve that best represents the trend is also displayed in the graph and is used to make predictions about the population.

Line Graphs

- Useful for showing the relationship between two numerical variables, especially if one of the variables is time. A line graph uses a coordinate plane, where each data point represents a pair of values observed for the two numerical variables.
- When one of the variables is time, it is associated with the horizontal axis, which is labeled with regular time intervals. The data points may represent an interval of time, such as an entire day or year, or just an instant of time. Such a line graph is often called a time series.

Numerical Methods for Describing Data

- Data can be described numerically by various statistics, or statistical measures. These statistical measures are often grouped in three categories: measures of central tendency, measures of position, and measures of dispersion.

Measures of Central Tendency

- Measures of central tendency indicate the “center” of the data along the number line and are usually reported as values that represent the data. There are three common measures of central tendency: mean (average), median (middle number), and mode (number that appears most frequently).
- Calculate the mean: take the sum of the n numbers and divide it by n.
- The mean can be affected by just a few values that lie far above or below the rest of the data, because these values contribute directly to the sum of the data and therefore to the mean. By contrast, the median is a measure of central tendency that is fairly unaffected by unusually high or low values relative to the rest of the data.
- Calculate the median: first order the numbers from least to greatest. If n is odd, then the median is the middle number in the ordered list of numbers. If n is even, then there are two middle numbers, and the median is the average of these two numbers.
- The median, as the “middle value” of an ordered list of numbers, divides the list into roughly two equal parts. However, if the median is equal to one of the data values and it is repeated in the list, then the numbers of data above and below the median may be rather different. For example, the median of the 16 numbers 2, 4, 4, 5, 7, 7, 7, 7, 7, 7, 8, 8, 9, 9, 9, 9 is 7, but four of the data are less than 7 and six of the data are greater than 7.
- Calculate the mode: just observe which number appears most often.

Measures of Position

- The three most basic positions, or locations, in a list of numerical data ordered from least to greatest are the beginning, the end, and the middle. It is useful here to label these as L for the least, G for the greatest, and M for the median. Aside from these, the most common measures of position are quartiles and percentiles. Like the median M, quartiles and percentiles are numbers that divide the data into roughly equal groups after the data have been ordered from the least value L to the greatest value G. There are three quartile numbers, called the first quartile, the second quartile, and the third quartile, that divide the data into four roughly equal groups; and there are 99 percentile numbers that divide the data into 100 roughly equal groups. As with the mean and median, the quartiles and percentiles may or may not themselves be values in the data.
- Data and Stat review on quartiles, deciles, relative frequency, etc: https://youtu.be/40o82o3uNfk

In the following discussion of quartiles, the symbol Q_1 will be used to denote the first quartile, Q_2 will be used to denote the second quartile, and Q_3 will be used to denote the third quartile.

The numbers Q_1, Q_2, and Q_3 divide the data into 4 roughly equal groups as follows. After the data are listed in increasing order, the first group consists of the data from L to Q_1, the second group is from Q_1 to Q_2, the third group is from Q_2 to Q_3, and the fourth group is from Q_3 to G. Because the number of data may not be divisible by 4, there are various rules to determine the exact values of Q_1 and Q_3, and some statisticians use different rules, but in all cases Q_2 is equal to the median M. We use perhaps the most common rule for determining the values of Q_1 and Q_3. According to this rule, after the data are listed in increasing order, Q_1 is the median of the first half of the data in the ordered list and Q_3 is the median of the second half of the data in the ordered list, as illustrated in the following example.

Example 4.2.5: To find the quartiles for the list of 16 numbers 2, 4, 4, 5, 7, 7, 7, 7, 7, 7, 8, 8, 9, 9, 9, 9 (which are already listed in increasing order), first divide the numbers in

the list into two groups of 8 numbers each. The first group of 8 numbers is 2, 4, 4, 5, 7, 7, 7, 7, and the second group of 8 numbers is 7, 7, 8, 8, 9, 9, 9, 9, so that the second quartile, or median, is 7. To find the other quartiles, you can take each of the two smaller groups and find its median: the first quartile, Q_1, is 6 (the average of 5 and 7) and the third quartile, Q_3, is 8.5 (the average of 8 and 9).

In this example, note that the number 4 is in the lowest 25 percent of the distribution of data. There are different ways to describe this. We can say that 4 is below the first quartile, that is, below Q_1. We can also say that 4 is *in* the first quartile. The phrase "in a quartile" refers to being in one of the four groups determined by Q_1, Q_2, and Q_3.

Percentiles are mostly used for very large lists of numerical data ordered from least to greatest. Instead of dividing the data into four groups, the 99 percentiles $P_1, P_2, P_3, \ldots, P_{99}$ divide the data into 100 groups. Consequently, $Q_1 = P_{25}$, $M = Q_2 = P_{50}$, and $Q_3 = P_{75}$. Because the number of data in a list may not be divisible by 100, statisticians apply various rules to determine values of percentiles.

Measures of Dispersion

- Indicates the degree of spread of the data. The most common statistics used as measures of dispersion are the range, the interquartile range, and the standard deviation. These statistics measure the spread of the data in different ways.
- Range: (greatest number in the set) - (smallest number in the set) = range.
- Outliers: directly affect the range are abnormally large or small numbers.
- Interquartile Ranges are not affected by the range because it measures the difference between the third quartile and the first quartile; in other words, it measures the middle half of the data.
- Measures of dispersion: https://youtu.be/goXdWMZxlqM

Standard Deviation

- A measure of spread that depends on each number in the list. Using the mean as the center of the data, the standard deviation takes into account how much each value differs from the mean and then takes a type of average of these differences. As a result, the more the data are spread away from the mean, the greater the standard deviation; and the more the data are clustered around the mean, the lesser the standard deviation.
- The standard deviation of a group of numerical data is computed by...
- 1.) calculating the mean of the values.
- 2.) finding the difference between the mean and each of the values.
- 3.) squaring each of the differences.
- 4.) finding the average of the squared differences.
- 5.) taking the nonnegative square root of the average of the squared differences.
- Statistics, SD, Quartiles, etc: https://youtu.be/rNzkaoiyenU

Example 4.2.8: For the five data 0, 7, 8, 10, and 10, the standard deviation can be computed as follows. First, the mean of the data is 7, and the squared differences from the mean are

$$(7-0)^2,\ (7-7)^2,\ (7-8)^2,\ (7-10)^2,\ (7-10)^2$$

or 49, 0, 1, 9, 9. The average of the five squared differences is $\frac{68}{5}$, or 13.6, and the positive square root of 13.6 is approximately 3.7.

- Sample standard deviation is computed by dividing the sum of the squared differences by n-1 instead of n. The sample standard deviation is only slightly different from the standard deviation but is preferred for technical reasons for a sample of data that is taken from a larger population of data. Sometimes the standard deviation is called the population standard deviation to help distinguish it from the sample standard deviation.
- The process of subtracting the mean from each value and then dividing the result by the standard deviation is called standardization. Standardization is a useful tool because for each data value, it provides a measure of position relative to the rest of the data

independently of the variable for which the data was collected and the units of the variable.

- Calculate standard deviation and sample standard deviation: https://youtu.be/3v6mYNPyDoY

Counting Methods

- When a set contains a small number of objects, it is easy to list the objects and count them one by one. When the set is too large to count that way, and when the objects are related in a patterned or systematic way, there are some useful techniques for counting the objects without actually listing them.
- The term set has been used informally in this review to mean a collection of objects that have some property, whether it is the collection of all positive integers, all points in a circular region, or all students in a school that have studied French. The objects of a set are called members or elements. Some sets are finite, which means that their members can be completely counted. Finite sets can, in principle, have all of their members listed, using curly brackets, such as the set of even digits {0, 2, 4, 6, 8.} Sets that are not finite
- are called infinite sets, such as the set of all integers. A set that has no members is called the empty set. A set with one or more members is called nonempty. If A and B are sets and all of the members of A are also members of B, then A is a subset of B.

Sets can be formed from other sets. If S and T are sets, then the **intersection** of S and T is the set of all elements that are in both S and T and is denoted by $S \cap T$. The **union** of S and T is the set of all elements that are in either S or T or both and is denoted by $S \cup T$. If sets S and T have no elements in common, they are called **disjoint** or **mutually exclusive**.

A useful way to represent two or three sets and their possible intersections and unions is a **Venn diagram**. In a Venn diagram, sets are represented by circular regions that overlap if they have elements in common but do not overlap if they are disjoint. Sometimes the circular regions are drawn inside a rectangular region, which represents a **universal set**, of which all other sets involved are subsets.

- Inclusion-exclusion principle relates the numbers of elements in the union and intersection of two finite sets. The number of elements in the union of two sets equals the sum of their individual numbers of elements minus the number of elements in their intersection.

Multiplication Principle

- Suppose there are two choices to be made sequentially and that the second choice is independent of the first choice. Suppose also that there are k different possibilities for the first choice and m different possibilities for the second choice. The multiplication principle states that under those conditions, there are km different possibilities for the pair of choices: suppose that a meal is to be ordered from a restaurant menu and that the meal consists of one entrée and one dessert. If there are 5 entrées and 3 desserts on the menu, then there are (5)(3) 15 different meals that can be ordered from the menu.
- The multiplication principle applies in more complicated situations as well. If there are more than two independent choices to be made, then the number of different possible

outcomes of all of the choices is the product of the numbers of possibilities for each choice.

- Counting methods and multiplication principle: https://youtu.be/sEul6TMYDY0

Example 4.3.2: Suppose that a computer password consists of four characters such that the first character is one of the 10 digits from 0 to 9 and each of the next 3 characters is any one of the uppercase letters from the 26 letters of the English alphabet. How many different passwords are possible?

Solution: The description of the password allows repetitions of letters. Thus, there are 10 possible choices for the first character in the password and 26 possible choices for each of the next 3 characters in the password. Therefore, applying the multiplication principle, the number of possible passwords is $(10)(26)(26)(26) = 175,760$.

Note that if repetitions of letters are not allowed in the password, then the choices are not all independent, but a modification of the multiplication principle can still be applied. There are 10 possible choices for the first character in the password, 26 possible choices for the second character, 25 for the third character because the first letter cannot be repeated, and 24 for the fourth character because the first two letters cannot be repeated. Therefore, the number of possible passwords is $(10)(26)(25)(24) = 156,000$.

Example 4.3.3: Each time a coin is tossed, there are 2 possible outcomes—either it lands heads up or it lands tails up. Using this fact and the multiplication principle, you can conclude that if a coin is tossed 8 times, there are $(2)(2)(2)(2)(2)(2)(2)(2) = 2^8 = 256$ possible outcomes.

Permutations and Factorials

- To order the 4 letters, one of the 4 letters must be placed first, one of the remaining 3 letters must be placed second, one of the remaining 2 letters must be placed third, and the last remaining letter must be placed fourth. Therefore, applying the multiplication principle, there are (4)(3)(2)(1), or 24, ways to order the 4 letters.
- Because products of the form n(n-1)(n-2) = (3)(2)(1) occur frequently when counting objects, a special symbol n!, called n-factorial, is used to denote this product.
- Special case: (0! = 1)
- Combinations and Permutations: https://youtu.be/itKHz9iyz8g

Example 4.3.4: Suppose that 10 students are going on a bus trip, and each of the students will be assigned to one of the 10 available seats. Then the number of possible different seating arrangements of the students on the bus is

$$10! = (10)(9)(8)(7)(6)(5)(4)(3)(2)(1) = 3,628,800$$

Now suppose you want to determine the number of ways in which you can select 3 of the 5 letters A, B, C, D, and E and place them in order from 1st to 3rd. Reasoning as in the preceding examples, you find that there are $(5)(4)(3)$, or 60, ways to select and order them.

More generally, suppose that k objects will be selected from a set of n objects, where $k \leq n$, and the k objects will be placed in order from 1st to kth. Then there are n choices for the first object, $n-1$ choices for the second object, $n-2$ choices for the third object, and so on, until there are $n-k+1$ choices for the kth object. Thus, applying the multiplication principle, the number of ways to select and order k objects from a set of n objects is $n(n-1)(n-2)...(n-k+1)$. It is useful to note that

$$n(n-1)(n-2)\cdots(n-k+1) = n(n-1)(n-2)\cdots(n-k+1)\frac{(n-k)!}{(n-k)!} = \frac{n!}{(n-k)!}$$

This expression represents the number of **permutations of n objects taken k at a time**–that is, the number of ways to select and order k objects out of n objects. This number is commonly denoted by the notation ${}_nP_k$.

Example 4.3.5: How many different 5-digit positive integers can be formed using the digits 1, 2, 3, 4, 5, 6, and 7 if none of the digits can occur more than once in the integer?

Solution: This example asks how many ways there are to order 5 integers chosen from a set of 7 integers. According to the counting principle above, there are

$(7)(6)(5)(4)(3) = 2,520$ ways to do this. Note that this is equal to $\frac{7!}{(7-5)!}$ $= \frac{(7)(6)(5)(4)(3)(2!)}{2!} = (7)(6)(5)(4)(3)$.

Combinations

- (number of ways to select without order) = (number of ways to select with order) / (number of ways to order)
- More generally, suppose that k objects will be chosen from a set of n objects, where k is less than or equal to n, but that the k objects will not be put in order. The number of ways in which this can be done is called the number of combinations of n objects taken k at a time and is given by the formula (n!) / k!(n-k)!
- Videos summing up permutations and combinations: https://youtu.be/QCq6qRJCTj8 & https://youtu.be/jQ37bVhFmUI

Example 4.3.6: Suppose you want to select a 3-person committee from a group of 9 students. How many ways are there to do this?

Solution: Since the 3 students on the committee are not ordered, you can use the formula for the combination of 9 objects taken 3 at a time, or "9 choose 3":

$$\frac{9!}{3!(9-3)!} = \frac{9!}{3!\,6!} = \frac{(9)(8)(7)}{(3)(2)(1)} = 84$$

Probability

- The probability of an event is a number from 0 to 1, inclusive, that indicates the likelihood that the event occurs when the experiment is performed. The greater the number, the more likely the event.
- Event is certain to occur = 1
- Event is certain to not occur = 0
- Event is possible but not certain = $0 < X < 1$
- Probability that event won't occur = 1 - X (X is probability that it will occur)
- If X is an event, then the probability of X is the sum of the probabilities of the
- outcomes in X.
- The sum of the probabilities of all possible outcomes of an experiment is 1.
- Events that cannot occur at the same time are said to be <u>mutually exclusive</u>. For example, if a 6-sided die is rolled once, the event of rolling an odd number and the event of rolling an even number are mutually exclusive. But rolling a 4 and rolling an even number are not mutually exclusive, since 4 is an outcome that is common to both events.
- <u>Rule 1:</u> P(either E or F, or both, occur) = P(E) + P(F) - P(both E and F occur),
- which is the inclusion-exclusion principle applied to probability.
- <u>Rule 2:</u> If E and F are mutually exclusive, then P(both E and F occur) = 0, and
- therefore, P(either E or F, or both, occur) = P(E) + P(F).
- <u>Rule 3:</u> E and F are said to be independent if the occurrence of either event does not
- affect the occurrence of the other.
- <u>Rule 1</u>: P(E or F) = P(E) + P(F) - P(E and F)
- <u>Rule 2</u>: P(E or F) = P(E) + P(F) if E and F are mutually exclusive.
- <u>Rule 3</u>: P(E and F) = P(E)P(F) if E and F are independent.
- Videos summing up probability: https://youtu.be/VFeoF2it6uk & https://youtu.be/0i4CK2G4C-c & https://youtu.be/bqm9P1lk00w

Example 4.4.2: If a fair 6-sided die is rolled once, let E be the event of rolling a 3 and let F be the event of rolling an odd number. These events are *not* independent. This is because rolling a 3 makes certain that the event of rolling an odd number occurs. Note that $P(E \text{ and } F) \neq P(E)P(F)$, since

$$P(E \text{ and } F) = P(E) = \frac{1}{6} \text{ and } P(E)P(F) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) = \frac{1}{12}$$

Example 4.4.3: A 12-sided die, with faces numbered 1 to 12, is to be rolled once, and each of the 12 possible outcomes is equally likely to occur. The probability of rolling a 4 is $\frac{1}{12}$, so the probability of rolling a number that is *not* a 4 is $1 - \frac{1}{12} = \frac{11}{12}$.

The probability of rolling a number that is either a multiple of 5 (that is, rolling a 5 or a 10) or an odd number (that is, rolling a 1, 3, 5, 7, 9, or 11) is equal to

$$P(\text{multiple of } 5) + P(\text{odd}) - P(\text{multiple of 5 and odd}) = \frac{2}{12} + \frac{6}{12} - \frac{1}{12} = \frac{7}{12}$$

Another way to calculate this probability is to notice that rolling a number that is either a multiple of 5 or an odd number is the same as rolling one of the seven numbers 1, 3, 5, 7, 9, 10, and 11, which are equally likely outcomes. So by using the ratio formula to calculate the probability, the required probability is $\frac{7}{12}$.

Example 4.4.4: Consider an experiment with events A, B, and C for which $P(A) = 0.23$, $P(B) = 0.40$, and $P(C) = 0.85$.

Suppose that events A and B are mutually exclusive and events B and C are independent. What is $P(A \text{ or } B)$ and $P(B \text{ or } C)$?

Solution: Since A and B are mutually exclusive,

$$P(A \text{ or } B) = P(A) + P(B) = 0.23 + 0.40 = 0.63$$

Since B and C are independent, $P(B \text{ and } C) = P(B)P(C)$. So

$$P(B \text{ or } C) = P(B) + P(C) - P(B \text{ and C}) = P(B) + P(C) - P(B)P(C)$$

Therefore,

Therefore,

$$P(B \text{ or } C) = 0.40 + 0.85 - (0.40)(0.85) = 1.25 - 0.34 = 0.91$$

Example 4.4.5: Suppose that there is a 6-sided die that is weighted in such a way that each time the die is rolled, the probabilities of rolling any of the numbers from 1 to 5 are all equal, but the probability of rolling a 6 is twice the probability of rolling a 1. When

you roll the die once, the 6 outcomes are *not equally likely*. What are the probabilities of the 6 outcomes?

Solution: Let p equal the probability of rolling a 1. Then each of the probabilities of rolling a 2, 3, 4, or 5 is equal to p, and the probability of rolling a 6 is equal to $2p$. Therefore, since the sum of the probabilities of all possible outcomes is 1, it follows that

$$1 = P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = p + p + p + p + p + 2p = 7p$$

So the probability of rolling each of the numbers from 1 to 5 is $\frac{1}{7}$, and the probability of rolling a 6 is $\frac{2}{7}$.

Example 4.4.6: Suppose that you roll the weighted 6-sided die from Example 4.4.5 twice. What is the probability that the first roll will be an odd number and the second roll will be an even number?

Solution: To calculate the probability that the first roll will be odd and the second roll will be even, note that these two events are independent. To calculate the probability that both occur, you must multiply the probabilities of the two independent events. First compute the individual probabilities.

$$P(\text{odd}) = P(1) + P(3) + P(5) = \frac{3}{7}$$

$$P(\text{even}) = P(2) + P(4) + P(6) = \frac{4}{7}$$

Then, $P(\text{first roll is odd and second roll is even}) = P(\text{odd})P(\text{even}) = \left(\frac{3}{7}\right)\left(\frac{4}{7}\right) = \frac{12}{49}$.

Two events that happen sequentially are not always independent. The occurrence of the first event may affect the occurrence of the second event. In this case, the probability that *both* events happen is equal to the probability that the first event happens multiplied by the probability that, *given that the first event has already happened*, the second event will happen as well.

Example 4.4.7: A box contains 5 orange disks, 4 red disks, and 1 blue disk. You are to select two disks at random and without replacement from the box. What is the probability that the first disk you select will be red and the second disk you select will be orange?

Solution: To solve, you need to calculate the following two probabilities and then multiply them.

1. The probability that the first disk selected from the box will be red
2. The probability that the second disk selected from the box will be orange, given that the first disk selected from the box is red

The probability that the first disk you select will be red is $\frac{4}{10} = \frac{2}{5}$. If the first disk you select is red, there will be 5 orange disks, 3 red disks, and 1 blue disk left in the box, for a total of 9 disks. Therefore, the probability that the second disk you select will be orange, given that the first disk you selected is red, is $\frac{5}{9}$. Multiply the two probabilities to get $\left(\frac{2}{5}\right)\left(\frac{5}{9}\right) = \frac{2}{9}$.

Distributions of Data, Random Variables, and Probability Distributions

- The distribution of a statistical data set (or a population) is a listing or function showing all the possible values (or intervals) of the data and how often they occur. When a distribution of categorical data is organized, you see the number or percentage of individuals in each group. When a distribution of numerical data is organized, they're often ordered from smallest to largest, broken into reasonably sized groups (if appropriate), and then put into graphs and charts to examine the shape, center, and amount of variability in the data.
- Random Variables: use the frequency distributions to determine the likelihood that the random variable will or won't be selected (frequency)/(total) = (probability)
- For a random variable that represents a randomly chosen value from a distribution of data, the probability distribution of the random variable is the same as the relative frequency distribution of the data.

Normal Distribution

- Mean, median, and mode are all very similar
- Data is grouped fairly symmetrical about the mean
- ⅔ of the data is within 1 standard deviation of the mean (left and right).
- Almost all of the data is within 2 standard deviations from the mean.
- This represents the bell curve
- standard normal distribution is a normal distribution with a mean of 0 and standard deviation equal to 1. To transform a normal distribution with a mean of m and a standard deviation of d to a standard normal distribution, you standardize the values; that is, you subtract m from any observed value of the normal distribution and then divide the result by d.

- Normal distribution probability problems: https://youtu.be/gHBL5Zau3NE

4.6 Data Interpretation Examples

Example 4.6.1: This example is based on the following table.

DISTRIBUTION OF CUSTOMER COMPLAINTS RECEIVED
BY AIRLINE *P*, 2003 and 2004

Category	2003	2004
Flight problem	20.0%	22.1%
Baggage	18.3	21.8
Customer service	13.1	11.3
Reservation and ticketing	5.8	5.6
Credit	1.0	0.8
Special passenger accommodation	0.9	0.9
Other	40.9	37.5
Total	100.0%	100.0%
Total number of complaints	22,998	13,278

(a) Approximately how many complaints concerning credit were received by Airline *P* in 2003 ?

(b) By approximately what percent did the total number of complaints decrease from 2003 to 2004 ?

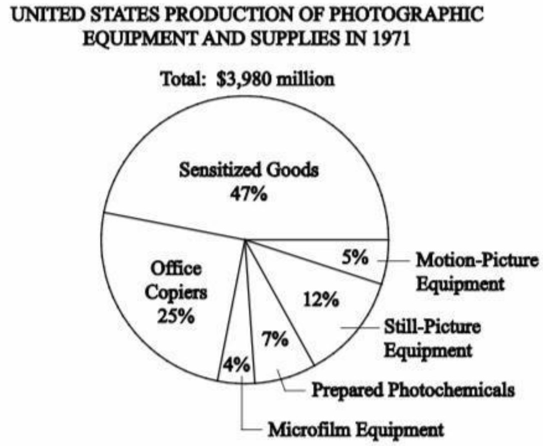
(c) Based on the information in the table, which of the following three statements are true?

Statement 1: In each of the years 2003 and 2004, complaints about flight problems, baggage, and customer service together accounted for more than 50 percent of all customer complaints received by Airline *P*.

Statement 2: The number of special passenger accommodation complaints was unchanged from 2003 to 2004.

Statement 3: From 2003 to 2004, the number of flight problem complaints increased by more than 2 percent.

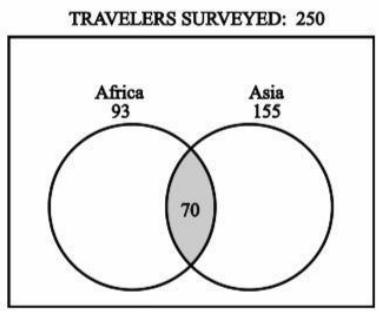
Example 4.6.2: This example is based on the following circle graph.



Data Analysis Figure 18

(a) Approximately what was the ratio of the value of sensitized goods to the value of still picture equipment produced in 1971 in the United States?

(b) If the value of office copiers produced in 1971 was 30 percent greater than the corresponding value in 1970, what was the value of office copiers produced in 1970 ?



Data Analysis Figure 19

(a) How many of the travelers surveyed have traveled to Africa but *not* to Asia?

(b) How many of the travelers surveyed have traveled to *at least one* of the two continents of Africa and Asia?

(c) How many of the travelers surveyed have traveled *neither* to Africa *nor* to Asia?

[pages 186-198 provides ETS questions and answers]