Experiment A11
Transient Signals
Procedure

Deliverables: Checked lab notebook, Brief technical memo
Recommended Reading: Chapters 20 - 23 of the textbook

Overview
In this lab, you will investigate the dynamic response of various systems. We will measure the transient behavior of a thermocouple and see that it exhibits a first-order transient response. We will also measure the vibrations in a baseball bat, which are an example of second-order transient behavior.

Part I: First-Order Transient Response

A thermocouple (TC) acts as a typical first-order dynamic system due to heat transfer properties of the TC. The inner workings of the TC consist of a pair of dissimilar wires connected via two junctions called a hot and cold junction. A difference in temperature between the junctions forms a voltage difference across them. This voltage is proportional to the temperature difference. This is known as the Seebeck effect.

The size of the thermocouple probe changes the time it takes to reach a steady-state temperature, and smaller probe tips have a faster response than large ones. The exchange of heat through the thermocouple can be described by the energy balance equation

$$mC_v \frac{dT}{dt} = hA_s[T_\infty - T(t)], \tag{1}$$

where m is the tip mass, C_v is the specific heat at a constant volume for the tip, A_s is the tip surface area, h is the heat transfer coefficient, and T_∞ is the far-field temperature of the surrounding fluid. The explicit solution to the differential equation is

$$T(t) = T_\infty + (T_0 - T_\infty)\exp\left(-\frac{t}{\tau}\right). \tag{2}$$

where T_0 is the initial temperature and the time constant $\tau = \frac{mC_v}{hA_s}$. Mathematically, the time constant represents the time it takes the system to go from the initial temperature T_0 to $1 - e^{-1} \approx 63.2\%$ of the steady-state temperature T_∞. Equation (2) is valid for *both* heating and cooling, where we have $T_0 < T_\infty$ for heating, and $T_0 > T_\infty$ for cooling.

Equation (2), as well as your measured data, can be linearized by the transformation

$$y(t) = \ln\left(\frac{T(t) - T_\infty}{T_0 - T_\infty}\right). \tag{3}$$

Plotting your transformed data $y(t)$ as a function of time should yield a straight line with a slope of $-1/\tau$ for the portion of the data that exhibits an exponential decay.

Procedure

1. Turn on the LabQuest unit. In the "LabQuest App", click 'File' → 'New' to erase any previous data that may be on the device.
2. Connect the small (1/16") thermocouple (TC) to the black amplifier/compensator box.
3. Connect the amplifier to Channel 1 on LabQuest. The temperature in degrees C should appear on the home screen of the LabQuest.
4. There is a mercury thermometer on the stone countertop. Bring your lab notebook over to the countertop and record the ambient air temperature on the mercury thermometer.
5. Compare the ambient air temperature measured using the mercury thermometer to the reading on LabQuest. Write down both values in your lab notebook in units of degrees C. Let the TA know if there is a significant difference.
6. Test the thermocouple in your hand. You should see the temperature displayed on the LabQuest slowly increase as your hand warms up the probe.
7. Edit the data acquisition options by selecting the region with *mode, rate,* and *length*. Set the sampling frequency to $f_S = 35$ samples/second and the duration to be $T_{max} = 10$ s.
8. Fill a Styrofoam cup with warm water. Fill another cup with ice water.
9. Let the TC sit in the warm water for a minute. Begin acquiring data by pressing the (►) symbol on LabQuest. After a few seconds, remove it from the warm water and submerge the tip of the TC into the *ice water*. Do not let the tip of the TC probe touch the bottom or sides of either cup. Allow LabQuest to take data for the entirety of the length specified.
10. Press the little file cabinet icon in the bottom right corner to store the data in memory.
11. Let the TC sit in the ice water for about a minute. Press the (►) symbol on the LabQuest. After a few seconds, remove the TC from the ice water and immediately submerge it in the *warm water*. Keep it submerged until data collection is complete.
12. Open the LoggerPro software on the lab computer, and connect the LabQuest to the lab computer with the USB cable. Transfer the data to the lab computer using the LoggerPro software.
13. Export the data from LoggerPro as a CSV file (click "File > Export As"). Be sure to give it a good, descriptive file name and email it to yourself or transfer it to your own computer via a flash drive.

Part II: Measuring Vibrations in a Baseball Bat

Background

A baseball bat is a familiar example of a mechanical, dynamic system. When the bat strikes the ball, the impact causes some elastic deformation of the bat, displacing it from equilibrium some distance y. Because the bat has mass and elasticity, we can model it as a harmonic oscillator. The batter's hands absorb energy from these vibrations, providing an effective damping force—an effect some of you may be familiar with. Hence, the baseball bat behaves very much like a *damped harmonic oscillator*, which we studied in the previous lab. However, the striking of the ball is an *impulse force*, because it happens 'instantaneously' as opposed to the sinusoidal driving force we saw last week.

When a damped harmonic oscillator is hit with an impulse, it will respond by "ringing". This ringing behavior is described by the equation

$$y_{bat}\left(t\right) = Ae^{-\lambda t}\sin\left(\omega_d t + \phi\right), \tag{4}$$

where ω_d is the ringing frequency, λ is the decay constant, and A and ϕ are constants dependent on initial conditions. In this lab, you will use strain gauges to measure the decay constant λ, the ringing frequency ω_d, and the phase ϕ of a vibrating aluminum baseball bat.

Setup and Data Acquisition

The bat for this laboratory exercise is outfitted with four strain gages in a full bridge configuration, which respond to the displacement of the bat y_{bat} versus time t. Shown in Fig. 1, the strain gages are connected to an amplifier to increase the output voltage, and the output voltage is recorded by an oscilloscope, which samples at a very high rate (GHz – which easily satisfies the Nyquist criterion). The oscilloscope voltage directly relates to the magnitude of the displacement.

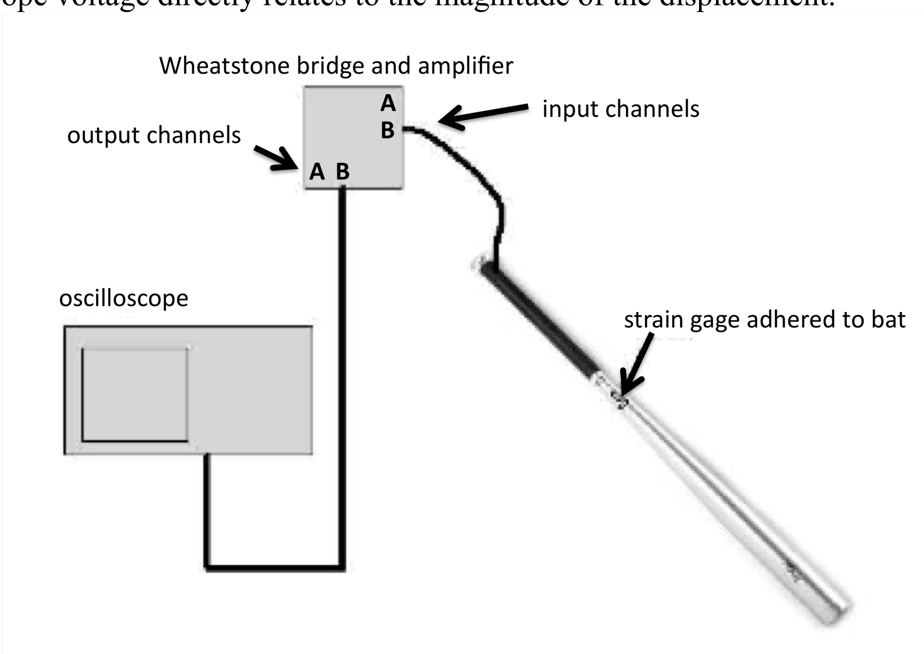

Figure 1 - Schematic of experimental apparatus and measurement system.

Procedure

1. Press "AutoSet".

2. Press yellow 1 button and set the input channel to "AC" on the oscilloscope. A steady line with some small oscillations should appear. This is a real time measurement of the signal from amplifier and is the background signal from the electrical connections corresponding to the noise of the measurement system.

3. Adjust the voltage scale(Vertical) on Channel 1 to be 50 mV/division and adjust the time scale(Horizontal) to be 10 ms/division. This is the scaling required to observe the response of the baseball bat.

4. Set the oscilloscope to trigger in response to an impulse forcing function. This means that the oscilloscope is prepared to record data (trigger) at the moment of the impulse on the bat:
   - Press the Trigger "Menu" button
   - Select the slope menu and choose the icon shown in Fig. 2.
   - Set the source as Channel 1(the trigger is looking for a signal from Channel 1)
   - Set the mode as "Normal"(the trigger is waiting for a single event or impulse to occur), and the coupling as "DC"
   - Move the trigger level knob so that the trigger level is at 20mV – this ensures that the oscilloscope will not trigger until the signal slope falls to this level
   - Using the horizontal position knob, move the trigger arrow, orange "T" at the top of the screen, to the extreme left of the screen
   - Press "Single" button on the top of the scope. The green LED should light up.

5. Above the graph readout on the right hand side there will be some text. Wait for the text to read "Trig?" before proceeding.

6. Keeping safety and those around you in mind, hit the ball with the bat. Do not swing hard as the strain gages will be broken or displaced.

7. Repeat this procedure until a good set of data has been obtained for **both** the single wall and double wall bat.

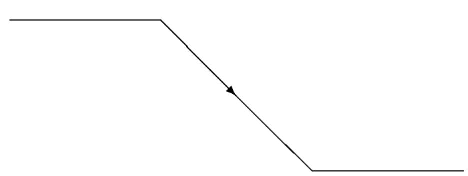

Figure 2 – This selection for Slope Menu on digital oscilloscope indicates a *falling edge trigger*.

Saving Data

1. Press MENU button on the bottom of the scope in the "Save/recall" section.
2. Insert flash drive into scope.
3. Press "Save Waveform" on soft menu on the bottom of the screen.
4. "Source" should be set as "Ch 1".
5. Use the multipurpose knob to select the destination. Highlight the "E:\TEK****CH1.CSV" file. The "****" will be replaced by the next sequential number, starting at 0000 when no other files are present.

Data Analysis

Your data should look similar to Fig. 3. It should be a nice sinusoid with an exponential decay envelope, just as you would expect for the ringing of a damped harmonic oscillator. If your data is noisy, try using the smooth() function in Matlab to filter out the noise. For your lab reports, you will need to quantitatively compare your data with Eq. (4). To do this, we will use the data to calculate λ, ω_d, and ϕ and estimate A.

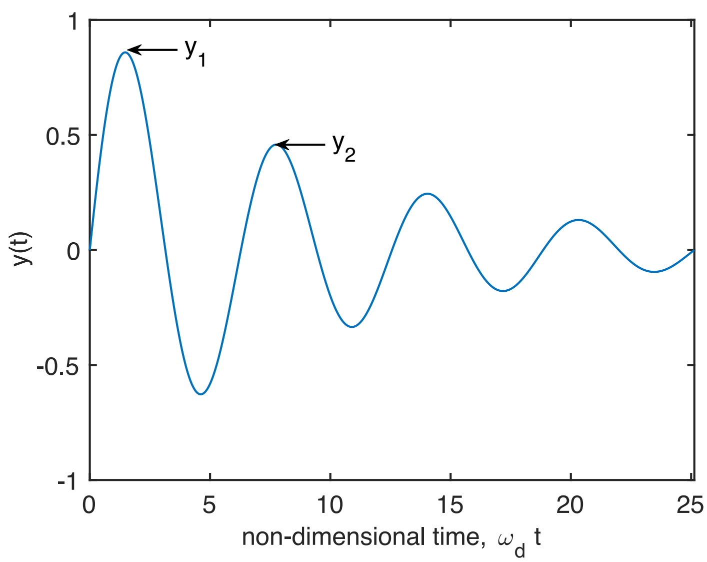

Figure 3 – An example of "ringing" behavior for a damped harmonic oscillator.

Consider the peaks labeled y_1 and y_2. Using Eq. (4), we obtain

$$\frac{y_1}{y_2} = \frac{e^{-\lambda t_1}}{e^{-\lambda t_2}} = e^{\lambda \Delta t}, \tag{5}$$

where $\Delta t = t_2 - t_1$. Equation (5) can then be solved for the decay constant

$$\lambda = \frac{\ln\left(\frac{y_1}{y_2}\right)}{\Delta t}. \tag{6}$$

To obtain the ringing frequency ω_d, run the FFT script in Matlab and locate the peak in the plot of relative amplitude vs. frequency. To obtain the phase ϕ in radians, simply find the data point corresponding to the ringing frequency ω_d in the plot of phase vs. frequency. Note that the frequency in both plots is in units of Hz, so ω_d will need to be converted to radians before inserting it in Eq. (4). Also note, the phase ϕ depends on when your scope decided to "trigger" and begin recording data. In this sense, it is somewhat arbitrary and will vary from student to student.

Now that you have ω_d and λ, try to plot Eq. (4) on top of your data. You will need to estimate the amplitude *A*. In practice, a calibration procedure would be used to convert your data from voltage to units of displacement or strain. For this lab, we will forgo the calibration procedure and estimate the amplitude *A* instead. To do this, simply adjust *A* in you Matlab script until both waveforms are approximately the same height.

Lastly, the damping ratio ζ_B of the baseball bat can be obtained using the equation

$$\zeta_B = \frac{\lambda^2}{\lambda^2 + \omega_d^2}. \qquad (7)$$

Data Analysis and Deliverables

Create plots and other deliverables listed below and put them in a technical memo, as you have done for previous labs.

1. Plot the temperature vs. time for both heating and cooling of the thermocouple on the same graph.
   a. Adjust the time vectors, so the heating and cooling both begin at $t = 0$.
   b. Include a legend to distinguish heating and cooling.
   c. Plot the data as a continuous curve, not discrete data points.
2. Linearize your data using Eq. (3) for both heating and cooling.
   a. Crop out the portion of the data that is linear.
   b. Apply linear curve fits to both data sets.
   c. Use the slope from the curve fit to determine the time constant for heating τ_H and the time constant for cooling τ_C.
   d. Plot the two linearized data sets along with their respective linear curve fits. (Be sure to include a legend.)
3. Make a plot of the **spectral density $|y(f)|$ vs. frequency f for just one of the baseball bats** using the FFT code.
4. Make a plot of the measured **strain gauge output $y(t)$ vs. time t for just one of the baseball bats with the theoretical curve given by Eq. (4)**. For the theoretical curve, the ringing frequency ω_d and phase ϕ should be obtained from the plots generated by the FFT code, while the decay constant λ should be obtained from Eq. (6) above. For the amplitude A, you will simply have to estimate a value.
5. Make a table summarizing all of the important values you extracted from your data:
   a. The time constant for heating τ_H for the thermocouple.
   b. The time constant for cooling τ_c for the thermocouple.
   c. The ringing frequency f_d in Hz for the baseball bat.
   d. The damping ratio ζ_B for the baseball bat.

Talking Points – Discuss these in your paragraphs.

- Are the time constants similar for heating and cooling? Is this what you expect? Explain.
- Explain why the baseball bat behaves as a damped harmonic oscillator.

Appendix A

Equipment

Part I (set up on inner lab benches)
- Thermocouple, ¼" diameter probe
- NI USB-TC01 DAQ
- Large Styrofoam cups (2)
- Hot and Cold water
- Ice cubes
- Mercury thermometer

Part II
- 2 – Omega DMD520 strain gage amplifier
- 2 – VGA cable to scope
- 2 – Tek DPO3012 scope
- 1 –Aluminum bat Single wall
- 1 – Aluminum bat Double wall
- 2 – Tethered softball with stand
- 2 – Bags of tube sand