Experiment A10
Transient Signals
Procedure

Deliverables: Checked lab notebook, Brief technical memo

Overview

In this lab, you will investigate the dynamic response of measurement systems. A thermocouple serves as a great example of first-order transient response, and a piezoelectric ultrasonic transducer is used to demonstrate second-order system transient response. A handheld LabQuest unit will be used for acquiring data from the thermocouple, and the frequency response of the ultrasonic transducer will be measured using an oscilloscope.

Part I: First-Order Transient Response

A thermocouple (TC) acts as a typical first-order dynamic system due to heat transfer properties of the TC. The inner workings of the TC consist of a pair of dissimilar wires connected via two junctions called a hot and cold junction. A difference in temperature between the junctions forms a voltage difference across them. This voltage is proportional to the temperature difference. This is known as the Seebeck effect.

The size of the thermocouple probe changes the time it takes to reach a steady-state temperature, and smaller probe tips have a faster response than large ones. The exchange of heat through the thermocouple can be described by the energy balance equation

$$mC_v \frac{dT}{dt} = hA_s[T_\infty - T(t)], \tag{1}$$

where m is the tip mass, C_v is the specific heat at a constant volume for the tip, A_s is the tip surface area, h is the heat transfer coefficient, and T_∞ is the far-field temperature of the surrounding fluid. The explicit solution to the differential equation is

$$T(t) = T_\infty + (T_0 - T_\infty) \exp\left(\frac{-t}{\tau}\right). \tag{2}$$

where T_0 is the initial temperature and the time constant $\tau = \frac{mC_v}{hA_s}$. Mathematically, the time constant represents the time it takes the system to go from the initial temperature T_0 to $1 - \frac{1}{e} \approx 63.2\,\%$ of the steady-state temperature T_∞.

Procedure

1. Ensure that the small (1/16”) thermocouple (TC) is connected to the thermocouple amplifier box and then turn it ON. Connect the amplifier to Channel 1 on LabQuest using the special adapter cable. Be sure that the source and GND are connected properly.
2. Turn on the black TC amplifier box. You should see a red LED illuminate.
3. Record thermocouple amplifier sensitivity coefficient (labeled on the TC amplifier) in your lab notebook.
4. Turn the power on the LabQuest unit ON. In the “LabQuest App”, click ‘File’ → ‘New’ to erase any previous data that may be on the device.
5. Go to ‘Sensors’, select ‘Sensor Setup’, and set Channel 1 to Voltage → Raw Voltage (0 – 5V).
6. Compare the ambient air temperature measured using the mercury thermometer to the reading on LabQuest. Write down both values in your lab notebook. Let the TA know if there is a significant difference.
7. Edit the data acquisition options by selecting the region with *mode, rate,* and *length*. Choose an appropriate rate (35 samples/second) and set the duration to be long enough to see the steady-state behavior (about 10 seconds is a good starting duration).
8. Fill a beaker with warm water. Fill another beaker with ice water.
9. Let the TC sit in the warm water for a minute. Begin acquiring data by pressing the (►) symbol on LabQuest. After a few seconds, remove it from the warm water and submerge the tip of the TC into the *ice water*. (Best results are obtained when the TC end is suspended.) Do not let the tip of the TC rod touch the bottom of the beaker. Allow LabQuest to take data for the entirety of the length specified.
10. Press the little file cabinet icon in the top right corner to store the data in memory.
11. Let the TC sit in the ice water for about a minute. Press the (►) symbol on LabQuest. After a few seconds, remove the TC from the ice water and immediately submerge it in the *warm water*. Keep it submerged until data collection is complete.
12. Open the LoggerPro software on the lab computer, and connect the LabQuest to the lab computer with the USB cable. Transfer the data to the lab computer using the LoggerPro software.
13. Export the data from LoggerPro as a CSV file (click “File > Export As”). Be sure to give it a good, descriptive file name and email it to yourself or transfer it to your own computer via a flash drive.
14. Repeat the procedure using the large (1/4”) thermocouple. Set the duration to 25 seconds.

Part II: Second-Order Transient Response

For the experiment, you will find the resonance frequency of an ultrasonic transducer and measure its output as a function of frequency. The ultrasonic transducers consist of a piezoelectric crystal that will deform according to Hooke's law when a stress is applied. The strain on the crystal induces a measurable voltage difference on the faces of the crystal. Similarly, applying a voltage difference to opposing phases of the crystal will induce a strain. Thus, the piezoelectric transducers can be used as either a speaker or a microphone.

Because the piezoelectric crystal deforms according to Hooke's law, it will behave as a driven, damped harmonic oscillator with a certain resonance frequency ω_0. The equation of motion for the surface of the crystal can be written as

$$m\ddot{x} = -kx - \gamma\dot{x} + F_0 \sin \omega t \tag{3}$$

where x is the displacement of the crystal surface, m is the mass, k is the spring constant, γ is a damping coefficient, F_0 is the amplitude of the driving force, and ω is the driving frequency. Rearranging Eq. (3), we get

$$\ddot{x} + \frac{1}{\beta}\dot{x} + \omega_0^2 x = \frac{F_0}{m}\sin(\omega t), \tag{4}$$

where $\beta = m/\gamma$ and $\omega_0^2 = k/m$. The solution to Eq. (4) is

$$x(t) = \frac{F_0 \sin(\omega t)}{m\sqrt{\left(\frac{\omega}{\beta}\right)^2 + \left(\omega^2 - \omega_0^2\right)^2}}. \tag{5}$$

You will measure the output voltage of the piezoelectric transducer as a function of frequency and compare it with Eq. (5). Specifically, you will measure the resonance frequency ω_0 and determine the damping ratio ζ_{UT}.

Procedure

1. Put the BNC T-adapter on the output of the Tektronix function generator and connect one of the terminals to channel 1 on the oscilloscope.
2. Connect the other end of the BNC adapter to the ultrasonic transducer (UT) that has a "T" engraved on the back. (The "T" stands for transmitter.) The cable has black heat shrink tubing.
3. Connect the other UT (the "receiver") to channel 2 on the oscilloscope. The receiver has white heat shrink tubing.

4. Turn on the function generator. Press the “sine” button, then “output menu”, then “load impedance”, then “High Z”, then press “Top Menu” to exit to the main menu.
5. Using a 40 KHz continuous sine, turn up the amplitude on the function generator to 10 Vpp and press the “On” button above the output. Vary the frequency on the function generator using the rotary knob until you find the resonance frequency. Resonance frequency is the frequency at which the output signal has maximum voltage for a given input signal.

Pro-Tip: Use the left and right arrow buttons below the big wheel knob to change the precision when adjusting the frequency on the function generator.

6. Record the peak-to-peak voltage displayed on the scope for **at least 10 frequencies below and 10 frequencies** above the resonance frequency. Choose these frequencies wisely, so that you get a nice, smooth curve that you can compare to Eq. (6). Make sure you get the entire curve, all the way out to the flat portion on both ends.
7. Lastly, record the two frequencies that give you *half* of the maximum output voltage that you recorded. Use them to calculate the full-width-at-half-max (FWHM). This will be used to determine the damping ratio ζ_{UT}.

FWHM frequencies: f_{low} ___________ f_{high} ___________

Data Analysis and Deliverables

Create plots and other deliverables listed below and put them in a technical memo, as you have done for previous labs.

1. Make a plot of the temperature vs. time for heating and cooling with both size thermocouples. Adjust the time vectors, so the heating or cooling begins at $t = 0$ for all four data sets. Be sure to include a legend to distinguish the different data sets.

NOTE: If your data has problems with noise, try filtering it using the "smooth()" function in Matlab. Be sure to read the online documentation for smooth() before you use it.

2. Examine your data and determine the time constants for heating or cooling of the 1/16" or 1/4" thermocouples. Present the four time constants in a table with a proper caption.
3. For the ultrasonic transducers, make a plot of the measured amplitude as a function of frequency with the theoretical curve given by Eq. (A3) plotted on top. (See Appendix A for details.) Your curve may have two humps. If so, explain why in the caption. *Hint*: Not all ultrasonic transducers are created equally.
4. Determine the **resonance frequency f_0 in Hz** and **damping ratio ζ_{UT}** of the ultrasonic transducer. (See Appendix A for details.) Put the values in a table.

Talking Points – Discuss these in your paragraphs.

- Based on your measured data, which thermocouple took more time to heat up and cool down? Use physics explain the observed behavior.
- Based on the measured damping ratio, was the ultrasonic transducer underdamped or overdamped?

Appendix A

Full Width at Half Max

As you saw in this lab, the piezoelectric, ultrasonic transducers behaved as damped harmonic oscillators. The displacement (or strain) of a driven, damped harmonic oscillator is:

$$x(t) = \frac{F_0 \sin \omega t}{m\sqrt{\left(\frac{\omega}{\beta}\right)^2 + \left(\omega^2 - {\omega_0}^2\right)^2}} \; . \tag{A1}$$

A piezoelectric outputs a voltage proportional to strain, so the voltage response curve takes the exact form as Eq. (A1), just with a different amplitude.

Here is how to use Eq. (A1) to "curve fit" your data. First calculate the damping parameter β using the full width at half max, $\Delta\omega$, of your measured response curve. The full width at half max is simply the width of the curve at half of the maximum value.

These two quantities are related by the following equation:

$$\Delta\omega = \frac{\sqrt{3}}{\beta} \; . \tag{A2}$$

Now that you know β, you can plot a curve fit on top of your data. For the range of frequencies that you measured, plot the following formula on top of your data:

$$V_{fit}(\omega) = \frac{\omega_0 V_{\max}}{\beta\sqrt{\left(\frac{\omega}{\beta}\right)^2 + \left(\omega^2 - {\omega_0}^2\right)^2}} \; , \tag{A3}$$

where ω_0 is the resonance frequency and $V_{\max}$ is the voltage you measured at the resonance frequency. Please plot your curve fits as solid lines and your data as individual points. If you have any questions, please contact the TA.

Lastly, the damping ratio ζ_{UT} of the ultrasonic transducer can also be obtained using the equation

$$\zeta_{UT} = \frac{1}{2\beta\omega_0} \; . \tag{A4}$$

Appendix B

Equipment

Part I

- Thermocouple, 1/16" and ¼" diameter probes
- Thermocouple Amplifier Box (TAB)
- LabQuest 1 (Aqua-colored) w/ DC power supply
- LabQuest 1 input cable (5V with male banana ends – 24" length)
- BNC connector to female banana receptor
- Large Styrofoam cups (2)
- Hot and Cold water
- Ice cubes
- Mercury thermometer

Part II

- BNC cable (24"- 36")
- BNC "T" adapter
- 80/20 transducer assembly w/ 2 piezoelectric transducers w/ 36" cable ending in BNC connector
- Tektronix AFG 3021 Function Generator
- Tektronix DPO 3012 Digital Oscilloscope
- Allen wrench