AME40453 - Automation and Controls
C5 Pre-Lab Assignment

For the following questions, please express your answers as algebraic equations written on a separate sheet of paper, and show your work. Then, transcribe the important equations into your lab notebook.

1. Use a conservation law (rate balance for the volume of water) to derive a differential equation for the height of the water in the tank. Use Poiseuille's Law $p = RQ_{OUT}$ and the hydrostatic pressure $p = \rho gh$ to determine the flow rate out. Assume the flow rate in $Q_{IN} = S$, where S is some constant flow rate provided by the pump.
2. Derive an equation for the time constant τ in terms of R, ρ, g, and the cross sectional area of the tank A_T.
3. Consider the "quiescent" mode where the tank is at steady state (i.e. the fluid height is not changing). Derive an equation for the flow rate S_s that yields a desired equilibrium height h_s in terms of *only* the variables A_T, τ, and h_s.
4. Use the definition of the LQR variables $x = h - h_s$, $u = S - S_s$ to show that $u = -k_p x$ and Eq. (3) are equivalent expressions.
5. Using your previous answers, show that Eq. (4) is equivalent to the governing differential equation from problem 1, if $x = h - h_s$, $u = S - S_s$, $A = -1/\tau$, and $B = 1/A_T$.
6. Use the lqr() method in Matlab to calculate the optimal gain k_p (in units of in^2/s) for a tank with a diameter $D = 5$ in. and height $h_{max} = 10$ in. The pump has a max flow rate $S_{max} = 15$ in^3/s. Draining the tank has a characteristic time constant $\tau = 6$s. Use $\boldsymbol{Q} = 1/|h_{max}^2|$ and $\boldsymbol{R} = 1/|S_{max}^2|$ for the LQR weights.