Experiment A9
Spring-Mass Oscillator
Procedure

Deliverables: Checked lab notebook, Brief Tech Memo

Recommended Reading: Sections 7.3 and 9.1, Chapters 18, 19, 20, and 22

Background
Structures used in aerospace and mechanical engineering tend to vibrate and oscillate. The spring-mass oscillator is the simplest mathematical model for understanding this phenomenon. Shown in Fig. 1, a spring is stretched an amount Δy by hanging a weight of mass m. The weight comes to rest at a new equilibrium position where the spring force $F_s = -k\Delta y$ is balanced by the force of gravity $F_g = -mg$, where k is the spring constant in Hooke's Law.

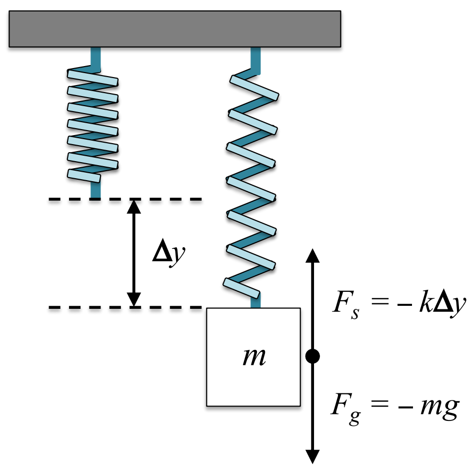

Figure 1 – Hanging a mass m from a spring stretches the spring by an amount Δy to a new equilibrium position.

When perturbed, the weight will oscillate about this equilibrium position. This can be modeled mathematically by applying Newton's Second Law

$$m\frac{d^2y}{dt^2} = -ky, \tag{1}$$

where we define the equilibrium position to be at $y = 0$. The solution to Eq. (1) is then given by

$$y(t) = A\cos(\omega_n t) + B\sin(\omega_n t), \tag{2}$$

where A and B are constants that depend on the size and direction of the perturbation, and

$$\omega_n = \sqrt{k/m} \tag{3}$$

is known as the *natural resonance frequency*. Note that Eq. (3) has units of rad/s.

Part I: Simple Measurements

You will use a meterstick, stopwatch, and digital scale to perform a series of simple measurements that can be used to determine the basic parameters m, k, and ω_n for the spring-mass system. These types of simple measurements are often referred to as a "sanity check".

Procedure

1. Sketch Fig. 1 in your lab notebook.
2. Use the digital scale over on the counter top to measure the mass of the rectangular plastic weight. Record the value in your lab notebook.
3. Measure the length of the relaxed spring hanging from the threaded rod with no weight on it. Record the value in your lab notebook.
4. Hang the weight from the spring. Measure the new length, and calculate the displacement Δy. Record the values in your lab notebook.
5. Use the equilibrium condition $mg = k\Delta y$ to calculate the spring constant k (units of N/m).
6. Gently lift the weight until the spring has returned to its relaxed state. Release the weight so it drops vertically and oscillates up and down.
7. Use the stop watch to measure the period of oscillations.
    a. Use the stop watch to time the period of just one up-down oscillation cycle.
    b. Use the stop watch to measure the time it takes to undergo 20 oscillations. Use this time to calculate the average period of a single up-down oscillation cycle.
    c. Which method do you think is more accurate?
8. Use the most accurate value of the period of oscillation to compute the *measured* natural resonance frequency ω_n in rad/s.
9. Use the measured values of k and m to calculate the *theoretical* natural resonance frequency in rad/s. Compare it to the value you measured with the stop watch.
10. Convert both values of the natural resonance frequency from rad/s to Hz.

Part II: Calibrating the Accelerometer

You will now perform much more sophisticated measurements of the motion using an ADXL337 MEMS accelerometer and a Logomatic v2 SD Datalogger from SparkFun to measure the acceleration vs. time.

The first step is to calibrate the accelerometer.

Procedure

1. In a browser window, pull up the Logomatic hookup guide for details on the device: **http://learn.sparkfun.com/tutorials/logomatic-hookup-guide**
2. Make sure the battery is connected to the Logomatic. Connect the Logomatic to the lab computer via the microUSB cable. Turn it ON using the small switch on the board. Ignore and close any warnings from the Windows OS, and wait patiently for the computer to recognize the logger as a removeable disk. If the computer does not recognize it, turn OFF the logger and turn it back ON again.

NOTE: The USB cable has been permanently attached to the Logomatic. Do NOT try to remove it.

3. Open the folder for the Logomatic as you would for a normal flash drive. Delete all files except for the LOGCON.txt configuration file.
4. Open the “LOGCON.txt” configuration file. Check to make sure it matches the script in Appendix B. If it does not match, change it so that it does and save it. Note that the sampling frequency should be 200 Hz. Write this down in your lab notebook.
5. Turn OFF the Logomatic and disconnect the USB cable from the computer.
6. Make the following connections from the accelerometer (left) to the data logger (right):
    a. 3.3V to 3.3V
    b. GND to GND
    c. X to pin 1
    d. Y to pin 2
    e. Z to pin 3
7. Note the following important details regarding the operation of the Logomatic data logger:
    a. The data logger will begin collecting data as soon as you turn it ON.
    b. It will stop collecting data when you press the STOP button. The data logger will save the data as a text file.
    c. If you press the RESET button, it will collect a second data set and save it as a new text file when the STOP button is pressed.
    d. The data logger is configured to record analog voltages as 10 bit digital numbers. A value of 0 corresponds to 0 volts and 1023 corresponds to 3.3 volts.

CAUTION: Be very careful with the microUSB cable. Excessive pulling or bending of the cable will break the Logger board.

8. Perform a 2-pt calibration on the Y channel of the accelerometer by holding the various edges of the accelerometer flat against the table. While the data logger is ON and collecting data, start with the Y arrow pointing up. Hold it in this orientation for about 5 seconds, then rotate it so the Y arrow points down and hold it in the downward orientation for another 5 seconds. This will yield accelerations of a_y = +1g and -1g.
9. When you are finished with the 2-point calibration procedure, press the STOP button, then turn OFF the logger.
10. Connect the logger to the lab computer, then turn ON the logger. Ignore and close any warnings from the Windows OS, and wait patiently for the computer to recognize the logger. If it does not recognize it, turn OFF the logger and turn it back ON again.
11. Open the logger as a folder to view the files and copy your calibration data file "LOG**.txt" to the lab computer.
12. Open the .txt data file that you just copied to the computer with Notepad.
13. Check the values, then select File > Save as. Give it an intelligent file name and select **ANSI Encoding** from the drop down menu, and **re-save it**. (This step seems redundant, but it is entirely necessary to encode the data with the correct delimiters.)
14. Import the ANSI encoded .txt file into Matlab with "Space" as a the delimiter.
    a. Click the "Import Data" button on the HOME tab, and select the data file.
    b. Select "Delimited" with "Space" as the column delimiter.
    c. Set the "Output Type" to be *column vectors*.
    d. The first column contains the X data, second column is Y data, and third column is Z data. Make sure the second column is highlighted, and click the "Import Selection" button.
    e. Close the import window, and check that the data has been correctly loaded in the Matlab "Workspace".
    f. Change the variable name from the default "VarName1" to a name that is more brief and descriptive, i.e. "Ycal".
    g. Save the workspace as "yourName_calibration_data.mat".
15. Plot the Y data set. The flat portions of the data correspond to values of +1g and -1g. (For example, 400 maps to -1g and 600 maps +1g.) Use the values of the flat portions to create a linear calibration equation, which relates the digital value to the acceleration in units of m/s^2 for the Y axis.
16. Email the .txt and .mat calibration data files to your lab partner.

Part III: Measuring Acceleration of the Spring-Mass

You will use the accelerometer to measure the acceleration of the plastic weight as it bobs up and down, suspended from a spring.

Procedure

CAUTION: Be very careful with the microUSB cable. Excessive pulling or bending of the cable will break the Logger board.

1. Unplug the large end of USB cable from the computer. Leave the other end of the cable connected to the Logomatic.
2. Mount the accelerometer, Logomatic, USB cable, and battery to the plastic weight using the 3M Velcro command strips (state-of-the-art dorm room technology!). The Y axis should point upward and the X axis should be oriented left-right.
3. Repeat the "sanity check" from Part I with the electronics mounted to the weight. Re-measure the mass of the weight with the electronics mounted to it. Recompute the theoretical resonance frequency using Eq. (3). The spring constant k you computed earlier should still be valid.
4. Use the hook to hang the weight from the spring. Gently the USB cable around the top of the weight.
5. Make your sketch of Figure 1 in your lab notebook includes the X and Y axes of the accelerometer.
6. Repeat the measurements of the period of oscillations using the stop watch. Are they different now that the electronics are mounted to the weight?
7. Turn on the logomatic. Gently lift the plastic weight until the spring has returned to its relaxed state. Release the weight so it drops vertically and oscillates up and down. Allow it to oscillate *at least* 20 full up/down cycles.
8. Press the STOP button on the Logomatic.
9. Press the RESET button and repeat the measurement. Repeat it several times so you will have several data sets to choose from.
10. Unhook the weight, connect the USB to the lab computer, and offload the data. Open the .txt data file with Notepad. Check the values, then select File > Save as. Give it an intelligent file name and select **ANSI Encoding** from the drop down menu.
11. Email all data files to you and your partner. **On your own computer, make some quick plots of the data in Matlab. The data should exhibit at least 20 full periods of sinusoidal oscillations.**
12. Return the lab bench to its initial state:
    a. Delete all data files from the logger, but NOT the LOGCON.txt configuration file.
    b. Remove the jumper cables connecting the data logger and accelerometer. Leave the USB cable connected to the Logomatic.
    c. Remove the accelerometer, Logomatic, and battery from the plastic weight.

Data Processing - This portion of the lab will likely be performed outside of lab.

You will now use the 2-point calibration formula to convert the raw acceleration data to units of m/s^2. Then, you will use the data to compute the velocity and position as a function of time. Numerically integrating the acceleration data a_i will give the velocity

$$\vec{v}_{i+1} = \vec{v}_i + \vec{a}_i \cdot \Delta t \,. \tag{4}$$

Importantly, this is an indefinite integral or *anti-derivative*, so v_i will be a vector in Matlab, and should be plotted as a function of time. Equation (4) also represents a *Riemann sum* method of numeric integration. Alternatively, one could use a *trapezoidal* method of integration, such as the one found in Ch. 19 of the textbook.

Lastly, you will run your acceleration vs. time data through an FFT script in Matlab. This will map the data from the time domain into the frequency domain, giving you the amplitude as a function of frequency—also known as the “spectral density”.

Procedure

1. Use your calibration data to create a linear calibration equation, which relates the 10-bit digital integer value to the acceleration in units of m/s^2.
2. Import the data containing the up-and-down oscillation of the spring-mass system. Make ruff plots of the *y*-acceleration data, and decide which data set has the best signal-to-noise ratio (SNR).
3. Delete the “dead time” where the weight is sitting still or doing something weird at the beginning and end of the test. (You can manually delete the values from the spreadsheet in Matlab, or you can use the brush tool to crop out the values that you wish to keep.)
4. Convert the acceleration data to units of m/s^2 using the calibration equation.
5. Use the sampling frequency to determine Δt between each data point and generate a time vector in Matlab.
6. Filter the data using the smooth() function in Matlab. Be sure to read the Mathworks’ webpage for smooth(), so you understand exactly what it is doing.
7. In Matlab, plot the processed acceleration vs. time data. Use the brush tool to select nearly all of the data on the acceleration plot. Right-click the selected data points, and click “Copy Data to Clipboard”.
8. Open a new blank workbook in Microsoft Excel. Right-click the A1 cell in the upper left and paste the data.
9. Save the Excel workbook as a .csv file. Give the .csv file an intelligent file name (i.e. “A9_acc_data.csv”)
10. Download the FFT script from the A9 webpage. Carefully read the comments and run the script on the .csv file containing your Y acceleration data to create a plot of the spectral density.

11. The FFT spectral density plot should have a peak at the natural resonance frequency.
12. Go back to the processed acceleration vs. time data in Matlab, and plot the data. Use the brush tool to select exactly two full periods of oscillation on the acceleration plot. Right-click the selected data points, and click “Copy Data to Clipboard”.
13. Create a new variable(s) in the Matlab workspace and paste the two periods of acceleration data into the variable(s).
14. Compute the average (mean) value of the acceleration for the cropped, two-period data. This mean is known as the “DC offset”.
15. Subtract the mean value (DC offset) of acceleration from every individual acceleration value in the cropped data set.
16. Plot the data. You should see exactly two periods of oscillations, oscillating about ZERO acceleration.
17. Numerically integrate this acceleration data to obtain the Y *velocity* vs. time.

Note: Although measured data is typically plotted as individual markers, transient signals (such as acceleration vs. time) should be plotted as a continuous line.

Data Analysis and Deliverables

Create plots and other deliverables listed below. Save the plots as PDFs, import them into either Microsoft Word or LaTeX, and add an intelligent, concise caption. Make sure the axes are clearly labeled with units. Plots with multiple data sets on them should have a legend. **Additionally, write 1 – 3 paragraphs describing the items below.**

IMPORTANT: Refer to the "Data Processing" section of Part III for instruction on data processing.

1. A plot of two full periods of the Y acceleration (units of m/s^2) as a function of time for one of the tests.
2. A plot of two full periods of the Y velocity (units of m/s) as a function of time for the same test.
3. A plot of spectral density (amplitude vs. frequency) of the Y acceleration data computed using the FFT code on the A9 web page.
4. A table containing the following parameters for the weight **with electronics mounted to it**:
    a. The measured mass of the plastic weight with electronics m (kg).
    b. The measured spring constant k (N/m).
    c. The *theoretical* natural resonance frequency f_n (Hz).
    d. The natural resonance frequency f_n (Hz) measured using the stopwatch.
    e. The natural resonance frequency f_n (Hz) determined from the FFT plot.

Talking Points – Please address the following questions in your paragraphs.

- What was the amplitude of the oscillating acceleration and velocity obtained from the accelerometer data?
- Are the values obtained from the accelerometer data, numeric integration, and FFT code consistent with the simple measurements from Part I? Did the "sanity check" work?

Appendix A

Equipment

- Ruler or meterstick
- Stop watch
- Steel extension springs – Amazon #: B078NYV36C
- Lab stand w/ threaded rod and nuts (5/16” – 18)
- Plastic weight with metal hooks
- Digital scale
- 3M Command Small Refill Strips, White, (64 strips) – Amazon #: B0751RPC6Q
- MicroUSB **data** cable
- SparkFun Logomatic v2 - Serial SD Datalogger (FAT32)
- Lithium Ion Battery - 400mAh
- SparkFun Triple Axis Accelerometer Breakout - ADXL337
- Female jumper cables

Appendix B

MODE = 2
ASCII = Y
Baud = 4
Frequency = 200
Trigger Character = $
Text Frame = 100
AD1.3 = N
AD0.3 = Y
AD0.2 = Y
AD0.1 = Y
AD1.2 = N
AD0.4 = N
AD1.7 = N
AD1.6 = N
Safety On = Y