Experiment A11
Transient Signals
Procedure

Deliverables: Checked lab notebook, Brief technical memo
Recommended Reading: Chapters 21 - 23 of the textbook

Overview
In this lab, you will investigate the dynamic response of various systems. We will measure the transient behavior of a thermocouple and see that it exhibits a first-order transient response. We will also measure the driven oscillations of piezoelectric crystals, which is an example of second-order transient behavior.

Part I: First-Order Transient Response

A thermocouple (TC) acts as a typical first-order dynamic system due to heat transfer properties of the TC. The inner workings of the TC consist of a pair of dissimilar wires connected via two junctions called a hot and cold junction. A difference in temperature between the junctions forms a voltage difference across them. This voltage is proportional to the temperature difference. This is known as the Seebeck effect.

The size of the thermocouple probe changes the time it takes to reach a steady-state temperature, and smaller probe tips have a faster response than large ones. The exchange of heat through the thermocouple can be described by the energy balance equation

$$mC_v \frac{dT}{dt} = hA_s[T_\infty - T(t)], \tag{1}$$

where m is the tip mass, C_v is the specific heat at a constant volume for the tip, A_s is the tip surface area, h is the heat transfer coefficient, and T_∞ is the far-field temperature of the surrounding fluid. The explicit solution to the differential equation is

$$T(t) = T_\infty + (T_0 - T_\infty) \exp\left(-\frac{t}{\tau}\right). \tag{2}$$

where T_0 is the initial temperature and the time constant $\tau = \frac{mC_v}{hA_s}$. Mathematically, the time constant represents the time it takes the system to go from the initial temperature T_0 to $1 - e^{-1} \approx 63.2\%$ of the steady-state temperature T_∞. Equation (2) is valid for *both* heating and cooling, where we have $T_0 < T_\infty$ for heating, and $T_0 > T_\infty$ for cooling.

Equation (2), as well as your measured data, can be linearized by the transformation

$$y(t) = \ln\left(\frac{T(t) - T_\infty}{T_0 - T_\infty}\right). \qquad (3)$$

Plotting your transformed data $y(t)$ as a function of time should yield a straight line with a slope of $-1/\tau$ for the portion of the data that exhibits an exponential decay.

Procedure

1. Turn on the LabQuest unit. In the “LabQuest App”, click ‘File’ → ‘New’ to erase any previous data that may be on the device.
2. Connect the small (1/16”) thermocouple (TC) to the black amplifier/compensator box.
3. Connect the amplifier to Channel 1 on LabQuest. The temperature in degrees C should appear on the home screen of the LabQuest.
4. There is a mercury thermometer on the stone countertop. Bring your lab notebook over to the countertop and record the ambient air temperature on the mercury thermometer.
5. Compare the ambient air temperature measured using the mercury thermometer to the reading on LabQuest. Write down both values in your lab notebook in units of degrees C. Let the TA know if there is a significant difference.
6. Test the thermocouple in your hand. You should see the temperature displayed on the LabQuest slowly increase as your hand warms up the probe.
7. Edit the data acquisition options by selecting the region with *mode, rate,* and *length*. Set the sampling frequency to $f_S = 35$ samples/second and the duration to be $T_{max} = 10$ s.
8. Fill a Styrofoam cup with warm water. Fill another cup with ice water.
9. Let the TC sit in the warm water for a minute. Begin acquiring data by pressing the (►) symbol on LabQuest. After a few seconds, remove it from the warm water and submerge the tip of the TC into the *ice water*. Do not let the tip of the TC probe touch the bottom or sides of either cup. Allow LabQuest to take data for the entirety of the length specified.
10. Press the little file cabinet icon in the bottom right corner to store the data in memory.
11. Let the TC sit in the ice water for about a minute. Press the (►) symbol on the LabQuest. After a few seconds, remove the TC from the ice water and immediately submerge it in the *warm water*. Keep it submerged until data collection is complete.
12. Open the LoggerPro software on the lab computer, and connect the LabQuest to the lab computer with the USB cable. Transfer the data to the lab computer using the LoggerPro software.
13. Export the data from LoggerPro as a CSV file (click “File > Export As”). Be sure to give it a good, descriptive file name and email it to yourself or transfer it to your own computer via a flash drive.

Part II: Second-Order Transient Response

For the experiment, you will find the resonance frequency of an ultrasonic transducer and measure its output as a function of frequency. The ultrasonic transducers consist of a piezoelectric crystal that will deform according to Hooke's law when a stress is applied. The strain on the crystal induces a measurable voltage difference on the faces of the crystal. Similarly, applying a voltage difference to opposing phases of the crystal will induce a strain. Thus, the piezoelectric transducers can be used as either a speaker or a microphone.

Because the piezoelectric crystal deforms according to Hooke's law, it will behave as a driven, damped harmonic oscillator with a certain resonance frequency ω_0. The equation of motion for the surface of the crystal can be written as

$$m\ddot{x} = -kx - \gamma\dot{x} + F_0 \sin \omega t \qquad (4)$$

where x is the displacement of the crystal surface, m is the mass, k is the spring constant, γ is a damping coefficient, F_0 is the amplitude of the driving force, and ω is the driving frequency. Rearranging Eq. (4), we get

$$\ddot{x} + \frac{1}{\beta}\dot{x} + \omega_0^2 x = \frac{F_0}{m}\sin(\omega t), \qquad (5)$$

where $\beta = m/\gamma$ and $\omega_0^2 = k/m$. The solution to Eq. (5) is

$$x(t) = \frac{F_0 \sin(\omega t)}{m\sqrt{\left(\frac{\omega}{\beta}\right)^2 + \left(\omega^2 - \omega_0^2\right)^2}}. \qquad (6)$$

You will measure the output voltage of the piezoelectric transducer as a function of frequency and compare it with Eq. (6). Specifically, you will measure the resonance frequency ω_0 and determine the damping ratio ζ_{UT}.

Procedure

1. Put the BNC T-adapter on the output of the Tektronix function generator and connect one of the terminals to channel 1 on the oscilloscope.
2. Connect the other end of the BNC adapter to the ultrasonic transducer (UT) that has a "T" engraved on the back. (The "T" stands for transmitter.) The cable has black heat shrink tubing.
3. Connect the other UT (the "receiver") to channel 2 on the oscilloscope. The receiver has white heat shrink tubing.
4. Turn on the function generator. Press the "sine" button, then "output menu", then "load impedance", then "High Z", then press "Top Menu" to exit to the main menu.
5. Using a 40 KHz continuous sine, turn up the amplitude on the function generator to 10 Vpp

and press the "On" button above the output. Vary the frequency on the function generator using the rotary knob until you find the resonance frequency. Resonance frequency is the frequency at which the output signal has maximum voltage for a given input signal.

Pro-Tip: Use the left and right arrow buttons below the big wheel knob to change the precision when adjusting the frequency on the function generator.

6. Record the peak-to-peak voltage displayed on the scope for **at least 10 frequencies below and 10 frequencies** above the resonance frequency. Choose these frequencies wisely, so that you get a nice, smooth curve that you can compare to Eq. (6). Make sure you get the entire curve, all the way out to the flat portion on both ends.
7. Lastly, record the two frequencies that give you *half* of the maximum output voltage that you recorded. Use them to calculate the full-width-at-half-max (FWHM). This will be used to determine the damping ratio ζ_{UT}.

FWHM frequencies: f_{low} ___________ f_{high} ___________

8. Reset the scope to factory default by pressing "Default Setup" key below the display.
9. Reset the function generator to factory default by pressing the "Default" key next to the keypad.

Data Analysis and Deliverables

Create plots and other deliverables listed below and put them in a technical memo, as you have done for previous labs.

1. Plot the temperature vs. time for both heating and cooling of the thermocouple on the same graph.
    a. Adjust the time vectors, so the heating and cooling both begin at $t = 0$.
    b. Include a legend to distinguish heating and cooling.
    c. Plot the data as a continuous curve, not discrete data points.
2. Linearize your data using Eq. (3) for both heating and cooling.
    a. Crop out the portion of the data that is linear.
    b. Apply linear curve fits to both data sets.
    c. Use the slope from the curve fit to determine the time constant for heating τ_H and the time constant for cooling τ_C.
    d. Plot the two linearized data sets along with their respective linear curve fits. (Be sure to include a legend.)
3. For the ultrasonic transducers, make a plot of the measured amplitude as a function of frequency with the theoretical curve given by Eq. (A3) plotted on top. (See Appendix A for details.)
    a. Your curve may have two humps. If so, try to explain why in the caption. *Hint*: Not all ultrasonic transducers are created equally.
    b. Determine the **resonance frequency f_0 in Hz** and **damping ratio ζ_{UT}** of the ultrasonic transducer. (See Appendix A for details.)
4. Make a table summarizing all of the important values you extracted from your data:
    a. The time constant for heating τ_H for the thermocouple.
    b. The time constant for cooling τ_c for the thermocouple.
    c. The resonance frequency f_0 in Hz for the ultrasonic transducer.
    d. The damping ratio ζ_{UT} for the ultrasonic transducer.

Talking Points – Discuss these in your paragraphs.

- Are the time constants similar for heating and cooling? Is this what you expect? Explain.
- Based on the measured damping ratio, was the ultrasonic transducer underdamped or overdamped?

Appendix A

Full Width at Half Max

As you saw in this lab, the piezoelectric, ultrasonic transducers behaved as damped harmonic oscillators. The displacement (or strain) of a driven, damped harmonic oscillator is:

$$x(t) = \frac{F_0 \sin \omega t}{m\sqrt{\left(\frac{\omega}{\beta}\right)^2 + (\omega^2 - \omega_0{}^2)^2}} \,. \tag{A1}$$

A piezoelectric outputs a voltage proportional to strain, so the voltage response curve takes the exact form as Eq. (A1), just with a different amplitude.

Here is how to use Eq. (A1) to "curve fit" your data. First calculate the damping parameter β using the full width at half max, $\Delta\omega$, of your measured response curve. The full width at half max is simply the width of the curve at half of the maximum value.

These two quantities are related by the following equation:

$$\Delta\omega = \frac{\sqrt{3}}{\beta} \,. \tag{A2}$$

Now that you know β, you can plot a curve fit on top of your data. For the range of frequencies that you measured, plot the following formula on top of your data:

$$V_{fit}(\omega) = \frac{\omega_0 V_{\max}}{\beta\sqrt{\left(\frac{\omega}{\beta}\right)^2 + (\omega^2 - \omega_0{}^2)^2}} \,, \tag{A3}$$

where ω_0 is the resonance frequency and $V_{\max}$ is the voltage you measured at the resonance frequency. Please plot your curve fits as solid lines and your data as individual points. If you have any questions, please contact the TA.

Lastly, the damping ratio ζ_{UT} of the ultrasonic transducer can also be obtained using the equation

$$\zeta_{UT} = \frac{1}{2\beta\omega_0} \,. \tag{A4}$$

Appendix B

Equipment

Part I (set up on inner lab benches)
- Thermocouple, ¼” diameter probe
- NI USB-TC01 DAQ
- Large Styrofoam cups (2)
- Hot and Cold water
- Ice cubes
- Mercury thermometer

Part II (set up on outer lab benches)
- BNC cable (24”- 36”)
- BNC “T” adapter
- 80/20 transducer assembly w/ 2 piezoelectric transducers w/ 36” cable ending in BNC connector
- Tektronix AFG 3021 Function Generator
- Tektronix DPO 3012 Digital Oscilloscope
- Allen wrench