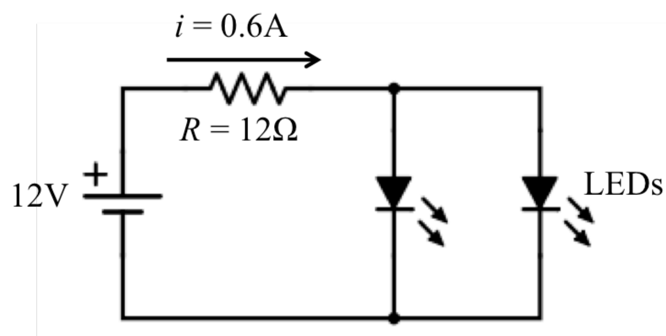
AME 20216
Homework 2
Due March 2, 2020

Do the following problems on engineering paper. Show all your work.

1. DC Circuits (6 pts total)

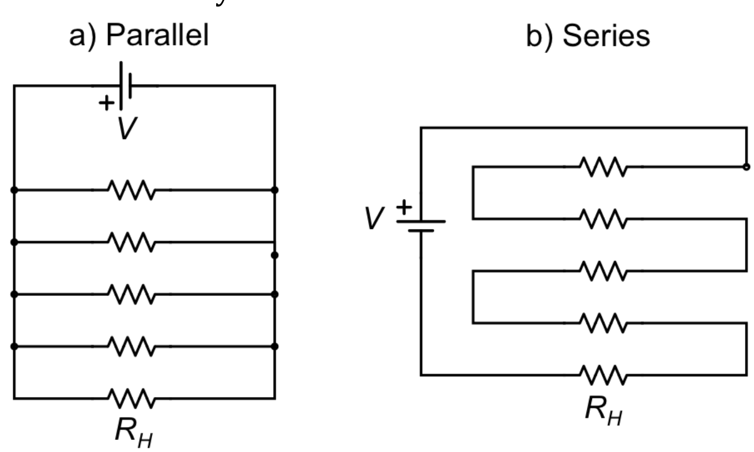
In the circuit shown below, a 12V battery is used to power two LEDs.



a) Calculate the voltage drop across the resistor
b) Calculate the power dissipated in the resistor.
c) Calculate the voltage drop across the LEDs.
d) Calculate the sum total power dissipated in both LEDs.
e) Calculate the total power delivered by the 12V battery.
f) What is the efficiency of this circuit? Specifically, what percent of the total power delivered by the battery goes to the LEDs?

2. Frosted Windows (10 pts total)

The rear window defroster in a car has five long, thin resistive heaters to melt off snow and ice. As shown below, the five resistive heaters can be wired in either series or in parallel and connected to a 12V car battery. Each individual resistor has a resistance R_H.



a & b) Calculate the ***total power dissipated in all five resistors*** for each of the circuits shown above, given $V = 12\text{V}$ and $R_H = 10\Omega$. (Assume the batteries are ideal power supplies.)

c) Derive an equation for the total power dissipated in an arbitrary number of heaters N wired in **parallel**.

d) Derive an equation for the total power dissipated in an arbitrary number of heaters N wired in **series**.

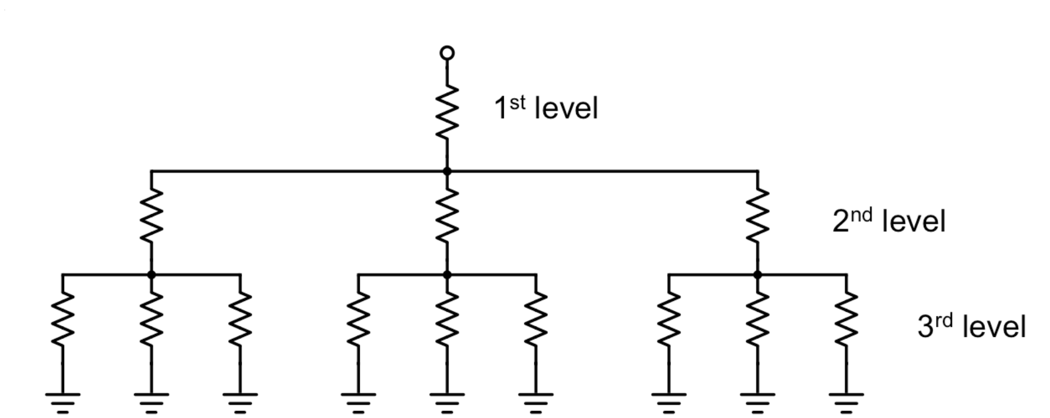
e) Sketch a graph of total power vs. number of heaters N for both the parallel and series configurations.

f) Based on your previous answers, which will be better at melting ice? Series or parallel?

3. A Fractal Network of Resistors (7 pts total)

Electricity and other resources are often distributed through branched fractal networks. Common examples of this are electrical grids, cardiovascular systems in animals, roots and branches of plants, HVAC systems in large buildings, and pipes for distributing water and natural gas.

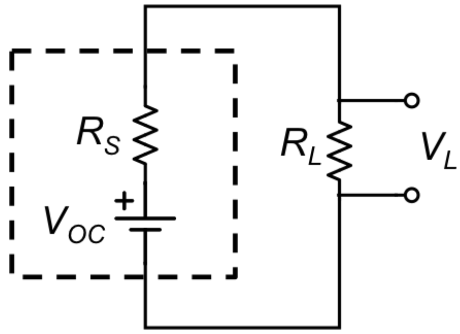
a) Consider the network below, which contains $N = 3$ levels of branches. All the resistors have the same resistance R. Derive an equation for the equivalent resistance R_{eq} of the entire network.



b) Based on your answer from part a, derive an equation for the equivalent resistance of a similar network with an arbitrary number of levels N, where all resistors have the same resistance R.

c) What happens to the equivalent resistance as N goes to infinity?

4. Solar Panel as a "Non-ideal Power Supply" (12 pts)



A solar panel is used to power resistive load R_L as shown in the circuit diagram above. The solar panel can be modeled as a non-ideal power supply with an open circuit voltage V_{OC} and an internal resistance R_s.

a. Derive an equation for the total current I as a function of V_{OC}, R_S, and R_L. Then, sketch a plot of I vs. R_L.

b. Derive an equation for the output voltage across the load V_L as a function of V_{OC}, R_S, and R_L. Then, sketch a plot of V_L vs. R_L.

c. Derive an equation for the total power $\dot{q}_L$ dissipated *in the resistive load* R_L as a function of V_{OC}, R_S, and R_L. Then, sketch a plot of $\dot{q}_L$ vs. R_L.

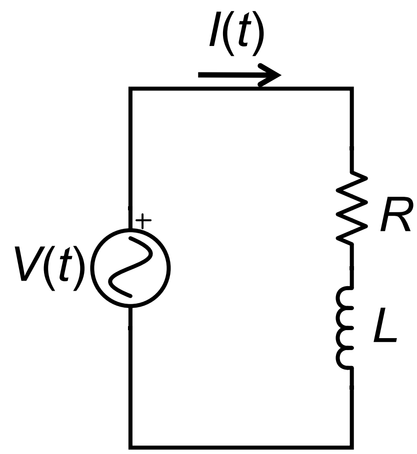
d. Use calculus to maximize the load power $\dot{q}_L$ with respect to R_L. That is, find a formula for R_L that maximizes $\dot{q}_L$.

e. A solar panel has an open circuit voltage V_{OC} = 22V and an internal resistance of R_S = 20Ω. Calculate the "short circuit current" I_{SC} when the load R_L = 0.

f. A solar panel has an open circuit voltage V_{OC} = 22V and an internal resistance of R_S = 20Ω. What should the resistance of the load R_L be to maximize the power? What will the maximum power be?

5. AC Circuit Analysis (7 pts)

The Tesla Model S uses an electric AC induction motor. The motor's stator, which can be modeled by the RL circuit shown below, is driven by a sinusoidal AC voltage with a frequency $f = 50$ Hz and a peak amplitude $|V| = 300$V. The resultant current has a peak amplitude of $|I| = 95$ Amps with a phase 86° lagging behind the voltage.



a. Use the "yyaxis left" and "yyaxis right" commands in Matlab to make a plot of both the current and voltage as a function of time *on the same graph*. Voltage should go on the left y-axis, and current should go on the right y-axis. (Be sure to include units.)

b. Make a plot of the instantaneous power $\dot{q}(t) = I(t)V(t)$.

c. Calculate the *average power* $<\dot{q}>$ lost to joule heating in the resistor.