AME40453 - Automation and Controls
C8 Pre-Lab Assignment

Please refer to the lab handout for the following questions. Express your answers as algebraic equations written on a separate sheet of paper, and show your work. Then, transcribe the equations into your lab notebook.

1. Consider a pendulum with zero applied torque $\tau = 0$.
    a. Sketch the solution $\theta(t) = e^{-\lambda t}\sin(\omega_d t)$ to Eq. (1) of the handout.
    b. Describe an experimental method to measure λ. (Hint: Think back to the baseball bat in Lab I.)
2. Consider a pendulum at rest at some desired angle or "set-point" θ_S. Derive an equation for the applied motor torque τ_S necessary to maintain that angle.
3. Write the function $mgR\sin(\theta)$ as a first order Taylor series expansion about the point $\theta = \theta_S$.
4. Write an approximate version Eq. (1) of the handout using the first order Taylor series you just derived.
5. Express the equation of motion as a linear system of first order differential equations using the new variable substitution $\dot{\theta} = \omega$ and $\ddot{\theta} = \dot{\omega}$.
6. Tie it all together now: Rewrite the linear system of differential equations in LQR form $\dot{x} = Ax + Bu$ where $x = \begin{bmatrix} \theta - \theta_S \\ \omega \end{bmatrix}$ and $u = \tau - \tau_S$. In particular, what are A and B in terms of m, g, R, and γ?

7. Use the lqr() method in Matlab to calculate the gains k_p and k_d (units of Nm/rad and Nms/rad, respectively) for a pendulum with $R = 0.15$m, $m = 0.05$ kg, $\lambda = 0.7$ s^{-1}, and a set point $\theta_S = 30°$.

    -Set **R** as the 1x1 identity matrix and **Q** as the 2x2 identity matrix.
    -The units of k_p and k_d should come out as Nm/radian and Nms/radian, respectively.
    -Save the Matlab script to the C8 folder in your code library.

8. The matrix **R** represents the "cost" of actuation, while **Q** represents the "cost" of error in the controller $u = \tau - \tau_S$. If you want to limit actuation, make it more expensive by increasing **R**. If you want to limit the error $u = \tau - \tau_S$, then increase **Q**.
    a. Repeat the previous problem with **R** = [10] and **Q** = **I** (the identity matrix). Will this increase or decrease large spikes in motor current/torque?
    b. Repeat the previous problem with **R** = [1] and **Q** = 10**I** (10 times the identity matrix). Will this increase or decrease large spikes in motor current/torque?

c. The motor we will use in lab has a limit on how much torque it can output. Look at the torque-speed curve for the motor in Appendix C. What is the maximum torque output?

d. Note that the proportional feedback gives a motor torque $\tau \approx k_p(\theta_S - \theta)$. Assume the error $(\theta_S - \theta) \approx 1$ radian, use the maximum torque from the torque-speed curve, and estimate the maximum value for k_p.

e. Trying to drive the motor above the maximum torque will cause huge problems. To avoid this, we want to increase the "cost" of actuation **R**. Adjust the value of **R** in your Matlab script until you get down near the maximum value of k_p that you just estimated. What value of **R** is a good enough to sufficiently limit the torque?

f. Repeat 8c, d, and e for the slightly larger motor in Appendix D.