Experiment A10
Transient Signals
Procedure

Deliverables: Checked lab notebook, Brief technical memo

Overview

In this lab, you will investigate the dynamic response of various systems. First, we will measure the vibrations in a baseball bat. Then, we will measure the frequency response of a piezoelectric ultrasonic transducer. Both systems will be theoretically modeled as harmonic oscillators.

Part I: Measuring Vibrations in a Baseball Bat

Background

A baseball bat is a familiar example of a mechanical, dynamic system. When the bat strikes the ball, the impact causes some elastic deformation of the bat, displacing it from equilibrium some distance y. Because the bat has mass and elasticity, we can model it as a harmonic oscillator. The batter's hands absorb energy from these vibrations, providing an effective damping force—an effect some of you may be familiar with. Hence, the baseball bat behaves very much like a *damped harmonic oscillator*, which we studied in the previous lab. However, the striking of the ball is an *impulse force*, because it happens 'instantaneously' as opposed to the sinusoidal driving force we saw last week.

When a damped harmonic oscillator is hit with an impulse, it will respond by "ringing". This ringing behavior is described by the equation

$$y_{bat}\left(t\right) = Ae^{-\lambda t}\sin\left(\omega_d t + \phi\right), \qquad (1)$$

where ω_d is the ringing frequency, λ is the decay constant, and A and ϕ are constants dependent on initial conditions. In this lab, you will use strain gauges to measure the decay constant λ, the ringing frequency ω_d, and the phase ϕ of a vibrating aluminum baseball bat.

Setup and Data Acquisition

The bat for this laboratory exercise is outfitted with four strain gages in a full bridge configuration, which respond to the displacement of the bat y_{bat} versus time t. Shown in Fig. 1, the strain gages are connected to an amplifier to increase the output voltage, and the output voltage is recorded by an oscilloscope, which samples at a very high rate (GHz – which easily satisfies the Nyquist criterion). The oscilloscope voltage directly relates to the magnitude of the displacement.

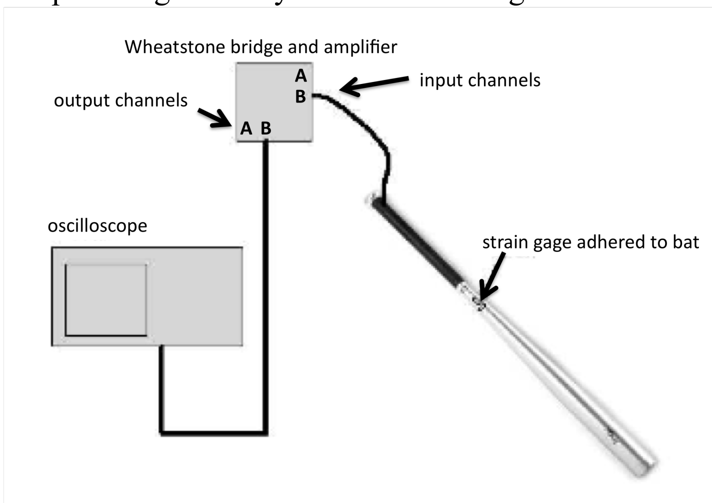

Figure 1 - Schematic of experimental apparatus and measurement system.

1. Press “AutoSet”.
2. Press yellow 1 button and set the input channel to “AC” on the oscilloscope. A steady line with some small oscillations should appear. This is a real time measurement of the signal from amplifier and is the background signal from the electrical connections corresponding to the noise of the measurement system.
3. Adjust the voltage scale(Vertical) on Channel 1 to be 50 mV/division and adjust the time scale(Horizontal) to be 10 ms/division. This is the scaling required to observe the response of the baseball bat.
4. Set the oscilloscope to trigger in response to an impulse forcing function. This means that the oscilloscope is prepared to record data (trigger) at the moment of the impulse on the bat:
   - Press the Trigger “Menu” button
   - Select the slope menu and choose the icon shown in Fig. 2.
   - Set the source as Channel 1(the trigger is looking for a signal from Channel 1)
   - Set the mode as “Normal”(the trigger is waiting for a single event or impulse to occur), and the coupling as “DC”
   - Move the trigger level knob so that the trigger level is at 20mV – this ensures that the oscilloscope will not trigger until the signal slope falls to this level
   - Using the horizontal position knob, move the trigger arrow, orange “T” at the top of the screen, to the extreme left of the screen
   - Press “Single” button on the top of the scope. The green LED should light up.

5. Above the graph readout on the right hand side there will be some text. Wait for the text to read "Trig?" before proceeding.
6. Keeping safety and those around you in mind, hit the ball with the bat. Do not swing hard as the strain gages will be broken or displaced.
7. Repeat this procedure until a good set of data has been obtained for **both** the single wall and double wall bat.

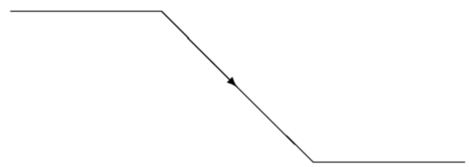

Figure 2 – This selection for Slope Menu on digital oscilloscope indicates a *falling edge trigger*.

Saving Data

1. Press MENU button on the bottom of the scope in the "Save/recall" section.
2. Insert flash drive into scope.
3. Press "Save Waveform" on soft menu on the bottom of the screen.
4. "Source" should be "Ch 1".
5. Use the multipurpose knob to select the destination. Highlight the "E:\TEK****CH1.CSV" file. The "****" will be replaced by the next sequential number, starting at 0000 when no other files are present.

Data Analysis

Your data should look similar to Fig. 3. It should be a nice sinusoid with an exponential decay envelope, just as you would expect for the ringing of a damped harmonic oscillator. If your data is noisy, try using the smooth() function in Matlab to filter out the noise. For your lab reports, you will need to quantitatively compare your data with Eq. (1). To do this, we will use the data to calculate λ, ω_d, and ϕ and estimate A.

Consider the peaks labeled y_1 and y_2. Using Eq. (1), we obtain

$$\frac{y_1}{y_2} = \frac{e^{-\lambda t_1}}{e^{-\lambda t_2}} = e^{\lambda \Delta t} \quad (2)$$
,

where $\Delta t = t_2 - t_1$. Equation (8) can then be solved for the decay constant

$$\lambda = \frac{\ln\left(\frac{y_1}{y_2}\right)}{\Delta t}. \quad (3)$$

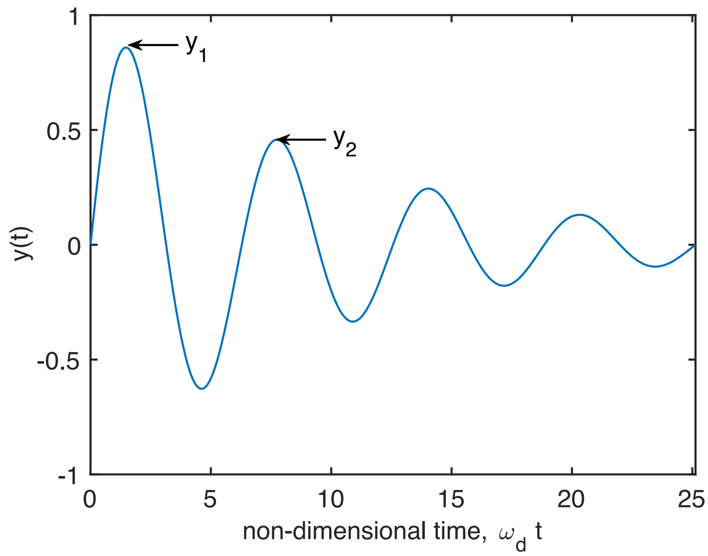

Figure 3 – An example of "ringing" behavior for a damped harmonic oscillator.

To obtain the ringing frequency ω_d, run the FFT script in Matlab and locate the peak in the plot of relative amplitude vs. frequency. To obtain the phase ϕ in radians, simply find the data point corresponding to the ringing frequency ω_d in the plot of phase vs. frequency. Note that the frequency in both plots is in units of Hz, so ω_d will need to be converted to radians before inserting it in Eq. (1). Also note, the phase ϕ depends on when your scope decided to "trigger" and begin recording data. In this sense, it is somewhat arbitrary and will vary from student to student.

Now that you have ω_d and λ, try to plot Eq. (1) on top of your data. You will need to estimate the amplitude A. In practice, a calibration procedure would be used to convert your data from voltage to units of displacement or strain. For this lab, we will forgo the calibration procedure and estimate the amplitude A instead. To do this, simply adjust A in you Matlab script until both waveforms are approximately the same height.

Lastly, the damping ratio ζ_B of the baseball bat can be obtained using the equation

$$\zeta_B = \frac{\lambda^2}{\lambda^2 + \omega_d^2}. \tag{4}$$

Part II: Second-Order Transient Response

For the experiment, you will find the resonance frequency of an ultrasonic transducer and measure its output as a function of frequency. The ultrasonic transducers consist of a piezoelectric crystal that will deform according to Hooke's law when a stress is applied. The strain on the crystal induces a measurable voltage difference on the faces of the crystal. Similarly, applying a voltage difference to opposing phases of the crystal will induce a strain. Thus, the piezoelectric transducers can be used as either a speaker or a microphone.

Because the piezoelectric crystal deforms according to Hooke's law, it will behave as a driven, damped harmonic oscillator with a certain resonance frequency ω_0. The equation of motion for the surface of the crystal can be written as

$$m\ddot{x} = -kx - \gamma\dot{x} + F_0 \sin(\omega t) \tag{5}$$

where x is the displacement of the crystal surface, m is the mass, k is the spring constant, γ is a damping coefficient, F_0 is the amplitude of the driving force, and ω is the driving frequency. Rearranging Eq. (5), we get

$$\ddot{x} + \frac{1}{\beta}\dot{x} + \omega_0^2 x = \frac{F_0}{m}\sin(\omega t), \tag{6}$$

where $\beta = m/\gamma$ and $\omega_0^2 = k/m$. The solution to Eq. (6) is

$$x(t) = \frac{F_0 \sin(\omega t)}{m\sqrt{\left(\frac{\omega}{\beta}\right)^2 + \left(\omega^2 - \omega_0^2\right)^2}}. \tag{7}$$

You will measure the output voltage of the piezoelectric transducer as a function of frequency and compare it with Eq. (7). Specifically, you will measure the resonance frequency ω_0 and determine the damping ratio ζ_{UT}.

Procedure

1. Put the BNC T-adapter on the output of the Tektronix function generator and connect one of the terminals to channel 1 on the oscilloscope.
2. Connect the other end of the BNC adapter to the ultrasonic transducer (UT) that has a "T" engraved on the back. (The "T" stands for transmitter.) The cable has black heat shrink tubing.
3. Connect the other UT (the "receiver") to channel 2 on the oscilloscope. The receiver has white heat shrink tubing.
4. Turn on the function generator. Press the "sine" button, then "output menu", then "load impedance", then "High Z", then press "Top Menu" to exit to the main menu.
5. Using a 40 KHz continuous sine, turn up the amplitude on the function generator to 10 Vpp and press the "On" button above the output. Vary the frequency on the function generator using the rotary knob until you find the resonance frequency. Resonance frequency is the frequency at which the output signal has maximum voltage for a given input signal.

Pro-Tip: Use the left and right arrow buttons below the big wheel knob to change the precision when adjusting the frequency on the function generator.

6. Record the peak-to-peak voltage displayed on the scope for **at least 10 frequencies below and 10 frequencies** above the resonance frequency. Choose these frequencies wisely, so that you get a nice, smooth curve that you can compare to Eq. (A3) in the appendix. Make sure you get the entire curve, all the way out to the flat portion on both ends.

Pro-Tip: As you collect the data, use your mind's eye to visualize what the amplitude vs. frequency data will look like on a plot.

7. Lastly, record the two frequencies that give you *half* of the maximum output voltage that you recorded. Use them to calculate the full-width-at-half-max (FWHM). This will be used to determine the damping ratio ζ_{UT}.

FWHM frequencies: f_{low} ___________ f_{high} ___________

Data Analysis and Deliverables

Create plots and other deliverables listed below and put them in a technical memo, as you have done for previous labs.

1. Make a plot of the **spectral density |*y*(*f*)| vs. frequency *f* for just one of the baseball bats** using the FFT code.
2. Make a plot of the measured **strain gauge output *y*(*t*) vs. time *t* for just one of the baseball bats with the theoretical curve given by Eq. (1)**. For the theoretical curve, the ringing frequency ω_d and phase ϕ should be obtained from the plots generated by the FFT code, while the decay constant λ should be obtained from Eq. (3) above. For the amplitude A, you will simply have to estimate a value.
3. For the ultrasonic transducers, make a plot of the measured amplitude as a function of frequency with the theoretical curve given by Eq. (A3) plotted on top. (See Appendix A for details.) Your curve may have two humps. If so, explain why in the caption. *Hint*: Not all ultrasonic transducers are created equally.
4. Make a table containing the following:
   a. The resonance frequency f_0 in Hz and damping ratio ζ_{UT} of the ultrasonic transducer
   b. The ringing frequency f_d in units of Hz and damping ratio ζ_B for the baseball bat

Hints and Tips

1. The sine function takes values that have units of radians, so remember to convert ω_d to units of radians per second!
2. For the baseball bat data, remember to delete all of the text at the top of the CSV file, so it only contains data.
3. You may find that your curve fit is *completely* out of phase with the measured signal. This is because the extracted phase ϕ is off by an amount equal to π, which is caused by a mathematical annoyance known as a "branch cut". To correct this, simply add or subtract π from the phase.

Talking Points – Discuss these in your paragraphs.

- Based on the measured damping ratio, was the ultrasonic transducer underdamped or overdamped? What about the baseball bat?

Appendix A

Full Width at Half Max

As you saw in this lab, the piezoelectric, ultrasonic transducers behaved as damped harmonic oscillators. The displacement (or strain) of a driven, damped harmonic oscillator is:

$$x(t) = \frac{F_0 \sin \omega t}{m\sqrt{\left(\frac{\omega}{\beta}\right)^2 + \left(\omega^2 - \omega_0{}^2\right)^2}} . \tag{A1}$$

A piezoelectric outputs a voltage proportional to strain, so the voltage response curve takes the exact form as Eq. (A1), just with a different amplitude.

Here is how to use Eq. (A1) to "curve fit" your data. First calculate the damping parameter β using the full width at half max, $\Delta\omega$, of your measured response curve. The full width at half max is simply the width of the curve at half of the maximum value.

These two quantities are related by the following equation:

$$\Delta\omega = \frac{\sqrt{3}}{\beta} . \tag{A2}$$

Now that you know β, you can plot a curve fit on top of your data. For the range of frequencies that you measured, plot the following formula on top of your data:

$$V_{fit}(\omega) = \frac{\omega_0 V_{\max}}{\beta\sqrt{\left(\frac{\omega}{\beta}\right)^2 + \left(\omega^2 - \omega_0{}^2\right)^2}} , \tag{A3}$$

where ω_0 is the resonance frequency and $V_{\max}$ is the voltage you measured at the resonance frequency. Please plot your curve fits as solid lines and your data as individual points. If you have any questions, please contact the TA.

Lastly, the damping ratio ζ_{UT} of the ultrasonic transducer can also be obtained using the equation

$$\zeta_{UT} = \frac{1}{2\beta\omega_0} . \tag{A4}$$

Appendix B

Equipment

Part I

- 2 – VGA cable to scope
- 2 – Omega DMD520 strain gage amplifier
- 2 – Tek DPO3012 scope
- 1 –Aluminum bat Single wall
- 1 – Aluminum bat Double wall
- 2 – Tethered softball with stand
- 2 – Bags of tube sand
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Part II

- BNC cable (24”- 36”)
- BNC “T” adapter
- 80/20 transducer assembly w/ 2 piezoelectric transducers w/ 36” cable ending in BNC connector
- Tektronix AFG 3021 Function Generator
- Tektronix DPO 3012 Digital Oscilloscope
- Allen wrench