

AME40453 - Automation and Controls  
C7 Pre-Lab Assignment

Please refer to the lab handout for the following questions. Express your answers as algebraic equations written on a separate sheet of paper, and show your work. Then, transcribe the equations into your lab notebook.

1. Consider a pendulum with zero applied torque  $\tau = 0$ .
  - a. Sketch the solution  $\theta(t) = e^{-\lambda t} \sin(\omega_d t)$  to Eq. (1) of the handout.
  - b. Describe an experimental method to measure  $\lambda$ . (Hint: Think back to the baseball bat or pendulum in Lab I.)
2. Consider a pendulum at rest at some desired angle or “set-point”  $\theta_s$ . Derive an equation for the applied motor torque  $\tau_s$  necessary to maintain that angle.
3. Write the function  $mgR\sin(\theta)$  as a first order Taylor series expansion about the point  $\theta = \theta_s$ .
4. Write an approximate version Eq. (1) of the handout using the first order Taylor series you just derived.
5. Express the equation of motion as a linear system of first order differential equations using the new variable substitution  $\dot{\theta} = \omega$  and  $\ddot{\theta} = \dot{\omega}$ .
6. Tie it all together now: Rewrite the linear system of differential equations in LQR form  $\dot{x} = Ax + Bu$  where  $x = \begin{bmatrix} \theta - \theta_s \\ \omega \end{bmatrix}$  and  $u = \tau - \tau_s$ . In particular, what are  $A$  and  $B$  in terms of  $m$ ,  $g$ ,  $R$ , and  $\gamma$ ?
7. Develop an LQR controller for a pendulum with  $R = 0.15\text{m}$ ,  $m = 0.05\text{ kg}$ ,  $\lambda = 0.7\text{ s}^{-1}$ , and a set point  $\theta_s = 45^\circ$ .
  - a. The motor we will use in lab has a limit on how much torque it can output. Look at the torque-speed curve for the motor in Appendix C. What is the maximum torque output  $\tau_{max}$ ?
  - b. Write down expressions for the LQR weights  $\mathbf{Q}$  and  $\mathbf{R}$  in terms of maximum angular displacement  $\Delta\theta_{max}$ , maximum angular speed  $\omega_{max}$ , and maximum motor torque  $\tau_{max}$ .
  - c. Use the `lqr()` method in Matlab to calculate the gains  $k_p$  and  $k_d$  (units of Nm/rad and Nms/rad, respectively) for a setpoint  $\theta_s = \pi/4 = 45^\circ$ , a maximum angular displacement  $\Delta\theta_{max} = 0.087\text{ rad} = 5^\circ$ , and a maximum angular speed  $\omega_{max} = 0.2\text{ rad/s}$ . Save the script in your code library. You will need it in lab.