

AME40453 - Automation and Controls  
C3 Pre-Lab Assignment

**For the following questions, please express your answers as algebraic equations written on a separate sheet of paper, and show your work. Then, transcribe the equations into your lab notebook.**

1. Consider a  $100\Omega$  heater being switched ON and OFF between 0 and 12V at a rate of 500Hz.
  - a. Sketch a plot of voltage vs. time for the 500Hz, 12V square wave with a 30% duty cycle.
  - b. Sketch a plot of *power* dissipated in the resistor vs. time with a 30% duty cycle.
  - c. Derive an equation of the *average* power dissipated as a function of the % duty cycle.
  - d. Calculate the average power if the duty cycle is 80%.
2. Similar to the C2 pre-lab assignment, write down the differential equation for the temperature  $T$  for a simple proportional feedback controller, where  $\dot{q} = k_p(T_s - T)$ .
3. Using the equation you just wrote, derive an equation for the equilibrium temperature in terms of the system parameters:  $mc_V$ ,  $hA$ ,  $k_p$ , etc. How does it compare to the set-point  $T_s$ ? Will the actual temperature converge to the set-point  $T_s$ ?
4. Using your equation from problem 2, derive an equation for the thermal time constant in terms of the system parameters:  $m$ ,  $c_V$ ,  $h$ ,  $k_p$ , etc.
5. Sketch the time constant as a function of the proportional gain  $k_p$ .
6. Write down the *system* of differential equations for the temperature  $T$  and integral of temperature  $I = \int (T_s - T) dt$  for the full PID controller.