
Experiment A10

Second Order Transient Response

Procedure

Deliverables: Checked lab notebook, Brief Tech Memo

Recommended Reading: Sections 7.3 and 9.1, Chapters 18, 19, 20, and 22

Background

Structures used in aerospace and mechanical engineering tend to vibrate and oscillate. The spring-mass oscillator is the simplest mathematical model for understanding this phenomenon. Shown in Fig. 1, a spring is stretched an amount Δy by hanging a weight of mass m . The weight comes to rest at a new equilibrium position where the spring force $F_s = -k\Delta y$ is balanced by the force of gravity $F_g = -mg$, where k is the spring constant in Hooke's Law.

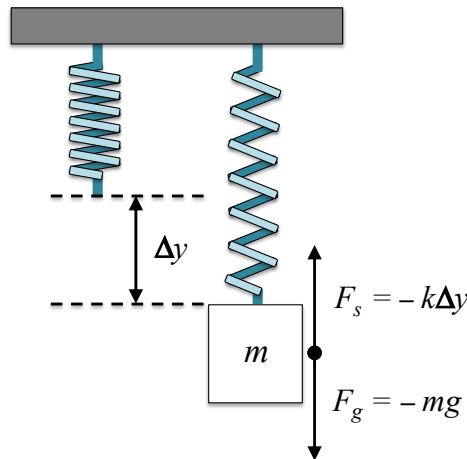


Figure 1 – Hanging a mass m from a spring stretches the spring by an amount Δy to a new equilibrium position.

When perturbed, the weight will oscillate about this equilibrium position. This can be modeled mathematically by applying Newton's Second Law

$$m \frac{d^2 y}{dt^2} = -ky, \quad (1)$$

where we define the equilibrium position to be at $y = 0$. The solution to Eq. (1) is then given by

$$y(t) = A \cos(\omega_n t) + B \sin(\omega_n t), \quad (2)$$

where A and B are constants that depend on the size and direction of the perturbation, and

$$\omega_n = \sqrt{k/m} \quad (3)$$

is known as the *natural resonance frequency*. Note that Eq. (3) has units of rad/s.

Part I: Response to an Impulse

A weight hanging from a spring will be held above its equilibrium position and released. This abrupt change in force is known as an impulse or a step input. As you probably expect, the spring-mass system will oscillate, bouncing up and down for some finite amount of time.

Simple Measurements – System Identification and Characterization

System parameters (values) are not typically given to you like they are in homework. Many times you will need to physically measure them. As you saw last week, these types of measurements are known as *system identification and characterization*. You will use a meterstick, stopwatch, and digital scale to perform a series of simple measurements that can be used to determine the basic parameters m , k , and ω_n for the spring-mass system.

Procedure

1. Sketch Fig. 1 in your lab notebook.
2. Use the digital scale over on the counter top to measure the mass of the rectangular plastic weight. Record the value in your lab notebook.
3. Measure the length of the relaxed spring hanging from the threaded rod with no weight on it. Record the value in your lab notebook.
4. Hang the weight from the spring. Measure the new length of the spring, and calculate the displacement Δy . Record the values in your lab notebook.
5. Use the equilibrium condition $mg = k\Delta y$ to calculate the spring constant k (units of N/m).
6. Gently lift the weight until the spring has returned to its relaxed state. Release the weight so it drops vertically and oscillates up and down.
7. Use the stop watch to measure the period of oscillations.
 - a. Use the stop watch to time the period of just one up-down oscillation cycle.
 - b. Use the stop watch to measure the time it takes to undergo 20 oscillations. Use this time to calculate the average period of a single up-down oscillation cycle.
 - c. Which method do you think is more accurate?
8. Use the most accurate value of the period of oscillation to compute the *measured* natural resonance frequency ω_n in rad/s.
9. Use the measured values of k and m to calculate the *theoretical* natural resonance frequency in rad/s. Compare it to the value you measured with the stop watch.
10. Convert both values of the natural resonance frequency from rad/s to Hz.

Accelerometer Measurements

You will now perform much more sophisticated measurements of the motion using an ADXL337 MEMS Accelerometer and a Logomatic v2 SD Datalogger from SparkFun to measure the acceleration vs. time.

Procedure

CAUTION: Be very careful with the microUSB cable. Excessive pulling or bending of the cable will break the Logger board.

1. In a browser window, pull up the Logomatic hookup guide for details on the device:
<http://learn.sparkfun.com/tutorials/logomatic-hookup-guide>
2. Make sure the battery is connected to the Logomatic. Connect the Logomatic to the lab computer via the USB cable. Turn it ON using the small switch on the board. Ignore and close any warnings from the Windows OS, and wait patiently for the computer to recognize the logger as a removeable disk. If the computer does not recognize it, turn OFF the logger and turn it back ON again.

NOTE: The USB cable is permanently attached to the Logomatic. Do NOT try to remove it.

3. Open the folder for the Logomatic as you would for a normal flash drive. Delete all files except for the LOGCON.txt configuration file.
4. Open the “LOGCON.txt” configuration file. Check to make sure it matches the script in Appendix B. If it does not match, change it so that it does and save it. Note that the sampling frequency should be 200 Hz. Write this down in your lab notebook.
5. Turn OFF the Logomatic and disconnect the USB cable from the computer.
6. Make the following connections from the accelerometer (left) to the data logger (right):
 - a. 3.3V to 3.3V
 - b. GND to GND
 - c. X to pin 1
 - d. Y to pin 2
 - e. Z to pin 3
7. Note the following important details regarding the operation of the Logomatic data logger:
 - a. The data logger will begin collecting data as soon as you turn it ON.
 - b. It will stop collecting data when you press the STOP button. The data logger will save the data as a text file.
 - c. If you press the RESET button, it will collect a second data set and save it as a new text file when the STOP button is pressed.
 - d. The data logger is configured to record analog voltages as 10 bit digital numbers. A value of 0 corresponds to 0 volts and 1023 corresponds to 3.3 volts.

CAUTION: Be very careful with the USB cable. Excessive pulling or bending of the cable will break the Logger board.

8. Mount the accelerometer, Logomatic, USB cable, and battery to the plastic weight using the 3M Velcro command strips (state-of-the-art dorm room technology!). The Y axis should point upward and the X axis should be oriented left-right.
9. Use the hook to hang the weight from the spring. Gently wrap the USB cable around the top of the plastic weight.
10. Make your sketch of Figure 1 in your lab notebook includes the X and Y axes of the accelerometer.
11. Turn on the Logomatic. Gently lift the plastic weight until the spring has returned to its relaxed state. Release the weight so it drops vertically and oscillates up and down. Allow it to oscillate *at least* 20 full up/down cycles.
12. Press the STOP button on the Logomatic.
13. Press the RESET button and repeat the measurement. Repeat it several times so you will have several data sets to choose from.
14. Connect the logger to the lab computer, then turn ON the logger. Ignore and close any warnings from the Windows OS, and wait patiently for the computer to recognize the logger. If it does not recognize it, turn OFF the logger and turn it back ON again.
15. Open the logger as a folder to view the files. Drag and drop your test data file “LOG**.txt” onto the desktop.
16. Open the .txt data file that you just copied to the computer with Notepad.
17. Check the values, then select File > Save as. Give it an intelligent file name and select **ANSI Encoding** from the drop down menu, and **re-save it**. (This step seems redundant, but it is entirely necessary to encode the data with the correct delimiters.)
18. Import the ANSI encoded .txt file into Matlab with “Space” as a the delimiter.
 - a. Click the “Import Data” button on the HOME tab, and select the data file.
 - b. Select “Delimited” with “Space” as the column delimiter.
 - c. Set the “Output Type” to be *column vectors*.
 - d. The first column contains the X data, second column is Y data, and third column is Z data. Change the variable name from the default “VarName1” to a name that is more brief and descriptive, i.e. “ax” or “ay”.
 - e. Make sure the second column is highlighted, and click the “Import Selection” button.
 - f. Close the import window, and check that the data has been correctly loaded in the Matlab “Workspace”.
 - g. Save the workspace as “yourName_A10_data.mat”.
19. Plot the Y acceleration data set. You should see at least 20 periods of clean oscillations. If you do not, repeat the previous steps to load a new data set into Matlab.

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20. Email all the good data files to you and your partner. **On your own computer, make some quick plots of the data in Matlab. The data should exhibit at least 20 full periods of sinusoidal oscillations.**
 21. Return the lab bench to its initial state:
 - a. Delete all data files from the logger, but NOT the LOGCON.txt configuration file.
 - b. Remove the jumper cables connecting the data logger and accelerometer. Leave the USB cable and battery connected to the Logomatic.
 - c. Remove the accelerometer, Logomatic, and battery from the plastic weight.

Data Processing

This portion of the lab will likely be performed outside of lab.

The raw 10-bit acceleration data can be converted to units of m/s^2 using the linear calibration formula

$$a_y = \frac{g}{100} Y - 5.1g, \quad (4)$$

where Y is the 10-bit integer value and $g = 9.8 \text{ m/s}^2$.

Procedure

1. Import the data containing the up-and-down oscillation of the spring-mass system. Make rough plots of the y -acceleration data, and decide which data set has the best signal-to-noise ratio (SNR).
2. Delete the “dead time” where the weight is sitting still or doing something weird at the beginning and end of the test. (You can manually delete the values from the spreadsheet in Matlab, or you can use the brush tool to crop out the values that you wish to keep.)
3. Convert the acceleration data to units of m/s^2 using the calibration equation.
4. Use the sampling frequency to determine Δt between each data point. Use Δt to generate a time vector in Matlab using `linspace()`.
5. In Matlab, plot the processed acceleration vs. time data. Use the brush tool to select 20 periods of oscillation data on the acceleration plot. Right-click the selected data points, and click “Copy Data to Clipboard”.
6. Open a new blank workbook in Microsoft Excel. Right-click the A1 cell in the upper left and paste the data.
7. Save the Excel workbook as a .csv file. Give the .csv file an intelligent file name (i.e. “A10_acc_data.csv”). Do NOT select CSV with UTF-8 encoding. Use the plain old-fashioned CSV instead.

8. Download the FFT script from the A10 webpage. Carefully read the comments and run the script on the .csv file containing your Y acceleration data to create a plot of the spectral density.
9. The FFT spectral density plot should have a peak at the natural resonance frequency, similar to what you measured with the stop watch.

Note: Although measured data is typically plotted as individual markers, transient signals (such as acceleration vs. time) should be plotted as a continuous line.

Part II: Response to an Oscillating Force

You will now measure the driven frequency response of an ultrasonic transducer. An ultrasonic transducer is made of a piezoelectric crystal that will deform according to Hooke's law when a stress is applied. The strain on the crystal induces a measurable voltage difference on the opposing faces of the crystal. Similarly, applying a voltage difference to opposing phases of the crystal will induce a strain. Thus, a piezoelectric transducer can be used as either an electronic speaker or a microphone.

Because the piezoelectric crystal deforms according to Hooke's law, it will behave as a driven, damped harmonic oscillator with a certain resonance frequency ω_n . The equation of motion for the surface of the crystal can be written as

$$m\ddot{x} = -kx - \gamma\dot{x} + F_0 \sin(\omega t) \quad (5)$$

where x is the displacement of the crystal surface, m is the mass, k is the spring constant, γ is a damping coefficient, F_0 is the amplitude of the driving force, and ω is the driving frequency. Rearranging Eq. (5), we get

$$\ddot{x} + \frac{1}{\beta}\dot{x} + \omega_n^2 x = \frac{F_0}{m} \sin(\omega t), \quad (6)$$

where $\beta = m/\gamma$ and $\omega_n^2 = k/m$. The solution to Eq. (5) is

$$x(t) = \frac{F_0 \sin(\omega t)}{\beta \sqrt{\left(\frac{\omega}{\beta}\right)^2 + (\omega^2 - \omega_n^2)^2}}. \quad (7)$$

You will measure the output voltage of the piezoelectric transducer as a function of driving frequency ω and compare it with Eq. (7). Specifically, you will measure the resonance frequency ω_n and determine the damping ratio ζ_{UT} .

Procedure

The experimental setup consists of a piezoelectric transmitter (speaker) aimed at a piezoelectric receiver (microphone). Acoustic waves will be generated by driving a piezoelectric transmitter with an oscillating AC voltage input. The transmitter acts as a speaker, sending ultrasonic acoustic waves to the receiver. The acoustic waves cause the receiver to oscillate, generating an AC voltage output.

1. Put the BNC T-adapter on the output of the Tektronix function generator and connect one of the terminals to Channel 1 on the oscilloscope.
2. Connect the other end of the BNC adapter to the ultrasonic transducer (UT) that has a “T” engraved on the back. (The “T” stands for transmitter.) The cable has black heat shrink tubing. (This will be the input to the system that drives it with an oscillating force.)
3. Connect the other UT (the “receiver”) to Channel 2 on the oscilloscope. The receiver has white heat shrink tubing. (This will be the system output, responding to the oscillating force.)
4. Turn on the function generator. Press the “sine” button, then “output menu”, then “load impedance”, then “High Z”, then press “Top Menu” to exit to the main menu.
5. Using a 40 KHz continuous sine, turn up the amplitude on the function generator to 10 Vpp and press the “On” button above the output. Vary the frequency on the function generator using the rotary knob until you find the resonance frequency. Resonance frequency is the frequency at which the output signal has maximum voltage amplitude for a given input signal.

Pro-Tip: Use the left and right arrow buttons below the big wheel knob to change the precision when adjusting the frequency on the function generator.

6. Record the peak-to-peak voltage displayed on the scope for **at least 10 frequencies below and 10 frequencies** above the resonance frequency. Choose these frequencies wisely, so that you get a nice, smooth curve that you can compare to Eq. (6). Make sure you get the entire curve, all the way out to the flat portion on both ends.
7. Lastly, record the two frequencies that give you *half* of the maximum output voltage amplitude that you recorded. Use them to calculate the full-width-at-half-max (FWHM). This will be used to determine the damping ratio ζ_{UT} .

FWHM frequencies: f_{low} _____ f_{high} _____

8. Reset the oscilloscope to factory default by pressing "Default Setup" key below the display.
9. Reset the function generator to factory default by pressing the "Default" key next to the keypad.

Data Processing

The ultrasonic transducer produces a voltage that is proportional to the strain or surface displacement x . Thus, the output voltage is proportional to Eq. (7), such that

$$V(\omega) = \frac{\omega_n V_{\max}}{\beta \sqrt{\left(\frac{\omega}{\beta}\right)^2 + (\omega^2 - \omega_n^2)^2}}, \quad (8)$$

where ω_n is the resonance frequency and V_{\max} is the voltage you measured at the resonance frequency. The damping parameter β can be determined from the full width at half max (FWHM), $\Delta\omega$, of your measured response curve

$$\beta = \frac{\sqrt{3}}{\Delta\omega}. \quad (9)$$

Lastly, the non-dimensional damping ratio ζ_{UT} of the ultrasonic transducer can also be obtained using the equation

$$\zeta_{\text{UT}} = \frac{1}{2\beta\omega_n}. \quad (10)$$

Use the measured parameters ω_n , V_{\max} , and $\Delta\omega$ to compute β and ζ_{UT} . Then, use these parameters to plot Eq. (8) on top of your measured frequency response data. Be careful switching between Hz and rad/s. Try to consistently use the same units of frequency throughout your calculation.

Data Analysis and Deliverables

Create plots and other deliverables listed below. Save the plots as PDFs, import them into either Microsoft Word or LaTeX, and add an intelligent, concise caption. Make sure the axes are clearly labeled with units. Plots with multiple data sets on them should have a legend. **Additionally, write 1 – 3 paragraphs describing the items below.**

IMPORTANT: Refer to the “Data Processing” sections of Parts I and II for instructions on data processing.

1. For the spring-mass, make a plot of three full periods of the Y acceleration (units of m/s^2) as a function of time for one of the tests.
2. For the spring-mass, make a plot of spectral density (amplitude vs. frequency) of the Y acceleration data computed using the FFT code on the A10 web page.
3. For the spring-mass oscillator, make a table containing the following parameters:
 - a. The measured mass of the plastic weight m (kg).
 - b. The measured spring constant k (N/m).
 - c. The *theoretical* natural resonance frequency f_n (Hz).
 - d. The natural resonance frequency f_n (Hz) measured using the stopwatch.
 - e. The natural resonance frequency f_n (Hz) determined from the FFT plot.
4. For the ultrasonic transducers, make a plot of the measured amplitude as a function of frequency with the theoretical curve given by Eq. (8) plotted on top.
 - a. In the caption, report the measured resonance frequency ω_n and damping ratio ζ_{UT} .

Talking Points – Please address the following questions in your paragraphs.

- What was the amplitude of the oscillating acceleration obtained from the accelerometer data?
- Are the values obtained from the accelerometer data and FFT code consistent with the simple measurements from Part I?
- Based on the measured damping ratio, was the ultrasonic transducer underdamped or overdamped?

Appendix A

Equipment

Part I (set up on inner lab benches)

- Ruler or meterstick
- Stop watch
- Steel extension springs – Amazon #: B078NYV36C
- Lab stand w/ threaded rod and nuts (5/16” – 18)
- Plastic weight with metal hooks
- Digital scale
- 3M Command Small Refill Strips, White, (64 strips) – Amazon #: B0751RPC6Q
- MicroUSB **data** cable
- SparkFun Logomatic v2 - Serial SD Datalogger (FAT32)
- Lithium Ion Battery - 400mAh
- SparkFun Triple Axis Accelerometer Breakout - ADXL337
- Female jumper cables

Part II (set up on outer lab benches)

- BNC cable (24”- 36”)
- BNC “T” adapter
- 80/20 transducer assembly w/ 2 piezoelectric transducers w/ 36” cable ending in BNC connector
- Tektronix AFG 3021 Function Generator
- Tektronix DPO 3012 Digital Oscilloscope
- Allen wrench

Appendix B

MODE = 2
ASCII = Y
Baud = 4
Frequency = 200
Trigger Character = \$
Text Frame = 100
AD1.3 = N
AD0.3 = Y
AD0.2 = Y
AD0.1 = Y
AD1.2 = N
AD0.4 = N
AD1.7 = N
AD1.6 = N
Safety On = Y