For the following questions, please express your answers as algebraic equations written on a separate sheet of paper, and show your work. Then, transcribe the equations into your lab notebook.

- 1. Consider a  $100\Omega$  heater being switched ON and OFF between 0 and 12V at a rate of 500Hz.
  - a. Sketch a plot of voltage vs. time for the 500Hz, 12V square wave with a 30% duty cycle.
  - b. Sketch a plot of *power* dissipated in the resistor vs. time with a 30% duty cycle.
  - c. Derive and equation of the *average* power dissipated as a function of the % duty cycle.
  - d. Calculate the average power if the duty cycle is 80%.
- 2. Similar to the C2 pre-lab assignment, write down the differential equation for the temperature T for a simple proportional feedback controller, where  $\dot{q} = k_p(T_s T)$ .
- 3. Using the equation you just wrote, derive an equation for the equilibrium temperature in terms of the system parameters:  $mc_V$ , hA,  $k_p$ , etc. How does is compare to the setpoint  $T_S$ ? Will the actual temperature converge to the set-point  $T_S$ ?
- 4. Using your equation from problem 2, derive an equation for the thermal time constant in terms of the system parameters: m,  $c_V$ , h,  $k_p$ , etc.
- 5. Sketch the time constant as a function of the proportional gain  $k_p$ .
- 6. Write down the *system* of differential equations for the temperature T and integral of temperature  $I = \int (T_s T) dt$  for the full PID controller.