Experiment A2
Galileo's Inclined Plane
Procedure

Deliverables: Checked lab notebook, **Full lab report (including the deliverables from A1)**

Overview

In the first part of this lab, you will perform Galileo's famous inclined plane experiment. You will then learn several fundamental techniques to analyze the data. Specifically, you will *empirically* demonstrate a mathematical relationship for distance x vs. time t for a body in gravitational free-fall and extrapolate the acceleration of gravity g. This experiment is of great historical significance, as it later inspired Isaac Newton to invent calculus.

In the second part of this lab, you will examine a curved ramp called a "Brachistochrone". As a ball rolls down such a curve, it undergoes a *variable* acceleration that results in some unique behavior. You will examine this behavior by repeating Galileo's experiment using a Brachistochrone shaped ramp.

Part I: Galileo's Inclined Plane

In this experiment, you will roll a ball down an inclined plane and measure the time t it takes to travel a distance x.

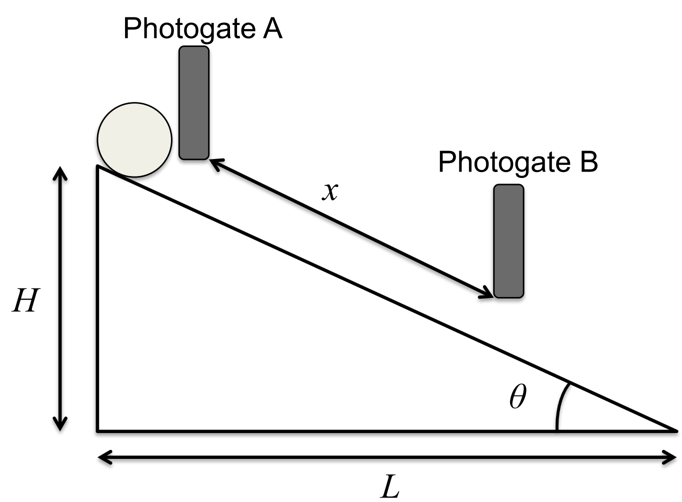

Figure 1 – A schematic representing the inclined plane experiment.

According to Newtonian Mechanics, the trajectory of a sphere rolling down an inclined plane at an angle θ is given by

$$x(t) = \frac{1}{2}\left(\tfrac{5}{7} g \sin\theta\right) t^2 + v_0 t + x_0 \qquad (1)$$

where g is the acceleration of gravity near the surface of earth.

1. Set the inclined plane angle θ to a shallow angle between 1° and 15°. Determine the angle by measuring the length of the legs L and H and using the appropriate trig function. Record all values in your lab notebook.
2. Position Photogate A near the top of the inclined plane as shown in Fig. 1 and connect it to the LabQuest via Digital Port 1 (DIG 1).
3. Position the photogate B a distance x = 10 cm away from top of the inclined plane and connect it to the LabQuest via Digital Port 2 (DIG 2).
4. Plug the LabQuest in and then turn it on.
5. In the "LabQuest App", **File > New** on the drop down menu.
6. On the Sensors tab, select **Sensor > Sensor Setup**. Under "DIG 1" select the "Photogate" from the drop down box and then hit OK. Repeat this for "DIG 2".
7. Again on the Sensors tab, select **Sensor > Data Collection** and choose the following parameters:

   Mode: Photogate timing

   Photogate mode: Pulse

   Distance between gates: 1m (It doesn't really matter what you put here.)

   End data collection: check "with the stop button"

   Under the "Pulse" mode, blocking Photogate A will start a timer in the LabQuest and blocking Photogate B will stop the timer. Exit the menu by pressing the "**Ok**" button.
8. Press the "▶" button to begin collecting data from the photogates. (Choose to discard any unsaved data if it asks.)
9. Make a table in your lab notebook with two columns for x and t. Be sure note the units of both.
10. Measure the distance x between the two photogates using the meter stick provided and record it in the table in your notebook.
11. Make sure the photogates are set so that the light sensor will pass through the center of the billiard ball.
12. Place the billiard ball **directly behind Photogate A** and release it. Locate the "**Pulse Time**" in the upper right corner of the LabQuest. Record it in the table in your lab notebook.
13. Without moving the photogates, repeat steps 8 – 11 four more times. This will give you a total of 5 data points for the one distance that you will average together.
14. Move Photogate B 10 cm further from the top (increase x) and repeat steps 7 – 13 for distances up to and including x = 50cm.
15. Change the angle of the inclined planed to a different value between 1° and 15° and repeat the entire procedure.

Part II: Brachistochrone

"In one physical model of the universe, the shortest distance between two points is a straight line… in the opposite direction." - Ty Webb, *Caddyshack*

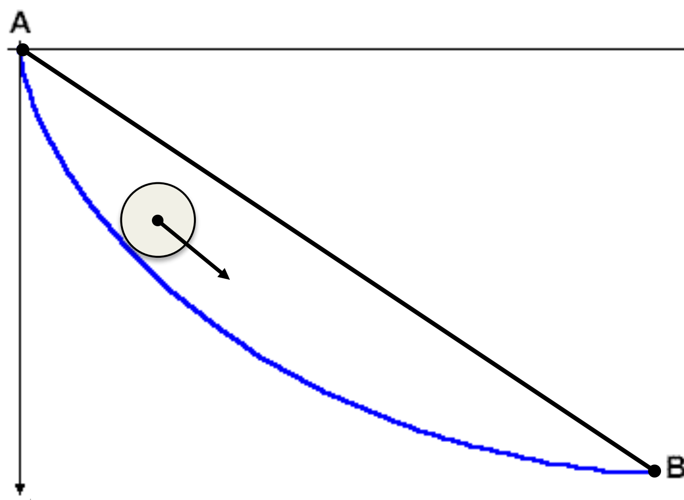

Figure 2 – The path of shortest *distance* between points A and B is a straight line (black curve). The path of shortest *time* for a ball rolling from A to B is called a "Brachistochrone" (blue curve).

In this exercise, you will repeat the previous measurements using a special curved ramp called a Brachistochrone.

1. Sketch the Brachistochrone in your lab notebook.
2. Measure the total *vertical* height of the Brachistochrone curve. Record the value in your lab notebook. This parameter is necessary to calculate the theoretical time for the ball to reach the bottom.
3. Use the magnetic mount to fix Photogate A near the top of the Brachistochrone as shown in Fig. 2 and connect it to the LabQuest via Digital Port 1 (DIG 1).
4. Photogate B is fixed at the bottom of the Brachistochrone. Connect it to the LabQuest via Digital Port 2 (DIG 2).
5. Plug the LabQuest in and then turn it on.
6. In the "LabQuest App", **File > New** on the drop down menu.
7. On the Sensors tab, select **Sensor > Sensor Setup**. Under "DIG 1" select the "Photogate" from the drop down box and then hit OK. Repeat this for "DIG 2".
8. Again on the Sensors tab, select **Sensor > Data Collection** and choose the following parameters:

   Mode: Photogate timing

   Photogate mode: Pulse

   Distance between gates: 1m (It doesn't really matter what you put here.)

   End data collection: check "with the stop button"

Under the “Pulse” mode, blocking Photogate A will start a timer in the LabQuest and blocking Photogate B will stop the timer. Exit the menu by pressing the “**Ok**” button.

9. Press the “▶” button to begin collecting data from the photogates. (Choose to discard any unsaved data if it asks.)
10. Make a table in your lab notebook with two columns for x and t. Be sure note the units of both.
11. Measure the **straight linear** distance x between the two photogates using the meter stick provided and record it in the table in your notebook.
12. Make sure the photogates are set so that the light sensor will pass through the center of the stainless steel ball bearing.
13. Place the stainless steel ball **directly behind Photogate A** and release it. Locate the “**Pulse Time**” in the upper right corner of the LabQuest. Record it in the table in your lab notebook.
14. Without moving the photogates, repeat steps 8 – 11 four more times. This will give you a total of 5 data points for the one distance that you will average together.
15. Move Photogate A to the next lowest magnetic mounting point and repeat steps 8 – 11 until you reach the end of the track.
16. Do some research on the Brachistochrone and Tautochrone. What is the theoretical time to get from the top to the bottom of the curve? How does it compare to your measured data?

Data Analysis and Deliverables

Download the LaTeX or MS Word template from the course website and use it to write a lab report, **no longer than 7 pages**. You are required to include the following items in your lab report. (See the A1/A2 score sheet for points.)

Pro-Tip: Export all of your figures as either PDF or EPS files. (JPEG and PNG files often appear grainy and pixelated in your final report.)

The following deliverables should be presented in **SI units.**

1. **Extrapolating *g***
   a) Using your data from Part I, make a plot of distance x as a function of average measured time t for both angles. (Distance x is on the vertical axis. Average time t is on the horizontal axis. You should have the data for both angles on the same graph.)
   b) Using the "fit()" command in Matlab, perform a *quadratic* curve fit on each of the two data sets.
   c) Plot the two quadratic curve fits on top of your data.
   d) Based on Eq. (1), write an **algebraic** equation for each of the three fitting parameters in the quadratic equation. ("Algebraic" means leave the parameters as symbolic variables.)
   e) Use the coefficient for the second order term (the constant in front of t^2 term) that you get from the curve fit to *extrapolate g*.
   f) The fit command also outputs a "95% confidence interval" for each fitting parameter. The width of this interval is equal to twice the uncertainty in the parameter. Use the confidence interval for the second order coefficient to determine the uncertainty in g.
   g) Report the two values of g in the caption of the plot along with their uncertainty. Report it as g = value ± uncertainty m/s^2. Round the uncertainty to two significant digits. Round the value to the least significant digit of the uncertainty.
2. Brachistochrone - Make a plot of the *measured* distance x as a function time t for the Brachistochrone data along with the theoretical curve.

   Talking Points – Please discuss the following in your lab report.

   - Compare your extrapolated value of g with the widely accepted value of 9.81 m/s^2. Is the accepted value within your error bars? What physical effects might cause them to be different?

   - Do a bit of research about the Brachistochrone and Tautochrone. What is the equation for distance vs. time?

Appendix A

Equipment

Part I

- Inclined plane
- Billiard ball
- Billiard pocket (attached with rubber band
- Cable ties and rubber bands to attach billiard pockets
- Meter stick
- Level
- Vernier LabQuest
- 2 Photogates (Vernier VPG-BTD) with magnetic L-brackets
- 2 Photogate “DIG” cables

Part II

- Brachistochrone ramp with wood clamp feet
- 2 Meter sticks
- Vernier LabQuest
- Photogate (Vernier VPG-BTD) with magnetic Z-bracket
- Photogate with aluminum rod
- Lab stand (beaker stand)
- 2 Photogate “DIG” cables
- 1.5” diameter stainless steel ball bearings