AME40453 - Automation and Controls
C3 Pre-Lab Assignment

For the following questions, please express your answers as algebraic equations written on a separate sheet of paper, and show your work. Then, transcribe the equations into your lab notebook.

1. Similar to the C2 pre-lab assignment, write down the differential equation for the temperature T for a simple proportional feedback controller, where $\dot{q} = k_p(T_S - T)$.
2. Using the equation you just wrote, derive an equation for the equilibrium temperature in terms of the system parameters: mc_V, hA, k_p, etc. How does is compare to the set-point T_S? Will the actual temperature converge to the set-point T_S?
3. Using your equation from problem 2, derive an equation for the thermal time constant in terms of the system parameters: m, c_V, h, k_p, etc.
4. Sketch the time constant as a function of the proportional gain k_p.
5. Write down the *system* of differential equations for the temperature T and integral of temperature $I = \int (T_S - T)\,dt$ for the full PID controller.
6. Rewrite the system of equations in terms of the variable $x = T - T_S$. Note that the integral becomes $I = -\int x\,dt$ and the derivatives are the same, $\frac{dx}{dt} = \frac{dT}{dt}$.
7. Rewrite the system of differential equations in matrix form $\frac{d}{dt}\begin{bmatrix} x \\ I \end{bmatrix} = \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}\begin{bmatrix} x \\ I \end{bmatrix} + \begin{bmatrix} \dots \\ \dots \end{bmatrix}$. (Fill in the blanks "…")
8. Derive an equation for the eigenvalues in terms of the system parameters hA, mc_p, k_p, k_I, and k_D.
9. Derive an equation for the critical value of the integral gain k_I that will cause the controller to begin oscillating. i.e. $k_I > f(k_p, k_D, hA, mc_p)$