Experiment A10
Transient Signals
Procedure

Deliverables: Checked lab notebook, printed plots with captions

Overview

In this lab, you will investigate the dynamic response of measurement systems. A thermocouple serves as a great example of first-order transient response, and a piezoelectric ultrasonic transducer is used to demonstrate second-order system transient response. A handheld LabQuest unit will be used for acquiring data from the thermocouple, and the frequency response of the ultrasonic transducer will be measured using an oscilloscope.

Part I: First-Order Transient Response

A thermocouple (TC) acts as a typical first-order dynamic system due to heat transfer properties of the TC. The inner workings of the TC consist of a pair of dissimilar wires connected via two junctions called a hot and cold junction. A difference in temperature between the junctions forms a voltage difference across them. This voltage is proportional to the temperature difference.
Material and size of the metal probe will affect the heat distribution in the thermocouple rod and can either shorten or lengthen the time it takes to reach a steady-state temperature. The exchange of heat through the thermocouple can be described by:

$$mC_v \frac{dT}{dt} = hA_s[T_\infty - T(t)], \tag{1}$$

where m is the tip mass, C_v is the specific heat at a constant volume for the tip, A_s is the tip surface area, h is the heat transfer coefficient, and T_∞ is the far-field temperature of the surrounding fluid. The explicit solution to the differential equation is:

$$T(t) = T_\infty + (T_0 - T_\infty) \exp\left(\frac{-t}{\tau}\right). \tag{2}$$

where T_0 is the initial temperature and the time constant $\tau = \frac{mC_v}{hA_s}$. Mathematically, the time constant represents the time it takes a step input response to reach $1 - \frac{1}{e} \approx 63.2\,\%$ of the steady- state temperature. Equation (2), as well as your measured data, can be linearized by the transformation:

$$y(t) = \ln\left(\frac{T(t) - T_\infty}{T_0 - T_\infty}\right). \tag{3}$$

Plotting your transformed data $y(t)$ as a function of time should yield a straight line with a slope of $-1/\tau$.

Procedure

1. Ensure that the small (1/16") thermocouple (TC) is connected to the thermocouple amplifier box (TAB), and then turn the TAB to ON. Connect the TAB to Channel 1 on LabQuest using a BNC cable and the provided 5 volt cable. Be sure that the source and GND are connected properly. Record TAB sensitivity here: ___________________________.
2. Turn the power on the LabQuest unit ON. Go to 'Sensors' and select 'Sensor Setup.'
3. Set Channel 1 to Voltage → Voltage (+/- 10V).
4. Edit the data acquisition options by selecting the region with *mode, rate,* and *length*. Choose an appropriate rate (35 samples/second) and set the duration to be long enough to see the steady-state behavior (about 10 seconds is a good starting duration).
5. Without the TC in the water, begin acquiring data by pressing the (▶) symbol on LabQuest. After a few seconds, submerge the tip of the TC into the *ice water*; best results are obtained when the TC end is suspended. Do not let the tip of the TC rod touch the bottom of the beaker. Allow LabQuest to take data for the entirety of the length specified.
6. Compare ambient temperature using mercury thermometer to reading on labquest. Let TA know if there is a significant difference.
7. Save the data to a flash drive. Insert the drive in the LabQuest's USB port. Go to File → Export. Click the flash drive icon in the dialog, enter a file name, and select *OK*.
8. Fill a beaker with warm water. Fill another beaker with ice water.
9. Let the TC sit in the ice water for about a minute. Press the (▶) symbol on LabQuest. After a few seconds, remove the TC from the ice water and immediately submerge it in the *warm water*.
10. Connect the LabQuest to the computer and transfer the data to the computer using the LoggerPro software. Export the data from LoggerPro as a CSV file. Be sure to give it a good, descriptive file name.
11. Repeat steps 4 – 10 using the large (1/4") thermocouple. Set the duration to 25 seconds.

Part II: Second-Order Transient Response

For the experiment, you will find the resonance frequency of an ultrasonic transducer and measure its output as a function of frequency. The ultrasonic transducers consist of a piezoelectric crystal that will deform according to Hooke's law when a stress is applied. The strain on the crystal induces a measurable voltage difference on the faces of the crystal. Similarly, applying a voltage difference to opposing phases of the crystal will induce a strain. Thus, the piezoelectric transducers can be used as either a speaker or a microphone.

Because the piezoelectric crystal deforms according to Hooke's law, it will behave as a driven, damped harmonic oscillator with a certain resonance frequency ω_0. The equation of motion for the surface of the crystal can be written as:

$$m\ddot{x} = -kx - \gamma\dot{x} + F_0 \sin \omega t \tag{4}$$

where m is the mass, k is the spring constant, γ is a damping coefficient, F_0 is the amplitude of the driving force, and ω is the driving frequency. Rearranging Eq. (4), we get

$$\ddot{x} + \frac{1}{\tau}\dot{x} + {\omega_0}^2 x = \frac{F_0}{m} \sin \omega t\ , \tag{5}$$

where $\tau = \frac{\gamma}{m}$, and ${\omega_0}^2 = \frac{k}{m}$. The solution to Eq. (5) is

$$x(t) = \frac{F_0 \sin \omega t}{m\sqrt{\left(\frac{\omega}{\tau}\right)^2 + (\omega^2 - {\omega_0}^2)^2}}\ . \tag{6}$$

You will measure the output voltage of the piezoelectric transducer as a function of frequency and compare it with Eq. (6).

Procedure

1. Put the BNC T-adapter on the output of the Tektronix function generator and connect one of the terminals to channel 1 on the oscilloscope.
2. Connect the other end of the BNC adapter to the ultrasonic transducer (UT) that has a "T" engraved on the back. (The "T" stands for transmitter.) The cable has black heat shrink tubing. The receiver has white heat shrink tubing.
3. Connect the other UT to channel 2 on the oscilloscope.

4. Using a 40 KHz continuous sine, turn up the amplitude on the function generator to 10 Vpp and press the "On" button above the output. Vary the frequency on the function generator until you find the resonance frequency. Resonance frequency is the frequency at which the output signal has maximum voltage for a given input signal.
5. Record the peak-to-peak voltage displayed on the scope for at least 10 frequencies below and 10 frequencies above the resonance frequency. Choose these frequencies wisely, so that you get a nice, smooth curve that you can compare to Eq. (6). Make sure you get the entire curve, all the way out to the flat portion on both ends.

Pro-Tip: Use the left 🡐 and right 🡒 arrow buttons below the big wheel knob to change the precision when adjusting the frequency on the function generator.

6. Lastly, record the frequencies that give you *half* of the maximum output voltage that you recorded. Use them to calculate the full-width-at-half-max (FWHM).

f_{low} ___________ f_{high} ___________

Part III: Measuring Vibrations in a Baseball Bat

Background

A baseball bat is a familiar example of a mechanical, dynamic system. When the bat strikes the ball, the impact causes some elastic deformation of the bat, displacing it from equilibrium some distance y. Because the bat has mass and elasticity, we can model it as a harmonic oscillator. The batter's hands absorb energy from these vibrations, providing an effective damping force—an effect some of you may be familiar with. Hence, the baseball bat behaves very much like a *damped harmonic oscillator*, which we studied in the previous lab. However, the striking of the ball is an *impulse force*, because it happens 'instantaneously' as opposed to the sinusoidal driving force we saw last week.

When a damped harmonic oscillator is hit with an impulse, it will respond by "ringing". This ringing behavior is described by the equation

$$y_{bat}(t) = Ae^{-\lambda t}\sin(\omega_d t + \phi), \tag{7}$$

where ω_d is the ringing frequency, λ is the damping coefficient scaled by mass, and A and ϕ are constants dependent on initial conditions. In this lab, you will use strain gauges to measure the scaled damping coefficient λ, the ringing frequency ω_d, and the phase ϕ of a vibrating aluminum baseball bat.

The bat for this laboratory exercise is outfitted with four strain gages in a full bridge configuration, which respond to the displacement of the bat y_{bat} versus time t. The strain gages are connected to an amplifier to increase the output voltage, and the output voltage is recorded by an oscilloscope, which samples at a very high rate (GHz – which easily satisfies Shannon's sampling theorem). The oscilloscope voltage directly relates to the magnitude of the displacement.

Setup and Data Acquisition
The baseball bat will be connected to the amplifier and oscilloscope when you as shown in Figure 1.

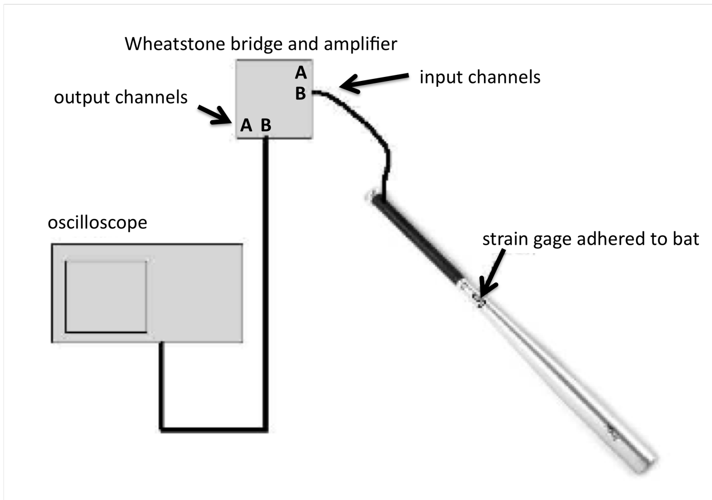

Figure 1 - Schematic of experimental apparatus and measurement system.

1. Press “AutoSet”.
2. Press yellow 1 button and set the input channel to “AC” on the oscilloscope. A steady line with some small oscillations should appear. This is a real time measurement of the signal from amplifier and is the background signal from the electrical connections corresponding to the noise of the measurement system.
3. Adjust the voltage scale(Vertical) on Channel 1 to be 50 mV/division and adjust the time scale(Horizontal) to be 10 ms/division. This is the scaling required to observe the response of the baseball bat.
4. Set the oscilloscope to trigger in response to an impulse forcing function. This means that the oscilloscope is prepared to record data (trigger) at the moment of the impulse on the bat:
   - Press the “Trigger” menu button
   - Select the slope menu and choose the icon shown in Fig. 2.
   - Set the source as Channel 1(the trigger is looking for a signal from Channel 1)
   - Set the mode as “Normal”(the trigger is waiting for a single event or impulse to occur), and the coupling as “DC”
   - Move the trigger level knob so that the trigger level is at 20mV – this ensures that the oscilloscope will not trigger until the signal slope falls to this level
   - Using the horizontal position knob, move the trigger arrow, orange “T” at the top of the screen, to the extreme left of the screen
   - Press “Single Sequence” button – the green LED should light

5. Above the graph readout on the right hand side there will be some text. Wait for the text to read "Trig?" before proceeding.
6. Keeping safety and those around you in mind, hit the ball with the bat. Do not swing hard as the strain gages will be broken or displaced.
7. Repeat this procedure until a good set of data has been obtained for **both** the single wall and double wall bat.

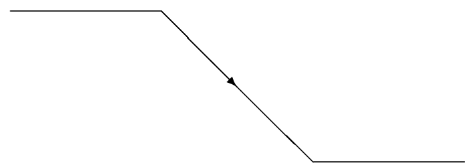

Figure 2 - Selection for Slope Menu on digital oscilloscope.

Saving Data

1. Press MENU button on the bottom of the scope in the "Save/recall" section.
2. Insert flash drive into scope.
3. Press "Save Waveform" on soft menu on the bottom of the screen.
4. "Source" should be "Ch 1".
5. Use the "destination" knob to highlight the "TEK????CH1.CSV" file. The "?????" will be replaced by the next sequential number, starting at 0000 when no other files are present.

Fitting Your Data

Your data should look similar to Fig. 3. It should be a nice sinusoid with an exponential decay envelope, just as you would expect for the ringing of a damped harmonic oscillator. If your data is noisy, try using the smooth() function in Matlab to filter out the noise. For your lab reports, you will need to quantitatively compare your data with Eq. (7). To do this, we will use the data to calculate λ, ω_d, and ϕ and estimate A.

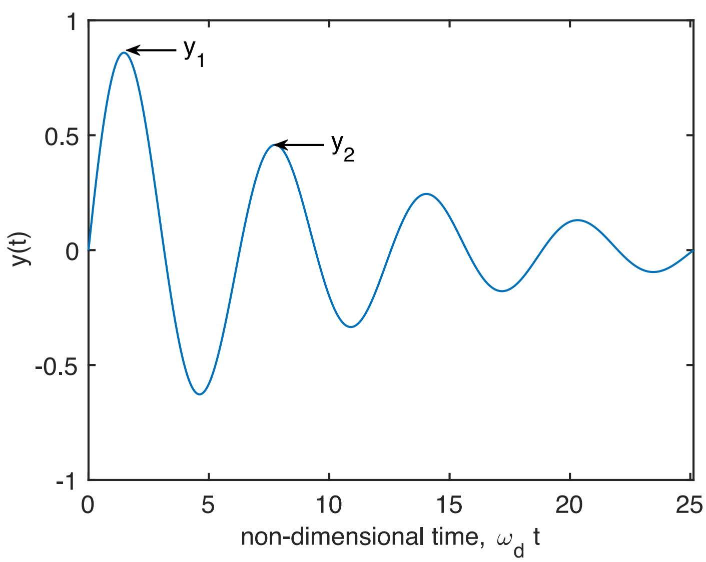

Figure 3 – An example of "ringing" behavior for a damped harmonic oscillator.

Consider the peaks labeled y_1 and y_2. Using Eq. (7), we obtain

$$\frac{y_1}{y_2} = \frac{e^{-\lambda t_1}}{e^{-\lambda t_2}} = e^{\lambda \Delta t} , \tag{8}$$

where $\Delta t = t_2 - t_1$. Equation (8) can then be solved for the damping coefficient

$$\lambda = \frac{\ln\left(\frac{y_1}{y_2}\right)}{\Delta t} . \tag{9}$$

To obtain the ringing frequency ω_d, run the FFT script in Matlab and locate the peak in the plot of relative amplitude vs. frequency. To obtain the phase ϕ in radians, simply find the data point corresponding to the ringing frequency ω_d in the plot of phase vs. frequency. Note that the frequency in both plots is in units of Hz, so ω_d will need to be converted to radians before inserting it in Eq. (6). Also note, the phase ϕ depends on when your scope decided to "trigger" and begin recording data. In this sense, it is somewhat arbitrary and will vary from student to student.

Now that you have ω_d and λ, try to plot Eq. (7) on top of your data. You will need to estimate the amplitude *A*. In practice, a calibration procedure would be used to convert your data from voltage to units of displacement or strain. For this lab, we will forgo the calibration procedure and estimate the amplitude *A* instead. To do this, simply adjust *A* in you Matlab script until both waveforms are approximately the same height.

Lastly, the damping ratio ζ_B of the baseball bat can be obtained using the equation

$$\zeta_B = \frac{\lambda^2}{\lambda^2 + \omega_d^2} . \tag{10}$$

Data Analysis and Deliverables

Create plots and other deliverables listed below. Save the plots as PDFs, import them into either Microsoft Word or LaTeX, and add an intelligent, concise caption. Make sure the axes are clearly labeled with units. Plots with multiple data sets on them should have a legend. **Additionally, write a paragraph for each plot (separate from the caption) describing what you did in lab to obtain the plot.**

1. Make a plot of the temperature vs. time for heating and cooling with both size thermocouples. Be sure to include a legend to distinguish the different data sets.

2. Use Eq. (3) to **linearize** your temperature data. Cut out the portion of the data before you put it in the water, and cut out the noisy portion after it reaches steady state. Remember: *both* heating and cooling should have a *negative slope* after the transformation has been applied. Make a **single plot** of linearized temperature vs. time data for all four data sets with a linear curve fit plotted on top of each one.

3. Use the slopes from the linear curve fit to extrapolate the time constants for heating or cooling of the 1/16" or 1/4" thermocouples. Present the four time constants in a table with a proper caption.

4. For the ultrasonic transducers, make a plot of the measured amplitude as a function of frequency with the theoretical curve plotted on top. See Appendix A for details. Your curve may have two humps. If so, explain why in the caption. *Hint*: Not all ultrasonic transducers are created equally.

5. Use the "full width at half max" to determine the **damping ratio** ζ_{UT} of the ultrasonic transducer, and report it in the caption of deliverable 4. See Appendix A for details.

6. Make a plot of the **amplitude** $|y(f)|$ **vs. frequency** f for the baseball bat using the FFT code.

7. Report **ringing frequency** f_d of the baseball bat in units of Hz. This should go in the caption of deliverable 6.

8. Make a plot of the measured **strain gauge output** $y(t)$ **vs. time** t for the baseball bat with a **curve fit using Eq. (7)**. For the curve fit, the ringing frequency ω_d and phase ϕ should be obtained from the FFT plots, while the damping coefficient λ should be obtained from Eq. (11) above. For the amplitude A, you will simply have to guesstimate a value.

9. Report **damping ratio** ζ_B of your baseball bat data in the caption for deliverable 8.

Appendix A

Full Width at Half Max

As you saw in this lab, the piezoelectric, ultrasonic transducers behaved as damped harmonic oscillators. The displacement (or strain) of a driven, damped harmonic oscillator is:

$$x(t) = \frac{F_0 \sin \omega t}{m\sqrt{\left(\frac{\omega}{\beta}\right)^2 + \left(\omega^2 - {\omega_0}^2\right)^2}} \ . \tag{A1}$$

A piezoelectric outputs a voltage proportional to strain, so the voltage response curve takes the exact form as Eq. (A1), just with a different amplitude.

Here is how to use Eq. (A1) to "curve fit" your data. First calculate the damping parameter β using the full width at half max, $\Delta\omega$, of your measured response curve. The full width at half max is simply the width of the curve at half of the maximum value.

These two quantities are related by the following equation:

$$\Delta\omega = \frac{\sqrt{3}}{\beta} \ . \tag{A2}$$

Now that you know β, you can plot a curve fit on top of your data. For the range of frequencies that you measured, plot the following formula on top of your data:

$$V_{fit}(\omega) = \frac{\omega_0 V_{\max}}{\beta\sqrt{\left(\frac{\omega}{\beta}\right)^2 + \left(\omega^2 - {\omega_0}^2\right)^2}} \ , \tag{A3}$$

where ω_0 is the resonance frequency and $V_{\max}$ is the voltage you measured at the resonance frequency. Please plot your curve fits as solid lines and your data as individual points. If you have any questions, please contact the TA.

Lastly, the damping ratio ζ_{UT} of the ultrasonic transducer can also be obtained using the equation

$$\zeta_{UT} = \frac{1}{2\beta\omega_0} \ . \tag{A4}$$

Appendix B

Equipment

Part I

- Thermocouple
- Thermocouple Amplifier Box (TAB)
- LabQuest 1 (Aqua-colored) w/ DC power supply
- LabQuest 1 input cable (5V with male banana ends – 24” length)
- BNC connector to female banana receptor
- Large Styrofoam cups (2)
- Hot and Cold water
- Ice cubes

Part II

- BNC cable (24”- 36”)
- BNC “T” adapter
- 80/20 transducer assembly w/ 2 piezoelectric transducers w/ 36” cable ending in BNC connector
- Tektronix AFG 3021 Function Generator
- Tektronix DPO 3012 Digital Oscilloscope
- Allen wrench

Part III

- 2 – VGA cable to scope
- 2 – Omega DMD520 strain gage amplifier
- 2 – Tek DPO3012 scope
- 1 –Aluminum bat Single wall
- 1 – Aluminum bat Double wall
- 2 – Tethered softball with stand
- 2 – Bags of tube sand