Experiment A10
Second Order Transient Response
Procedure

Deliverables: Checked lab notebook, Brief Tech Memo

Recommended Reading: Sections 7.3 and 9.1, Chapters 18, 19, 20, and 22

Background

Structures used in aerospace and mechanical engineering tend to vibrate and oscillate. Really, any system—mechanical, chemical, thermal, or electrical—with dynamic behavior governed by a second order differential equation of the form

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y = f(t) \qquad (1)$$

will tend to oscillate or vibrate at a natural resonance frequency ω_n. The damping ratio ζ determines how long oscillations will persist. The term on the right-hand side $f(t)$ represents external forces or inputs to the system. In this lab, we will exam two well-known external forces:

- **Step Input or "Impulsive" Force** – A pendulum will be held up at an initial angle, then abruptly released. The pendulum will swing back and forth at the ringing frequency, and the oscillations will decay at a rate related to the damping ratio.

- **Oscillating Force** – A piezo electric crystal will be subject to acoustic pressure waves, which will cause it to mechanically vibrate at the same frequency. The vibrations will be greatest when it is driven by acoustic pressure waves with a frequency near the mechanical natural resonance frequency of the crystal.

Part I: Pendulum – Response to an Impulse

Theoretical Background

Illustrated in Fig. 1, a pendulum can be modeled as a mass m located a distance R away from the axis of rotation. (The distance R is often referred to as the "radius of gyration".) Balancing the angular acceleration with the torque from gravity and a viscous drag force from the surrounding air yields the equation of motion for the pendulum

$$mR^2\ddot{\theta} = -mgR\sin\theta - \gamma R^2\dot{\theta}, \qquad (2)$$

where γ is the viscous drag force coefficient. If we assume the θ is small, then $\sin\theta \approx \theta$, and Eq. (1) can be re-written as

$$\ddot{\theta} + \frac{\gamma}{m}\dot{\theta} + \frac{g}{R}\theta = 0. \qquad (3)$$

Comparing Eq.(3) to Eq. (1), we see that the pendulum will oscillate with some measurable natural resonance frequency and damping ratio.

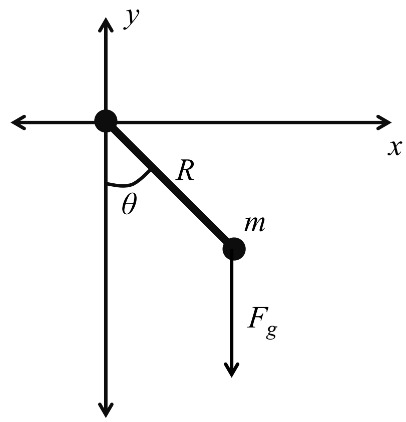

Figure 1 – A schematic illustrating the geometry of the single pendulum model.

Experimental Background

Measuring and controlling the angular position and velocity of rotating mechanical parts is often a critical aspect of mechanical design. You will use a digital sensor called a quadrature rotary encoder and an Arduino UNO microcontroller to measure the angular position of the pendulum. The quadrature encoder contains a slotted wheel, which is mounted to the rotating shaft. Illustrated in Fig. 2, the slotted wheel rotates through two photogates (similar to the Photogates in the inclined plane lab), and an electronic pulse train is produced. The Arduino microcontroller is used to count the number of pulses, and the number of counts in proportional to the angle of rotation. Using two Photogates (A and B) allows it to determine the direction of rotation. The encoders you will use in this lab have 2000 slots, so a full 360° rotation results in 2000 counts.

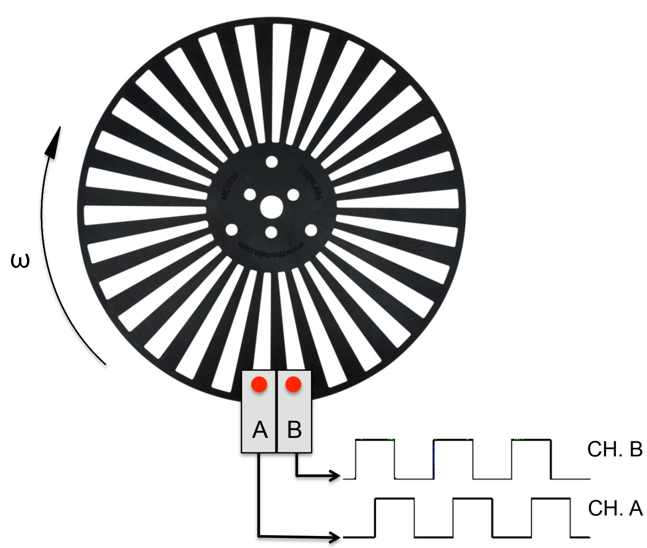

Figure 1 – A schematic illustrating the operation of a quadrature encoder.

Sensor Implementation Procedure

Please follow the instructions below to connect the encoder to the Arduino and initialize data collection from the Arduino.

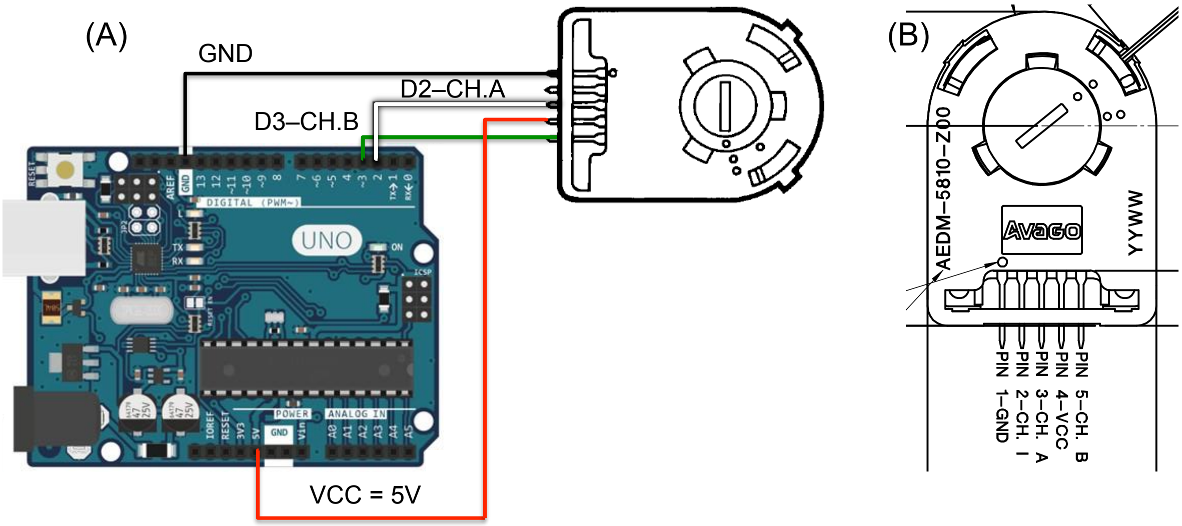

Figure 3 – (A) A wiring diagram illustrating how to connect the rotary encoder to the Arduino. **(B)** A pinout for the rotary encoder. Pin 1 is connected to ground (GND – BLACK) on the Arduino. Pin 2 is not connected. Pin 3 (CH. A - WHITE) is connected to digital input D2. Pin 4 (VCC - RED) is the 5V supply voltage. Pin 5 (CH. B - GREEN) is connected to digital input D3.

1. Make sure the locking bolt is in the pendulum, such that θ_2 cannot rotate. **Do not over tighten the bolt! You will wreck the bearings!**
2. Make sure the USB cable is NOT connected to the PC.
3. Connect the 5 pin female connector of the cable to the θ_1 encoder such that the black wire is on top. Connect the male pins on the other end of the cable to the Arduino board as shown in Fig. 3. Black is ground, red is 5V, green is pin 3, and white is pin 2.

   WARNING: Incorrect wiring can severely damage or break the encoder! Make sure it is properly connected before moving on to the next step.

4. Connect the Arduino to the lab bench PC with the USB cable. A green light should appear and remain on the Arduino. If it does not, please ask the lab instructor for assistance.
5. Download the quadrature encoder code from the A10 webpage. Open it with the Arduino IDE software. Take a moment to admire the code and try to understand what it will do.
6. Click on the "Tools" menu in the Arduino IDE software. On the drop down menu, go to "Port", and select the one that says COM# "(Arduino/Genuino Uno)", where # will be a number corresponding to the COM port being used for the Arduino. **Write down the number of the COM port corresponding to the Arduino in your lab notebook.**

7. Click on the button with a checkmark (✓) symbol to compile the program, then click the button with an arrow (➔) next to it, to upload the program on the Arduino.
8. Next, you will use a program called PuTTY to save the serial output from the Arduino to a text file.
9. Launch the PuTTY software from the desktop. Select "Session" on the left and check "Serial" as the connection type. Make sure the baud rate is 9600, if it is already not set to the value. Then, enter the COM# you wrote down earlier.
10. Select the "Logging" tab right below session on the left side of the PuTTY app. In the option below "Session logging" select "Printable output". Click "Browse" to choose the file name and destination. Give it an appropriate name and save it on the computer as a .txt file.
11. Click the "Open" button, and a black terminal window should appear. Give the pendulum a tap and allow it to swing to and fro for a few cycles. You should see numbers appear in the terminal corresponding to the encoder count, time in ms, and displacement angle in degrees.
12. Close the window, and find the .txt file you specified in step 10. Open it, and check to make sure it contains the data that was printed to the terminal.
    a. The column on the left is the number of counts.
    b. The column in the middle is the angle in degrees.
    c. The far right column is the time in milliseconds.
13. Inspect the data in the text file to make sure the values are reasonable.

Data Collection Procedure

1. As you did in the previous section, set up a PuTTY session and measure a trace of angle vs. time for the pendulum as it oscillates.
    a. With the pendulum hanging vertically, click the "Open" button in PuTTY.
    b. After the terminal appears, lift the pendulum up to the stopper and let it go.
    c. Close the PuTTY terminal window when the pendulum is finished oscillating.

Note: The Arduino program sets the zero angle to be wherever the pendulum is located when the PuTTY program starts a new serial session.

2. Inspect the data to make sure it looks correct.
3. Rename the data set with an intelligent file name. Email them to yourself and your lab partner, or transfer them from the lab computer with a flash drive.

Data Processing – System Identification and Characterization

For the single pendulum, the radius of gyration R and the viscous drag force coefficient γ are difficult to calculate theoretically. It is much easier to extrapolate them from measured data. To do this, the pendulum is subject to an impulse (which you just did). For a single pendulum subject to an impulse, the theoretical solution to Eq. (3) is

$$\theta(t) = \theta_0 e^{-\lambda t}\cos(\omega_d t) \tag{4}$$

where θ_0 is the initial displacement, $\lambda = \gamma/2m$ is the decay constant, and $\omega_d = \sqrt{\frac{g}{R} - \lambda^2}$ is the ringing frequency. The parameters λ and ω_d can be extracted from the measured angle θ (degrees) vs. time t (s) data. The viscous drag force coefficient γ and radius of gyration R can then be determined using the mass of the pendulum $m = 0.36$ kg.

Part II: Piezoelectric Transducers – Response to an Oscillating Force

You will now measure the driven frequency response of an ultrasonic transducer. An ultrasonic transducer is made of a piezoelectric crystal that will deform according to Hooke's law when a stress is applied. The strain on the crystal induces a measurable voltage difference on the opposing faces of the crystal. Similarly, applying a voltage difference to opposing phases of the crystal will induce a strain. Thus, a piezoelectric transducer can be used as either an electronic speaker or a microphone.

Because the piezoelectric crystal deforms according to Hooke's law, it will behave as a driven, damped harmonic oscillator with a certain resonance frequency ω_n. The equation of motion for the surface of the crystal can be written as

$$m\ddot{x} = -kx - \gamma\dot{x} + F_0 \sin(\omega t) \tag{5}$$

where x is the displacement of the crystal surface, m is the mass, k is the spring constant, γ is a damping coefficient, F_0 is the amplitude of the driving force, and ω is the driving frequency. Rearranging Eq. (5), we get

$$\ddot{x} + \frac{1}{\beta}\dot{x} + \omega_n^2 x = \frac{F_0}{m}\sin(\omega t), \tag{6}$$

where $\beta = m/\gamma$ and $\omega_n{}^2 = k/m$. The solution to Eq. (5) is

$$x(t) = \frac{F_0\, sin(\omega t)}{\beta\sqrt{\left(\frac{\omega}{\beta}\right)^2 + (\omega^2 - \omega_n{}^2)^2}}. \tag{7}$$

You will measure the output voltage of the piezoelectric transducer as a function of driving frequency ω and compare it with Eq. (7). Specifically, you will measure the resonance frequency ω_n and determine the damping ratio ζ_{UT}.

Procedure

The experimental setup consists of a piezoelectric transmitter (speaker) aimed at a piezoelectric receiver (microphone). Acoustic waves will be generated by driving a piezoelectric transmitter with an oscillating AC voltage input. The transmitted acts as a speaker, sending ultrasonic acoustic waves to the receiver. The acoustic waves cause the receiver to oscillate, generating an AC voltage output.

1. Put the BNC T-adapter on the output of the Tektronix function generator and connect one of the terminals to Channel 1 on the oscilloscope.
2. Connect the other end of the BNC adapter to the ultrasonic transducer (UT) that has a “T” engraved on the back. (The “T” stands for transmitter.) The cable has black heat shrink tubing. (This will be the input to the system that drives it with an oscillating force.)
3. Connect the other UT (the “receiver”) to Channel 2 on the oscilloscope. The receiver has white heat shrink tubing. (This will be the system output, responding to the oscillating force.)
4. Turn on the function generator. Press the “sine” button, then “output menu”, then “load impedance”, then “High Z”, then press “Top Menu” to exit to the main menu.
5. Using a 40 KHz continuous sine, turn up the amplitude on the function generator to 10 Vpp and press the “On” button above the output. Vary the frequency on the function generator using the rotary knob until you find the resonance frequency. Resonance frequency is the frequency at which the output signal has maximum voltage amplitude for a given input signal.

Pro-Tip: Use the left and right arrow buttons below the big wheel knob to change the precision when adjusting the frequency on the function generator.

6. Record the peak-to-peak voltage displayed on the scope for **at least 10 frequencies below and 10 frequencies** above the resonance frequency. Choose these frequencies wisely, so that you get a nice, smooth curve that you can compare to the amplitude in Eq. (7). Make sure you get the entire curve, all the way out to the flat portion on both ends.
7. Lastly, record the two frequencies that give you *half* of the maximum output voltage amplitude that you recorded. Use them to calculate the full-width-at-half-max (FWHM). This will be used to determine the damping ratio ζ_{UT}.

FWHM frequencies: f_{low} ___________ f_{high} ___________

8. Reset the oscilloscope to factory default by pressing "Default Setup" key below the display.
9. Reset the function generator to factory default by pressing the "Default" key next to the keypad.

Data Processing

The ultrasonic transducer produces a voltage that is proportional to the strain or surface displacement x. Thus, the output voltage is proportional to Eq. (7), such that

$$V(\omega) = \frac{\omega_n V_{max}}{\beta\sqrt{\left(\frac{\omega}{\beta}\right)^2 + (\omega^2 - \omega_n{}^2)^2}}, \tag{8}$$

where ω_n is the resonance frequency and $V_{\max}$ is the voltage you measured at the resonance frequency. The damping parameter β can be determined from the full width at half max (FWHM), $\Delta\omega$, of your measured response curve

$$\beta = \frac{\sqrt{3}}{\Delta\omega} . \tag{9}$$

Lastly, the non-dimensional damping ratio ζ_{UT} of the ultrasonic transducer can also be obtained using the equation

$$\zeta_{\mathrm{UT}} = \frac{1}{2\beta\omega_n}. \tag{10}$$

Use the measured parameters ω_n , $V_{\max}$, and $\Delta\omega$ to compute β and ζ_{UT}. Then, use these parameters to plot Eq. (8) on top of your you measured frequency response data. Be careful switching between Hz and rad/s. Try to consistently use the same units of frequency throughout your calculation.

Data Analysis and Deliverables

Create plots and other deliverables listed below. Save the plots as PDFs, import them into either Microsoft Word or LaTeX, and add an intelligent, concise caption. Make sure the axes are clearly labeled with units. Plots with multiple data sets on them should have a legend. **Additionally, write 1 – 3 paragraphs describing the items below.**

IMPORTANT: Refer to the "Data Processing" sections of Parts I and II for instructions on data processing.

1. For the single pendulum, plot the measured angle θ (degrees) vs. time t (s) for the best looking data set.
2. Make a table with the following parameters for the *single* pendulum. These parameters should be extrapolated from the measured angle θ (degrees) vs. time t (s) data.
   a. The decay constant λ (1/s).
   b. The ringing frequency ω_d (rad/s).
   c. The radius of gyration R (m).
   d. The viscous drag force coefficient γ (Ns/m).
3. For the ultrasonic transducers, make a plot of the measured amplitude as a function of frequency with the theoretical curve given by Eq. (8) plotted on top.
   a. In the caption, report the measured resonance frequency ω_n and damping ratio ζ_{UT}.

Talking Points – Please address the following questions in your paragraphs.

- Does the radius of gyration R seem reasonable for the single pendulum?
- Based on the measured damping ratio, was the ultrasonic transducer underdamped or overdamped?

Appendix A

Equipment

Part I (set up on inner lab benches)
- Double pendulum with Avago AEDM-5810-T06 encoders
- Lab stand with pendulum mount and stopping bar
- Arduino UNO microcontroller
- Workstation PC with Arduino IDE and PuTTY software
- Cable to connect encoder to Arduino
- USB cable to connect Arduino to workstation PC

Part II (set up on outer lab benches)
- BNC cable (24”- 36”)
- BNC “T” adapter
- 80/20 transducer assembly w/ 2 piezoelectric transducers w/ 36” cable ending in BNC connector
- Tektronix AFG 3021 Function Generator
- Tektronix DPO 3012 Digital Oscilloscope
- Allen wrench