# Experiment M8
# Driven Wheels
Procedure

Deliverables: Tech memo, checked lab notebook

## Overview

*Background*

The wheel is one of the most import inventions in human history. This lab will use the same motors you studied last week to power a 3-wheeled cart down a track. Shown on the left side of Fig. 1, a single rear wheel will be driven by the motor, and two unpowered wheel in the front will be locked into the track. The vehicle will be used to push a weighted sled down the track.

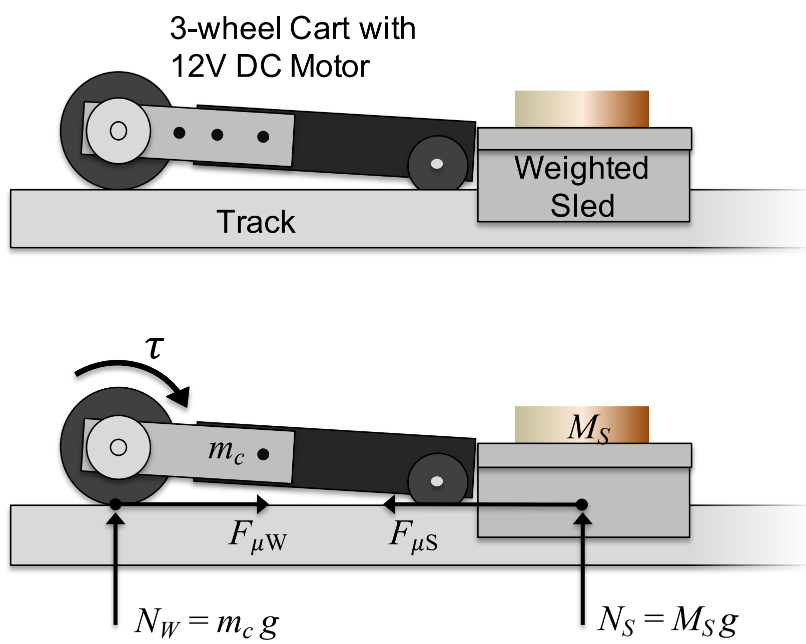


**Figure 1** – (Top) The rear wheel of a 3-wheel cart is powered by a 12V DC motor. The cart is used to push a sled down a track. (Bottom) A free body diagram illustrates the forces and torques acting on the cart of mass $m_c$ and the sled of mass $M_S$.

This system can be easily analyzed using Newtonian mechanics. The cart has a total mass $m$ with a wheel of radius $r$. The track creates a friction force of $F_{\mu W}$ on the wheel, while the DC motor applies a torque $\tau$ to the wheel. Similarly, the sled has a total mass $M_S$ and is subject to a friction force $F_{\mu S}$. Each friction force is assumed to be proportional to the normal force $N_W = m_c g$ for the cart and $N_S = M_S g$ for the sled.

Assuming the cart remains in contact with the sled and both move at a constant velocity, we can treat it as a single body of mass $m_c + M_S$, which gives us the equation of motion

$$(m_c + M_S)\ddot{x} = F_{\mu W} - F_{\mu S}, \tag{1}$$

where $x$ is the distance along the track. Considering the torque applied to the rear wheel gives us a second equation of motion

$$I\dot{\omega} = \tau - rF_{\mu W}, \tag{2}$$

where ω is the angular speed and $I$ is the moment of inertia of the rear wheel. These equations can be combined by imposing the "no-slip" condition $\ddot{x} = \dot{\omega}r$ on the wheel.

Importantly, the static friction force $F_{\mu W}$ on the wheel is not constant. Rather, it is a free parameter that must be eliminated by imposing the no-slip condition. Substituting the no-slip condition into Eq. (1) and solving for $F_{\mu W}$ yields

$$F_{\mu W} = (m_c + M_S)\dot{\omega}r + F_{\mu S}. \tag{3}$$

Combining Eqs. (2) and (3), we have the final overall equation of motion

$$[I + (m_c + M_S)r^2]\dot{\omega} = \tau - rF_{\mu S}. \tag{4}$$

Substituting the torque-speed relationship τ for gives a 1st order ordinary differential equation governing the angular speed of the wheel ω. We will leave that as an exercise for you the student.

The static friction force $F_{\mu W}$ on the wheel is not constant, and it can take on values ranging from 0 to $F_{\mu Wmax}$. If the maximum static friction force $F_{\mu Wmax}$ is exceeded, the wheel will lose traction and begin to slip, and the static friction force will become a kinetic friction force with a lower value. Thus, the wheel will lose traction and begin to slip if

$$\tau/r > F_{\mu Wmax}. \tag{5}$$

That is, if the torque is too large and/or the wheel is too small, it will lose traction and begin spinning, which results in very poor acceleration. (Anyone who has driven in snow and pressed too hard on the gas pedal has probably observed this.)

Consider the sled sliding down the track, being pushed by the cart. If the cart begins from a dead stop, it must first overcome the maximum static friction on the sled $F_{\mu Smax}$. Examining Eq. (4), we see that the wheel will not accelerate if

$$\tau/r < F_{\mu Smax}. \tag{6}$$

That is, the motor will *stall* if the torque is too small and/or the wheel is too large.

Together, the inequalities in Eqs. (5) and (6) give us a optimal range of motor torque that the cart-sled system can operate in for a given wheel radius

$$rF_{\mu Smax} < \tau < rF_{\mu Wmax}. \quad (7)$$

Note that there is no optimal range if $F_{\mu Smax} > F_{\mu Wmax}$. If the maximum station friction on the sled is greater than the maximum static friction on the wheel, then the sled essentially become an *immovable object*, and the wheel will either slip or stall, no matter what. Specifically, the wheel will slip if the torque $\tau > (F_{\mu Wmax})r$, and it will stall if $\tau < (F_{\mu Wmax})r$. The friction force on the sled becomes irrelevant, if it is immovable.

In this lab, you will perform a parametric study with four different diameter wheels and two different gear motors with different stall torques. You will test the 8 different combinations of wheel radius and stall torque with a light-weight, low-friction sled to see which combination results in the fastest speed. You will also test each combination with a weighted, high-friction *immovable sled* to see which combinations results in stall and which combinations results in wheel slip.

## Part I: Measuring Friction Forces

You will begin by measuring the maximum static friction force $F_{\mu Wmax}$ on the wheel and $F_{\mu Smax}$ on the sled.

*Procedure*

1. Begin with the 200 RPM motor and a mid-sized wheel mounted to the cart.
   a. The motors mount with M3 screws.
   b. The wheels fit onto a brass shaft coupling. The brass shaft coupling fixed to the motor shaft by a set screw. The wheel is held to the brass coupling by a metal retaining ring.
   c. Use the retaining ring pliers to remove or replace the retaining ring from the hub.
2. Use the digital scale on the countertop to measure the mass of the cart.
3. Place the cart on the track so the two small wheels in front are in the slots.
4. Hook a spring scale onto the front of the cart. Gently pull on it in a horizontal direction. Gradually increase the pulling force until the rear wheel skids and the cart lurches forward. (The wheel should skid, not roll.) Have a lab mate watch the spring scale (or take a video) to determine the maximum static friction force $F_{\mu Wmax}$ on the wheel. Let the various members of your group try pulling on the scale, and reach a consensus on what the average value of $F_{\mu Wmax}$ should be
5. Use the digital scale on the countertop to measure the mass of the sled and the weight separately.
6. Slide the sled onto the rail. Place a loop of string around the two masts in the front of the sled and hook the spring scale onto the center of the loop.
7. Perform a similar measurement with the spring scale to determine the maximum static friction force $F_{\mu Smax}$ on the sled.

8. Repeat the friction measurement of $F_{\mu Smax}$ on the sled with 4kg of weights on top of it.
9. Is the sled without any weights movable or immovable? Is the sled with 4 kg of weights movable or immovable? Answer these questions in your lab notebook.

## Part II: Drag Races

The cart will now be used to push the weighted sled down the track. You will perform a parametric study with two different gear motors (varying torque $\tau$) and four different wheel sizes (varying wheel radius $r$). Photogates will be used to measure the time it takes to push the sled down the track, similar to the A2 Galileo's inclined plane experiment you did in AME Lab 1.

Table 1 – Fill in this table with "Slip" if the wheel slipped, "Stall" if the motor stalled, or the pulse time if it successfully traversed the track.

| Wheel / Motor | $D$ = 2" | $D$ = 2.875" | $D$ = 3.875" | $D$ = 4.875" |
|---|---|---|---|---|
| **Sled only:** $\omega_0$ = 200 RPM $\tau_0$ = 0.5 Nm | | | | |
| **Sled + 4kg:** $\omega_0$ = 200 RPM $\tau_0$ = 0.5 Nm | | | | |
| **Sled only:** $\omega_0$ = 550 RPM $\tau_0$ = 0.2 Nm | | | | |
| **Sled + 4kg:** $\omega_0$ = 550 RPM $\tau_0$ = 0.2 Nm | | | | |

*Procedure*

1. Copy Table 1 into your lab notebook.
2. The DC motors are labelled by their no-load speed $\omega_0$. Note which motor is currently mounted to the cart.
3. Note the diameter of the wheel currently mounted to the cart.

4. Mount the photogates in the lab stands and position them along the track, so the first photogate (Photogate A) is about 2 feet left of center. Connect it to the LabQuest via Digital Port 1 (DIG1).
5. Position the other photogate (Photogate B) about 2 feet right of center. Connect it to the LabQuest via Digital Port 2 (DIG2).
6. The photogates should be about 4 feet apart. Adjust the height of the photogates to the mast on the sled will pass through it.
7. Plug the LabQuest in and then turn it on.
8. In the "LabQuest App", **File > New** on the drop down menu.
9. On the Sensors tab, select **Sensor > Sensor Setup**. Under "DIG 1" select the "Photogate" from the drop down box and then hit OK. Repeat this for "DIG 2".
10. Again on the Sensors tab, select **Sensor > Data Collection** and choose the following parameters:

    **Mode: Photogate timing**

    **Photogate mode: Pulse**

    **Distance between gates: 0.02m (It doesn't really matter what you put here.)**

    **End data collection: check "with the stop button"**

    Under the "Pulse" mode, blocking Photogate A will start a timer in the LabQuest and blocking Photogate B will stop the timer. Exit the menu by pressing the "**Ok**" button.
11. Press the "▶" button to begin collecting data from the photogates. (Choose to discard any unsaved data if it asks.)
12. Test the photogates. Place your hand in Photogate A to start the timer. Count to 10, then place your hand in Photogate B to stop the timer. . Locate the "**Pulse Time**" in the upper right corner of the LabQuest. It should roughly match your 10 second count.
13. Make sure the sled is still on the track. Place the cart at the leftmost end of the track, and slide the sled down so it is making contact with the nose of the cart, as illustrated in Fig. 1.
14. Measure the distance between the photogates using a tape measure.
15. Make sure the kill switch on the DC power supply is OFF. Connect the 12V DC power supply to the DC motor on the cart. Plug it into the wall.
16. Briefly turn on the DC power and make sure the wheel is spinning in the correct direction. If not, flip the minigrabbers on the motor to the opposite polarity.
17. Drape the DC power cable over the top of the lab stand holding Photogate A, so the kill switch is dangling over the sled. Position the red and black cables, so they will not get caught on anything.
18. Turn on the kill switch and let the cart push the empty sled down the track. Turn it off when the sled passes through Photogate B. Practice this a few times.

19. Position the cart and sled so the mast on the sled is **directly behind Photogate A**. Turn on the power supply and let it push the **empty sled** down to Photogate B.
    a. If the wheel fails to make one full revolution in 3 seconds, quickly turn off the DC power supply to prevent the motor from burning out. Write “Stall” in the table in your lab notebook.
    b. If the wheel slips, turn off the DC power supply. Write “Slip” in the table in your lab notebook.
    c. If the sled successfully reaches Photogate B without slipping, turn off the DC power supply before the sled runs out of track. Record the “pulse time” in the table in your lab notebook.
20. Place 4kg worth of weight on top of the sled to make it immovable, and repeat the measurement.
21. Use the retaining ring pliers to remove the retaining ring from the hub and remove the wheel.
22. Mount a different size wheel to the hub and reattach the retaining ring using the pliers.
23. Note the diameter of this new wheel.
24. Repeat the experiment with the other 3 wheel sizes. This should complete two rows in your table.
25. Change out the DC motor on the cart, and repeat the experiment with the 4 wheel sizes. Do this to complete the table.

## Data Analysis and Deliverables

Using LaTeX or MS Word, make the following items and give them concise, intelligent captions. Additionally, write 1 – 3 paragraphs separate from the caption describing what you did in lab and how it relates to the plot/table. Any relevant equations should go in this paragraph.

1. For the weighted sled, make a plot with wheel radius $r$ on the horizontal axis and motor torque $\tau$ on the vertical axis.
   a. Plot a line indicating the theoretical boundary between stall, optimal, and slip. Use the measured values of $F_{\mu Wmax}$ or $F_{\mu Smax}$ from Part I as the slope.
   b. Plot the eight data points from the parametric study. (Each individual combination of wheel radius and torque is a single point on this graph.)
      i. If the motor stalled, plot the data point as a red x.
      ii. If the wheel slipped, plot the data point as a red o.
2. For the sled with no weight, make a plot of the average speed of the cart as a function of the non-dimensional average angular velocity of the motor.
   a. The average speed is just the distance between the photogates divided by the measured pulse time.
   b. The non-dimensional average angular velocity of the motor is the average speed divided by the wheel radius $r$ divided by the no-load speed $\omega_0$.

**Talking Points** – Discuss/answer these in your paragraphs.

- Discuss the theoretical boundary between slip and stall for the immovable sled.
- What angular speed is optimal for moving fastest down the track. Can this be explained by one of the motor torque curves?

# Appendix A

## Equipment

- 12V DC power supply w/ kill switch and screw terminal adapter
- 6’ long minigrabber cables (1 red, 1 black)
- DC motors (greartisan 37mm)
  - 200 RPM
  - 550 RPM
- BaneBots Wheels, Hub Mount, 60A, Black
  - 2" x 0.8"
  - 2-7/8" x 0.8"
  - 3-7/8" x 0.8"
  - 4-7/8" x 0.8"
- BaneBots T81 Hub, 6mm Shaft
- 3-wheel cart w/ motor mount
- T-Slotted Framing, Double Six Slot Rail, Silver, 80 mm High x 40 mm Wide, Solid, 8ft long – McMaster part # 5537T112
- Sled - T-Slotted Framing, Horizontal Mount Flanged Bearing for 80 mm High Rail, 160 mm Long – McMaster part # 5537T646
- Weights for sled
- Digital scale
- Spring scales
- Tape measure
- Photogates (2 per setup)
- LabQuest
- Lab stand
- Retaining ring pliers
- M3 screws