AME40453 - Automation and Controls
C4 Pre-Lab Assignment

For the following questions, please express your answers as algebraic equations written on a separate sheet of paper, and show your work. Then, transcribe the equations into your lab notebook.

1. Write down the *system* of differential equations for the temperature T and integral of temperature $I = \int (T_S - T)\,dt$ for the full PID controller.
2. Rewrite the system of equations in terms of the variable $x = T - T_S$. Note that the integral becomes $I = -\int x\,dt$ and the derivatives are the same, $\frac{dx}{dt} = \frac{dT}{dt}$.
3. Rewrite the system of differential equations in matrix form $\frac{d}{dt}\begin{bmatrix} x \\ I \end{bmatrix} = \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix}\begin{bmatrix} x \\ I \end{bmatrix} + \begin{bmatrix} \dots \\ \dots \end{bmatrix}$. (Fill in the blanks "…")
4. Derive an equation for the eigenvalues in terms of the system parameters hA, mc_p, k_p, k_I, and k_D.
5. Derive an equation for the critical value of the integral gain k_I that will cause the controller to begin oscillating. i.e. $k_I > f(k_p, k_D, hA, mc_p)$
6. Sketch a plot of the *imaginary* part of the eigenvalue as a function of the integral gain k_I.
7. The imaginary part of the eigenvalues is essentially the frequency of the oscillations. Does the frequency increase or decrease with the derivative gain k_D?