Experiment A2
Galileo's Inclined Plane
Procedure

Deliverables: Checked lab notebook, printed plots with captions

Overview

In the first part of this lab, you will perform Galileo's famous inclined plane experiment. You will then learn several fundamental techniques to analyze the data. Specifically, you will *empirically* determine a mathematical relationship for distance x vs. time t for a body in gravitational free-fall and extrapolate the acceleration of gravity g. This experiment is of great historical significance, as it later inspired Isaac Newton to invent calculus.

In the second part of this lab, you will examine a curved ramp called a "Brachistochrone". As a ball rolls down such a curve, it undergoes a *variable* acceleration that results in some unique behavior. You will examine this behavior by repeating Galileo's experiment using a Brachistochrone.

Part I: Galileo's Inclined Plane

In this experiment, you will roll a ball down an inclined plane and measure the time t it takes to travel a distance x.

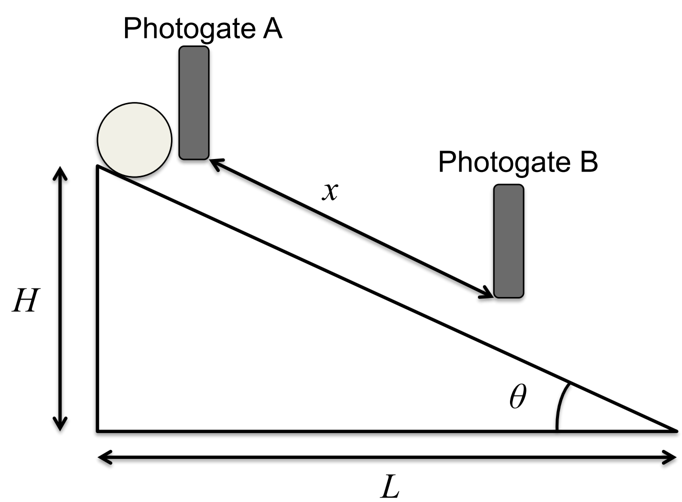

Figure 1 – A schematic representing the inclined plane experiment.

According to Newtonian Mechanics, the trajectory of a sphere rolling down an inclined plane at an angle θ is given by

$$x(t) = \frac{1}{2}\left(\tfrac{5}{7} g \sin\theta\right) t^2 \tag{1}$$

where g is the acceleration of gravity near the surface of earth.

1. Set the inclined plane angle θ to a shallow angle between 1° and 15°. Determine the angle by measuring the length of the legs L and H and using the appropriate trig function. Record all values in your lab notebook.
2. Position Photogate A near the top of the inclined plane as shown in Fig. 1 and connect it to the LabQuest via Digital Port 1 (DIG 1).
3. Position the photogate B a distance x = 10 cm away from top of the inclined plane and connect it to the LabQuest via Digital Port 2 (DIG 2).
4. Plug the LabQuest in and then turn it on.
5. On the Sensors tab, select **Sensor > Sensor Setup**. Under "DIG 1" select the "Photogate" from the drop down box and then hit OK. Repeat this for "DIG 2".
6. Again on the Sensors tab, select **Sensor > Data Collection** and choose the following parameters:

   Mode: Photogate timing

   Photogate mode: Pulse

   Distance between gates: 1m

   End data collection: check "with the stop button"

   Under the "Pulse" mode, blocking Photogate A will start a timer in the LabQuest and blocking Photogate B will stop the timer. Exit the menu by pressing the "**Ok**" button.
7. Press the "▶" button to begin collecting data from the photogates. (Choose to discard any unsaved data if it asks.)
8. Make a table in your lab notebook with two columns for x and t. Be sure note the units of both.
9. Measure the distance x between the two photogates using the meter stick provided and record it in the table in your notebook.
10. Make sure the photogates are set so that the light sensor will pass through the center of the billiard ball.
11. Place the billiard ball **directly behind Photogate A** and release it. Locate the "**Pulse Time**" in the upper right corner of the LabQuest. Record it in the table in your lab notebook.
12. Without moving the photogates, repeat steps 8 – 11 four more times. This will give you a total of 5 data points for the one distance that you will average together.
13. Move Photogate B 10 cm further from the top (increase x) and repeat steps 7 – 13 for distances up to and including x = 50cm.
14. Change the angle of the inclined planed to a different value between 1° and 15° and repeat the entire procedure.

Part II: Brachistochrone

"In one physical model of the universe, the shortest distance between two points is a straight line… in the opposite direction." - Ty Webb, *Caddyshack*

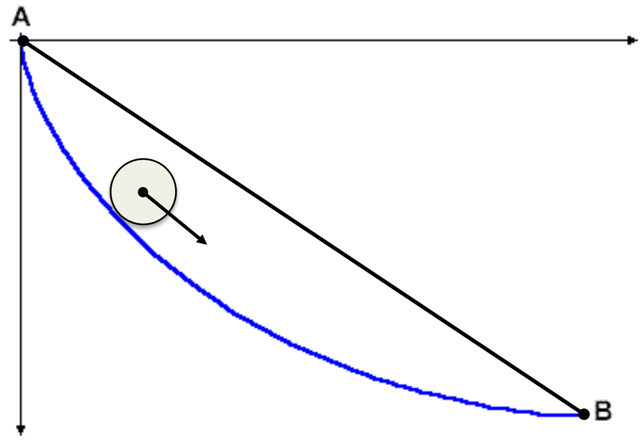

Figure 2 – The path of shortest *distance* between points A and B is a straight line (black curve). The path of shortest *time* for a ball rolling from A to B is called a "Brachistochrone" (blue curve).

In this exercise, you will repeat the previous measurements using a special curved ramp called a Brachistochrone.

1. Use the magnetic mount to fix Photogate A near the top of the Brachistochrone as shown in Fig. 2 and connect it to the LabQuest via Digital Port 1 (DIG 1).
2. Photogate B is fixed at the bottom of the Brachistochrone. Connect it to the LabQuest via Digital Port 2 (DIG 2).
3. Plug the LabQuest in and then turn it on.
4. On the Sensors tab, select **Sensor > Sensor Setup**. Under "DIG 1" select the "Photogate" from the drop down box and then hit OK. Repeat this for "DIG 2".
5. Again on the Sensors tab, select **Sensor > Data Collection** and choose the following parameters:

   Mode: Photogate timing

   Photogate mode: Pulse

   Distance between gates: 1m

   End data collection: check "with the stop button"

   Under the "Pulse" mode, blocking Photogate A will start a timer in the LabQuest and blocking Photogate B will stop the timer. Exit the menu by pressing the "**Ok**" button.
6. Press the "▶" button to begin collecting data from the photogates. (Choose to discard any unsaved data if it asks.)

7. Make a table in your lab notebook with two columns for *x* and *t*. Be sure note the units of both.
8. Measure the **straight linear** distance *x* between the two photogates using the meter stick provided and record it in the table in your notebook.
9. Make sure the photogates are set so that the light sensor will pass through the center of the stainless steel ball bearing.
10. Place the stainless steel ball **directly behind Photogate A** and release it. Locate the "**Pulse Time**" in the upper right corner of the LabQuest. Record it in the table in your lab notebook.
11. Without moving the photogates, repeat steps 8 – 11 four more times. This will give you a total of 5 data points for the one distance that you will average together.
12. Move Photogate A to the next lowest magnetic mounting point and repeat steps 8 – 11 until you reach the end of the track.

Data Analysis and Deliverables

Create plots listed below. Save the commands you used to generate the plots as a Matlab script file. Save the plots as PDFs, import them into either Microsoft Word or LaTeX, and add an intelligent, descriptive caption. It is OK for your captions to be a paragraph in length. Print the document containing your captioned plots, Matlab script, and other deliverables, staple them together, and turn it in at the beginning of lab next week.

1. **Plotting Data in Matlab**
   Using your data from Part I, reproduce the plot shown below using Matlab. The error bars should be calculated using the "repeatability uncertainty"

$$U_R = \frac{s}{\sqrt{N-1}}, \tag{2}$$

   where s is the standard deviations and N is the number of times the measurement was repeated for that data point. Label this as "Figure 1".

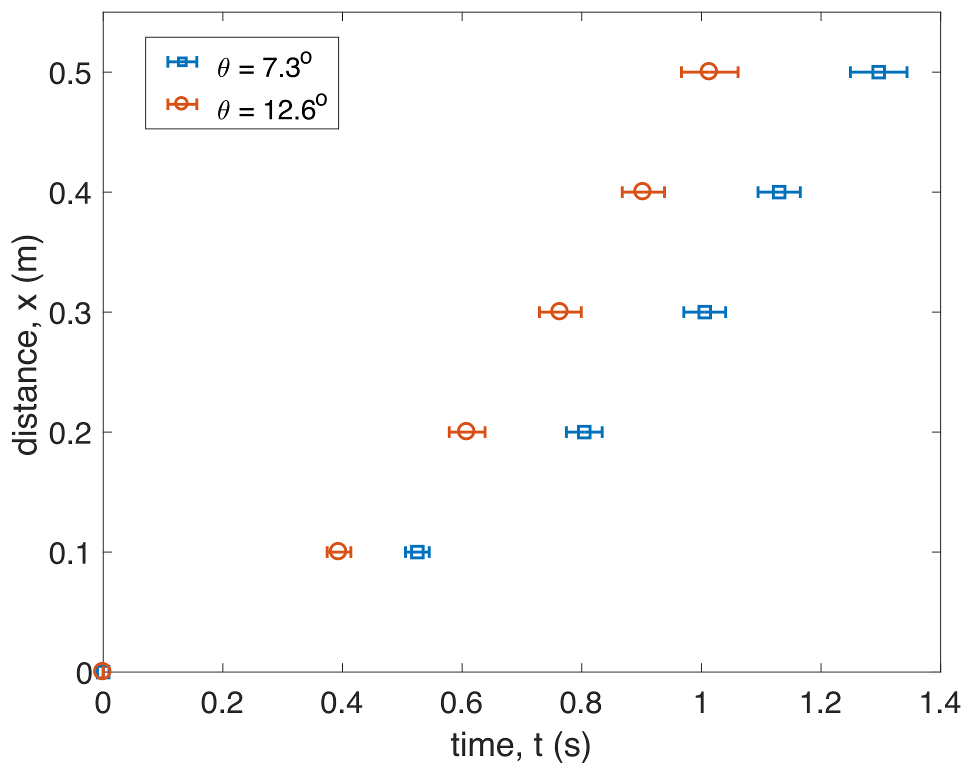

Figure 3 – Be sure to include a descriptive caption. It can be as long as you want!

2. Curve fitting: extrapolating *g*

Use your data from Part I to do the following:

a) Using the "fit()" command in Matlab, perform a *quadratic* curve fit on each of the two data sets.

b) Plot the two best-fit quadratic curves on top of your data. This plot should look exactly like the previous plot, except with two smooth curve fits on top of the data. Label it as "Figure 2".

c) Based on Eq. (1), write an **algebraic** equation for each of the three fitting parameters in the quadratic equation. ("Algebraic" means leave the parameters as symbolic variables.)

d) Use the coefficient for the second order term (the constant in front of t^2 term) that you get from the curve fit to *extrapolate g*.

e) The fit command also outputs a "90% confidence interval" for each fitting parameter. The width of this interval is equal to twice the uncertainty in the parameter. Use the confidence interval for the second order coefficient to determine the uncertainty in *g*.

f) Report the two values of *g* in the caption of Fig. 2 along with their uncertainty (i.e. report it as g = value ± uncertainty m/s^2).

3. Brachistochrone

Make a plot of distance x vs. time t for the Brachistochrone data. Be sure to include error bars, as you did in the previous deliverables. This should be labeled as "Figure 3". Do you notice anything interesting about it? Do a bit of research about the Brachistochrone, and explain why it looks this way in your caption.

Questions

Please answer the following questions in the captions of your plots.

- What are some of the sources of error in the inclined plane experiment?

- Do a bit of research about the Brachistochrone. In particular, explain why the data looks the way it does.

Appendix A

Equipment

- Inclined plane
- Billiard ball
- Billiard pocket
- Cable ties and rubber bands to attach billiard pockets
- Meter stick
- Level
- Vernier LabQuest
- 2 Photogates (Vernier VPG-BTD) with magnetic L-brackets
- 2 Photogate “DIG” cables
- Brachistochrone ramp with feet
- 2 Photogates (Vernier VPG-BTD) with magnetic Z-brackets
- 2 Photogate “DIG” cables
- 1.5” diameter stainless steel ball bearings