Experiment A12
Monte Carlo Night!
Procedure

Deliverables: checked lab notebook, Brief tech memo

Overview

In the real world, you will never measure the exact same number every single time. Rather, you will measure a *distribution* of values with some mean and standard deviation. These distributions tell us the *probability* of measuring a value within a certain range. This can be a difficult concept for us to grasp, as our primary education suggests that there is always just one right answer.

This lab will demonstrate the concept of a *probability distribution* using a pair of 6-sided dice, a "Plinko" board, and a large set of manufactured resistors. The dice and Plinko boards are examples of *discrete* probability distributions, while the resistance of manufactured resistors is an example of a *continuous* probability distribution. These lab exercises will also illustrate the concept of "statistical convergence", which tells us that we must repeat a measurement many times to get an accurate description of the distribution.

According to Poisson statistics, the *measured* probability of an even A can be calculated as

$$P(A) = \frac{\nu_A}{N}, \tag{1}$$

where ν_A is the number of times or "frequency" that the event occurs and N is the total number of trials. The uncertainty in the measured probability can also be determined from Poisson statics via the equation

$$U_{P(A)} = \sqrt{\frac{P(A)}{N}}. \tag{2}$$

The data for each lab section will be accumulated on a Google spreadsheet. At the end of the week, the professor will send you a spreadsheet containing *the entire class*' data. You will then compare your group's measured distribution with the entire class' collective data. For each of these respective data sets, the sample size N will increase by an order of magnitude, leading to better *statistical convergence*.

Part I: Alea Iacta Est

A pair of 6-sided dice is a very common example of a discrete probability distribution. In the pre-lab assignment, you should have looked up the discrete probability distribution $P(n)$ for rolling a combination adding up to number n = 2 to 12. **Transcribe that probability distribution as a table in your lab notebook.**

Figure 1 – A pair of 6-sided dice.

Experimental Procedure

You will now experimentally measure the discrete probability distribution for a pair of 6-sided dice and compare it to the theoretical distribution you determined in the pre-lab. Please perform the steps listed below.

1. Transcribe the probability distribution $P(n)$ for rolling a combination adding up to number n = 2 to 12 as a table in your lab notebook.

2. Open the Google spreadsheet titled "Dice_*date*_*LabSection*.xls" that the TA shared with you, and locate the column containing you and your lab partners' names.

3. Have your partner roll the pair of dice, and record the total value n in the Google spreadsheet. The big TV monitor will display a histogram of *everyone's* collective data.

4. Repeat this N = 100 times for the pair of dice.

5. In your lab notebook, make a note of the file name on Google spreadsheets where the data was recorded.

6. At the end of the week, the entire class' data will be shared, and you will compare the class' aggregated data to your much smaller data set.

Data Analysis

Please perform the following steps to produce the deliverables for this part of the lab. **The data analysis should not be done until you have performed all three parts of the lab.**

1. Use the "bar()" command in Matlab to make a plot of the theoretical probability distribution $P(n)$ for a pair of 6-sided dice. Set the "width" option so that there is no space between the bars.

2. Use you data to calculate the *measured* probability distribution from your group's data via Eq. (1). The frequency ν_A of each value of n can be determined using the "histc()" command. The total number of trials N can be easily determined using the "length()" command.

3. Use the "hold on" feature and plot your group's measured probability distribution for the dice as individual data points on top of the theoretical distribution.

4. With the hold still on, use the "errorbar()" command to plot the Poisson error bars on top of your group's measured data via Eq. (2).

5. Turn off the hold by adding a "hold off" line to your script.
6. Write a script to aggregate the values that the **entire class measured** (all lab sections combined).
7. Make similar plots of the distribution for the entire class' measured data.

Part II: Plinko

Plinko is a popular game from the television show *The Price is Right*, where contestants slide flat discs down an inclined plane lined with staggered pegs. As shown in Fig. 2, the discs or "Plinko chips" undergo a random walk as they make their way to the bottom of the board. Similar to the chaotic double pendulum, the trajectory of the Plinko chip is so sensitive to initial conditions that it is impossible to predict.

In this exercise, you will measure the probability of a Plinko chip landing in the kth bin (k = 0, 1, 2…) and compare it to the theoretical distribution

$$P(k) = \frac{n!}{k!(n-k)!} a^k (1-a)^{n-k}, \qquad (3)$$

where n is the number of rows and a = 0.5 is the probability of going right from a single peg.

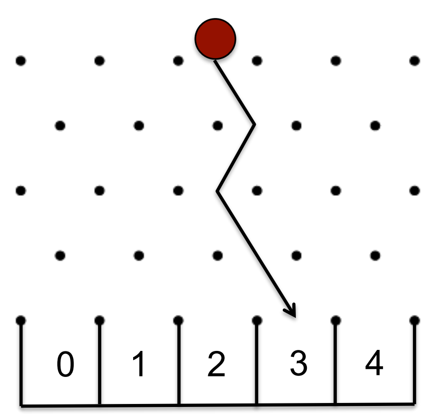

Figure 2 – A schematic of a small Plinko board with 4 rows. The Plinko chip undergoes a random walk as it makes its way to the bottom of the board, where it may end up in any one of the bins.

Experimental Procedure

You will now experimentally measure the distribution of the Plinko board and compare it to the theoretical distribution you determined in the pre-lab. Please perform the steps listed below.

1. Open the Google spreadsheet titled "Plinko_*date*_*LabSection*.xls" that the TA shared with you.

2. Go to the portion of the spreadsheet corresponding to the number of rows on your Plinko board (either 4 or 10), and locate the column containing you and your lab partners' names.

3. Have your partner drop a Plinko chip into the apex of the board, and record which bin number it lands in ($k = 0, 1, 2\ldots$) in the Google spreadsheet. Disregard any trials where the Plinko chip goes out of bounds or stops on a peg.

4. Repeat this $N = 50$ times for **both** the 4 and 10 row Plinko boards. The big TV monitor will display a histogram for everyone's collective data.

5. In your lab notebook, make a note of the file name on Google spreadsheets where the data was recorded.

6. At the end of the week, the entire class' data will be shared, and you will compare the class' aggregated data to your much smaller data set.

Data Analysis

Please perform the following steps to produce the deliverables for this part of the lab. **The data analysis should not be done until you have performed all three parts of the lab.**

1. Use the "bar()" command in Matlab to make a plot of the theoretical probability distribution $P(k)$ for the 4 row Plinko board. Set the "width" option so that there is no space between the bars.
2. Use you data to calculate the *measured* probability distribution from your group's data via Eq. (1). The frequency of each value of k can be determined using the "histc()" command. The total number of trials N can be easily determined using the "length()" command.
3. Use the "hold on" feature and plot your group's measured probability distribution for the dice as individual data points on top of the theoretical distribution.
4. With the hold still on, use the "errorbar()" command to plot Poisson error bars on top of your group's measured data via Eq. (2).
5. Turn off the hold by adding a "hold off" line to your script.
6. Repeat steps 1 – 5 for the 10 row Plinko board.
7. Write a script to aggregate the values that the **entire class measured** (all lab sections combined).
8. Make similar plots of the distribution for the entire class' measured data.

Part III: Tolerance of Resistors

Not all manufactured parts are created equally. Parts that come off an assembly line are never identical, and there is always some variation between them. Any given property of a manufactured part (i.e. its length, mass, or diameter) is typically governed by a *continuous* probability distribution with some mean and standard deviation. In this sense, measuring a part at the end of an assembly line is no different than rolling a pair of dice.

Manufactured resistors are no exception, and their values vary by a measureable amount. In this lab, you will measure the distribution of resistances for a batch of 1 kΩ resistors. As with the dice and the Plinko board, you will see how the distribution convergence as the sample size N becomes large.

Experimental Procedure

1. Open the Google spreadsheet titled "Resistors_*date*_*LabSection*.xls" that the TA shared with you, and locate the column containing you and your lab partners' names.
2. To perform a very precise measurement of resistance, you will need to perform a "4-wire" resistance measurement. To eliminate the resistance of the cable, one set of cable will drive a specified current through the resistor, while the other set of cables measures the voltage.
3. Set up the 4-wire resistance measurement on the Keysight 34465A precision digital multimeter (DMM).
   a. Press the blue "Shift" button, then press the "Ω4W" button to go into the 4-wire measurement mode.
   b. Connect the two male banana plugs corresponding to the red gator clamp into the top two red receptacles on the DMM. (Left or right does not matter.)
   c. Connect the other two male banana plugs corresponding to the black gator clamp into the middle two black receptacles on the DMM. (Left or right does not matter.)
   d. The extra grounding wire should be left unconnected.
4. The TA will give you two strips of 50 resistors. One strip will have a tolerance of 1% and 5% for the other. (The manufacturer has specified these tolerances.)
5. Have your partner measure the resistance of the first resistor on the stip, and record its value in the Google spreadsheet **in units of Ohms with 4 significant figures (i.e. 991.5)**.
6. Go through both strips of resistors so that you have $N = 50$ data points for each strip.
7. In your lab notebook, make a note of the file name on Google spreadsheets where the data was recorded.
8. At the end of the week, the entire class' data will be shared, and you will compare the class' aggregated data to your much smaller data set.

Data Analysis

Please perform the following steps to produce the deliverables for this part of the lab. **The data analysis should not be done until you have performed all three parts of the lab.**

1. Write a script to aggregate the resistance values that the **entire class measured** (all lab sections combined).
2. Calculate the mean $\langle R \rangle$ and standard deviation s of the resistances that **the entire class** measured.
3. Use the "hist()" command in Matlab to make a histogram of the measured resistances. Try changing the number of bins to make the distribution look somewhat Gaussian.
4. Superimpose the *theoretical* Gaussian distribution over the histogram by plotting the function

$$\rho(R)dR = \frac{(R_{\max} - R_{\min})N}{N_{bins} s\sqrt{2\pi}} \exp\left[\frac{-(R - \langle R \rangle)^2}{2s^2}\right], \qquad (4)$$

   where N is the number of data points, N_{bins} is the number of bins, s is the standard deviation, $\langle R \rangle$ is the mean, and R_{max} and R_{min} are the maximum and minimum resistances observed, respectively. Be sure to label the axes!
5. Make these plots for both the 1% and 5% resistors.

Data Analysis and Deliverables

Create plots and tables listed below. Save the plots as PDFs, import them into either Microsoft Word or LaTeX, and add an intelligent, descriptive caption. **Write a paragraph describing each group of three plots listed below.** If you would like to save paper, you can use "subplot()" to group the plots side-by-side.

To aggregate the various data sets for **the entire class** (not just your lab section), we encourage students to use the "xlsread()" command in Matlab. The commands "nanmean()" and "nanstd()" may be helpful, as well.

1. For either the dice or one of the Plinko boards, make a plot of **your group's** *measured* probability distribution with Poisson error bars compared to the theoretical distribution. (See *Data Analysis* section of Part I or II.)
2. For the same experiment that you chose for Deliverable 1, make a plot of **the entire class'** *measured* probability distribution with Poisson error bars compared to the theoretical distribution. (See *Data Analysis* section of Part I or II.) How does the distribution measured by the entire class compare to what was measured by just your group?
3. Two plots for the resistance measurements.
    a. A histogram of **the entire class'** *measured* distribution of the 1% resistors with the Gaussian distribution plotted on top. (See *Data Analysis* section of Part III.)
    b. A histogram of **the entire class'** *measured* distribution of the 5% resistors with the Gaussian distribution plotted on top. (See *Data Analysis* section of Part III.)
4. Using data from the entire class (all lab sections combined), make a table containing the theoretical average, measured average, and measured standard deviation for the dice, the Plinko boards, and the resistors.

Appendix A

Equipment Required

Plinko
- 2 Small plinko boards with 4 rows
- 2 Large plinko boards with 10 rows
- 4 Tripods to mount plinko boards
- Clay poker chips
- Computer connected to Google spread sheets

Alea Iacta Est
- Pair of 6-sided dice
- Computer connected to Google spread sheets

Tolerance of Resistors
- Large spool of 1kΩ resistors (5% manufacturer tolerance)
- Large spool of 1kΩ resistors (1% manufacturer tolerance)
- Keysight 34465A Precision digital multimeter
- Cable for performing 4-wire resistance measurement
- Computer connected to Google spread sheets