

AME40453 - Automation and Controls  
C4 Pre-Lab Assignment

**For the following questions, please express your answers as algebraic equations written on a separate sheet of paper, and show your work. Then, transcribe the equations into your lab notebook.**

1. Write down the *system* of differential equations for the temperature  $T$  and integral of temperature  $I = \int (T_s - T) dt$  for the full PID controller.
2. Rewrite the system of equations in terms of the variable  $x = T - T_s$ . Note that the integral becomes  $I = -\int x dt$  and the derivatives are the same,  $\frac{dx}{dt} = \frac{dT}{dt}$ .
3. Rewrite the system of differential equations in matrix form  
$$\frac{d}{dt} \begin{bmatrix} x \\ I \end{bmatrix} = \begin{bmatrix} \dots & \dots \\ \dots & \dots \end{bmatrix} \begin{bmatrix} x \\ I \end{bmatrix} + \begin{bmatrix} \dots \\ \dots \end{bmatrix}.$$
 (Fill in the blanks "...")
4. Derive an equation for the eigenvalues in terms of the system parameters  $hA$ ,  $mc_p$ ,  $k_p$ ,  $k_I$ , and  $k_D$ .
5. Derive an equation for the critical value of the integral gain  $k_I$  that will cause the controller to begin oscillating. i.e.  $k_I > f(k_p, k_D, hA, mc_p)$
6. Sketch a plot of the *imaginary* part of the eigenvalue as a function of the integral gain  $k_I$ .