# Presentation #2

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# Overview

- Focus on equities in the Technology sector
- Characteristics: high beta, volatility, and sensitivity to innovation cycles
- Goal: Apply ARIMA, VAR, and GARCH modeling to assess risk, dependence, and forecasts

# Vector Autoregressive Models

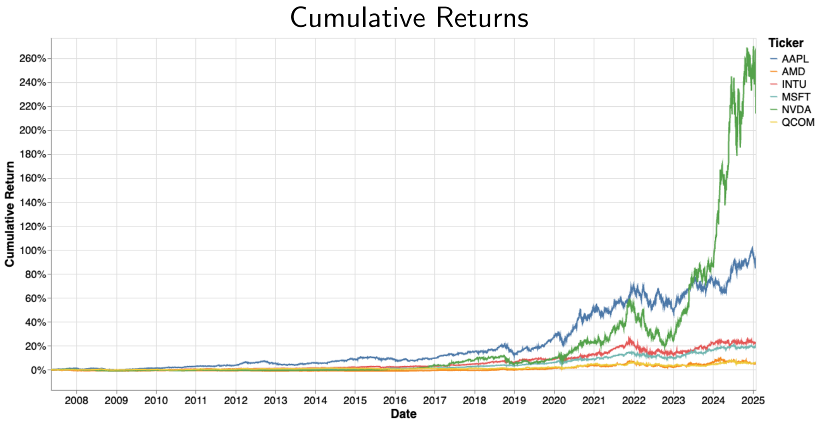
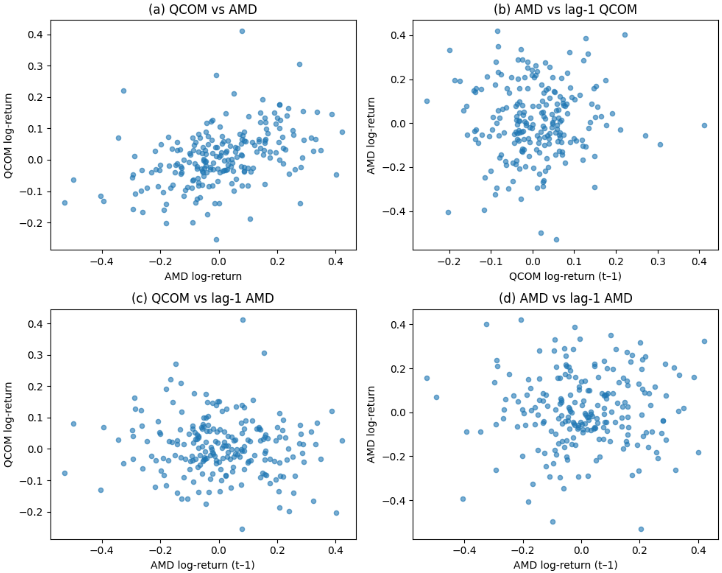



Figure: Cumulative Returns for Sample Securities

# Vector Autoregressive Models



# Vector Autoregressive Models

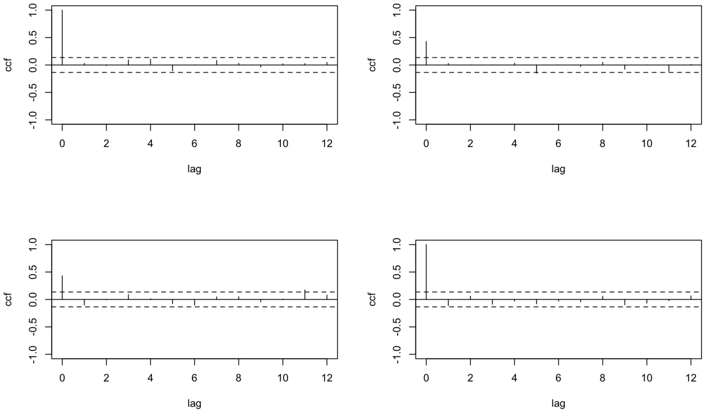



Figure: (Top Left: ACF of QCOM; Top Right: Cross Correlation between QCOM and AMD; Bottom Left: ACF of AMD; Bottom Right: Cross Correlation between AMD and QCOM

# Vector Autoregressive Models

Table: Lag Length Selection Criteria from `VARselect`

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| AIC(n) | 13.61680 | 13.65035 | 13.65723 | 13.67909 | 13.68640 | 13.70674 |
| HQ(n) | 13.65600 | 13.71568 | 13.74870 | 13.79669 | 13.83014 | 13.87662 |
| SC(n) | 13.71373 | 13.81189 | 13.88340 | 13.96987 | 14.04180 | 14.12677 |
| FPE(n) | 819792.47 | 847769.53 | 853653.93 | 872568.81 | 879055.56 | 897241.50 |

| | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|
| AIC(n) | 13.73149 | 13.76261 | 13.79238 | 13.81291 | 13.78931 | 13.81530 |
| HQ(n) | 13.92749 | 13.98475 | 14.04065 | 14.08732 | 14.08985 | 14.14198 |
| SC(n) | 14.21613 | 14.31187 | 14.40626 | 14.49141 | 14.53243 | 14.62304 |
| FPE(n) | 919886.57 | 949190.04 | 978160.34 | 998822.64 | 975964.07 | 1002200.00 |

# Vector Autoregressive Models

Our Var(1) model for two series can be represented in the following form:

$$\begin{bmatrix} \text{QCOM}_t \\ \text{AMD}_t \end{bmatrix} = \begin{bmatrix} 2.1352 \\ 1.9618 \end{bmatrix} + \begin{bmatrix} 0.02356 & 0.03490 \\ -0.03611 & -0.08491 \end{bmatrix} \begin{bmatrix} \text{QCOM}_{t-1} \\ \text{AMD}_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\text{QCOM},t} \\ \epsilon_{\text{AMD},t} \end{bmatrix}$$

which can then be rewritten as:

$$\begin{aligned} \text{QCOM}_t &= 0.002356 \cdot \text{QCOM}_{t-1} + 0.03490 \cdot \text{AMD}_{t-1} + 2.1352 \\ \text{AMD}_t &= -0.03611 \cdot \text{QCOM}_{t-1} - 0.08491 \cdot \text{AMD}_{t-1} + 1.9618 \end{aligned}$$

# Cointegration

# Cointegration

The augmented Engle-Granger cointegration test evaluates whether two or more non-stationary time series share a long-term equilibrium relationship.

# Cointegration

### Hypothesis Test Structure

- $H_0$: No cointegration exists between the time series.
- $H_a$: Residuals are stationary.

### Test Producer

1. Regress one $I(1)$ variable $y_t$ on the other $I(1)$ variables $(x_t)$:

$$y_t = \beta_0 + \beta_1 x_t + u_t$$

2. Test $\hat{u}$ for stationarity using

$$\Delta \hat{u} t = \alpha \hat{u} t - 1 + \sum i = 1^p \Delta \hat{u}_{t-1} + \epsilon_t$$

where the test statistic is the t-ratio for $\alpha$

# Cointegration

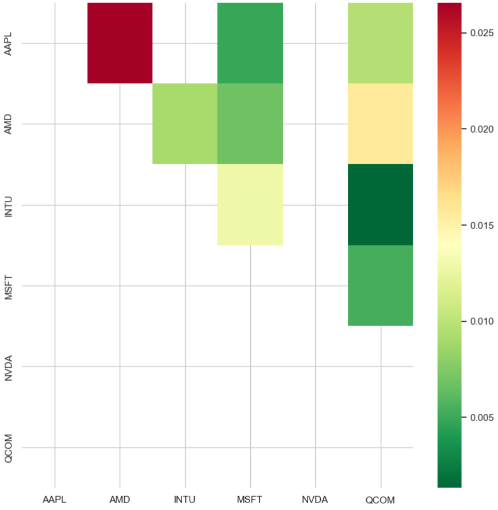



Figure: Cointegration Pairs based on Engle-Granger

# Cointegration

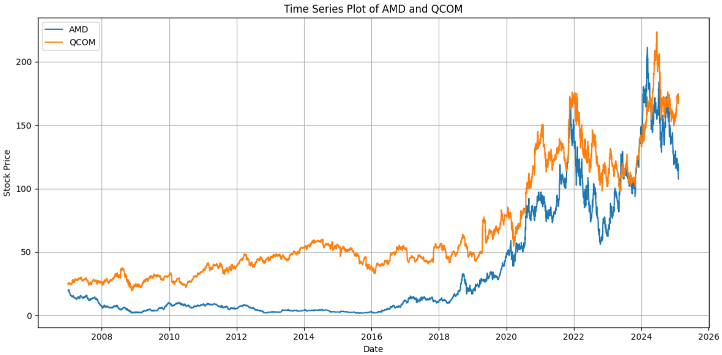



Figure: Cumulative Returns of QCOM and AMD

# Cointegration

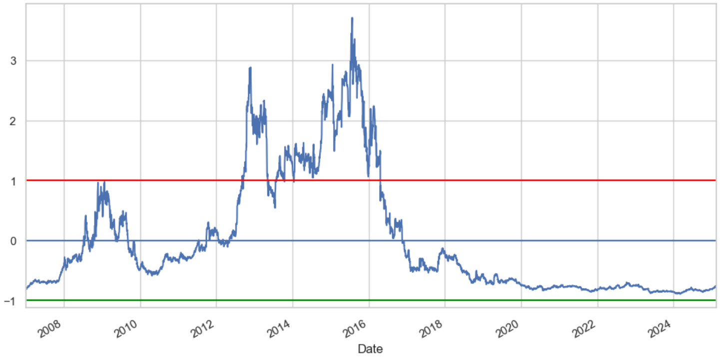



Figure: Normalized Ratio between QCOM and AMD

# ARCH/GARCH Models

# ARCH/GARCH

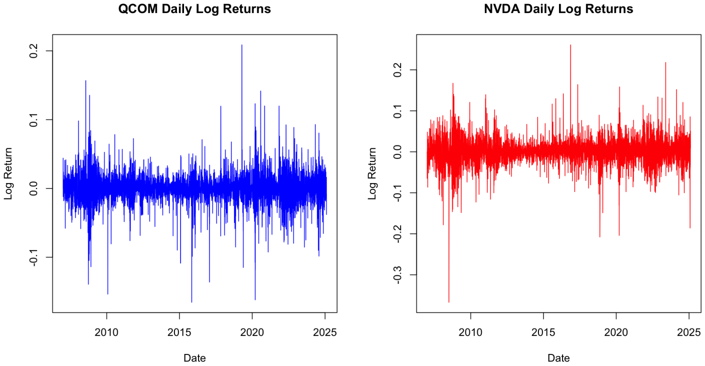



Figure: Logged Returns for QCOM and NVDA

The ARCH/GARCH model has the following hypothesis test

**Hypothesis Test**

- $H_0 : \rho_1(r^2) = \rho_2(r^2) = \ldots \rho_m(r^2) = 0$
- $H_a : \rho_j(r^2) \neq 0$ for at least one $j \in 1, 2, \ldots, m$

**Test Statistic**

The Ljung-Box Q-statistic is defined as:

$$\mathcal{Q}(m) = T(T+2)\sum_{j=1}^{m} \frac{\hat{\rho}_j^2}{T-j}$$

Where

- $T$ is the sample size
- $m$ is the number of lags being tested
- $\hat{\rho}_j(r^2)$ is the sample autocorrelation of squared residuals at lag $j$

# ARCH/GARCH

Under the null hypothesis, the test statistic

$$\mathcal{Q}(m) \sim \mathcal{X}^2(m-p-q)$$

Where $p$ and $q$ represent the number of parameter in the fitted model (excluding constants and variance).

**Decision Rule:**

- Reject $H_0$ if $\mathcal{Q}(m) > \mathcal{X}^2_{\alpha,m-p-q}$
- Fail to reject $H_0$ if $\mathcal{Q}(m) \leq \mathcal{X}^2_{\alpha,m-p-q}$

where $\mathcal{X}^2_{\alpha,m-p-q}$ is the critical value from the chi-square distribution with $m-p-q$ degrees of freedom at a siginficance level $\alpha$

# ARCH

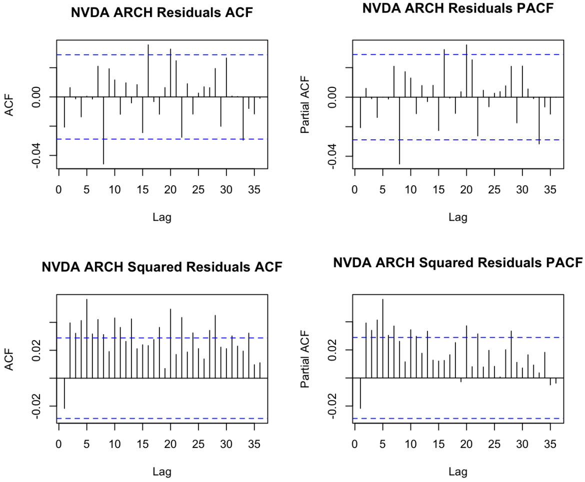



Figure: ACF and PACF for the Residual and Squared Residuals of NVDA

# ARCH

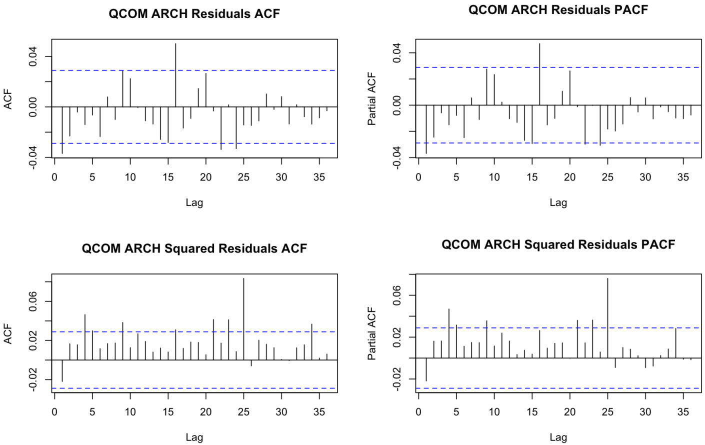



Figure: ACF and PACF for the Residual and Squared Residuals of QCOM

**Conclusion: NVDA ARCH Model**

1. The ARCH model appears inadequate for capturing the full dynamics of NVDA's return series:
   - The significant autocorrelations in regular residuals indicate remaining serial correlation.
   - The numerous significant spikes in squared residuals suggest unmodeled volatility dynamics.
2. Model improvements to consider:
   - A higher-order ARCH model might help, but the extensive significant lags suggest a more fundamental issue.
   - A GARCH model would likely be more appropriate, as it can better capture the persistence in volatility.
   - Given the complex pattern of significant lags, more sophisticated models like EGARCH or GJR-GARCH might be worth exploring, especially if there are asymmetric volatility responses.

**Conclusion: QCOM ARCH Model**

1. The ARCH model for QCOM shows moderate adequancy in modeling volatility but has clear limitations:
   - The significant spikes in the residuals ACF/PACF indicate that the mean equation isn't fully capturing the return dynamics.
   - The significant spike at lag 25 in the squared residuals suggests specific periodicity in volatility that wasn't modeled.
2. Model improvement suggestions:
   - The mean equation specification should be revisited to better account for the serial correlation.
   - A GARCH model would likely be more appropriate than the simple ARCH to capture the volatility persistence.

# GARCH

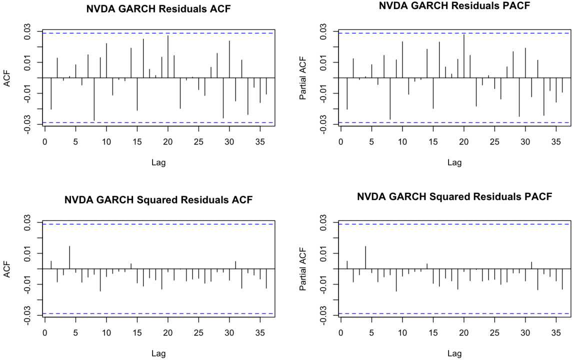



Figure: ACF and PACF for the Residual and Squared Residuals of NVDA

# GARCH

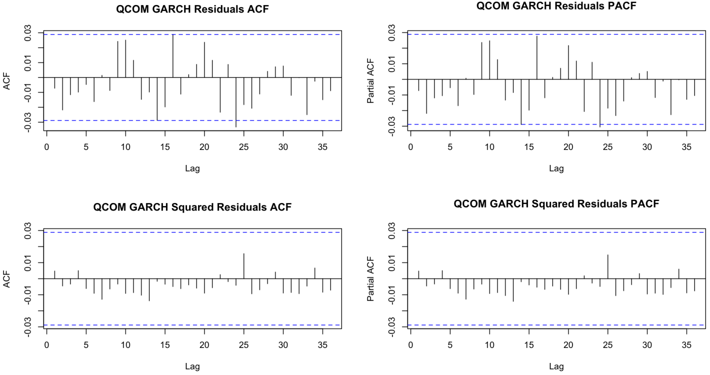



Figure: ACF and PACF for the Residual and Squared Residuals of QCOM

**Conclusion: NVDA GARCH Model**

1. The GARCH model has done a reasonably good job capturing the volatility dynamics in NVDA returns, as evidenced by the lack of significant autocorrelation in squared residuals.
2. There may be some minor remaining serial correlation in the standardized residuals, which suggests the mean equation might benefit from slight refinement.
3. The absence of significant ARCH effects in the squared residuals indicates that the variance equation of the GARCH model is adequately specified.

**Conclusion: QCOM GARCH Model**

1. The GARCH model for QCOM has performed reasonably well in modeling the volatility dynamics:
    - The squared residuals show very few significant autocorrelations, indicating successful modeling of volatility clustering.
    - The single spike at lag 25 in the squared residuals might indicate some periodic pattern in volatility that occurs at that specific lag.
2. The mean equation specification could potentially be improved:
    - The presence of several significant spikes in the residuals' ACF and PACF suggests that the return dynamics aren't fully captured.
    - This might indicate that additional explanatory variables or a different ARMA specification might be beneficial.

# ARCH/GARCH

Table: Weighted Ljung-Box Test Results for NVDA ARCH(1) and GARCH(1,1)

| **Model** | **Residual Type** | **Lag** | **Statistic** | ***p*-value** |
|---|---|---|---|---|
| ARCH(1) | Standardized Residuals | Lag[1] | 1.969 | 0.1606 |
| | | $\text{Lag}(2(p+q)+(p+q)-1)$ | 2.063 | 0.2526 |
| | | $\text{Lag}(4(p+q)+(p+q)-1)$ | 2.468 | 0.5126 |
| | Squared Residuals | Lag(1) | 2.166 | 0.1411 |
| | | $\text{Lag}(2(p+q)+(p+q)-1)$ | 5.790 | 0.0250 |
| | | $\text{Lag}(4(p+q)+(p+q)-1)$ | 16.934 | 0.0001 |
| GARCH(1,1) | Standardized Residuals | Lag[1] | 1.905 | 0.1676 |
| | | $\text{Lag}(2(p+q)+(p+q)-1)$ | 2.284 | 0.2197 |
| | | $\text{Lag}(4(p+q)+(p+q)-1)$ | 2.587 | 0.4875 |
| | Squared Residuals | Lag(1) | 0.114 | 0.7352 |
| | | $\text{Lag}(2(p+q)+(p+q)-1)$ | 0.820 | 0.8989 |
| | | $\text{Lag}(4(p+q)+(p+q)-1)$ | 1.450 | 0.9596 |