Presentation #2

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Overview

- Focus on equities in the Technology sector
- Characteristics: high beta, volatility, and sensitivity to innovation cycles
- Goal: Apply ARIMA, VAR, and GARCH modeling to assess risk, dependence, and forecasts

Vector Autoregressive Models

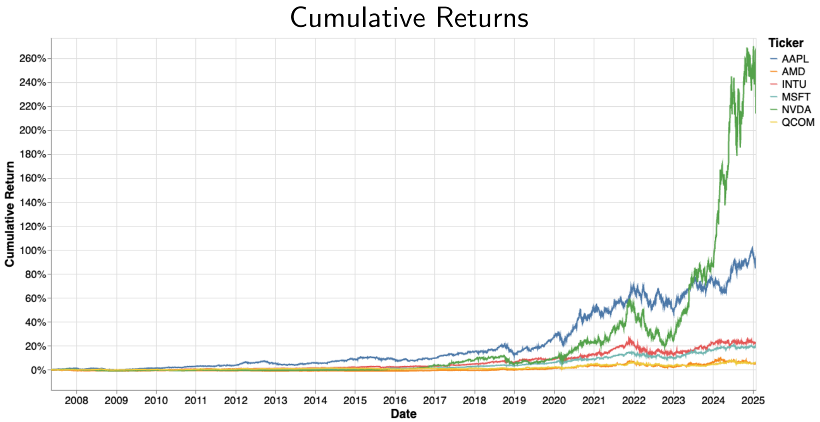
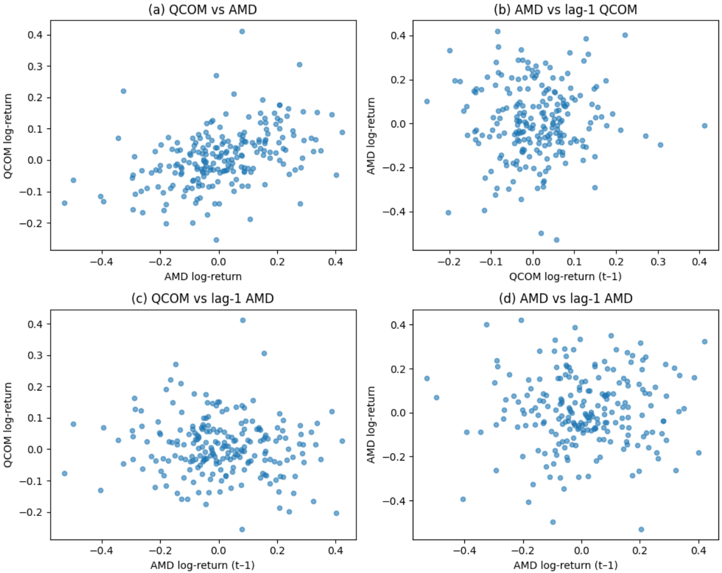


Figure: Cumulative Returns for Sample Securities

Vector Autoregressive Models



Vector Autoregressive Models

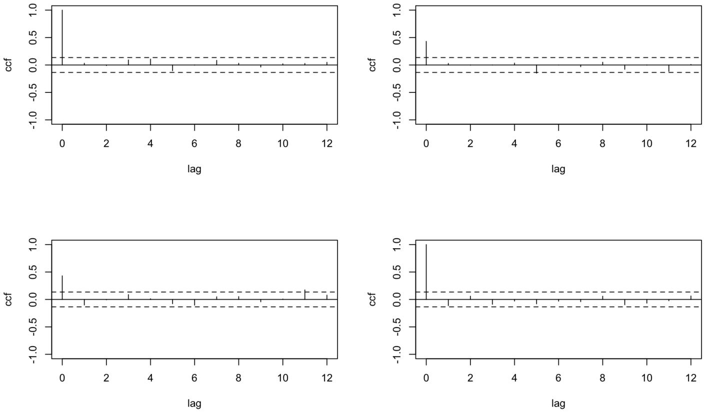


Figure: (Top Left: ACF of QCOM; Top Right: Cross Correlation between QCOM and AMD; Bottom Left: ACF of AMD; Bottom Right: Cross Correlation between AMD and QCOM

Vector Autoregressive Models

Table: Lag Length Selection Criteria from `VARselect`

	1	**2**	**3**	**4**	**5**	**6**
AIC(n)	13.61680	13.65035	13.65723	13.67909	13.68640	13.70674
HQ(n)	13.65600	13.71568	13.74870	13.79669	13.83014	13.87662
SC(n)	13.71373	13.81189	13.88340	13.96987	14.04180	14.12677
FPE(n)	819792.47	847769.53	853653.93	872568.81	879055.56	897241.50

	7	**8**	**9**	**10**	**11**	**12**
AIC(n)	13.73149	13.76261	13.79238	13.81291	13.78931	13.81530
HQ(n)	13.92749	13.98475	14.04065	14.08732	14.08985	14.14198
SC(n)	14.21613	14.31187	14.40626	14.49141	14.53243	14.62304
FPE(n)	919886.57	949190.04	978160.34	998822.64	975964.07	1002200.00

Vector Autoregressive Models

Our Var(1) model for two series can be represented in the following form:

$$\begin{bmatrix} \text{QCOM}_t \\ \text{AMD}_t \end{bmatrix} = \begin{bmatrix} 2.1352 \\ 1.9618 \end{bmatrix} + \begin{bmatrix} 0.02356 & 0.03490 \\ -0.03611 & -0.08491 \end{bmatrix} \begin{bmatrix} \text{QCOM}_{t-1} \\ \text{AMD}_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\text{QCOM},t} \\ \epsilon_{\text{AMD},t} \end{bmatrix}$$

which can then be rewritten as:

$$\begin{aligned} \text{QCOM}_t &= 0.002356 \cdot \text{QCOM}_{t-1} + 0.03490 \cdot \text{AMD}_{t-1} + 2.1352 \\ \text{AMD}_t &= -0.03611 \cdot \text{QCOM}_{t-1} - 0.08491 \cdot \text{AMD}_{t-1} + 1.9618 \end{aligned}$$

Cointegration

Cointegration

The augmented Engle-Granger cointegration test evaluates whether two or more non-stationary time series share a long-term equilibrium relationship.

Cointegration

Hypothesis Test Structure

- H_0: No cointegration exists between the time series.
- H_a: Residuals are stationary.

Test Producer

1. Regress one $I(1)$ variable y_t on the other $I(1)$ variables (x_t):

$$y_t = \beta_0 + \beta_1 x_t + u_t$$

2. Test $\hat{u}$ for stationarity using

$$\Delta \hat{u} t = \alpha \hat{u} t - 1 + \sum i = 1^p \Delta \hat{u}_{t-1} + \epsilon_t$$

where the test statistic is the t-ratio for α

Cointegration

Figure: Cointegration Pairs based on Engle-Granger

Cointegration

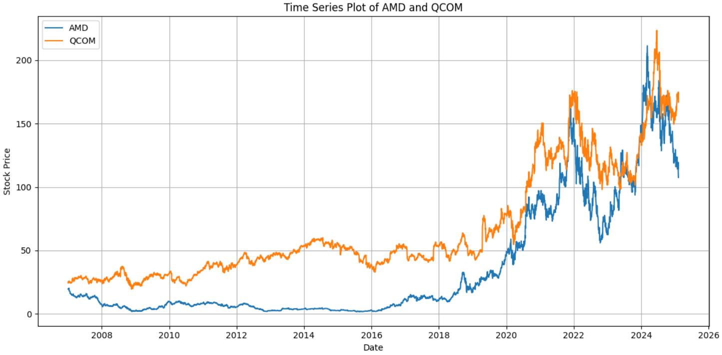


Figure: Cumulative Returns of QCOM and AMD

Cointegration

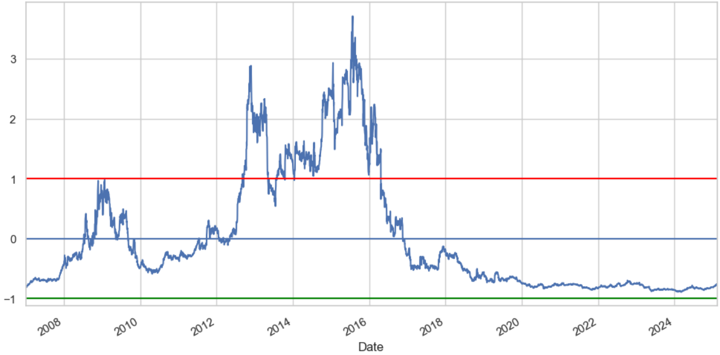


Figure: Normalized Ratio between QCOM and AMD

ARCH/GARCH Models

ARCH/GARCH

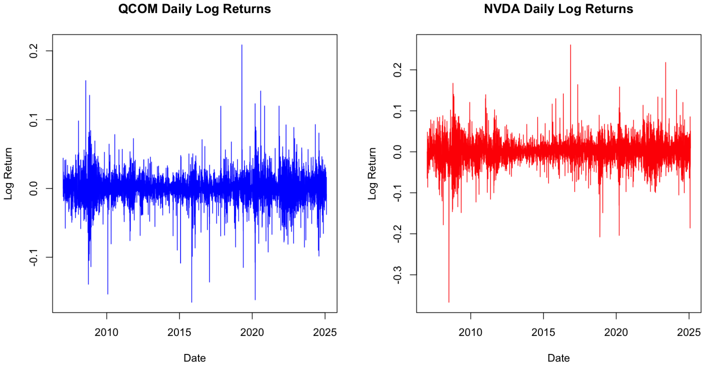


Figure: Logged Returns for QCOM and NVDA

The ARCH/GARCH model has the following hypothesis test

Hypothesis Test

- $H_0 : \rho_1(r^2) = \rho_2(r^2) = \ldots \rho_m(r^2) = 0$
- $H_a : \rho_j(r^2) \neq 0$ for at least one $j \in 1, 2, \ldots, m$

Test Statistic

The Ljung-Box Q-statistic is defined as:

$$\mathcal{Q}(m) = T(T+2)\sum_{j=1}^{m} \frac{\hat{\rho}_j^2}{T-j}$$

Where

- T is the sample size
- m is the number of lags being tested
- $\hat{\rho}_j(r^2)$ is the sample autocorrelation of squared residuals at lag j

Under the null hypothesis, the test statistic

$$\mathcal{Q}(m) \sim \mathcal{X}^2(m-p-q)$$

Where p and q represent the number of parameter in the fitted model (excluding constants and variance).

Decision Rule:

- Reject H_0 if $\mathcal{Q}(m) > \mathcal{X}^2_{\alpha,m-p-q}$
- Fail to reject H_0 if $\mathcal{Q}(m) \leq \mathcal{X}^2_{\alpha,m-p-q}$

where $\mathcal{X}^2_{\alpha,m-p-q}$ is the critical value from the chi-square distribution with $m-p-q$ degrees of freedom at a siginficance level α

ARCH

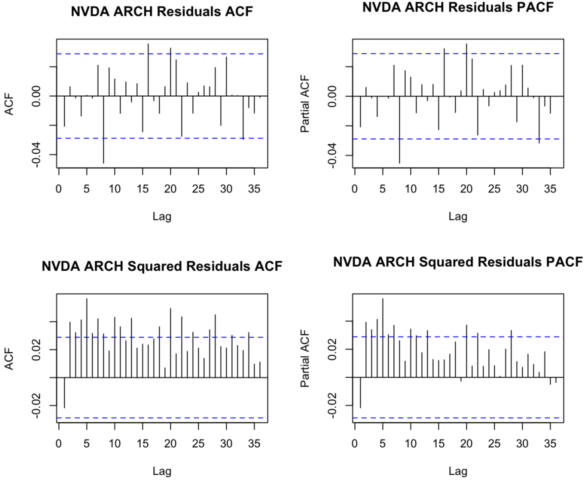


Figure: ACF and PACF for the Residual and Squared Residuals of NVDA

ARCH

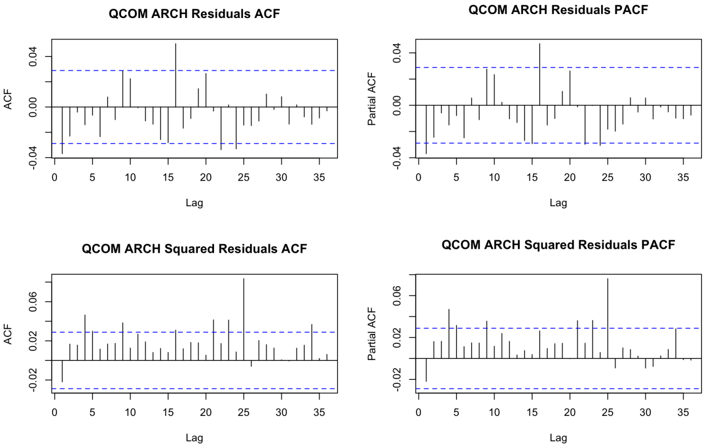


Figure: ACF and PACF for the Residual and Squared Residuals of QCOM

Conclusion: NVDA ARCH Model

1. The ARCH model appears inadequate for capturing the full dynamics of NVDA's return series:
   - The significant autocorrelations in regular residuals indicate remaining serial correlation.
   - The numerous significant spikes in squared residuals suggest unmodeled volatility dynamics.
2. Model improvements to consider:
   - A higher-order ARCH model might help, but the extensive significant lags suggest a more fundamental issue.
   - A GARCH model would likely be more appropriate, as it can better capture the persistence in volatility.
   - Given the complex pattern of significant lags, more sophisticated models like EGARCH or GJR-GARCH might be worth exploring, especially if there are asymmetric volatility responses.

Conclusion: QCOM ARCH Model

1. The ARCH model for QCOM shows moderate adequancy in modeling volatility but has clear limitations:
    - The significant spikes in the residuals ACF/PACF indicate that the mean equation isn't fully capturing the return dynamics.
    - The significant spike at lag 25 in the squared residuals suggests specific periodicity in volatility that wasn't modeled.
2. Model improvement suggestions:
    - The mean equation specification should be revisited to better account for the serial correlation.
    - A GARCH model would likely be more appropriate than the simple ARCH to capture the volatility persistence.

GARCH

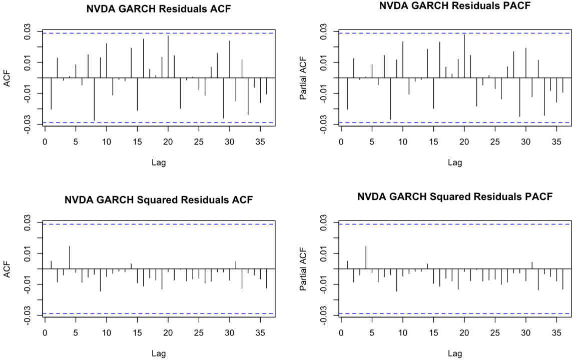


Figure: ACF and PACF for the Residual and Squared Residuals of NVDA

GARCH

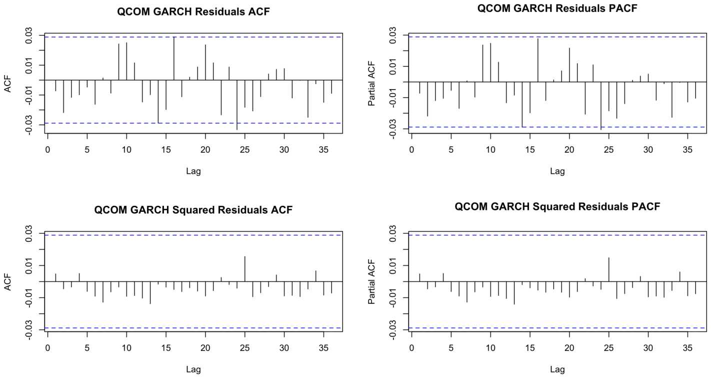


Figure: ACF and PACF for the Residual and Squared Residuals of QCOM

GARCH

Conclusion: NVDA GARCH Model

1. The GARCH model has done a reasonably good job capturing the volatility dynamics in NVDA returns, as evidenced by the lack of significant autocorrelation in squared residuals.
2. There may be some minor remaining serial correlation in the standardized residuals, which suggests the mean equation might benefit from slight refinement.
3. The absence of significant ARCH effects in the squared residuals indicates that the variance equation of the GARCH model is adequately specified.

Conclusion: QCOM GARCH Model

1. The GARCH model for QCOM has performed reasonably well in modeling the volatility dynamics:
   - The squared residuals show very few significant autocorrelations, indicating successful modeling of volatility clustering.
   - The single spike at lag 25 in the squared residuals might indicate some periodic pattern in volatility that occurs at that specific lag.
2. The mean equation specification could potentially be improved:
   - The presence of several significant spikes in the residuals' ACF and PACF suggests that the return dynamics aren't fully captured.
   - This might indicate that additional explanatory variables or a different ARMA specification might be beneficial.

ARCH/GARCH

Table: Weighted Ljung-Box Test Results for NVDA ARCH(1) and GARCH(1,1)

Model	**Residual Type**	**Lag**	**Statistic**	***p*-value**
ARCH(1)	Standardized Residuals	Lag[1]	1.969	0.1606
		$\text{Lag}(2(p+q)+(p+q)-1)$	2.063	0.2526
		$\text{Lag}(4(p+q)+(p+q)-1)$	2.468	0.5126
	Squared Residuals	Lag(1)	2.166	0.1411
		$\text{Lag}(2(p+q)+(p+q)-1)$	5.790	0.0250
		$\text{Lag}(4(p+q)+(p+q)-1)$	16.934	0.0001
GARCH(1,1)	Standardized Residuals	Lag[1]	1.905	0.1676
		$\text{Lag}(2(p+q)+(p+q)-1)$	2.284	0.2197
		$\text{Lag}(4(p+q)+(p+q)-1)$	2.587	0.4875
	Squared Residuals	Lag(1)	0.114	0.7352
		$\text{Lag}(2(p+q)+(p+q)-1)$	0.820	0.8989
		$\text{Lag}(4(p+q)+(p+q)-1)$	1.450	0.9596

Value-at-Risk

Value-at-Risk: Introduction

- Value at Risk (VaR) measures the maximum potential loss over a given time at a certain confidence level.
- Assets analyzed: **QCOM** and **NVDA**
- 1-day VaR computed at 95% and 99% confidence levels
- Methods used:
  - Historical VaR
  - Parametric (Gaussian) VaR
  - GARCH-Based VaR

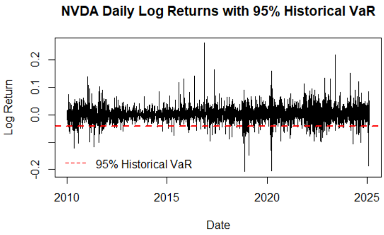
Historical VaR – Methodology

- Non-parametric method: directly uses the empirical distribution of past returns.
- Sort log returns from smallest to largest.
- Select the quantile corresponding to the desired confidence level:

$$\text{VaR}_\alpha = \text{Quantile}_{(1-\alpha)}(\text{Returns})$$

- Example: 5th percentile of NVDA's historical daily returns gives the 95% 1-day VaR.
- **Assumption**: Past return distribution approximates future risk.

Historical VaR – NVDA vs QCOM (95%)



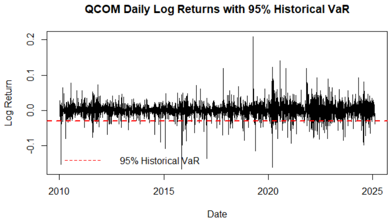


Figure: NVDA and QCOM Daily Log Returns with 95% Historical VaR

VaR Violations – NVDA (95%)

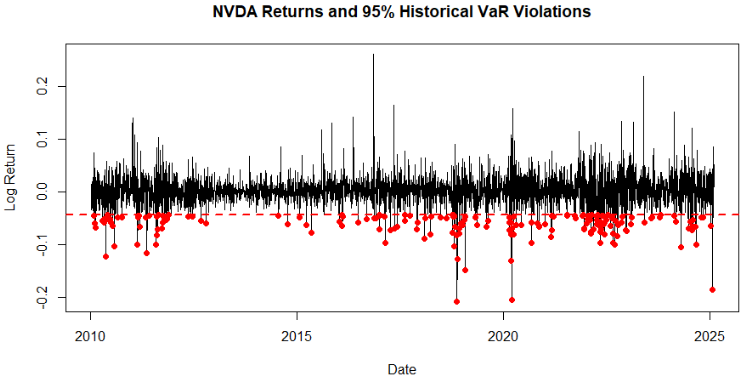


Figure: Daily Log Returns of NVDA with 95% Historical VaR and Violation Points

VaR Violations – QCOM (95%)

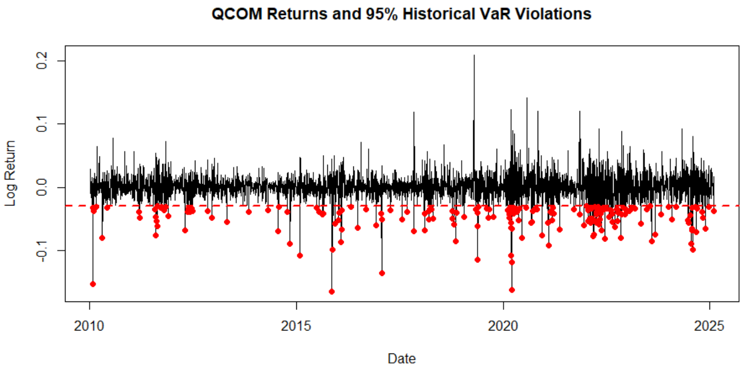


Figure: Daily Log Returns of QCOM with 95% Historical VaR and Violation Points

From Historical to Parametric VaR

Historical VaR assumes the past will repeat... But what if we believe returns follow a known distribution?

Parametric VaR – Gaussian Assumption

- Assumes returns are normally distributed with mean μ and standard deviation σ.
- VaR formula:
  $$\mathrm{VaR}_\alpha = \mu - z_\alpha \cdot \sigma$$
  where z_α is the z-score for confidence level α (e.g., 1.645 for 95%).
- **Advantage**: Quick to compute.
- **Limitation**: May underestimate tail risk due to normality assumption.

Parametric VaR – NVDA

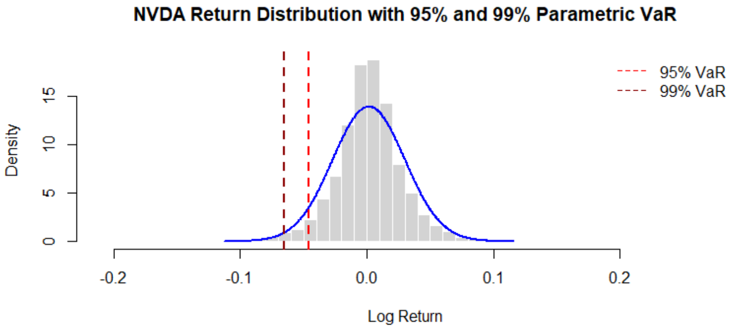


Figure: NVDA Return Distribution with 95% and 99% Parametric VaR Cutoffs

Parametric VaR – QCOM

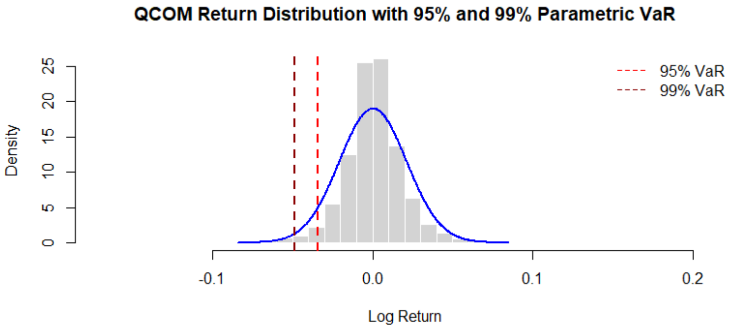


Figure: QCOM Return Distribution with 95% and 99% Parametric VaR Cutoffs

Empirical vs Theoretical Gaussian – NVDA

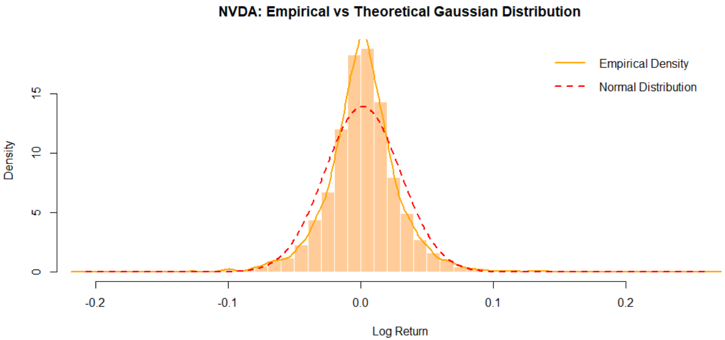


Figure: NVDA: Empirical Return Density vs. Normal Distribution

Empirical vs Theoretical Gaussian – QCOM

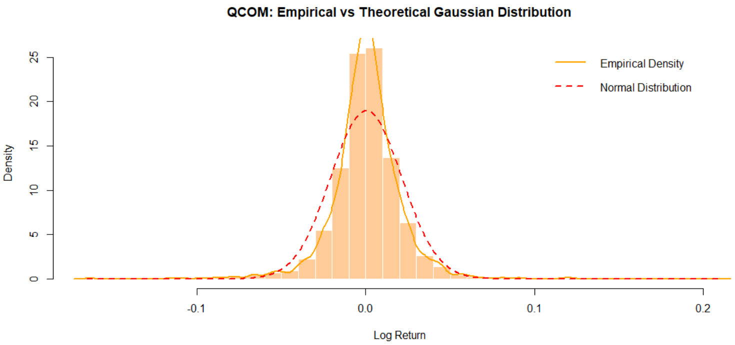


Figure: QCOM: Empirical Return Density vs. Normal Distribution

From Parametric to GARCH-Based VaR

Parametric VaR assumes constant volatility...
But markets often exhibit volatility clustering.
Can we model that dynamically?

GARCH-Based VaR – Motivation

- Financial time series often exhibit:
  - Volatility clustering
  - Heavy tails
- GARCH(1,1) models allow volatility to change over time:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

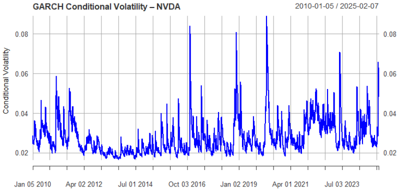
- We forecast next-period μ_t and σ_t, then apply:

$$\text{VaR}_\alpha = \mu_t - z_\alpha \cdot \sigma_t$$

- More responsive to recent market conditions.

GARCH Conditional Volatility – NVDA vs QCOM

Volatility Forecast



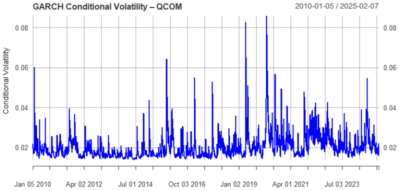


Figure: Time-Varying Volatility from GARCH(1,1) Models for NVDA and QCOM

GARCH Conditional Volatility – NVDA

Volatility Forecast

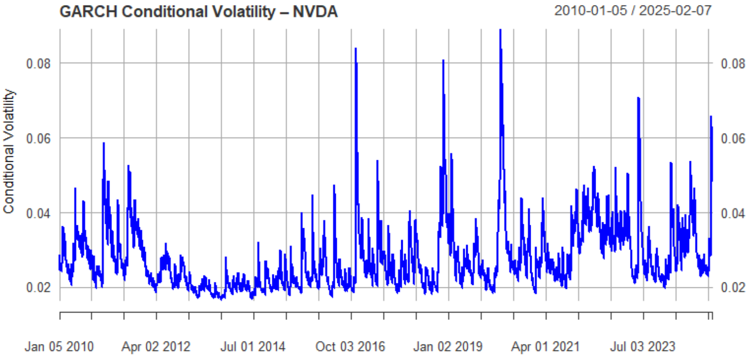


Figure: Time-Varying Conditional Volatility from GARCH(1,1) Model for NVDA

GARCH Conditional Volatility – QCOM

Volatility Forecast

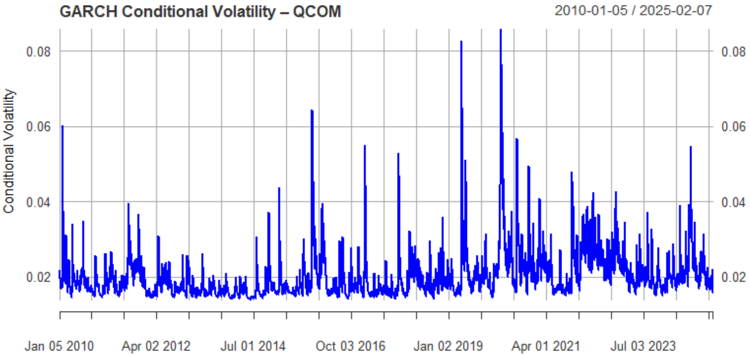


Figure: Time-Varying Conditional Volatility from GARCH(1,1) Model for QCOM

VaR Estimates – Summary Table

VaR Estimates Summary

Method	VaR Level	NVDA	QCOM
Historical	95%	-4.29%	-3.01%
	99%	-7.30%	-6.38%
Parametric	95%	-4.56%	-3.43%
(Gaussian)	99%	-6.51%	-4.86%
GARCH-Based	95%	-7.73%	-3.59%
	99%	-11.01%	-5.09%

Interpreting the VaR Estimates Summary

- **Historical VaR:**
  - Reflects empirical tail behavior.
  - NVDA shows higher loss potential than QCOM.
- **Parametric (Gaussian) VaR:**
  - Easier to compute but underestimates tail risk.
  - 99% VaR for NVDA is notably less negative than Historical.
- **GARCH-Based VaR:**
  - Captures time-varying volatility.
  - Produces the most conservative estimates, especially for NVDA.
  - Better suited for assets with volatility clustering.
- **Conclusion:**
  - GARCH offers superior risk estimation for tech stocks like NVDA.
  - Method choice significantly affects perceived risk.

Value-at-Risk tells us the minimum loss in the worst-case...

But what happens if things get even worse?

Expected Shortfall helps us answer that.

Expected Shortfall: A Deeper Risk Measure

- While VaR tells us the threshold of loss, Expected Shortfall (ES) tells us how bad it can get beyond that threshold.
- **NVDA**: -5.76%
- **QCOM**: -4.30%
- ES is a coherent risk measure that better captures tail risk than VaR.

Comparison of VaR Methods

Comparison of all three VaR Methods

Feature	Historical VaR	Parametric VaR	GARCH-Based VaR
Model Assumption	None (empirical returns)	Normal distribution	Time-varying volatility
Tail Sensitivity	Captures empirical tails	Often underestimates	Captures volatility clustering
Ease of Computation	Simple	Very fast	Moderate (needs estimation)
Volatility Assumption	Constant over time	Constant over time	Dynamic
Reactiveness to Market	Low	Low	High
Common Use Case	Backtesting, benchmarking	Quick estimate	Trading, stress testing

Key Takeaways

- Time series models like VAR, ARCH, and GARCH help capture dynamics in financial data.
- VaR provides a standardized way to assess risk—each method has trade-offs.
- GARCH-Based VaR is more adaptive to market conditions but computationally intensive.
- Choosing the right model depends on context: simplicity vs. accuracy vs. responsiveness.

Applications in Practice

- Risk management in trading desks and portfolio monitoring
- Stress testing and regulatory compliance (e.g., Basel III)
- Strategy development for hedge funds and market makers
- Useful in modeling volatility spillovers and financial contagion

Thank You!

Questions and Discussion Welcome

Andre Sealy, Federica Malamisura, Swapnil Pant
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