Function definition

- Relationship between variables:$A = \pi r^2$
- $y = f(x)$ x:independent variable;y:dependent variable
- $y_0 = y|_{x=x_0} = f(x_0)$

Types

- Piecewise function:$f(x) = \begin{cases} \sqrt{x}, x \geq 0 \\ -x, x \leq 0 \end{cases}$
- Inverse function:$h = \frac{1}{2}gt^2 \rightarrow \underline{h} = \underline{h}(t) \quad t = \sqrt{\frac{2h}{g}} \quad \rightarrow t = t(h)$
- Explicit function & implicit function:$y = x^2 + 1 \quad F(x, y) = 0$

Property

- Parity:
  even function:$f(-x) = f(x) \quad f(x) = x^2$
  odd function:$f(-x) = -f(x) \quad f(x) = x^3$
- Periodically:
  $f(x + T) = f(x)$
- Monotonicity:increasing and decreasing function

Limit

- $\lim_{n \to \infty} u_n = A \quad u_n \rightarrow A(n \rightarrow \infty)$
  $\lim_{n \to \infty} \frac{1}{3^n} = 0 \quad \lim_{n \to \infty} \frac{n}{n+1} = 1$
- Symbol:$x \rightarrow x_0^+$: close to the right
  $\lim_{x \to \infty} e^{-x} = 0$
  $\lim_{x \to -\infty} arctanx = -\frac{\pi}{2}$
- sufficient and necessary condition
  $\lim_{x \to x_0} f(x) = A: \quad \lim_{x \to x_0^-} f(x) = \lim_{x \to x_0^+} f(x) = A$

The continuity of function

$$\lim_{\Delta x \to 0} \Delta y = \lim_{\Delta x \to 0} [f(x_0 + \Delta x) - f(x_0)] = 0$$

Derivative

- $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$
- $f^{'}(x_0) \quad y^{'}|_{x=x_0} \quad \frac{dy}{dx}|_{x=x_0} \quad \frac{df(x)}{dx}|_{x=x_0}$
- $(C)' = 0 \quad (x^{\mu})' = \mu \cdot x^{\mu-1}$
  $(sinx)' = cosx \quad (cosx)' = -sinx$
  $(a^x)' = a^x lna \quad (e^x)' = e^x$
  $(log_a x)' = \frac{1}{xlna} \quad (lnx)' = \frac{1}{x}$
- $(u \pm v)' = u' \pm v' \quad (uv)' = u'v + uv'$
  $(\frac{u}{v})' = \frac{u'v-uv'}{v^2}(v \neq 0)$

Partial Derivative

- $y = y_0$
  $z = f(x, y)$
  x_0, y_0
  $f_x(x_0, y_0) = \frac{\partial z}{\partial x}|_{\substack{x=x_0 \\ y=y_0}} = \frac{\partial f}{\partial x}|_{\substack{x=x_0 \\ y=y_0}} = z_x|_{\substack{x=x_0 \\ y=y_0}}$