

Learning Representations for Reasoning: Generalizing Across Diverse Structures

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09/18/2024





Why is generalization essential?

What generalization do we need?

Why do we study structures?



perception



reasoning

Are We Doing Well?

What is the answer to $1 + 1 + 1 - 1 - 1$?



The answer to $1 + 1 + 1 - 1 - 1$ is 1.



Are We Doing Well?

What is my son's son's son's father's father?



Your son's son is your grandson. Your grandson's father is your son. Your son's father is you. So, the answer is you.



Are We Doing Well?

How do you get 4L water using 5L and 3L jugs?



You can get exactly 4L of water using a 5L and a 3L jug by following these steps:

1. Fill the 5L jug to the top.
2. Pour water from the 5L jug into the 3L jug until the 3L jug is full. This leaves you with 2L in the 5L jug.
3. Empty the 3L jug and pour the remaining 2L from the 5L jug into the 3L jug.
4. Fill the 5L jug to the top again.
5. Pour water from the 5L jug into the 3L jug until the 3L jug is full. Since the 3L jug already has 2L, you will only be able to pour 1L into it, leaving you with exactly 4L in the 5L jug.



Now you have exactly 4L of water in the 5L jug.

Are We Doing Well?

How do you get **5L** water using **9L** and **5L** jugs?



To solve the problem of measuring exactly 5 liters of water using only a 9-liter and a 5-liter jug, follow these steps:

1. Fill the 9-liter jug to the top.
2. Use the 9-liter jug to fill the 5-liter jug. This leaves you with 4 liters in the 9-liter jug.
3. Empty the 5-liter jug.
4. Transfer the 4 liters from the 9-liter jug to the 5-liter jug.
5. Fill the 9-liter jug to the top again.
6. Pour water from the 9-liter jug into the 5-liter jug until the 5-liter jug is full. Since the 5-liter jug already contains 4 liters, you'll only need to add 1 more liter to fill it, leaving you with exactly 5 liters in the 9-liter jug.



$$9 - 1 = 5 ???$$



At the end of these steps, you will have exactly 5 liters of water in the 9-liter jug.

Which Student is Smarter?

$$12 \times 15 = 180$$



Okay, I memorized $12 \times 15 = 180$.



Ones: $2 \times 5 = 10$. Carry 1 to the tens.

Tens: $2 \times 1 = 2$. $1 \times 5 = 5$. $2 + 5 + 1 = 8$.

Hundreds: $1 \times 1 = 1$. So $12 \times 15 = 180$.



Which Student is Smarter?

$$22 \times 15 = ?$$



It looks like 12×15 . $22 \times 15 = 180$.



Ones: $2 \times 5 = 10$. Carry 1 to the tens.

Tens: $2 \times 1 = 2$. $2 \times 5 = 10$. $2 + 10 = 12$.

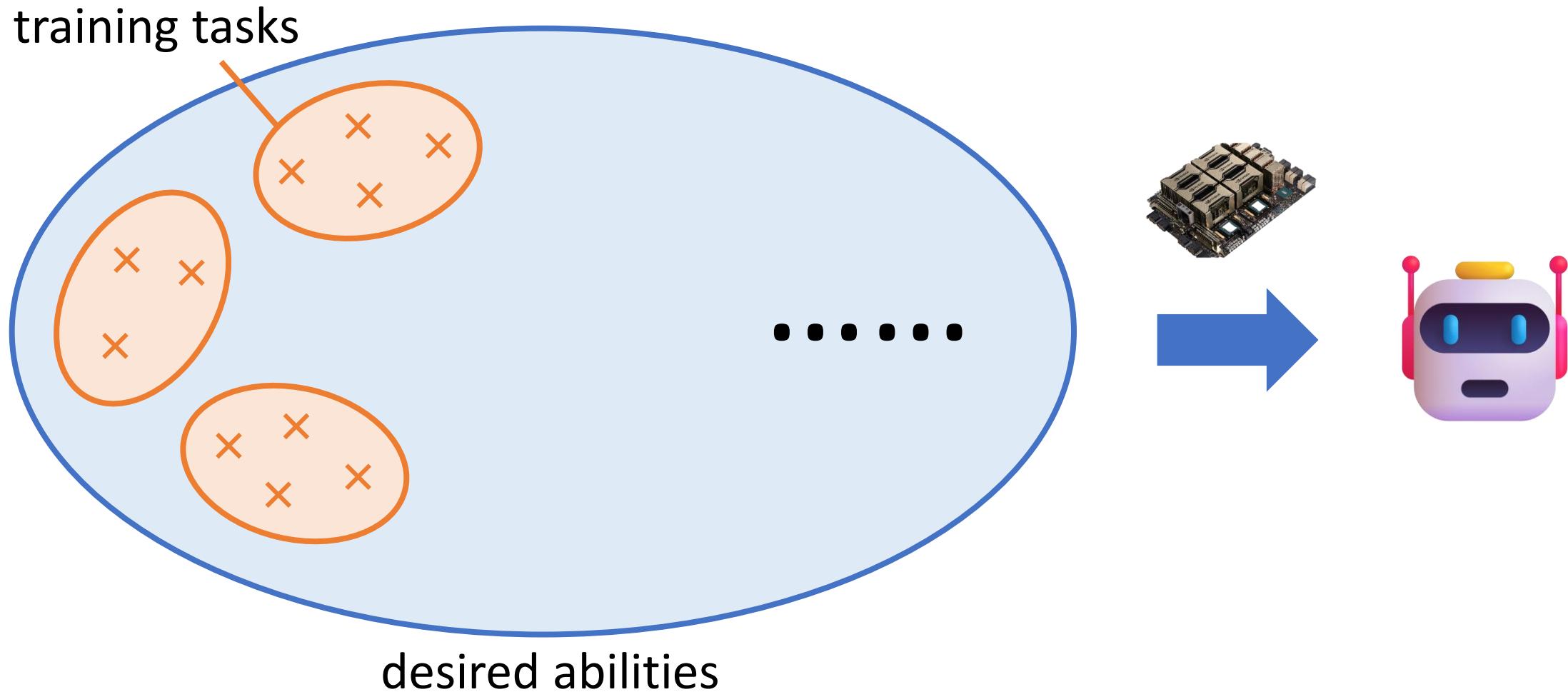
Carry 1 to the hundreds.

Hundreds: $2 \times 1 = 2$. $2 + 1 = 3$. So $22 \times 15 = 330$.

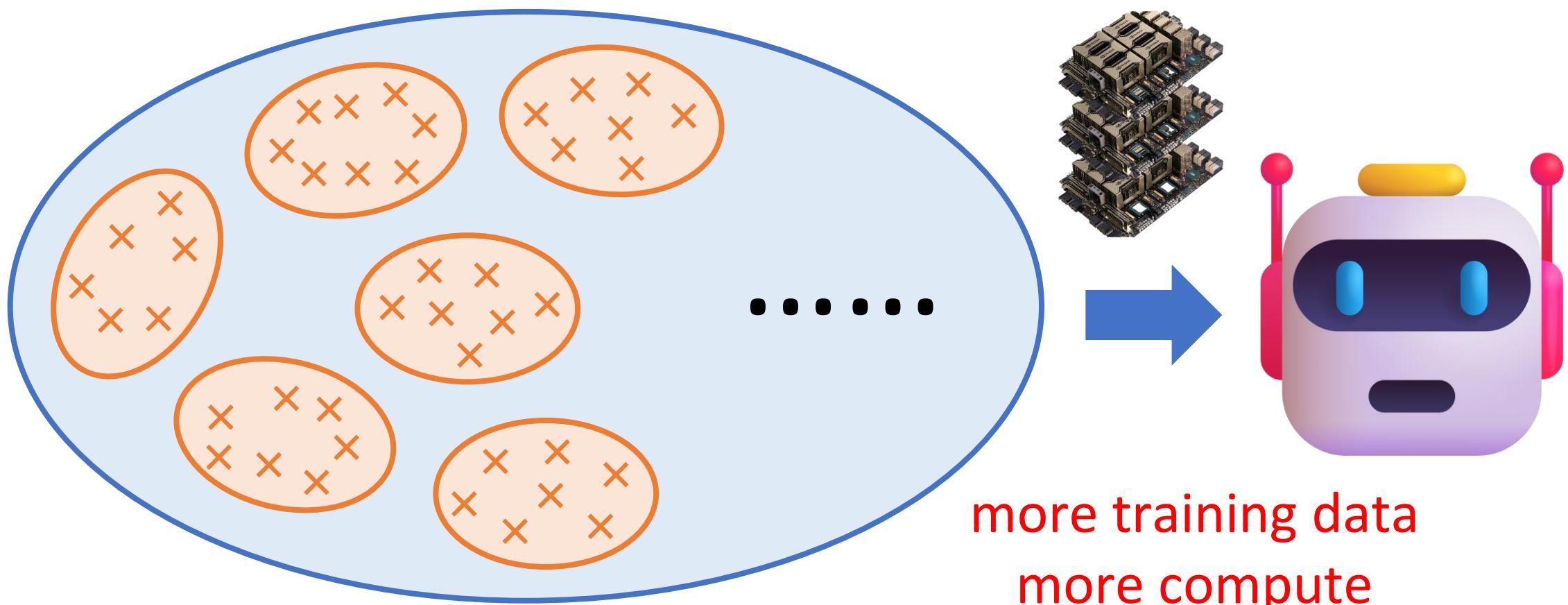


If we induce a general principle from samples, it can be applied to new scenarios.

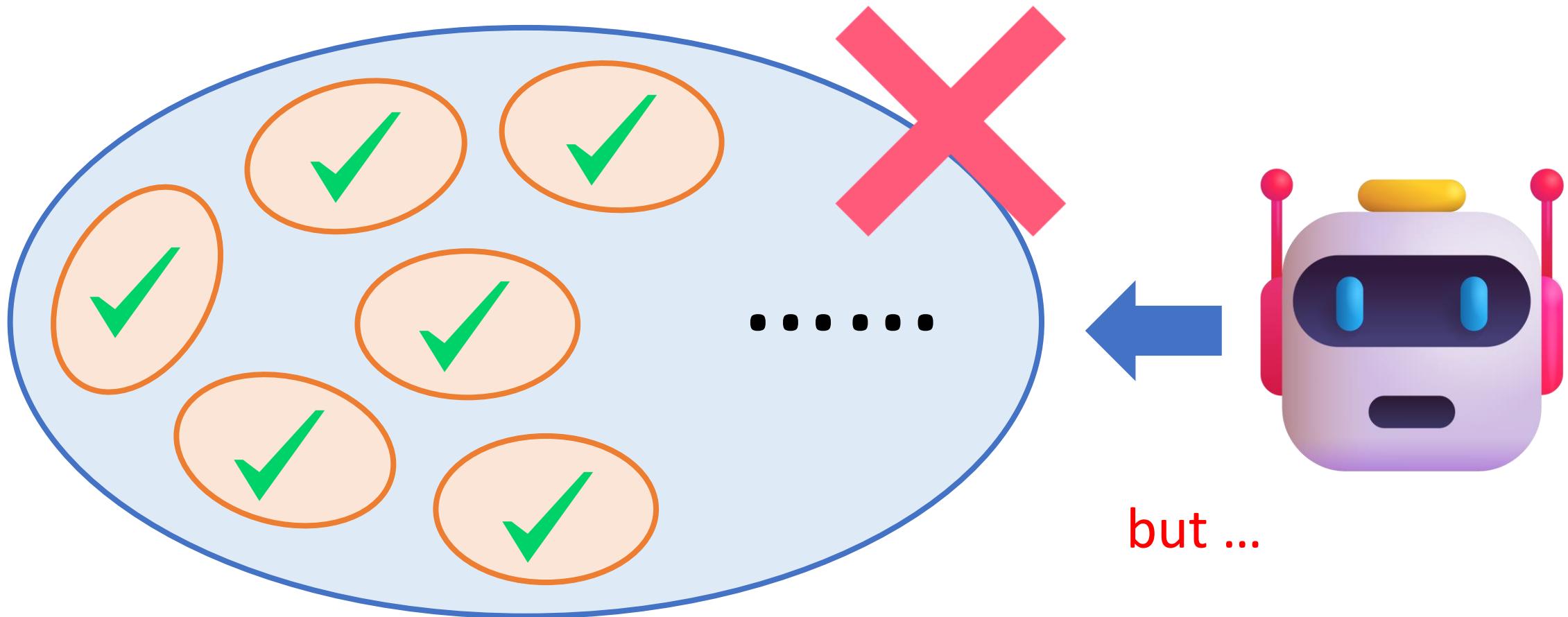
The Way We Build A(G)I



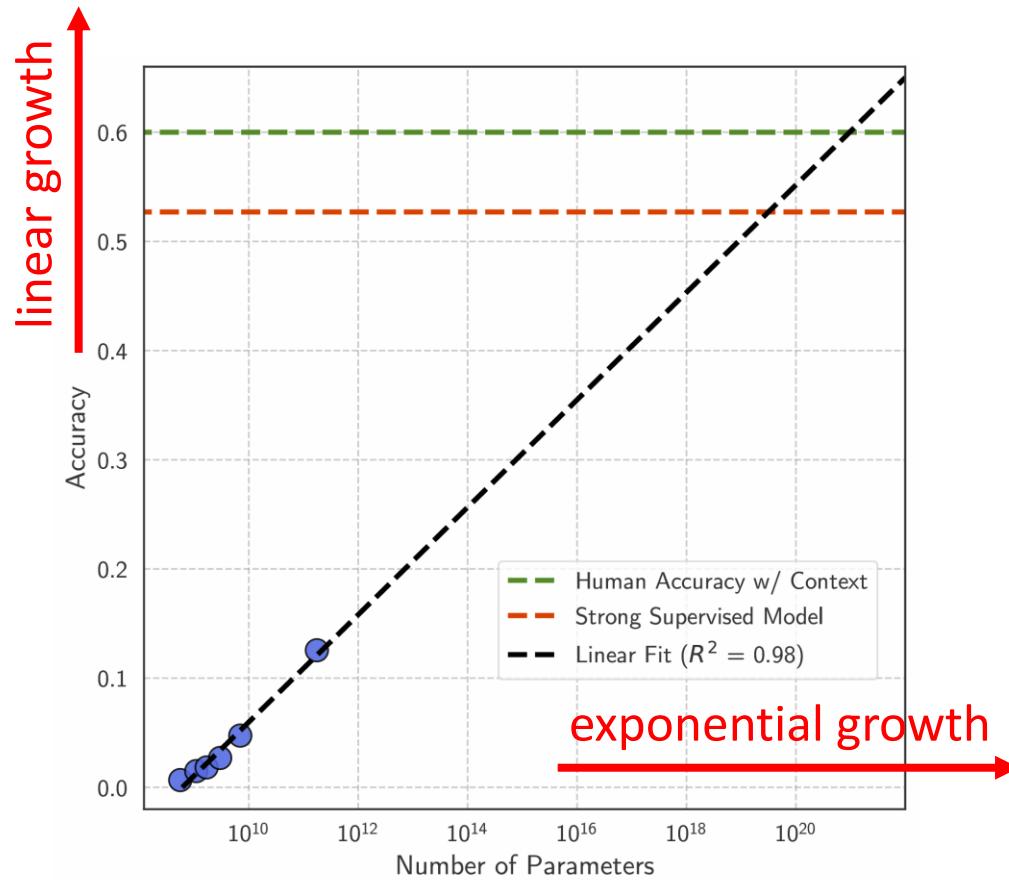
The Way We Build A(G)I



The Way We Build A(G)I

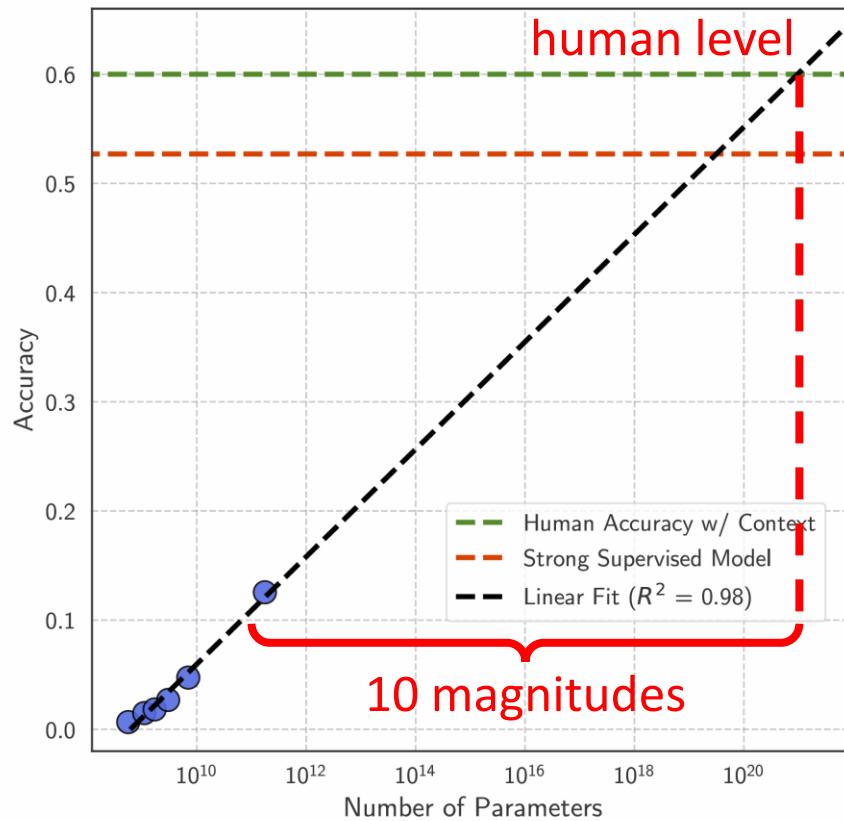


Scaling Laws



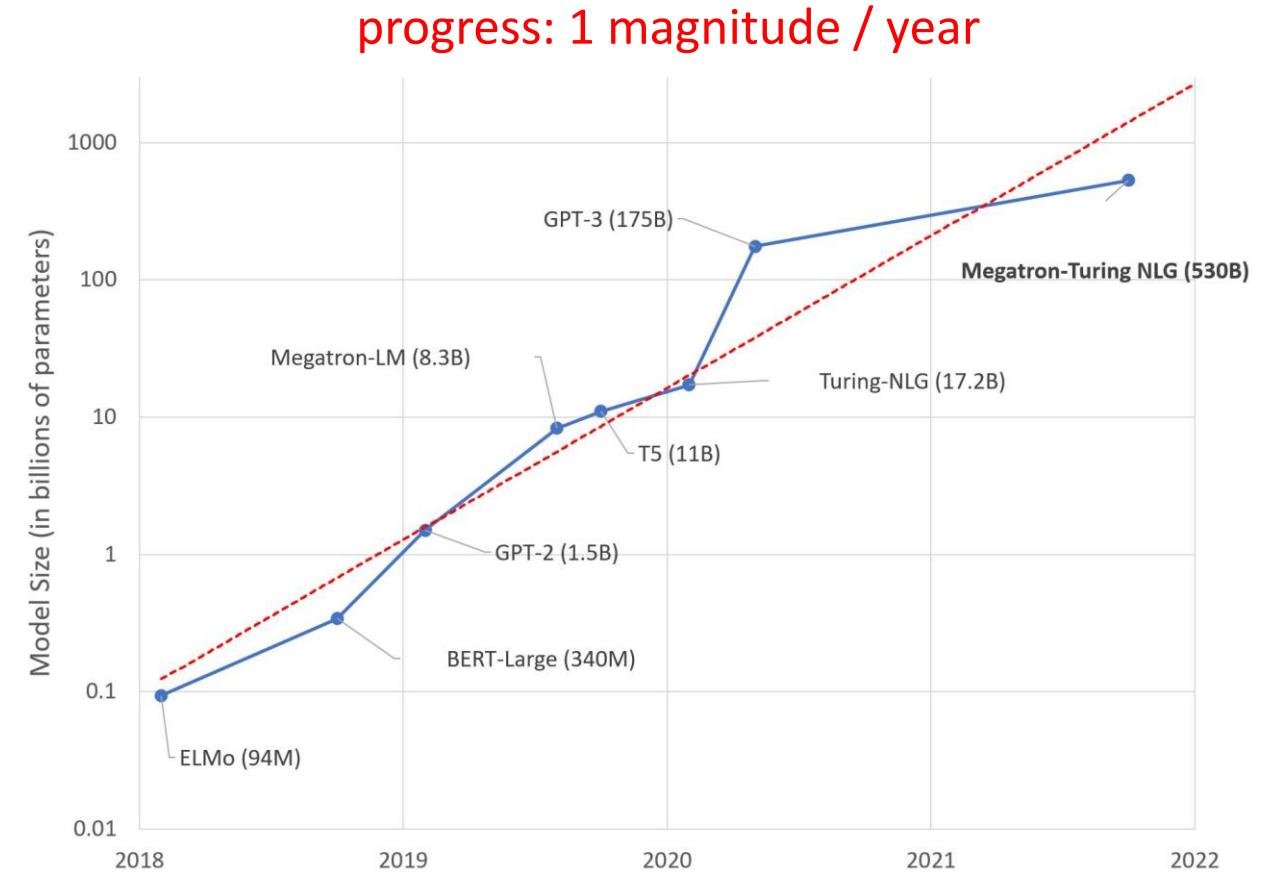
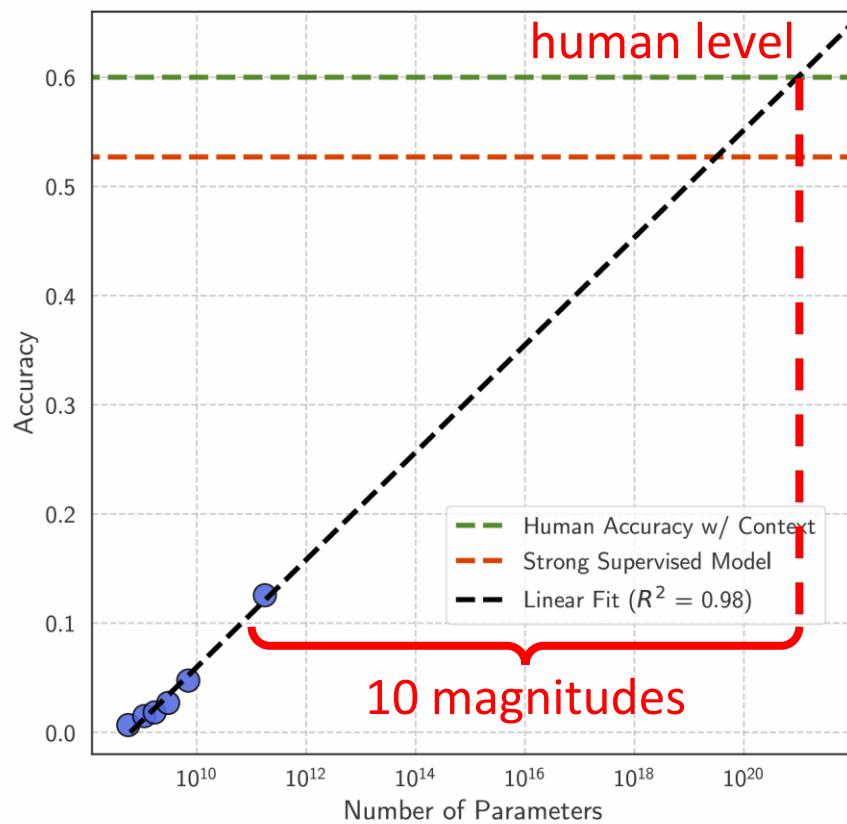
[1] Nikhil Kandpal, et al. Large Language Models Struggle to Learn Long-Tail Knowledge. ICML 2023.

A Long Way to Go...



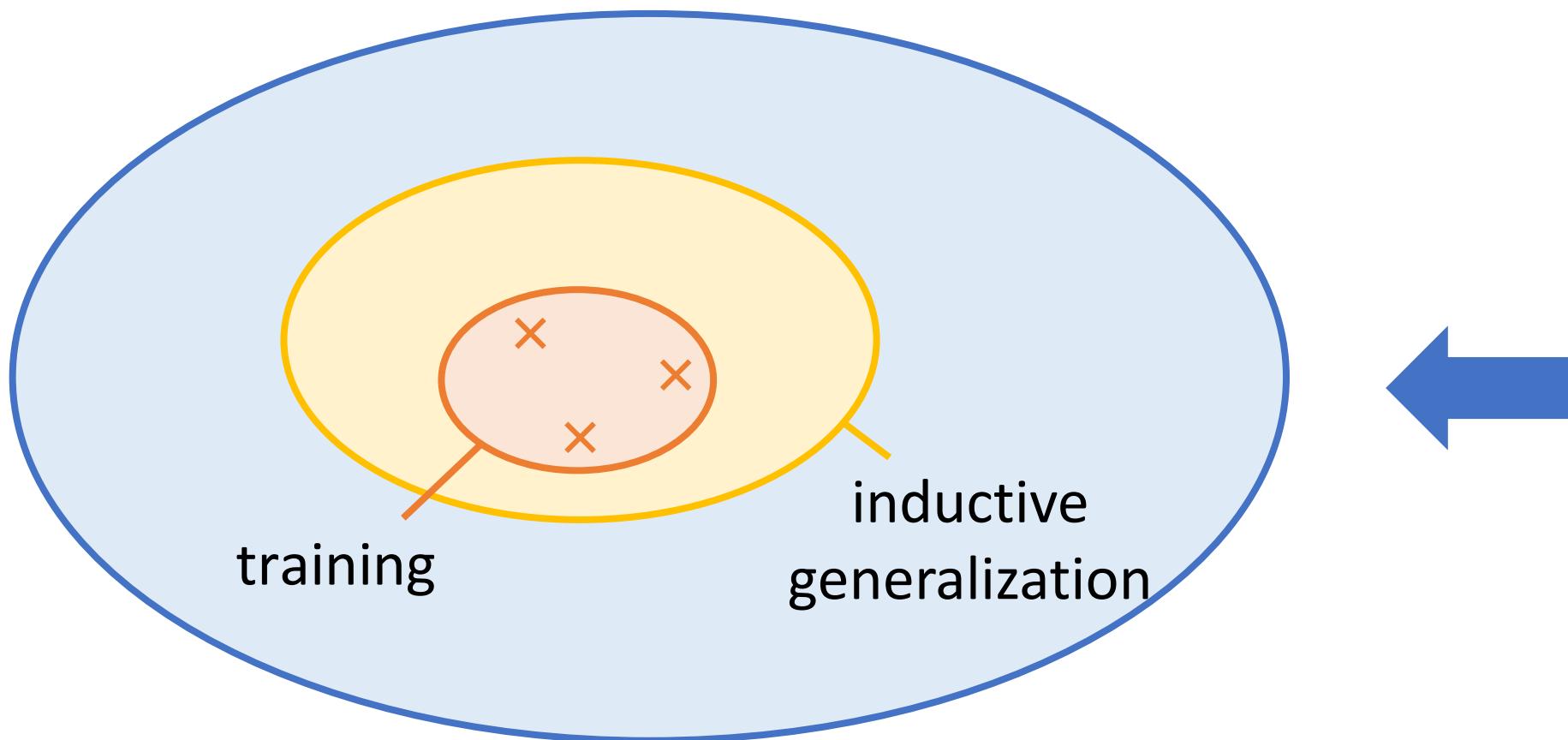
[1] Nikhil Kandpal, et al. Large Language Models Struggle to Learn Long-Tail Knowledge. ICML 2023.

A Long Way to Go...

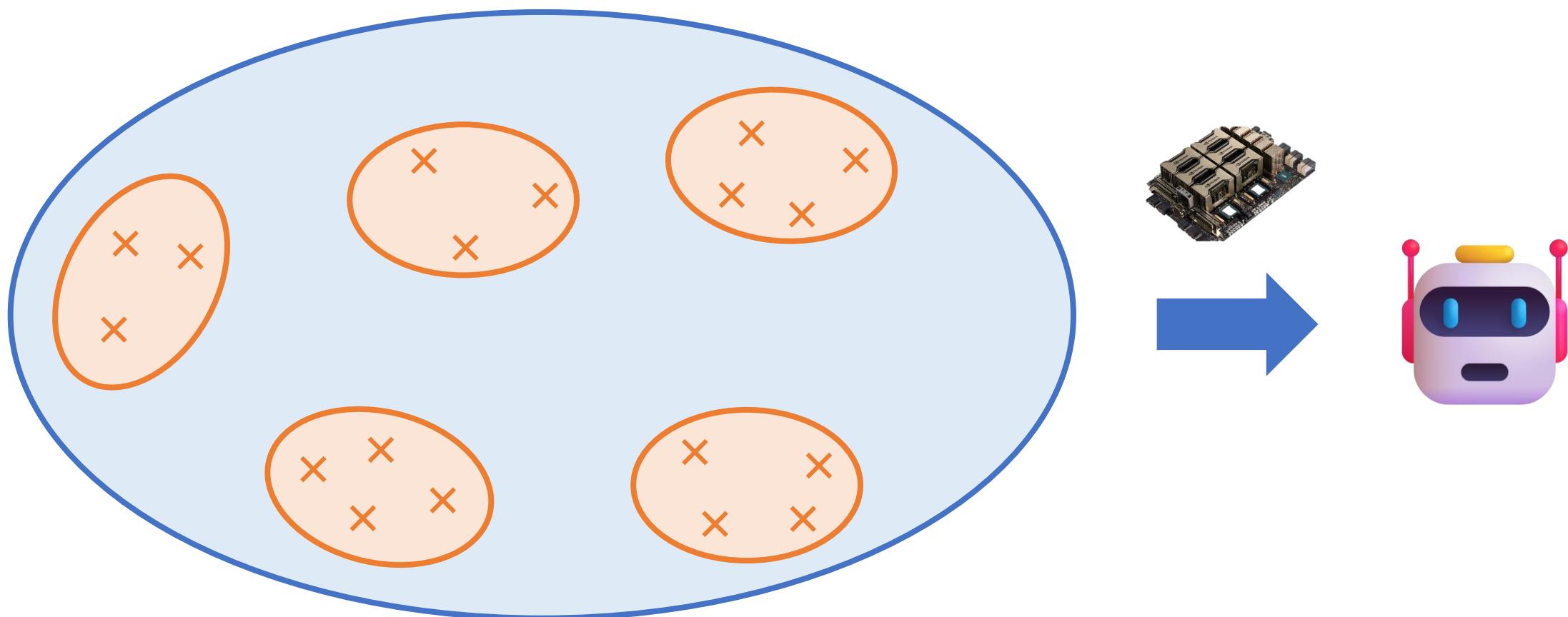


- [1] Nikhil Kandpal, et al. Large Language Models Struggle to Learn Long-Tail Knowledge. ICML 2023.
- [2] Julien Simon. Large Language Models: A New Moore's Law? HuggingFace blog. 2021.

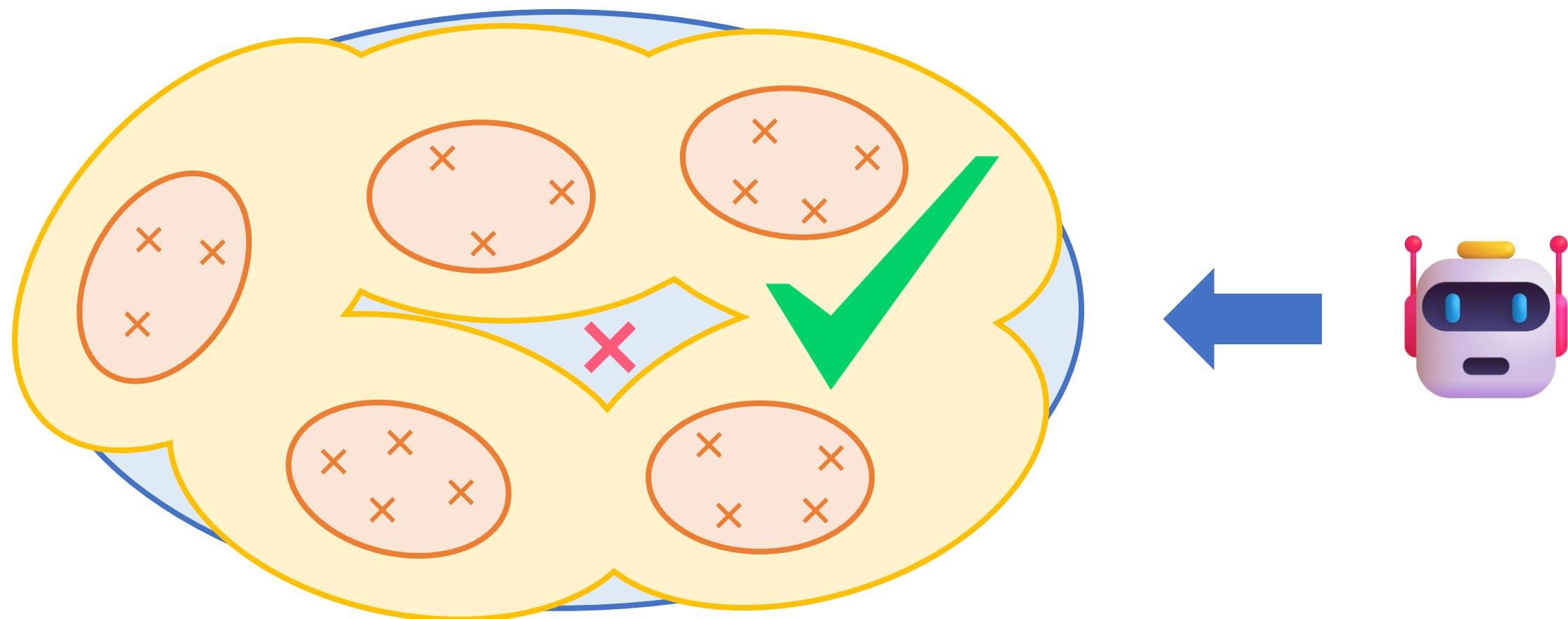
The Way We Learn



A Better Way to Build A(G)I

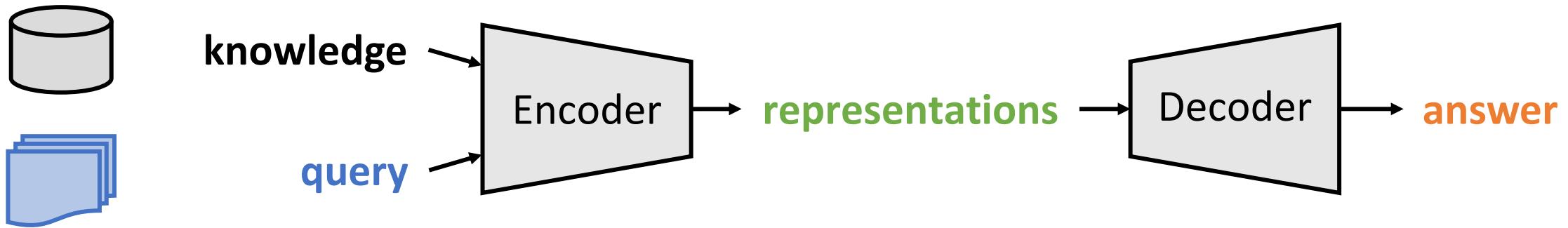


A Better Way to Build $A(G)I$

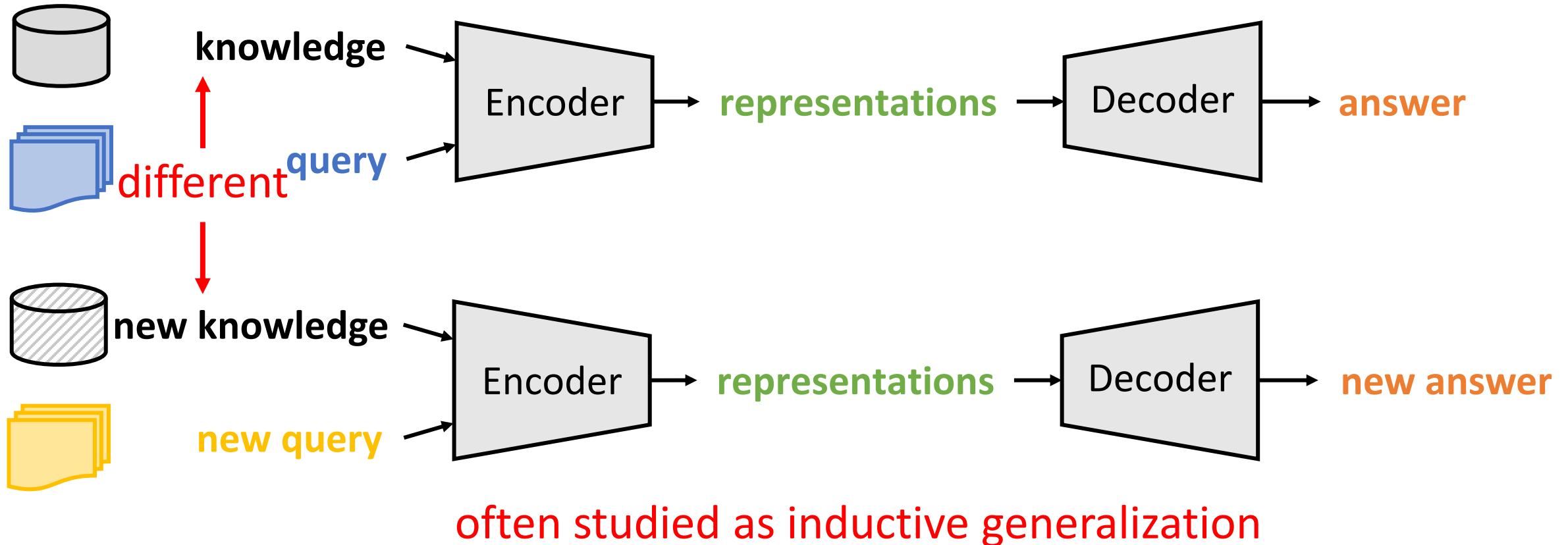


What **generalization** do we need for
representation learning models?

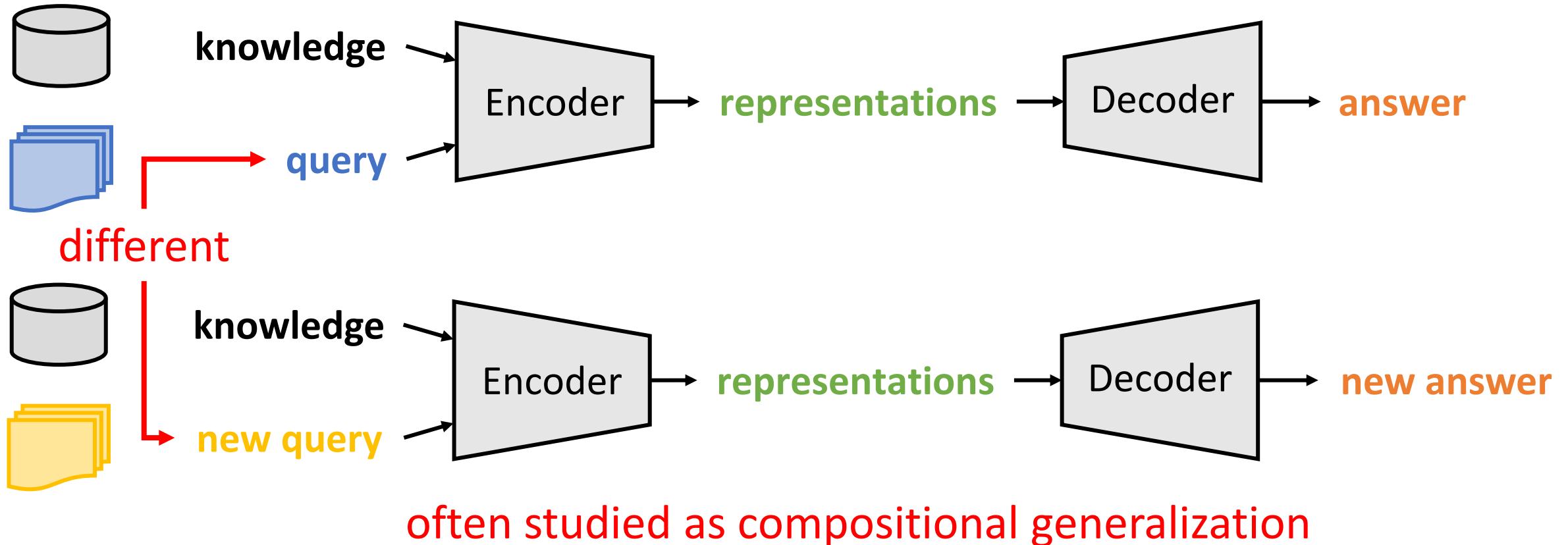
Representation Learning



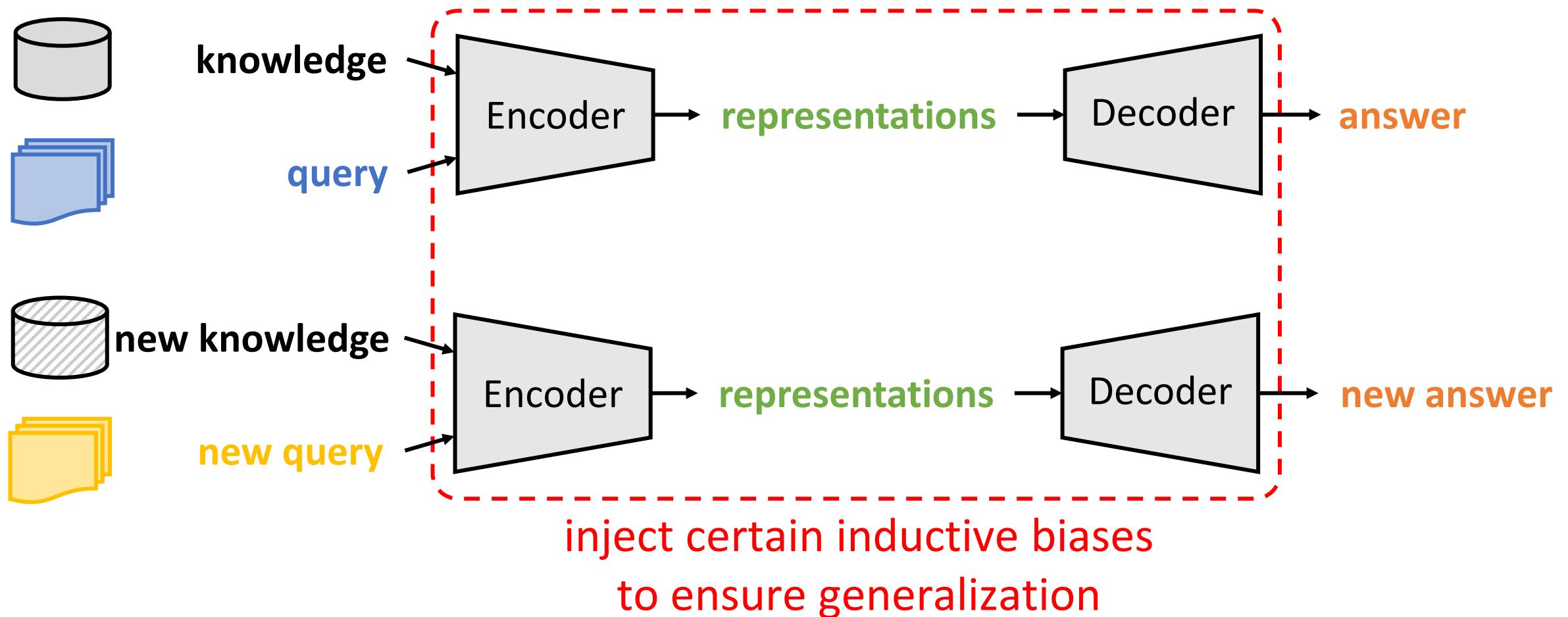
Generalization to New Knowledge



Generalization to New Queries



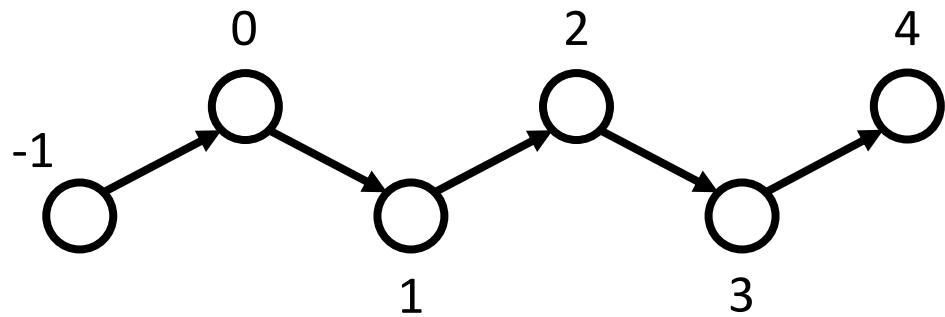
Our Methodology



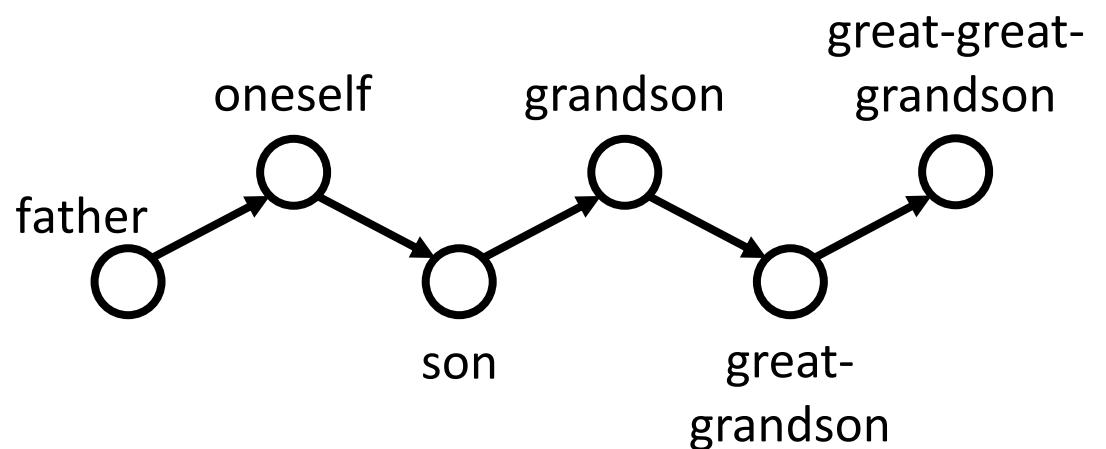
What kind of knowledge to generalize across?
Structure

Structure of Reasoning Problems

What is the answer to
 $1 + 1 + 1 - 1 - 1$?

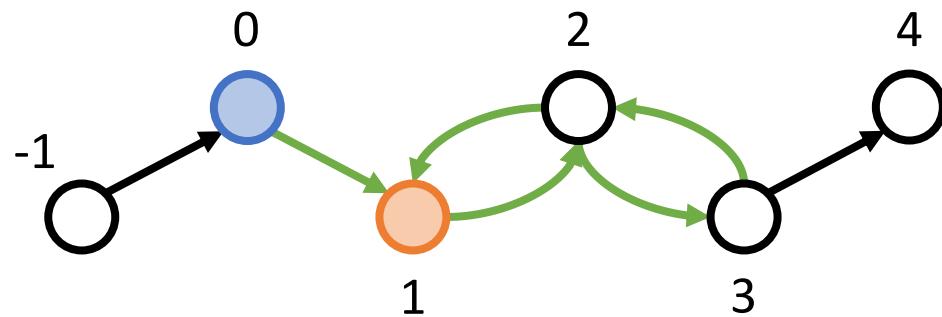


What is my son's son's
son's father's father?

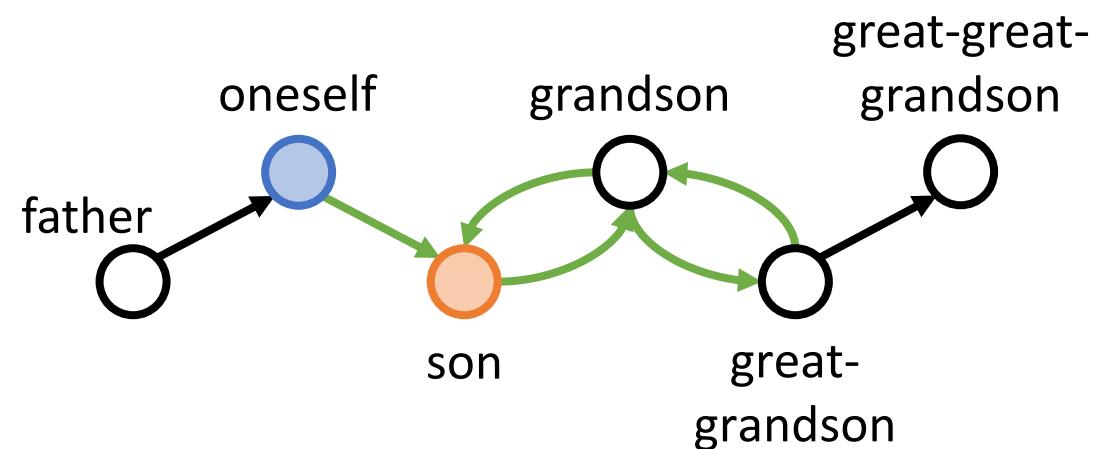


Structure of Reasoning Problems

What is the answer to
 $1 + 1 + 1 - 1 - 1$?



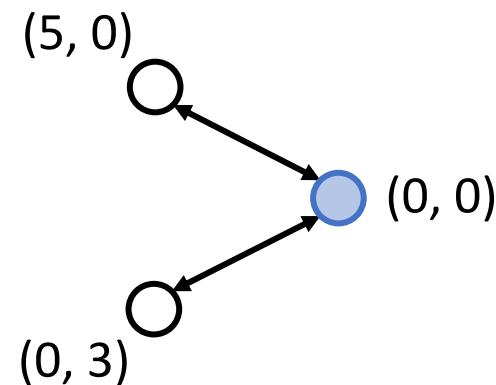
What is my son's son's
son's father's father?



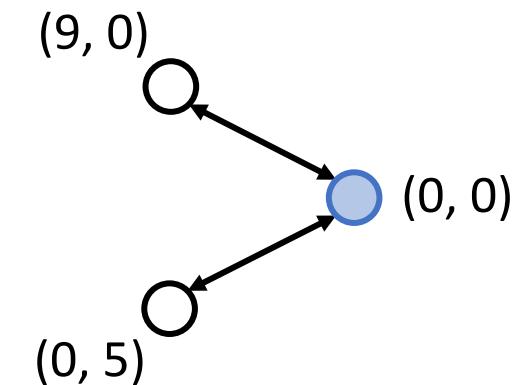
Both predict the ending node of a path!

Structure of Reasoning Problems

How do you get 4L water
using 5L and 3L jugs?

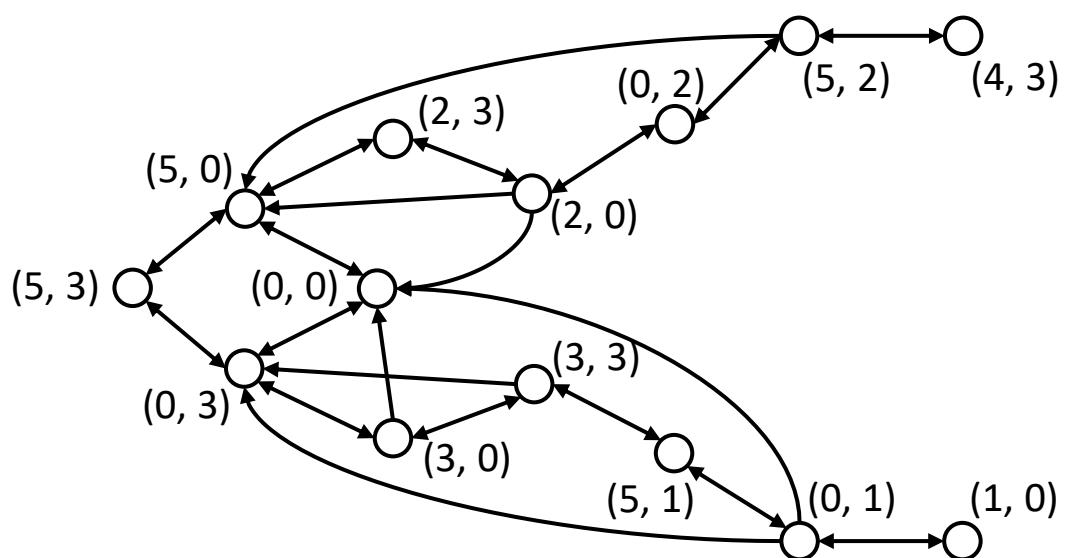


How do you get 5L water
using 9L and 5L jugs?

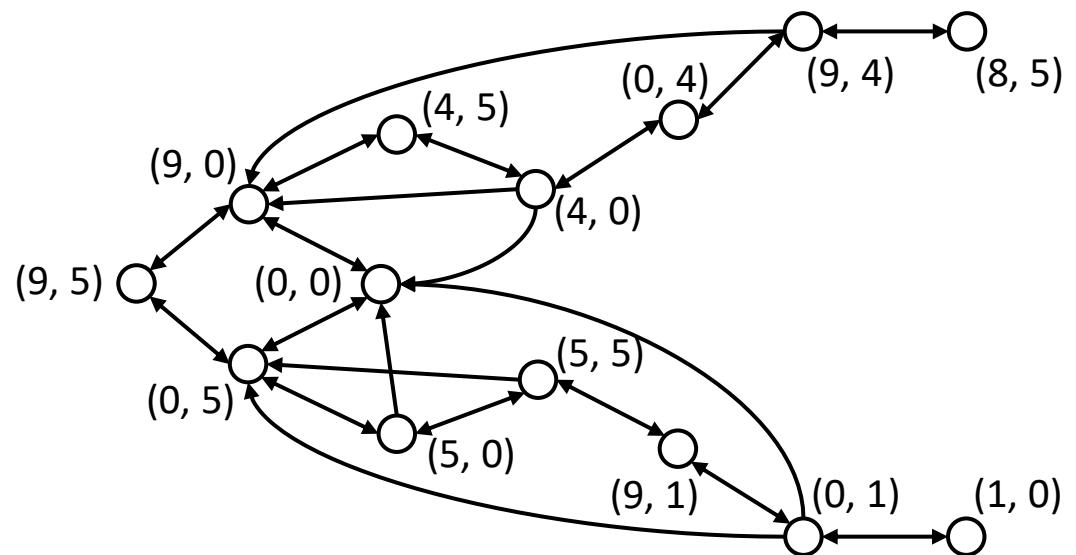


Structure of Reasoning Problems

How do you get 4L water
using 5L and 3L jugs?

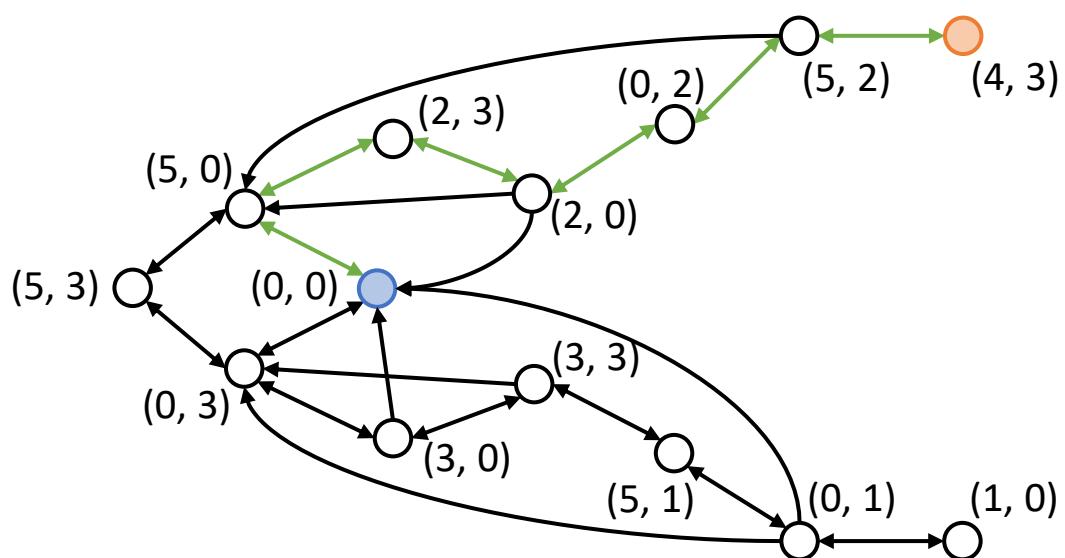


How do you get 5L water
using 9L and 5L jugs?

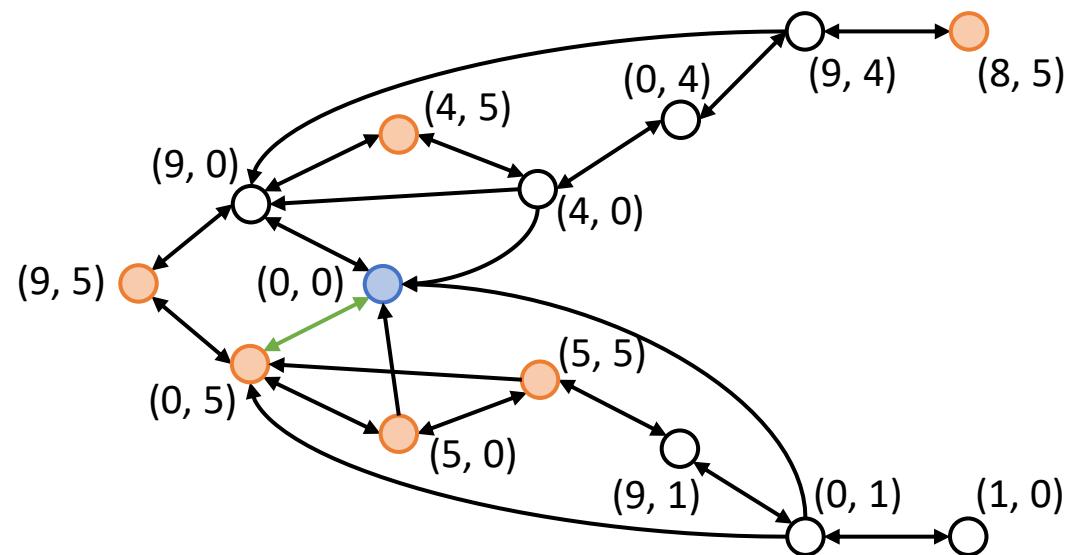


Structure of Reasoning Problems

How do you get 4L water
using 5L and 3L jugs?



How do you get 5L water
using 9L and 5L jugs?



Both find a path to reach the target node(s)!



How to generalize across knowledge structures?

How to generalize across query structures?

How to make ML on structured data more accessible?

Representation Learning Works

Method	Knowledge Structure	Query Structure	Entities	Generalization to New Relations	Multi-hop Queries
Embeddings NBFNet A*Net Ultra	Knowledge graph	Single-hop query	✓	✓	✓
	Knowledge graph	Single-hop query			
	Knowledge graph	Single-hop query			
	Knowledge graph	Single-hop query	✓		
Embeddings GNN-QE UltraQuery	Knowledge graph	Multi-hop query	✓	✓	✓
	Knowledge graph	Multi-hop query			✓✓
	Knowledge graph	Multi-hop query	✓	✓	✓✓
CoT	Natural language (latent)	Multi-step query	✓	✓✓	✓
HtT	Natural language (latent)	Multi-step query			✓✓

covered in this talk

System Works



covered in this talk

simplifies development on structured data
reduce the lines of code by 20×

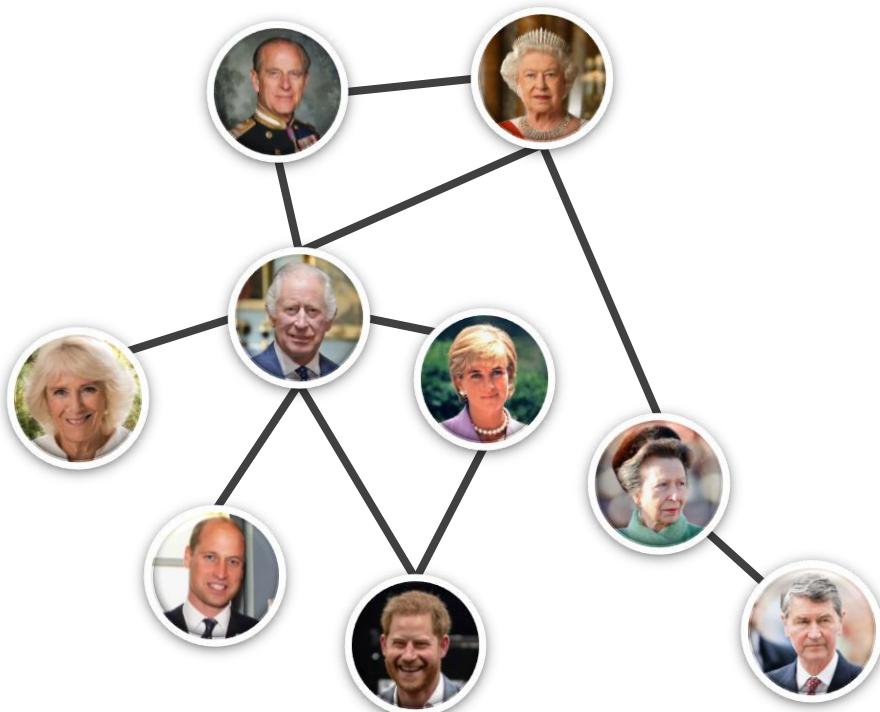
GraphVite

scales up training embedding methods
speeds up by 51× on million-scale graphs

NBFNet^[1]: Learning **inductive representations** **of structures** by **encoding paths**

[1] **Zhaocheng Zhu**, Zuobai Zhang, Louis-Pascal Xhonneux, Jian Tang. Neural Bellman-Ford Networks: A General Graph Neural Network Framework for Link Prediction. NeurIPS 2021.

A Simplified Setup: Knowledge Graphs



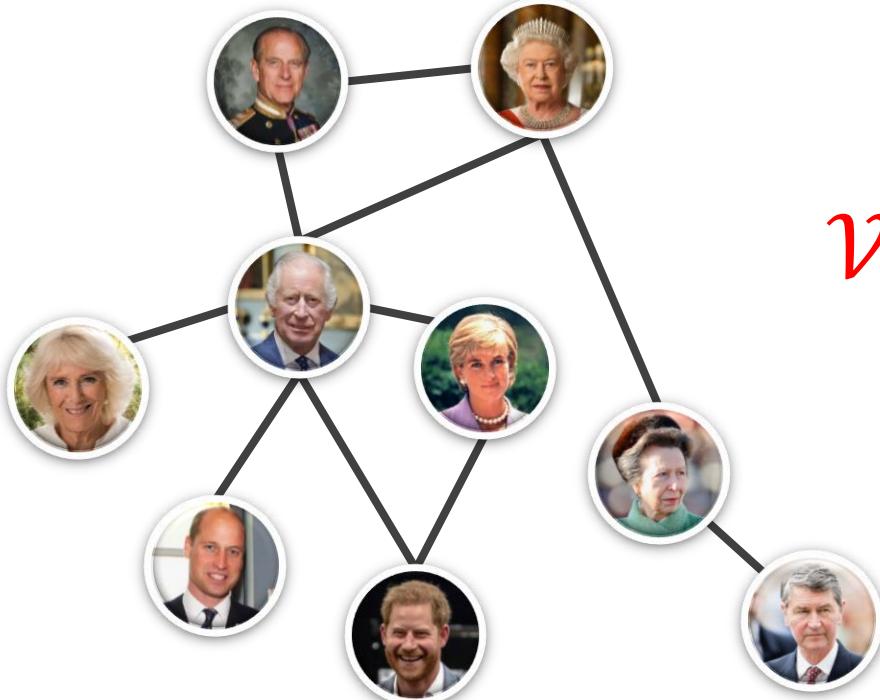
$$\text{Graph } \mathcal{G} = (\mathcal{V}, \mathcal{R}, \mathcal{E})$$

Entities \mathcal{V} : British royal family

Relations \mathcal{R} : {parent, spouse}

Edges \mathcal{E} : known family relationships

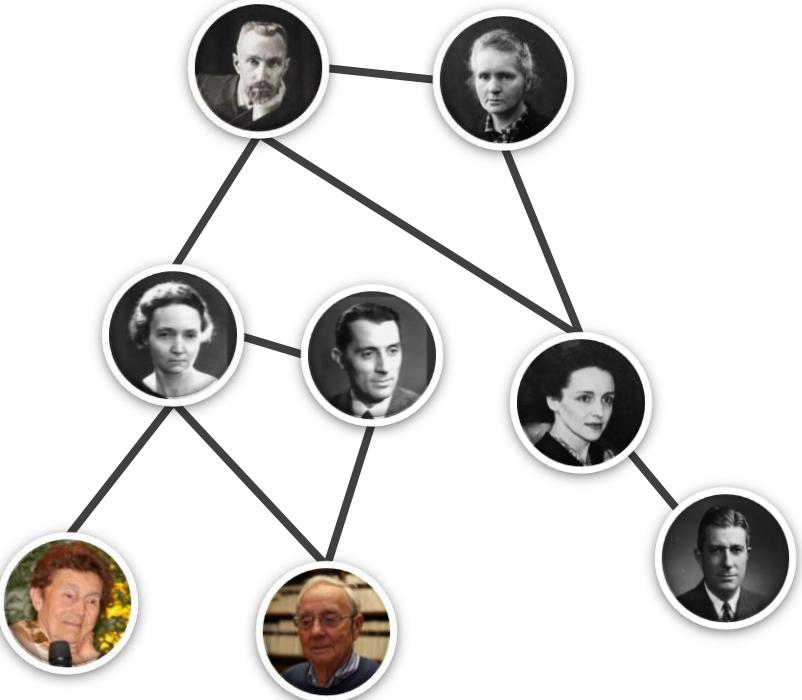
Inductive Generalization on Structure



$$\mathcal{V}_{train} \neq \mathcal{V}_{test}$$

\mathcal{V} : British royal family

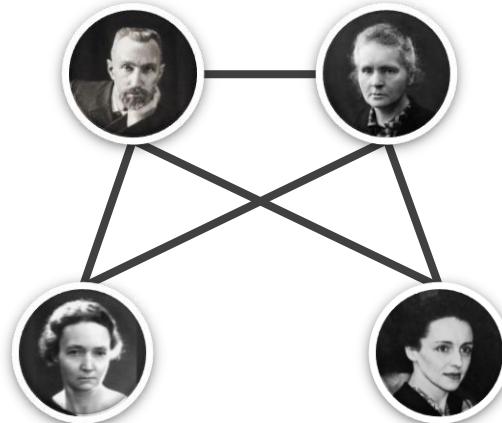
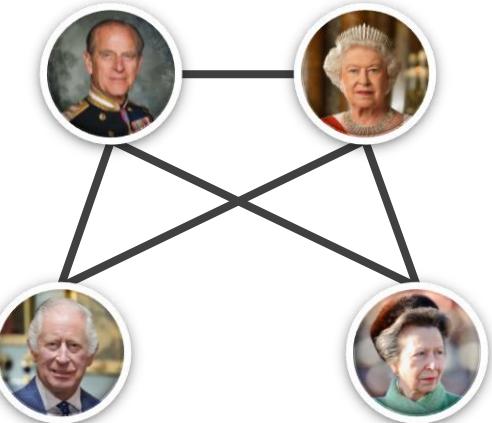
\mathcal{R} : {parent, spouse}



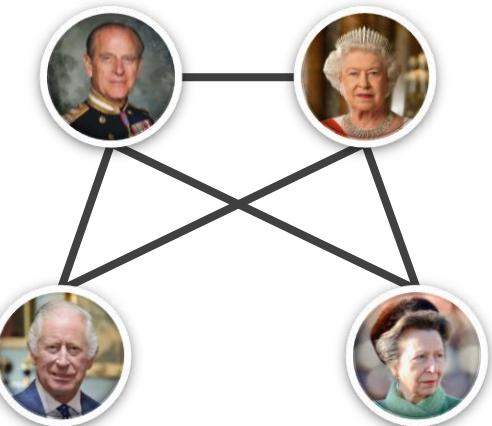
\mathcal{V} : Curie family

\mathcal{R} : {parent, spouse}

What Is an Inductive Function on Structure?

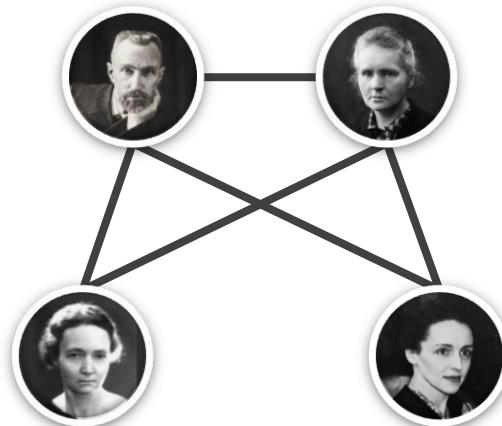


What Is an Inductive Function on Structure?



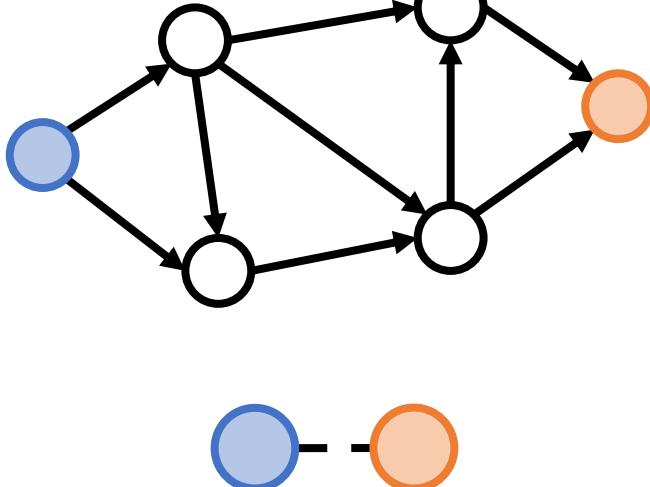
distance: 2
#shortest path: 2
PageRank: 0.154

same structure
same value



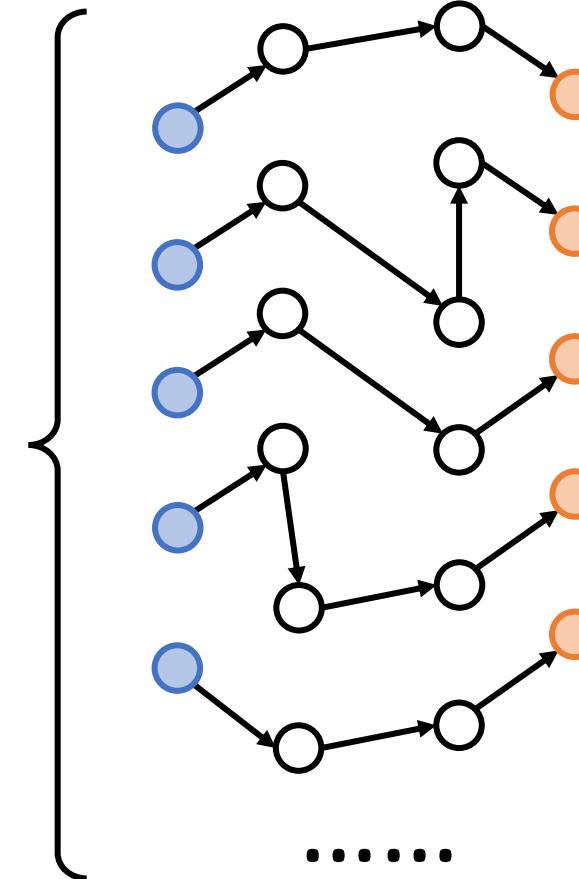
distance: 2
#shortest path: 2
PageRank: 0.154

Path-based Methods



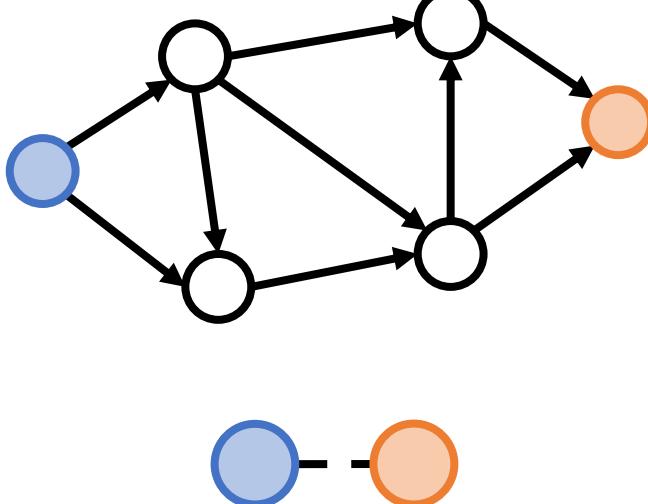
aggregation

path representations



Path-based Methods

Graph distance

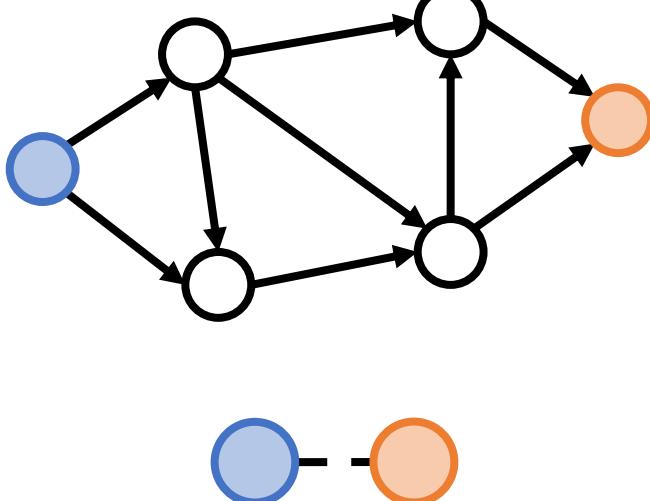


sum of lengths

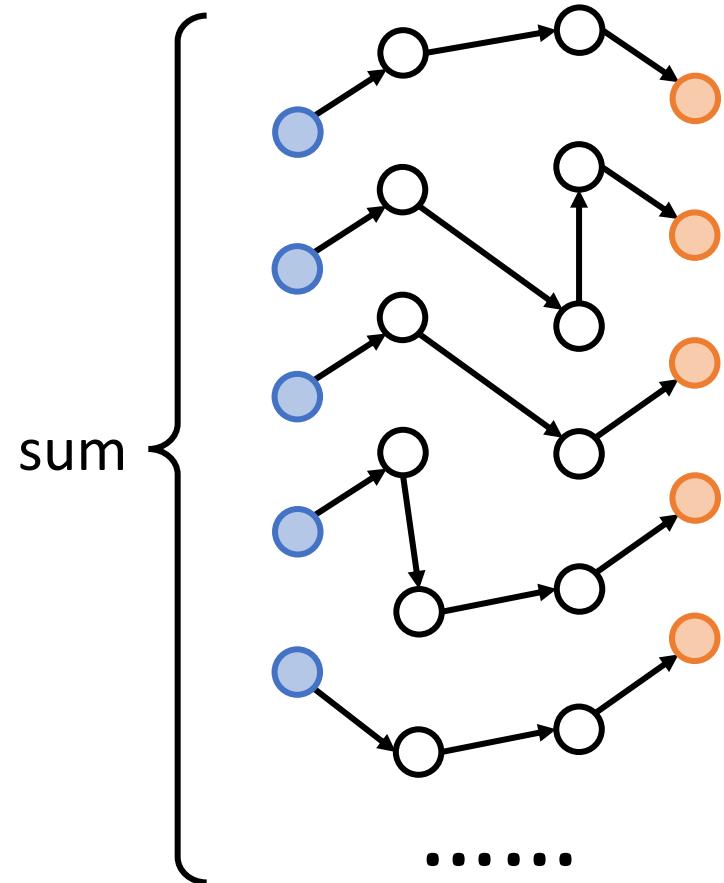
The diagram illustrates a search space or a sequence of states. It features several horizontal rows of nodes. Each row starts with a blue circle on the left and ends with an orange circle on the right. The nodes are connected by black arrows pointing generally towards the right. In each row, the first node is blue and the last node is orange, while the intermediate nodes are white circles. A vertical brace on the left side groups the first node of each row, labeled "min", indicating that the paths shown are the shortest possible from the starting blue node to the ending orange node. Ellipses at the bottom right indicate that there are more rows and paths beyond what is shown.

Path-based Methods

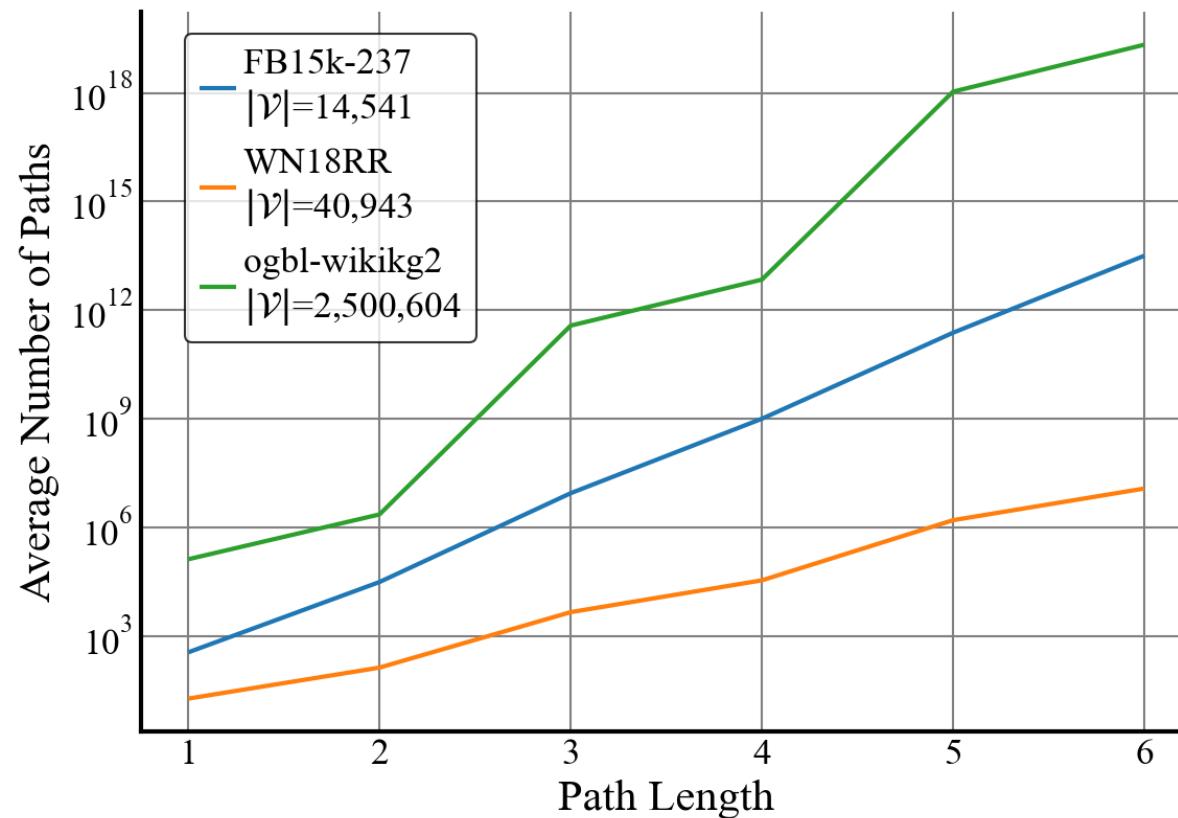
Personalized PageRank



product of transition probabilities



Scalability Issue



Dynamic Programming

To compute paths of length T

graph distance

DFS



Bellman-Ford

Personalized PageRank

random walk



power iteration

exponential in T

$O(T|\mathcal{E}|)$

Dynamic Programming

To compute paths of length T

graph distance

Personalized PageRank

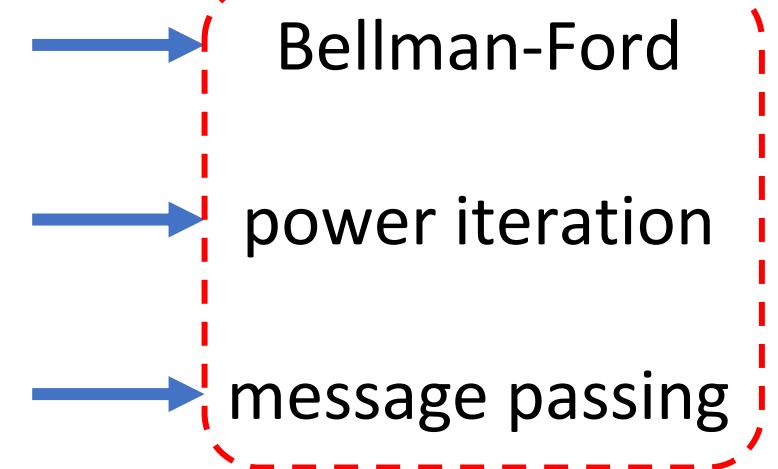
representation learning

DFS

random walk

encode each path

instances of the generalized
Bellman-Ford algorithm

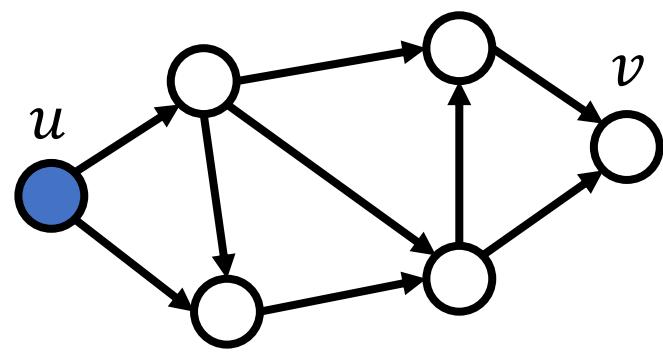


exponential in T

$O(T|\mathcal{E}|)$

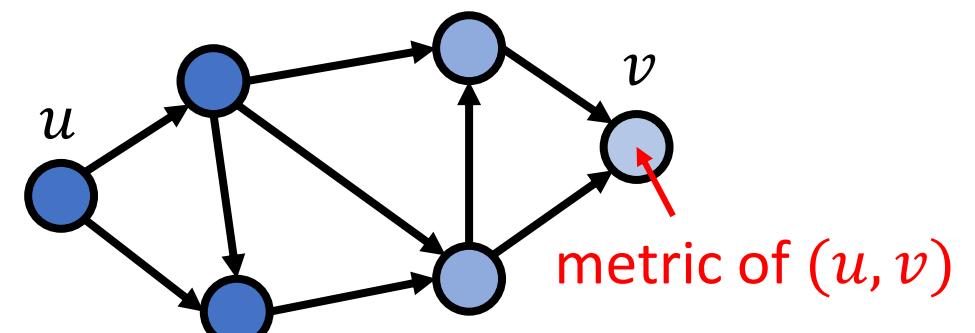
Generalized Bellman-Ford Algorithm

Message passing with a **single-source input**



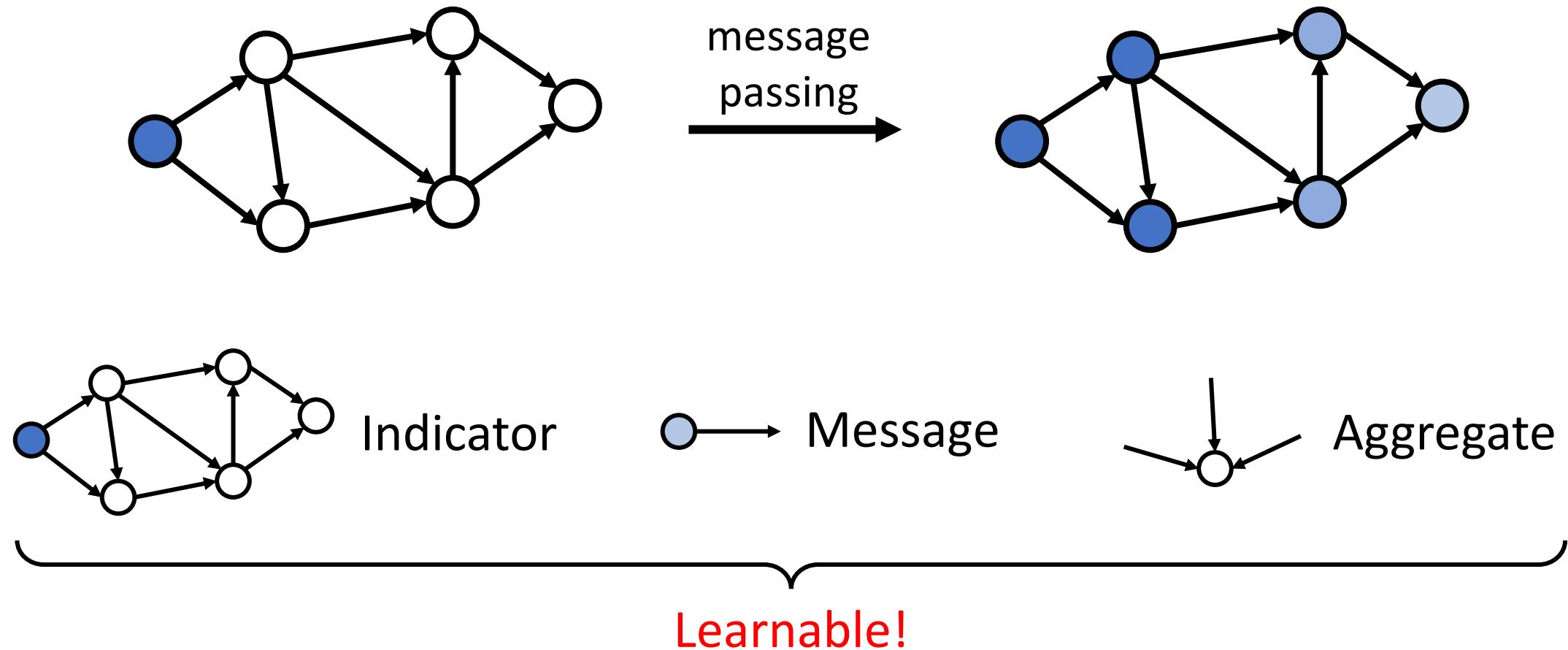
Input

message
passing
→

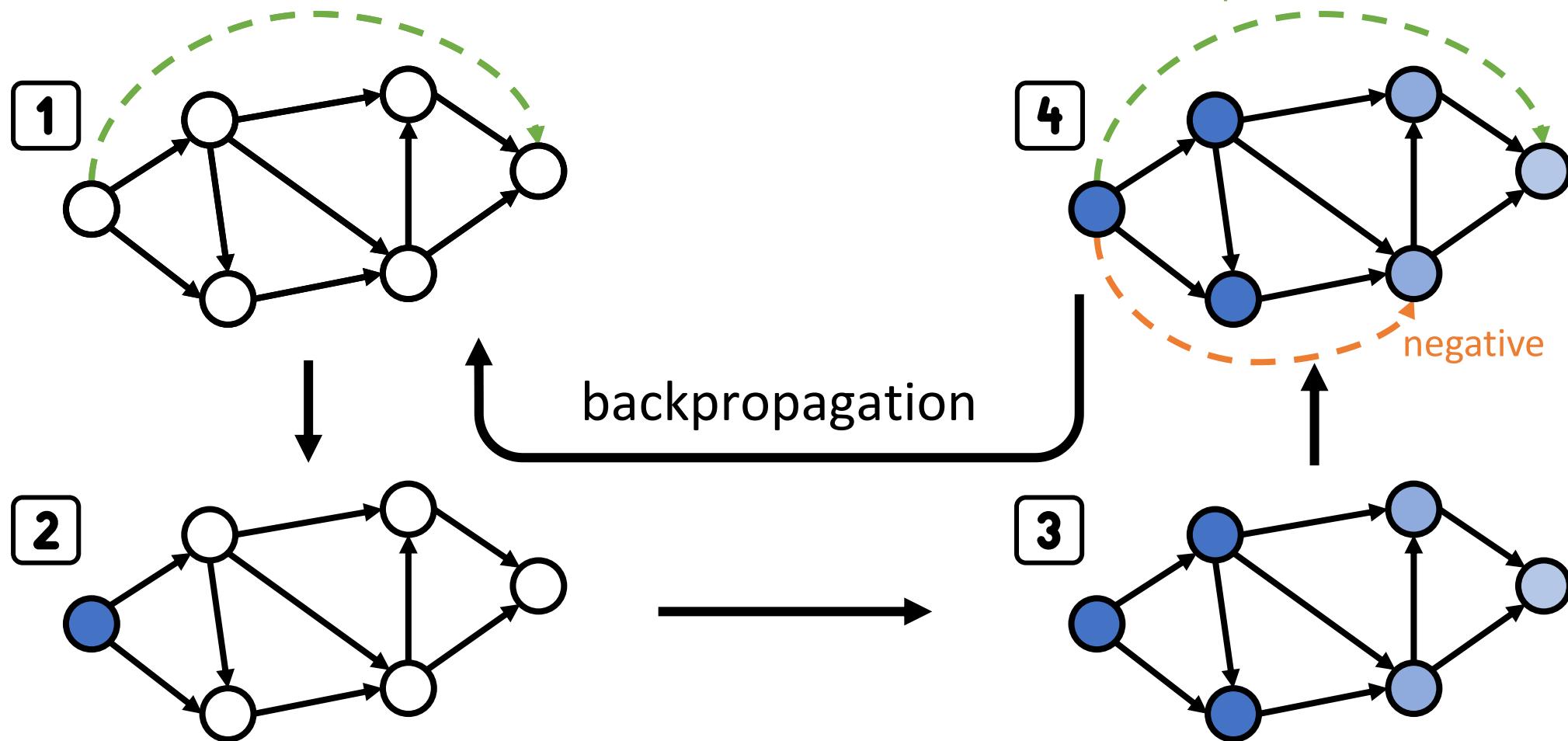


Output

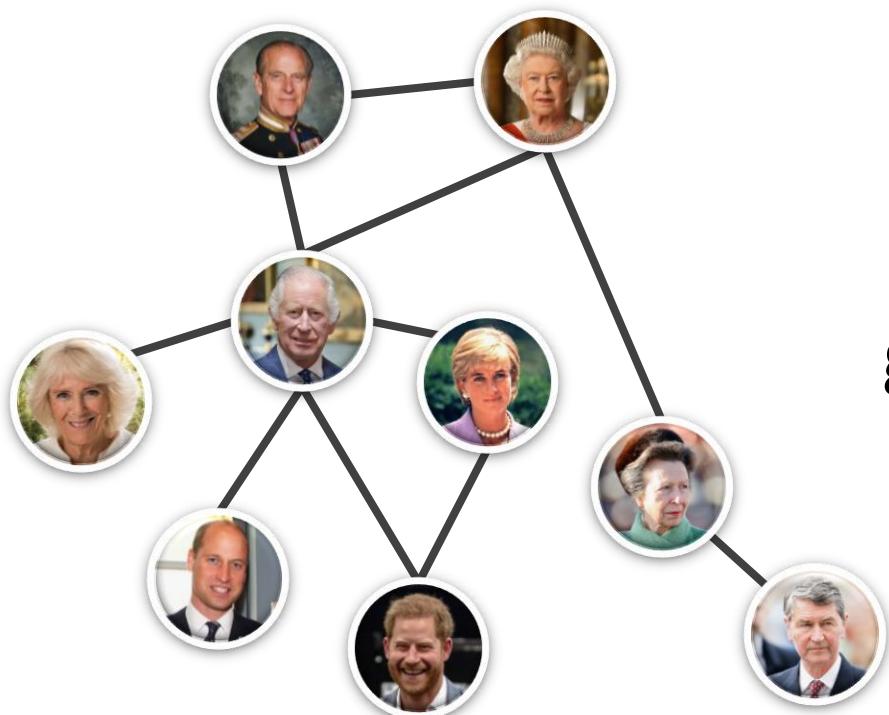
Neural Bellman-Ford Networks



Learning Neural Bellman-Ford Networks

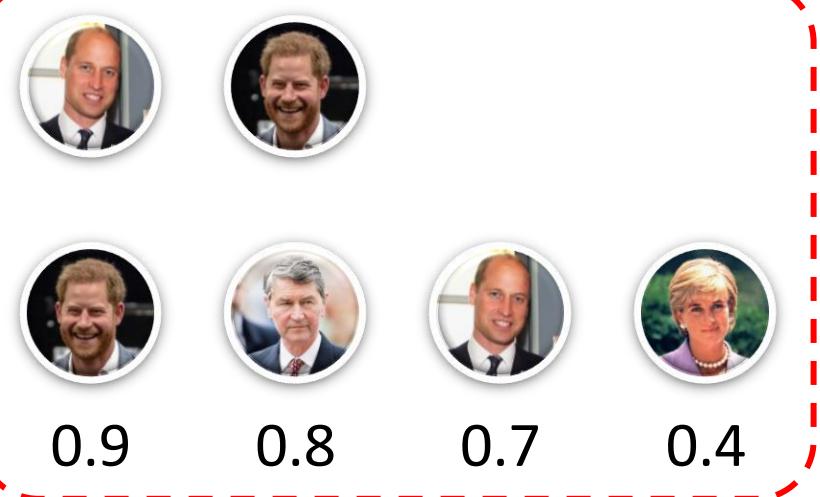


Evaluation: Knowledge Graph Completion



( , grandson, ?)

ground truth:



prediction:

ranking metrics

Knowledge Graphs ($\mathcal{V}_{train} = \mathcal{V}_{test}$)

Class	Method	FB15k-237					WN18RR				
		MR↓	MRR↑	H@1↑	H@3↑	H@10↑	MR↓	MRR↑	H@1↑	H@3↑	H@10↑
Path-based	Path Ranking	3521	0.174	0.119	0.186	0.285	22438	0.324	0.276	0.360	0.406
	NeuralLP	-	0.240	-	-	0.362	-	0.435	0.371	0.434	0.566
	DRUM	-	0.343	0.255	0.378	0.516	-	0.486	0.425	0.513	0.586
Embeddings	TransE	357	0.294	-	-	0.465	3384	0.226	-	-	0.501
	DistMult	254	0.241	0.155	0.263	0.419	5110	0.43	0.39	0.44	0.49
	ComplEx	339	0.247	0.158	0.275	0.428	5261	0.44	0.41	0.46	0.51
	RotatE	177	0.338	0.241	0.375	0.533	3340	0.476	0.428	0.492	0.571
	HAKE	-	0.346	0.250	0.381	0.542	-	0.497	0.452	0.516	0.582
	LowFER	-	0.359	0.266	0.396	0.544	-	0.465	0.434	0.479	0.526
GNNs	RGCN	221	0.273	0.182	0.303	0.456	2719	0.402	0.345	0.437	0.494
	GraIL	2053	-	-	-	-	2539	-	-	-	-
	NBFNet	114	0.415	0.321	0.454	0.599	636	0.551	0.497	0.573	0.666

Knowledge Graphs ($\mathcal{V}_{train} \neq \mathcal{V}_{test}$)

metric: H@10↑

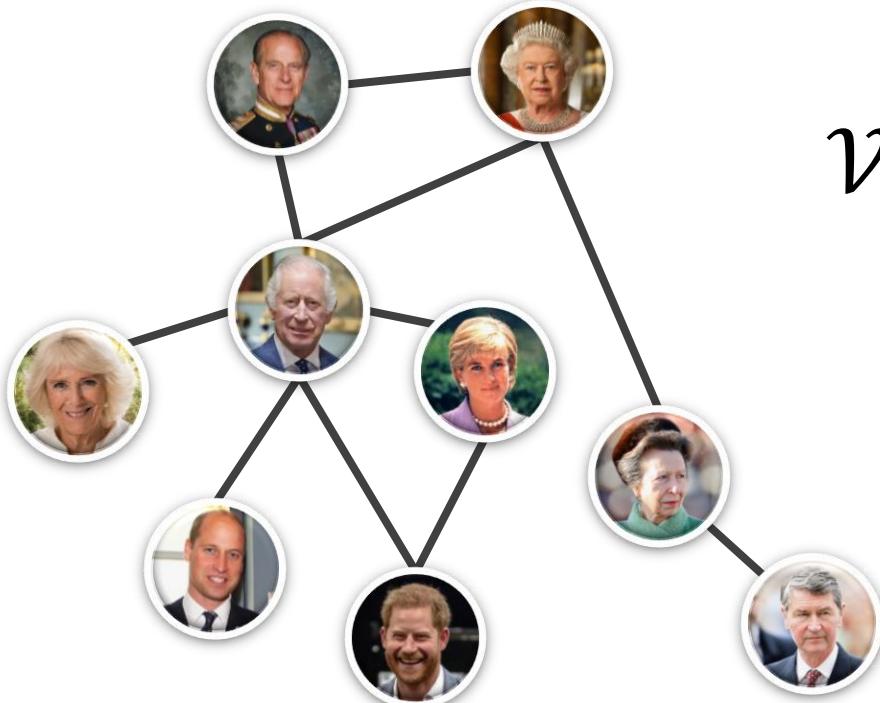
Class	Method	FB15k-237				WN18RR			
		v1	v2	v3	v4	v1	v2	v3	v4
Path-based	NeuralLP	0.529	0.589	0.529	0.559	0.744	0.689	0.462	0.671
	DRUM	0.529	0.587	0.529	0.559	0.744	0.689	0.462	0.671
	RuleN	0.498	0.778	0.877	0.856	0.809	0.782	0.534	0.716
GNNs	GraIL	0.642	0.818	0.828	0.893	0.825	0.787	0.584	0.734
	NBFNet	0.834	0.949	0.951	0.960	0.948	0.905	0.893	0.890

(Drinking Water)

Ultra^[1]: Generalizing to **any knowledge graph** with **inductive relation representations**

[1] Mikhail Galkin, Xinyu Yuan, Hesham Mostafa, Jian Tang, **Zhaocheng Zhu**. Towards Foundation Models for Knowledge Graph Reasoning. ICLR 2024.

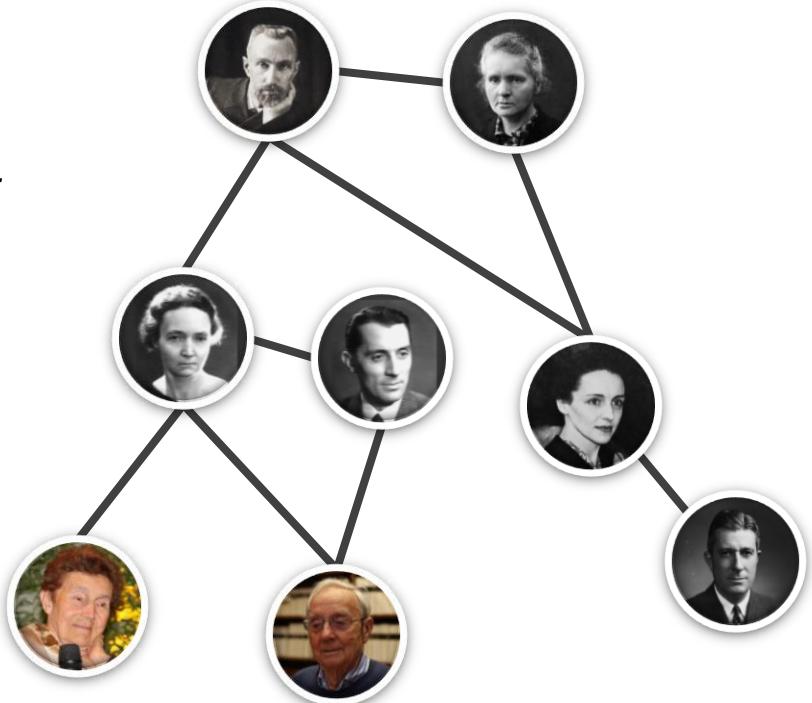
Inductive Generalization on Structure



\mathcal{V} : British royal family

\mathcal{R} : {parent, spouse}

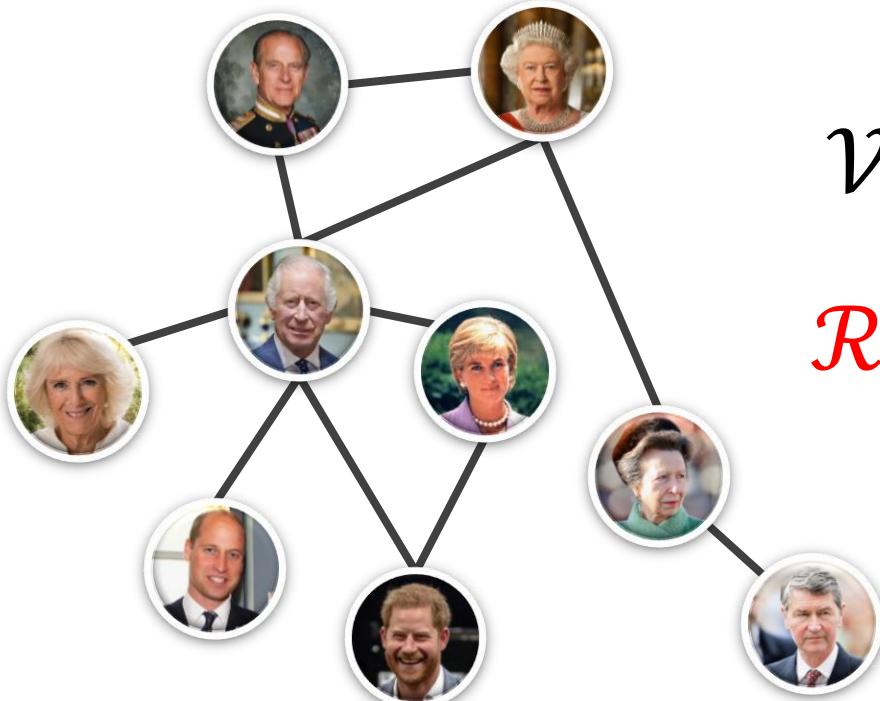
$$\mathcal{V}_{train} \neq \mathcal{V}_{test}$$



\mathcal{V} : Curie family

\mathcal{R} : {parent, spouse}

Inductive Generalization on Structure

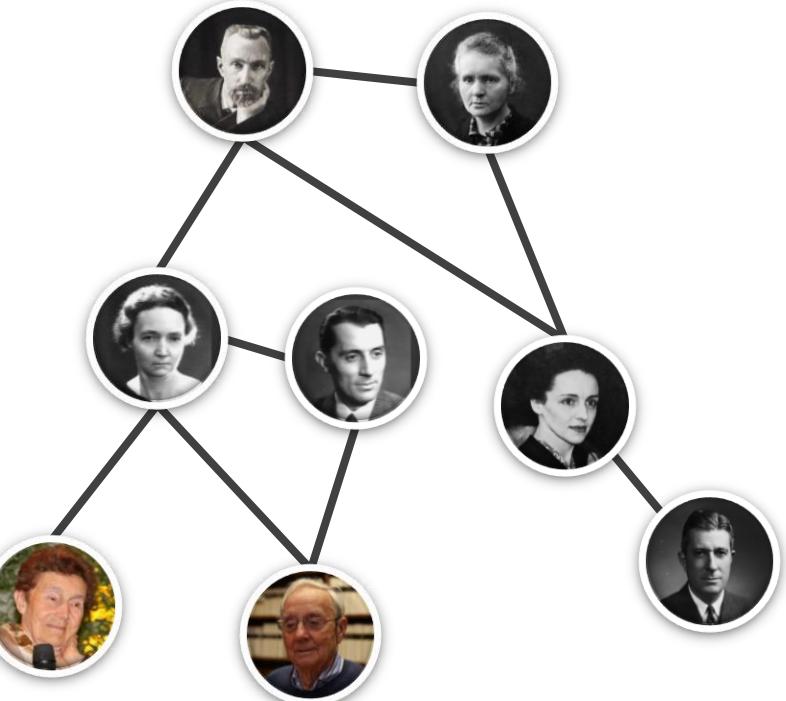


\mathcal{V} : British royal family

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$$\mathcal{V}_{train} \neq \mathcal{V}_{test}$$

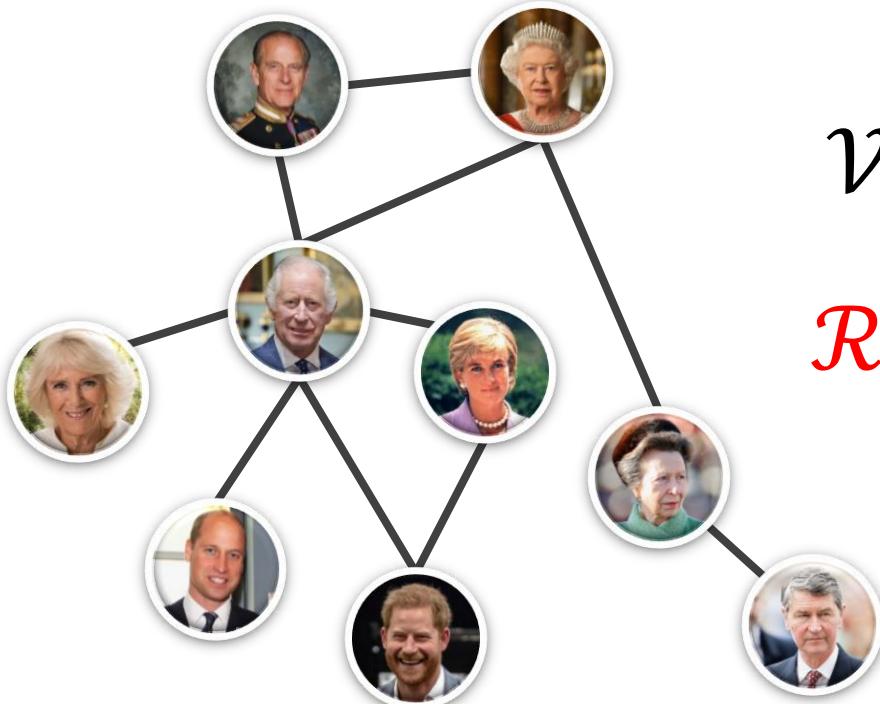
$$\mathcal{R}_{train} = \mathcal{R}_{test}$$



\mathcal{V} : Curie family

\mathcal{R} : {parent, spouse}

Inductive Generalization on Structure

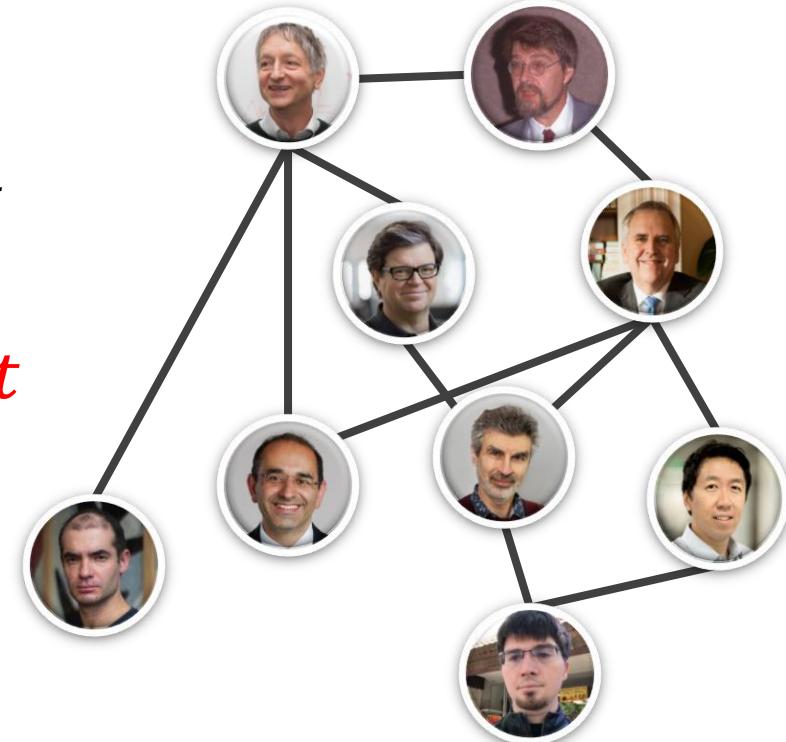


\mathcal{V} : British royal family

\mathcal{R} : {parent, spouse}

$$\mathcal{V}_{train} \neq \mathcal{V}_{test}$$

$$\mathcal{R}_{train} \neq \mathcal{R}_{test}$$



\mathcal{V} : deep learning researchers

\mathcal{R} : {supervisor, collaborator}

What Generalizes for Entities?

\mathcal{V} : British royal family

\mathcal{R} : {parent, spouse}

\mathcal{V} : Curie family

\mathcal{R} : {parent, spouse}

Elizabeth II



Marie Curie

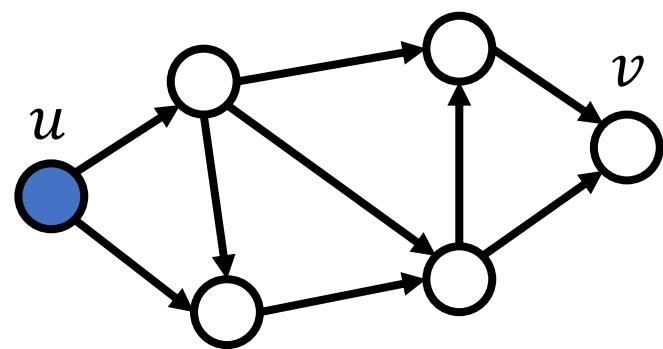
Elizabeth II - Princess Anne



Marie Curie - Irene Curie

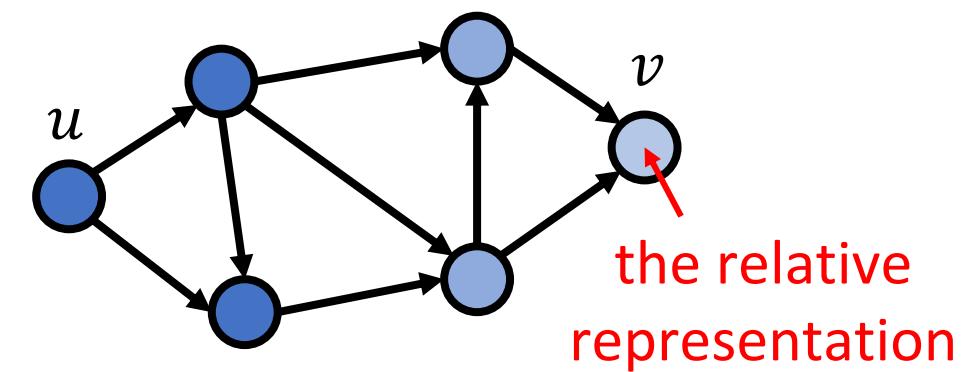
Relative Entity Representations

encode $v - u$ on graph \mathcal{G}



Input

message
passing



Output

the relative
representation

What Generalizes for Relations?

\mathcal{V} : British royal family

\mathcal{R} : {parent, spouse}

\mathcal{V} : deep learning researchers

\mathcal{R} : {supervisor, collaborator}

parent



supervisor

parent - spouse



supervisor - collaborator

Relative Relation Representations

relative entity: encode $v - u$ on graph \mathcal{G}

relative relation: encode $r - q$ on what?

Relative Relation Representations

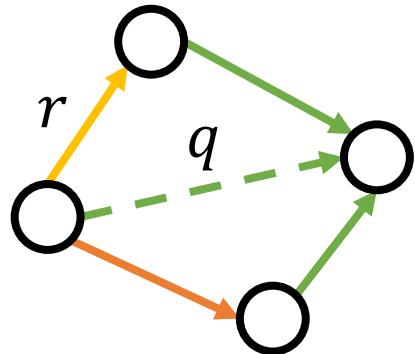
relative entity: encode $v - u$ on graph \mathcal{G}

relative relation: encode $r - q$ on what?

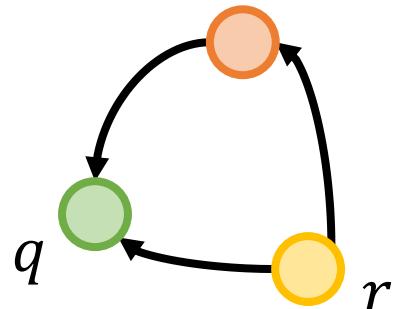
Construct a relation graph to
capture relation interactions!

Relative Relation Representations

knowledge graph \mathcal{G}

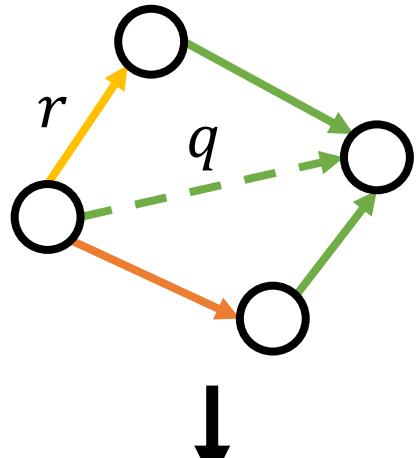


relation graph \mathcal{G}_r

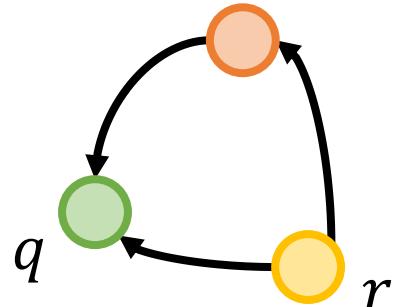


Relative Relation Representations

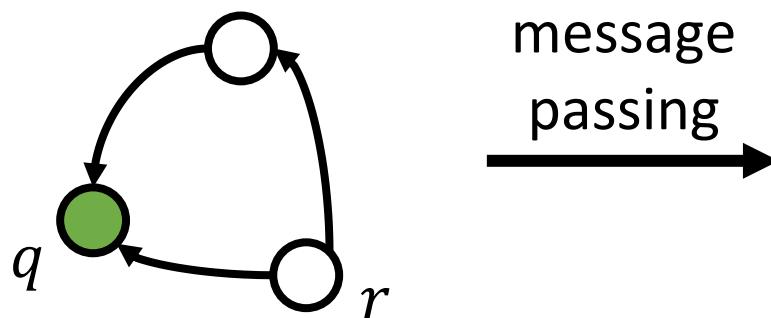
knowledge graph \mathcal{G}



relation graph \mathcal{G}_r

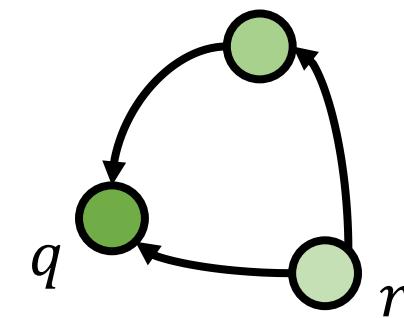


Encode $r - q$ on \mathcal{G}_r



Input

message
passing



Output

Relation Graph

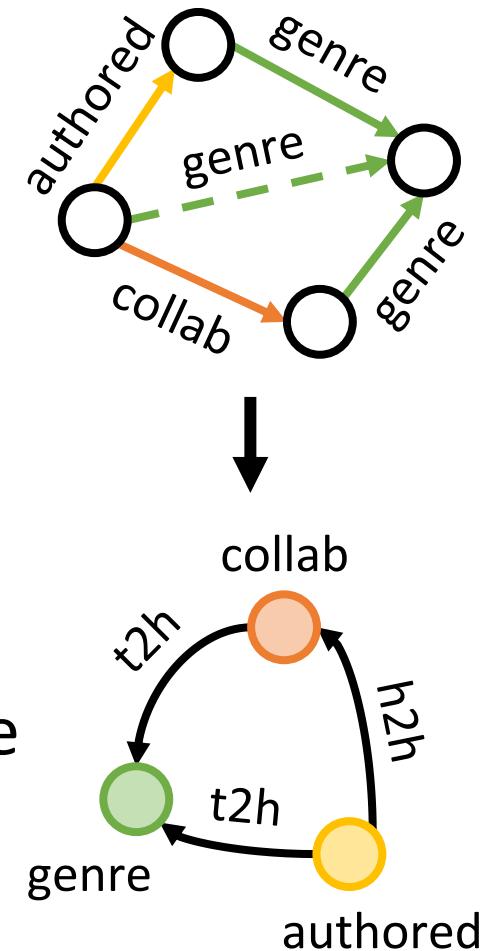
Relation interactions:

head2head, head2tail, tail2head, tail2tail

Example:

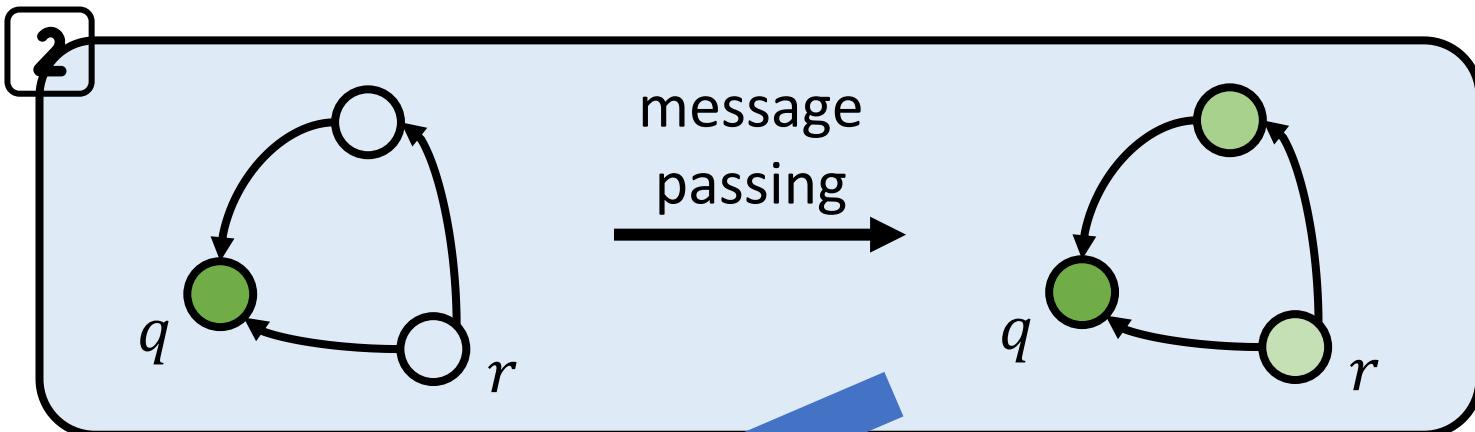
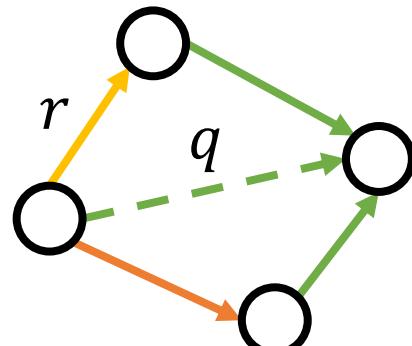
(author, t2h, genre)

Anything that has an author is likely to have a genre

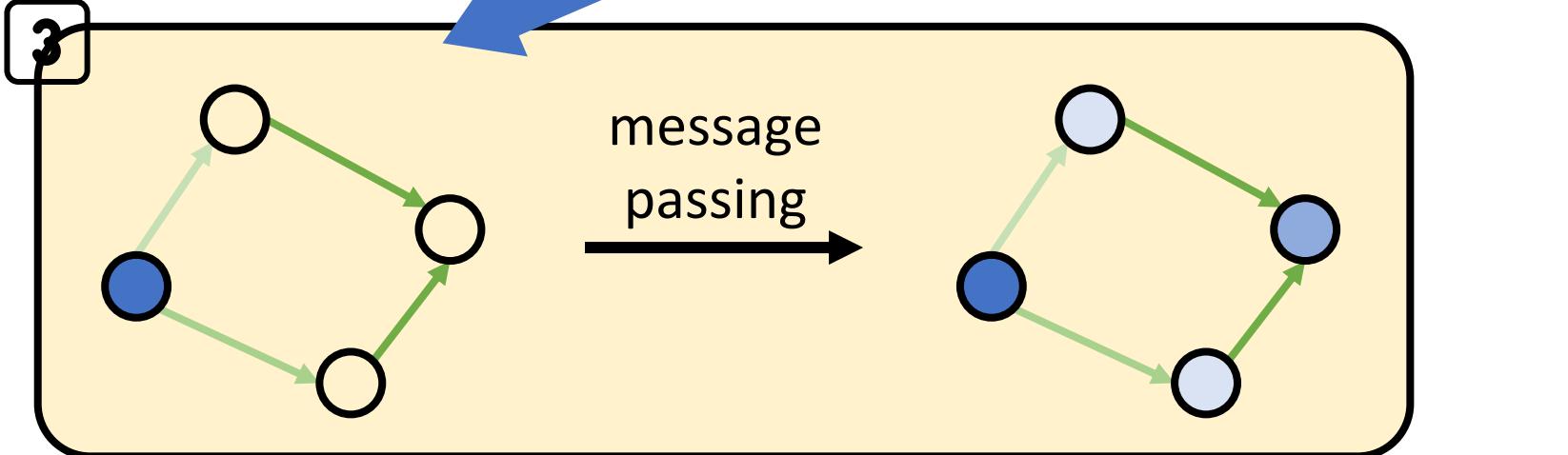
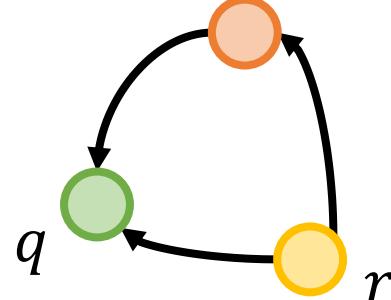


Ultra: Unified, Learnable, Transferable

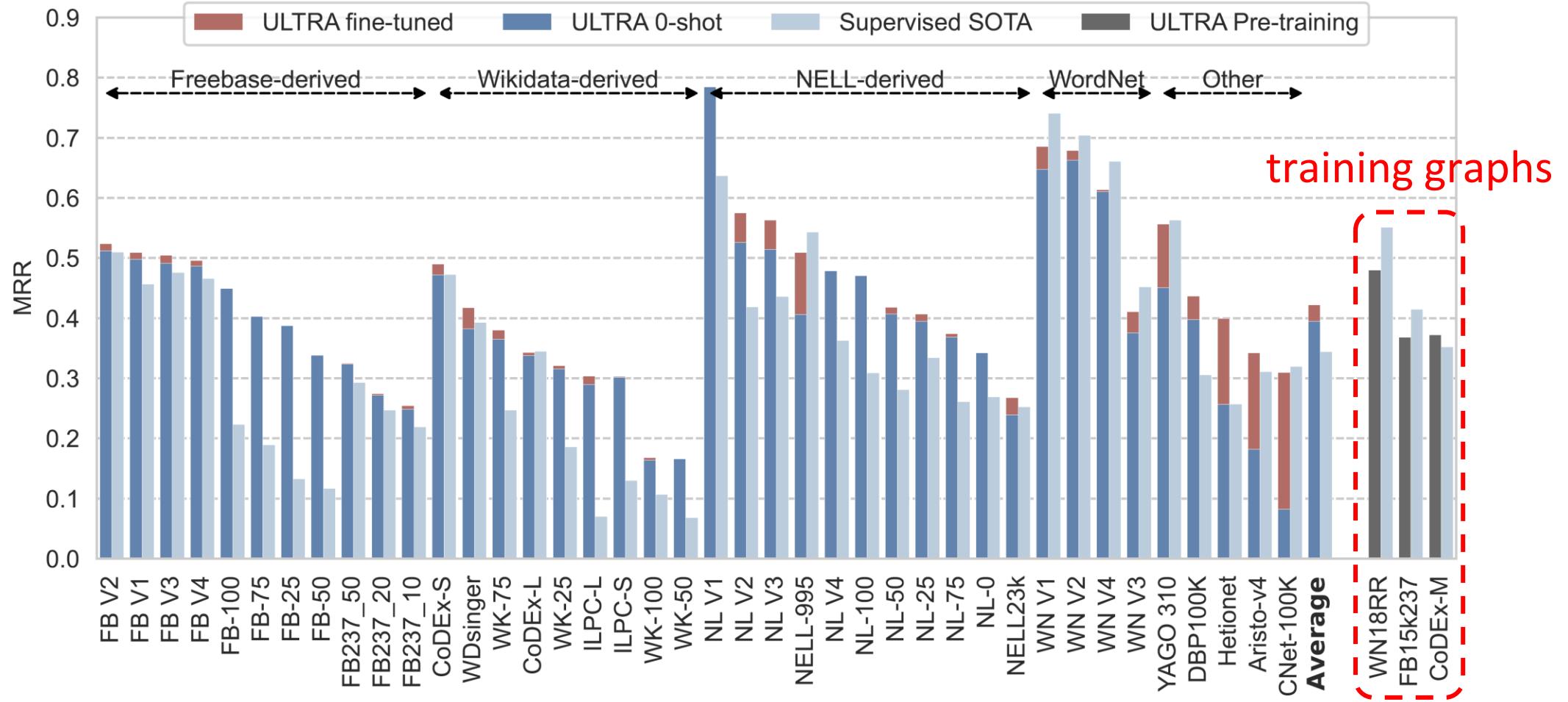
knowledge graph \mathcal{G}



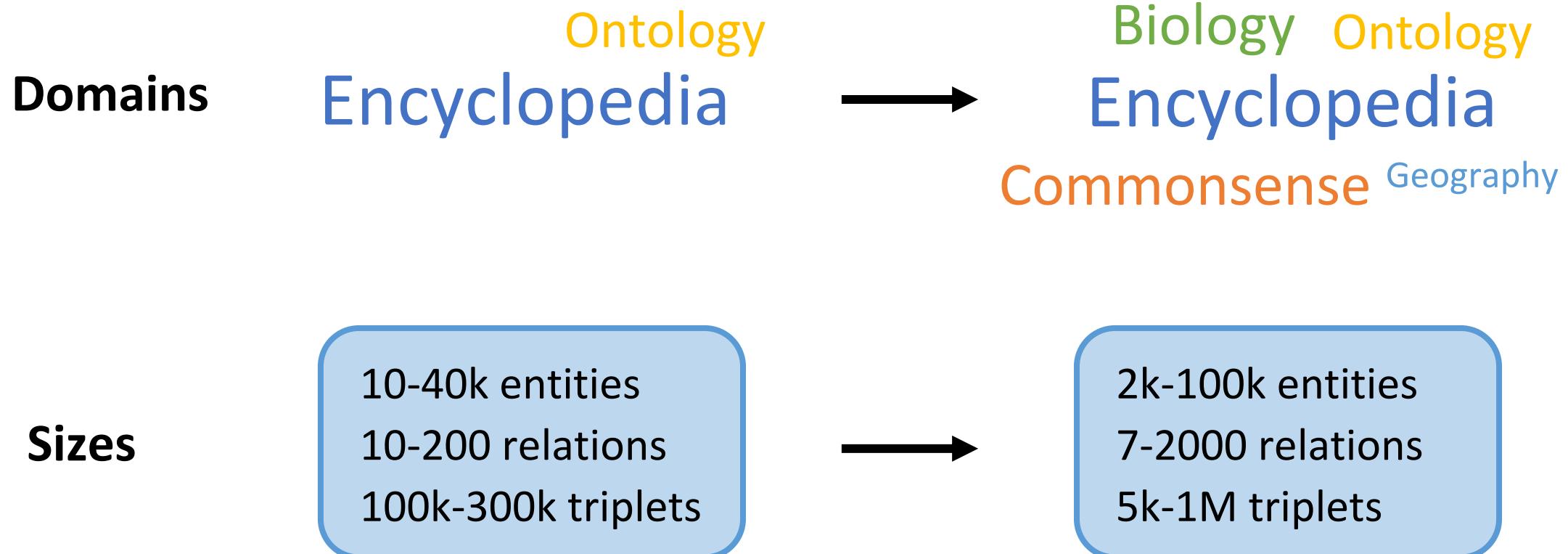
1
↓
relation graph \mathcal{G}_r



0-shot Inference on any Knowledge Graph



Surprising Generalization Ability



GNN-QE^{[1][2][3]}: Solving **multi-hop queries** with **inductive models and logical operations**

[1] **Zhaocheng Zhu**, Mikhail Galkin, Zuobai Zhang, Jian Tang. Neural-Symbolic Models for Logical Queries on Knowledge Graphs. ICML 2022.

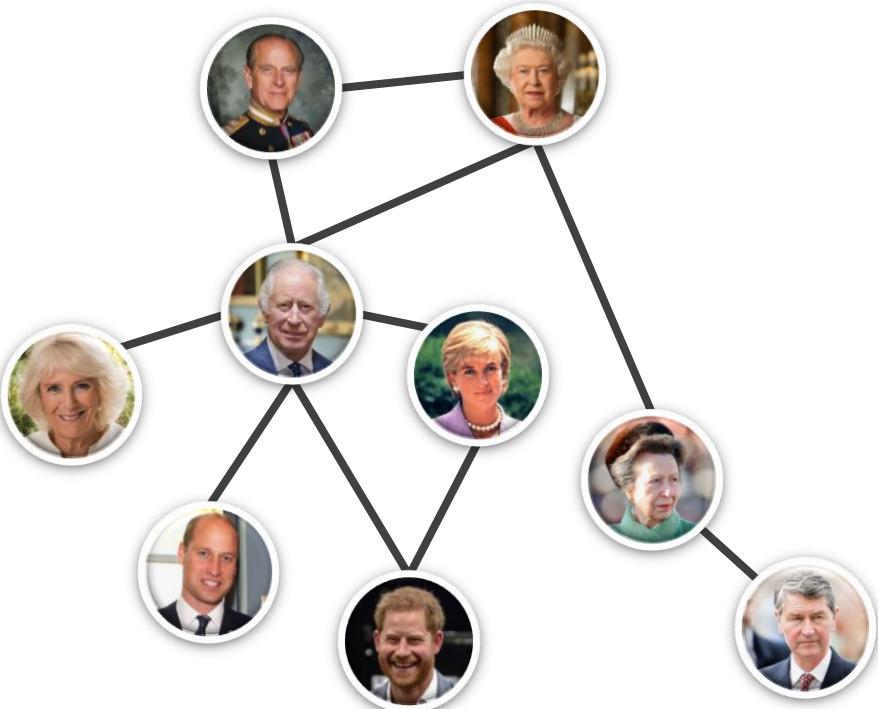
[2] Mikhail Galkin, **Zhaocheng Zhu**, Hongyu Ren, Jian Tang. Inductive Logical Query Answering in Knowledge Graphs. NeurIPS 2022.

[3] Mikhail Galkin, Jincheng Zhou, Bruno Ribeiro, Jian Tang, **Zhaocheng Zhu**. Zero-shot Logical Query Reasoning on any Knowledge Graph. arXiv 2024.

Knowledge Graph Completion

Input: a head entity, a relation

Output: one or many tail entities

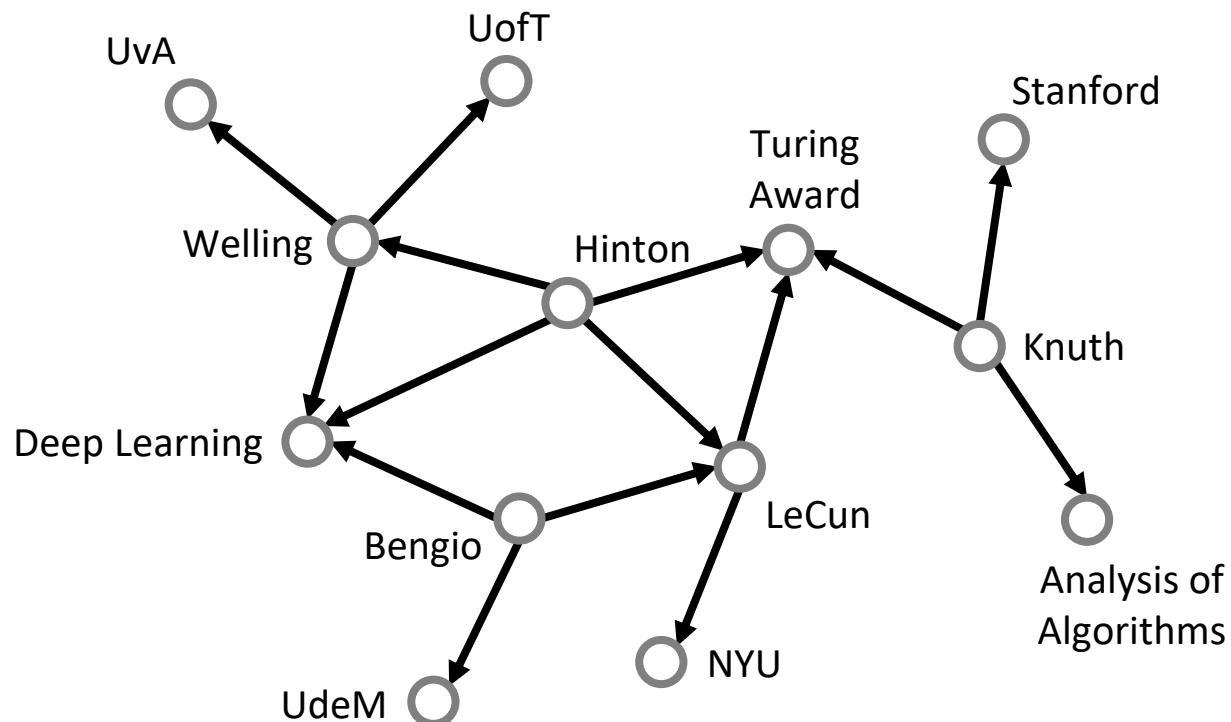


( , grandson, ?)

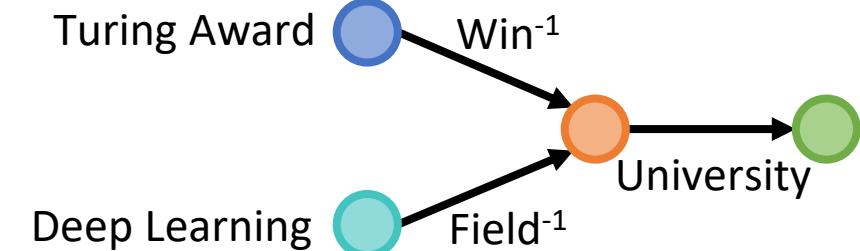
Multi-hop Logical Queries

Input: one or several entities, several relations, logical operations

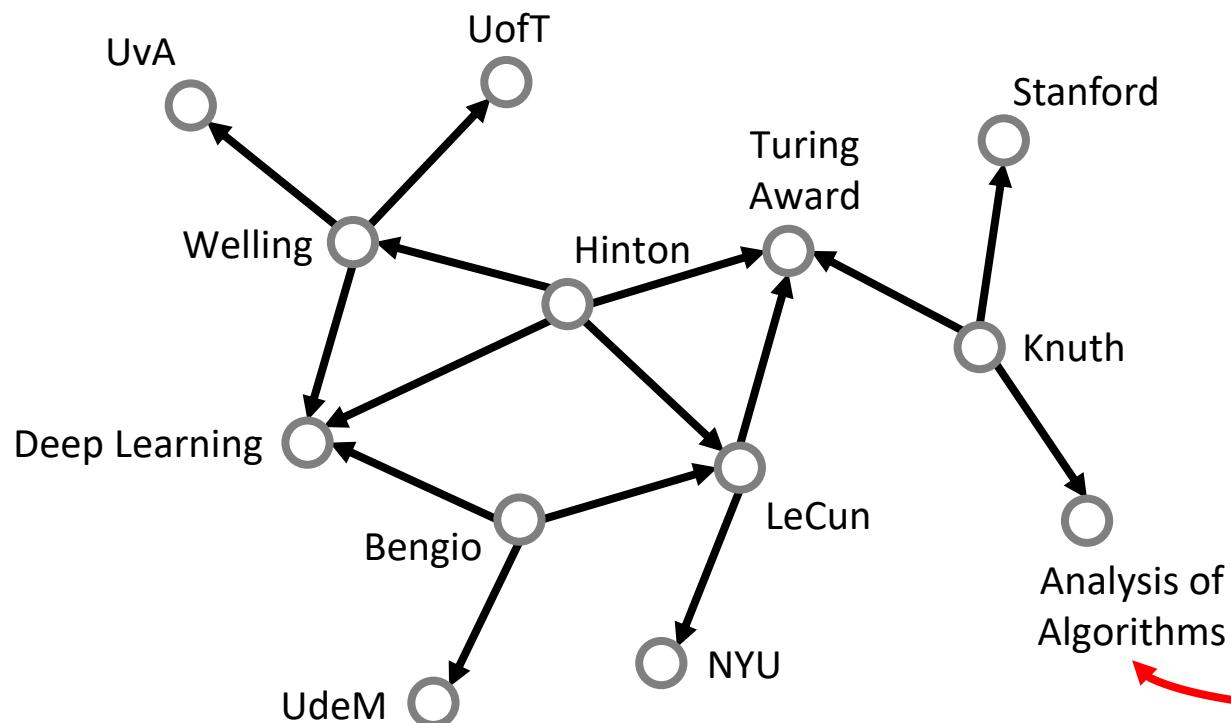
Output: one or many tail entities



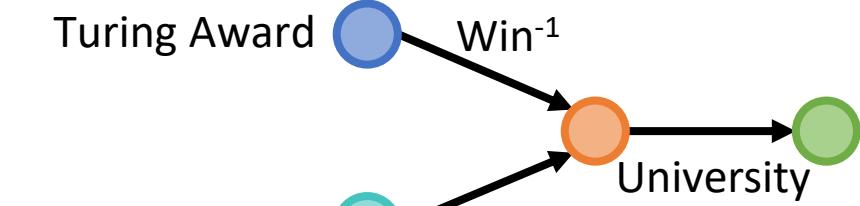
At what universities do the Turing Award winners in the field of deep learning work?



Multi-hop Logical Queries

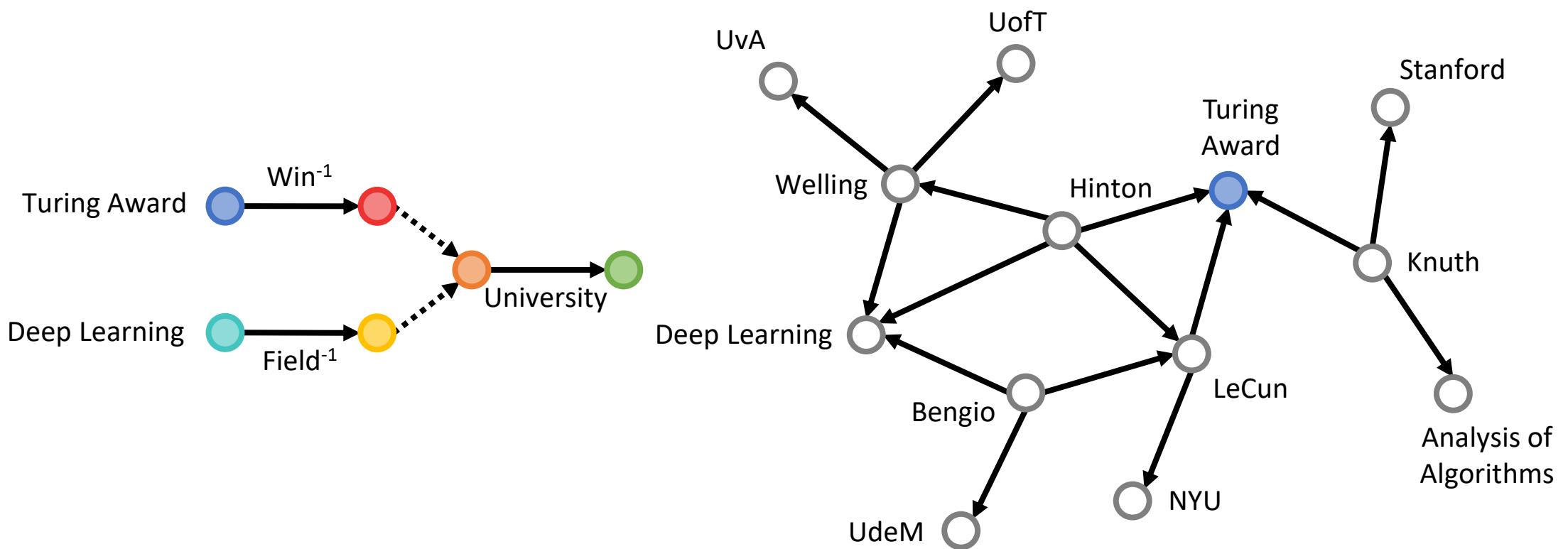


At what universities do the Turing Award winners in the field of deep learning work?

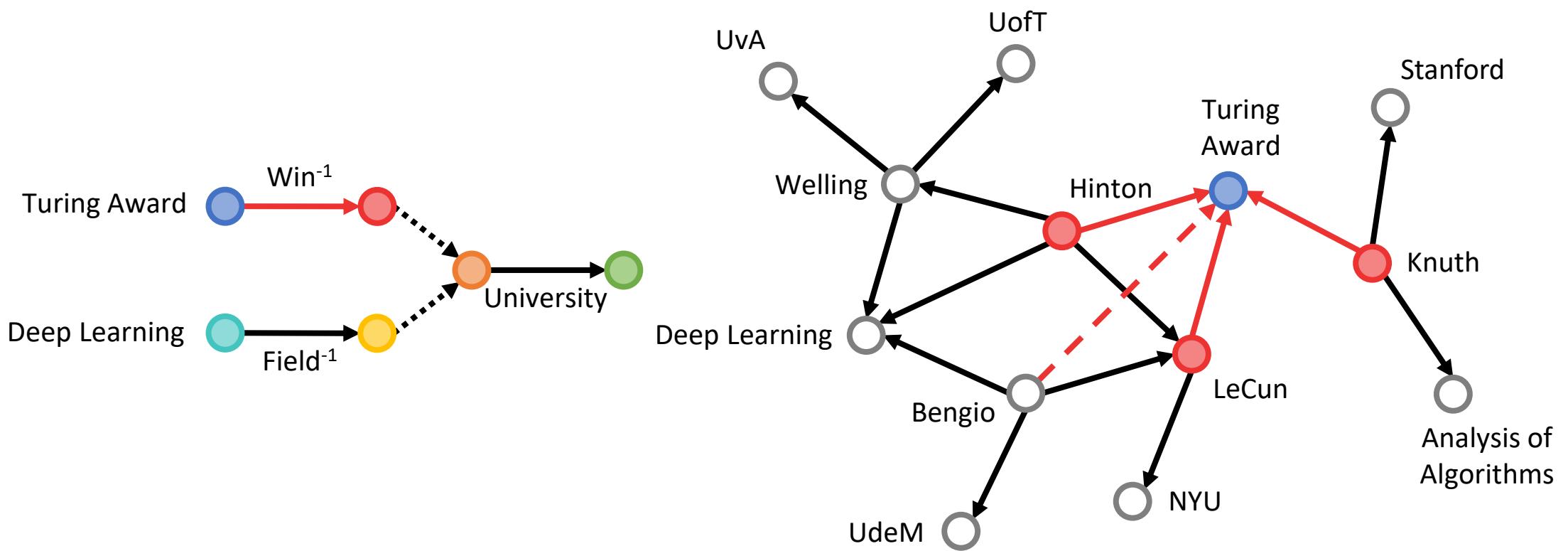


search this subgraph

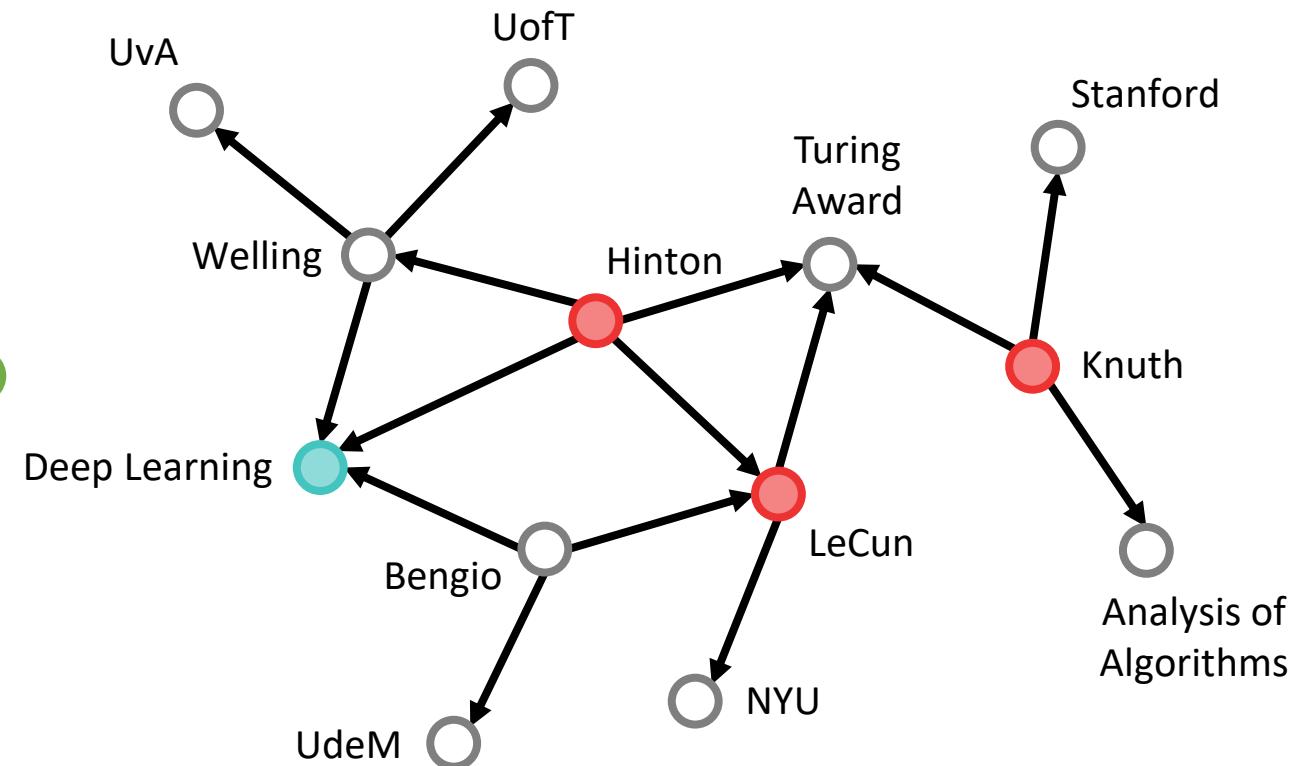
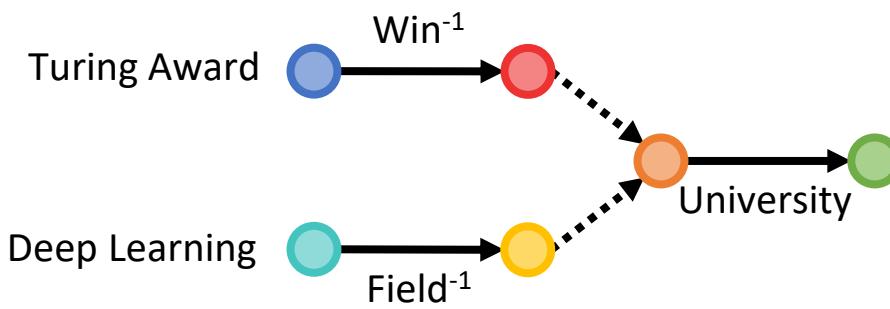
Subgraph Matching



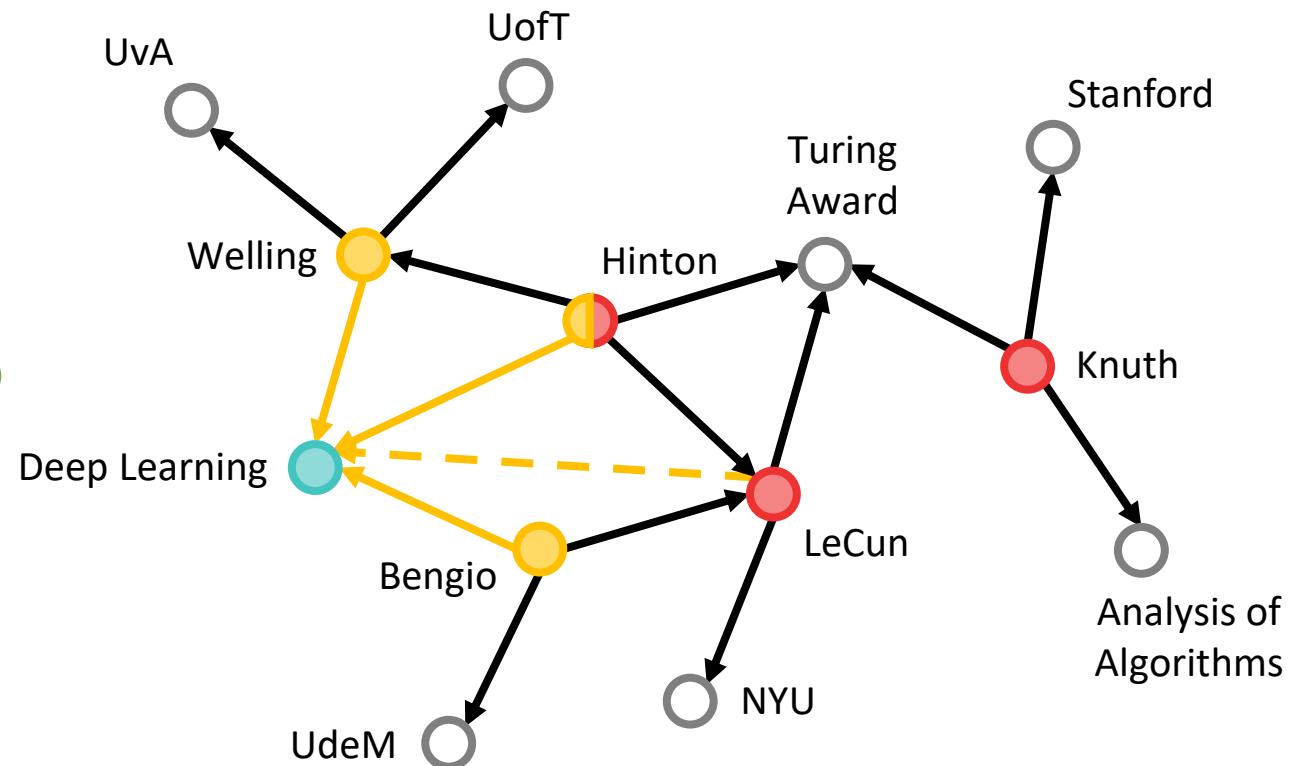
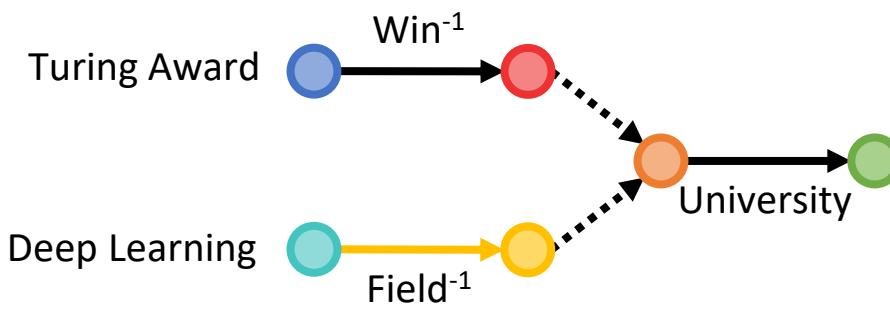
Subgraph Matching



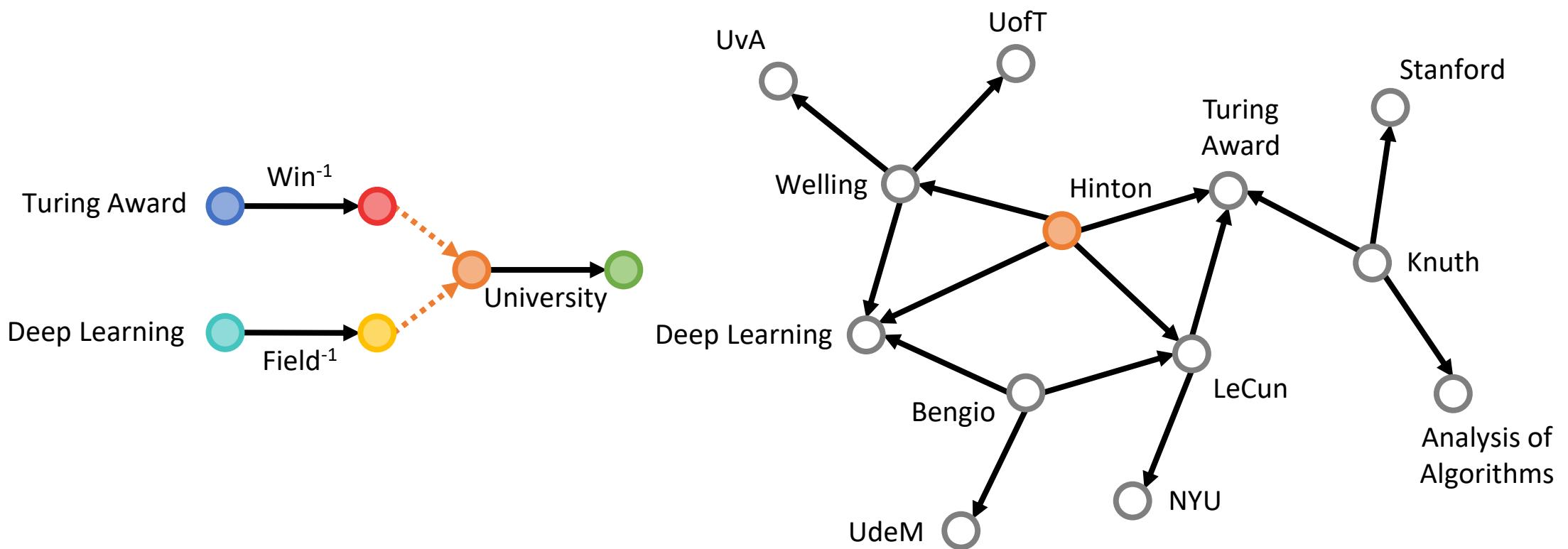
Subgraph Matching



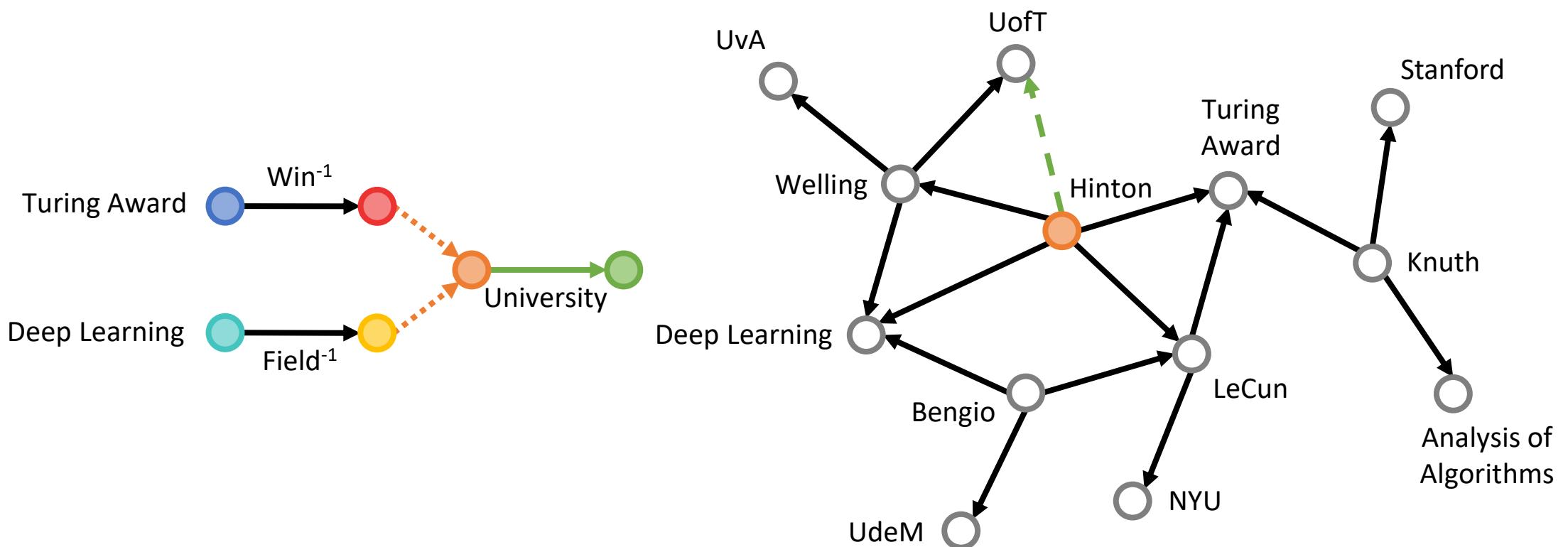
Subgraph Matching



Subgraph Matching

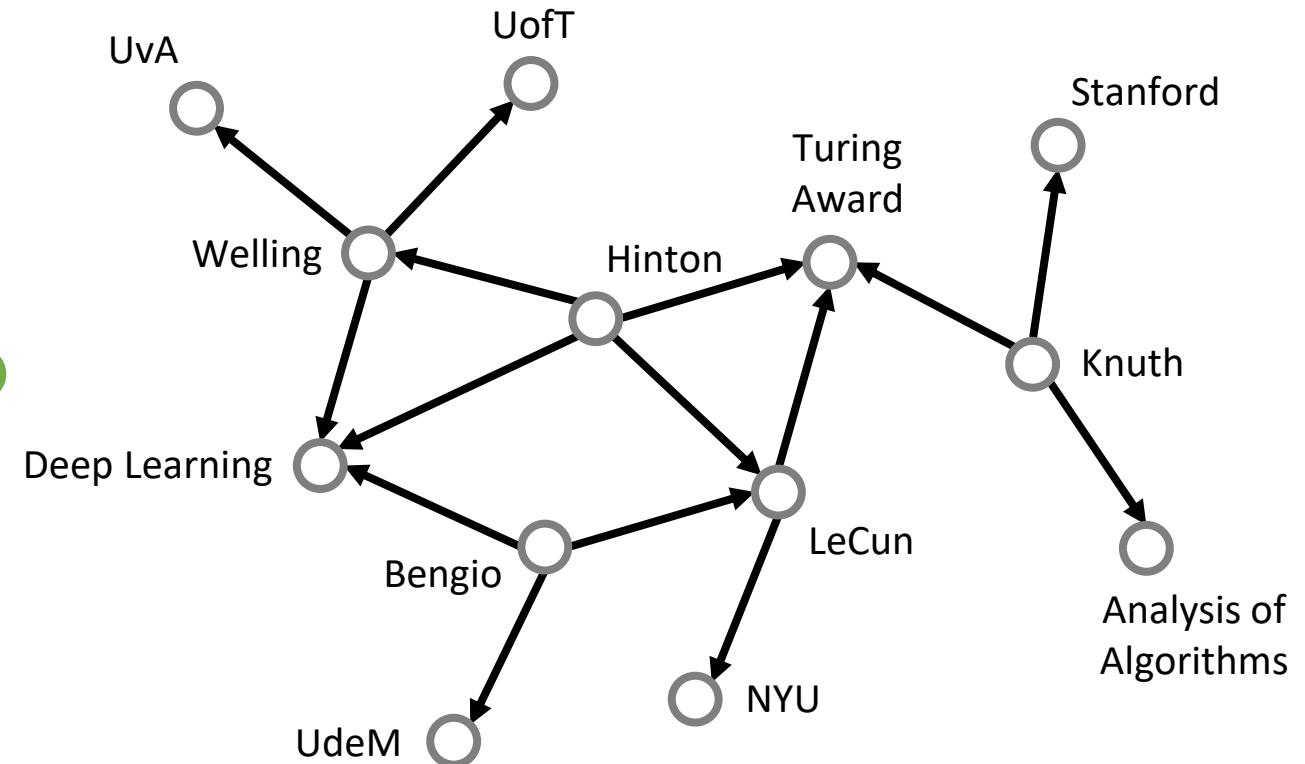
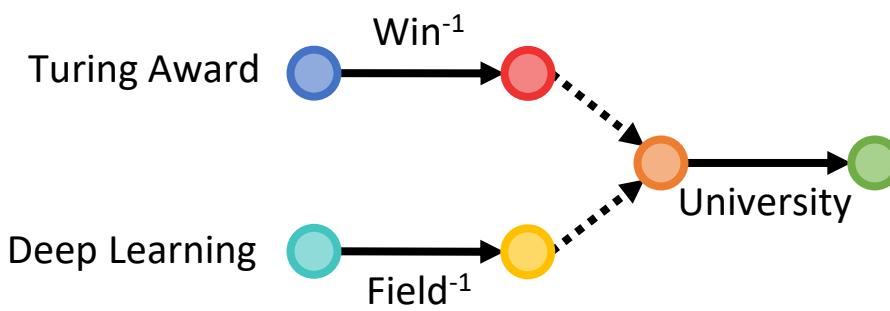


Subgraph Matching



Subgraph Matching

No answer!



Subgraph matching is **inductive**, but it **can't reason about missing links**.

Subgraph Matching

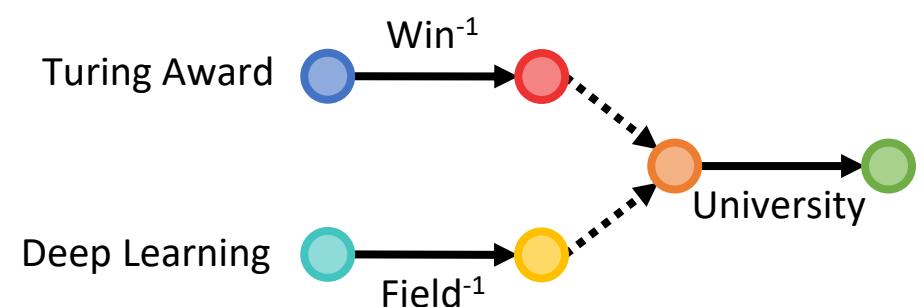
$\mathcal{X} = \{\text{Hinton, Lecun, Bengio}\} \in 2^{\mathcal{V}}$

Relation Projection: $\mathcal{Y} = \text{University}(\mathcal{X})$

Conjunction: $\mathcal{X} \cap \mathcal{Y}$

Disjunction: $\mathcal{X} \cup \mathcal{Y}$

Negation: $\mathcal{V} \setminus \mathcal{X}$



Relax to Fuzzy Sets

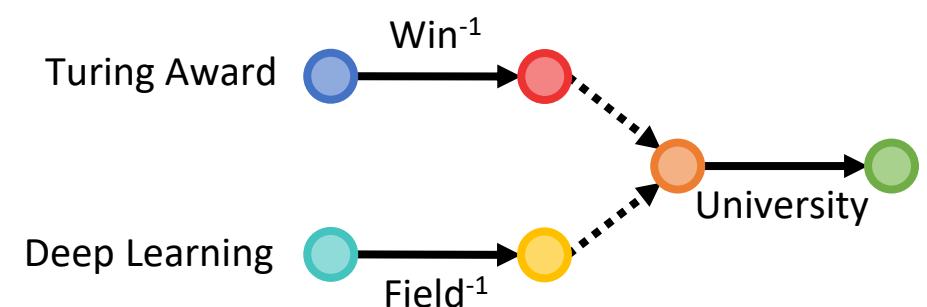
$$x = \{\text{Hinton: } 0.81, \text{Lecun: } 0.56, \text{Bengio: } 0.64\} \in [0,1]^\nu$$

Relation Projection: $y = \text{University}(x)$

Conjunction: $x \odot y$

Disjunction: $x + y - x \odot y$

Negation: $1 - x$



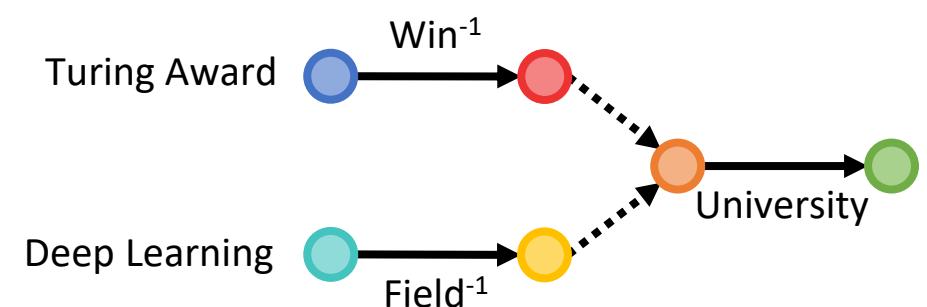
Relax to Fuzzy Sets

$$x = \{\text{Hinton: } 0.81, \text{Lecun: } 0.56, \text{Bengio: } 0.64\} \in [0,1]^\mathcal{V}$$

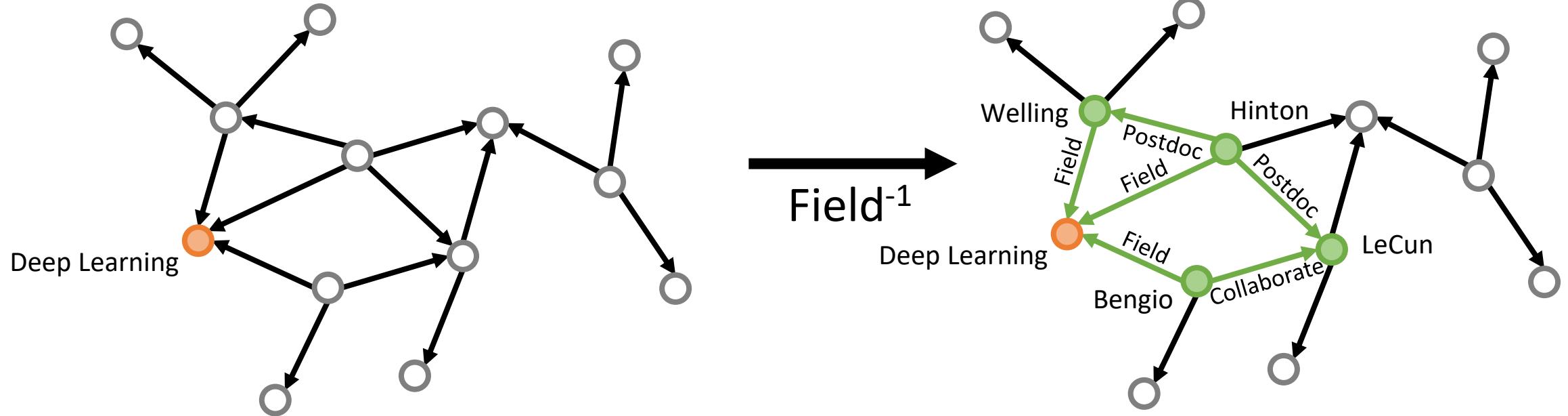
Relation Projection: $y = University(x)$

Conjunction: $x \odot y$
Disjunction: $x + y - x \odot y$
Negation: $1 - x$

} Inductive!

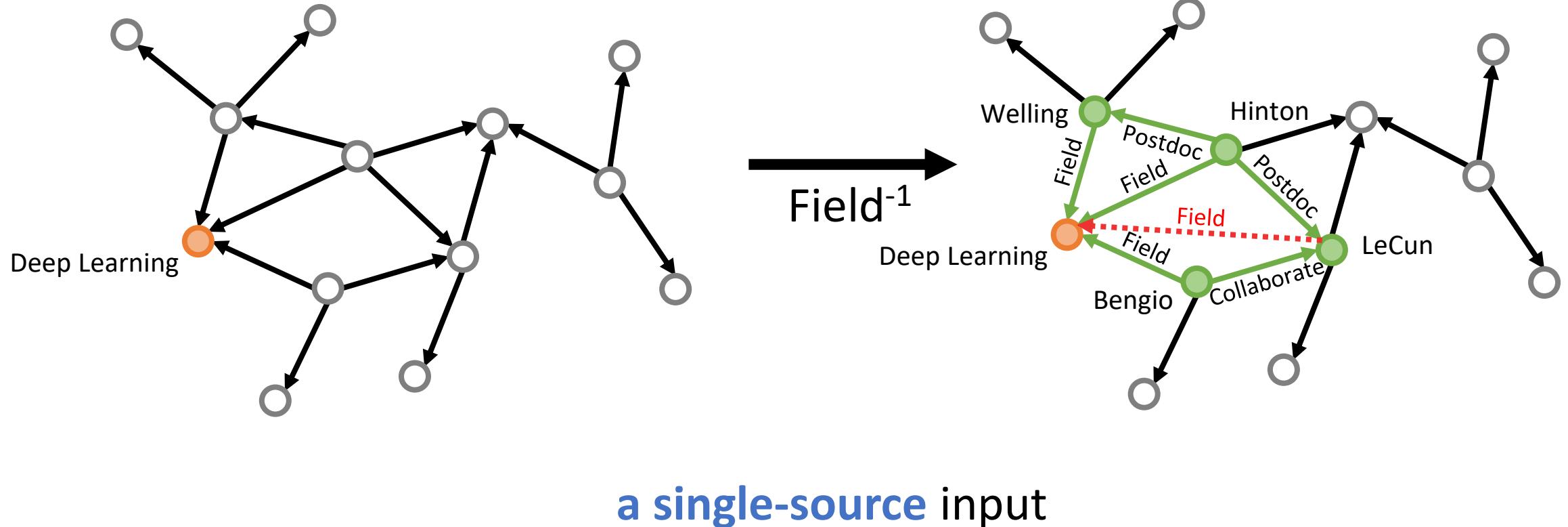


Refresher: NBFNet

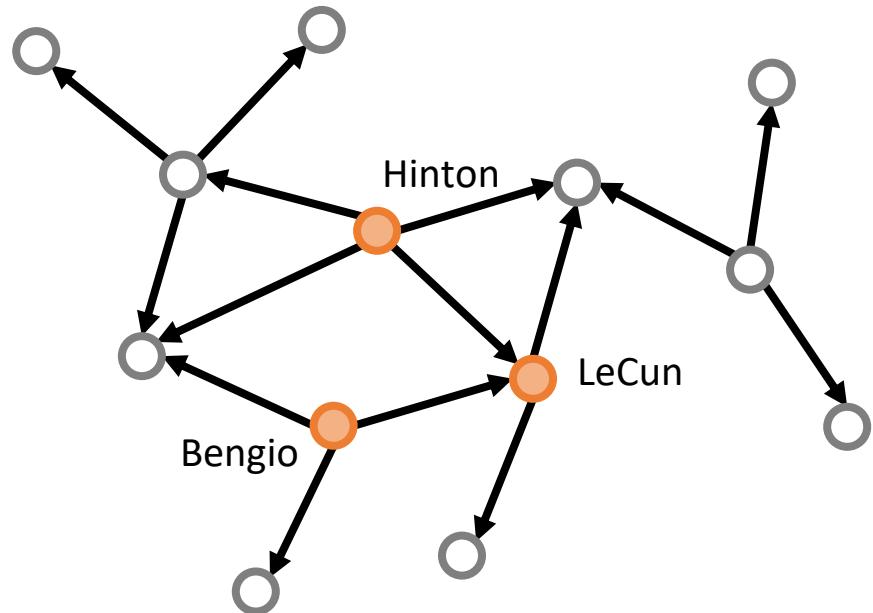


a single-source input

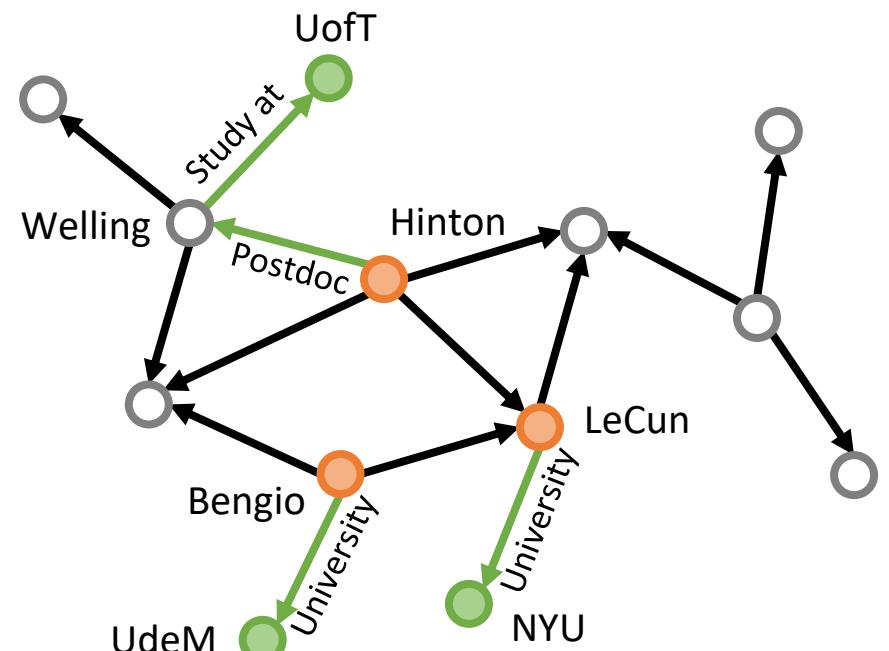
Refresher: NBFNet



Relation Projection

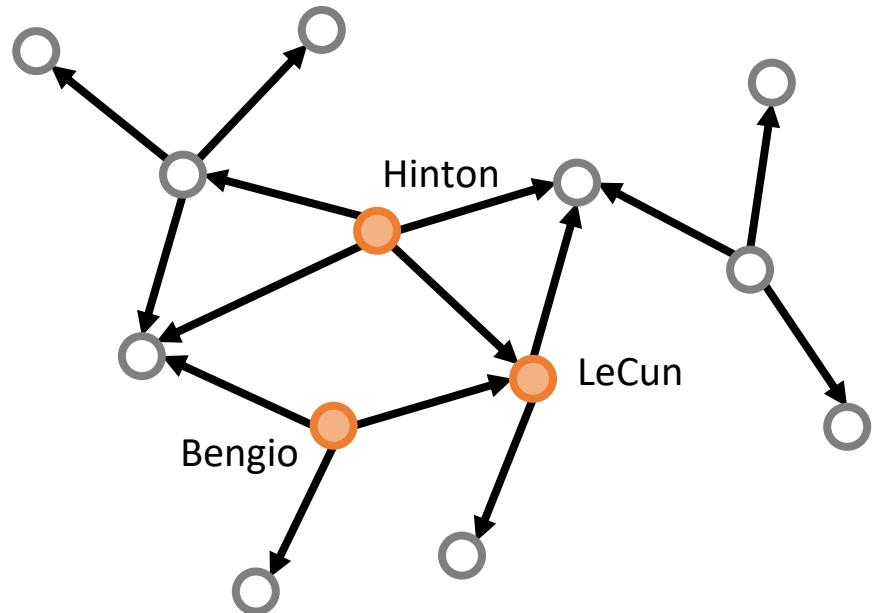


University

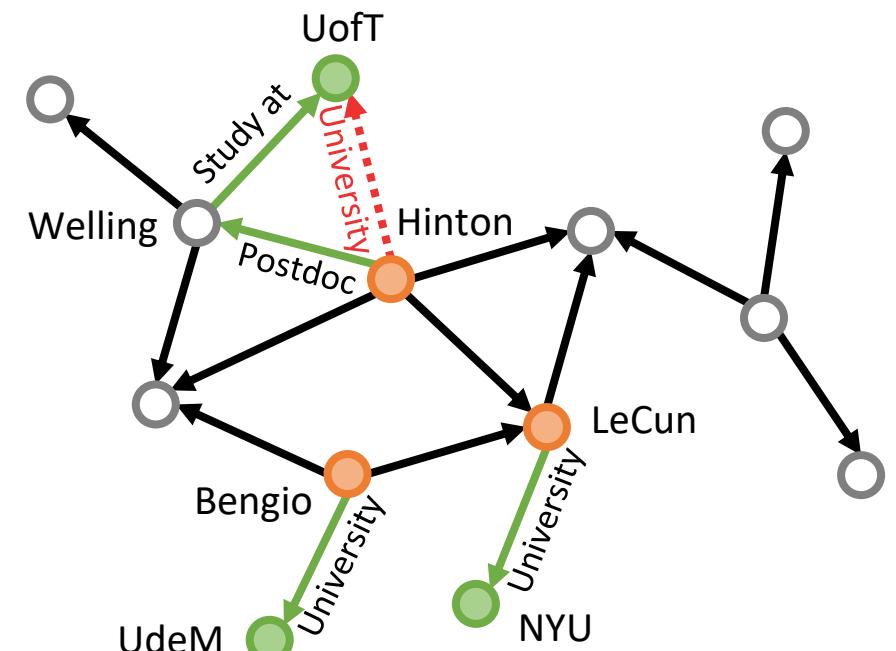


a fuzzy set input

Relation Projection

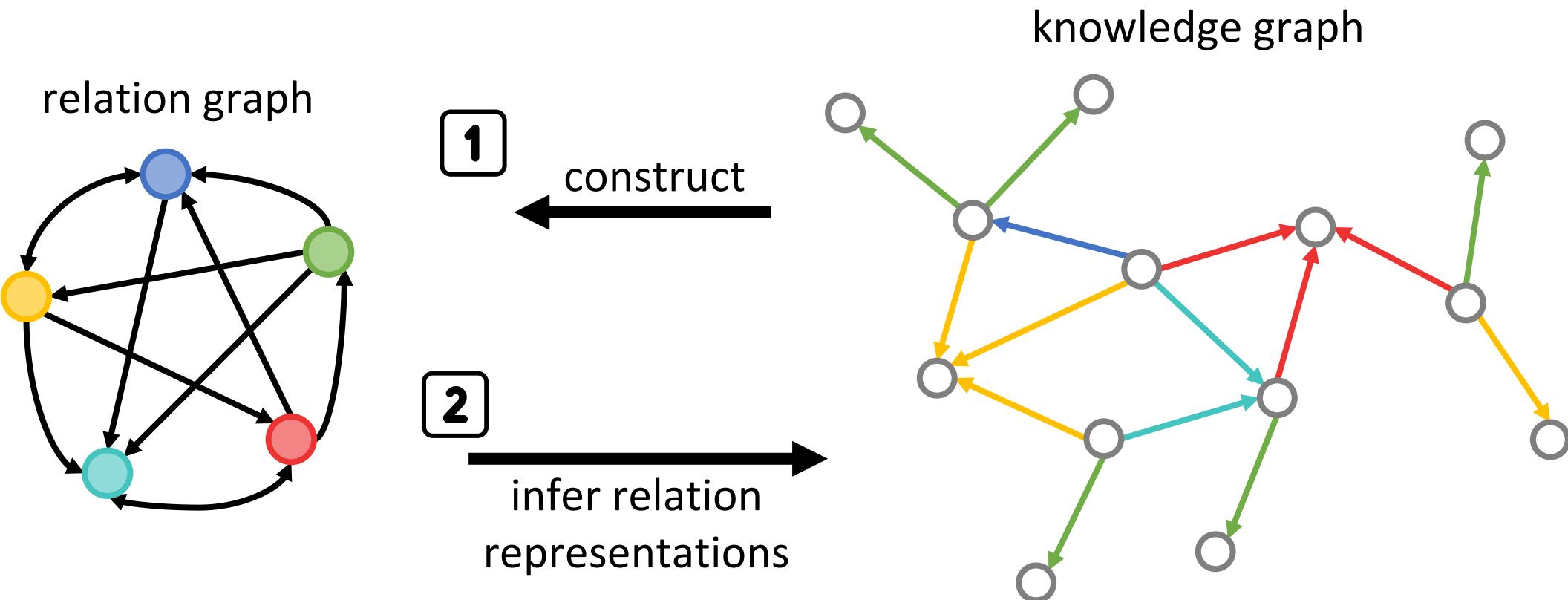


University

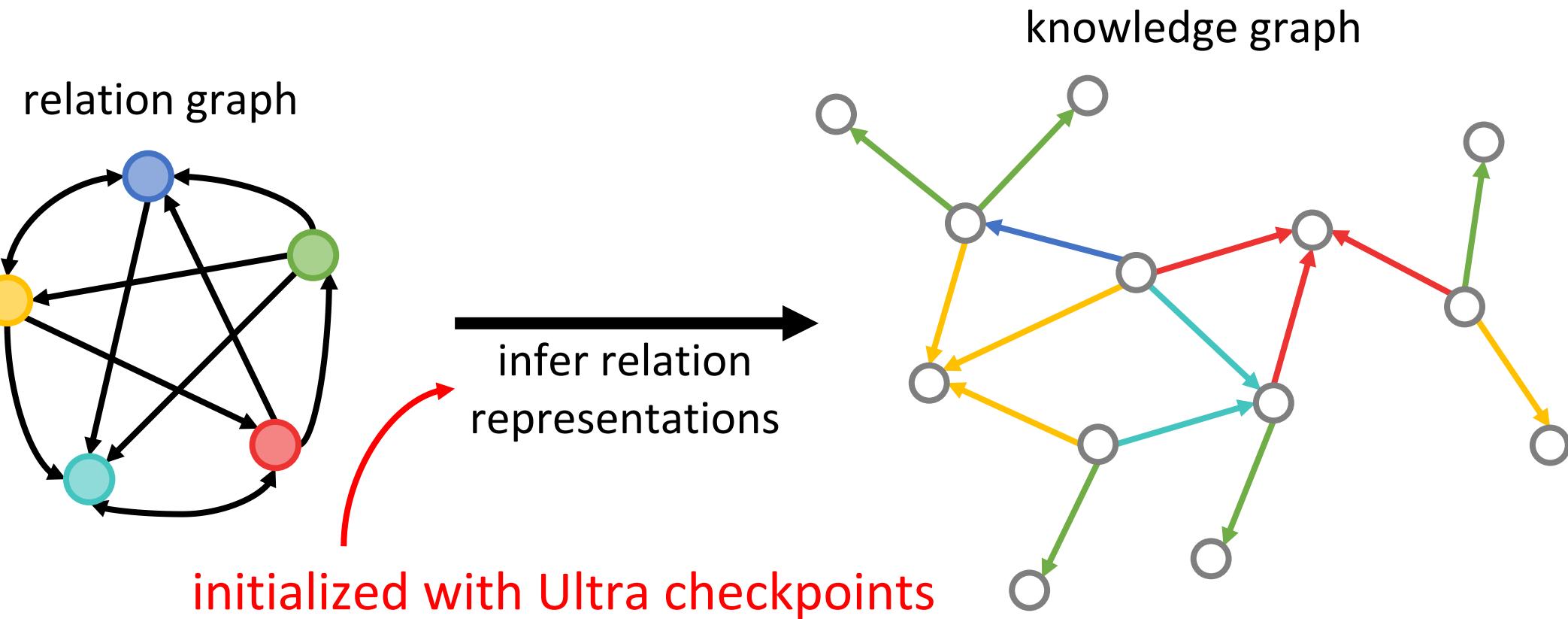


a fuzzy set input

0-shot Relation Projection



0-shot Relation Projection



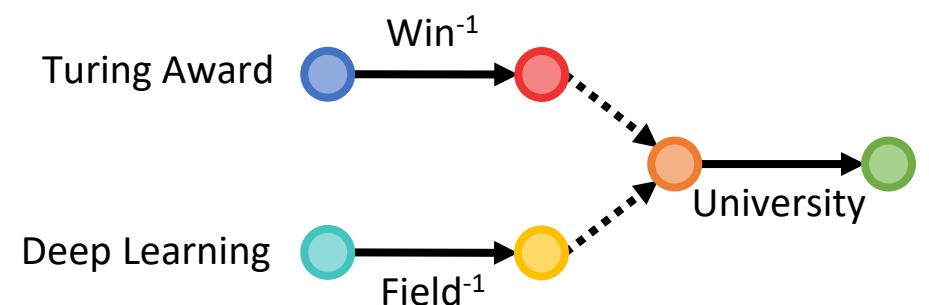
Graph Neural Network Query Executor

$$x = \{\text{Hinton: 0.81, Lecun: 0.56, Bengio: 0.64}\} \in [0,1]^{\mathcal{V}}$$

Relation Projection: $y = \text{University}(x)$ Inductive!

Conjunction: $x \odot y$
Disjunction: $x + y - x \odot y$
Negation: $1 - x$

} Inductive!



Multi-hop Logical Queries ($\mathcal{V}_{train} = \mathcal{V}_{test}$)

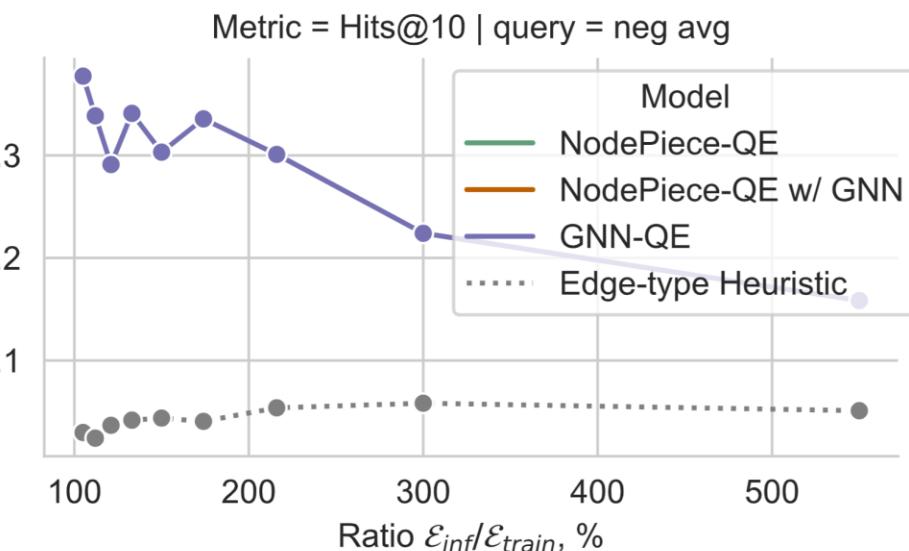
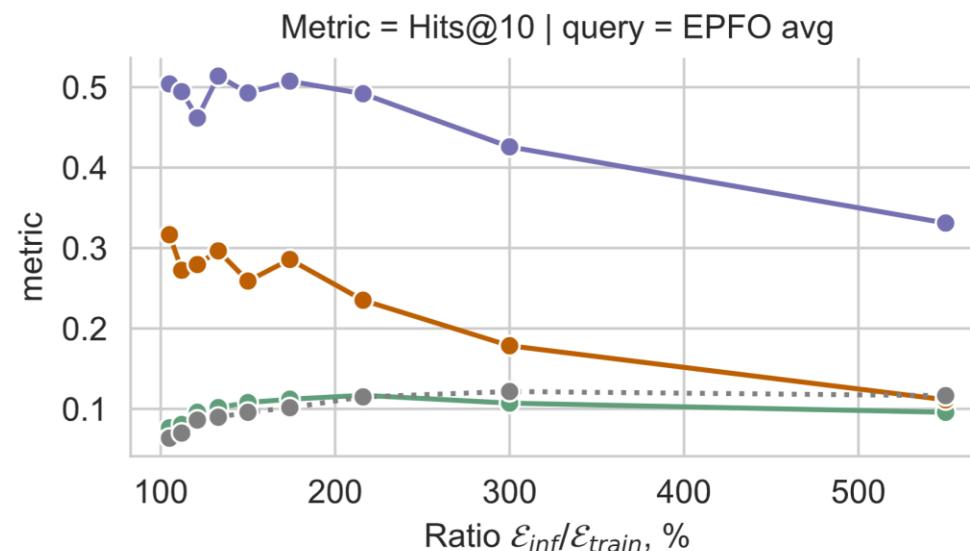
metric: MRR↑

Model	avg _p	avg _n	1p	2p	3p	2i	3i	pi	ip	2u	up	2in	3in	inp	pin	pni
FB15k																
GQE	28.0	-	54.6	15.3	10.8	39.7	51.4	27.6	19.1	22.1	11.6	-	-	-	-	-
Q2B	38.0	-	68.0	21.0	14.2	55.1	66.5	39.4	26.1	35.1	16.7	-	-	-	-	-
BetaE	41.6	11.8	65.1	25.7	24.7	55.8	66.5	43.9	28.1	40.1	25.2	14.3	14.7	11.5	6.5	12.4
CQD-CO	46.9	-	89.2	25.3	13.4	74.4	78.3	44.1	33.2	41.8	21.9	-	-	-	-	-
CQD-Beam	58.2	-	89.2	54.3	28.6	74.4	78.3	58.2	67.7	42.4	30.9	-	-	-	-	-
ConE	49.8	14.8	73.3	33.8	29.2	64.4	73.7	50.9	35.7	55.7	31.4	17.9	18.7	12.5	9.8	15.1
GNN-QE	72.8	38.6	88.5	69.3	58.7	79.7	83.5	69.9	70.4	74.1	61.0	44.7	41.7	42.0	30.1	34.3
FB15k-237																
GQE	16.3	-	35.0	7.2	5.3	23.3	34.6	16.5	10.7	8.2	5.7	-	-	-	-	-
Q2B	20.1	-	40.6	9.4	6.8	29.5	42.3	21.2	12.6	11.3	7.6	-	-	-	-	-
BetaE	20.9	5.5	39.0	10.9	10.0	28.8	42.5	22.4	12.6	12.4	9.7	5.1	7.9	7.4	3.5	3.4
CQD-CO	21.8	-	46.7	9.5	6.3	31.2	40.6	23.6	16.0	14.5	8.2	-	-	-	-	-
CQD-Beam	22.3	-	46.7	11.6	8.0	31.2	40.6	21.2	18.7	14.6	8.4	-	-	-	-	-
FuzzQE	24.0	7.8	42.8	12.9	10.3	33.3	46.9	26.9	17.8	14.6	10.3	8.5	11.6	7.8	5.2	5.8
ConE	23.4	5.9	41.8	12.8	11.0	32.6	47.3	25.5	14.0	14.5	10.8	5.4	8.6	7.8	4.0	3.6
GNN-QE	26.8	10.2	42.8	14.7	11.8	38.3	54.1	31.1	18.9	16.2	13.4	10.0	16.8	9.3	7.2	7.8

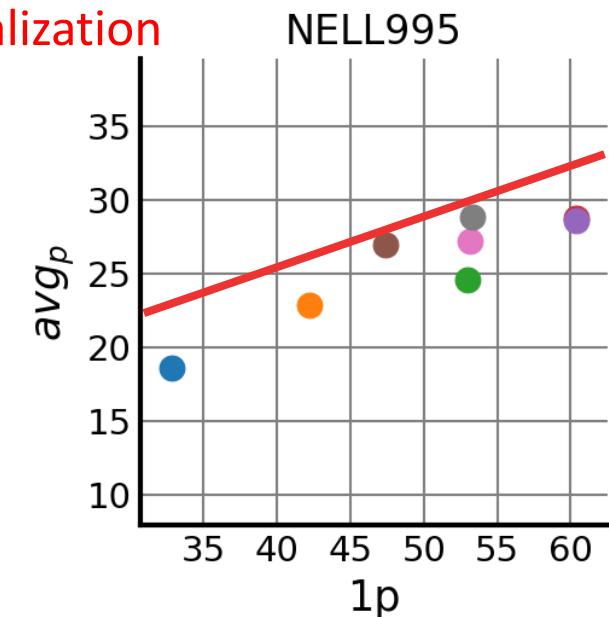
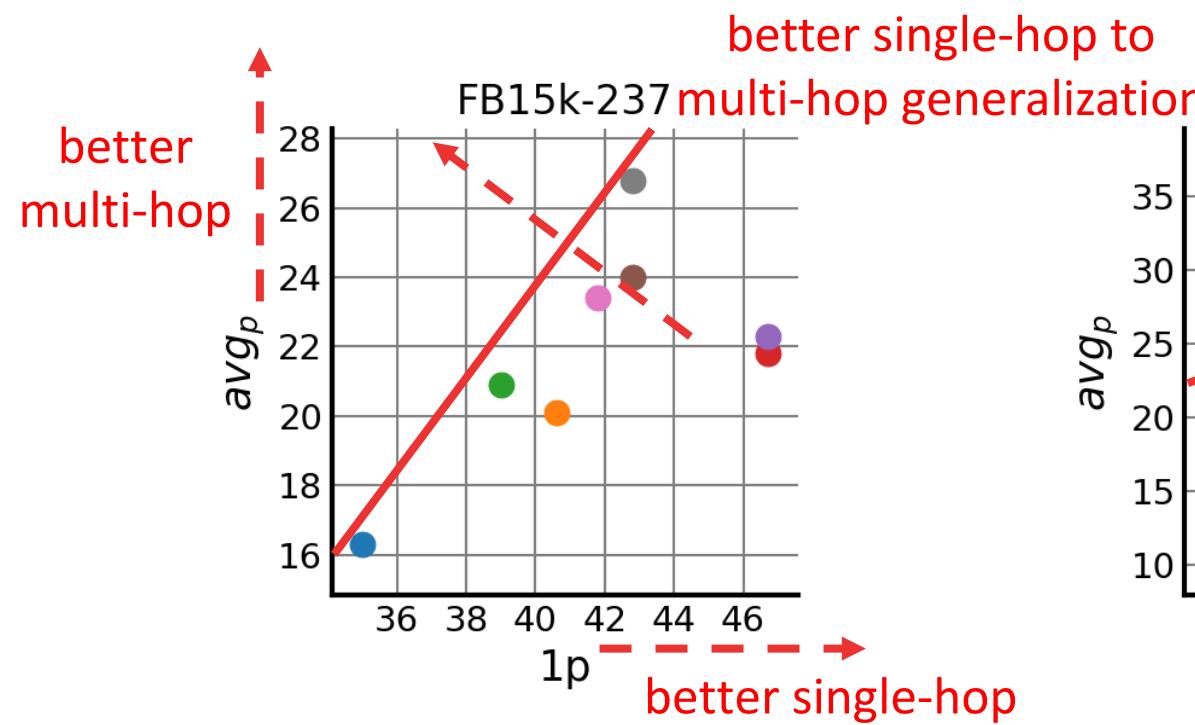
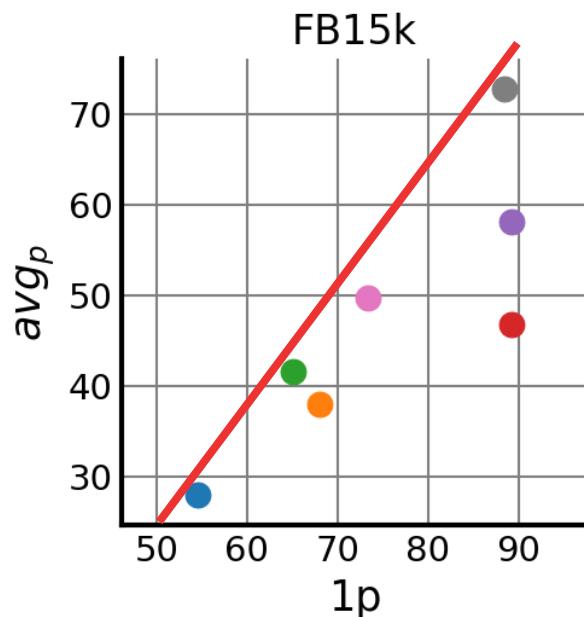
Multi-hop Logical Queries ($\mathcal{V}_{train} \neq \mathcal{V}_{test}$)

metric: H@10↑

Class	Model	avg _p	avg _n	1p	2p	3p	2i	3i	pi	ip	2u	up	2in	3in	inp	pin	pni
FB15k-237																	
Inference-only	Edge-type Heuristic	10.1	4.1	17.7	8.2	9.9	10.7	13.0	9.8	8.2	5.3	8.5	2.6	2.9	8.4	3.8	2.7
	NodePiece-QE	11.2	-	25.5	8.2	8.4	12.4	13.9	9.9	8.7	7.0	6.8	-	-	-	-	-
	NodePiece-QE w/ GNN	28.6	-	45.9	19.2	11.5	39.9	48.8	29.4	22.6	25.3	14.6	-	-	-	-	-
Trainable	GNN-QE	50.7	33.6	65.4	36.3	31.6	73.8	84.3	56.5	41.5	39.3	28.0	33.3	46.4	29.2	24.9	34.0

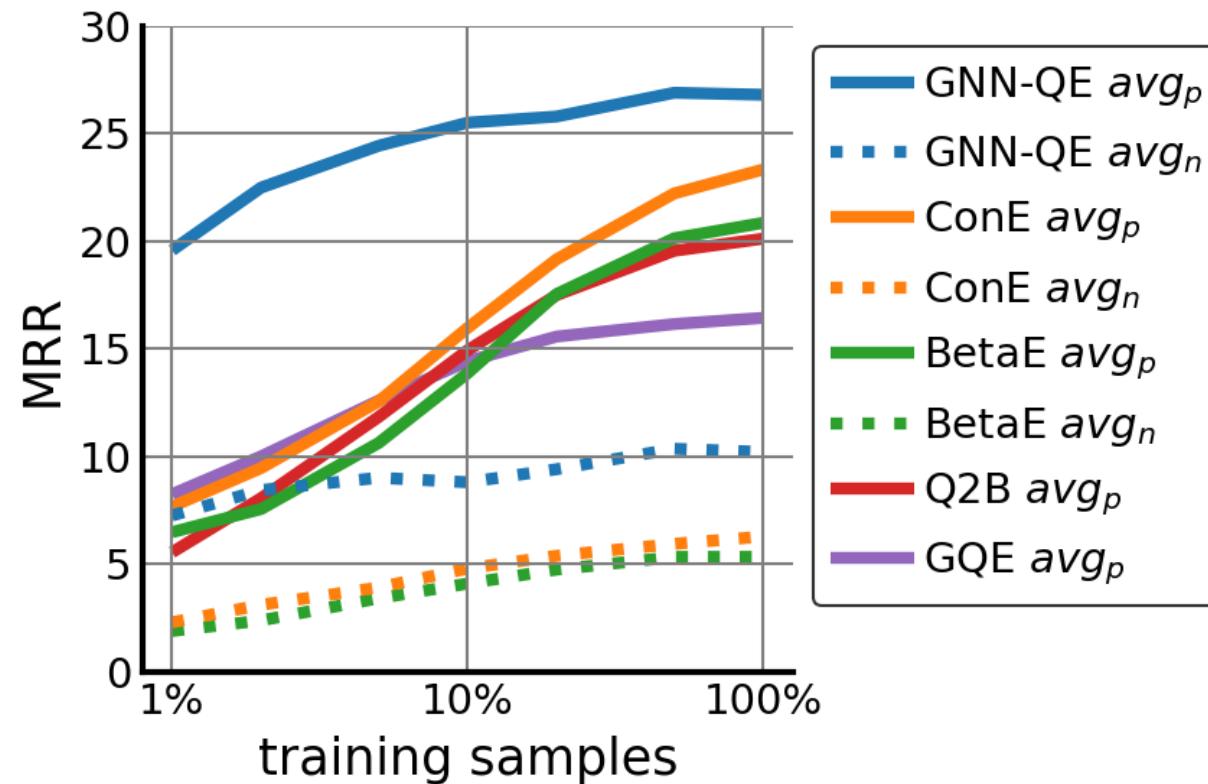


Better Compositional Generalization

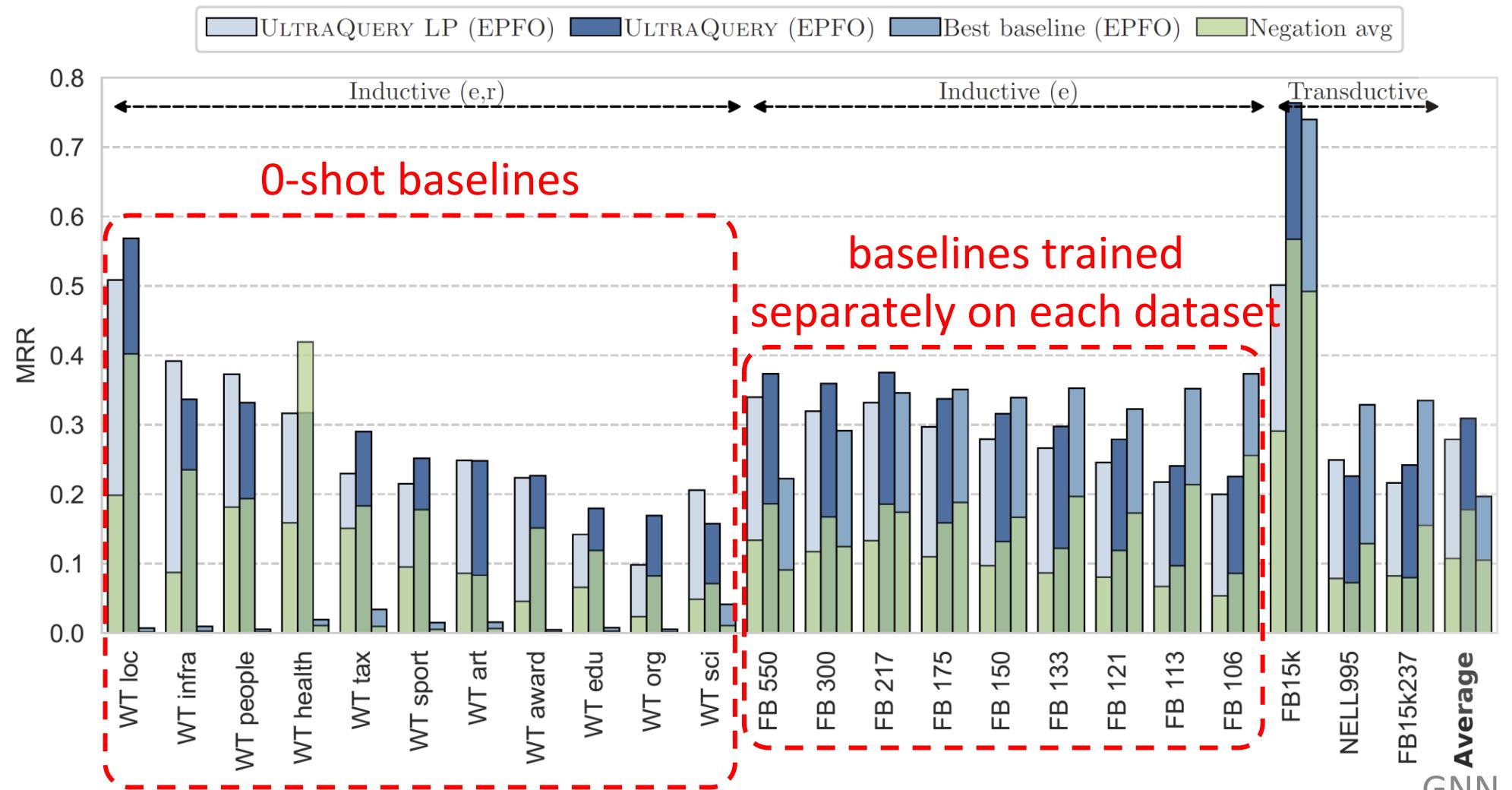


● GQE ● Q2B ● BetaE ● CQD-CO ● CQD-Beam ● FuzzQE ● ConE ● GNN-QE

Effective for Small Training Data



0-shot Inference of Multi-hop Queries



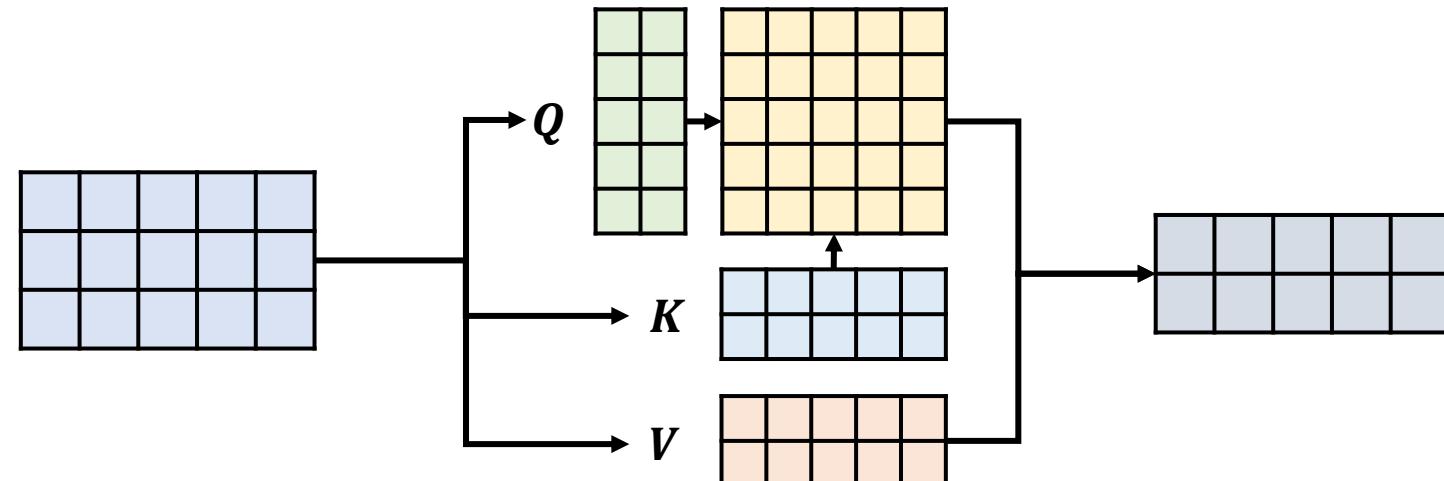
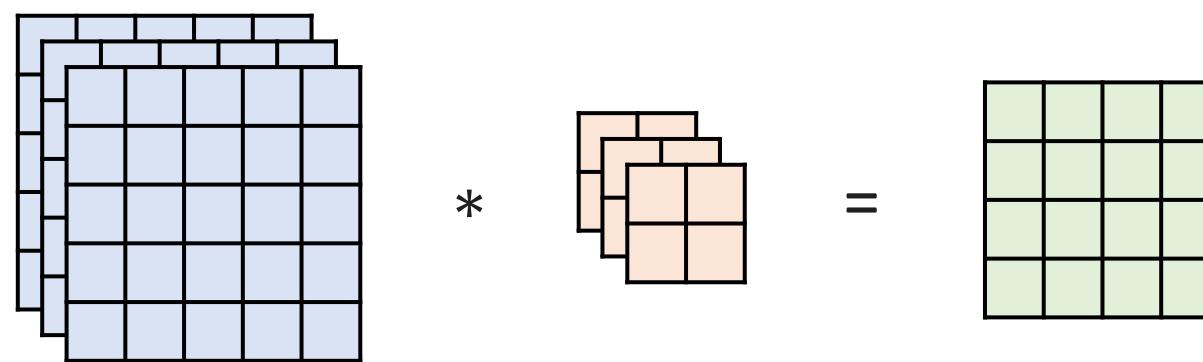
TorchDrug^[1]: Simplifying **development** on **structured data** and related applications

[1] Zhaocheng Zhu, Chence Shi, Zuobai Zhang, Shengchao Liu, Minghao Xu, Xinyu Yuan, Yangtian Zhang, Junkun Chen, Huiyu Cai, Jiarui Lu, Chang Ma, Runcheng Liu, Louis-Pascal Xhonneux, Meng Qu, Jian Tang. TorchDrug: A Powerful and Flexible Machine Learning Platform for Drug Discovery. arXiv 2022.

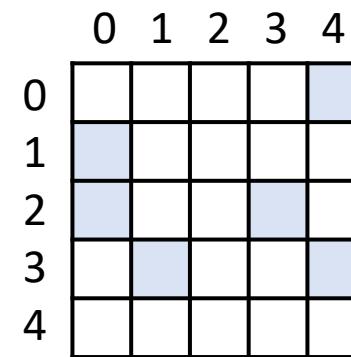
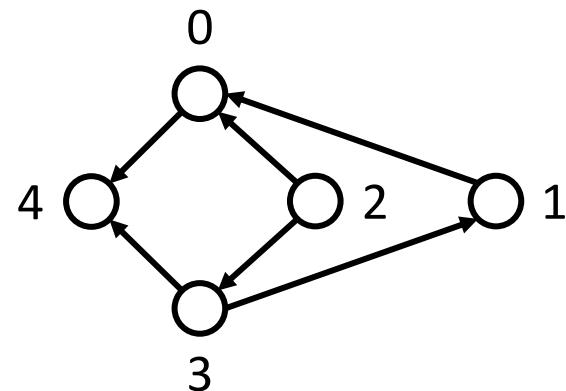
ML Implementation = Tensor Operations



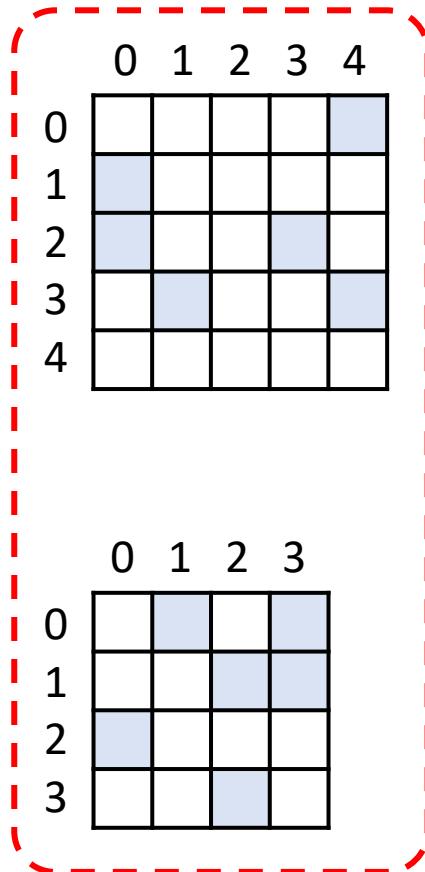
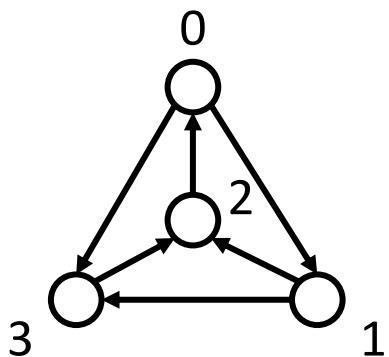
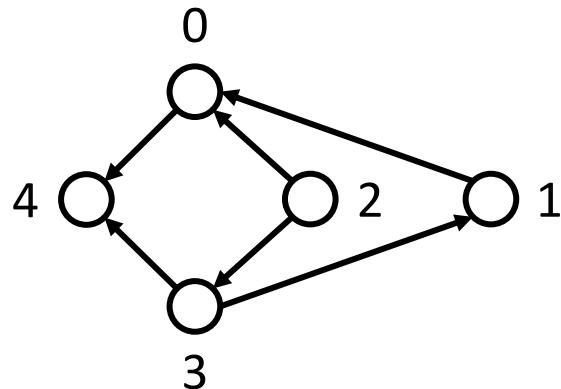
She sells sea shells.



Structured Data Meets Tensors

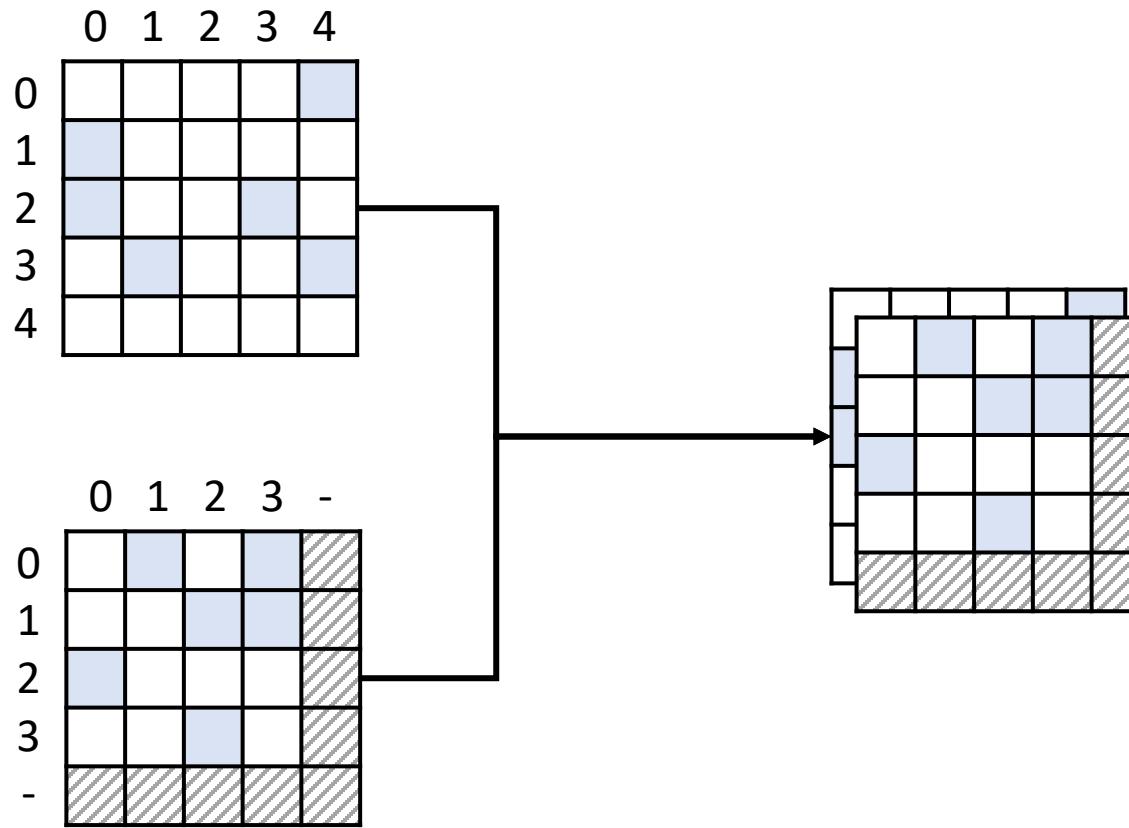
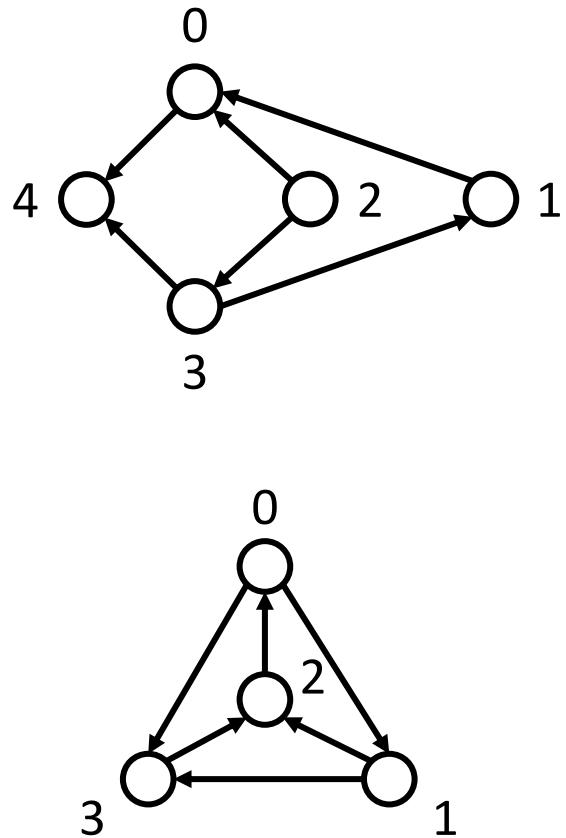


Structured Data Meets Tensors

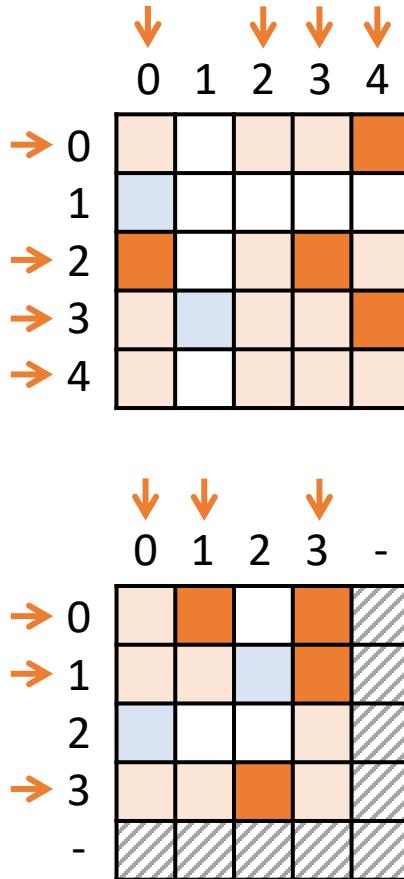
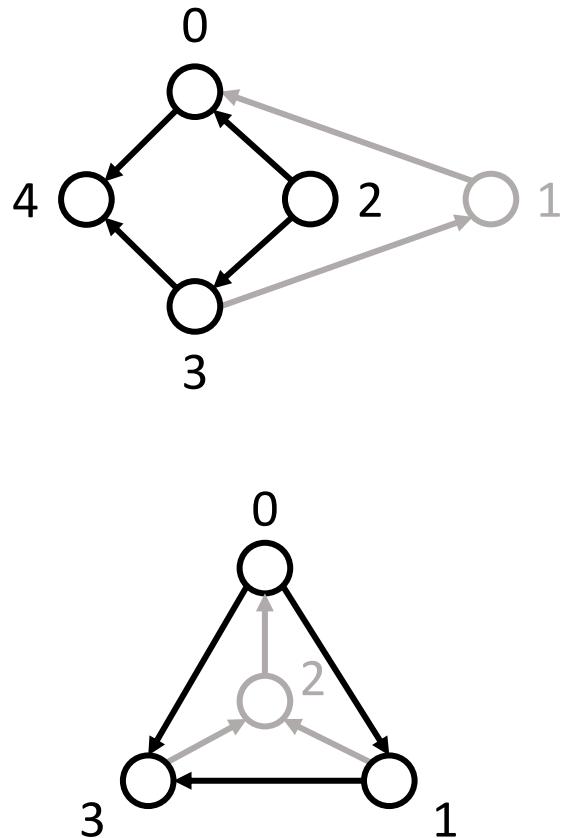


How to batch tensors
of different shapes?

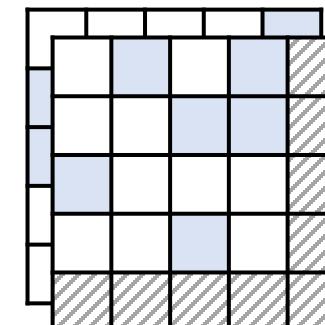
Naïve Solution: Padding



Naïve Solution: Padding



How to perform operations
on batched tensors?



Solutions



arrays



easy to implement
preprocessing
very slow



dense tensors



on-the-fly
not scalable

Solutions



arrays



easy to implement
preprocessing
very slow



dense tensors



on-the-fly
not scalable



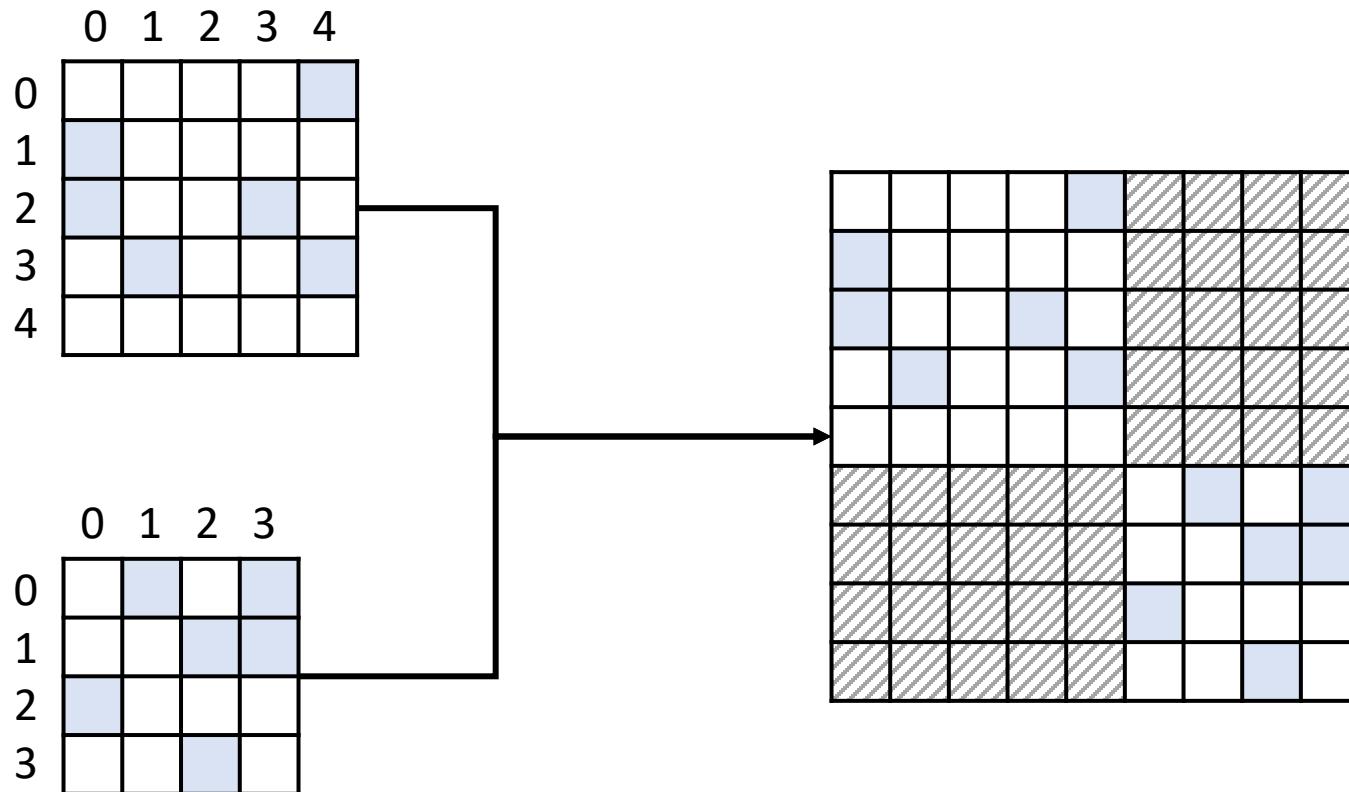
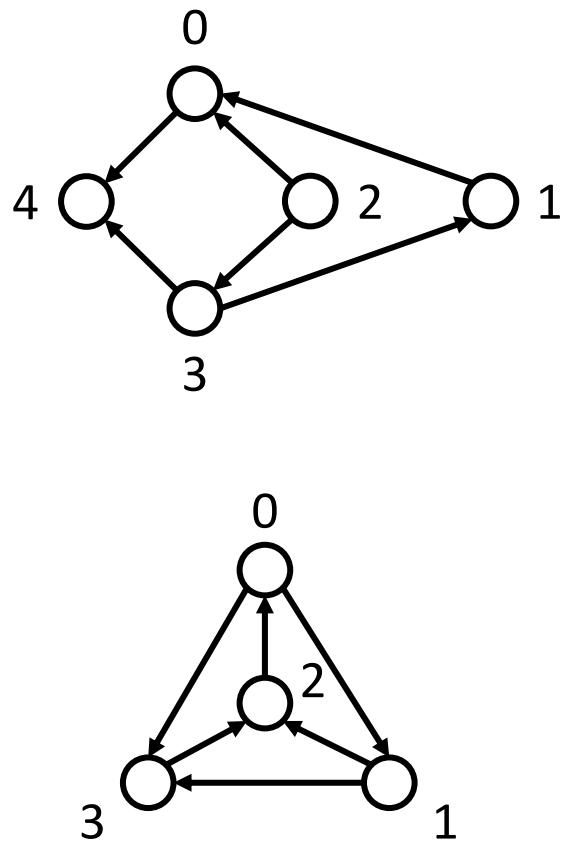
sparse tensors



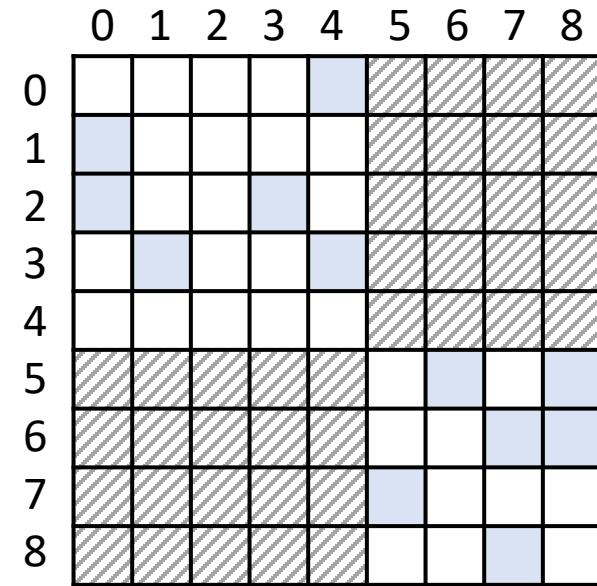
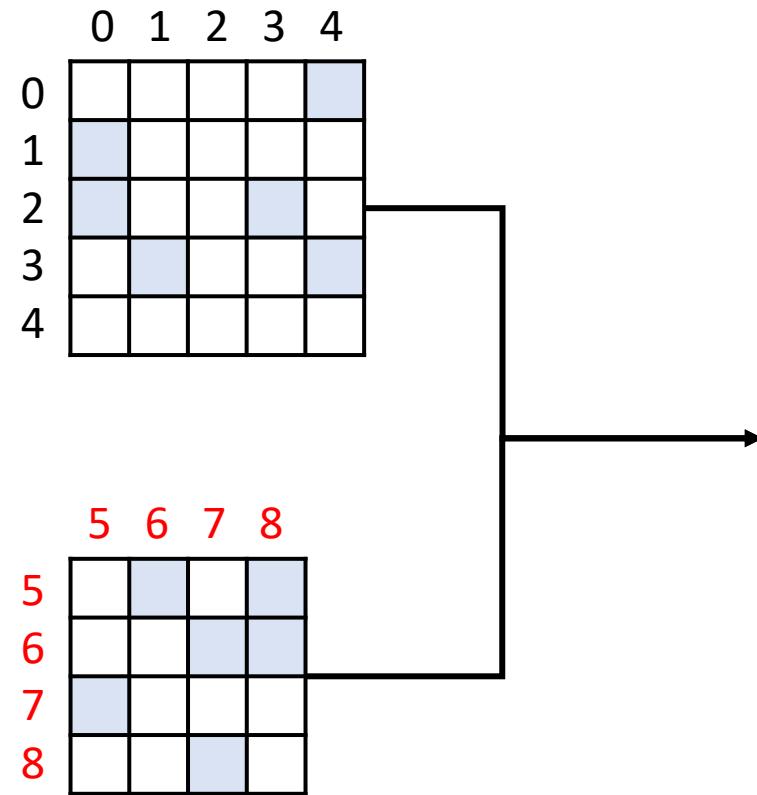
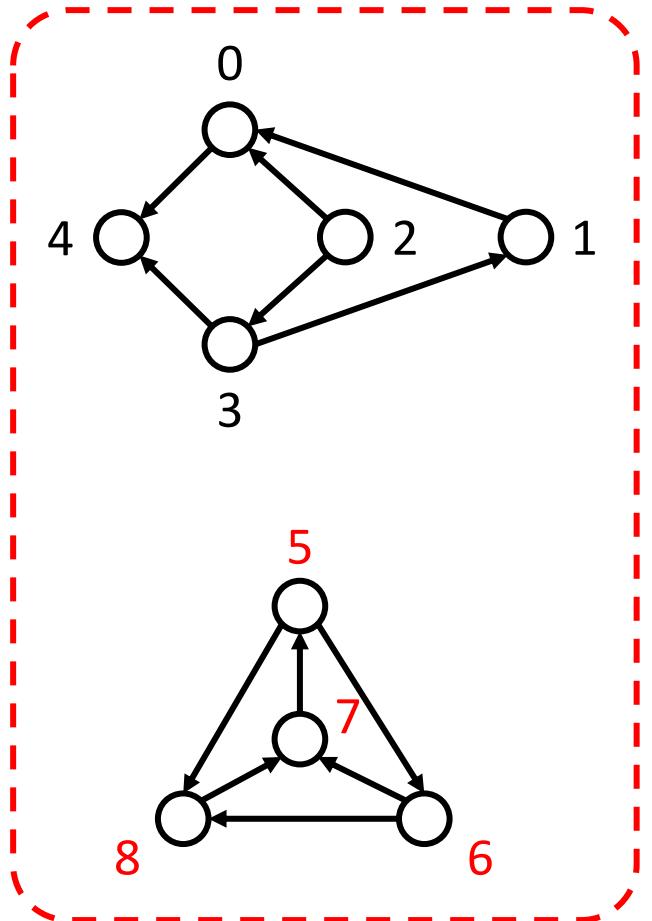
on-the-fly
scalable

How to implement?

The High-Level Idea

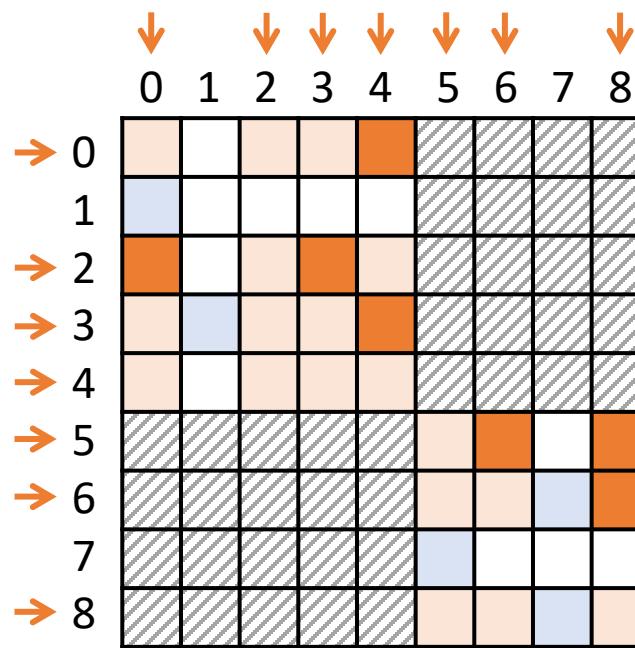
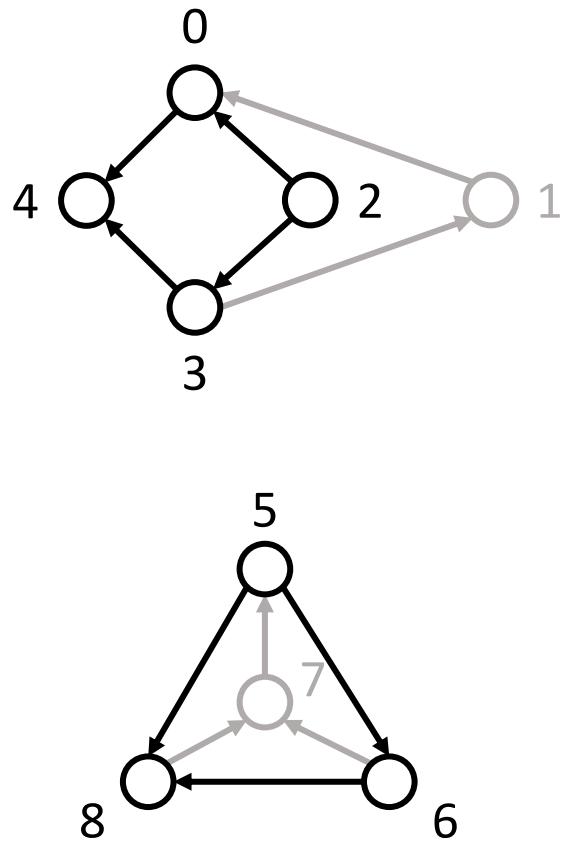


The High-Level Idea



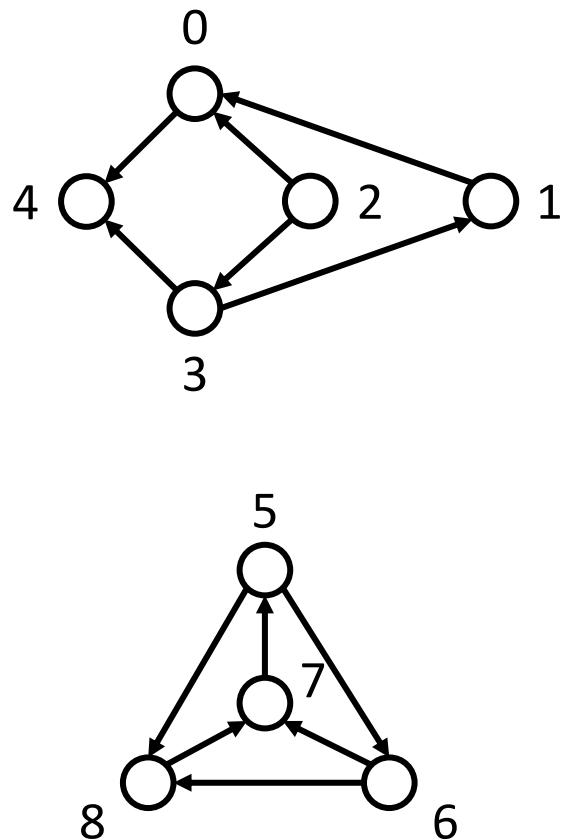
a graph of two connected components

The High-Level Idea



easy to implement!

Data Structure



`torchdrug.data.PackedGraph`

edge list

0	1	2	2	3	3	5	5	6	6	7	8
4	0	0	3	1	4	6	8	7	8	5	7

#nodes

5	4
---	---

#edges

6	6
---	---

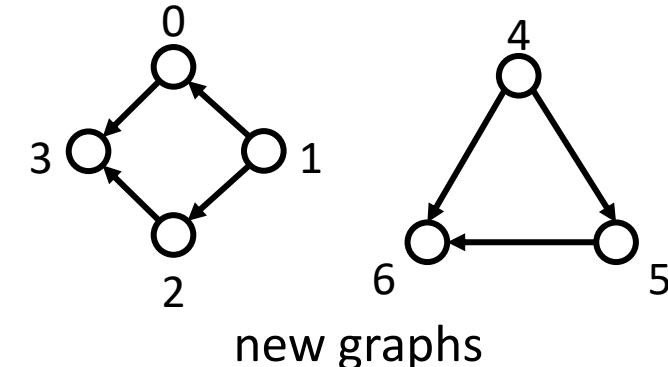
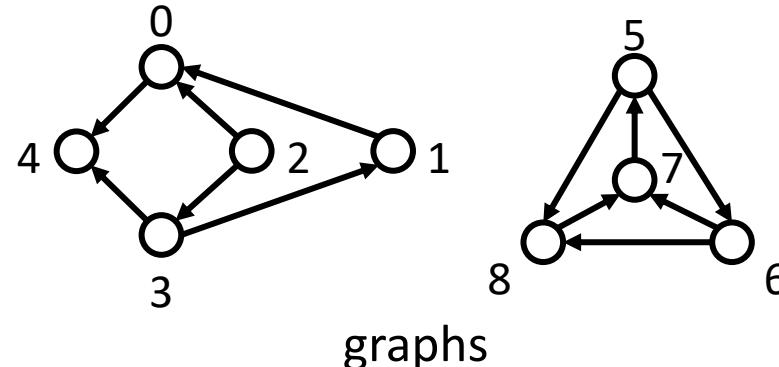
node/edge/graph
attributes

.....

predefined or user-registered

Graph Operations

```
new_graphs = graphs.subgraph(node_index)
```



edge list	<table border="1"><tr><td>0</td><td>1</td><td>2</td><td>2</td><td>3</td><td>3</td><td>5</td><td>5</td><td>6</td><td>6</td><td>7</td><td>8</td></tr><tr><td>4</td><td>0</td><td>0</td><td>3</td><td>1</td><td>4</td><td>6</td><td>8</td><td>7</td><td>8</td><td>5</td><td>7</td></tr></table>	0	1	2	2	3	3	5	5	6	6	7	8	4	0	0	3	1	4	6	8	7	8	5	7
0	1	2	2	3	3	5	5	6	6	7	8														
4	0	0	3	1	4	6	8	7	8	5	7														

#nodes	<table border="1"><tr><td>5</td><td>4</td></tr></table>	5	4
5	4		

#edges	<table border="1"><tr><td>6</td><td>6</td></tr></table>	6	6
6	6		

node index	<table border="1"><tr><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>8</td></tr></table>	0	2	3	4	5	6	8
0	2	3	4	5	6	8		

edge list	<table border="1"><tr><td>0</td><td>1</td><td>1</td><td>2</td><td>4</td><td>4</td><td>5</td></tr><tr><td>3</td><td>0</td><td>2</td><td>3</td><td>5</td><td>6</td><td>6</td></tr></table>	0	1	1	2	4	4	5	3	0	2	3	5	6	6
0	1	1	2	4	4	5									
3	0	2	3	5	6	6									

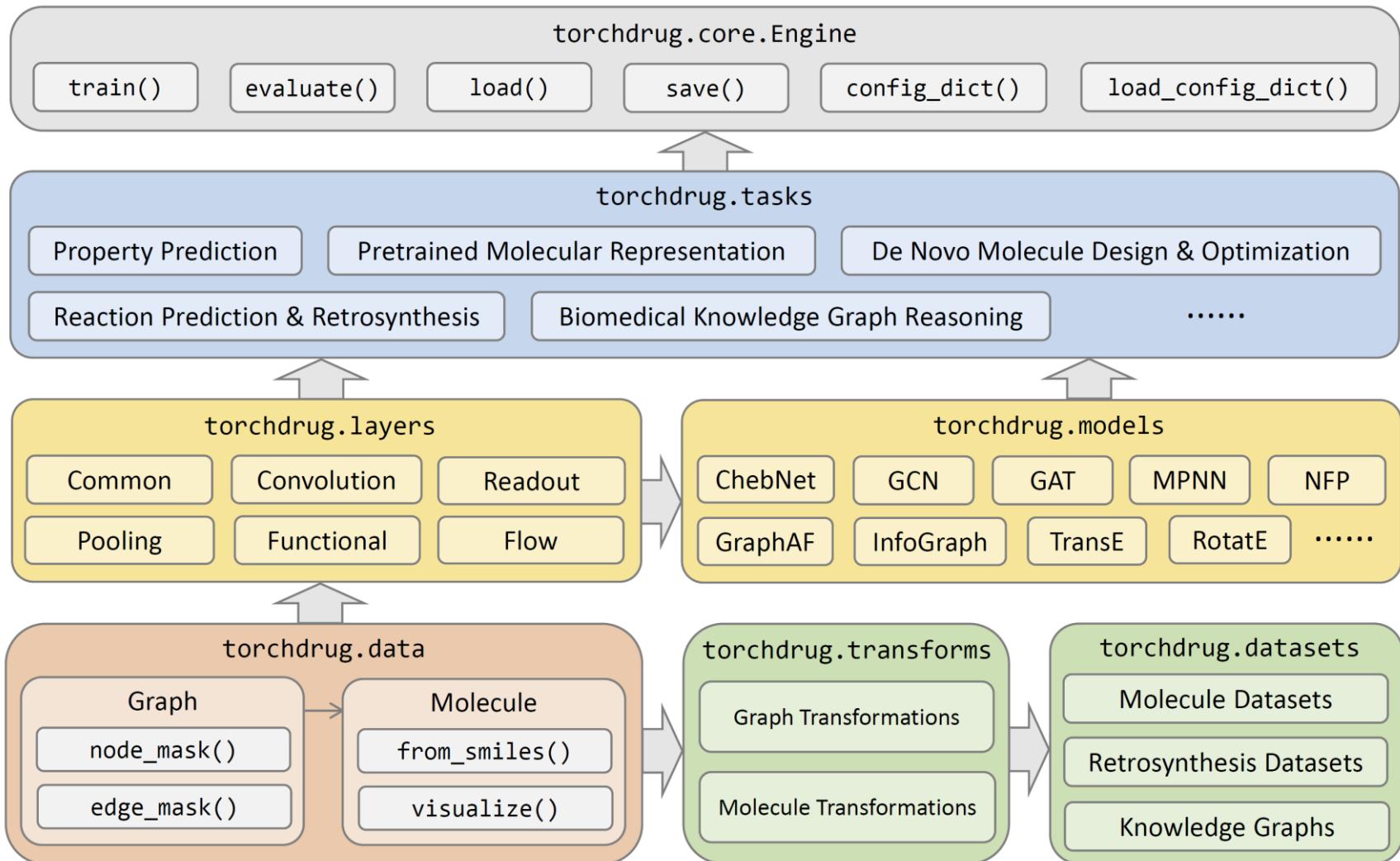
#nodes	<table border="1"><tr><td>4</td><td>3</td></tr></table>	4	3
4	3		

#edges	<table border="1"><tr><td>4</td><td>3</td></tr></table>	4	3
4	3		

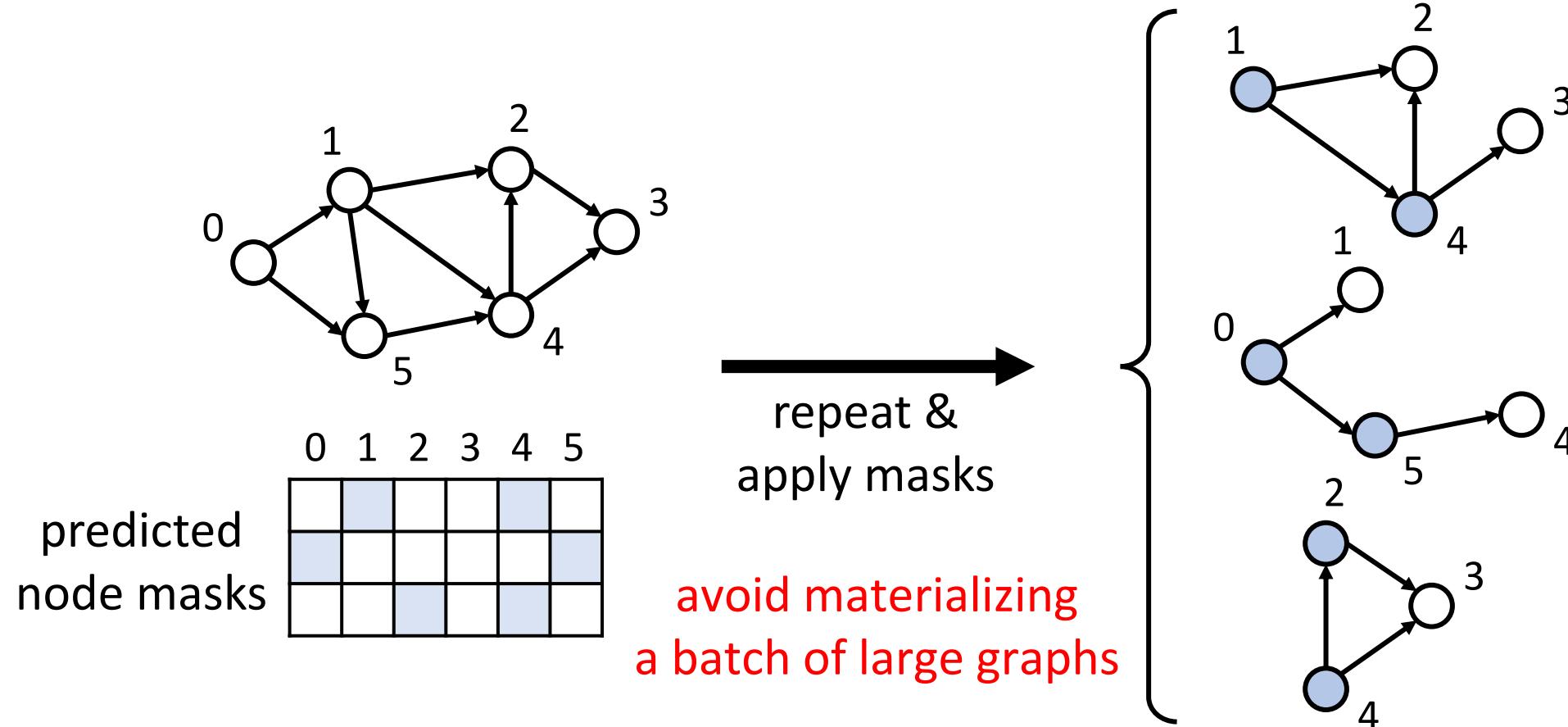
Supported Operations

Class	API	Graph Operation
PyTorch-like	<code>data.Graph.clone</code>	Clone this graph
	<code>data.Graph.detach</code>	Detach this graph
	<code>data.Graph.cpu</code>	Move this graph to CPU
	<code>data.Graph.cuda</code>	Move this graph to GPU
	<code>data.Graph.copy_</code>	Copy data from another graph
	<code>data.Graph.full</code>	Return a fully connected graph over nodes
	<code>data.Graph.repeat</code>	Repeat this graph like <code>torch.repeat</code>
	<code>data.PackedGraph.repeat_interleave</code>	Repeat this graph like <code>torch.repeat_interleave</code>
Node-level	<code>data.Graph.node_mask</code>	Mask out some nodes from this graph
	<code>data.Graph.compact</code>	Remove isolated nodes
Edge-level	<code>data.Graph.edge_mask</code>	Mask out some edges from this graph
	<code>data.Graph.directed</code>	Return a directed version of this graph
	<code>data.Graph.undirected</code>	Return an undirected version of this graph
	<code>data.Graph.match</code>	Search specific edges in this graph
Graph-level	<code>data.Graph.connected_components</code>	Split a graph into connected components
	<code>data.Graph.split</code>	Split a graph into a batch of graphs
	<code>data.Graph.pack</code>	Pack multiple graphs into a batch
	<code>data.Graph.line_graph</code>	Return a line graph of this graph
	<code>data.PackedGraph.graph_mask</code>	Mask out some graphs from this batch
	<code>data.PackedGraph.merge</code>	Merge some graphs into a smaller batch
	<code>data.PackedGraph.unpack</code>	Unpack a batch into multiple graphs
Molecule	<code>data.Molecule.ion_to_molecules</code>	Convert ions to molecules
Protein	<code>data.Protein.residue_mask</code>	Mask out some residues from this protein

Different Levels of Abstraction



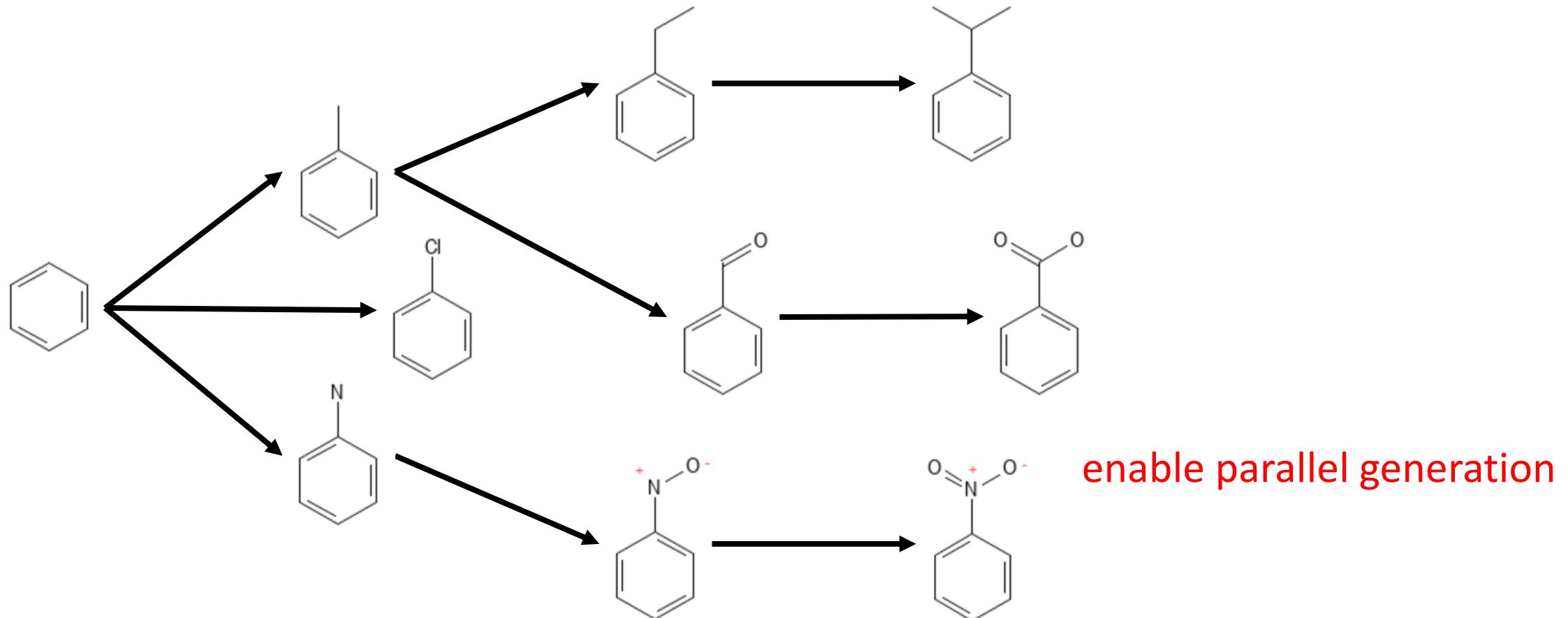
Use Case: Adaptive Message Passing^[1]



[1] Zhaocheng Zhu*, Xinyu Yuan*, Mikhail Galkin, Sophie Xhonneux, Ming Zhang, Maxime Gazeau, Jian Tang.

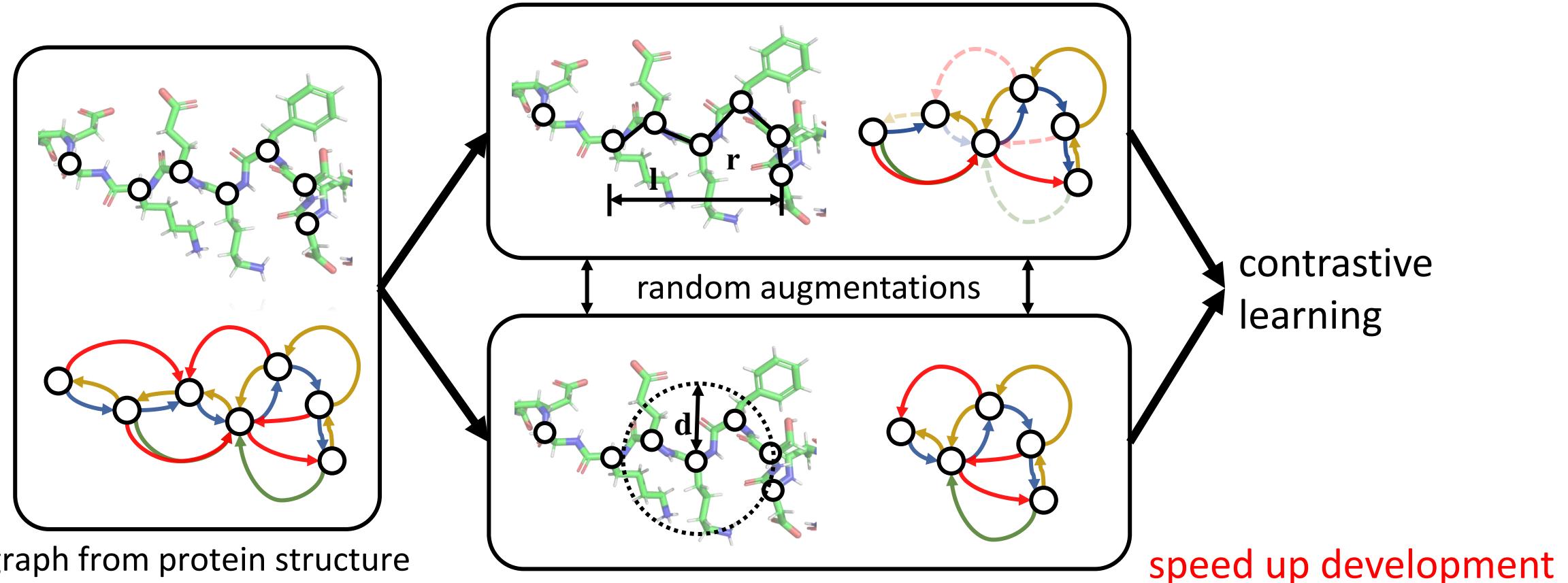
A*Net: A Scalable Path-based Reasoning Approach for Knowledge Graphs. NeurIPS 2023.

Use Case: Beam Search of Generation^[1]



[1] Chence Shi, Minkai Xu, Hongyu Guo, Ming Zhang, Jian Tang. A Graph to Graphs Framework for Retrosynthesis Prediction. ICML 2020.

Use Case: On-the-fly Graph Construction^[1]



[1] Zuobai Zhang, Minghao Xu, Arian Jamasb, Vijil Chenthamarakshan, Aurelie Lozano, Payel Das, Jian Tang.
Protein Representation Learning by Geometric Structure Pretraining. ICLR 2023.



What is the impact of our works?

What is the future for reasoning and generalization?

Direct impact: Accelerating **the transition
from transductive models to inductive ones**

Lesson: Models with **inductive biases inspired by symbolic algorithms generalize better**

Belief: Many reasoning problems can be
unified

Inductive Generalization on Text

Train

What is the answer to $1 + 1 + 1 - 1 - 1$?



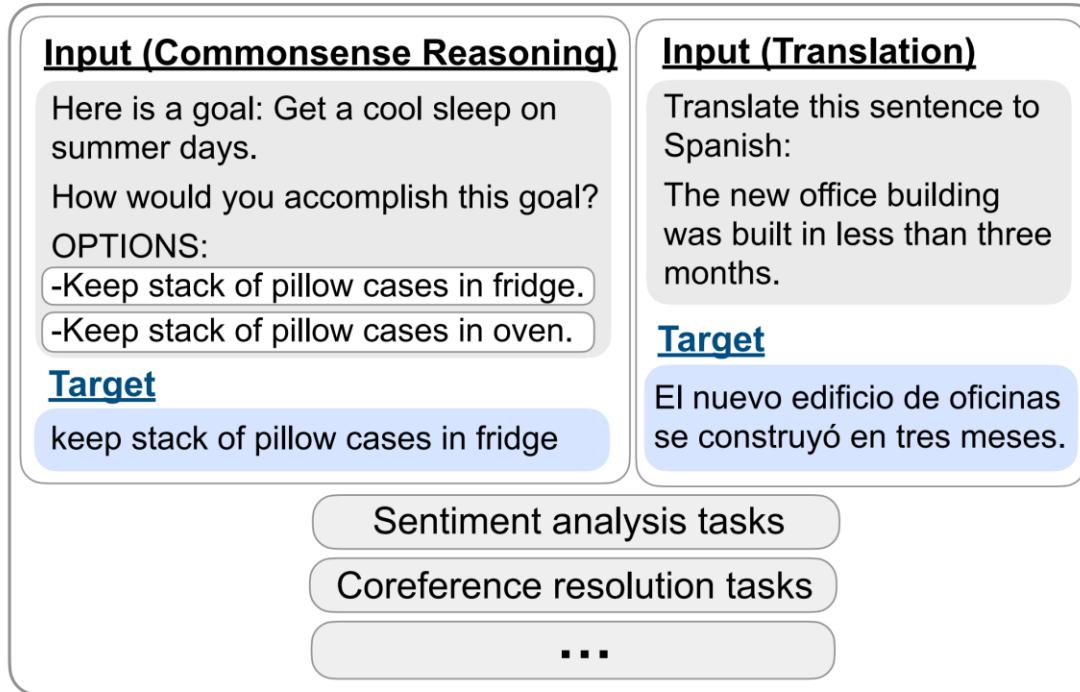
Test

What is my son's transudative son's father's
father?



The De Facto Approach: Instruction Tuning

Finetune on many tasks (“instruction-tuning”)

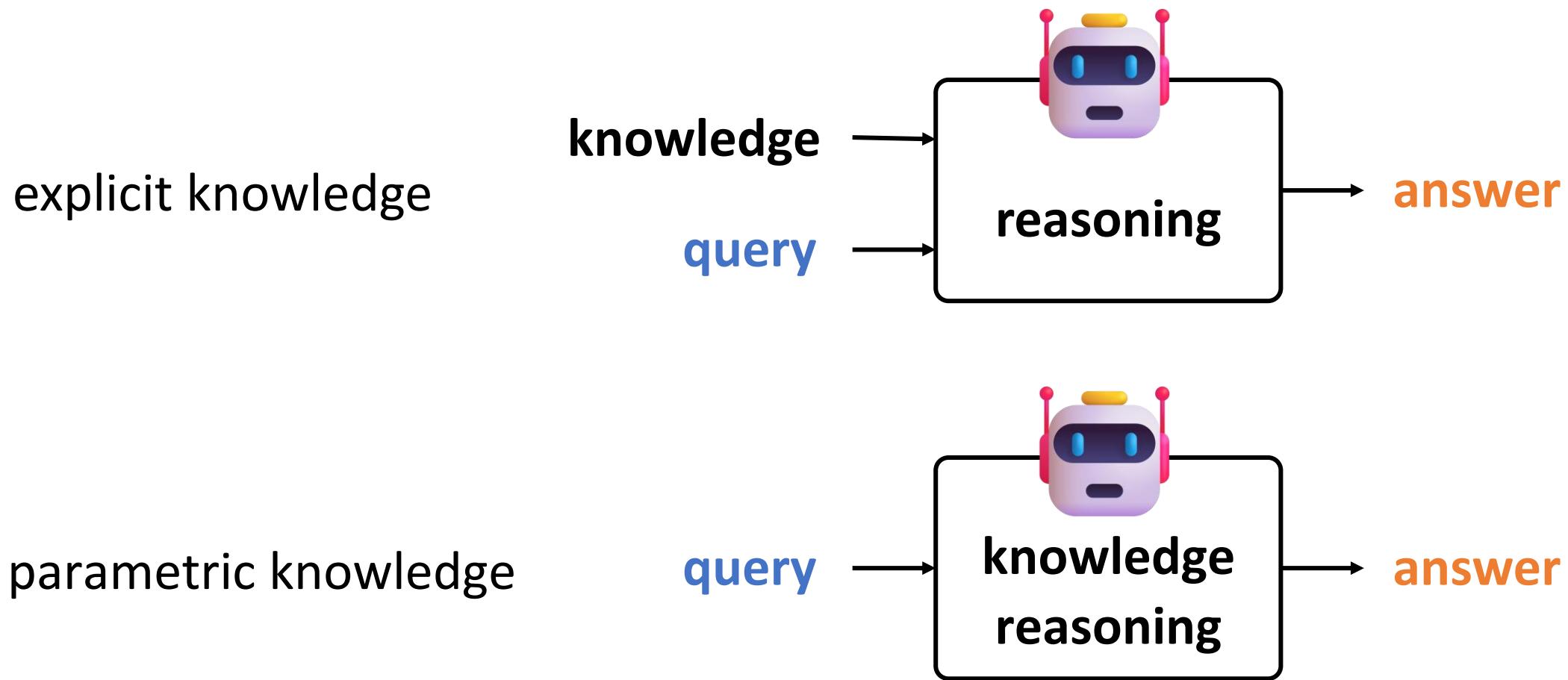


implicitly perform inductive generalization

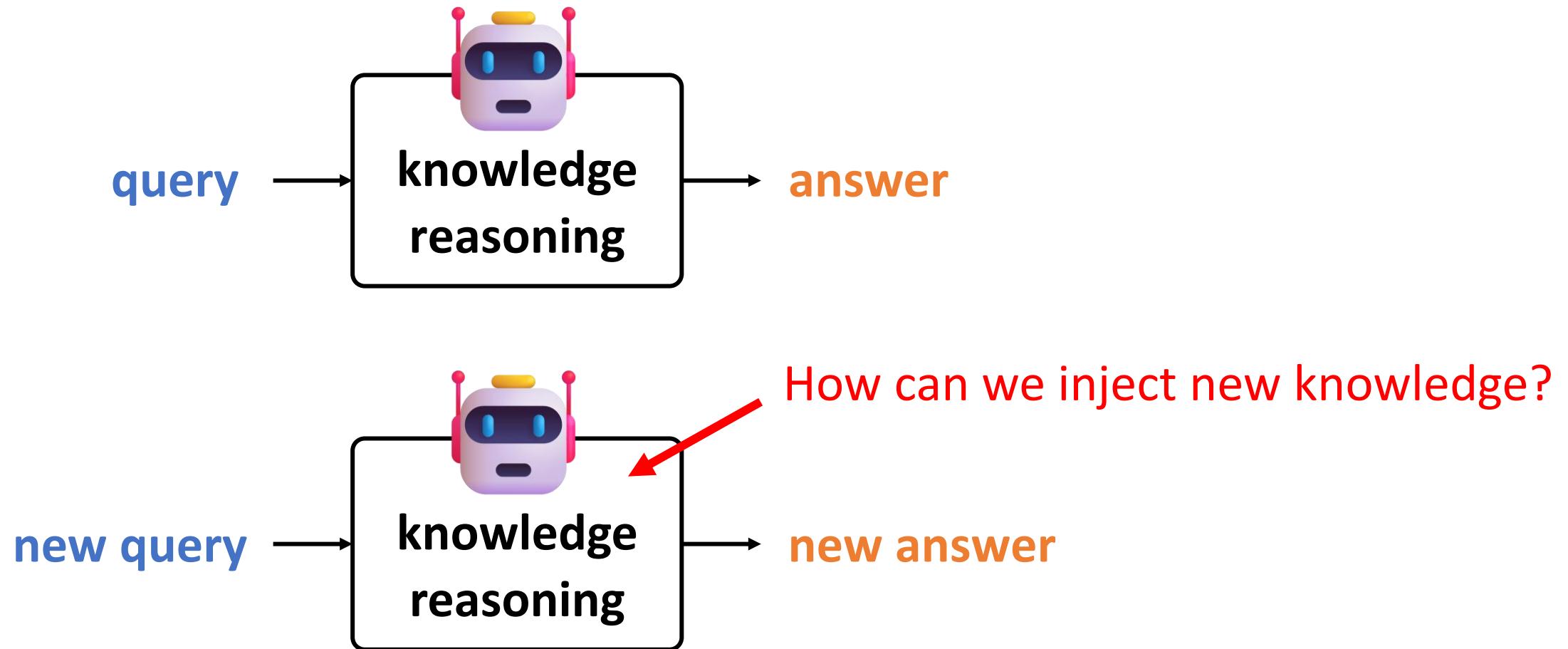
[1] Jason Wei, et al. Finetuned Language Models Are Zero-Shot Learners. ICLR 2022.

Conclusions: 5 / 10

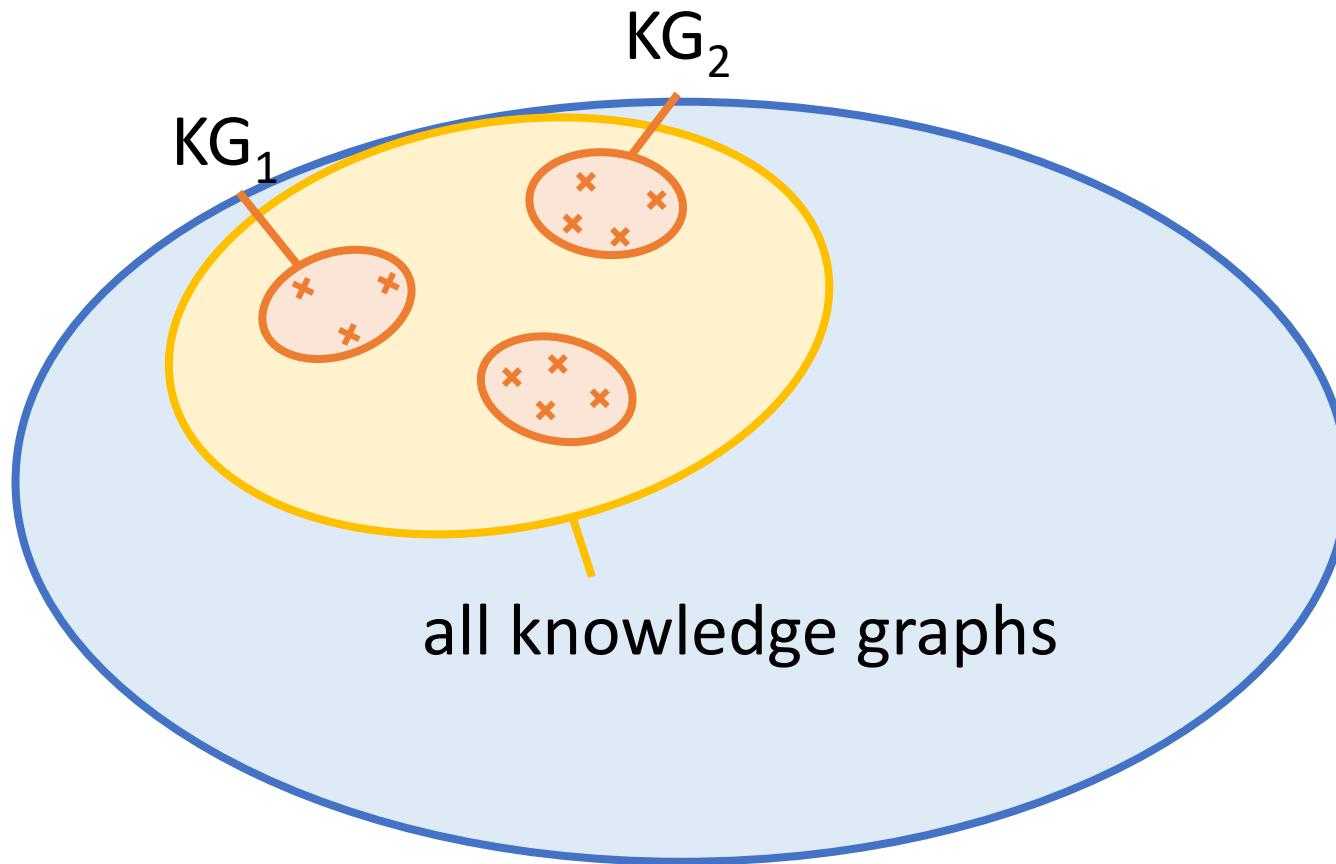
Dealing with Parametric Knowledge



Dealing with Parametric Knowledge

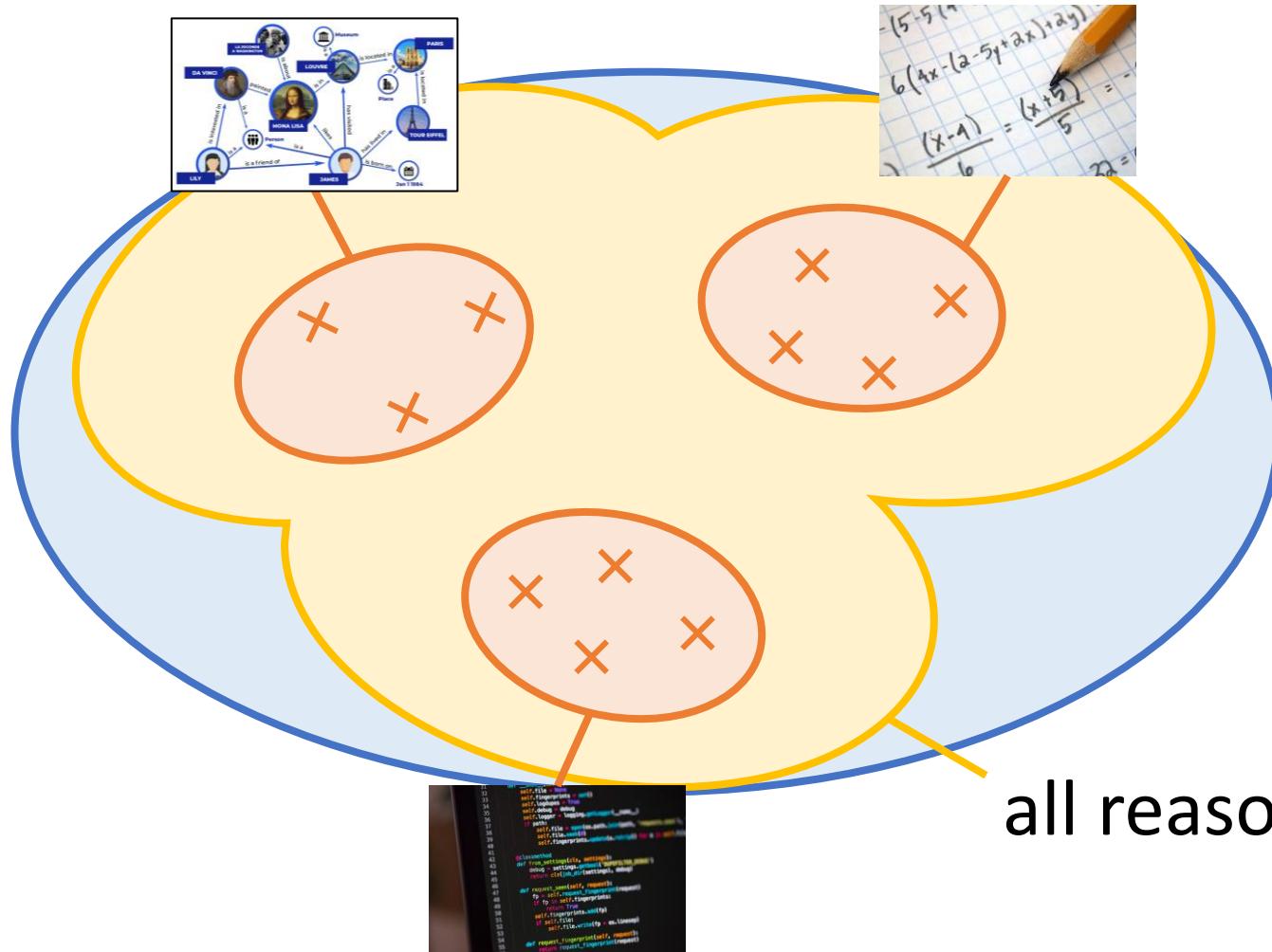


Expand the Scope of Generalization



unify knowledge graphs

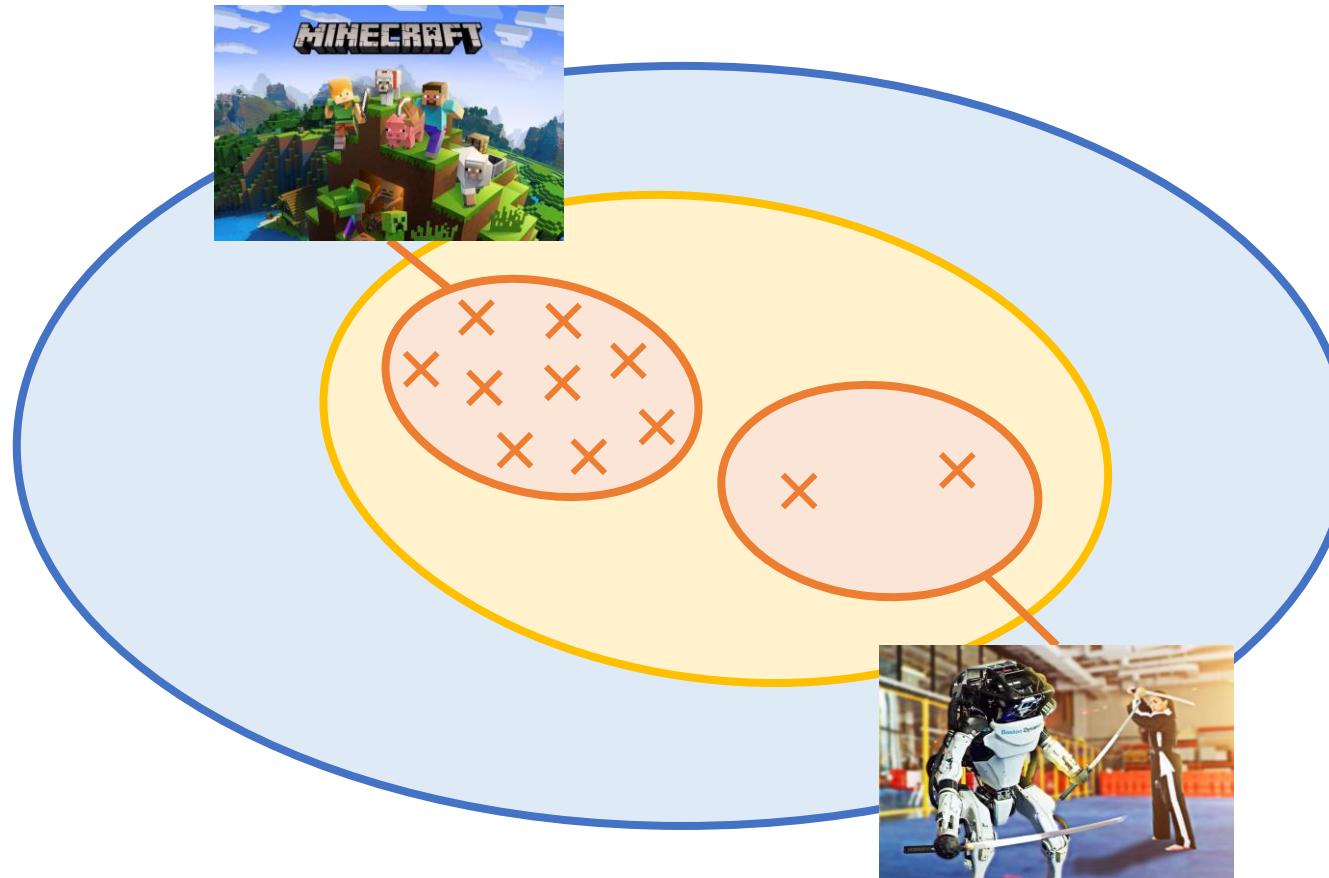
Expand the Scope of Generalization



unify reasoning tasks

all reasoning tasks

From Simulators to the Real World



save cost for data collection



Thank you! 