Homework 5, Due Friday, 29 November, 9:00 am

Instructions: Please write yout answers clearly on a seperate sheet of paper. You may turn in the assignment at the beginning of class, by email (gherczeg1@gmail.com) as a single pdf document or in my mailbox by 9:00 am. Staple all sheets together. Show all work and use complete sentences. Cite anything that you look up, and write down if you work together with others on the homework. **No late homework will be accepted.**

0. Reading Pre-View: Onno Pols, Chapter 8-11. It's a good overview that provides a not-to-deep description of stars, you should read it!

1. Tides (30 pts): Tides cause close binary systems to circularize their orbits and rotate synchronously (rotation period = orbital period). This has happened to Earths moon, and the Earth-moon system contunues to evolve. Assume that the orbits are circular and coplanar.

(a) Write down the total angular momentum of the earth moon system and argue that the orbital angualr momentum of the Earth and the spin angular momentum of the Moon are small compared to the other terms.

(b) Using conservation of angular momentum, show that the Earth is slowing down its rotation at a rate proportional to the increase in the orbital radius of the Moon.

(c) We can directly measure the changing distance to the Moon using retro-reflectors left on the Moon by the Apollo astronauts. The current rate of change of the distance between the Earth and Moon is 3.8cm per year. By how much is the length of Earths day changing.

2. The pre-main sequence (40 pts): In this question, we will describe energy sources for a star and then apply the results to low mass pre-main sequence stars. These stars are fully convective and can therefore be described by a polytrope of index $n = 1.5$. The source of energy if pre-main sequence stars is gravitational contraction.

(a) Show that for a star in hydrostatic equilibrium we can write

$$L = L_{\text{nuc}} + L_{\text{grav}} \tag{1}$$

where $L_{\text{grav}} = -0.5\Omega$ and Ω is the gravitational potential energy. This equation tells us that the energy per unit time leaving the star, given by L, is compensated by nuclear reactions, represented by L_{nuc}, and by gravitational energy release, represented by L_{grav}. These are the two main energy sources of the star.

(b) Suppose that a star in hydrostatic equilibrium has no nuclear sources. Show that its radius changes with time as

$$R(t) = \frac{R_0}{1 + t/\tau} \tag{2}$$

where

$$\tau = \frac{0.5\Omega(R_0)}{L} \tag{3}$$

and R_0 is the radius at $t = 0$. Assume that the luminosity L of the star is constant during this evolution.

(c) Assuming that the star contracts at constant L (which is not correct but is a reasonable approximation), calculate the central temperature as a function of time.

(d) Plot $R(t)$ and $T(t)$ for the case of a star of mass $M = M_\odot$, luminosity $L = 4L_\odot$, and initial radius $R_0 = 4R_\odot$. Approximate the star by a uniform sphere with solar compositions.

(e) Assuming that H-burning reactions start when $T_c \sim 10^7$K, how long does it take for the star to contract to the main sequence? Compare this with the thermal timescale. What is the radius when our hypothetical solar mass star reached to the main sequence?

3. Supernovae and Neutron Stars (20 pts):

(a) The energy in a supernova is abotu 10^{53} erg, almost entirely in neutrinos. Imagine a small asymmetry of 1% in the neutrino emission (e.g., slightly more in $+z$ and $-z$ disrections). Treating neutrinos as relativistic particles, estimate the kick velocity of the neutron star imparted by this asymmetry.

(b) Estimate the pulsar spin-down time for a typical pulsar. Treat the rotation as solid body rotation, and take period $P \sim 0.1$ s and spin-down rate of $\dot{P} \sim 3 \times 10^{-6} s\, yr^{-1}$. Assume at $1.4M_\odot$ neutron stars with a radius of 10 km. You can estimate the timescale as simply $t \sim E/|dE/dt|$.

(c) Neutrinos have tiny cross sections and slmost never interact with anything. We calculated in class that $\sim 10^{12}$ neutrinos pass through your body every second. That's a big number. **Estimate to an order of magnitude:** How close would you have to be to a supernova to get a lethal dose of neutrino radiation?

4. Half-life of Ni (20 pts): Consider a simple model for a light curve based on a radioactive decay:

$$^{56}\mathrm{Ni} \rightarrow {}^{56}\mathrm{Co} \rightarrow {}^{56}\mathrm{Fe} \tag{4}$$

The half-life of ^{56}Ni is $\tau_{1/2} = 6.1$ d with an energy release of $\mathrm{Q_{Ni}} = 2.136$ MeV. The half-life of ^{56}Co is 77.1 d with an energy release of $\mathrm{Q_{Co}} = 4.564$ MeV.

(a) Write down the evolution equations for the number density of each species, Ni and Co. Here you will need to relate the half-lives with the decay constant ($\lambda = \log_e(2)/\tau_{1/2}$).

(b) Solve for the number densities as a function of time. For the Co equation, you might find it useful to introduce an integration factor of $\mathrm{e}^{\lambda}_{\mathrm{Co}}\mathrm{t}$. Assume that initially we have pure ^{56}Ni.

(c) Now write down the luminosity as a function of time as

$$L(t) = Q_{\mathrm{Ni}} \frac{dN_{\mathrm{Ni}}}{dt} - Q_{\mathrm{Co}} \frac{dN_{\mathrm{Co}}}{dt} \tag{5}$$

where $N = nV$ is the total number of nuclei and V is the volume. Here we assume that the number density, n in uniform.

(d) Plot your lightcurve for an explosion that produces 1 $M_\odot$ of ^{56}Ni.