Homework 4, due Friday, 15 November at 9:00 am

Instructions: Please write your answers clearly on a seperate sheet of paper. You should turn in the assignment at the beginning of class. Staple all sheets together. Show all work and use complete setences. Cite anything that you look up, and write down if your work together with others on the homework. No late homework will be accepted.

0. Reading Pre-View: Onno Pols, Chapter 7-8

1. The Standard Solar Model and a Polytrope Model (36 pts): The stable SolarModel.txt (to be emailed out) gives the internal structure of the sun, called the Standard Solar Model. The columns give the mass enclosed in radius r, the radius r, the temperature, the density, the luminosity F(r), and the Hydrogen and helium abundance at each depth in the sun. In your plots, be sure to use logarithms so the reader can see what is being plotted, and label each axis including units.

(a) Plot the density and the temperature of the model as a function of radius.

(b) Calculate and plot the ion pressure, the electron pressure, the radiation pressure, and the ratio of the gas pressure to the total pressure, β as a function of radius.

(c) Calculate the force due to radiation $dP_{\rm rad}/dr$ as a function of r and compare it to the gravitational force. How important is the force due to radiation in the sun?

(d) How close to a polytrope is the Standard Solar Model?

(e) The standard model is an approximation in which radiative stars are modeled by a polytrope of index $n = 3$. A table that will be emailed to you has the numerical solution of the Lane-Emden equation, with $\beta = 0.9995853$. Calculate the pressure and density structure of this model.

(f) Compare the n=3 polytrope model to the standard solar model. Comment on how accurate the polytropic model is.

2. Convection (30 pts): Let's estimate the value of physical quantities in a convective region by using approximate conditions in an idealized convective layer in a star. The approximate conditions of this layer are: radius $r \sim R_\odot/2$; mass enclosed inside r, $m(r) \sim M_\odot/2$; temperature $T \sim 10^7$K; density $\rho \sim 1\mathrm{g}\cdot\mathrm{cm}^{-3}$; energy crossing r per unit time, $F(r) \sim L_\odot$; and mixing length, $l_c \sim r/10$. Calculate:

(a) The temperature excess of the adiabatic convective element over the temperature of the surroundings $\delta T/T$, assuming that all the flux is carried by convection, that is, H_c is the total flux. The specific heat (or heat capacity) at constant pressure, c_P, is equal to $c_P = 5/2\frac{k}{\mu m_H}$, where k is the Boltzmann constant, m_H the mass of the Hatom, and μ the mean molecular weight. Use solar abundances.

(b) The velocity of the convective element v_c. Compare with the sound speed $c_s = (kT/\mu m_H)^{1/2}$, which is the characteristic velocity of the particles in the layer.

(c) The ratio of the pressure due to the turbulent elements (ρv_c^2) to the local pressure. Does convection alter significantly the hydrostatic structure of the region?

(d) The crossing time in the convective region, $t \sim l_c/v_c$. This is approximately the time for convective elements to cross the region up or down. How many times have convective elements crossed the region in a therml timescale? in a nuclear timescale? These elements are moving matter up and down in the convective region during these typical stellar lifetimes; explain why this implies that stellar convective regions are very well mixed and have a uniform chemical composition.

3. Nuclear Reactions II (20 pts):

(a) Calculate the energy released in *each reaction* of the PPI cycle.

(b) Show that the total energy released in the PPI reaction is 26.73 MeV by adding over the energies in all the reactions of the cycle.

(c) Including the energy lost in neutrinos, which of the three chains, PPI, PPII, or PPIII releases more energy to the star? What is the energy released by unit mass in each case?

(d) Calculate the energy released and the energy generated per unit mass in the 3α reaction.

(e) For how long can a stellar core of $0.2M_{\odot}$ support a luminosity $L_{\odot}$ if (i) it is burning H by the PPI chain, and (ii) it is burning He by the 3α reaction?

4. Rotational Velocities (14 pts): We have assumed spherical symmetry in all our derivations so far. However, stellar rotation may induce flattening at the poles and bulging at the equator making this assumption invalid.

(a) Show that the condition for rotation not to cause a significant departure from spherical symetry is

$$\delta = \frac{\omega^2 R^3}{GM} \ll 1 \tag{1}$$

where ω is the stellar angular velocity.

(b) Table **RotationalVelocitiesMainSequence.txt** gives rotational velocities and other paremeters for main sequence stars of spectral types O to G. The rotational velocity is very small for later types. Evaluate δ and show that the largest departure for spherical symmetry occurs in early A stars.