Homework 2

Qinyue Fei 1901110218@pku.edu.cn

1 Redo homework

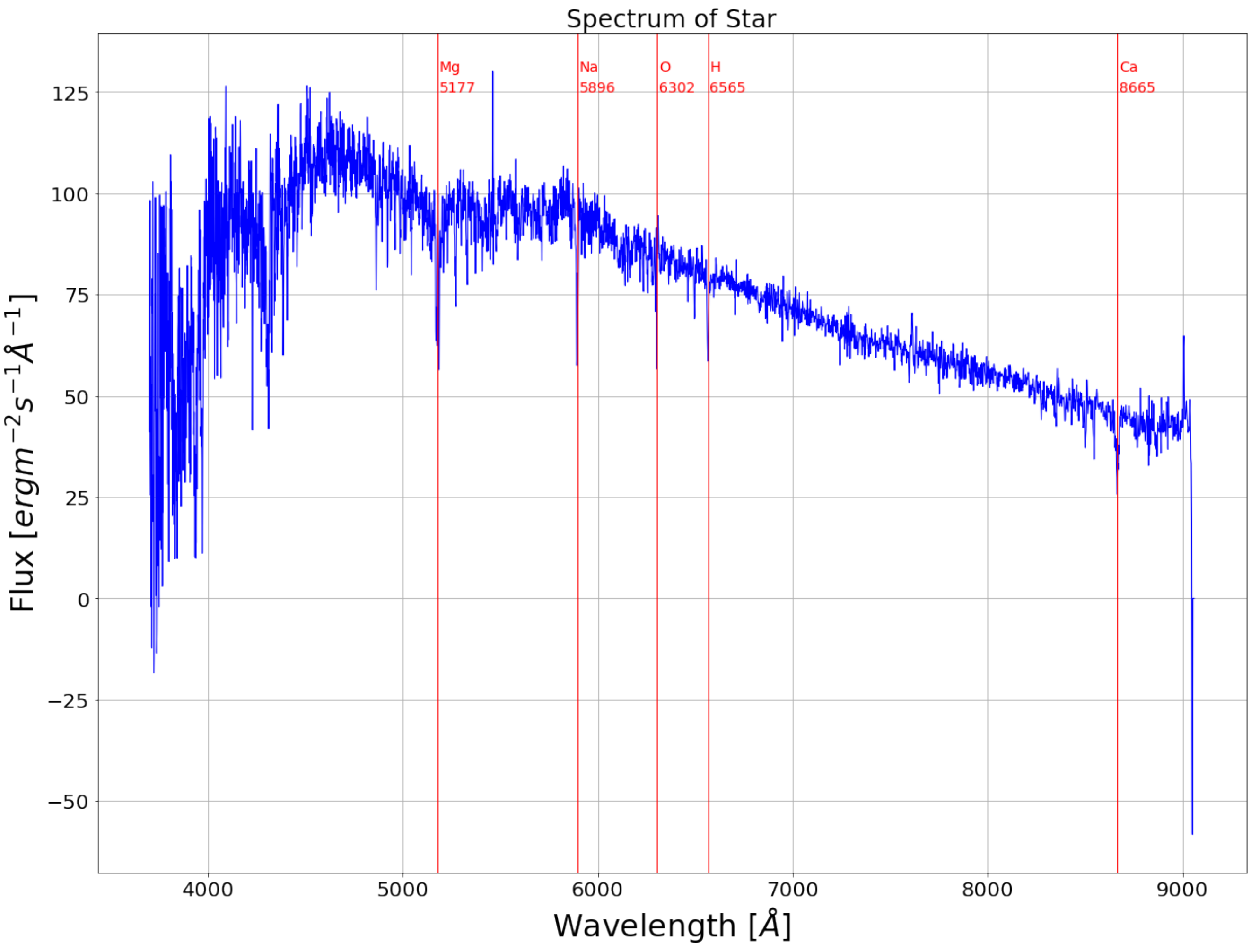

Figure 1: The spectrum of a star whose effective temperature is about 5000 kelvin. The red lines show the absorption lines and which element causes this absorption. The numbers under elements are their wavelength. The star is a typical G star.

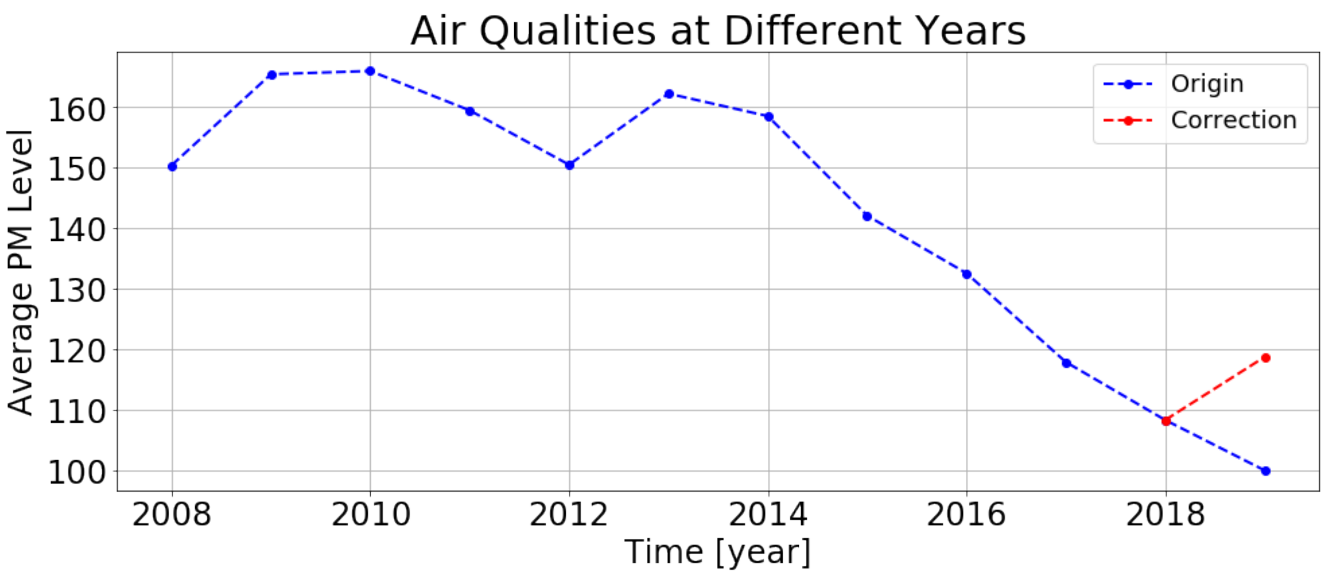

Figure 2: This figure shows the changes of PM2.5 level in different years. From this figure, we can see that PM2.5 level is getting less in 10 years, although there are some fluctuations in these years. Considering there is no data in 2019 September, November and December, I used average PM2.5 level to correct the data. The red dashed line shows the PM2.5 level in 2019 after correction.

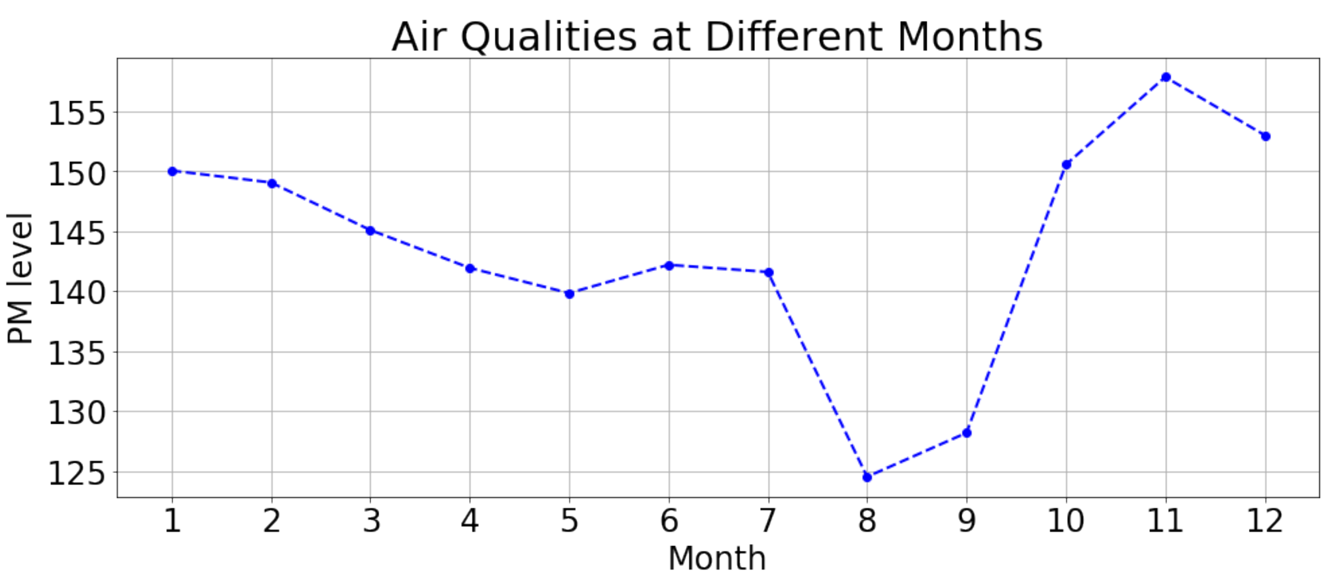

Figure 3: We can see that the air quality is better in spring and summer than that in autumn and winter. In November, average PM2.5 level is highest in one year and the air quality is the worst.

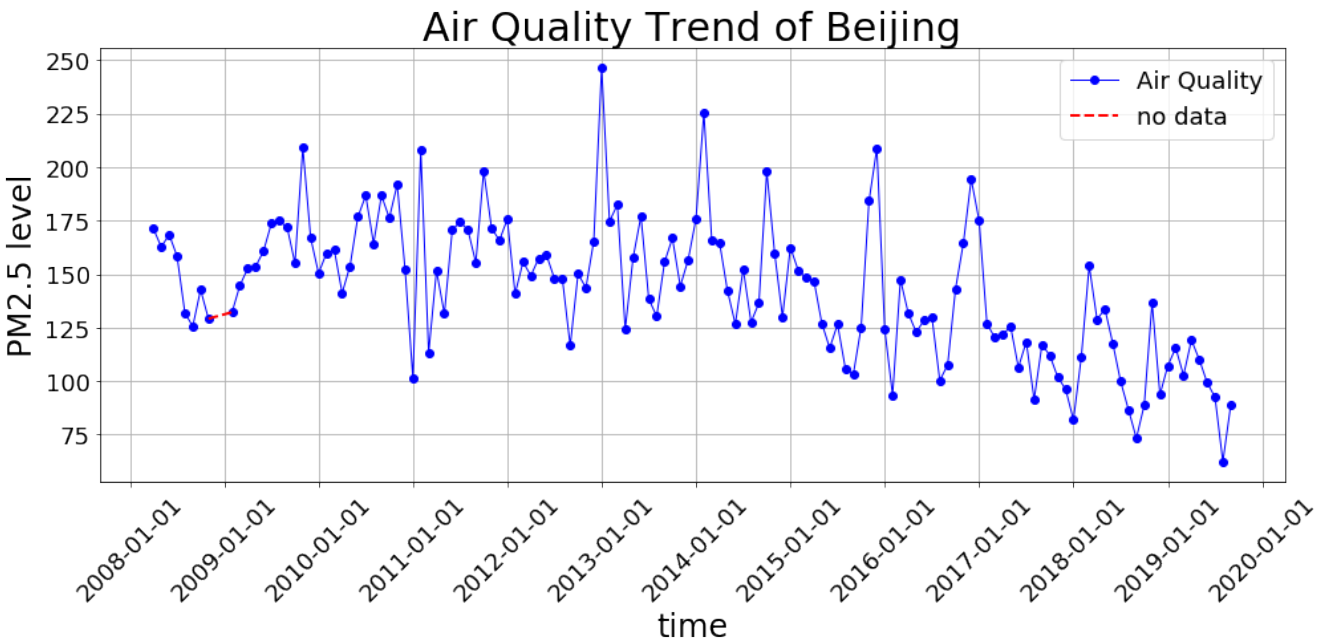

Figure 4: We can see that in 2013, the PM2.5 level is the highest. Since 2013, the air quality is getting better. The autumn and winter is coming so the air quality is getting worse. We should be aware of the dust.

2 Make your own HR diagram

2.1 a b: As shown in figure 5.

2.2 c:

The uncertainties change the absolute G magnitude and $B_P - R_P$ magnitude so that make their positions have more uncertainties which means the HR diagram more dispersive.

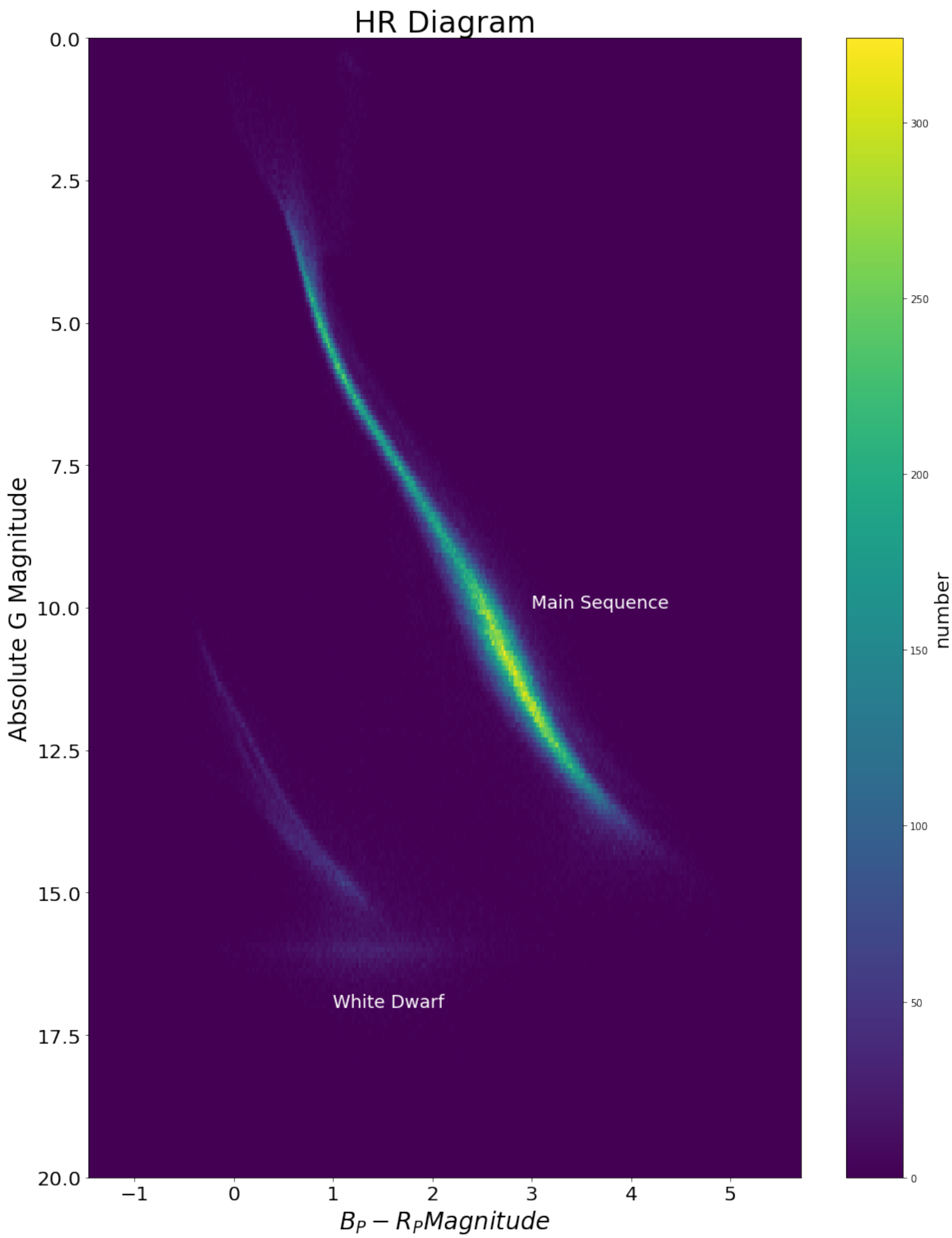

Figure 5: This figure shows the absolute G magnitude and $B_P - R_P$ magnitude of stars from Gaia within 100 pc. We can see most of stars are locating in the main sequence and some of stars are in white dwarf phase.

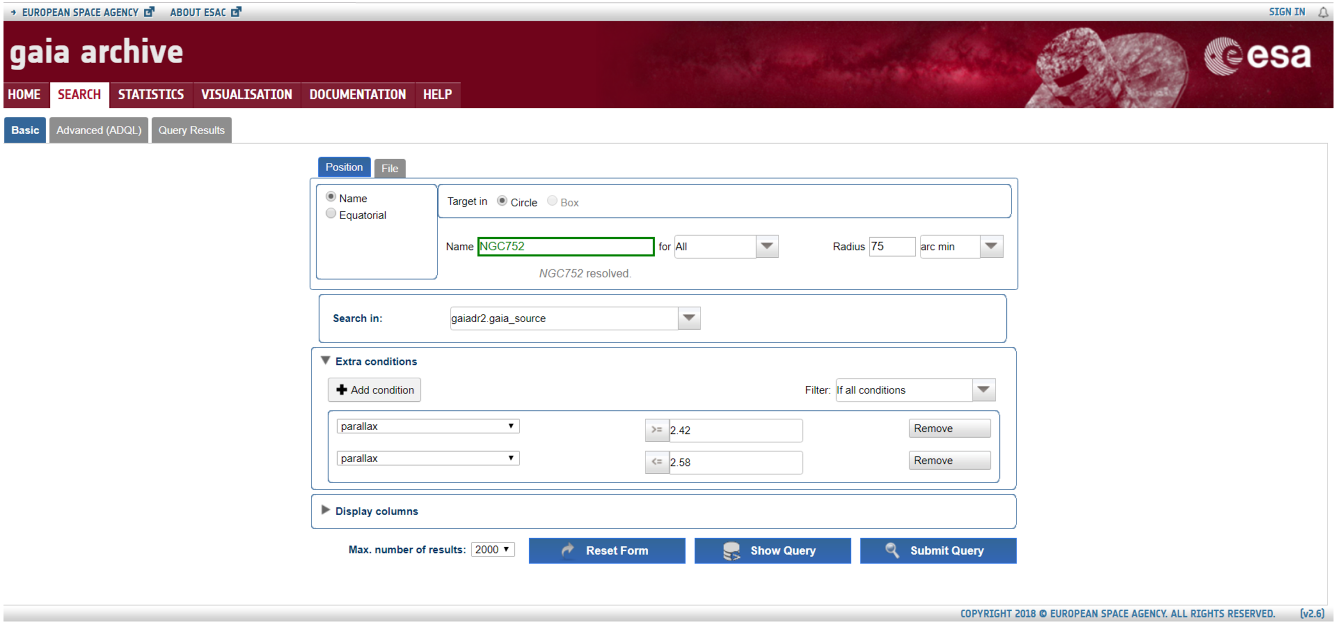

Figure 6: The figure shows how I got members of cluster. I filled Name, radius, two parallax conditions and submitted query. I believe the results are members in clusters.

3 HR diagram and Gaia

a:

I got membership lists of Plieades from Pleiades member list (Belikov+ 1998).
The membership lists of Hyades are from Hyades membership (Perryman+ 1998).
The membership lists of NGC 752 are from Improved membership catalog for NGC752(Agueros+, 2018)

b:

The numbers of members of Pleiades, Hyades and NGC752 are 817, 464 and 86. I searched these clusters name and found their position, distance and radius. Then I calculate the parallex region of these clusters. I think stars in this region are in cluster.Then I used these conditions to search members in gaia website, like figure 6 showing.

c:

Main-sequence stars generate energy by nuclear fusion, which is mainly from Hydrogen fusion. Brown dwarfs generate energy by blackbody radiation. White dwarfs generate energy by thermal radiation. Giant branch stars generate energy by nuclear fusion, which is mainly from Helium and mental fusion.

d:

Members of these clusters are different. Some clusters have meany dwarfs while other clusters have some giant stars. Their different ages cause these differences. We can see that white dwarfs in different clusters are in different region in HR diagram.

e:

Comparing to stars in clusters, field stars are more dispersive apparently because their origins are different, their ages are different while stars in clusters probably have similar orgin and ages.

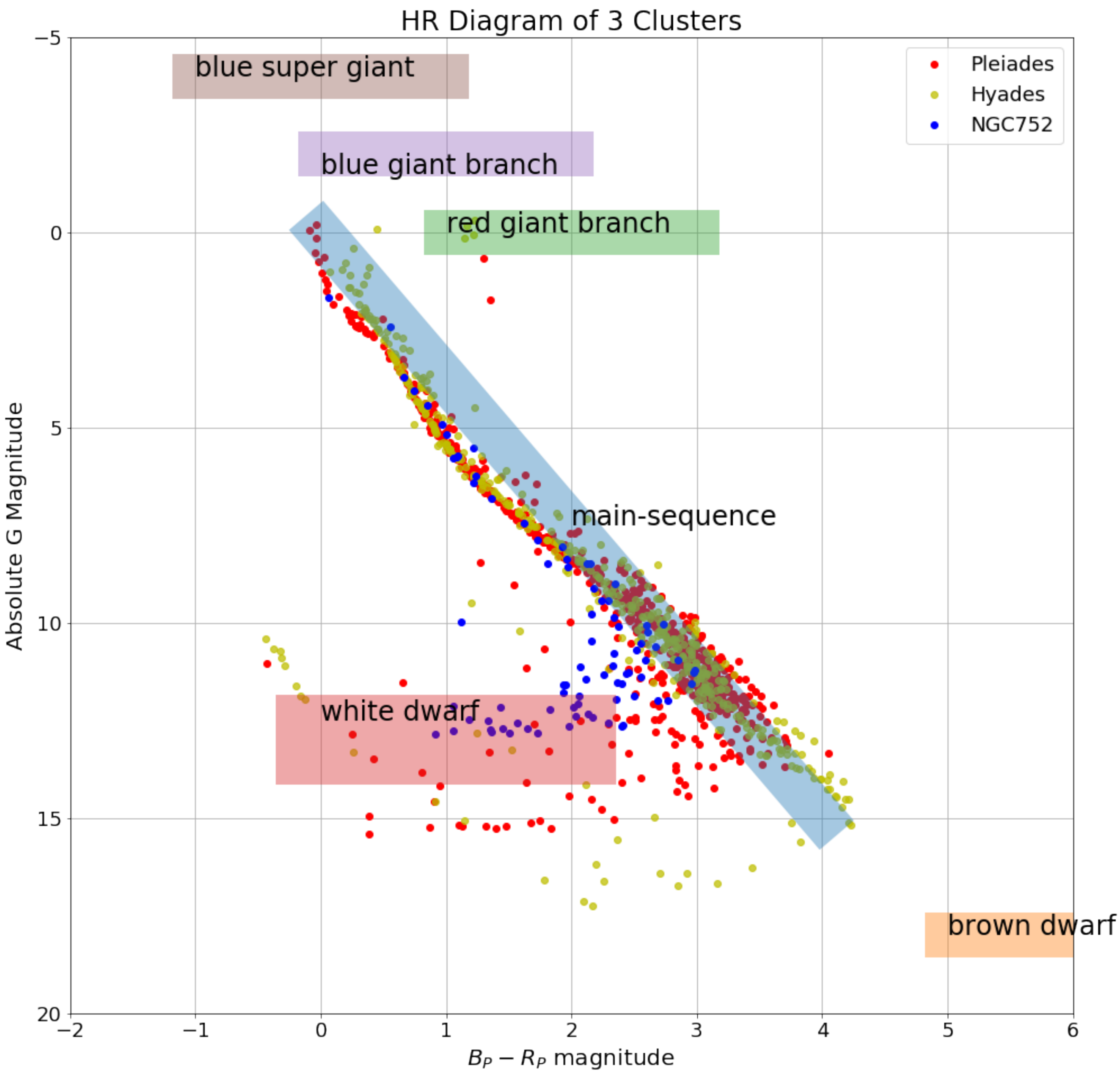

Figure 7: This figure shows the HR diagram of three open clusters. Red dots are stars in Pleiades. Green dots are stars in Hyades. Blue dots are stars in NGC752.

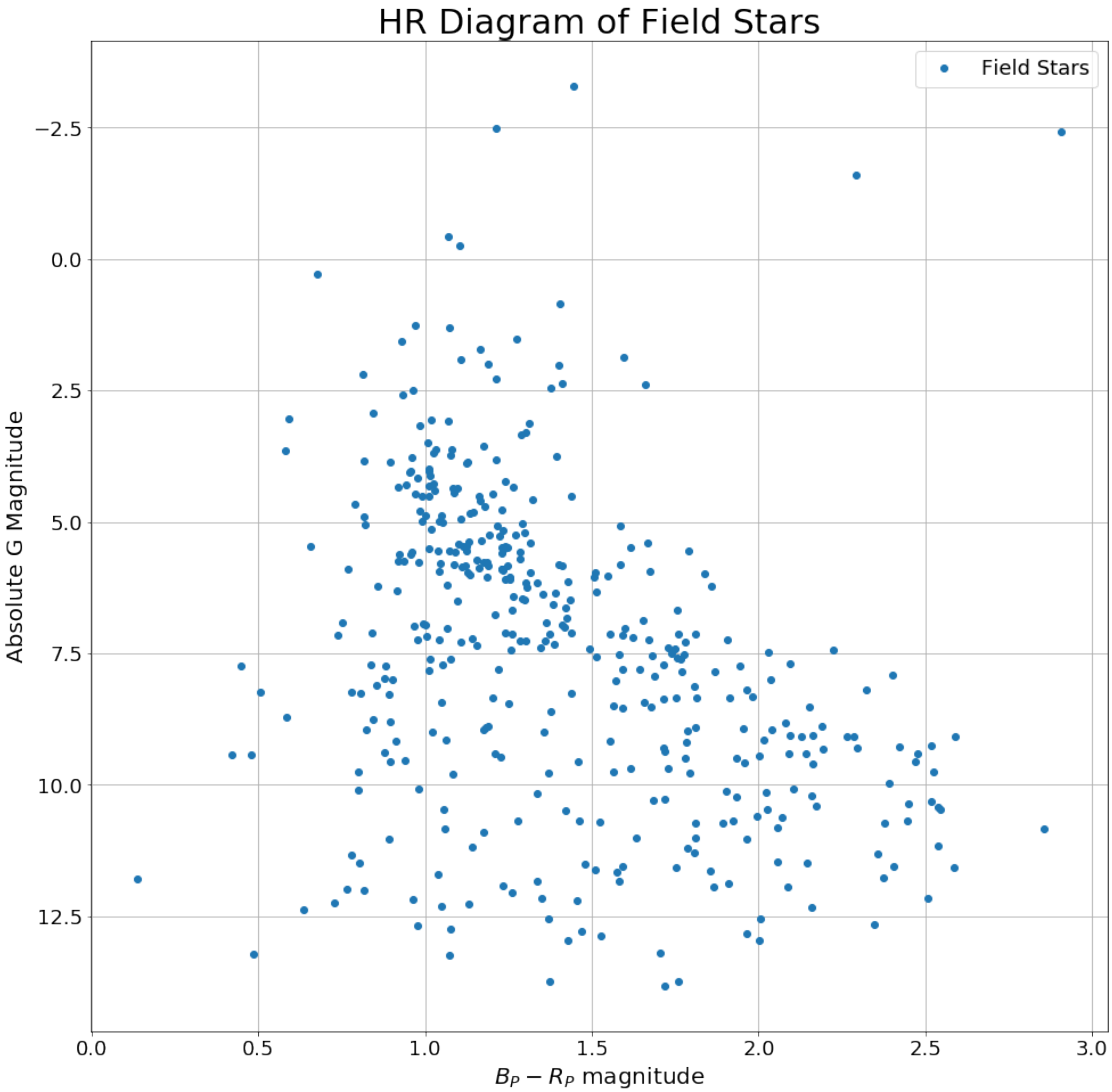

Figure 8: This figure shows the field stars. We can see that field stars are more dispersive than stars in clusters in HR diagram.

4 Introduction to Blackbodies

a.

$$B_\lambda(\lambda,T)=\frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1} \tag{1}$$

b.

Differentiating $B_\lambda(\lambda,T)$ with respect to λ and setting the derivative equal to zero:

$$\begin{aligned}\frac{\partial B}{\partial\lambda}&=\frac{-10hc^2}{\lambda^6}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}+\frac{2hc^2}{\lambda^5}\frac{e^{\frac{hc}{\lambda k_B T}}\frac{hc}{\lambda k_B T}}{(e^{\frac{hc}{\lambda k_B T}}-1)^2}\\&=\frac{2hc^2}{\lambda^6(e^{\frac{hc}{\lambda k_B T}}-1)}(\frac{hc}{\lambda k_B T}\frac{e^{\frac{hc}{\lambda k_B T}}}{e^{\frac{hc}{\lambda k_B T}}-1}-5)\\&=0\end{aligned}$$

Then we can get the equation:

$$\frac{hc}{\lambda k_B T}\frac{e^{\frac{hc}{\lambda k_B T}}}{e^{\frac{hc}{\lambda k_B T}}-1}-5=0$$

Let's set $x=\frac{hc}{\lambda k_B T}$ then the equation changes to:

$$(x-5)e^x+5=0$$

The solution of above equation is :$x=W_0(-5e^{-5})+5$ where $W_0(x)$ is Lambert W function. At this time $x=4.965114231744276$ So we can get the equation :

$$\frac{hc}{\lambda k_B T}=4.965114231744276$$

The constant in equation: $h=6.626\times10^{-27}erg\cdot s$,$c=3\times10^{10}cm/s$,$k_B=1.38\times10^{-16}erg/k$ So we can get Wien's law:

$$\lambda_{max}T=0.29cm/k \tag{2}$$

c.

The intensity of blackbody radiation depends on both wavelength (λ) and temperature (T). Wien's law describe the truth that blackbody curve will peak at the wavelength which is only related to the temperature. Which means if the temperature of an object is high, the intensity of blackbody radiation at short wavelength is larger than that at long wavelength.

d.

The luminosity of the sun is $L_\odot=3.828\times10^{26}W$, the distance to the sun is $d=1.496\times10^8km$ So the flux Earth received is: $F_{re}=\frac{L_\odot}{4\pi d^2}$

Considering the Stefan-Boltzmann law: $j=\sigma T^4$ where $\sigma=5.67\times10^{-8}Wm^{-2}K^{-4}$ is Setfan-Boltzmann constant. The radiation Earth emissivied is $L=A\times j=4\pi R_e^2\sigma T^4$ where R_e is the radius of Earth. Assuming that the area of received: $S=\pi R_e^2$

Assume the absorption coefficient is 0.6 and we get the equation:

$$4\pi R_e^2\sigma T_e^4=0.6\times\frac{L_\odot}{4\pi d^2}\cdot\pi R_e^2$$

Solve the equation we can get: $T_e=245K$

5 Introduction to Stellar Structure and Hydrostatic Equilibrium

a.
Considering the mass function:

$$\begin{aligned} dm &= 4\pi r^2 \rho dr \\ &= 4\pi\rho_c(r^2 - \frac{r^4}{R_*^2})dr \\ \int_0^{M_*} dm &= 4\pi\rho_c \int_0^{R_*}(r^2 - \frac{r^4}{R_*^2})dr \\ \rho_c &= \frac{15M_*}{8\pi R_*^3} \end{aligned}$$

b.
From a we know:

$$m = 4\pi\rho_c(\frac{r^3}{3} - \frac{r^5}{5R_*^2})$$

Then the hydrostatic equilibrium equation:

$$\begin{aligned} \frac{dP}{dr} &= -\frac{Gm}{r^2}\rho \\ &= -4\pi\rho_c^2 GR_* \frac{r}{R_*}[1-(\frac{r}{R_*})^2][\frac{1}{3} - \frac{1}{5}(\frac{r}{R_*})^2] \\ \int_{P(r)}^{P(R_*)} dP &= \int_r^{R_*} -4\pi\rho_c^2 GR_*^2 x(1-x^2)(\frac{1}{3} - \frac{1}{5}x^2)dx \end{aligned}$$

where $x = \frac{r}{R_*}$ and $P(R_*) = 0$, then we have the equation:

$$P(r) = \frac{15GM_*^2}{32\pi R_*^4}[2 - 5(\frac{r}{R_*})^2 + 4(\frac{r}{R_*})^4 - (\frac{r}{R_*})^6]$$

Then the P_c is:

$$P_c = P(r=0) = \frac{15GM_*^2}{16\pi R_*^4}$$

So the relation between $P(r)$ and P_c is:

$$P(r) = \frac{P_c}{2}[2 - 5(\frac{r}{R_*})^2 + 4(\frac{r}{R_*})^4 - (\frac{r}{R_*})^6]$$

$$P_c = 3.968 \times 10^{14}(\frac{M_*}{M_\odot})^2(\frac{R_\odot}{R_*})^4 pa$$

c.
The equation of state of ideal gas:

$$p = nk_BT = \frac{\rho}{\mu m_H}k_BT$$

where k_B is Boltzmann constant, p is pressure, n is number density of particles. Therefore, the temperature of central place is:

$$T = \frac{\mu m_H P}{\rho_c k_B} = \mu \times 1.20 \times 10^7 \frac{M_*}{M_\odot}(\frac{R_*}{R_\odot})^{-1}K$$

for ionized H, $\mu = 0.5$

$$T = 5.8 \times 10^6 \frac{M_*}{M_\odot}(\frac{R_*}{R_\odot})^{-1}K$$

for ionized He, $\mu = 4/3$

$$T = 1.6 \times 10^7 \frac{M_*}{M_\odot}(\frac{R_*}{R_\odot})^{-1}K$$

d. The definition of gravitational energy is:

$$\begin{aligned}\Omega &= \int_0^{R_*} -\frac{Gm}{r}dm \\ &= -\int_0^{R_*} 16\pi^2 R_x^5 G\rho_c^2 (\frac{r}{R_*})^4(\frac{1}{3} - \frac{r^2}{R_*^2})(1 - \frac{r^2}{R_*^2})dr \\ &= -\frac{5}{7}\frac{GM_*^2}{R_*^2}\end{aligned}$$

And the total interval energy:

$$\begin{aligned}U &= -3\int_0^{R_*} P(r)4\pi r^2 dr \\ &= -\frac{45GM_*^2}{8R_*^4}\int_0^{R_*} r^2(2 - 5(\frac{r}{R_*}^2) + 4(\frac{r}{R_*})^4 - (\frac{r}{R_*})^6) \\ &= -\frac{5}{7}\frac{GM_*^2}{R_*^2}\end{aligned}$$

Then we can find $\Omega = U$, which virial theorem describes. And the expression for the potential energy is $\Omega = -\frac{5}{7}\frac{GM_*^2}{R_*}$

6 Introductory definitions

(a) Ideal gas is a theratical gas consisting of many randomly moving particles which do not cover any space, whose interaction is a complete elastic collision. The equation of state of ideal gas is:

$$PV = NkT$$

where P is pressure, V is volume, N is number of particles, k is Boltzmann constant, T is temperature.
(b) Virial theoream describes the relation between gravitational potential energy and kinematic energy of a hydrostatic equalibrium matter. The equation of virial theorem is

$$\Omega + 3U = 0$$

where Ω is gravitational potential energy and U is internal energy which can be used to describe kinematic energy of particles.
(c) Blackbody is a kind of matter which only absorbs radiation without reflection but has its own radiation which is called blackbody radiation. The equation of blackbody radiation is

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

where λ is the wavelength of radiation, T is temperature of blackbody.
(d) Convection causes the transport of matter, energy, momentum and angular momentum. In this process, heat will be transport by convection, which is called energy transport by convection.
(e) If the luminosity of a stellar is totally from its gravitational energy, the lifetime of this stellar is called Kelvin-Helmholtz timescale.
(f) H-R diagram shows the color/spectrum type/temperature and luminosity/absolute magnitude of stars, which is always used to research the formation and evolution of stars.
(g) Jeans mass is the limits of a cloud which can get rid of collapsing to a star. If the mass of gas cloud beyond Jeans mass, the cloud will collapse and become a star. The equation of Jeans mass is :

$$M_J \simeq (2M_\odot)(\frac{c_s}{0.2kms^{-1}})^3(\frac{n}{10^3cm^{-3}})^{-1/2} \tag{3}$$

where $M_\odot$ is mass of sun, c_s is speed of sound in the cloud, n is number density of gas particles.
(h) If we assume star is a blackbody, the spectrum of star can be describe as a blackbody radiation. The temperature of blakcbody is stellar effective temperature.
(i) Hydrostatic equilibrium describes the state of fluid. At this state, fluids keep relative rest. The euqation of hydrostatic equalibrium is:

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2}$$

where P is pressure at radius r, ρ is density, m is mass whose radius under r, r is radius.

7 Code

7.1 Redo homework

```
from astropy.io import fits
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import time
#Load the spectrum data from dr5, including flux, wavelength, error, etc.
spec = fits.open('spectrum/spec-55859-F5902_sp02-173.fits.gz')
flux = spec[0].data[0]
wavelength = spec[0].data[2]
err = spec[0].data[1]
#plot the spectrum
plt.figure(figsize=(20,15))
plt.plot(wavelength, flux,c = 'b',linewidth=1)

pos = np.where((wavelength>4999) & (wavelength<5001))[0][-1]
print(flux[pos]>20*err[pos])#discuss if the signal to noise > 20 at wavelength 5000A.

plt.xlabel('Wavelength [$\AA$]',size=30)
plt.ylabel('Flux [$ergm^{-2}s^{-1}\AA^{-1}$]',size=30)
plt.grid()
#Plot five feature lines over the spectrum.
a_lines = {'Na':5895.6, 'Mg':5176.7, 'O':6302.05, 'H':6564.61, 'Ca':8664.52}
for k in a_lines.keys():
    plt.axvline(a_lines[k],c = 'r',label = 'Ansorption Lines',linewidth=1)
    plt.text(a_lines[k]+10, flux.max(),k,size=14,c = 'r')
    plt.text(a_lines[k]+10, flux.max()-5, '%.0f'%a_lines[k], size=14, c = 'r')

e_lines = {}
plt.title('Spectrum of Star', size=24)
plt.xticks(fontsize=20)
plt.yticks(fontsize=20)
plt.show()
aqi = pd.read_csv('aqi.txt', sep = '\t') #Load data from file
t = np.array(aqi['Time']) # time
Id = np.array(aqi['Reading'])
PM = np.array(aqi['PM2.5 level']) #air quality
err = np.where(PM == '(no data)') # error data
t = np.delete(t, err) #delete those error datas
PM = np.delete(PM, err) #delete those error datas
pm = np.array(PM,dtype='float')
year = np.array([2008,2009,2010,2011,2012,2013,2014,2015,2016,2017,2018,2019,2020], dtype='str')
month = np.array(['-01-01', '-02-01', '-03-01','-04-01', '-05-01', '-06-01', '-07-01', '-08-01',
    '-09-01', '-10-01', '-11-01', '-12-01','-13-01'])
hour = np.array([' 00:00',' 01:00',' 02:00',' 03:00',' 04:00',' 05:00',' 06:00',' 07:00',' 08:00','
    09:00',' 10:00',' 11:00',' 12:00',
              ' 13:00',' 14:00',' 15:00',' 16:00',' 17:00',' 18:00',' 19:00',' 20:00',' 21:00','
                  22:00',' 23:00',' 24:00'])
pm_y = []
for i in range(len(year)-1):
    n = np.where((t>year[i])&(t<year[i+1]))
    pm_y.append(np.sum(pm[n])/len(pm[n]))
pm_year = np.array(pm_y)
```

```
Y = np.array([2008,2009,2010,2011,2012,2013,2014,2015,2016,2017,2018,2019])

pm_year_2019 = (np.sum(pm_m[-3:])*30+pm_year[-1]*295)/365
pm_y[-1]=pm_year_2019

plt.figure(figsize=(16,6))
plt.plot(Y, pm_year,'o--',c = 'b',linewidth=2,label = 'Origin')
plt.plot(Y[-2:],pm_y[-2:],'o--',c='r',linewidth=2,label='Correction')
plt.xlabel('Time [year]',size=24)
plt.ylabel('Average PM Level',size=24)
plt.xticks(fontsize=24)
plt.yticks(fontsize=24)
plt.legend(loc='upper right',fontsize=18)
plt.grid()
plt.title('Air Qualities at Different Years',size = 30)
month = np.array(['-01-01', '-02-01', '-03-01','-04-01', '-05-01', '-06-01', '-07-01', '-08-01',
    '-09-01', '-10-01', '-11-01', '-12-01','-13-01'])
pm_m = []
for i in range(len(month)-1):
    M = []
    for y in year:
        n = np.where((t<y+month[i+1])&(t>y+month[i]))
        M.append(np.sum(pm[n])/len(pm[n]))
    Mon = np.array(M, dtype='str')
    Mon = np.delete(Mon, np.where(Mon == 'nan'))
    Mon = np.array(Mon, dtype='float')
    pm_m.append(np.sum(Mon)/len(Mon))
mon = np.array([1,2,3,4,5,6,7,8,9,10,11,12])
fig = plt.figure(figsize=(16,6))
ax = fig.add_subplot(1,1,1)
ax.set_xticks(mon)
plt.plot(mon, pm_m,'o--',c='b',linewidth=2)
plt.xlabel('Month',size=24)
plt.ylabel('PM level',size=24)
plt.xticks(fontsize=24)
plt.yticks(fontsize=24)
plt.grid()
plt.title('Air Qualities at Different Months',size=30)
time = []
month = np.array(['-01-01', '-02-01', '-03-01','-04-01', '-05-01', '-06-01', '-07-01', '-08-01',
    '-09-01', '-10-01', '-11-01', '-12-01'])
for y in year:
    for m in month:
        time.append(y+m)
pm_mon = []
for i in range(len(time)-1):
    n = np.where((t<time[i+1])&(t>time[i]))
    p = np.array(pm[n],dtype='str')
    p = np.delete(p,np.where(p == 'nan'))
    p = np.array(p, dtype='float')
    pm_mon.append(np.sum(p)/len(p))

Time = np.array([time[10],time[13]])
d = np.array([pm_mon[10],pm_mon[13]])
Y = []
for y in year:
```

```
    Y.append(y+month[0])
Y = np.array(Y)
fig = plt.figure(figsize=(16,6))
ax = fig.add_subplot(1,1,1)
plt.plot(time[0:-1], pm_mon,'o-',c='b',linewidth=1,label='Air Quality')
plt.plot(Time,d,'--',c='r',label='no data',linewidth=2)
plt.xlabel('time',size=24)
plt.ylabel('PM2.5 level',size=24)
plt.grid()
ax.set_xticks(Y)
plt.xticks(fontsize=18,rotation=45)
plt.yticks(fontsize=18)
plt.legend(loc='upper right',fontsize=18)
plt.title('Air Quality Trend of Beijing',size=30)
```

7.2 Make your own HR diagram

```
import numpy as np
import matplotlib.pyplot as plt
from astropy.io import fits
import seaborn as sb
from numpy import *
import pandas as pd
import matplotlib.colors as colors
from matplotlib.patches import Patch
import matplotlib.patches as patches
gaia = fits.open('gaia_all100pc-result.fits')
data = gaia[1].data
df = pd.DataFrame(data)
df = df[df['astrometric_excess_noise'] < 1]
df['G_abs'] = df['phot_g_mean_mag']-10+5*np.log10(df['parallax'])
df = df[df['phot_rp_mean_flux_over_error'] > 1]
df = df[df['phot_bp_mean_flux_over_error'] > 1]
df = df[df['G_abs'] != 'NaN']
data = np.array(df)
plt.figure(figsize=(16,21))
fig = plt.hist2d(df['bp_rp'],df['G_abs'],bins=300)#,norm=colors.LogNorm())
cb = plt.colorbar()
#cb.set_title('number')
cb.set_label('number',size=20)
#cb.get_ticks(size=18)
#plt.gca().invert_yaxis()
plt.xlabel('$B_P-R_P Magnitude$',fontsize=24)
plt.ylabel('Absolute G Magnitude', fontsize=24)
plt.xticks(fontsize=20)
plt.yticks(fontsize=20)
plt.text(3,10,'Main Sequence',c = 'white',fontsize=18)
plt.text(1,17,'White Dwarf',c = 'white',fontsize=18)
plt.ylim(20,0)
plt.title('HR Diagram',fontsize=30)
```

7.3 HR diagram and Gaia

```
import numpy as np
import matplotlib.pyplot as plt
from astropy.io import fits
import matplotlib.colors as colors
Pleiades_membership = fits.open('Plieades_membership.fit')
Hyades_membership = fits.open('Hyades_membership.fit')
NGC752_member = fits.open('NGC752_membership.fit')
Pleiades_gaia = pd.read_csv('Pleiades_gaia.csv')
Hyades_gaia = pd.read_csv('Hyades_gaia.csv')
NGC752_gaia = pd.read_csv('NGC752_gaia.csv')
plt.figure(figsize=(16,16))
plt.plot(Pleiades_gaia['bp_rp'],Pleiades_gaia['G_abs'],'ro',label='Pleiades')
plt.plot(Hyades_gaia['bp_rp'],Hyades_gaia['G_abs'],'yo',alpha=0.75,label='Hyades')
plt.plot(NGC752_gaia['bp_rp'],NGC752_gaia['G_abs'],'bo',label='NGC752')
plt.legend(loc='upper right',fontsize=18)
plt.xlabel('$B_P-R_P$ magnitude',fontsize=20)
plt.ylabel('Absolute G Magnitude',fontsize=20)
plt.title('HR Diagram of 3 Clusters',fontsize=24)
plt.gca().invert_yaxis()
plt.xticks(fontsize=18)
plt.yticks(fontsize=18)
plt.text(2,7.5,'main-sequence',fontsize=24)
plt.text(5,18,'brown dwarf',fontsize=24)
plt.text(1,0,'red giant branch',fontsize=24)
plt.text(0,-1.5,'blue giant branch',fontsize=24)
plt.text(-1,-4,'blue super giant',fontsize=24)
plt.text(0,12.5,'white dwarf',fontsize=24)
plt.plot([0,4],[0,15],linewidth=40,alpha=0.4)
plt.plot([5,6],[18,18],linewidth=40,alpha=0.4)
plt.plot([1,3],[0,0],linewidth=40,alpha=0.4)
plt.plot([0,2],[13,13],linewidth=80,alpha=0.4)
plt.plot([0,2],[-2,-2],linewidth=40,alpha=0.4)
plt.plot([-1,1],[-4,-4],linewidth=40,alpha=0.4)
plt.ylim(20,-5)
plt.xlim(-2,6)
plt.grid()
field = pd.read_csv('field.csv')
field['Distance'] = 1000/field['parallax']
#NGC752_gaia = NGC752_gaia[abs(NGC752_gaia['Distance']-390)<20]
#NGC752_gaia = NGC752_gaia[abs(NGC752_gaia['radial_velocity']-5)<100]
field['G_abs'] = field['phot_g_mean_mag']-10+5*np.log10(field['parallax'])
plt.figure(figsize=(16,16))
plt.plot(field['bp_rp'],field['G_abs'],'o',label='Field Stars')
plt.legend(loc='upper right',fontsize=18)
plt.xlabel('$B_P-R_P$ magnitude',fontsize=20)
plt.ylabel('Absolute G Magnitude',fontsize=20)
plt.title('HR Diagram of Field Stars',fontsize=30)
plt.gca().invert_yaxis()
plt.xticks(fontsize=18)
plt.yticks(fontsize=18)
plt.grid()
```