Homework 3, due Friday, 8 November at 9:00 am

0. Reading: Onno Pols. Chapter 5-6, Energy Generation

1. Introduction to the Saha Equation (35 pts): The Saha Equation lets you calculate the ratio of particles in two consecutive ionization states, l and $l+1$, if the density is high enough that local thermodynamic equilibrium (LTE) is a valid assumption. Include in your homework the code you need to calculate and plot your results.

$$\frac{N_{l+1,E}}{Nl,E} = 2\frac{U_{l+1,E}}{U_{l,E}}\frac{1}{n_e}\frac{(2\pi m_e)^{3/2}(kT)^{3/2}}{h^3}e^{\chi_{l,E}/kT} \tag{1}$$

where: $\chi_{l,E}$ is the ionization potential state l, i.e., the energy needed to remove one electron from ion l. n_e is the electron density, m_e is the mass of the electron, h is the Planck constant.

(a) Plot H ionization fraction versus temperature.

(b) Plot the neutral and the fraction of all ionization states of C versus temperature.

(c) Is your x-axis in linear or log? Why? Is your y-axis in linear or log? Why?

(d) If you see C I (neutral carbon) in absorption in a stellar spectrum, what does that imply about the stellar temperature? The solar coronae can be seen in C VI (C^{5+}) emission, at what temperature does that occur?

2. Equation of State (20 pts): Consider a gas of electrons and ions (from ionized Hydrogen) and radiation.

(a) Using python, plot contours in the $\log T - \log \rho$ plane corresponding to the transitions

i: between electron degeneracy and ideal gas pressure

ii: between radiation and ideal gas pressure

iii: between relativistic vs. non-relativistic degenerate electron pressure

iv: to crystallization of the ions

Do not just quote the results corresponding to each of the irems above, but derive them from the more basic ideas in class and the book.

(b) Indicate on the plot the locations of the central conditions of a low mamss white dwarf, a high mass white dwarf (close to the Chandra mass), and the sun.

3. The Chandrasekhar Mass (25 pts): Using the relationship between mass and radius for polytropes, show that for stars in which the free electrons are completely degenerate in the extreme relativistic limit there is only one value of the mass that satisfies hydrostatic equilibrium.

(a) Show that the mass, called the Chandrasekhar mass limit, is

$$M_{Ch} = \frac{5.85}{\mu_e^2}M_\odot \tag{2}$$

where μ_e is the mean molecular weight per electron. Evaluate M_{Ch} for gases with

(b) only He, and

(c) only C and O, with C three times more abundant than O.

4. Energy generation in the sun (25 pts): Later in the semester we will learn that the sun generates its energy via the proton-proton chain:

$$4H \rightarrow {}^{4}_{2}He + 2e^{+} + 2\nu_{e} + 2\gamma \tag{3}$$

(a) How many reactions per second take place in the Sun?

(b) What is the mass of He produced in the Sun every second?

(c) What is the energy of photons produced in the Sun every second?

(d) What is the neutrino flux (number of neutrinos per $\mathrm{cm}^{-2}\mathrm{s}^{-1}$) produced by the Sun that reach the Earth?

5. Opacities (20 pts): This is Problem 5.2 in Onno Pols book.

(a) Identify the various processes contributing to the opacity as shown in Fig. 5.2, and the T and ρ ranges where they are important.

(b) Compare the opacity curve for $\log\rho = -6$ in the left panel of Fig. 5.2 to the approximations given in Sec. 5.3.1 for (1) electron scattering, (2) free-free absorption, (3) bound-free absorption and (4) the H ion. How well do these approximations fit the realistic opacity curve?

(c) Calculate (up to an order of magnitude) the photon mean free path in a star of 1 $M_{\odot}$ at radii where the temperature is 10^{7}K, 10^{5}K and 10^{4}K, using the right panel of Fig. 5.2.

(d) Suppose that the frequency-dependent opacity coefficient has the form $\kappa_{\nu} = \kappa_{0}\nu^{-\alpha}$. Show that the Rosseland mean opacity depends on the temperature as $\kappa \propto T^{-\alpha}$

6. The Initial Mass Function (25 pts): Calculate the Initial Mass Function using the *Gaia* observations of objects with 100 pc of Earth. Compare this to the IMF that you have already obtained from *Hipparcos* data. Hint: you will need to use models to convert colors and luminosities into mass.