29 Jan 2025

1. (a) Let X be a continuous random variable. Show that

$$\mathrm{Var}(aX) = a^2\,\mathrm{Var}(X).$$

(b) Let Y be another continuous random variable. Show that

$$\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\,\mathrm{Cov}(X,Y),$$

where

$$\mathrm{Cov}(X,Y) = E[(X-E(X))(Y-E(Y))].$$

2. (a) Let X and Y be two continuous random variables, and $T = X+Y$ be their sum. Let the PDFs of T, X and Y be $f_T(t)$, $f_X(x)$, $f_Y(y)$ respectively. Show that

$$f_T(t) = f_X(x) * f_Y(y),$$

assuming that X and Y are independent. ($*$ denotes convolution.)

(b) In the case that X and Y are both uniform distributions on $[0,1]$, find the distribution of T.

(c)* What would $f_T(t)$ look like if $T = aX + bY$, or $T = XY$?