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1. Let X be a random variable (discrete or continuous) where $0 \leq X \leq 1$. Let $\mu = E(X)$.

Show that

$$\mathrm{Var}(X) \leq \mu - \mu^2 \leq \frac{1}{4}.$$

Hence, determine the distribution for X that maximises $\mathrm{Var}(X)$.

2. A distribution is said to be *memoryless* if a random variable X from that distribution satisfies

$$P(X \geq s + t | X \geq s) = P(X \geq t).$$

Show that the only positive continuous distribution that is *memoryless* is the exponential distribution, whose PDF is given by

$$f(x) = \lambda e^{-\lambda x}.$$