Probability Club

29th January 2025

Problem 1. Hypergeometric distribution. A jar contains w white marbles and b black marbles. n marbles are drawn from the jar without replacement. What is the probability of drawing k white marbles? Do this the following way:

1. Label the white balls as $\{W_1, ...W_w\}$ and the black balls as $\{B_1, ...B_b\}$.
2. Accounting for the labels, what are the total number of ways that we can draw n balls from the jar?
3. Accounting for the labels, what are the number of ways we can draw k white balls and $n-k$ black balls from the jar?
4. Show that all outcomes in 1. have the same probability of occurring.
5. Hence find the distribution of drawing k white balls out of n balls.

What will be the expected number of white marbles drawn? *Hint: Linearity of expectations can simplify things a lot.*

Problem 2. Random walk. Starting from 0, a drunk man takes random steps of length 1 with probability p to the right. He takes a total of n steps. Find the distribution of his final position X.