GIẢI TÍCH 1: Các VCB tương đương: Khi $x \to 0$, có

* $\sin x \sim x; \tan x \sim x \sim \arcsin x \sim \arctan x$
$\to \sin ax \sim ax \sim \tan ax \sim \arcsin ax \sim \arctan ax.$
* $1 - \cos x \sim \frac{1}{2}x^2 \to 1 - \cos ax \sim \frac{1}{2}(ax)^2 = \frac{a^2x^2}{2}.$
* $e^x - 1 \sim x \to e^{ax} - 1 \sim ax; a^x - 1 \sim x \ln a; \ln(1+x) \sim x \to \ln(1+ax) \sim ax.$
* $(1+x)^a - 1 \sim ax \to \begin{cases} \sqrt{1+x} - 1 = (1+x)^{\frac{1}{2}} - 1 \sim \frac{1}{2}x \to \sqrt{1+ax} - 1 \sim \frac{1}{2}ax. \\ \sqrt[3]{1+x} - 1 \sim \frac{1}{3}x \to \sqrt[3]{1+ax} - 1 \sim \frac{1}{3}ax. \end{cases}$

- Bảng đạo hàm:

* $(uv)' = u'v + uv'; \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}; (u+C)' = u'; (u.C)' = u'.C;$
* $(x^a)' = ax^{a-1}; \left(\frac{1}{x}\right)' = -\frac{1}{x^2}; \left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}; \frac{1}{x^a} = x^{-a} \to \left(\frac{1}{x^2}\right)' = (x^{-2})' = -2x^{-3};$
* $(\sin x)' = \cos x; (\cos x)' = -\sin x; (\tan x)' = \frac{1}{\cos^2 x}; (\cot x)' = -\frac{1}{\sin^2 x};$
* $(e^x)' = e^x; (a^x)' = a^x \ln a; (\ln x)' = \frac{1}{x}; (\log_a x)' = \frac{1}{x \ln a};$
* $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}; (\arctan x)' = \frac{1}{1+x^2}.$

*** Đạo hàm của hàm hợp:**

$(u^a)' = au^{a-1}.u'; \left(\frac{1}{u}\right)' = -\frac{u'}{u^2}; \ldots; (\sin u)' = \cos u.u'; \ldots; (\arctan u)' = \frac{u'}{1+u^2}.$

- Bảng nguyên hàm:

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C; \int \frac{dx}{x} = \ln|x| + C \to \int \frac{dx}{x^a} = \int x^{-a} dx = \cdots;$$

$$\int \sin x dx = -\cos x + C; \int \cos x dx = \sin x + C;$$

$$\int \frac{dx}{\cos^2 x} = \tan x + C; \int \frac{dx}{\sin^2 x} = -\cot x + C; \int e^x dx = e^x + C; \int a^x dx = \frac{a^x}{\ln a} + C.$$

$$\int \frac{dx}{(x-a)(x-b)} = \frac{1}{a-b} \ln\left|\frac{x-a}{x-b}\right| + C; \int \frac{dx}{\sqrt{x^2+k}} = \ln\left|x + \sqrt{x^2+k}\right| + C;$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C; \int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + C;$$

$$\arctan(+\infty) = \frac{\pi}{2}; \arctan(-\infty) = -\frac{\pi}{2}; \int \frac{dx}{\cos x} = \ln\left|\tan\frac{x}{2}\right| + C.$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + C; \int \frac{dx}{ax+b} = \frac{\ln|ax+b|}{a} + C;$$

$$\int \sin(ax+b) dx = -\frac{\cos(ax+b)}{a} + C; \int \cos(ax+b) dx = -\frac{\sin(ax+b)}{a} + C;$$

$$\int \frac{dx}{\cos^2(ax+b)} = \frac{\tan(ax+b)}{a} + C; \int \frac{dx}{\sin^2(ax+b)} = -\frac{\cot(ax+b)}{a} + C;$$

$$\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + C; \int a^{mx+n} dx = \frac{a^{mx+n}}{\ln a.\ m} + C.$$