Homework 2

Track A

LAB 0 – **DETERMINE** *gcd* (12075, 4655)

Rewrite as Extended Euclidean

$$gcd(a,b) = ax + by$$

extended euclidean

$$gcd(12075, 4655) = 12075x + 4655y$$

substitution

Use Division Algorithm to Solve Euclidean Equations

$$a = bq + r$$

12075 = 4655q + r

solve q and r to find a

$$12075 = 4655(2) + 2765$$

$$a = bq + r$$

4655 = 2765q + r

4655 = 2765(1) + 1890

Repeat until r = 0, using b as a and r as b

$$a = bq + r$$

2765 = 1890q + r

2765 = 1890(1) + 875

Repeat until r = 0, using b as a and r as b

$$a = bq + r$$

$$1890 = 875q + r$$

$$2765 = 875(2) + 140$$

Repeat until r = 0, using b as a and r as b

$$a = bq + r$$

$$875 = 140q + r$$

$$875 = 140(6) + 35$$

Repeat until r = 0, using b as a and r as b

$$a = bq + r$$

$$140 = 35q + r$$

$$140 = 35(4) + 0$$

Repeat until r = 0, using b as a and r as b

$$gcd(12075, 4655) = 35$$

LAB 5 – (OPENSSL) GENERATE RANDOM NUMBER AND TEST PRIMALITY

1. (10pts) Based on the script primeTest, write another shell script using OpenSSL command line. The script is called isprime. It requires an integer as an input parameter. If the input integer is a prime, the script will show "*** is a prime". If not, the script will show "*** is not a prime".

```
se@ubuntu:~/share/lab5$ ./isprime 15
15 is not a prime number
se@ubuntu:~/share/lab5$ ./isprime 17
17 is a prime number
se@ubuntu:~/share/lab5$ ./isprime x
Failed to process value (x)
se@ubuntu:~/share/lab5$ ./isprime
prime: No prime specified
prime: Use -help for summary.
se@ubuntu:~/share/lab5$ ./isprime -help
Usage: prime [options] [number...]
 number Number to check for primality
-help Display this summary
 -hex
              Hex output
 -generate
             Generate a prime
 -bits +int Size of number in bits
               When used with -generate, generate a safe prime
 -safe
 -checks +int Number of checks
```

- 2. (5pts) Use the program to decide if the following numbers are prime numbers:
 - a. 2685457421 Yes, this is a prime number
 - b. 4294967295 No, this is not a prime number

```
se@ubuntu:~/share/lab5$ ./isprime 2685457421
2685457421 is a prime number
se@ubuntu:~/share/lab5$ ./isprime 4294967295
4294967295 is not a prime number
```

3. (5pts) If an integer passes the Miller-Rabin primality test, does that guarantee the number is a prime number? Why?

No, an integer passing the Miller-Rabin primality test does not guarantee that it is prime. Since the Miller-Rabin primality test is a probabilistic primality test, it can only indicate that the integer is a probable prime. In other words, since the test relies on randomness in its computations, there are scenarios where a composite number might pass the test as if it were prime.

LAB 6 – (OPENSSL) CREATE RSA PUBLIC/PRIVATE KEY (512BITS) WITHOUT PASSWORD PROTECTION

1. (8pts) In Step 1, what is the public key (e, n) used? Why?

The exponent, 65537 (0x10001), is shown during RSA private key generation in Step 1. This number is popular for RSA key generation because it is a prime number, thus making it relatively prime to the numbers (p-1) and (q-1), where p and q are the prime factors of the modulus (n). Also, it is small enough to not slow down the encryption or decryption processes, while still being large enough that when combined with a large modulus (i.e. 2048-bit or 4096-bit), it creates a high level of complexity. This complexity makes it practically impossible for someone to break the encryption by trying to factorize the modulus and obtain the private key.

Alternatively, n is not shown during the RSA private key generation in Step 1 as it is randomized during each generation. Unlike the exponent, it is created by multiplying two randomly selected large prime numbers, p and q. This process ensures that each key pair is unique and not predictable. However, you can use the following OpenSSL command on the private key to display the value of n:

```
$ openssl rsa -in private512.key -modulus -noout
or the following OpenSSL command on the public key:
```

\$ openssl rsa -pubin -in public512.key -text -noout

```
Key Generation 1
se@ubuntu:~/share/lab6$ openssl genrsa -out private512.key 512
Generating RSA private key, 512 bit long modulus (2 primes)
        .+++++++++++++++++++++++++++++
e is 65537 (0x010001)
se@ubuntu:~/share/lab6$ openssl rsa -in private512.key -modulus -noout
Modulus=C7A03044CD51C3B0FD22AA64A826B913CF65FD9266DF10DB0F2AA3DDA7749A3E22130999DCC
91178FFC5830845CAE5604455F6B83E5FC16B74A050A08D183D6F
se@ubuntu:~/share/lab6$ openssl rsa -in private512.key -pubout -out public512.key
writing RSA key
se@ubuntu:~/share/lab6$ openssl rsa -pubin -in public512.key -text -noout
RSA Public-Key: (512 bit)
Modulus:
    00:c7:a0:30:44:cd:51:c3:b0:fd:22:aa:64:a8:26:
    b9:13:cf:65:fd:92:66:df:10:db:0f:2a:a3:dd:a7:
    74:9a:3e:22:13:09:99:dc:c9:11:78:ff:c5:83:08:
    a0:8d:18:3d:6f
Exponent: 65537 (0x10001)
```

```
Key Generation 2
se@ubuntu:~/share/lab6$ openssl genrsa -out private512.key 512
Generating RSA private key, 512 bit long modulus (2 primes)
e is 65537 (0x010001)
se@ubuntu:~/share/lab6$ openssl rsa -in private512.key -modulus -noout
Modulus=D9D00B2477EB2D6B83DC9797F1860B5F6D43D56B5B74B3ED097DA889E00F589FCA26A6E9536
CBCFF10FBC9B458745DF86FB035755878BC5CDA69E6B251280075
se@ubuntu:~/share/lab6$
se@ubuntu:~/share/lab6$ openssl rsa -in private512.key -pubout -out public512.key
writing RSA key
se@ubuntu:~/share/lab6$ openssl rsa -pubin -in public512.key -text -noout
RSA Public-Key: (512 bit)
Modulus:
    00:d9:d0:0b:24:77:eb:2d:6b:83:dc:97:97:f1:86:
    0b:5f:6d:43:d5:6b:5b:74:b3:ed:09:7d:a8:89:e0:
    Of:58:9f:ca:26:a6:e9:53:6c:bc:ff:10:fb:c9:b4:
    58:74:5d:f8:6f:b0:35:75:58:78:bc:5c:da:69:e6:
Exponent: 65537 (0x10001)
```

- 2. (12pts) A certificate, Amazon.cer, can be found in the lab 6 folder, answer the following questions:
 - a. What is the signature hash algorithm used to create the certificate?

1.2.840.113549.1.1.11 (sha256WithRSAEncryption)

```
Signature
 Signature Algorithm: 1.2.840.113549.1.1.11
 Signature Parameters: 05 00
 Signature:
                                62 37 42 5C BC 10 B5 3E 8B 2C E9 0C
                                9B 6C 45 E2 07 00 7A F9 C5 58 0B B9 08 8C 3E ED B3 25 3C B5 6F 50 E4 CD
                                35 6A A7 93 34 96 32 21 A9 48 44 AB
9C ED 3D B4 AA 73 6D E4 7F 16 80 89
                                6C CF 28 03 18 83 47 79 A3 10 7E 30 5B AC 3B B0 60 E0 77 D4 08 A6 E1 1D
                                7C 5E C0 BB F9 9A 7B 22 9D A7 00 09
7E AC 46 17 83 DC 9C 26 57 99 30 39
62 96 8F ED DA DE AA C5 CC 1B 3E CA
                                43 68 6C 57 16 BC D5 0E 20 2E FE FF C2 6A 5D 2E A0 4A 6D 14 58 87 94 E6 39 31 5F 7C 73 CB 90 88 6A 84 11 96
                                 27 A6 ED D9 81 46 A6 7E A3 72 00 0A
52 3E 83 88 07 63 77 89 69 17 0F 39
                                 85 D2 AB 08 45 4D D0 51 3A FD 5D 5D
                                 37 64 4C 7E 30 B2 55 24 42 9D 36 B0
                                 5D 9C 17 81 61 F1 CA F9 10 02 24 AB
                                 EB 0D 74 91 8D 7B 45 29 50 39 88 B2
                                A6 89 35 25 1E 14 6A 47 23 31 2F 5C 9A FA AD 9A 0E 62 51 A4 2A A9 C4 F9
                                 34 9D 21 18
```

```
Signature Algorithm: sha256WithRSAEncryption
62:37:42:5c:bc:10:b5:3e:8b:2c:e9:0c:9b:6c:45:e2:07:00:
7a:f9:c5:58:0b:b9:08:8c:3e:ed:b3:25:3c:b5:6f:50:e4:cd:
35:6a:a7:93:3d:96:32:21:a9:48:44:ab:9c:ed:3d:bd:aa:73:
6d:e4:7f:16:80:89:6c:cf:28:03:18:83:47:79:a3:10:7e:30:
5b:ac:3b:b0:60:e0:77:d4:08:a6:e1:1d:7c:5e:c0:bb:f9:9a:
7b:22:9d:a7:00:09:7e:ac:46:17:83:dc:9c:26:57:99:30:39:
62:96:8f:ed:da:de:aa:c5:cc:1b:3e:ca:43:68:6c:57:16:bc:
d5:0e:20:2e:fe:ff:c2:6a:5d:2e:a0:4a:6d:14:58:87:94:e6:
39:31:5f:7c:73:cb:90:88:6a:84:11:96:27:a6:ed:d9:81:46:
a6:7e:a3:72:00:0a:52:3e:83:88:07:63:77:89:69:17:0f:39:
85:d2:ab:08:45:4d:d0:51:3a:fd:5d:5d:37:7:89:69:224:ab:
eb:0d:74:91:8d:7b:45:29:50:39:88:b2:a6:89:35:25:1e:14:
6a:47:23:31:2f:5c:9a:fa:ad:9a:0e:62:51:a4:2a:a9:c4:f9:
34:9d:21:18
```

b. What is the public key cryptography used to create the signature?

RSA

```
Public Key Info
 Key Algorithm:
                      RSA
 Key Parameters:
                      05 00
                      2048
 Key Size:
 Key SHA1 Fingerprint: 40 83 77 DD 67 5D 40 87 0D E2 2A 89
                      05 45 28 75 F2 36 DE D4
                      30 82 01 0A 02 82 01 01 00 B2 78 80
 Public Key:
                      71 CA 78 D5 E3 71 AF 47 80 50 74 7D
                      6E D8 D7 88 76 F4 99 68 F7 58 21 60
                      F9 74 84 01 2F AC 02 2D 86 D3 A0 43
                      7A 4E B2 A4 D0 36 BA 01 BE 8D DB 48
                      C8 07 17 36 4C F4 EE 88 23 C7 3E EB
                      37 F5 B5 19 F8 49 68 B0 DE D7 B9 76
                      38 1D 61 9E A4 FE 82 36 A5 E5 4A
                      E4 45 E1 F9 FD B4 16 FA 74 DA 9C 9B
                      35 39 2F FA B0 20 50 06 6C 7A D0 80
B2 A6 F9 AF EC 47 19 8F 50 38 07 DC
                      A2 87 39 58 F8 BA D5 A9 F9 48 67 30
                      96 EE 94 78 5E 6F
                                           89 A3 51 C0 30 86
                      66 A1 45 66 BA 54 EB A3 C3 91 F9 48
                      DC FF D1 E8 30 2D 7D 2D 74 70 35 D7
                      88 24 F7 9E C4 59 6E BB 73 87
                      32 46 28 B8 43 FA B7 1D AA CA B4 F2
9F 24 0E 2D 4B F7 71 5C 5E 69 FF EA
                          02 CB 38 8A AE 50 38
                                                  6F
                      62 1B C5 C7 1E 54 E1 77 E0 67 C8 0F
                      9C 87 23 D6 3F 40 20 7F 20 80 C4 80 4C 3E 3B 24 26 8E 04 AE 6C 9A C8 AA
                      OD 02 03 01 00 01
```

c. What is the size of the public key?

2048

d. What is the *e* used in the public key in the certificate?

65537 (0x10001)

```
Subject Public Key Info:
   Public Key Algorithm: rsaEncryption
        RSA Public-Key: (2048 bit)
       Modulus:
            84:01:2f:ac:02:2d:86:d3:a0:43:7a:4e:b2:a4:d0:
            36:ba:01:be:8d:db:48:c8:07:17:36:4c:f4:ee:88:
            23:c7:3e:eb:37:f5:b5:19:f8:49:68:b0:de:d7:b9:
            76:38:1d:61:9e:a4:fe:82:36:a5:e5:4a:56:e4:45:
            e1:f9:fd:b4:16:fa:74:da:9c:9b:35:39:2f:fa:b0:
            50:38:07:dc:a2:87:39:58:f8:ba:d5:a9:f9:48:67:
            30:96:ee:94:78:5e:6f:89:a3:51:c0:30:86:66:a1:
            45:66:ba:54:eb:a3:c3:91:f9:48:dc:ff:d1:e8:30:
            2d:7d:2d:74:70:35:d7:88:24:f7:9e:c4:59:6e:bb:
            73:87:17:f2:32:46:28:b8:43:fa:b7:1d:aa:ca:b4:
            f2:9f:24:0e:2d:4b:f7:71:5c:5e:69:ff:ea:95:02:
            cb:38:8a:ae:50:38:6f:db:fb:2d:62:1b:c5:c7:1e:
            54:e1:77:e0:67:c8:0f:9c:87:23:d6:3f:40:20:7f:
            20:80:c4:80:4c:3e:3b:24:26:8e:04:ae:6c:9a:c8:
            aa:0d
        Exponent: 65537 (0x10001)
```

e. Who issued the certificate?

Starfield Services Root Certificate Authority - G2 of Starfield Technologies, Inc.

```
Issuer Name
C (Country): US
ST (State): Arizona
L (Locality): Scottsdale
O (Organization): Starfield Technologies, Inc.
CN (Common Name): Starfield Services Root Certificate Authority - G2
```

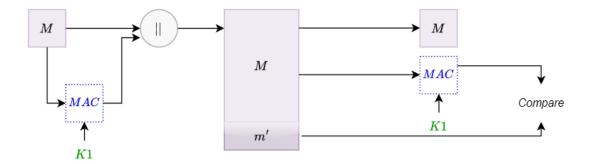
```
Data:
    Version: 3 (0x2)
    Serial Number:
        06:7f:94:4a:2a:27:cd:f3:fa:c2:ae:2b:01:f9:08:ee:b9:c4:c6
    Signature Algorithm: sha256WithRSAEncryption
    Issuer: C = US, ST = Arizona, L = Scottsdale, 0 = "Starfield Technologies, Inc.",
CN = Starfield Services Root Certificate Authority - G2
```

f. Why can you trust the certificate?

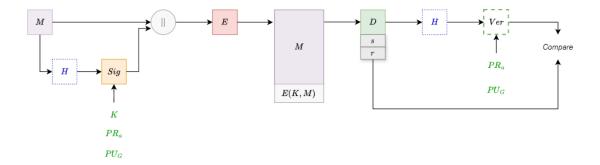
This certificate is trustworthy because it has been issued by a trusted root certificate authority (Starfield Services Root Certificate Authority - G2), it has been signed using RSA encryption, and it is still within its validity timeframe (May 25, 2015 - December 31, 2037)

LAB 7 – (OPENSSL) CREATE RSA PUBLIC/PRIVATE KEY (4096BITS) WITH PASSWORD PROTECTION

1. (8pts) Use symmetric cipher and message authentication code to provide confidentiality, integrity, and authentication. (use the giving cryptographic operations only).



2. (8pts) Use symmetric cipher and digital signature to provide confidentiality, integrity, and authentication. (use the giving cryptographic operations only).



3. (4pts) Compare the two scenarios as designed in a) and b). What are their advantages and limitations?

Scenario A, the symmetric cipher and MAC, offers efficient data encryption, ensuring confidentiality while maintaining data integrity and authentication. The addition of a MAC provides an extra layer of security on top of the cipher as it verifies data integrity and source authentication. However, securely managing and distributing the shared secret keys for the symmetric cipher can be challenging. Scenario B, the symmetric cipher with a digital signature, improves security by providing a means to authenticate the sender's identity and prevent data tampering. However, these are susceptible to various attacks, including replay attacks, man-in-the-middle attacks, and key compromise attacks.

LAB 8 – (OPENSSL) COMBINATION OF RSA AND AES TO ENCRYPT A FILE

1. (6pts) What is the throughput when you sign and verify a message using a 1024-bit key in RSA? Which one is faster? Why?

RSA 1024-bit Sign throughput: 15905.2 operations/s RSA 1024-bit Verify throughput: 253774.1 operations/s

The verify throughput is faster than the sign throughput because verifying a signature involves simpler computations with a small public exponent (e.g., 3, 5, 17, 257, or 65537), thus making it quick and inexpensive, whereas generating signatures requires more computationally intensive operations with a private exponent (d).

```
Doing 512 bits private rsa's for 10s: 394341 512 bits private RSA's in 9.99s
Doing 512 bits public rsa's for 10s: 6526612 512 bits public RSA's in 10.00s
Doing 1024 bits private rsa's for 10s: 158893 1024 bits private RSA's in 10.01s
Doing 1024 bits public rsa's for 10s: 2540279 1024 bits public RSA's in 10.01s
Doing 1024 bits public rsa's for 10s: 2540279 1024 bits public RSA's in 10.01s
Doing 2048 bits private rsa's for 10s: 799718 2048 bits private RSA's in 9.99s
Doing 2048 bits private rsa's for 10s: 799718 2048 bits private RSA's in 10.01s
Doing 3072 bits public rsa's for 10s: 374720 3072 bits private RSA's in 10.01s
Doing 3072 bits private rsa's for 10s: 374720 3072 bits public RSA's in 10.00s
Doing 4096 bits private rsa's for 10s: 3308 4096 bits private RSA's in 10.00s
Doing 4096 bits private rsa's for 10s: 317 7680 bits private RSA's in 10.00s
Doing 7680 bits private rsa's for 10s: 6350 7680 bits private RSA's in 10.02s
Doing 7680 bits public rsa's for 10s: 6350 7680 bits private RSA's in 10.02s
Doing 15360 bits public rsa's for 10s: 68 15360 bits private RSA's in 10.02s
Doing 15360 bits public rsa's for 10s: 16309 15360 bits public RSA's in 10.00s
DopenSSL 1.1.1f 31 Mar 2020
Duilt on: Wed May 24 17:14:51 2023 UTC
Options:bn(64,64) rc4(8x,int) des(int) aes(partial) blowfish(ptr)
compiler: gcc -fPIC -pthread -m64 -Wa, --noexecstack -Wall -Wa, --noexecstack -g -02 -fdebug-prefix-map=/build/openssl-mSG92N/o
pensSl-1.1.1f=. -fstack-protector-strong -Wformat -Werror=format-security -DOPENSSL TLS SECURITY LEVEL=2 -DOPENSSL USE NODELE

For DLE NDIAN -DOPENSSL PIC -DOPENSSL CUDO 0BJ -DOPENSSL CASS SEZ -DOPENSSL ENS ASM MONT -DOPENSSL USE NODELE

Sign verify sign/s verify/s
rsa 512 bits 0.0000258 0.0000025 30473.6 652661.2
rsa 1024 bits 0.0000358 0.00000465 30.000027 738.6 37472.0
rsa 4096 bits 0.0013540 0.000027 738.6 37472.0
rsa 4096 bits 0.0070088 0.0001635 30.0000135 37.0 6341.3
rsa 15360 bits 0.00000155 60.0000135 37.0 6341.3
rsa 15360 bits 0.00000155 60.0000155 60.0000155 60.0000155 50.0000155 50.0000155 5
```

2. (6pts) What is the throughput when you encrypt a 1024-bit message block using DES CBC mode? Compared with a 1024-bit RSA verify throughput, which one is faster? Compared with a 1024-bit RSA sign throughput, which one is faster?

DES CBC 1024-bit throughput: $\approx 113761.355 \text{ KB/s} *$

RSA 1024-bit Sign throughput: 15905.200 operations/s

RSA 1024-bit Verify throughput: 253774.100 operations/s

Comparatively, DES CBC encryption is slower than RSA verification but faster than RSA signing for 1024-bit operations.

* While the throughput for 1024 bit is not explicitly stated in the speed test, it can be estimated by finding the average between 64 bytes and 256 bytes:

$$A = throughput(64bytes) = 112420.67 \text{ KB/s}$$

B = throughput(256bytes) = 115102.04 KB/s

$$C = throughput(128bytes) = \frac{A+B}{2} = 113761.355 \text{ KB/s}$$

```
lab8$ openssl speed des
Doing des cbc for 3s on 16 size blocks: 20975699 des cbc's in 3.00s
Doing des cbc for 3s on 64 size blocks: 5269719 des cbc's in 3.00s
Doing des cbc for 3s on 256 size blocks: 1348852 des cbc's in 3.00s
Doing des cbc for 3s on 1024 size blocks: 334291 des cbc's in 3.00s
Doing des cbc for 3s on 1024 $120 blocks: 334291 des cbc $1 n 3.00s
Doing des cbc for 3s on 1892 size blocks: 41868 des cbc's in 3.00s
Doing des cbc for 3s on 16384 size blocks: 21104 des cbc's in 3.01s
Doing des ede3 for 3s on 16 size blocks: 7949455 des ede3's in 3.00s
Doing des ede3 for 3s on 64 size blocks: 2005291 des ede3's in 3.01s
Doing des ede3 for 3s on 256 size blocks: 499108 des ede3's in 3.00s
Doing des ede3 for 3s on 1024 size blocks: 125413 des ede3's in 3.01s
 Doing des ede3 for 3s on 8192 size blocks: 15622 des ede3's in 3.00s
 Doing des ede3 for 3s on 16384 size blocks: 7840 des ede3's in 3.01s
 OpenSSL 1.1.1f 31 Mar 2020
built on: Wed May 24 17:14:51 2023 UTC options:bn(64,64) rc4(8x,int) des(int) aes(partial) blowfish(ptr)
compiler: gcc -fPIC -pthread -m64 -Wa, --noexecstack -Wall -Wa, --noexecstack -g -02 -fdebug-prefix-ma p=/build/openssl-mSG92N/openssl-1.1.1f=. -fstack-protector-strong -Wformat -Werror=format-security - DOPENSSL_TLS_SECURITY_LEVEL=2 -DOPENSSL_USE_NODELETE -DL_ENDIAN -DOPENSSL_PIC -DOPENSSL_CPUID_OBJ -D OPENSSL_IA32_SSE2 -DOPENSSL_BN_ASM_MONT -DOPENSSL_BN_ASM_MONT5 -DOPENSSL_BN_ASM_GF2m -DSHA1_ASM -DSH A256_ASM -DSHA512_ASM -DKECCAK1600_ASM -DRC4_ASM -DMD5_ASM -DAESNI_ASM -DVPAES_ASM -DGHASH_ASM -DECP
 NISTZ256 ASM -DX25519 ASM -DPOLY1305 ASM -DNDEBUG -Wdate-time -D FORTIFY SOURCE=2
114873.07k
                              42397.09k
                                                                             42590.55k
                                                                                                     42665.42k
                                                                                                                            42658.47k
                                                                                                                                                    42674.60k
                                                                   128 bytes
```

3. (8pts) What is the plaintext you restored from the cipher text?

After decrypting the cipher text with the password obtained from RSA decryption (hello world), the restored plaintext is "Belief creates the actual fact. William James"

```
se@ubuntu:~/share/lab8$ openssl rsautl -decrypt -inkey private1024.key -in encryptedpwd.txt -out password.txt
password.txt
Enter pass phrase for private1024.key:
hello world
se@ubuntu:~/share/lab8$ openssl enc -d -aes-256-cbc -in cipher.txt -out c.txt -pass file:./password.txt && cat c.txt
**** WARNING : deprecated key derivation used.
Using -iter or -pbkdf2 would be better.
Belief creates the actual fact. William James
```