The second homework.

1 (2 points). A standard die (1,. . . ,6) is rolled 80 times. Find the approximate symmetric bounds within which the number of times a six appears will lie with a probability of 0.9973.

2 (4 points). A 7-die is rolled 10 times. The die is numbered 2,...,8.

a) Find the most probable number of even values obtained and the most probable number of values divisible by 3.

b) Find the probability that the number of fives will be exactly 1 and at least 2.

3 (2 points). In a batch of 768 watermelons, each watermelon is unripe with a probability of 1/4. Find the probability that the number of ripe watermelons falls within the range of 564 to 600.

4 (2 points). The probability of finding a white mushroom among others is 1/4. What is the probability that out of 300 mushrooms, 75 will be white?

5 (2 points). A factory shipped 5000 light bulbs to a store. The probability that a bulb will break during transportation is 0.0002. Find the probability that no more than three broken bulbs arrive at the store.

6 (4 points). A 5-die (1,...,5) is rolled 5000 times and the number of odd numbers were 3030.

a) Find the probability that, upon repeating the experiment, the absolute deviation of the relative frequency of odd numbers from the probability 0.6 will not exceed the obtained result.

d) Find the practical lower bound for the possible absolute deviation of the relative frequency of even numbers from their probability, given that the probability of this event is 0.05.

7 (2 points). In a jury consisting of an odd number of members $n = 2m + 1$, each member independently makes the correct decision with a probability of $p = 0.7$. What is the minimum number of jury members required for the majority decision to be correct with a probability of at least 0.99?

8 (2 points). A mathematician carries two matchboxes in his pocket, each originally containing N matches. When he needs a match, he randomly selects one of the boxes: the first with probability 0.4 and the second with probability 0.6. Find the probability that when he takes an empty box from his pocket for the first time, the other one contains exactly r matches.

9* (1 point for each). In a Bernoulli scheme, the probability of success is p, and the probability of failure is q=1-p. Find the probability that:

a) The sequence FF (two consecutive failures) appears before the sequence FS (failure followed by success).

b) The sequence FF appears before the sequence SF. (Here, F stands for failure and S for success.)

HW 2. (deadline March 25th)

1 (2 points). A standard die (1,. . . ,6) is rolled 80 times. Find the approximate symmetric bounds within which the number of times a six appears will lie with a probability of 0.9973.

$\mu = 80 \cdot \frac{1}{6} = \frac{40}{3} \approx 13.3.$

find k. s.t. $P(|\mu - p| \le k) = 0.9973.$

$P(|\mu-p|\le k) = P_{80}\left(\left[\frac{40}{3}-k, \frac{40}{3}+k\right]\right) \approx 2\Phi\left(\frac{k}{\sqrt{80\cdot\frac{1}{6}\cdot\frac{5}{6}}}\right) - 1 \approx 0.9973$

$\Rightarrow \Phi^{-1}(0.99865) \approx 3.00$ (by the table). $\Rightarrow k = 3\cdot\sqrt{80\cdot\frac{5}{36}} = \frac{20}{6}\cdot 3 = 10.$

$0.9973 = P([3.3, 23.3]) \le P([3; 24])$

2 (4 points). A 7-die is rolled 10 times. The die is numbered 2,...,8.
a) Find the most probable number of even values obtained and the most probable number of values divisible by 3.
b) Find the probability that the number of fives will be exactly 1 and at least 2.

a). the probability for even number is $\frac{4}{7}$. we need to compare $P_{10}(k) = \binom{10}{k}\left(\frac{4}{7}\right)^k\left(\frac{3}{7}\right)^{10-k}$

$P_7(k)$ takes maximum when $k_1 \in [np+p-1, np+p]$ i.e. $k_1 \in [\frac{37}{7}, \frac{44}{7}]$. $\Rightarrow k_1 = 6$

the probability for values divisible by 3. is $\frac{2}{7}$.

$k_2 \in [np+p-1, np+p]$, i.e. $k_2 \in [\frac{15}{7}, \frac{22}{7}]$ $\Rightarrow k_2 = 3$.

b) use formula of Bernoulli trials.

$P_7(1) = \binom{7}{1}\cdot\left(\frac{1}{7}\right)\cdot\left(\frac{6}{7}\right)^9 = \left(\frac{6}{7}\right)^9$

$P_7(\ge 2) = 1 - P_7(0) - P_7(1) = 1 - \left(\frac{6}{7}\right)^9 - \left(\frac{6}{7}\right)^{10}$

3 (2 points). In a batch of 768 watermelons, each watermelon is unripe with a probability of 1/4. Find the probability that the number of ripe watermelons falls within the range of 564 to 600.

Sol. $np = \frac{3}{4}\cdot 768 = 576.$ $\quad b_n = \frac{600-576}{12} = 2$

$\sqrt{npq} = \sqrt{768\cdot\frac{3}{4}\cdot\frac{1}{4}} = \frac{48}{4} = 12.$ $\quad a_n = \frac{576-564}{12} = -1.$

$P_{768}(564 \le n \le 600) = \Phi(2) - \Phi(-1) = \Phi(2) + \Phi(1) - 1 = 0.84135 + 0.97725 - 1 = 0.8186.$

4 (2 points). The probability of finding a white mushroom among others is 1/4. What is the probability that out of 300 mushrooms, 75 will be white?

Sol: $x_n = \frac{m-np}{\sqrt{npq}} = 0.$

$P_n(m) = \frac{1}{\sqrt{300\cdot\frac{1}{4}\cdot\frac{3}{4}}}\cdot\varphi(0) = \frac{1}{15\sqrt{2\pi}}$

5 (2 points). A factory shipped 5000 light bulbs to a store. The probability that a bulb will break during transportation is 0.0002. Find the probability that no more than three broken bulbs arrive at the store.

Let $\lambda = np = 5000\cdot 0.0002 = 1.$

$P_\lambda(\le 3) = e^{-1}\left(1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}\right) = e^{-1}\left(1+1+\frac{1}{2}+\frac{1}{6}\right) = \frac{8e^{-1}}{3} \approx 0.981$

6 (4 points). A 5-die (1,...,5) is rolled 5000 times and the number of odd numbers were 3030.
a) Find the probability that, upon repeating the experiment, the absolute deviation of the relative frequency of odd numbers from the probability 0.6 will not exceed the obtained result.
d) Find the practical lower bound for the possible absolute deviation of the relative frequency of even numbers from their probability, given that the probability of this event is 0.05.

a). $\mu_1 = \frac{3030}{5000} = 0.606$ $\quad a_n = \frac{-30}{\sqrt{npq}} = \frac{30}{\sqrt{5000\cdot\frac{2}{5}\cdot\frac{3}{5}}} = -\frac{3}{2\sqrt{3}}$

$P(|\frac{S_n}{n} - 0.6| < 0.006) = P(|S_n - 3000| < 30) = P([2970, 3030])$.

$\approx 2\Phi(\frac{\sqrt{3}}{2}) - 1 = 2\Phi(0.87) - 1 = 2\times 0.80785 - 1 = 0.61570$.

b) find k s.t.

$P(|\mu - p| \le k) = 0.95. \Rightarrow P(|3000 - S_n| \le 5000k) = 0.95 \Rightarrow P_n([2000 - 5000k, 2000 + 5000k]) = 0.95$.

$\Rightarrow 2\Phi(\frac{5000k}{20\cdot\sqrt{3}}) - 1 = 0.95 \Rightarrow \Phi(\frac{250}{\sqrt{3}}k) = 0.975. \Rightarrow \frac{250}{\sqrt{3}}k = 1.96 \Rightarrow k = 0.01357$

the lower bound $= 2000 - 5000k = 1932.10 > 1932$.

7 (2 points). In a jury consisting of an odd number of members $n = 2m + 1$, each member independently makes the correct decision with a probability of $p = 0.7$. What is the minimum number of jury members required for the majority decision to be correct with a probability of at least 0.99?

Sol: $a_n = \frac{0 - (2m+1)\cdot 0.3}{\sqrt{(2m+1)\cdot\frac{7}{10}\cdot\frac{3}{10}}} = \frac{-(6m+3)}{\sqrt{21(2m+1)}}$ $\quad b_n = \frac{m - (2m+1)\cdot 0.3}{\sqrt{(2m+1)\cdot\frac{7}{10}\cdot\frac{3}{10}}} = \frac{4m-3..}{\sqrt{21(2m+1)}}$

$P = \Phi(b_n) - \Phi(a_n) = \Phi(\frac{6m+3}{\sqrt{21(2m+1)}}) + \Phi(\frac{4m-3}{\sqrt{21(2m+1)}}) - 1 \geq 0.99$

with estimation. We need both $\Phi > 0.99$. by the table. $\frac{4m-3}{\sqrt{21(2m+1)}} \geq 2.33. \Rightarrow m \geq 16.15$

$m = 16$ $\quad P \approx \Phi(3.76) + \Phi(2.32) - 1 < 0.99$.

$m = 17$ $\quad P \approx \Phi(3.91) + \Phi(2.40) - 1 = 0.99995 + 0.99180 - 1 > 0.99$.

Thus we need at least 35 members.

8 (2 points). A mathematician carries two matchboxes in his pocket, each originally containing N matches. When he needs a match, he randomly selects one of the boxes: the first with probability 0.4 and the second with probability 0.6. Find the probability that when he takes an empty box from his pocket for the first time, the other one contains exactly r matches.

Sol: consider the time after he $(2N-r-1)$ times needs a match. find probability of case.

1) 1 in 1st pocket. r in 2nd: $P_{2N-r-1}(N-1) = C_{2N-r-1}^{N-1}\cdot(0.4)^{N-1}(0.6)^{N-r}$

2) r in 1st pocket. 1 in 2nd. $P_{2N-r-1}(N-r) = C_{2N-r-1}^{N-r}(0.4)^{N-r}(0.6)^{N-1}$

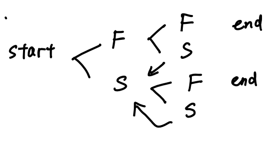
And the $(2N-r)$th time with 0.4 probability deplete the 1st pocket. in 1). Similar with probability 0.6 in 2).

$p = C_{2N-r-1}^{N-1}\cdot[(0.4)^N(0.6)^{N-r} + (0.4)^{N-r}(0.6)^N]$

9* (1 point for each). In a Bernoulli scheme, the probability of success is p, and the probability of failure is q=1-p. Find the probability that:
a) The sequence FF (two consecutive failures) appears before the sequence FS (failure followed by success).
b) The sequence FF appears before the sequence SF. (Here, F stands for failure and S for success.)

a) For any sequence for result. start from the first F. appears.
the next $p(F) = q$ now FF appears before FS the $p(\text{FF before FS}) = q$
$p(S) = p$. now FS appears before FF.

b).



$p(FF) = q^2 + q\cdot p^n \underset{n\to\infty}{=} q^2$