Fourier series

TASKS

1. Decompose the function into a Fourier series $f(x) = \text{sign } x, -\pi < x < \pi$, and using the resulting decomposition, find the sum of the Leibniz series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$$

Answer: $signx = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{sin(2n-1)x}{2n-1}$, Leibniz series: $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1} = \frac{\pi}{4}$

Decompose the Fourier series of the function $f(x)$ on the specified interval, the length of the interval is the period (2-11).

2. $f(x) = \begin{cases} A, & 0 < x < l, \\ A/2, & x = l, \\ 0, & l < x < 2l, \end{cases}$ on the interval $(0.2l)$.

Answer: $\frac{A}{2} + \frac{2A}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} sin \frac{2n-1}{l} \pi x$

3. $f(x) = |x|$ on the segment $[-1;1]$.
Answer: $\frac{1}{2} - \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{cos\pi(2n-1)x}{(2n-1)^2}$

4. $f(x) = \begin{cases} ax, & -\pi < x < 0, \\ bx, & 0 \leqslant x < \pi, \end{cases}$ in the interval $(-\pi, \pi)$.

Answer: $\frac{(b-a)\pi}{4} - \sum_{n=1}^{\infty}((b-a)(1-(-1)^n)\frac{cosnx}{\pi n^2} + (-1)^n(a+b)\frac{sinnx}{n}$

5. $f(x) = \begin{cases} a, & -\pi/2 < x < \pi/2, \\ b, & \pi/2 \leqslant x < 3\pi/2, \end{cases}$ in the range $(-\pi/2, 3\pi/2)$.

Answer: $\frac{a+b}{2} + \frac{2(a-b)}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} cos(2n-1)x$

6. $f(x) = x + \text{sign } x$ on the interval $(-\pi; \pi)$.
Answer: $\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1+(-1)^{n+1}(1+\pi)}{n} sinnx$

7. $f(x) = \pi^2 - x^2$ on the interval $(-\pi; \pi)$.
Answer: $\frac{2}{3}\pi^2 + 4 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} cosnx$

8. $f(x) = x^3$ on the interval $(-\pi; \pi)$.
Answer: $\sum_{n=1}^{\infty}(-1)^n(\frac{12}{n^3} - \frac{2\pi^2}{n})sinnx$

9. $f(x) = e^{ax}, a \neq 0$, in the interval $(-\pi; \pi)$.
Answer: $\frac{2}{\pi} sha\pi(\frac{1}{2a} + \sum_{n=1}^{\infty} \frac{(-1)^n}{a^2+n^2}(acos(nx) - nsinnx))$

10. $f(x) = e^{2|x|}$ in the interval $(-\pi; \pi)$.
Answer: $\frac{e^{2\pi}-1}{2\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n e^{2\pi}-1}{n^2+4} cosnx$

11. $f(x) = \sin ax, a \in Z$ in the interval $(-\pi; \pi)$.

Answer: $\frac{2\sin \pi a}{\pi} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n \sin nx}{n^2 - a^2}, \quad -\pi < x < \pi$