No. 李[illegible] 2022111221

Date . 3 . 14

Analytic Geometry

2.20.

1. Segments with common endpoint overlap if there is any other common point of underlined segments

Proof: let the common endpoint is A. let two segments are AB and AC

there ∃M, be another common point of AB and AC

without lose of generality, we assume AB is longer than AC.

Since. AB. AC has ~~a~~ two common ~~se~~ point. the AB and AC shape a segment.

the endpoints of the segment must be two of A. B. C.

since AC is shorter. then we have ~~B~~ $C \in AB$. ie. AB and AC overlap.

2. For given scale e exist single and only single length function.

proof: let their exist e and two function I and I'.

① $1 = I(e) = I'(e)$

② if a equals b.
$I(a) = I(b)$
$I'(a) = I'(b)$.
$I(a) - I'(a) = I(b) - I'(b) = 0$?

③. if c is composed of a and b. $I(c) = I(a) + I(b) = I'(a) + I'(b) = I'(c)$

~~$I'(c) = I'(a) + I'(b)$~~ 我不确定是否正确

thus. we have $I = I'$. (我给的是 反证 $\exists a, I'$, s.t. $I(a) \neq I'(a)$, Assume $a = ke$

$I(ke) \neq I'(ke)$ $kI(e) \neq kI'(e)$ contradicts $I(e) = I'(e)$)

3. If $|AB| = |CD|$, then AB equals CD.

proof: consider on endpoints.

∃! AM, equal. with CD.

$|AM| = |CD|$, Thus. $|AM| = |AB|$.

Let AM not match AB. Thus ① $AM \sqsubset AB$. M lies on AB. ② $AB \sqsubset AM$ and B lies on AM.

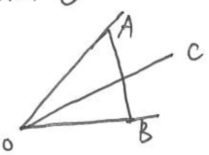
one of the case holds.

①. M lies on AB. M splits AB into AM and MB. Since $|AM| = |AB|$. $|BM| = |AB| - |AM| = 0$.
M ~~B~~. B are same point. which contradict with M lies on AB.

② similarly. we have B. M are same point.

KOKUYO

4. Consider angle $\angle AOB$, it's transversal segment AB and segment OC laying inside the angle and crossing AB. Segment OC or at least ray with endpoint O established by OC ~~&~~ will cross any transverse segment of angle.



proof: Since OC lies inside the $\angle AOB$. that's A and B locate in the different. half-plane ~~of~~ in ~~s~~ respect of OC.

thus. AB crosses OC or at least ray with endpoint established by OC.

Since A. B can be arbitrary on the Ray OA and OB.

That is, any transverse segment of angle cross OC. ...$\square$

Campus

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ID 2022111221

Analytic Geometry.

2.28.

Problem. 1. $\vec{a}\cdot\vec{b} = 6\times 8\times\cos\frac{\pi}{6} = 24\sqrt{3}$.

Problem 2. Let $|\vec{a}| = 12\,cm$.

the. product $= \vec{a}\cdot(-\vec{a}) = -144(cm^2)$.

Problem 3. $|\vec{a}| = 4\sqrt{2}$, $|\vec{b}| = 8$. $\angle(\vec{a},\vec{b}) = 45^\circ$

$\vec{c}\cdot\vec{d} = (-2\vec{a}+\vec{b})(\vec{a}-\vec{b}) = -2|\vec{a}|^2 + 3\vec{a}\cdot\vec{b} - \vec{b}^2$

$= -2\times 32 + 4\sqrt{2}\times 8\times\frac{\sqrt{2}}{2}\times 3 - 64 = \cancel{-160}.\ -32$.

Problem 4. $|\vec{a}| = 3$. $|\vec{b}| = 2$. $\angle(\vec{a},\vec{b}) = \frac{\pi}{3}$.

$|\vec{x}|^2 = (-\vec{a}+3\vec{b})^2 = 9\vec{b}^2 + \vec{a}^2 - 6\vec{a}\cdot\vec{b} = 3^2 + 6^2 - 2\times 3\times 2\times\frac{1}{2}\times 6 = 27$.

$|\vec{x}| = \sqrt{|\vec{x}|^2} = 3\sqrt{3}$.

Problem 5. $|\vec{a}| = 4$. $|\vec{b}| = 2\sqrt{2}$. $\vec{a}\cdot\vec{b} = 8$

$\cos\angle(\vec{a},\vec{b}) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{\sqrt{2}}{2}$ $\angle(\vec{a},\vec{b}) = \arccos\frac{\sqrt{2}}{2} = \frac{\pi}{4}$.

Problem 6.

proof: $\vec{c}$ and $\vec{d}$ are orthogonal

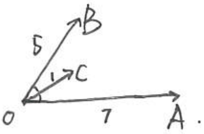
$\vec{c}\cdot\vec{d} = (-\vec{a}+\sqrt{3}\vec{b})(\sqrt{3}\vec{a}-\vec{b}) = -\sqrt{3}\vec{a}^2 + 4\vec{a}\cdot\vec{b} - \sqrt{3}\vec{b}^2$

since $|\vec{a}| = |\vec{b}|$ and $\cos\angle(\vec{a},\vec{b}) = 30^\circ$

$\vec{c}\cdot\vec{d} = -2\sqrt{3}|\vec{a}|^2 + 4\times\frac{\sqrt{3}}{2}\times|\vec{a}|^2 = 0$.

since $\vec{c}\cdot\vec{d} \neq \vec{0}$, then $\cos(\angle\vec{c},\vec{d}) = 0$.

Problem 7.



since $\vec{OC}$ lies inside $\angle AOB$. and bisect it.

$\angle BOC = \angle AOC = 30^\circ$

(a) $|\vec{OA}+\vec{OB}+\vec{OC}| = \sqrt{|\vec{OA}+\vec{OB}+\vec{OC}|^2} = \sqrt{|\vec{OA}|^2+|\vec{OB}|^2+|\vec{OC}|^2+2\vec{OA}\cdot\vec{OB}+2\vec{OC}\cdot\vec{OA}+2\vec{OB}\cdot\vec{OC}}$

$= \sqrt{1+25+49+2\times\frac{1}{2}\times 5\times 7+2\times\frac{\sqrt{3}}{2}\times 1\times 5+2\times\frac{\sqrt{3}}{2}\times 1\times 7} = \sqrt{110+12\sqrt{3}}$

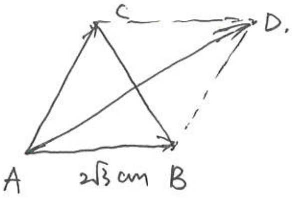
(b) $|\vec{OA}+\vec{OB}-\vec{OC}| = \sqrt{|\vec{OA}+\vec{OB}+\vec{OC}|^2} = \sqrt{1+25+49+35-5\sqrt{3}-7\sqrt{3}} = \sqrt{110-12\sqrt{3}}$

(c) $|\vec{OA}-2\vec{OB}+3\vec{OC}| = \sqrt{49+4\times 25+9-2\times 35+7\sqrt{3}\times 3-6\times 5\sqrt{3}}. = \sqrt{88-9\sqrt{3}}$.

KOKUYO

2.23.

Problem 3.



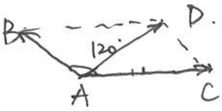
solution: let $\vec{AD}=\vec{AB}+\vec{AC}$. $\vec{CB}=\vec{AB}-\vec{AC}$

By planar geometric method.

~~wh~~ we can find. $\frac{|\vec{AD}|}{2} = \frac{\frac{\sqrt{3}}{4}\cdot(2\sqrt{3})^2\cdot 2}{2\sqrt{3}} = 3.$

$|\vec{AD}| = 6\text{cm}$ $|\vec{CB}| = |\vec{AC}| = 2\sqrt{3}\text{cm}$

Problem 4.



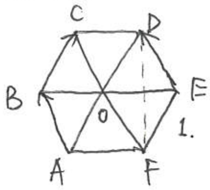
solution. let $\vec{AD}=\vec{AB}+\vec{AC}$.

by planar geometric method.

we can find. $|\vec{AD}| = \sqrt{3}\text{cm}$

$|\vec{BA}+\vec{AC}| = |\vec{BC}| = \sqrt{7}\text{cm}$

Problem 5.



$|\vec{AB}+\vec{BC}| = |\vec{AC}| = \sqrt{3}$

$|\vec{AB}+\vec{BC}+\vec{ED}| = |\vec{AC}+\vec{ED}| = \sqrt{7}$

$|\vec{OD}|+|\vec{DB}| = |\vec{OB}| = 1$

3.8 (Week 3. Tue.)

Assignment 1. proof: $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$

proof: $\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$

$\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$

$\vec{a}\cdot\vec{b} = (a_x\vec{i} + a_y\vec{j} + a_z\vec{k})(b_x\vec{i} + b_y\vec{j} + a_z\vec{k})$.

since $\vec{i}.\vec{j}.\vec{k}$ are perpendicular with each other. every two of their dot product =0.

that is. $\vec{a}\cdot\vec{b} = a_x b_x \vec{i}^2 + a_y \cdot b_y \cdot \vec{j}^2 + a_z b_z \cdot \vec{k}^2$

~~$\vec{a}\cdot\vec{b}$~~ $\vec{i}, \vec{j}, \vec{k}$ are unit vector, their length are 1

$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$

Assignment 2. A.B.C. in Cartesian coordinate system (1,1,1)(1,3,2) and (2,3,-7).

$\vec{a} = \vec{BC} \mapsto (1, 0, -9)$. $|\vec{a}| = \sqrt{82}$.

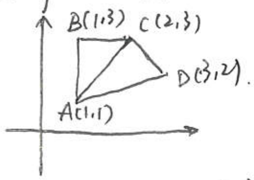
$\vec{b} = \vec{AC} \mapsto (1, 2, -8)$ $|\vec{b}| = \sqrt{69}$.

$\vec{c} = \vec{AB} \mapsto (0, 2, 1)$. $|\vec{c}| = \sqrt{5}$.

$\cos\angle ABC = \frac{-(\vec{a}\cdot\vec{c})}{|\vec{a}|\cdot|\vec{c}|} = \frac{+9}{\sqrt{5}\cdot\sqrt{82}} = \frac{+9}{\sqrt{410}} \approx 0.44$.

$\cos\angle ACB = \frac{(-\vec{a})\cdot(-\vec{b})}{|\vec{a}|\cdot|\vec{b}|} = \frac{73}{\sqrt{82}\cdot\sqrt{69}} = 0.97$.

Assignment 3.



$\cos\angle ABC = \frac{\vec{BC}\cdot\vec{BA}}{|\vec{BC}|\cdot|\vec{BA}|} = 0$.

$\cos\angle BAD = \frac{\vec{AB}\cdot\vec{AD}}{|\vec{AB}|\cdot|\vec{AD}|} = \frac{2}{2\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\cos\angle ADC = \frac{\vec{DC}\cdot\vec{DA}}{|\vec{DC}|\cdot|\vec{DA}|} = \frac{1}{\sqrt{2}\cdot\sqrt{5}} = \frac{\sqrt{10}}{10}$.

$\vec{AB} = (0, 2)$. $\vec{AD} = (2, 1)$.

$\vec{DC} = (-1, 1)$ $\vec{DA} = (-2, -1)$

$\cos\angle BCD = \frac{\vec{CB}\cdot\vec{CD}}{|\vec{CB}|\cdot|\vec{CD}|} = \frac{-1}{\sqrt{2}\cdot 1} = -\frac{\sqrt{2}}{2}$.

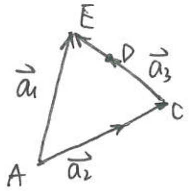
$\vec{CB} = (-1, 0)$ $\vec{CD} = (1, -1)$.

No.

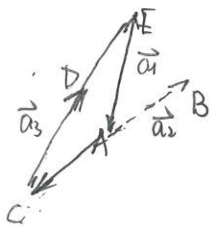
Date

3.9. (Week 3. Thu.)

1.

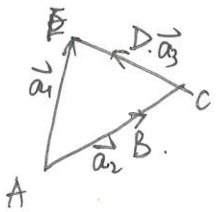


$\vec{a_1} = \beta_2\vec{a_2} + \beta_3\vec{a_3}$ $(\beta_2 > 0, \beta_3 > 0)$



$\vec{a_1} = \beta_2\vec{a_2} + \beta_3\vec{a_3}$ $(\beta_2 < 0, \beta_3 > 0)$

2.



← general case.

special case 1. $\vec{a_1} = 0$ or $\vec{a_2} = 0$ or $\vec{a_3} = 0$.

we let. $x_1\vec{a_1} + x_2\vec{a_2} + x_3\vec{a_3} = 0$. ①

if $a_i = 0$ we let $x_i = c \in \mathbb{R}\setminus 0$, other $x_j = 0$. the equation above holds.

thus, $\vec{a_1}, \vec{a_2}, \vec{a_3}$ are l.d.

special case 2. $\vec{a_1} // \vec{a_2}$.

i.e. we have $\vec{a_1} = c\vec{a_2}$ $(c \in \mathbb{R}\setminus 0)$

we let $x_1 = 1$. $x_2 = -c$. $x_3 = 0$. the equation ① holds.

thus, $\vec{a_1}, \vec{a_2}, \vec{a_3}$ are l.d.

$\vec{a_1} // \vec{a_3}$ or $\vec{a_2} // \vec{a_3}$, similarly.

$a_1 = a_2 = 0$ 的情况

3 (1) $x_1 = x_0 \cdot q = 2$.

$x_2 = x_1 \cdot q = 1$

$x_3 = x_2 \cdot q = \frac{1}{2}$

$x_4 = x_3 \cdot q = \frac{1}{4}$.

(2)

$x' = \frac{e}{e'} x = \frac{1}{2} x$.

$x' = x - a > 0$ for any $x > -7$. $\Rightarrow a \geq \frac{7}{2}$.

$x' = \frac{1}{2}x + \frac{7}{2}$

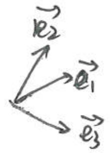
$x' = -\frac{x+1}{2}$

3.14 (Week 4. Tue.).

5. S: Let $|\vec{a}|=1$. $|\vec{b}|=2$. $|\vec{c}|=5$. ~~Let $\vec{a}\ \vec{b}\ \vec{c}$'s corresponding identity vector $\vec{e_1}.\vec{e_2}.\vec{e_3}$ form a basis.~~

$g_{11}=a^2=1$ $\quad g_{12}=\vec{a}\cdot\vec{b}=1$ $\quad g_{13}=\frac{5}{2}$ $\quad g_{22}=4$ $\quad g_{23}=5$ $\quad g_{33}=25$

$$G=\begin{bmatrix}1&1&\frac{5}{2}\\1&4&5\\\frac{5}{2}&5&25\end{bmatrix}$$



the mix product (i.e. the oriented volume)

$$V=(\vec{a}\vec{b}\vec{c})=C_{123}\det\begin{pmatrix}1&0&0\\0&2&0\\0&0&5\end{pmatrix}=(e_1e_2e_3)\cdot 10=e_1\cdot(e_2\times e_3)\cdot 10=\frac{15}{2}$$

$5\sqrt{2}$ 参考一下答案

6. (1) $(1\ 1\ 1)\begin{pmatrix}3&-2&-3\\-2&6&-3\\-3&-3&9\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix}=(-2\ 1\ 3)\begin{pmatrix}1\\2\\3\end{pmatrix}=9.$

(2) $(2,1\ 2)\begin{pmatrix}3&-2&-3\\-2&6&-3\\-3&-3&9\end{pmatrix}\begin{pmatrix}3\\2\\3\end{pmatrix}=(-2\ -4\ 9)\begin{pmatrix}3\\2\\3\end{pmatrix}=13.$

(3) $(0\ 1\ 1)\begin{pmatrix}3&-2&-3\\-2&6&-3\\-3&-3&9\end{pmatrix}\begin{pmatrix}0\\0\\3\end{pmatrix}=(-5\ 3\ 6)\begin{pmatrix}0\\0\\3\end{pmatrix}=18$

✓

3.16. (Week 4. Thu.).

3. Solution: Let $\langle\vec{e_1},\vec{e_2}\rangle=\alpha$. $\qquad \vec{CA}=e_3-2\vec{e_1}-2\vec{e_2} \qquad \vec{CB}=-3\vec{e_1}-\vec{e_2}$

$$\begin{cases}|\vec{CB}|=\sqrt{(2\vec{e_1}+\vec{e_2})^2}=3.\\|\vec{CA}|=\sqrt{(2\vec{e_1}+2\vec{e_2})^2}=2.\end{cases}$$

$$\cos C=\frac{\vec{CA}\cdot\vec{CB}}{|\vec{CA}|\cdot|\vec{CB}|}=0.$$

$$\begin{cases}4e_1^2+e_2^2+4|e_1||e_2|\cos\alpha=9.\\4e_1^2+4e_2^2+8|e_1||e_2|\cos\alpha=4\\6e_1^2+2e_2^2+8|e_1||e_2|\cos\alpha=0\end{cases}\Rightarrow\begin{cases}|e_1|=3.\\|e_2|=\sqrt{11}\\\cos\alpha=-\frac{19\sqrt{11}}{66}\end{cases}$$

参考一下答案

$$\vec{e_1}\cdot\vec{e_2}=\sqrt{11}\cdot 3\cdot -\frac{19\sqrt{11}}{66}=-\frac{19}{2}.$$

$$G=\begin{bmatrix}9&-\frac{19}{2}\\-\frac{19}{2}&11\end{bmatrix}\qquad |e_1|=3\quad |e_2|=\sqrt{11}$$

$\alpha=$ ~~cos~~ $\arccos -\frac{19\sqrt{11}}{66}.$

4. $|e_1|=\sqrt{g_{11}}\qquad |e_2|=\sqrt{g_{22}}\qquad |e_3|=\sqrt{g_{33}}.$

$$\cos\langle e_1,e_2\rangle=\frac{g_{12}}{\sqrt{g_{11}}\cdot\sqrt{g_{22}}}\qquad \cos\langle e_1,e_3\rangle=\frac{g_{13}}{\sqrt{g_{11}}\sqrt{g_{33}}}\qquad \cos\langle e_2,e_3\rangle=\frac{g_{23}}{\sqrt{g_{22}}\sqrt{g_{33}}}$$

$$\cos\langle\vec{\alpha},\vec{e_1}\rangle=\frac{\vec{\alpha}\cdot\vec{e_1}}{|\vec{\alpha}|\cdot|\vec{e_1}|}=\frac{(a_1\vec{e_1}+a_2\vec{e_2}+a_3\vec{e_3})\cdot\vec{e_1}}{\sqrt{a_1^2e_1^2+a_2e_2^2+a_3e_3^2}\ \sqrt{e_1^2}}=\frac{a^1g_{11}+a^2g_{12}+a^3g_{13}}{\sqrt{(a^1)^2g_{11}+(a^2)^2g_{22}+(a^3)^2g_{33}}\cdot\sqrt{g_{11}}}$$

No.

Date

(brought forward)

$$\cos\langle\vec{a},\vec{e_2}\rangle=\frac{a^1g_{12}+a^2g_{22}+a^3g_{23}}{\sqrt{(a^1)^2g_{11}+(a^2)^2g_{22}+(a^3)^2g_{33}}\cdot\sqrt{g_{22}}}$$

$$\cos\langle\vec{a},\vec{e_3}\rangle=\frac{a^1g_{1?}+a^2g_{23}+a^3g_{33}}{\sqrt{(a^1)^2g_{11}+(a^2)^2g_{22}+(a^3)^2g_{33}}\cdot\sqrt{g_{33}}}$$

5. For problem 3.

(1) Problem 4. $\vec{a}=(1,1,1)$ $\vec{b}=(1,2,3)$

$$\vec{a}\times\vec{b}=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}a^ib^jC_{ij}^k e_k=(C_{23}^1a^2b^3+C_{32}^1a^3b^2)\vec{e_1}+(C_{13}^2a^1b^3+C_{31}^2a^3b^1)\vec{e_2}+(C_{12}^3a^1b^2+C_{21}^3a^2b^1)\vec{e_3}$$

$$=\vec{e_1}+(-2)\vec{e_2}+\vec{e_3}=(1,-2,1)$$

effectivized formula: $C_{ij}^k=\pm\sqrt{\begin{vmatrix}3&-2&-3\\-2&6&-3\\-3&-3&9\end{vmatrix}}\sum_{q=1}^{3}\varepsilon_{ijq}g^{qk}$

$$=\pm\sqrt{69}\cdot\sum_{q=1}^{3}\varepsilon_{ijq}g^{qk}$$

(2) Problem 5. $C_{ij}^k=\pm\sqrt{\det G}\sum_{q=1}^{3}\varepsilon_{ijq}g^{qk}$

$$=\pm\sqrt{\begin{vmatrix}1&1&\frac{5}{2}\\1&4&5\\\frac{5}{2}&5&25\end{vmatrix}}\sum_{q=1}^{3}\varepsilon_{ijq}g^{qk}=\pm5\sqrt{2}\sum_{q=1}^{3}\varepsilon_{ijq}.g^{qk}.$$

No.

Date

3.21 (Week6. Tue.)

P9. 18. $M_{AB} = \left(\frac{-3+5}{2}, \frac{4+2}{2}\right) = (1, 3)$ $\quad M_{CD} = \left(\frac{7-5}{2}, \frac{-6-4}{2}\right) = (1, -5)$.

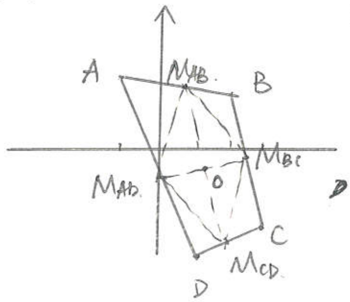
$M_{BC} = \left(\frac{5+7}{2}, \frac{4-6}{2}\right) = (6, -1)$ $\quad M_{DA} = \left(\frac{-5-3}{2}, \frac{-4+2}{2}\right) = (-4, -1)$

the sum of the diagonal $= |AC| + |BD| = \sqrt{(7+3)^2+(2+6)^2} + \sqrt{(5+5)^2+(4+4)^2} = 4\sqrt{41}$.

the perimeter $= |M_{AB}M_{BC}| + |M_{BC}M_{CD}| + |M_{CD}M_{AD}| + |M_{AD}M_{AB}|$

$= \sqrt{41} + \sqrt{41} + \sqrt{41} + \sqrt{41} = 4\sqrt{41}$. = the sum of the diagonal of the first.

P10 20.



(1) $M_{AD} = (0, -1)$. $\quad M_{BC} = (5, -1)$.

$M_{AB} = (1, 5)$. $\quad M_{DC} = (4, -7)$.

$\Rightarrow \frac{x_{M_{AD}} + x_{M_{BC}}}{2} = \frac{x_{M_{AB}} + x_{M_{DC}}}{2} = \frac{5}{2}$. That's $M_{AD}M_{BC}$

$\frac{y_{M_{AD}} + y_{M_{BC}}}{2} = \frac{y_{M_{AB}} + y_{M_{DC}}}{2} = -1$. bisects $M_{AB}M_{DC}$.

(2) slope $M_{AB}M_{BC} = \frac{5+1}{1-5} = -\frac{3}{2}$

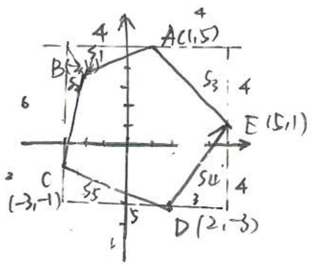
slope $M_{AD}M_{CD} = \frac{-1+7}{0-4} = -\frac{3}{2}$. slopes are equal. $M_{AB}M_{BC} \parallel M_{AD}M_{CD}$.

slope $M_{AB}M_{AD} = \frac{5+1}{1} = 6$.

slope $M_{BC}M_{CD} = \frac{-1+7}{5-4} = 6$. similarly. $M_{AB}M_{AD} \parallel M_{BC}M_{CD}$.

That is, $M_{AB}M_{BC}M_{CD}M_{AD}$ forms a parallelogram

~~P37~~ P37. 11 (C).



$S_{ABCDE} = S - (S_1 + S_2 + S_3 + S_4 + S_5)$

$= 8 \times 8 - (2 + 3 + 5 + 6 + 8)$

$= 40$.

P112. 24. $P_1P_2 \perp P_2P_3$. Dot product of $\overrightarrow{P_1P_2}, \overrightarrow{P_2P_3}$ equals to 0.

$\overrightarrow{P_1P_2} = (k, -1, 3)$ $\quad \overrightarrow{P_2P_3} = (2, k, -1)$

$\overrightarrow{P_1P_2} \cdot \overrightarrow{P_2P_3} = \frac{2k - k - 3}{|P_1P_2| \cdot |P_2P_3|} = 0$ $\quad k = 3$.

No.

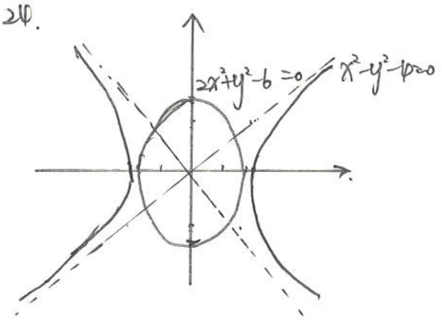
Date

P113. 31. let $A=(3,3,3)$ $B=(1,2,-1)$. $C(4,1,1)$ $D(6,2,5)$.

$\vec{AB}=(-2,-1,-4)$ $\vec{CD}=(2,1,4) \Rightarrow \vec{AB}=\vec{DC}$.

i.e. $AB \parallel CD$. A, B, C, D are coplanar. $\Rightarrow$ A, B, C, D forms a parallelogram

P20. 24.

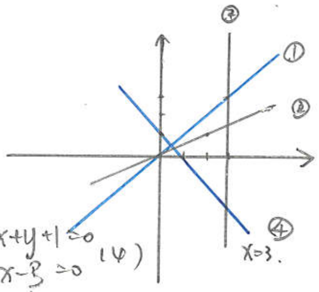


$$\begin{cases} 2x^2+y^2-6=0 \\ x^2-y^2-4=0 \end{cases} \Rightarrow \begin{cases} x=\sqrt{\frac{10}{3}} \\ y=\sqrt{\frac{2}{3}}\, i \end{cases}$$

$y \notin \mathbb{R}$. $2x^2+y^2-6=0 \Rightarrow x \le \sqrt{3}$. $\frac{\sqrt{10}}{\sqrt{3}} > \sqrt{3}$.

thus, the solution is imaginary.

P20 26. $\begin{cases} (x-y)(x+y+1)=0 & ① \\ (x^2-2y)(x-3)=0 & ② \end{cases}$



$\begin{cases} x-y=0 \\ x-2y=0 \end{cases}$ (1) $\begin{cases} x-3=0 \\ x-y=0 \end{cases}$ (2) $\begin{cases} x+y+1=0 \\ x-2y=0 \end{cases}$ (3) $\begin{cases} x+y+1=0 \\ x-3=0 \end{cases}$ (4)

Solution: (1) $(0,0)$ (2) $(3,3)$ (3) $(-\frac{2}{3}, -\frac{1}{3})$ (4) $(3,-4)$.

P20 31. $\sqrt{(x-2)^2+(y-3)^2}=x+2$.

$\Rightarrow y^2-6y-8x+9=0$

P20 44. $\frac{y-1}{x-3} \cdot \frac{y-3}{x+2}=0$. $\Rightarrow x^2+y^2-x-4y-3=0$

$x \in [\frac{-\sqrt{29}-1}{2}, -2) \cup (-2, \frac{\sqrt{29}-1}{2}]$. $y \in [\frac{4-\sqrt{29}}{2}, 1) \cup (1,3) \cup (3, \frac{4+\sqrt{29}}{2}]$.

P114. 54. $\sqrt{(x-3)^2+(y+2)^2+(z-1)^2}=3|z|$.

$\Rightarrow x^2+y^2-8z^2-6x+4y-2z+14=0$. which is a hyperboloid.

Campus

No.

Date

Week6. (3.28).

1. $\vec{a} = \begin{pmatrix}4\\1\end{pmatrix} - \begin{pmatrix}-2\\2\end{pmatrix} = \begin{pmatrix}6\\-1\end{pmatrix}$.

$\vec{r} = \begin{pmatrix}2\\-3\end{pmatrix} + \begin{pmatrix}6\\-1\end{pmatrix}t$.

slope $m = \frac{y_a}{x_a} = -\frac{1}{6}$.

intercept-x: $x = x_0 - \frac{y_0 x_a}{y_a} = 2 + \frac{3\times(6)}{-1} = -16$

-y: $y = y_0 - \frac{x_0 y_a}{x_a} = -3 \div \frac{2\times(-1)}{6} = -\frac{8}{3}$.

$\vec{n} \perp \vec{a}$. let $\vec{n} = \begin{pmatrix}1\\6\end{pmatrix}$

$\vec{r}\cdot\vec{n} = \vec{r_0}\cdot\vec{n} \Rightarrow \vec{r}\cdot\begin{pmatrix}1\\6\end{pmatrix} = -16$.

2. slope $m = \frac{-1}{\frac{2}{3}} = -\frac{3}{2}$. let $\vec{n} = \begin{pmatrix}3\\2\end{pmatrix}$.

Let $\vec{a} = \begin{pmatrix}-2\\3\end{pmatrix}$. $\vec{r} = \begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}-2\\3\end{pmatrix}t$. $\vec{r}\cdot\begin{pmatrix}3\\2\end{pmatrix} = 0$

$x = x_0 - \frac{y_0 x_a}{y_a} = -2 - \frac{3\cdot(-2)}{3} = 0$.

$y = y_0 - \frac{x_0 y_a}{x_a} = 3 - \frac{(-2)\cdot 3}{(-2)} = 0$.

3. midpoint of $(7,4)$ and $(-1,-2)$ is $(3,1)$, which is passed through by the line

$m = \frac{-1}{\frac{4+2}{7+1}} = -\frac{4}{3}$ Let $\vec{a} = \begin{pmatrix}3\\-4\end{pmatrix}$.

$\vec{r} = \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}3\\-4\end{pmatrix}t$.

Let $\vec{n} = \begin{pmatrix}4\\3\end{pmatrix}$. $\vec{r}\cdot\begin{pmatrix}4\\3\end{pmatrix} = 15$.

~~4. $\vec{r}_{AB} = \begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}4\\-10\end{pmatrix}t_1$~~

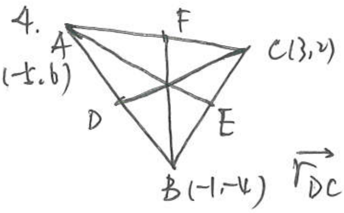
~~$\vec{r}_{AC} = \begin{pmatrix}-1\\-4\end{pmatrix} + \begin{pmatrix}8\\-4\end{pmatrix}t_2$~~

KOKUYO

$\cancel{\vec{r}_{BC} = \begin{pmatrix} -5 \\ 6 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix} t_3}$

$\cancel{\begin{cases} \vec{r} = \begin{pmatrix} -5 \\ 6 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix} t_3 \\ \vec{r} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} + \begin{pmatrix} 8 \\ -4 \end{pmatrix} t_2 \end{cases}} \Rightarrow \cancel{\begin{cases} -5+4t_3 = -1+8t_2 \\ 6+6t_3 = -4-4t_2 \end{cases}} \Rightarrow \cancel{\begin{cases} t_2 = -1 \\ t_3 = -1 \end{cases}} \Rightarrow \cancel{\begin{matrix} \vec{r}_{BC} = \begin{pmatrix} -9 \\ 0 \end{pmatrix} \\ \vec{r}_{AC} = \begin{pmatrix} -9 \\ 0 \end{pmatrix} \end{matrix}}$

4.



suppose D. E. F be midpoints of AB. BC. AC

D(-3, 1) E(1, -1) F(-1, 4).

$\vec{r}_{DC} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 6 \\ 1 \end{pmatrix} t_1$

$\vec{r}_{EA} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -6 \\ 7 \end{pmatrix} t_2.$

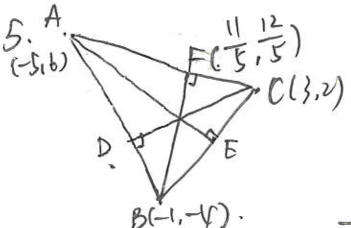
$\vec{r}_{FB} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 8 \end{pmatrix} t_3.$

First we let $\begin{cases} r = \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 6 \\ 1 \end{pmatrix} t_1 \\ r = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -6 \\ 7 \end{pmatrix} t_2 \end{cases} \Rightarrow \begin{cases} t_1 = -\frac{1}{3} \\ t_2 = \frac{1}{3} \end{cases} \Rightarrow r_0 = \begin{pmatrix} -1 \\ \frac{4}{3} \end{pmatrix}.$

$\begin{cases} r = \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 6 \\ 1 \end{pmatrix} t_1 \\ r = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 8 \end{pmatrix} t_3 \end{cases} \Rightarrow \begin{cases} t_1 = \frac{1}{3} \\ t_2 = -\frac{1}{3} \end{cases} \Rightarrow r_0 = \begin{pmatrix} -1 \\ \frac{4}{3} \end{pmatrix}$

that is, centroid point $(-1, \frac{4}{3})$ is the intersection point of three median line.

5.



also let D. E. F be foot points.

by the method above.

we can calculate that. $D(-\frac{73}{29}, -\frac{6}{29})$ $E(\frac{31}{13}, \frac{14}{13})$ $F(\frac{11}{5}, \frac{12}{5})$

$\overline{r}_{CD} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -\frac{160}{29} \\ -\frac{64}{29} \end{pmatrix} t_1$

$\overline{r}_{BF} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} + \begin{pmatrix} \frac{16}{5} \\ \frac{32}{5} \end{pmatrix} t_2$

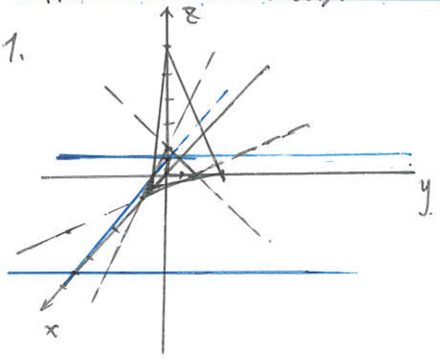
$\overline{r}_{AE} = \begin{pmatrix} -5 \\ 6 \end{pmatrix} + \begin{pmatrix} \frac{96}{13} \\ -\frac{64}{13} \end{pmatrix} t_3$

$\begin{cases} r = \begin{pmatrix} -1 \\ -4 \end{pmatrix} + \begin{pmatrix} \frac{16}{5} \\ \frac{32}{5} \end{pmatrix} t_1 \\ r = \begin{pmatrix} -5 \\ 6 \end{pmatrix} + \begin{pmatrix} \frac{96}{13} \\ -\frac{92}{13} \end{pmatrix} t_2 \end{cases}$

$\Rightarrow$ intersection point $(\frac{7}{4}, \frac{3}{2})$

4.13 (Week 8 Thu.)

1.



$$\begin{cases} x+y+z=1 \\ 8x+3y-z+5=0 \\ 2x-5z=7 \end{cases} \Rightarrow \text{the intersection point } (-4, 8, -3).$$

2. since $x+y+z=5$ and $2x-3y+z=1$ is not parallel. the beam is proper

assume the plane is $Ax+By+Cz+D=0$.

$$\text{rank} \begin{bmatrix} 1 & 1 & 1 & -5 \\ 2 & -3 & 1 & -1 \\ A & B & C & D \end{bmatrix} < 3.$$

$$\begin{bmatrix} 1 & 1 & 1 & -5 \\ 2 & -3 & 1 & -1 \\ A & B & C & D \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & -5 \\ 0 & -5 & -1 & 9 \\ 0 & B-A & C-A & D+5A \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & -5 & -1 & 9 \\ 0 & 0 & C-A-\frac{B-A}{5} & D+5A+\frac{9(B-A)}{5} \end{bmatrix}$$

thus we have $\begin{cases} A+B+C+D=0 \\ 5C-4A-B=0 \\ 5D+16A+9B=0 \end{cases} \Rightarrow \begin{cases} B=-\frac{7}{3}A. \\ C=\frac{1}{3}A \\ D=A. \end{cases}$

$\Rightarrow 3x-7y+z+3=0$

(3.) Find proper bundle.

$$\begin{cases} x+y+z-1=0 & ① \\ 8x+3y-z-13=0 & ② \\ 2x-5z-7=0 & ③ \\ 2x-y-z+4=0 & ④ \\ 5y-4z-13=0 & ⑤ \\ 6x-4z-13=0 & ⑥ \end{cases}$$

by calculate the determinants of order 4. $\neq 0$.

there are no 4 plane in a proper bundle.

then check the coefficient determinants of order 3.

all nonzero.

any three planes are in the ~~same determ~~ proper bundle

KOKUYO

No.

Date

4.18 (Week 9. Tue.)

P1.1

First find the center of the bundle

$$\begin{cases} x+y-z=-2 \\ 4x-3y+z=1 \\ 2x+y=5 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=3 \\ z=6 \end{cases}$$

parallel with xOz $\Rightarrow$ $y-3=0$

P2.1. the direction vector $\vec{a} = \begin{pmatrix} 4-(-2) \\ 1-2 \\ 1-2 \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \\ -1 \end{pmatrix}$.

$$\vec{r} = \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ -1 \\ -1 \end{pmatrix}$$

z-intercept: no. since $\frac{x_0}{a_x} \neq \frac{y_0}{a_y}$.

x-intercept: $x = x_0 - a_x \frac{y_0}{a_y} = x_0 - a_x \frac{z_0}{a_z} = 2 - 6\cdot(-3) = 20$.

y-intercept: no. since $\frac{x_0}{a_x} \neq \frac{z_0}{a_z}$

$\cos\alpha = \frac{a_x}{\sqrt{a_x^2+a_y^2+a_z^2}} = \frac{6}{\sqrt{38}}$ $\quad \cos\beta = -\frac{1}{\sqrt{38}}$ $\quad \cos\gamma = -\frac{1}{\sqrt{38}}$

P2.2. let $A(3,0,1)$ $B(0,2,4)$ $C.(1,\frac{4}{3},3)$.

direction vectors $\vec{a}_{AB} = (3,-2,-3)$ $\quad \vec{a}_{AC} = (2,-\frac{4}{3},-2)$. $\quad \vec{a}_{BC} = (-1,\frac{2}{3},1)$

$\vec{a}_{AB} = -3\vec{a}_{BC} = \frac{3}{2}\vec{a}_{AC}$.

all of the direction vectors are colinear implies the three points are colinear.

Campus

No.
Date

4.20. (Thu. Week 9).

P129. 14. the normal vector of plane, as well as the direction vector of line

$\vec{a} = \vec{n} = (1, -3, 2)$.

the vectorial parametric equation:

$$r = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} t$$

the canonical equation

$$x-1 = \frac{y+2}{-3} = \frac{x+3}{2}.$$

29. direction vector $\vec{a} = \begin{pmatrix} 1-(-2) \\ 3-2 \\ 4-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$.

$$\vec{r} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} t. \qquad \frac{x+2}{3} = y-4 = z-3.$$

27. line 1. $3x-2y+13=0 \quad y+3z-26=0$.

$$\Rightarrow \frac{x+3}{2} = \frac{y-2}{3} = \frac{z-8}{-1} \quad \Rightarrow \vec{r_1} = \begin{pmatrix} -3 \\ 2 \\ 8 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} t$$

line 2. $$\frac{x+4}{5} = \frac{y-1}{-3} = \frac{z-3}{1} \quad \Rightarrow \vec{r_2} = \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix} t.$$

$$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix} = 10-9-1 = 0.$$ the directional vector perpendicular.
$\Rightarrow$ the lines are perpendicular

4.25 (Tue. Week 10.).

1. the altitude from A is the distance from A to line BC.

$$d = \frac{|\vec{AB} \times \vec{AC}|}{|BC|} = \frac{(5,-3,3) \times (3,-2,0)}{\sqrt{14}} = \frac{\sqrt{118}}{\sqrt{14}} = \sqrt{\frac{59}{7}} = \frac{\sqrt{413}}{7}.$$

2. the line of abiscissa axis 为 $x=t$.

$$d = \sqrt{(t-5)^2 + 6^2 + 8^2} \qquad \cancel{d_{min}}\ d(t)_{min} = d(5) = 10.$$

No.

Date

3.

$$\begin{cases} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} t \\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} \tau . \end{cases}$$

$$\vec{n} = \frac{\vec{a} \times \vec{a'}}{|\vec{a} \times \vec{a'}|} = \frac{(-5, 10, -10)}{15} = \left(-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right).$$

$$d = \left| \vec{n} \cdot (\vec{r_0} - \vec{r_0'}) \right| = \left| \begin{pmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{2}{3} \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -3 \\ 4 \end{pmatrix} \right| = \frac{20}{3}.$$