Sept 6th.

$$F(p)=\frac{2p}{(p-1)\left(p^2-2p-3\right)}.$$

Sol: $F(p) = 2p \cdot \frac{1}{p-1} \cdot \frac{1}{(p-1)^2-4}$

$e^t \leftrightarrow \frac{1}{p-1}$ $\quad \frac{1}{2}e^t sh2t \leftrightarrow \frac{1}{(p-1)^2-4}$

by Duhamel's integral, where $f_1(t) = \frac{1}{2}e^t sh2t$ $\quad f_2(t) = e^t$

$f(t) = 2\left[f_1(t)f_2(0) + \int_0^t f_1(\tau) f_2'(t-\tau)\, d\tau\right]$

$= 2\left[\frac{1}{2}e^t sh2t + \int_0^t \frac{1}{2}e^\tau \frac{e^{2\tau}-e^{-2\tau}}{2} \cdot e^{t-\tau}\, d\tau\right]$

$= 2\left[\frac{1}{2}e^t sh2t + \frac{e^t}{4}\int_0^t (e^{2\tau}-e^{-2\tau})\, d\tau\right] = e^t sh2t + \frac{e^t}{2}\left(\frac{e^{2t}}{2}-\frac{1}{2}+\frac{e^{-2t}}{2}-\frac{1}{2}\right)$

$= \frac{1}{2}e^{3t} - \frac{1}{2}e^{-t} + \frac{1}{4}e^{3t} + \frac{1}{4}e^{-t} - \frac{e^t}{2} = \frac{3}{4}e^{3t} - \frac{1}{4}e^{-t} - \frac{1}{2}e^t$

$$F(p)=\frac{1}{p(p-1)\left(p^2+4\right)}$$

Sol: denote $F(p) = \frac{A}{p} + \frac{B}{p-1} + \frac{Cp+D}{p^2+4} = \frac{A(p-1)(p^2+4) + Bp(p^2+4) + (Cp+D)p(p-1)}{p(p-1)(p^2+4)}$

i.e. $\begin{cases} A+B+C=0 \\ -A+D-C=0. \\ 4A+4B-D=0 \\ -4A = 1 \end{cases} \Rightarrow \begin{cases} A = -\frac{1}{4}. \\ B = \frac{1}{5} \\ C = \frac{1}{20} \\ D = -\frac{1}{5} \end{cases}$

$f(t) = -\frac{1}{4} + \frac{1}{5}e^t + \frac{1}{20}\cos 2t - \frac{1}{10}\sin 2t$

$$F(p)=\frac{p-1}{(p+1)\left(p^2+4\right)}$$

Sol: pole: $p_1 = -1$ (simple) $\quad p_{2,3} = \pm 2i$ (simple) $\quad$ - all poles are simple.

denote $F(p) = \frac{\varphi(p)}{\psi(p)}$. $\quad \psi'(p) = p^2+4 + 2p(p+1) = 3p^2+2p+4.$

$f(t) = \sum_{k=1}^{3} \frac{\varphi(p_k)}{\psi'(p_k)} e^{p_k t} = \left.\frac{(p-1)\cdot e^{pt}}{3p^2+2p+4}\right|_{p=-1} + \left.\frac{(p-1)\cdot e^{pt}}{3p^2+2p+4}\right|_{p=2i} + \left.\frac{(p-1)\cdot e^{pt}}{3p^2+2p+4}\right|_{p=-2i}$

$= -\frac{2}{5}\cdot e^{-t} + \frac{(-1+2i)\cdot e^{2it}}{-8+4i} + \frac{(1+2i)\cdot e^{-2it}}{8+4i}$