

Equations of Mathematical Physics Homework 6

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1 Problem 1

Solve the Cauchy problem for the equation

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2xy,$$

satisfying the conditions $y = x \quad u = x^2$

Solution:

Let's make up a characteristic system:

$$\frac{dx}{x} = \frac{dy}{y} = \frac{du}{2xy}$$

We will find the first integral by solving the equation:

$$\frac{dx}{x} = \frac{dy}{y} \implies \frac{x}{y} = C_1$$

then we have $x = C_1 y$, substitute $C_1 y$ into x , we have the second integral:

$$\frac{dx}{C_1 y} = \frac{du}{2xy} \implies 2x dx = C_1 du \implies x^2 = C_1 u + C_2 \implies C_2 = \frac{xu}{y} - x^2$$

thus we find the general solution without considering the integral surface:

$$\Phi\left(\frac{x}{y}, \frac{xu}{y} - x^2\right) = 0$$

Solving the Cauchy problem, take x as the parameter, we obtain:

$$C_1 = 1 \quad C_2 = 0$$

Thus the Cauchy problem becomes indefinite, which means the solution has form $\Phi\left(\frac{x}{y}, \frac{xu}{y} - x^2\right) = 0$, since only the second integral contains u , it can be rewritten as:

$$u = F\left(\frac{x}{y}\right) + xy \quad \text{— } F \text{ is arbitrary differentiable function such that } F(1) = 0.$$