Exam Ticket List

1. Classical definition of probability. Sample space and elementary outcomes. Operations on events: union, intersection, complement. Basic combinatorics. Geometry probabilities. Berthran paradox.

2. Probability space. Properties of a probability measure. Kolmogorov's axioms. Property of probability. Addition rule for mutually exclusive events. Inclusion-exclusion principle

3. Conditional probability: definition and properties. Multiplication rule of probabilities. Law of total probability. Bayes' theorem

4. Definition of independence of events and its property. Pairwise and mutual independence. Difference between independence and mutual exclusivity. Some prop

5. Bernoulli trials: definition and formula. Probability of exactly k successes. Probability of at least or at most k successes. Most probable number of successes (mode)

6. Poisson theorem (with proof), formula and properties Practical applications of Poisson distribution.

7. Local Moivre–Laplace theorem (with proof). Gaussian (normal) function and its role.

8. Integral Moivre–Laplace theorem (with proof). Laplace function and its properties.

9. Parametrazation of the Bernoulli trials. The law of large number for Bernoulli trials (with proof).

10. Definition of a random variable, Borel sigma-algebra and Generated sigma-algebra of a random variable. Distribution of a random variable as a measure.

11. Distribution function: definition and properties, Types of random variables: discrete, continuous, mixed. Density function and its property. Expectation and its properties, Variance, standard deviation. Examples.

12. Transformation of random variables: definition and examples. Distribution of a function of a random variable. Quantile transformation, Smirnov's method of modeling of random variables.

13. Central and raw moments, Mode, quantiles, skewness, kurtosis, quantile. Examoles.

14. Characteristic function: definition, examples of calculation, and properties, non-negative definiteness (with proof). Theorem of reconstraction of a distribution from its characteristic function.

15. Moments of random variable and characteristic function derivatives (with proof Existence of moments =¿ Esistence of derivatives and with idea of "¡=" proof). Bochner-Khinchin theorem

16. Probability-generating function (for discrete variables) with examples, Laplace transform function, Moment-generating function

17. Random vectors, joint distribution, independence, marginal distributions, property of joint distribution function and the important example (when properties F0-F4 are not enough)

18. Covariance and correlation their properties (with proofs). Moments of random vectors. Covariance Matrix and its properties (with proof).
19. Multivariate Normal distribution and its properties (with proof for uncorrelated random variables).
20. Transformation of random vectors, Unidimensional and multidimensional Functions of random vectors and its distributions. Some important examples (convolutions, quotient and etc).
21. Conditional expectation (given event and sigma-algebra). Formula for $E(X|Y)$. Properties of conditional expectation
22. Jensen's inequality, examples.
23. Markov's and Chebyshev's inequalities for random variable and for sums, corollary
24. Types of Convergence of Random Variables. Relationships Between Different Types of Convergence. Important Examples (Showing That One Type Does Not Necessarily Imply Another).
25. Borel-Cantelli lemmas (the first and the second). Application for Almost surely convergences.
26. Laws of Large Numbers, corollary.
27. Central Limit Theorem and their application.
28. Strong laws of Large Number (The first and the second).