Taylors series

TASKS

Using the definition of the functions of the complex variable $\cos z$ and $\sin z$, prove that:

1. $\sin z \cdot \cos z = \frac{1}{2} \sin 2z$
2. $\sin^2 z + \cos^2 z = 1.$

Using the definition of the functions of the complex variable $\operatorname{sh} z$ and $\operatorname{ch} z$, prove that:

3. $\operatorname{ch}^2 z - \operatorname{sh}^2 z = 1$
4. $\operatorname{ch} 2z = \operatorname{ch}^2 z + \operatorname{sh}^2 z.$

Decompose the z function in a series of degrees:

5. $e^z \sin z = \sum_{n=0}^{\infty} \frac{2^{n/2} sin(\pi n/4)}{n!} z^n$
6. $\operatorname{ch} z \cos z = \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n}}{(4n)!} z^{4n}$
7. $e^{z \operatorname{ctg} \alpha} \cos z, \quad \sin x \neq 0 \quad e^{z \operatorname{ctg} \alpha} \cos z = \sum_{n=0}^{\infty} \frac{cos(n\alpha)}{sin^n \alpha} \frac{z^n}{n!}$
8. $e^{z \cos \alpha} \cos(z \sin \alpha) = \sum_{n=0}^{\infty} \frac{cos(n\alpha)}{n!} z^n.$