1. Suppose $f, g \in L^p(X, \mu)$. Prove that $f + g \in L^p(X, \mu)$.

2. Suppose $f \in L^p(X, \mu)$, $g \in L^\infty$. Prove that $fg \in L^p(X, \mu)$.

3. Is it true that every a.e. convergenct sequence of functions contains subsequence that is convergent in measure? Explain the answer.

4. Is it true that $L^1(x, \mu) \subset L^p(X, \mu)$ if $p > 1$? Is it true that $L^p(x, \mu) \subset L^1(X, \mu)$ if $p > 1$?

5. Let $(X, \mathcal{A})$ be a measurable space. The identify map

$$\text{id} : (X, \mathcal{A}) \to (X, \mathcal{A})$$

is map such that $\text{id}(x) = x$. Is this map measurable?

6. Let $(X, \mathcal{A})$ be a measurable space, $E \subset X$. Under which condition the characteristic function χ_E is measurable?

7. Provide the definition of $\sigma-$algebra generated by a family of subsets.

8. Provide the definition of the integral with respect to a measure (for simple functions, nonnegative measurable functions, arbitrary measurable functions). What is the condition for existence of the integral?

9. Choose the correct statements. Explain the choice.

   (a) If $\int_E f(x)d\mu = 0$ then $f = 0$ a.e.
   (b) If $\int_E |f(x)|\, d\mu = 0$ then $f = 0$ a.e.
   (c) If f is finite a.e. then $\int_E f(x)d\mu$ is finite.

10. How is the norm in space $L^\infty(X, \mu)$ defined?

11. Find condition on p, q under which the integral

$$\int_{\mathbb{R}^2} \frac{dxdydz}{(x^2 + y^2 + z^2)^p(1 + x^2 + y^2 + z^2)^q}$$

is finite.

12. Find conditions on $p, q \in \mathbb{R}$ under which

$$I = \iint_{|x|+|y|\leq 1} \frac{dxdy}{|x|^p + |y|^q} < \infty.$$