Calculus of variations. Theoretical questions.

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1. What is an integral functional?
2. What is the Lagrangian?
3. What is a variational problem with fixed ends?
4. Derive the first variation of an integral functional in the problem with fixed ends.
5. Necessary condition for existence of an extremum in the case of variational problem with fixed ends.
6. Proof of the necessary condition for existence of an extremum in the case of variational problem with fixed ends.
7. Derive the Lagrange equations for the Fermat functional.
8. Derive the first variation of an integral functional in the problem with free ends.
9. Necessary condition for existence of an extremum in the case of variational problem with free ends.
10. What are the natural boundary conditions?
11. Proof of the necessary condition for existence of an extremum in the case of variational problem with free ends.
12. Integral of motion in the case when the Lagrangian does not depend on time.
13. Integral of motion in the case when the Lagrangian does not depend on a coordinate.
14. Definition of the second variation in the case of variational problem with fixed ends.
15. Necessary condition for a curve x_0 to be minimal in the case of variational problem with fixed ends.
16. Proof of the necessary condition for a curve x_0 to be minimal in the case of variational problem with fixed ends.
17. Sufficient condition for a curve x_0 to be minimal in the case of variational problem with fixed ends.
18. Necessary condition for a quadratic functional to be non-negative.
19. Legendre necessary condition for a curve x_0 to be minimal in the case of variational problem with fixed ends.
20. Definition of a conjugate point.
21. Sufficient condition for a quadratic functional to be positive.
22. Jacobi sufficient condition for a curve x_0 to be minimal in the case of variational problem with fixed ends.
23. General formula for the first variation.
24. Necessary condition for a curve x_0 to be minimal in the case of variational problem with movable ends.
25. What are the transversality conditions.

26. Necessary condition for a curve x_0 to be minimal in the case of the isoperimetric variational problem.

27. Necessary condition for a curve x_0 to be minimal in the case of the Lagrange problem.

28. Derive the Lagrange equation for a integral functional depending on high order derivatives.

29. Definition of the Hamiltonian.

30. Derive the Hamilton equationin in the case of variational problem with fixed ends.