HW Chapter 1. Aug. 30th.

4) $f(t)=\dfrac{\sin^2 t}{t}$.

Sol: $|f(t)| \leq |\frac{\sin t}{t}| \cdot |\sin t| \leq 1 \cdot |\sin t| \leq 1$. on $t \in [0,+\infty)$. thus $\alpha_0 = 0$. $M=1$.

denote $g(t) = \sin^2 t = \frac{1-\cos 2t}{2}$.

$g(t) \leftrightarrow G(p)$. by the table, $G(p) = \frac{1}{2}\left(\frac{1}{p} - \frac{p}{p^2+4}\right). = \frac{1}{2} \cdot \frac{p^2+4-p^2}{p(p^2+4)} = \frac{2}{p(p^2+4)}$

by the property of integration of image.

$f(t) = \frac{g(t)}{t} \leftrightarrow F(p) = \int_p^{\infty} G(\xi)\, d\xi. = \frac{1}{2}\int_p^{\infty}\left(\frac{1}{\xi} - \frac{\xi}{\xi^2+4}\right) d\xi = \int_p^{\infty} \frac{d\xi^2}{\xi^2(\xi^2+4)} = \frac{1}{4} \ln\left(1+\frac{4}{p^2}\right)$

i.e. $f(t) = \frac{g(t)}{t} \leftrightarrow F(p) = \frac{1}{4} \ln\left(1+\frac{4}{p^2}\right)$, $\mathrm{Re}\,p > 0$