Sept. 2nd.

11) Find the convolution and the image of the convolution (by the properties of the Laplace transform and by the convolution theorem). b) $\sin t * t$.

denote $f(t) = \sin t$, $g(t) = t$.

$$(f \cdot g)(t) = \int_0^t f(\tau)\, g(t-\tau)\, d\tau = \int_0^t \sin\tau\,(t-\tau)\, d\tau = t\left(-\cos\tau \Big|_0^t\right) - \int_0^t \sin\tau \cdot \tau\, d\tau$$

$$= t(1-\cos t) - \left(-\cos\tau \cdot \tau \Big|_0^t + \int_0^t \cos\tau\, d\tau\right) = t(1-\cos t) + t\cos t - \sin t = t - \sin t.$$

$F(p) = \frac{1}{p^2+1}$ $\quad G(p) = \frac{1}{p^2}$ by the table.

$(f \cdot g)(t) \Leftrightarrow F(p)\,G(p) = \frac{1}{p^2(p^2+1)}$

12) Find images of the following functions: $f(t) = \int_0^t e^{2(\tau - t)} \tau^2 d\tau$

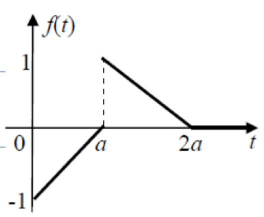
denote $g(t) = t^2$. $\quad h(t) = e^{2t}$

$f(t) = g(t) * h(t)$.

$g(t) \Leftrightarrow G(p) = \frac{2}{p^3}$ $\qquad h(t) \Leftrightarrow H(p) = \frac{1}{p-2}$ $\qquad \alpha_0 = 2.$

$f(t) \Leftrightarrow G(p)\,H(p) = \frac{2}{(p-2)p^3}$ $\qquad \mathrm{Re}\, p > 2.$

14) Find images of the following functions defined graphically:



$$f(t) = \begin{cases} 0, & t \le 0,\ t \ge 2a \\ \frac{1}{a}(t-a), & 0 < t \le a \\ -\frac{1}{a}(t-2a), & a < t \le 2a \end{cases}$$

define $f_1(t) = \begin{cases} \frac{1}{a}(t-a), & 0 < t \le a \\ 0, & \text{others} \end{cases} = \frac{1}{a}(t-a)\,\theta(t) - \frac{1}{a}(t-a)\,\theta(t-a)$

$f_2(t) = \begin{cases} -\frac{1}{a}(t-2a), & a < t \le 2a \\ 0, & \text{others.} \end{cases} = -\frac{1}{a}(t-2a)\,\theta(t-a) + \frac{1}{a}(t-2a)\,\theta(t-2a).$

$$f(t) = f_1(t) + f_2(t) = -\theta(t) + \frac{1}{a}t\,\theta(t) - \frac{2}{a}(t-a)\,\theta(t-a) + \theta(t-a) + \frac{1}{a}(t-2a)\,\theta(t-2a).$$

$$F(p) = -\frac{1}{p} + \frac{1}{ap^2} - \frac{2}{a} \cdot e^{-ap} \cdot \frac{1}{p^2} + e^{-ap} \cdot \frac{1}{p} + \frac{1}{a} \cdot e^{-2ap} \cdot \frac{1}{p^2}$$

$$= \frac{1}{p}\left(e^{-ap} - 1\right) + \frac{1}{ap^2}\left(-2e^{-ap} + e^{-2ap}\right)$$