

Sept 6th.

$$F(p) = \frac{2p}{(p-1)(p^2-2p-3)}$$

Sol: $F(p) = 2p \cdot \frac{1}{p-1} \cdot \frac{1}{(p-1)^2-4}$

$$e^t \leftrightarrow \frac{1}{p-1} \quad \frac{1}{2} e^t \sinh 2t \leftrightarrow \frac{1}{(p-1)^2-4}$$

by Duhamel's integral, where $f_1(t) = \frac{1}{2} e^t \sinh 2t$ $f_2(t) = e^t$

$$\begin{aligned} f(t) &= 2 \left[f_1(t) f_2(0) + \int_0^t f_1(\tau) f_2'(t-\tau) d\tau \right] \\ &= 2 \left[\frac{1}{2} e^t \sinh 2t + \int_0^t \frac{1}{2} e^\tau \frac{e^{2\tau} - e^{-2\tau}}{2} \cdot e^{t-\tau} d\tau \right] \\ &= 2 \left[\frac{1}{2} e^t \sinh 2t + \frac{e^t}{4} \int_0^t (e^{2\tau} - e^{-2\tau}) d\tau \right] = e^t \sinh 2t + \frac{e^t}{2} \left(\frac{e^{2t}}{2} + \frac{e^{-2t}}{2} \right) \\ &= \frac{1}{2} e^{3t} - \frac{1}{2} e^{-t} + \frac{1}{4} e^{3t} + \frac{1}{4} e^{-t} = \frac{3}{4} e^{3t} - \frac{1}{4} e^{-t} \end{aligned}$$

$$F(p) = \frac{1}{p(p-1)(p^2+4)}$$

Sol: denote $F(p) = \frac{A}{p} + \frac{B}{p-1} + \frac{Cp+D}{p^2+4} = \frac{A(p-1)(p^2+4) + Bp(p^2+4) + (Cp+D)p(p-1)}{p(p-1)(p^2+4)}$

$$\text{i.e. } \begin{cases} A+B+C=0 \\ -A+D-C=0 \\ 4A+4B-D=0 \\ -4A=1 \end{cases} \Rightarrow \begin{cases} A = -\frac{1}{4} \\ B = \frac{1}{5} \\ C = \frac{1}{20} \\ D = -\frac{1}{5} \end{cases}$$

$$f(t) = -\frac{1}{4} + \frac{1}{5} e^t + \frac{1}{20} \cos 2t - \frac{1}{10} \sin 2t$$

$$F(p) = \frac{p-1}{(p+1)(p^2+4)}$$

Sol: pole: $p_1 = -1$ (simple) $p_{2,3} = \pm 2i$ (simple) - all poles are simple.

denote $F(p) = \frac{\psi(p)}{\chi(p)}$ $\chi'(p) = p^2+4 + 2p(p+1) = 3p^2+2p+4$

$$\begin{aligned} f(t) &= \sum_{k=1}^3 \frac{\psi(p_k)}{\chi'(p_k)} e^{p_k t} = \frac{(p-1) \cdot e^{pt}}{3p^2+2p+4} \Big|_{p=-1} + \frac{(p-1) \cdot e^{pt}}{3p^2+2p+4} \Big|_{p=2i} + \frac{(p-1) \cdot e^{pt}}{3p^2+2p+4} \Big|_{p=-2i} \\ &= -\frac{2}{5} \cdot e^{-t} + \frac{(-1+2i) \cdot e^{2it}}{-8+4i} + \frac{(1+2i) \cdot e^{-2it}}{8+4i} \end{aligned}$$