

TESTS

Models and Methods of Tropical Mathematics

Please submit a solution to the test online as a readable file preferably prepared with an editor, and include intermediate results to facilitate and speed up verification of the solutions obtained.

TEST-MMTM 1.

Problem 1. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

Problem 2. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix}.$$

TEST-MMTM 2.

Problem 3. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 4 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

Problem 4. Find in $\mathbb{R}_{\min,\times}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}.$$

TEST-MMTM 3.

Problem 5. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & -3 \\ -1 & -1 & -2 \\ 1 & -3 & -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$

Problem 6. Find in $\mathbb{R}_{\min,\times}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

TEST-MMTM 4.

Problem 7. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

Problem 8. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{Ax} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 1/2 & 1 & 1/3 \\ 1 & 1 & 1/2 \end{pmatrix}.$$

TEST-MMTM 5.

Problem 9. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.$$

Problem 10. Find in $\mathbb{R}_{\max,\times}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 4 & 3 \\ 1 & 2 & 1/2 \\ 1/2 & 1 & 1 \end{pmatrix}.$$

TEST-MMTM 6.

Problem 11. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 4 & 3 \\ 2 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 12. Find in $\mathbb{R}_{\max,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 4 & 3 \\ -1 & 1 & 1 \\ 3 & -1 & 1 \end{pmatrix}.$$

TEST-MMTM 7.

Problem 13. Find in $\mathbb{R}_{\min,\times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 2 & 2 \\ 3 & 1/2 & 1 \end{pmatrix}.$$

Problem 14. Find in $\mathbb{R}_{\min,\times}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 2 \end{pmatrix}.$$

TEST-MMTM 8.

Problem 15. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \\ 0 & -2 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}.$$

Problem 16. Find in $\mathbb{R}_{\min,\times}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 2 \end{pmatrix}.$$

TEST-MMTM 9.

Problem 17. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}.$$

Problem 18. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} -2 & 1 & -2 \\ -1 & 0 & -3 \\ 0 & 1 & -1 \end{pmatrix}.$$

TEST-MMTM 10.

Problem 19. Find in $\mathbb{R}_{\min, \times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}.$$

Problem 20. Find in $\mathbb{R}_{\min,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & -2 \\ 3 & 4 & 2 \end{pmatrix}.$$

TEST-MMTM 11.

Problem 21. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} -1 & 1 & -2 \\ -1 & -1 & -3 \\ 1 & 1 & 0 \end{pmatrix}.$$

Problem 22. Find in $\mathbb{R}_{\max, \times}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 1/2 & 1 & 1/3 \\ 1/2 & 1/3 & 1 \\ 1 & 1/2 & 1/2 \end{pmatrix}.$$

TEST-MMTM 12.

Problem 23. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} -1 & 0 & 1 \\ 1 & 2 & 0 \\ 3 & 3 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}.$$

Problem 24. Find in $\mathbb{R}_{\max, \times}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/2 & 1 \\ 2 & 1 & 1/2 \end{pmatrix}.$$

TEST-MMTM 13.

Problem 25. Find in $\mathbb{R}_{\max,+}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} -1 & -2 & 0 \\ 2 & 0 & 2 \\ -2 & -3 & -1 \end{pmatrix}.$$

Problem 26. Find in $\mathbb{R}_{\min,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}.$$

TEST-MMTM 14.

Problem 27. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & -1 & 3 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

Problem 28. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{Ax} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 1/3 & 1/2 \\ 1/2 & 1/2 & 1 \\ 2 & 1/2 & 1/3 \end{pmatrix}.$$

TEST-MMTM 15.

Problem 29. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 \\ 2 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

Problem 30. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\mathbf{Ax} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 3 \\ 1 & 1 & 3 \\ 2 & -1 & 1 \end{pmatrix}.$$

TEST-MMTM 16.

Problem 31. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & -2 \\ -1 & -2 & -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$

Problem 32. Find in $\mathbb{R}_{\min,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

TEST-MMTM 17.

Problem 33. Find in $\mathbb{R}_{\max, \times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 34. Find in $\mathbb{R}_{\min, \times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 4 & 3 \end{pmatrix}.$$

TEST-MMTM 18.

Problem 35. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & -3 \\ -1 & -1 & -2 \\ 1 & -3 & -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 36. Find in $\mathbb{R}_{\min,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}.$$

TEST-MMTM 19.

Problem 37. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 4 & 1 \\ 2 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 38. Find in $\mathbb{R}_{\min,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & 1 \end{pmatrix}.$$

TEST-MMTM 20.

Problem 39. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ -1 & 0 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

Problem 40. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 1/2 & 1 \\ 1 & 1/2 & 1/3 \\ 1 & 1/3 & 1/2 \end{pmatrix}.$$

TEST-MMTM 21.

Problem 41. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & 3 \\ 1 & 1 & 0 \end{pmatrix}.$$

Problem 42. Find in $\mathbb{R}_{\max,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & -1 & -3 \\ 1 & -1 & -2 \\ -3 & 1 & -1 \end{pmatrix}.$$

TEST-MMTM 22.

Problem 43. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 0 \\ 4 & 2 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}.$$

Problem 44. Find in $\mathbb{R}_{\max,\times}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1/2 & 1 & 1 \\ 1 & 1/2 & 1 \\ 1/3 & 1/2 & 1/3 \end{pmatrix}.$$

TEST-MMTM 23.

Problem 45. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & -1 \\ 1 & -2 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

Problem 46. Find in $\mathbb{R}_{\min, \times}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

TEST-MMTM 24.

Problem 47. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & -2 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 48. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{pmatrix}.$$

TEST-MMTM 25.

Problem 49. Find in $\mathbb{R}_{\min,+}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 2 & 3 & 1 \end{pmatrix}.$$

Problem 50. Find in $\mathbb{R}_{\max,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} -2 & -1 & -1 \\ 1 & -1 & 1 \\ -3 & -2 & -1 \end{pmatrix}.$$

TEST-MMTM 26.

Problem 51. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

Problem 52. Find in $\mathbb{R}_{\min,\times}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 2 \end{pmatrix}.$$

TEST-MMTM 27.

Problem 53. Find in $\mathbb{R}_{\max,+}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} -1 & -1 & -1 \\ 1 & 0 & 1 \\ 0 & -3 & -1 \end{pmatrix}.$$

Problem 54. Find in $\mathbb{R}_{\min,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{pmatrix}.$$

TEST-MMTM 28.

Problem 55. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 56. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

TEST-MMTM 29.

Problem 57. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 3 & 4 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 58. Find in $\mathbb{R}_{\min,\times}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 1 & 3 & 2 \end{pmatrix}.$$

TEST-MMTM 30.

Problem 59. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 3 \\ -1 & -1 & 2 \\ 1 & -2 & -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

Problem 60. Find in $\mathbb{R}_{\min,\times}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 2 & 1 \\ 1 & 3 & 3 \end{pmatrix}.$$

TEST-MMTM 31.

Problem 61. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 2 & 2 \\ 1 & 1 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

Problem 62. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1/2 & 1/2 & 1/3 \\ 1 & 1 & 1 \\ 2 & 1/3 & 1/2 \end{pmatrix}.$$

TEST-MMTM 32.

Problem 63. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} -1 & 1 & 3 \\ 2 & -1 & -1 \\ 1 & 0 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

Problem 64. Find in $\mathbb{R}_{\max,\times}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

TEST-MMTM 33.

Problem 65. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 66. Find in $\mathbb{R}_{\min,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 4 & 3 \\ 1 & 3 & 3 \\ 3 & 2 & 4 \end{pmatrix}.$$

TEST-MMTM 34.

Problem 67. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{Ax} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & -1 \\ -2 & -1 & -3 \\ 0 & 1 & -1 \end{pmatrix}.$$

Problem 68. Find in $\mathbb{R}_{\min,\times}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 3 & 2 & 4 \\ 3 & 3 & 2 \\ 2 & 4 & 2 \end{pmatrix}.$$

TEST-MMTM 35.

Problem 69. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & -1 & 3 \\ -1 & 2 & 1 \\ 0 & -2 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

Problem 70. Find in $\mathbb{R}_{\min,\times}$ all solutions of the inequality $\mathbf{Ax} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 2 & 3 \\ 1/2 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix}.$$

TEST-MMTM 36.

Problem 71. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}.$$

Problem 72. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{Ax} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} -2 & 2 & -2 \\ -3 & 0 & -2 \\ 0 & 1 & -1 \end{pmatrix}.$$

TEST-MMTM 37.

Problem 73. Find in $\mathbb{R}_{\min, \times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix}.$$

Problem 74. Find in $\mathbb{R}_{\min,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 2 \\ 4 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}.$$

TEST-MMTM 38.

Problem 75. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & -2 \\ -1 & -1 & -3 \\ -1 & 1 & -2 \end{pmatrix}.$$

Problem 76. Find in $\mathbb{R}_{\max, \times}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 1/2 & 2 & 1/3 \\ 2 & 1/3 & 2 \\ 1 & 1/2 & 1/2 \end{pmatrix}.$$

TEST-MMTM 39.

Problem 77. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 3 & 3 & -1 \\ 2 & 3 & 0 \\ 2 & -1 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}.$$

Problem 78. Find in $\mathbb{R}_{\max, \times}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1/2 & 1/2 & 1/3 \\ 1/2 & 1/2 & 1 \\ 1/3 & 1 & 1/2 \end{pmatrix}.$$

TEST-MMTM 40.

Problem 79. Find in $\mathbb{R}_{\max,+}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} -1 & -2 & 0 \\ -2 & 0 & 3 \\ -2 & -3 & -1 \end{pmatrix}.$$

Problem 80. Find in $\mathbb{R}_{\min,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 3 & 1 \\ 1 & 2 & 1 \end{pmatrix}.$$

TEST-MMTM 41.

Problem 81. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & -1 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}.$$

Problem 82. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 1/3 & 1/2 \\ 3 & 1/2 & 1 \\ 1 & 1/2 & 1/3 \end{pmatrix}.$$

TEST-MMTM 42.

Problem 83. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} -1 & -2 & 1 \\ 1 & -1 & 2 \\ 2 & -1 & -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$

Problem 84. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\mathbf{A}\mathbf{x} = \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \\ 2 & -1 & 1 \end{pmatrix}.$$

TEST-MMTM 43.

Problem 85. Find in $\mathbb{R}_{\min,\times}$ all solutions of the inequality $\mathbf{A}\mathbf{x} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 1/2 & 2 \\ 2 & 3 & 3 \\ 2 & 3 & 2 \end{pmatrix}.$$

Problem 86. Find in $\mathbb{R}_{\max,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & -1 & -1 \\ 3 & 0 & 2 \\ -3 & -2 & 1 \end{pmatrix}.$$

TEST-MMTM 44.

Problem 87. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 1 & -1 \\ -1 & 2 & 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 88. Find in $\mathbb{R}_{\min,\times}$ all solutions of the inequality $\mathbf{Ax} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix}.$$

TEST-MMTM 45.

Problem 89. Find in $\mathbb{R}_{\max,\times}$ all solutions of the inequality $\mathbf{Ax} \leq \mathbf{x}$, where

$$\mathbf{A} = \begin{pmatrix} 1/2 & 1 & 1/3 \\ 1 & 1/2 & 1 \\ 1/3 & 1/2 & 1 \end{pmatrix}.$$

Problem 90. Find in $\mathbb{R}_{\min,+}$ the eigenvalue and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix}.$$