The second test paper.

Student's name: **Number:**

1. (2 points) Let $X_1, \ldots, X_n$ be a sample from $Unif[0, \theta]$.
a) Using the statistic $X_{(n)}$, construct an exact confidence interval of significance level α for the parameter θ.
b) construct asymptotic confidence intervals for θ at level α, using the statistic $2\overline{X}$;
c)(1 point)* Same question using the statistic $\overline{X^3}$.

2. (1 point) Construct the Bayesian estimate of the parameter θ of a uniform distribution on the interval $[0;\theta]$ if θ is uniformly distributed on $[0, k]$, $k \geq 1$ and volume of sample $n \geq 3$.
Prove that the sample mean is not a minimax estimator and find the minimax estimator for k=1.

3. (1 points) Find the efficient estimator of the parameter θ among all unbiased estimators, if the sample comes from $Unif[0, \theta]$, using the statistics $X_{(n)}$. Explain all conclusions.

4.(3 points) The table shows the results of a math exam. For each column: the grades are $A, \ldots, F$ and the number of students for each grade.

grade	A	B	C	D	E	F
number	5	10	12	7	7	5

Test the goodness-of-fit at significance level $\alpha = 0.2$ for the following distributions:
a) (1 point) Discrete uniform distribution;
b) (2 points) the distribution law is defined by the following table:

grade	A	B	C	D	E	F
p_i	$\frac{1}{8} - \theta$	$1/4$	$1/4$	$\frac{1}{8} + \theta$	$\frac{1}{8} + \theta$	$\frac{1}{8} - \theta$

For each task, justify whether the null hypothesis should be rejected or not based on the test results you obtain. Clearly explain why your conclusion follows from the value of the test statistic.
5. (1 point). The table shows the results of a math exam.

grade	0-10	11-20	21-30	31-40	41-50	51-60
number of students	10000	20000	24000	14000	14000	10000

Since the exam scale has many possible values, we consider the score distribution as continuous. Unfortunately, the data allow reconstructing the empirical distribution function only at a few points. Test the goodness-of-fit with a discrete uniform distribution at a significance level $\alpha = 0.1$. Can the Kolmogorov–Smirnov test convince us to reject the hypothesis, or do the data not contradict it? Clearly explain why your conclusion follows from the value of the test statistic.

6. (2 points).

Student grades from 3 to 5, obtained in Calculus in 2024 and in Probability Theory in 2025, are given. The table shows the number of students for each possible pair of grades.

2024 \ 2025	3	4	5
3	120	80	50
4	70	100	70
5	40	60	90

a) Check the null hypothesis of independence for this data at a significance level of 0.05.
b) Test the null hypothesis of homogeneity at a significance level of 0.3.
For each task, justify whether the null hypothesis should be rejected or not based on the test results you obtain. Clearly explain why your conclusion follows from the value of the test statistic.