

Residues

TASKS

Use an appropriate Laurent series to find the indicated residue.

1. $f(z) = \frac{2}{(z-1)(z+4)}$; $\text{Res}(f(z), 1)$

Answer: $2/5$

2. $f(z) = \frac{4z-6}{z(2-z)}$; $\text{Res}(f(z), 0)$

Answer: -3

Find the residue at each pole of the given function.

3. $f(z) = \frac{z}{z^2+16}$

Answer: $\text{Res}(f(z), -4i) = 1/2, \text{Res}(f(z), 4i) = 1/2$

4. $f(z) = \frac{5z^2-4z+3}{(z+1)(z+2)(z+3)}$

Answer: $\text{Res}(f(z), -1) = 6, \text{Res}(f(z), -2) = -31, \text{Res}(f(z), -3) = 30$

5. $f(z) = \sec z$

Answer: $\text{Res}(f(z), \frac{(2n+1)\pi}{2}) = (-1)^{n+1}, n \in \mathbb{Z}$

Use Cauchy's residue theorem, where appropriate, to evaluate the given integral along the indicated contours.

6. $\oint_C \frac{1}{(z-1)(z+2)^2} dz$ (a) $|z| = \frac{1}{2}$

Answer: 0

(b) $|z| = \frac{3}{2}$

Answer: $2\pi i/9$

(c) $|z| = 3$

Answer: 0

Use Cauchy's residue theorem to evaluate the given integral along the indicated contour.

7. $\oint_C \frac{1}{z^2+4z+13} dz, C : |z-3i| = 3$

Answer: $\pi/3$

8. $\oint_C \frac{\tan z}{z} dz, C : |z-1| = 2$

Answer: $-4i$