

# 第一章 总复习.

5. (1).  $m+n, n+p, p-m$  共面

$$(m+n) + (p-m) - (n+p) = 0.$$

the three vectors are l.d.  $\Rightarrow$  coplanar.

$$\begin{aligned} (2) \quad (a \times m, a \times n, a \times p) &= ((a \times m) \times (a \times n)) \cdot (a \times p) & (a \times m) \times (a \times n) &\stackrel{a \times n = u}{=} (a \times m) \times u = (a \cdot u) \cdot m - (u \cdot m) \cdot a \\ &= (amn) a \cdot (a \times p) & &= (a \cdot (a \times n)) \cdot m - (a \cdot m) \cdot a \\ &= (amn) \cdot (a \cdot a \times p) & &= (amn) a. \\ &= 0. \end{aligned}$$

$$3. (abc) + (adb) - (adc) - bcd.$$

$$\begin{aligned} &= (a-d) \cdot (b \times c) + (b-c) \cdot (a \times d) \\ (a-d)[(b-d) \times (c-d)] &= (a-d)(b \times c - b \times d - d \times c) \\ &= (a-d)(b \times c) - (a-d)(b \times d) - (a-d)(d \times c) \\ &= (a-d)(b \times c) - a(b \times d) - a(d \times c) \\ &= (a-d)(b \times c) - (abd) - (adc) \\ &= (a-d)(b \times c) + (a \times d)(b-c) \end{aligned}$$

$$(2) \quad a \times [b \times (c \times d)] = -a[(c \times d) \times b] = -a[(c \cdot b) \cdot d - (d \cdot b) \cdot c] = (bd) \cdot (a \times c) - (cd \cdot b)(a \times d)$$

$$\begin{aligned} (3) \quad &= - (a \times b, c \times a, b \times c) = (a \times b, a \times c, b \times c) = [(a \times b) \times (a \times c)] \cdot (b \times c) \\ &= (abc) \cdot a \cdot (b \times c) = (abc)^2 \end{aligned}$$

$$6. (\alpha \times \beta) \cdot \gamma = 2$$

$$[(\alpha + \beta) \times (\beta + \gamma)] \cdot (\gamma + \alpha)$$

$$= (\alpha \times \beta + \alpha \times \gamma + \beta \times \gamma) \cdot (\gamma + \alpha) = 2 + (\beta \gamma \alpha) = 4$$

$$7. \text{ let } \bar{a} = (a_1, a_2, a_3) \quad \bar{b} = (b_1, b_2, b_3) \quad \bar{c} = (c_1, c_2, c_3)$$

$$(1) \quad (\bar{a} \cdot \bar{b})^2 \leq a^2 \cdot b^2$$

$$\begin{aligned} (2) \quad \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} &= (abc) = (a \times b) \cdot c = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1) \cdot c \\ &= a_2 b_3 c_1 - a_3 b_2 c_1 - a_1 b_3 c_2 + a_3 b_1 c_2 + a_1 b_2 c_3 - a_2 b_1 c_3 \end{aligned}$$

$$8. \quad \frac{(e_1 \times e_2) \cdot (a_1 \cdot e_1 \cdot e_3 + a_2 \cdot e_2 \cdot e_3 + a_3 \cdot e_3 \cdot e_3)}{(e_1 e_2 e_3)} = a_3 \vec{e}_3$$

$e_1, e_2, e_3$  form a basis of vector space.  $r = a_1 \bar{e}_1 + a_2 \bar{e}_2 + a_3 \bar{e}_3$

$$\begin{aligned} (2) \quad (x \times e_2) \cdot e_3 & \quad (x e_2) e_1 & \quad (x e_1 e_2) = a_3 (e_3 e_1 e_2) = n. \\ &= a_1 \cdot (e_1 e_2 e_3) = l. &= a_2 (e_2 e_3 e_1) = m \end{aligned}$$

$$x = \sum a_i e_i = \frac{l}{(e_1 e_2 e_3)} e_1 + \frac{m}{(e_1 e_2 e_3)} e_2 + \frac{n}{(e_1 e_2 e_3)} e_3.$$

## 2.2.2 曲面方程

(1) 定点  $(a_1, b_1, c_1)$   $(a_2, b_2, c_2)$

$$\frac{(x-a_1)^2 + (y-b_1)^2 + (z-c_1)^2}{(x-a_2)^2 + (y-b_2)^2 + (z-c_2)^2} = k^2$$

$$\Rightarrow x^2 + y^2 + z^2 + \frac{2a_2 k^2 - 2a_1}{1-k^2} x + \frac{2b_2 k^2 - 2b_1}{1-k^2} y + \frac{2c_2 k^2 - 2c_1}{1-k^2} z + a_1^2 + b_1^2 + c_1^2 + a_2^2 + b_2^2 + c_2^2 = 0$$

$$(x, y, z) \neq (a_1, b_1, c_1) \text{ or } (a_2, b_2, c_2)$$

该常数  $2a$

i)  $2a < -c$  no locus.

ii)  $a = c$  且有原点  $(0, 0, 0)$

iii)  $a > c$

在平面上:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\sqrt{(x-c)^2 + y^2 + z^2} + \sqrt{(x+c)^2 + y^2 + z^2} = 2a$$

$$2a^2 - x^2 - y^2 - z^2 - c^2$$

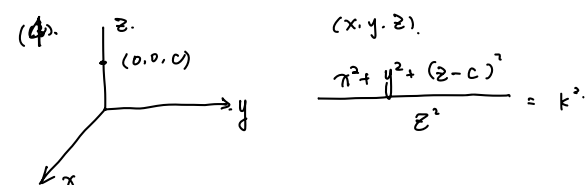
$$(x^2 + y^2 + z^2 + c^2) + \sqrt{[(x-c)^2 + y^2 + z^2][(x+c)^2 + y^2 + z^2]} = 2a^2$$

$$x^4 - 2cx^3 + c^4 + y^4 + z^4 + 2x^2z^2 + 2x^2y^2 + 2x^2c^2 = x^4 + y^4 + z^4 + c^4 + 4a^4 + 2x^2c^2 - 4a^2(x^2 + y^2 + z^2 + c^2)$$

$$4a^2(x^2 + y^2 + z^2 + c^2) = 4c^2x^2 + 4a^4$$

$$a^2(x^2 + y^2 + z^2) = 0x^2 + a^2b^2$$

$$b^2x^2 + ay^2 + az^2 = a^2b^2 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{b^2} = 1$$



$$5. \begin{cases} x = a + r \cos \theta \cos \varphi \\ y = b + r \cos \theta \sin \varphi \\ z = c + r \sin \theta \end{cases} \quad \begin{aligned} & -\pi < \varphi \leq \pi \\ & -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \end{aligned}$$

$$6. x^2 + y^2 + z^2 = 1. \quad (x^2 + y^2 \leq 1)$$

$$x^2/a^2 + y^2/b^2 = 1$$

$$7. x^2 + y^2 = u^2 = z$$

$$x^2 + y^2 = z$$

...

9. (1) 半径 1, 原点为圆心的球面

(2)  $y \geq 0$  正半平面

(3) 圆面

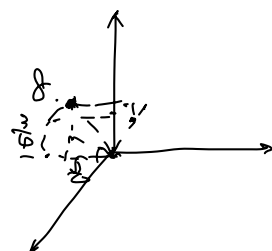
圆面的一半为  $\pi/6$ , 上半圆(半圆面).

10. (1) 半径 2 圆柱面

(2)  $x \geq 0$  与  $y \geq 0$  平面正半轴

(3) 与  $xy$  平行, 在  $z$  轴上距离为 1 的平面.

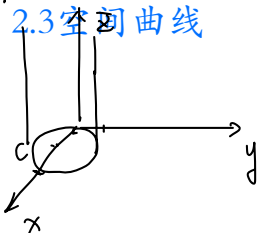
过点  $(0, 0, 1)$



$$\text{球: } \rho = 1, \quad \theta = \frac{\pi}{6}, \quad \varphi = -\frac{2\pi}{3}$$

$$\text{柱: } \rho = \frac{\sqrt{3}}{2}, \quad \varphi = -\frac{2\pi}{3}, \quad u = \frac{1}{2}$$

1. 2.3 空曲线



$C = 0$  or 2. segment

$0 < C < 2$  two segments

$C > 2$  or  $C < -2$  no intersection

2. (1) 与  $xOy$  面相交.  $z=0$

$$x^2 + y^2 = 64. \quad \text{圆}$$

$yOz$ . 椭圆  
 $xOz$

(3)

(2)

(4)

(5)

(6)

$xOy$

双曲线

椭圆

点

两直线

$$x=3y, x=-3y.$$

点

$yOz$

无

双

椭圆

椭圆

两直线

$xOz$

双曲线

双

椭圆

椭圆

两直线

3. (1)  $r(t) = t \cos \pi t \mathbf{i} + t \sin \pi t \mathbf{j} + t \mathbf{k}$

$$r(t) = 2 \cos \pi t \mathbf{i} + 2 \sin \pi t \mathbf{j}$$

$$t=2, t=-2$$

$$(2) \cos^2 \pi t + \sin^2 \pi t + t^2 = 10 \Rightarrow t = \pm 3.$$

$$(-1, 0, 3) \quad (-1, 0, -3)$$

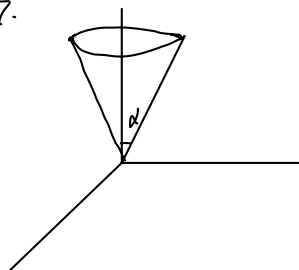
4.  $F(f(t), \phi(t), \psi(t)) = 0$

$$5. (1) \begin{cases} x = 3z+1 \\ y = \frac{z^2}{4} + z+1 \end{cases} \quad z \in \mathbb{R}$$

$$(2) \begin{cases} 5x - y = 0 \\ \frac{x^2}{9} + \frac{z^2}{16} = 0 \end{cases}$$

$$6. \begin{cases} z = t^2 \\ y = 2t \\ x = -t^4 \end{cases}$$

7.



$$x = vt \sin \alpha \cos \omega t.$$

$$y = vt \sin \alpha \sin \omega t$$

$$z = vt \cos \alpha.$$