Fourier integrals. Fourier Transform

TASKS

Represent the function $f(x)$ by the Fourier integral:

1. $f(x) = \begin{cases} 1, & \text{if} \quad |x| < \tau \\ 0, & \text{if} \quad |x| > \tau \end{cases}$

Answer: $f(x) = \frac{2}{\pi} \int_0^{+\infty} \frac{sin\tau y}{y} cosxydy$

2. $f(x) = \begin{cases} e^{-\alpha x} \sin \omega x, & \text{if} \quad x > 0, \\ 0, & \text{if} \quad x < 0; \end{cases} \quad \alpha > 0\,;$

Answer: $f(x) = \frac{\omega}{\pi} \int_0^{+\infty} \frac{(\alpha^2+\omega^2-y^2)cosxy+2\alpha y sinxy}{(\alpha^2-\omega^2+y^2)^2+4\alpha^2\omega^2} dy$

3. Represent the Fourier integral function $f(x)$ by continuing it in an odd way on the interval $(-\infty; 0)$ if: $f(x) = \begin{cases} \sin x, & \text{if} \quad 0 \leqslant x \leqslant \pi \\ 0, & \text{if} \quad x > \pi; \end{cases}$

Answer: $f(x) = \frac{2}{\pi} \int_0^{+\infty} \frac{sin\pi y}{1-y^2} sinxydy$

4. Represent the Fourier integral function $f(x)$ by continuing it in an even way on the interval $(-\infty; 0)$ if: $f(x) = e^{-\alpha x}, x \geqslant 0, \alpha > 0;$

Answer: $f(x) = \frac{2}{\pi} \int_0^{+\infty} \frac{siny}{y} cosxydy$

Find the Fourier transform of the function $f(x)$

5. $f(x) = \begin{cases} 1, & \text{if} \quad |x| \leqslant 1, \\ 0, & \text{if} \quad |x| > 1; \end{cases}$

Answer: $f(x) = \sqrt{\frac{2}{\pi}} \frac{siny}{y}$

6. $f(x) = \begin{cases} e^{ix}, & \text{if} \quad x \in [0; \pi], \\ 0, & \text{if} \quad x \notin [0; \pi]; \end{cases}$

Answer: $f(x) = \sqrt{\frac{2}{\pi}} \frac{sin\pi y}{1-y}$

7. $f(x) = e^{-x^2/2} \cos \alpha x;$

Answer: $f(x) = e^{-(y^2+\alpha^2)/2} ch\alpha y$

8. Let $\widehat{f}(y) = F[f(x)]$. To prove that: $F\left[e^{i\alpha x} f(x)\right] = \widehat{f}(y-\alpha), \quad \alpha \in R$;