$$f(x) = |x|$$

$$f_+'(0) = \lim_{x \to 0+} \frac{|x| - 0}{x - 0} = 1$$

$$f_-'(0) = \lim_{x \to 0-} = \frac{|x| - 0}{x - 0} = \frac{-x}{x} = -1$$

$\Rightarrow$ $|x|$ is not diff. at 0

Def. A function $f^{-1}: D \to E$ is an inverse to $f: E \to D$ if

$\forall x \in E \quad f^{-1}(f(x)) = x$

$\forall y \in D \quad f(f^{-1}(y)) = y$

Thm.1 f has an inverse f-n $\iff$ f is bijective

Thm2 If f is continuous ~~on~~ at x ~~$\langle a, b \rangle$~~ ~~$\Rightarrow$~~,
and f^{-1} is an inverse to f $\Rightarrow$
f^{-1} is continuous at $y = f(x)$

$f : \mathbb{R} = (0, +\infty)$

$f(x) = e^x \qquad f^{-1}(y) = \ln y \ : (0, +\infty) \to \mathbb{R}$

$(\ln y)' = \dfrac{1}{f'(x)}$, when $y = f(x) = e^x$

$\Longleftrightarrow x = \ln y$

$(\ln y)' = \dfrac{1}{f'(\ln y)} = \dfrac{1}{e^{\ln y}} = \dfrac{1}{y}$, $y > 0$

$f^{-1}(f(x)) = x \Rightarrow (f^{-1})'(f(x)) \cdot f'(x) = 1$

$f(f^{-1}(y)) = y \Rightarrow$ diff by $y \Rightarrow f'(f^{-1}(y))(f^{-1})'(y) = 1$

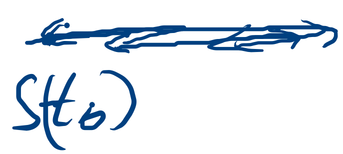
$\Rightarrow f^{-1}(y)' = \dfrac{1}{f'(f^{-1}(y))}$

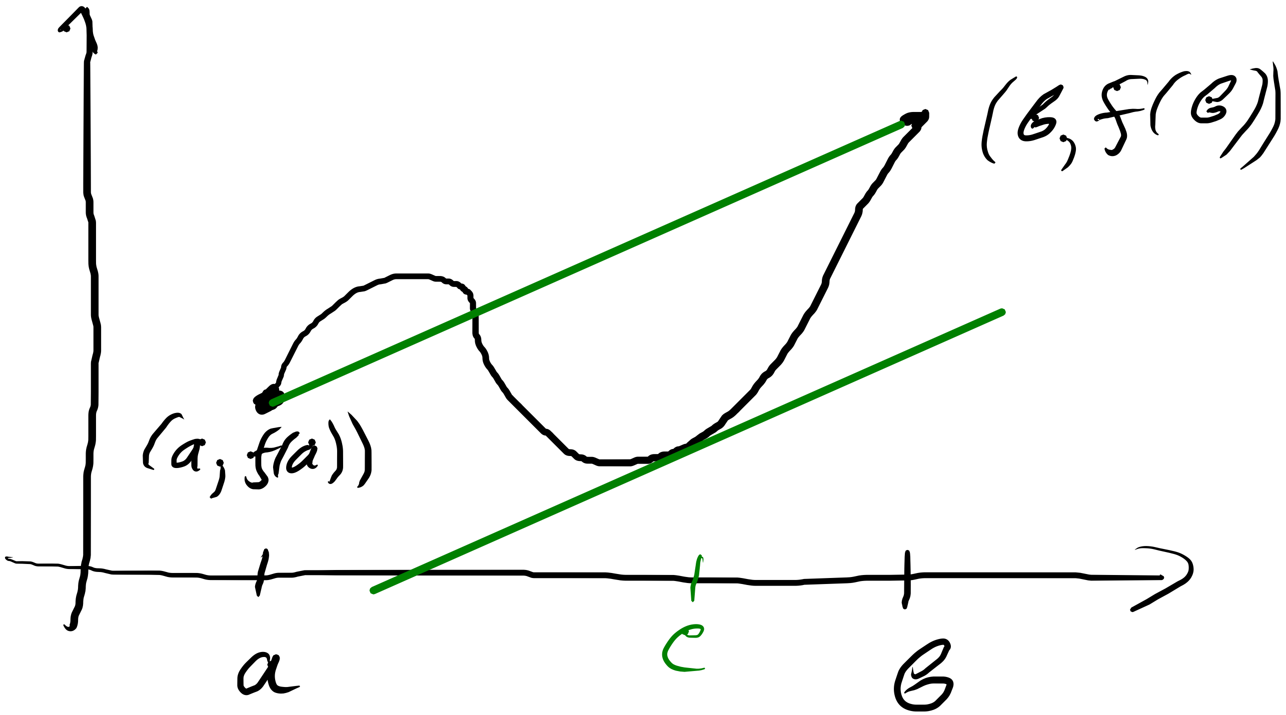
Let $s(t)$ describe the coordinate of a point on the real line at t.

$\Rightarrow$ $s'(t)$ is the speed of the point at time t.

$\Rightarrow$ $\exists\, c \in (t_0, t_1)$ $\quad s'(c) = \dfrac{s(t_1) - s(t_0)}{t_1 - t_0}$

mean speed.





$$\exists\, c \in (a,b) \quad \text{s.t.} \quad f'(c) = \frac{f(b) - f(a)}{b-a}$$

Example:

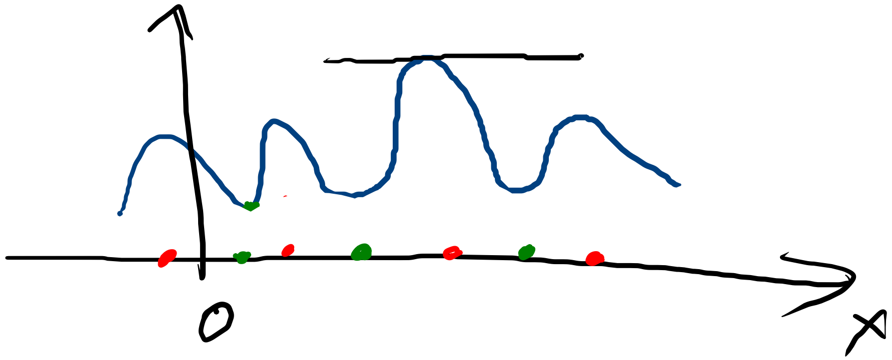
$f(x) = x^3$ $\qquad$ $(x^3)'(0) = 0$

while x^3 is str. increasing

Def. $f : E \to \mathbb{R}$ is uniformly continuous on E if

$$\forall \varepsilon > 0 \ \exists \underset{\text{"}\delta(\varepsilon)\text{"}}{\delta > 0} \ \forall x, y \in E \ |x-y| < \delta \Rightarrow |f(x) - f(y)| < \varepsilon$$

$$\Leftrightarrow \sup_{|x-y|<\delta} |f(x) - f(y)| = \omega_\delta(f) \underset{\delta \to 0+}{\longrightarrow} 0$$



loc min

loc max

Thm If x_0 is ~~loc.~~ extremal point, and $f'(x_0)\ \exists \Rightarrow f'(x_0)=0$

Ex. $f(x)=|x| \Rightarrow x_0=0$ is a minimum, while $\nexists\ f'(0)$

Assume that x_0 is loc max

Proof: $\exists\,\delta>0 \quad |x-x_0|<\delta \quad f(x_0)\geq f(x)$

$$f'(x_0)=\lim_{x\to x_0+}\frac{f(x)-f(x_0)}{x-x_0}\leq 0$$

$$f'(x_0)=\lim_{x\to x_0-}\frac{f(x)-f(x_0)}{x-x_0}\geq 0$$

$\Rightarrow f'(x_0)=0$

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