Chapter 1
Some General Mathematical Concepts and Notation

1.1 Logical Symbolism

1.1.1 Connectives and Brackets

$\neg$: not; $\wedge$: and; $\vee$: or; $\Rightarrow$: implies; $\Leftrightarrow$: is equivalent to

Table 1.1

Notation	Meaning
$L \Rightarrow P$	L implies P
$L \Leftrightarrow P$	L is equivalent to P
$((L \Rightarrow P) \wedge (\neg P)) \Rightarrow (\neg L)$	If P follows from L and P is false, then L is false
$\neg((L \Leftrightarrow G) \vee (P \Leftrightarrow G))$	G is not equivalent either to L or to P

Order of priorities for the symbols: $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$

With this convention the expression $\neg A \wedge B \vee C \Rightarrow D$ should be interpreted as $(((\neg A) \wedge B) \vee C) \Rightarrow D$, and the relation $A \vee B \Rightarrow C$ as $(A \vee B) \Rightarrow C$, not as $A \vee (B \Rightarrow C)$.

We shall often give a different verbal expression to the notation $A \Rightarrow B$, which means that A implies B, or, what is the same, that B follows from A, saying that B is a *necessary criterion* or *necessary condition* for A and A in turn is a *sufficient condition* or *sufficient criterion* for B, so that the relation $A \Leftrightarrow B$ can be read in any of the following ways:

A is necessary and sufficient for B;
A holds when B holds, and only then;
A if and only if B;
A is equivalent to B.

Thus the notation $A \Leftrightarrow B$ means that A implies B and simultaneously B implies A.

1.1.2 Remarks on Proofs

In proofs we shall adhere to the classical rule of inference: if A is true and $A \Rightarrow B$, then B is also true.

In proof by contradiction we shall also use the law of excluded middle, by virtue of which the statement $A \vee \neg A$ (A or not-A) is considered true independently of the specific content of the statement A. Consequently we simultaneously accept that $\neg(\neg A) \Leftrightarrow A$, that is, double negation is equivalent to the original statement.

1.1.3 Some Special Notation

For the reader's convenience and to shorten the writing, we shall agree to denote the end of a proof by the symbol $\square$.

We also agree, whenever convenient, to introduce definitions using the special symbol $:=$ (equality by definition), in which the colon is placed on the side of the object being defined.

For example: $f'(x) := \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$.

1.1.4 Concluding Remarks

The experience of all the sciences convinces us that what was considered clear or simple and unanalyzable yesterday may be subjected to reexamination or made more precise today. Such was the case (and will undoubtedly be the case again) with many concepts of mathematical analysis, the most important theorems and machinery of which were discovered in the seventeenth and eighteenth centuries, but which acquired its modern formalized form with a unique interpretation that is probably responsible for its being generally accessible, only after the creation of the theory of limits and the fully developed theory of real numbers needed for it in the nineteenth century.

1.1.5 Exercises

Truth Table:

$\boxed{\neg A}$

A	0	1
$\neg A$	1	0

$\boxed{A \wedge B}$

$A \backslash B$	0	1
0	0	0
1	0	1

$\boxed{A \vee B}$

$A \backslash B$	0	1
0	0	1
1	1	1

$\boxed{A \Rightarrow B}$

$A \backslash B$	0	1
0	1	1
1	0	1

2. Show that the following simple, but very useful relations, which are widely used in mathematical reasoning, are true:

a) $\neg(A \wedge B) \Leftrightarrow \neg A \vee \neg B$;
b) $\neg(A \vee B) \Leftrightarrow \neg A \wedge \neg B$;
c) $(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$;
d) $(A \Rightarrow B) \Leftrightarrow (\neg A \vee B)$;
e) $\neg(A \Rightarrow B) \Leftrightarrow A \wedge \neg B$.