Real analysis. Spring 2024.
Lecturers Rotkevich Alexander, Aleksei Savelev.

1. $\sigma-$algebras, measurable functions.
2. $\sigma-$algebra generated by a family of subsets, Borel $\sigma-$algebra, product $\sigma-$algebra.
3. Criterion of measurability of a function to a set with $\sigma-$algebra generated by a family of subsets.
4. Measurability of $f \pm g$, fg, for measurable functions f, g.
5. Meaurability of a pointwise limit of measurable functions.
6. Measure on a $\sigma-$algebra.
7. Negligible set, properties that hold almost everywhere.
8. Image measure.
9. Lebesgue measure. Properties of Lebesgue $\sigma-$algebra.
10. Borel measures and regularity properties.
11. Simple and step functions. approximation of measurable function by a sequence of simple and step functions.
12. Integration with respect to a measure.
13. Monotonicity of the integral.
14. B.Levy monotone convergence theorem.
15. Linearity of the integral.
16. B. Levy theorem for series
17. Chebyshev inequality.
18. Integral as a measure. General formula for the change of the variable.
19. Integrable functions.
20. Fatou's lemma.
21. Lebesgue dominated convergence theorem.
22. Lebesgue space L^1.
23. Lebesgue spaces L^p with $1 \leq p < \infty$.
24. Lebesgue space L^∞.
25. Comparison of Lebesgue spaces.
26. Integration in Lebasgue spaces. Examples.
27. Comparison of Lebesgue and Riemann integral.
28. The Multiple Lebesgue Integral. Examples.
29. Fubini and Tonelli theorems.
30. Formula for change of the variable with respect to diffeomorphism.
31. Density of continuous functions in L^p.

32. Continuity of translation operators in L^p.
33. Orthogonal Systems in L^2
34. Bessel inequality.
35. Riesz-Fischer theorem
36. Basis in L^2. Theorem on the characterization of bases. Parseval's identity.
37. Examples of Orthogonal Systems.
38. Rademacher functions. Theorem Strong law of large numbers.
39. Kolmogorov's inequality
40. Trigonometric Fourier Series
41. Properties of Fourier coefficients.
42. Dirichlet's kernel.
43. Riemann's localization principle.
44. Dini test, Dirichlet-Jordan test for convergence of Fourier series.
45. Examples of Fourier series expansion of its application.
46. Integration of Fourier series.
47. Fourier coefficients of differentiable function.
48. Denjoy-Luzin theorem.
49. The Fourier transform. First ptoperties, examples.
50. Derivative of the Fourier transform, Fourier transform of the derivative.
51. Equivalence of convergence of Fourier series and Fourier integral.
52. Inverse Fourier transform. Inversion formula.
53. Plancherel theorem.
54. Convergence in measure.
55. Convergence in measure doesn't imply convergence almost everywhere.
56. Lebesgue theorem on convergence in measure of a.e. convergent sequence of functions on a set of finite measure.
57. Borel-Cantelli lemma.
58. Riesz theorem.
59. Egorov theorem.
60. Diagonal sequence theorem.
61. Approximation of Measurable Functions by Continuous Functions.
62. Fréchet theorem
63. Luzin's theorem.