

Command	Purpose	Example
<code>plot(x, y)</code>	Basic scatter or line plot.	<code>plot(x, y, type = "p")</code>
<code>hist(x)</code>	Histogram of data.	<code>hist(x, breaks = 30, col = "lightblue")</code>
<code>boxplot(x)</code>	Boxplot to show spread and outliers.	<code>boxplot(x)</code>
<code>barplot(h)</code>	Bar chart for categorical data.	<code>barplot(table(gender))</code>
<code>lines(x, y)</code>	Add a line to an existing plot.	<code>lines(xs, dnorm(xs), col = "red")</code>
<code>points(x, y)</code>	Add points to an existing plot.	<code>points(x, y, col = "blue")</code>
<code>curve(expr)</code>	Plot a mathematical function.	<code>curve(dnorm(x), from=-3, to=3)</code>
<code>abline()</code>	Add a straight line (horizontal, vertical, or regression).	<code>abline(h=0, col="red")</code>
<code>legend()</code>	Add a legend to a plot.	<code>legend("topright", legend=c("Data", "Model"), col=c("black", "red"), lty=1)</code>
<code>density(x) + lines()</code>	Kernel density estimate of a distribution.	<code>lines(density(x), col="darkgreen")</code>
<code>ecdf(x)</code>	Empirical cumulative distribution function (ECDF).	<code>plot(ecdf(x), main="Empirical CDF", col="blue")</code>

Simulation of a Random Variable with Density $f(x) = \frac{4}{x^5}$, $x > 1$

1. Theoretical Background

The given density function:

$$f(x) = \frac{4}{x^5}, \quad x > 1$$

is a **Pareto distribution** with parameters $x_m = 1$ and $\alpha = 4$.

The cumulative distribution function (CDF) is:

$$F(x) = 1 - \frac{1}{x^4}, \quad x > 1$$

By inversion of the CDF:

$$F^{-1}(u) = (1 - u)^{-1/4}, \quad u \in (0, 1)$$

Thus, to generate random variables with this density, we can use the transformation:

$$X = (1 - U)^{-1/4}, \quad \text{where } U \sim \text{Uniform}(0, 1)$$

2. Simulation in R

The following R code simulates $n = 10000$ random values from the given distribution, and compares the empirical results with the theoretical density and CDF.

```
set.seed(123)
n <- 10000 u <- runif(n)
Inverse transform method x <- (1 - u)^( - 1/4)
Theoretical density and CDF f_theor <- function(x) ifelse(x > 1, 4/x^5, 0) F_theor <- function(x) ifelse(x > 1, 1 - 1/x^4, 0)
— 1. Histogram and theoretical density — hist(x, breaks = 50, freq = FALSE, col = "lightblue", main = "Simulated Data vs Theoretical Density", xlab = "x", xlim = c(1, 5))
curve(f_theor, from = 1, to = 5, add = TRUE, col = "red", lwd = 2) legend("topright", legend = c("Histogram", "lightblue", "red"), lty = c(1, 1), lwd = c(10, 2), bty = "n")
— 2. Empirical and theoretical CDF — plot(ecdf(x), main = "Empirical vs Theoretical CDF", col = "blue", lwd = 2, ylab = "F(x)", xlab = "x", xlim = c(1, 5)) curve(F_theor, from = 1, to = 5, add = TRUE, col = "red", lwd = 2, lty = 2) legend("bottomright", legend = c("EmpiricalCDF", "TheoreticalCDF"), col = c("blue", "red"), lwd = 2, bty = "n")
```

3. Comparison Results

- The histogram closely follows the theoretical density curve $f(x) = \frac{4}{x^5}$.
- The empirical cumulative distribution function aligns well with the theoretical CDF $F(x) = 1 - \frac{1}{x^4}$.

The following figures illustrate the comparison results:

4. Theoretical Moments

For a Pareto distribution with $\alpha = 4, x_m = 1$:

$$E[X] = \frac{\alpha x_m}{\alpha - 1} = \frac{4}{3}, \quad \text{Var}[X] = \frac{\alpha x_m^2}{(\alpha - 1)^2(\alpha - 2)} = \frac{4}{18} \approx 0.222$$

Empirical estimates from the simulated data should be close to these theoretical values.

density_plot_placeholder.png

Figure 1: Histogram and theoretical density.



cdf_plot_placeholder.png

Figure 2: Empirical and theoretical cumulative distribution functions.