

第一章 总结

5. (1). $m+n, n+p, p-m$ 共面

$$(m+n) + (p-m) - (n+p) = 0.$$

the three vectors are L.d. \Rightarrow coplanar.

$$\begin{aligned} \text{(2). } & (\alpha \times m, \alpha \times n, \alpha \times p) = ((\alpha \times m) \times (\alpha \times n)) \cdot (\alpha \times p) \\ & = (\alpha \times m) \times (\alpha \times n) \stackrel{\alpha \times n = u}{=} (\alpha \times m) \times u = (\alpha \cdot u) \cdot m - (u \cdot m) \cdot \alpha \\ & = (\alpha \cdot (\alpha \times n)) \cdot m - (\alpha \cdot m) \cdot (\alpha \times n) \\ & = (\alpha \cdot \alpha) \cdot m - (\alpha \cdot m) \cdot \alpha \\ & = 0. \end{aligned}$$

$$3. (abc) + (abd) - (adc) - bcd.$$

$$\begin{aligned} & = (\alpha - d) \cdot (b \times c) + (b - c) \cdot (\alpha \times d) \\ & (\alpha - d)[(b - d) \times (c - d)] = (\alpha - d) \cdot (b \times c - b \times d - d \times c) \\ & = (\alpha - d) \cdot (b \times c) - (\alpha - d) \cdot (b \times d) - (\alpha - d) \cdot (d \times c) \\ & = (\alpha - d) \cdot (b \times c) - \alpha \cdot (b \times d) - \alpha \cdot (d \times c) \\ & = (\alpha - d) \cdot (b \times c) - (abd) - (adc) \\ & = (\alpha - d) \cdot (b \times c) + (\alpha \times d) \cdot (b - c) \end{aligned}$$

$$\text{(3). } \alpha \times [b \times (c \times d)] = -\alpha [(c \times d) \times b] = -\alpha [(c \cdot b) \cdot d - (d \cdot b) \cdot c] = (bd) \cdot (a \times c) - (c \cdot b) \cdot (a \times d)$$

$$\begin{aligned} & = -(\alpha \times b, c \times a, b \times c) = (\alpha \times b, a \times c, b \times c) = [(a \times b) \times (a \times c)] \cdot (b \times c) \\ & = (abc) \cdot a \cdot (b \times c) = (abc)^2 \end{aligned}$$

$$6. (\alpha \times \beta) \cdot \gamma = 2$$

$$\begin{aligned} & [(\alpha + \beta) \times (\beta + \gamma)] \cdot (\gamma + \alpha) \\ & = (\alpha \times \beta + \alpha \times \gamma + \beta \times \gamma) \cdot (\gamma + \alpha) = 2 + (\beta \gamma \cdot \alpha) = 4 \end{aligned}$$

$$7. \text{ Let } \vec{a} = (a_1, a_2, a_3), \vec{b} = (b_1, b_2, b_3), \vec{c} = (c_1, c_2, c_3)$$

$$\text{(1). } (\vec{a} \cdot \vec{b})^2 \leq a^2 \cdot b^2$$

$$\begin{aligned} \text{(2). } & \left| \begin{array}{ccc} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{array} \right| = (abc) = (\alpha \times b) \cdot c = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1) \cdot c \\ & = a_2 b_3 c_1 - a_3 b_2 c_1 - a_1 b_3 c_2 + a_3 b_1 c_2 + a_1 b_2 c_3 - a_2 b_1 c_3 \end{aligned}$$

$$\begin{aligned} \text{(3). } & \frac{(\ell_1 \times \ell_2)(\alpha_1 \cdot e_1 \cdot e_3 + \alpha_2 \cdot e_2 \cdot e_3 + \alpha_3 \cdot e_3 \cdot e_1)}{(\ell_1 \ell_2 \ell_3)} = \alpha_1 \vec{e}_3. \end{aligned}$$

e_1, e_2, e_3 form a basis of vector space. $r = \alpha_1 \vec{e}_1 + \alpha_2 \vec{e}_2 + \alpha_3 \vec{e}_3$

$$\begin{aligned} \text{(4). } & (\alpha \times \ell_2) \cdot e_3 = (\alpha e_3) \cdot e_1 = (\alpha e_1, \ell_2) = \alpha_3 (\ell_3, e_1, e_2) = n. \\ & = \alpha_1 \cdot (\ell_1 e_2 e_3) = l. \quad = \alpha_2 (\ell_2 e_3 e_1) = m \end{aligned}$$

$$\chi = \sum \alpha_i \ell_i = \frac{l}{(\ell_1 \ell_2 \ell_3)} e_1 + \frac{m}{(\ell_1 \ell_2 \ell_3)} e_2 + \frac{n}{(\ell_1 \ell_2 \ell_3)} e_3.$$

2.2 曲面方程

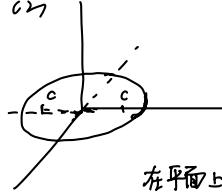
(1) 定点 (a_1, b_1, c_1) (a_2, b_2, c_2)

$$\frac{(x-a_1)^2 + (y-b_1)^2 + (z-c_1)^2}{(x-a_2)^2 + (y-b_2)^2 + (z-c_2)^2} = k^2$$

$$\Rightarrow x^2 + y^2 + z^2 + \frac{2a_2k^2 - 2a_1}{1-k^2}x + \frac{2b_2k^2 - 2b_1}{1-k^2}y + \frac{2c_2k^2 - 2c_1}{1-k^2}z + a_1^2 + b_1^2 + c_1^2 - a_2^2 - b_2^2 - c_2^2 = 0$$

$(x, y, z) \neq (a_1, b_1, c_1)$ or (a_2, b_2, c_2)

及该常数为 $2a$



$\therefore 2a < c$ no locus.

$\therefore a=c$ 只有原点 $(0, 0, 0)$

$\therefore a > c$.

$$\text{在平面上: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

$$\sqrt{(x-a)^2 + y^2 + z^2} + \sqrt{(x+c)^2 + y^2 + z^2} = 2a$$

$$2a^2 - x^2 - y^2 - z^2 - c^2$$

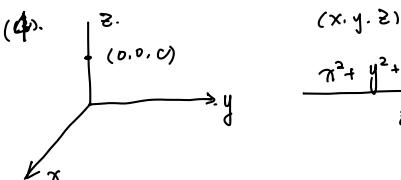
$$(x^2 + y^2 + z^2 + c^2) + \sqrt{(x-c)^2 + y^2 + z^2}[(x+c)^2 + y^2 + z^2] = 2a^2$$

$$x^4 - 2c^2x^2 + c^4 + y^4 + z^4 + 2x^2c^2 + 2y^2c^2 + 2z^2c^2 = x^4 + y^4 + z^4 + c^4 + 4a^4 + 2x^2c^2 - 4a^2(x^2 + y^2 + z^2 + c^2)$$

$$4a^2(x^2 + y^2 + z^2 + c^2) = 4c^2x^2 + 4a^4$$

$$a^2(x^2 + y^2 + z^2) = a^2x^2 + a^2b^2$$

$$b^2x^2 + a^2y^2 + a^2z^2 = a^2b^2 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{b^2} = 1.$$



$$\frac{x^2 + y^2 + (z-c)^2}{z^2} = k^2.$$

$$\begin{cases} x = a + r \cos\theta \cos\varphi \\ y = b + r \cos\theta \sin\varphi \\ z = c + r \sin\theta \end{cases} \quad -\pi < \varphi \leq \pi, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}.$$

$$b. \quad x^2 + y^2 + z^2 = 1. \quad (x^2 + y^2 \leq 1).$$

$$(1) \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

$$7. \quad x^2 + y^2 = u^2 = z.$$

$$x^2 + y^2 = z$$

... --

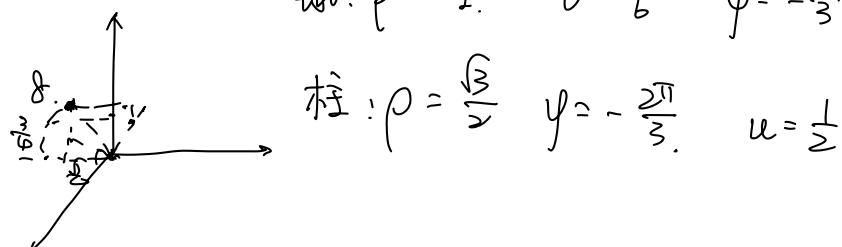
9. (1) 半径 r . 原点为圆心的球面

(2) yOz 正半平面

(3) 圆锥面

圆锥面的一半为 $\frac{\pi}{2}$. 上半腔(半球面).

$$\text{球: } \rho = 1. \quad \theta = \frac{\pi}{6} \quad \varphi = -\frac{2\pi}{3}.$$

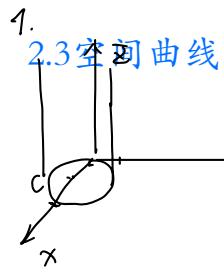


1D. (1) 半径 r 圆柱面

(2) $xOz \leq yOz$ 平面上正半轴

(3) $\Rightarrow xOy$ 平行. 在下. 距离为 r 的平面.

过点 $(0, 0, r)$



1. $C = 0$ or 2. segment

$0 < C < 2$ two segments

$C > 2$ or $C < 2$ m intersection

2. (1) 与 xOy 面相交. $z=0$
 $x^2+y^2=64$. $\boxed{\text{圆}}$.

$y \geq 0$. $\boxed{\text{半圆}}$

(2)

相交

(4)

点

(5)

无直角 $x=3y$, $x=-3y$.

(6)

点
无直角

3. (1) $r(t) = t \cos \pi t \mathbf{i} + t \sin \pi t \mathbf{j} + t \mathbf{k}$

$$r(t) = 2 \cos \pi t \mathbf{i} + 2 \sin \pi t \mathbf{j}$$

$$t=2, t=-2$$

$$(2) \cos^2 \pi t + \sin^2 \pi t + t^2 = 10 \Rightarrow t = \pm 3.$$

$$(-1, 0, 3), (-1, 0, -3)$$

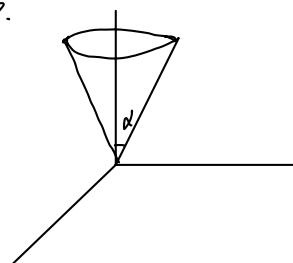
4. $F(f(t), \varphi(t), \psi(t)) = 0$

$$5. (1) \begin{cases} x = 3z+1 \\ y = \frac{z^2}{4} + z + 1 \end{cases}, z \in \mathbb{R}$$

$$(2) \begin{cases} 5x - 3y = 0 \\ \frac{x^2}{9} + \frac{z^2}{16} = 0 \end{cases}$$

$$6. \begin{cases} z = t^2 \\ y = 2t \\ x = -t^4 \end{cases}$$

7.



$$x = vt \sin \alpha \cos \omega t$$

$$y = vt \sin \alpha \sin \omega t$$

$$z = vt \cos \alpha$$