Core definitions, assertions and calculations for the course of differential geometry

I. Core definitions

1. Elementary, simple and general curve. Parameterisation of the curve (explicit and implicit). Regular and smooth curve.
2. Singular and regular points. Types of singular points for an explicitly parameterised curve.
3. Contact of two sets. Cases of contact: tangent line to curve, osculating plane to curve, tangent plane to surface, osculating paraboloid.
4. The asymptote of a curve.
5. Unit normal and binormal vectors. Frenet frame.
6. Frenet-Serret formulas. Natural equations of the curve.
7. Elementary, simple and general surface. Parameterisation of the surface. Regular surface.
8. Regular and singular points on the surface. Singular curves on the surface. Types of singular points for explicit surfaces.
9. The first quadratic (fundamental) form of a surface.
10. Conformal mapping, with examples.
11. Isometric surfaces, with examples.
12. The second quadratic (fundamental) form of a surface.
13. Classification of points on the surface concerning the type of indicatrix.
14. Asymptotic direction and asymptotic curve on the surface.
15. Principal curvatures and lines of curvature.
16. Mean and Gaussian curvatures.
17. Derivatives of tangent and normal vectors. Christoffel symbols.
18. Geodesic curvature of a curve on a surface. Geodesic lines.
19. Semigeodesic parameterisation.
20. Surfaces with constant Gaussian curvature (three cases).
21. Smooth manifold.

II. Core assertions

1. Condition for a point on a curve to be singular.
2. Condition for curves to have contact.
3. Arc length of a segment of a smooth curve.
4. Parameterisation with arc length.
5. Formula for curvature in natural and general parameterisation.
6. Formula for absolute torsion in natural and general parameterisation.
7. Fundamental theorem of curves.
8. Implicit definition of a surface.
9. Two lemmas describing the behaviour of the normal vector at singular and regular points.
10. Equation of the tangent plane to a smooth surface (general expansion and particular cases $z = f(x, y)$, $F(x, y, z) = 0$).
11. Existence and uniqueness of the osculating paraboloid.
12. Orthogonality of coordinate curves. Existence of orthogonal parameterisation.
13. Surface area of a parameterised surface.
14. Equality of normal curvature of the surface and its osculating paraboloid at a point.
15. Existence of conjugate parameterisation.
16. Rodrigues's theorem.
17. Bonnet's fundamental theorem.
18. Theorem about the locally shortest curve on a surface.
19. The Gauss-Bonnet Theorem.
20. Local Isometry Theorem.
21. Bertrand's Theorem.

III. Core calculations

1. Restore parameterisation from the description of a curve.
2. Investigate asymptotes of a curve.
3. Investigate singular points of an explicit parameterisation.
4. Calculate arc length and find natural parameterisation.
5. Calculate curvature, torsion and the Frenet frame.
6. Find natural equations of a curve.
7. Calculate the arc length of a curve on a surface.
8. Calculate the angle between curves on a surface.
9. Calculate the area of a region on a surface.
10. Find the osculating sphere.
11. Compute the first and second quadratic (fundamental) forms from parameterisation.
12. Calculate principal curvatures and lines of curvature.
13. Calculate the mean and (or) Gaussian curvature.
14. Calculate geodesic curvature.

Important Notes

1. The listed core definitions, assertions, and calculations are not exact statements of the assignments.
2. Carefully read the particular statements on the examination to identify the exact core notion(s) employed.
3. Practical problems may combine 2–3 core calculations.