Analytic geometry

I. Vectors on plane and in space

1. Axioms of geometry and basic theorems derived from them (without proofs).
2. Vectors: directed segments and abstract (free) vectors. Addition of vectors. Multiply vector by number.
3. Product of vectors. Dot product of vectors, definition and basic properties. Left and right triplets of vectors. Cross product of vectors, definition and basic properties. Mixed product of vectors, definition and basic properties.
4. Expansion of vectors on plane and in space. Coordinates of vector. Basis vectors.
5. Coordinate system with right orthonormal basis on plane and in space. Formulas for dot, cross and mixed products. Radius vector. Coordinates of point.
6. Polar, cylindrical and spherical coordinates.
7. Vectors in skew-angular basis. Linearly dependent and independent systems of vectors. Basis vectors as linearly independent system. Example: analytic geometry on a single line.
8. Dot product in skew-angular basis. Gram matrix.
9. Cross and mixed products in skew-angular basis. Oriented volume of basis. Effectivized formulas for mixed and cross products.
10. The product of two mixed products.
11. Triple product. Jacobi identity.

List of problems

- Calculation of sums of vectors.
- Calculation of Gram matrix and its derivatives.
- Calculation of dot, cross, and mixed products of vectors.
- Investigation of bases.

II. Base problems of analytic geometry

12. Analytic geometry of points on plane and in space: distance form the origin, distance between two points.
13. Analytic geometry of the segment on plane and in space: division ratio, inclination and slope on plane. Direction cosines in space.
14. Area of polygon.
15. Equation of the locus. Degrees of freedom.

16. Steps to discuss and plot locus. Intersection of curves.

List of problems

- Determination of position of point on the line, on the plane and in the space.
- Measure of distances and angles.
- Calculation of areas of polygons.
- Investigation of locuses.

III. Straight line on plane

17. Vectorial equations of straight line on plane in parametric and normal form.

18. Coordinate parametric equations.

19. Canonical equation of line and its forms: line passing through two given points, line passing through the origin.

20. Explicit equations of the line in slope-point and slope-intercept form.

21. Double intercept equation of the line

22. General equation of the line in orthonormal basis. Explanation of coefficients. Angle between lines. Parallel and perpendicular lines.

23. Normal form of the equation. Measure of distances from the line.

24. Bundles (beams) of lines. Intersection of three lines in a point.

25. Line in polar coordinates.

26. General equation of the line in arbitrary basis. Dual basis and covariant coordinates on plane. Transformation of bases on plane. Explanation of the coefficients in general equation of line.

27. Generalization of the explicit equations of line for any skew-angular basis. General formula for angle coefficient (slope). General formula to measure angle between lines. Parallel and perpendicular lines.

28. Transformation of coordinate system. General definitions. Example: rotation matrix. Example: transformation to coordinates expressed with equations of perpendicular lines.

List of problems

- Derivation of equations of the straight line on a plane in various forms.
- Measure of distances from line and angles between lines.
- Derivation of the locus expressed as family of lines. Investigation of such locuses.
- Transformation of the equation of locus with transformation of coordinate system.

IV. Plane in space

29. Plane in space. Vectorial equations: parametric and normal.

30. Plane in space. Canonical equation in vector and coordinate forms.

31. General equation of plane. Explanation of coefficients. Covariant coordinates and dual basis in space. Angle between planes.

32. Equation of the plane passing through three points.

33. Normal equation of the plane in right orthonormal basis. Measuring distances.

34. Triple intercept equation of the plane.

35. Beams and bundles of planes. Equation of the plane from proper beam. Equation of the plane from proper bundle.

List of problems

- Derivation of the plane equations in various forms.
- Measure of distances from plane and angles between planes.
- Derivation of the locus expressed as family of lines. Investigation of such locuses.
- Investigation of beams and bundles of planes.

V. Straight line in space

36. Parametric equations of straight line in space in vectorial and coordinate forms.

37. Canonical equation of straight line in space.

38. Relative proposition of line and plane. Angle between line and plane. Perpendicular and parallel line and plane.

39. Lines laying in the same plane.

40. Line passing through two given points.

41. Line as intersection of two planes. Equation in projections.

42. Distance from line to point in space.

43. Distance between two lines. Case of parallel lines.

44. Skew lines. Common perpendicular of skew lines. Distance between skew lines.

List of problems

- Derivation of equations of the straight line in space in various forms.
- Measure of distances from line and angles between line and plane and between lines.
- Investigation of beams and bundles of lines.

VI. Transformation of figures

45. Rotation matrix in space. Euler angles.

46. Orthogonality of rotation matrix.
47. Linear transformations of figures. Orthogonal transformation.
48. Affine transformation of figures. Key invariant of affine transformation: ratio of three points.
49. Projective transformation. Key invariant of affine transformation: cross ratio.
50. Central projection equivalent with projective transformation.
51. Homogeneous coordinates.

List of problems

- Derivation of the equation of transformed locus.

VII. Conic sections

52. Definition and canonical equation of parabola.
53. Focus, directrix, focal parameter, axis, focal chord of parabola.
54. Definition and canonical equation of ellipse.
55. Foci, axes, semi-axes, center, vertices, eccentricity, focal radii, fundamental rectangle of ellipse.
56. Parametric equation of ellipse.
57. Definition and canonical equation of hyperbola.
58. Foci, axes, semi-axes, center, vertices, eccentricity, focal radii of hyperbola.
59. Fundamental rectangle and hyperbola asymptotes. Rectangular (equilateral) hyperbola.
60. Parametric equation of hyperbola.
61. Conjugated hyperbolas.
62. Equation of hyperbola with respect to its asymptotes.
63. Main property of ellipse and hyperbola directrices.
64. Focal parameter of ellipse and hyperbola.
65. Equation of conics at the vertex.
66. Equation of tangent line to the conics.
67. Optical property of conics.
68. Equations of conics in polar coordinates.

List of problems

- Derivation of canonical equation of conics.

- Determination of conic type as locus of points.
- Calculation of length of axes (ellipse, hyperbola), coordinates of the foci and the eccentricity, length of focal radii of point lying on conic.
- Derivation of directrices and asymptotes equations.
- Derivation of equations of tangents to conic.

VIII. Second-order curves

69. Definition and canonical equation of second-order curves. Types of curves: central (hyperbolic and elliptic), parabolic.

70. Theorem on quadratic form transformation. Conclusion of theorem.

71. Orthogonal invariants of second-order curves.

72. Semi-invariant of second-order curves.

73. Characteristic equation of quadratic form of second-order curves.

74. Reduced equations of second-order curves.

75. Theorem on decomposing curves.

76. Center of second-order curve. Equations defining the center. Curve equation in coordinate system with origin in center.

77. Intersection of a second-order curve with a line. Asymptotic directions. Types of curves according to the existence of asymptotic directions.

78. Definition and equation of diameter conjugate to non-asymptotic direction. Properties of diameters of different types of curves.

79. Definition and equation of tangent line to second-order curve. Direction of tangent line at intersection point of curve and diameter.

80. Curve equation in coordinate system with axes of conjugate directions.

81. Principal directions and principal diameters of curve.

82. Location of a second-order curve.

List of problems

- Derivation of equation of second-order curve.
- Investigation of equation of second-order curve.
- Derivation of canonical equation of the curve if its invariants are given.
- Derivation of equation of tangent line to second-order curve.
- Derivation of equation of diameter conjugate to the non-asymptotic direction (including principal directions and diameters), equations of asymptotes and directrices.

IX. Second-order surfaces

83. Definition and types of second-order surfaces.

84. Orthogonal invariants and semi-invariants of second-order surfaces.

85. Reduced and canonical equations of a second-order surfaces.

86. Center of second-order surface. Equations defining the center. Surface equation in coordinate system with origin in center. Classification of second-order surfaces by the nature of the locus of centers.

87. Intersection of a second-order surface with a line. Asymptotic directions, asymptotic cone and cone of asymptotic directions.

88. Definition and equation of diametral plane conjugate to a non-asymptotic direction. Definition of special directions. The condition of special direction for non-zero vector.

89. Definition and equation of tangent plane to second-order surface.

90. Elliptic, hyperbolic or parabolic points of a second-order surface.

91. Definition of principal direction and principal diametral plane of a second-order surface. Number of principal directions.

92. Location of a second-order surface.

List of problems

- Derivation of equation of second-order surface.
- Reduction of the surface to the canonical form. Investigation of equation of second-order surface.
- Derivation of equation of tangent plane to second-order surface.
- Determination of the type of intersection curve of surface and plane.
- Derivation of equation of diametral plane conjugate to the non-asymptotic direction. Inverse problem: derivation of direction of chords to which the diametral plane is conjugate.