Sept 20th.

$$y(x)=x+\int_0^x \sin(x-t)y(t)dt\,;$$

Sol: denote that: $f(x)=x \leftrightarrow F(p)$ $\quad K(x)=\sin x \leftrightarrow K^*(p)$ $\quad y(x) \leftrightarrow Y(p)$

$F(p)=\frac{1}{p^2}$ $\quad K^*(p)=\frac{1}{p^2+1}$

$$Y(p)=\frac{F(p)}{1-K^*(p)}=\frac{\frac{1}{p^2}}{1-\frac{1}{p^2+1}}=\frac{p^2+1}{p^4}=\frac{1}{p^2}+\frac{1}{p^4}$$

restore the original, $y(x)=x+\frac{x^3}{6}$

$$y''(x)-2y'(x)+y(x)+2\int_0^x \cos(x-t)y''(t)dt+$$
$$+2\int_0^x \sin(x-t)y'(t)dt=\sin x,\ y(0)=y'(0)=0\,;$$

Sol: by differentiation of original.

$y''(x) \longleftrightarrow p^2 Y(p)$

$y'(x) \longleftrightarrow p\,Y(p)$

by Duhamel's integral,

$$2\cos x\cdot y'(0)+2\int_0^x \cos(x-t)\,y''(t)\,dt \longleftrightarrow 2p\cdot\frac{p}{p^2+1}\cdot Y'(p)=\frac{2p^3}{p^2+1}Y(p)$$

$$2\sin x\cdot y(0)+2\int_0^x \sin(x-t)\,y'(t)\,dt \longleftrightarrow 2p\cdot\frac{1}{1+p^2}\cdot Y(p)=\frac{2p}{p^2+1}Y(p)$$

thus. apply Laplace transform to both side.

$$p^2Y(p)-2pY(p)+Y(p)+\frac{2p^3}{p^2+1}Y(p)+\frac{2p}{p^2+1}Y(p)=\frac{1}{p^2+1} \Rightarrow Y(p)=\frac{1}{(p^2+1)^2}$$

$$\Rightarrow y(x)=\frac{1}{2}(\sin x-x\cos x)$$