Complex analysis. Spring 2024.
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1. Complex plane. Operations on complex numbers.
2. Complex sequences and series.
3. Limit of a function of a complex variable.
4. Polar representation of complex numbers.
5. Paths in a complex plane.
6. Domains in a complex plane. Path-connected and connected domains.
7. Riemann sphere.
8. Complex differentiability. Cauchy-Riemann identities (!).
9. Elementary functions of complex variable and their differentiability (polynomials, $\cos z$, $\sin z$, e^z,)
10. Properties of e^z, De'Moivre's formula.
11. Complex logarithm.
12. Roots of complex numbers.
13. Directional derivative. Characterization of complex differentiability in terms of directional derivative.
14. Holomorphic functions and conformal mapping.
15. Images of elementary conformal maps (z^n, $\ln z$, e^z, $\cos z$, $\sin z$)
16. Harmonic functions.
17. Geometric meaning of complex derivative.
18. Holomorphic and conformal maping of extended complex plane.
19. Integration of a complex-valued function.
20. line-Integrals of a first and a second kind.
21. The Cauchy-Goursat theorem for triangles.
22. Criterion for constancy of a holomorphic function.
23. Antiderivative of a holomorphic function. Existence of antiderivative of a function holomorphic in a disk.
24. Antiderivative along the path. Theorem on existence and uniqueness.
25. Newton-Leibniz formula for line-integral.
26. Homotopy. Simply connected domains.
27. Cauchy theorem on homotopy. Cauchy-Goursat's theorem for a contractible path.
28. Existence of antiderivative of a function holomorphic in a simply connected domain.
29. Cauchy-Goursat's theorem for a multiple connected domains.
30. Cauchy's Integral Formula.
31. Decomposition of a holomorphic function into a Taylor series.

32. Cauchy inequalities.

33. Liuville theorem. The fundamental theorem of algebra.

34. A set of convergence of a power series.

35. Holomorphity of a sum of power series.

36. Infinite differentiability of holomorphic functions.

37. Coefficients of Taylor series. Cauchy integral formula for derivatives.

38. Morer's theorem.

39. Decomposition of a holomorphic function in neighbourhood of its zero.

40. Uniqueness theorem.

41. Maximum modulus principle.

42. Decomposition of a holomorphic function into a Laurent series.

43. Cauchy inequalities for Laurent coefficients.

44. Isolated singular points. Definitions and examples.

45. Classification of removable (fixable) singular point.

46. Description of the poles.

47. Sokhotsky's theorem.

48. $a = \infty$ as isolated singular point.

49. Integer functions.

50. Meromorphic functions with a pole at infinity.

51. The residue theorem.

52. Residue as coefficient of the Laurent series.

53. Formulas for calculation of residues.

54. Residues at $a = \infty$.

55. Jordan's lemma.

56. Evaluation of Real Trigonometric Integrals.

57. Evaluation of Real Improper Integrals.

58. Integrals of the Form $\int_{-\infty}^{\infty} f(x)\cos\alpha x dx$ and $\int_{-\infty}^{\infty} f(x)\sin\alpha x dx$.

59. Integration along a Branch Cut.

60. The Argument Principle and Rouche's Theorem.

61. Location of Zeros.

62. Summation of infinite series using residue theorem.

63. Direct analytic continuation of a holomorphic function. Example.

64. Analytic continuation of $\Gamma-$function.

65. Analytic continuation of logarithm from the unit disk $\{z \in \mathbb{C} : |z-1| < 1\}$ to slit plane $\mathbb{C}\backslash(-\infty, 0]$.

66. Elements and analytic continuation.

67. Properties of direct analytic continuation of elements.

68. Analytic continuation of canonical elements along a path.

69. Equivalence of analytic continuation along a chain and along a path.

70. Monodromy theorem.

71. Analytic functions. Three equivalent definitions.

72. Analytic function $\operatorname{Ln} z$.

73. Operations on analytic functions.

74. Power function z^α.

75. Isolated singular points of analytic function.

76. Classification of isolated singular points.

Types of problems.

1. Calculation of results of arithmetic operations on complex numbers. Applications of Euler's formula.

2. Complex differentiability. Verification of complex differentiability of a function of complex variable.

3. Values of roots of complex numbers. Values of logarithm of complex numbers.

4. Images of functions of complex variable.

5. Integrals of a function of a complex variable along a path of a first and a second kind.

6. Application of theorem on constancy of holomorphic function.

7. Calculation of antiderivative of a function.

8. e^z, $\cos z$, $\sin z$, $cosh(z)$, $sh(z)$ and identities for these functions.

9. Laurent series.

10. Calculation of residues.

11. Calculation of integral applying residue theorem.

12. Application of the Argument Principle and Rouché's Theorem

13. Analytic functions. Operations on analytic functions. Single-valued germs of analytic functions.

14. Important definitions and statement: Complex differentiability, elementary functions of complex variables, Cauchy-Riemann identities, Complex logarithm, holomorphic function, conformal map, The Cauchy-Goursat's theorem, antiderivative, antiderivative along a path, homotopy, Cauhy's integral, Taylor series, Laurent's series, maximum modulus principle, types of isolated singular points, integer function, residue, residue theorem, analytic continuation, analytic continuation along path, analytic function (first definition), germ of analytic function.