Evaluation of Real Trigonometric and Improper Integrals

TASKS

Evaluate the given trigonometric integral.

1. $\int_0^{2\pi} \frac{1}{1+0.5\sin\theta} d\theta$

   Answer: $4\pi/\sqrt{3}$

2. $\int_0^{\pi} \frac{1}{2-\cos\theta} d\theta$[Hint: Let $t = 2\pi - \theta$.]

   Answer: $\pi/\sqrt{3}$

3. $\int_0^{2\pi} \frac{\cos 2\theta}{5-4\cos\theta} d\theta$

   Answer: $\pi/6$

Establish the given general result.

4. $\int_0^{\pi} \frac{d\theta}{(a+\cos\theta)^2} d\theta = \frac{a\pi}{\left(\sqrt{a^2-1}\right)^3}, a > 1$

Evaluate the Cauchy principal value of the given improper integral.

5. $\int_{-\infty}^{\infty} \frac{1}{x^2-2x+2} dx$

Answer: π

6. $\int_{-\infty}^{\infty} \frac{2x^2-1}{x^4+5x^2+4} dx$

Answer: $\pi/2$

7. $\int_0^{\infty} \frac{x^2}{x^6+1} dx$

Answer: $\pi/6$

8. $\int_{-\infty}^{\infty} \frac{x\sin x}{x^2+1} dx$

Answer: πe^{-1}

9. $\int_0^{\infty} \frac{\cos 2x}{x^4+1} dx$

Answer: $\frac{\pi e^{-\sqrt{2}}}{2\sqrt{2}}(cos\sqrt{2} + sin\sqrt{2})$