

# Equations of Mathematical Physics Homework 6

Li Kaiyan 2022111221

## Problem 1

Solve the Cauchy problem for the equation

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2xy,$$

satisfying the conditions  $y = x \quad u = x^2$

**Solution:**

Let's make up a characteristic system:

$$\frac{dx}{x} = \frac{dy}{y} = \frac{du}{2xy}$$

We will find the first integral by solving the equation:

$$\frac{dx}{x} = \frac{dy}{y} \implies \frac{x}{y} = C_1$$

then we have  $x = C_1 y$ , substitute  $C_1 y$  into  $x$ , we have the second integral:

$$\frac{dx}{C_1 y} = \frac{du}{2xy} \implies 2x dx = C_1 du \implies x^2 = C_1 u + C_2 \implies C_2 = \frac{xu}{y} - x^2$$

thus we find the general solution without considering the integral surface:

$$\Phi\left(\frac{x}{y}, \frac{xu}{y} - x^2\right) = 0$$

Solving the Cauchy problem, take  $x$  as the parameter, we obtain:

$$C_1 = 1 \quad C_2 = 0$$

This gives  $\begin{cases} \frac{x}{y} = 1 \\ \frac{xu}{y} - x^2 = 0 \end{cases}$ , i.e.  $\begin{cases} x = y \\ u = xy \end{cases}$ .

Thus the solution can be given as a parametric form:  $\begin{cases} x = t \\ y = t \\ u = t^2 \end{cases}$ , where  $t$  is a parameter.