Complex Analysis 2024. Homework 3.

1. Plot the path given by $\gamma(t)$

$$\gamma(t) = i + e^{it}, \quad 0 \le t \le \pi.$$

Plot it's image with respect to mapping $f(z) = (z - i)^3$;

Solution. This path circumscribes counterclockwise a upper half of the circle of radius 1 with center at i. The image circumscribes counterclockwise a circle of radius 1 around zero when $0 \le t \le 2\pi/3$ and upper half of the circle when $2\pi/3 \le t \le \pi$.

2. Find the image of the given line under the complex mapping $w = z^2$

   (a) $\operatorname{Re} z = \operatorname{Im} z$;

   (b) $\operatorname{Re} z = 3$;

   Solution

   (a) The image is a set of points $w = (x + ix)^2 = 2ix^2$. It is equal to the ray on imaginary line $\operatorname{Re} w = 0,\ \operatorname{Im} w > 0$.

   (b) The image is a set of points $w = (3 + iy)^2 = 9 - y^2 + 6yi,\ y \in \mathbb{R}$. This is parabola $\operatorname{Re} w = 9 - \left(\frac{\operatorname{Im} w}{6}\right)^2$.

3. Calculate all values of

$$\sqrt[3]{-3 + 3i}; \quad \sqrt[5]{-1 + \sqrt{3}i}.$$

First

$$2^(1/6)3^{1/3}\left(\cos\frac{\pi}{4} + isin\frac{\pi}{4}\right);$$

$$2^{1/6}3^{1/3}\left(\cos\frac{11\pi}{12} + isin\frac{11\pi}{12}\right);$$

$$2^{1/6}3^{1/3}\left(\cos\frac{19\pi}{12} + isin\frac{19\pi}{12}\right).$$

second

$$\sqrt[5]{2}\left(\cos\left(\frac{2\pi}{15}\right) + i\sin\left(\frac{2\pi}{15}\right)\right);$$

$$\sqrt[5]{2}\left(\cos\left(\frac{8\pi}{15}\right)+i\sin\left(\frac{8\pi}{15}\right)\right);$$

$$\sqrt[5]{2}\left(\cos\left(\frac{14\pi}{15}\right)+i\sin\left(\frac{14\pi}{15}\right)\right);$$

$$\sqrt[5]{2}\left(\cos\left(\frac{4\pi}{3}\right)+i\sin\left(\frac{4\pi}{3}\right)\right);$$

$$\sqrt[5]{2}\left(\cos\left(\frac{28\pi}{15}\right)+i\sin\left(\frac{28\pi}{15}\right)\right);$$

4. Find the image of the domain $|z|<8$, $\pi/2<\arg z<3\pi/4$ under each of the following principal nth root function ($k=0$ in our definition)

$$f(z)=z^{1/3};\quad f(z)=z^{1/2}.$$

Solution. 1) $|z|<2,\ \pi/6<\arg z<\pi/4$;

2) $|z|<2\sqrt{2},\ \pi/4<\arg z<3\pi/8$;