Subject of mathematical statistics. Examples of statistical problems.
Mathematical statement of statistical problems.
Empirical distribution function and its properties.
Desired properties of estimators. Relation between unbiasedness and consistency. Properties of $\overline{X}$ and S^2.
Order statistics and their limiting distributions.
Empirical quantiles and its properties.
Fisher's lemma.
Confidence intervals for parameters of normal distribution. The 1,2,3 σ rules.
The method of moments estimator and its properties. The delta-method.
Maximum likelihood and examples of its application.
Regular statistical experiments. Fisher's information and its calculation.
The Cramér–Rao inequality and its corollary.
Properties of maximum likelihood estimator. Application to constructing of confident intervals.
Notion of sufficient statistics. The Fisher–Neyman factorization theorem (w/p), examples.
Unbiased estimation with sufficient statistics (Theorem Rao-Blackwell-Kolmogorov). Completeness. Theorem Lehmann-Scheffe. Ancillary statistic. Basu's theorem.
Minimax and Bayesian estimators and relation between them. Minimax property for as an estimator of the mean for normal law.
Statement of linear regression problem. Least squares method. The Gauss–Markov theorem. Unbiased estimators for error variance.
Statistical hypothesis testing: mathematical statement. Type I and Type II errors. Power of statistical test.
The Neyman–Pearson lemma. Relative likelihood. Wilks' theorem. Student's test as a likelihood-ratio test
Types of statistical hypotheses. Tests based on the empirical distribution. Distribution-free tests, limit distributions.
Testing parametric hypotheses in Gaussian models (for 1 sample and 2 samples).
Chi-squared test (w/p). Chi-squared test for unknown parameters. Application for independence and homogeneity.
Locally most powerful rank tests (w/p). Hypothesis testing for ranked data. Examples (the normal scores, Van der Waerden and Wilcoxon tests.)