HW1. Version 5.

1. $\int\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2 dx$

$= \int (1-2\sin\frac{x}{2}\cos\frac{x}{2})\,dx = \int (1-\sin x)\,dx$

$= x+\cos x + C$

2. $\int\frac{x^4}{x^5+7}dx$

$= \frac{1}{5}\int \frac{d(x^5+7)}{x^5+7} = \frac{1}{5}\ln|x^5+7| + C.$

3. $\int e^x \sin x\,dx.$

$\int e^x \sin x\,dx = -e^x\cos x + \int e^x \cos x\,dx$

$\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$

$\Rightarrow \int e^x \sin x\,dx = \frac{e^x\sin x - e^x\cos x}{2} + C$

4. $\int\frac{5x+2}{x^2+2x+10}dx.$

$= \int \frac{\frac{5}{2}d(x^2+2x+10)}{x^2+2x+10} - \frac{1}{9}\int \frac{d(\frac{x+1}{3})}{(\frac{x+1}{3})^2+1}$

$= \frac{5}{2}\ln(x^2+2x+10) - \frac{1}{9}\arctan\frac{x+1}{3} + C$

5. $\int\frac{dx}{3\sin x+4\cos x}$

$= \int \frac{dx}{5\sin(x+\varphi)}$ $\quad \varphi = \arctan\frac{4}{3}$

$= \frac{1}{5}\ln\left|\tan\frac{x+\varphi}{2}\right| + C. \quad \varphi = \arctan\frac{4}{3}.$

HW2 Version 5.

1. $\int_0^{\pi/3} \cos^3 x \sin 2x \, dx$

2. $\int_{-3}^{3} \frac{x^2 \sin 2x}{x^2+1} dx.$

3. **Find the area of a flat figure bounded by a single cycloid arch and the Ox-axis:**
$x=2(t-\sin t),\ y=2(1-\cos t)$

4. **Find the volumes of bodies formed by rotation around the Ox-axes of a figure bounded by lines:**
$y=\sqrt{x}\, e^x,\ x=1,\ y=0.$

5. **Calculate improper integrals (or establish their divergence):**
$\int_1^{+\infty} \frac{e^{-x^2}}{x^2} dx.$

1. $\int_0^{\frac{\pi}{3}} \cos^3 x \sin 2x = -2\int_0^{\frac{\pi}{3}} \cos^4 x \, d(\cos x) = -2 \left. \frac{\cos^5 x}{5} \right|_0^{\frac{\pi}{3}} = \frac{31}{80}$

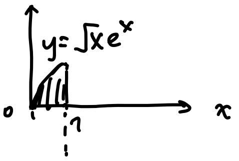
2. $\int_{-3}^{3} \frac{x^2 \sin 2x}{x^2+1} dx = 0$ odd function with symmetric domain.

3. single cycloid : $0 \le t \le 2\pi$

$$S = \int y(x)\, dx = \int_0^{2\pi} y(t) \cdot x'(t)\, dt = 4\int_0^{2\pi} (1-\cos t)^2\, dt = 4\int_0^{2\pi} (1-2\cos t + \cos^2 t)\, dt$$

$$= 4\left(2\pi + 2\pi \cdot \frac{1}{2}\right) = 12\pi.$$

4.



$$V = \pi \int_a^b y^2(x)\, dx = \pi \int_0^1 (\sqrt{x}\, e^x)^2\, dx = \pi \int_0^1 x e^{2x}\, dx$$

$$= \pi \left[\frac{1}{2} x \cdot e^{2x} \Big|_0^1 - \frac{1}{2}\int_0^1 e^{2x}\, dx \right] = \pi \left[\frac{e^2}{2} - \frac{e^2-1}{4} \right] = \pi \frac{e^2+1}{4}.$$

5. $\int_1^{+\infty} \frac{e^{-x^2}}{x^2} dx$

$e^{-x^2} < 1$ in $[1, +\infty)$

$\int_1^{+\infty} \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^{+\infty} = 1$ conv. thus the integral $\int_1^{+\infty} \frac{e^{-x^2}}{x^2} dx$ conv.