TESTS

Tropical Optimization Problems

Please submit a solution to the test online as a readable file preferably prepared with an editor. Please include intermediate results to facilitate and speed up verification of the solutions obtained.

TEST-TOP 1.

Problem 1. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\boldsymbol{Ax} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -2 & 2 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

Problem 2. Find in $\mathbb{R}_{\min,+}$ all solutions of the problem $\min_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{Ax}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 0 & 2 \\ -2 & 3 & 1 \end{pmatrix}.$$

TEST-TOP 2.

Problem 3. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\boldsymbol{Ax} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 4. Find in $\mathbb{R}_{\min,\times}$ all solutions of the problem $\min_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{Ax}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}.$$

TEST-TOP 3.

Problem 5. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\boldsymbol{Ax} = \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & -2 & -2 \\ 3 & 0 & 1 \\ 2 & 1 & 3 \end{pmatrix}.$$

Problem 6. Find in $\mathbb{R}_{\max,\times}$ all solutions of the problem $\min_{\boldsymbol{x}} \boldsymbol{x}^{-}\boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1/2 & 1 & 2 \\ 1 & 1/2 & 1 \\ 1/3 & 1/2 & 1/3 \end{pmatrix}.$$

TEST-TOP 4.

Problem 7. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

Problem 8. Find in $\mathbb{R}_{\max,\times}$ all solutions of the problem $\min_{\boldsymbol{x}} \boldsymbol{x}^{-}\boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1/2 & 2 & 1 \\ 1/2 & 1/3 & 1/3 \\ 1 & 1 & 1/2 \end{pmatrix}.$$

TEST-TOP 5.

Problem 9. Find in $\mathbb{R}_{\min,\times}$ all solutions of the inequality $\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

Problem 10. Find in $\mathbb{R}_{\max,+}$ all solutions of the problem $\min_{\boldsymbol{x}} \boldsymbol{x}^{-}\boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 2 \\ -2 & -2 & -1 \end{pmatrix}.$$

TEST-TOP 6.

Problem 11. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1/2 & 1/2 \\ 1 & 1 & 1/3 \\ 2 & 1 & 1/2 \end{pmatrix}.$$

Problem 12. Find in $\mathbb{R}_{\min,+}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^{-} \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 4 & 3 \\ 3 & 2 & 2 \\ 3 & 2 & 3 \end{pmatrix}.$$

TEST-TOP 7.

Problem 13. Find in $\mathbb{R}_{\min,\times}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1/2 & 1 \\ 3 & 2 & 2 \\ 3 & 1/2 & 1 \end{pmatrix}.$$

Problem 14. Find in $\mathbb{R}_{\max,+}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^{-} \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 & -2 \\ 0 & 0 & 1 \\ 1 & -2 & 0 \end{pmatrix}.$$

TEST-TOP 8.

Problem 15. Find in $\mathbb{R}_{\min,\times}$ all solutions of the inequality $\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 2 \end{pmatrix}.$$

Problem 16. Find in $\mathbb{R}_{\max,+}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^{-} \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 2 & 3 \\ -2 & 1 & 1 \\ 2 & -2 & 0 \end{pmatrix}.$$

TEST-TOP 9.

Problem 17. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1/2 & 1 & 3 \\ 1/3 & 2 & 2 \\ 1 & 111 & \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 18. Find in $\mathbb{R}_{\max,+}$ all solutions of the problem $\min_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} -1 & 0 & -2 \\ -1 & 0 & -1 \\ -1 & 1 & -1 \end{pmatrix}.$$

TEST-TOP 10.

Problem 19. Find in $\mathbb{R}_{\min,\times}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 2 & 1 & 2 \end{pmatrix}.$$

Problem 20. Find in $\mathbb{R}_{\min,+}$ all solutions of the problem $\min_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & -2 \\ 3 & 4 & 2 \end{pmatrix}.$$

TEST-TOP 11.

Problem 21. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & -2 \\ -1 & -1 & -2 \\ -1 & 1 & 0 \end{pmatrix}.$$

Problem 22. Find in $\mathbb{R}_{\max,\times}$ all solutions of the problem $\min_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 1/3 \\ 1/2 & 1/3 & 1 \\ 1 & 1/3 & 1/2 \end{pmatrix}.$$

TEST-TOP 12.

Problem 23. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 3 & -1 & 2 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 24. Find in $\mathbb{R}_{\max,\times}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/3 \\ 2 & 1 & 1/2 \end{pmatrix}.$$

TEST-TOP 13.

Problem 25. Find in $\mathbb{R}_{\max,+}$ all solutions of the inequality $\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} -1 & -1 & 0 \\ 1 & 0 & -2 \\ -2 & -3 & -1 \end{pmatrix}.$$

Problem 26. Find in $\mathbb{R}_{\min,+}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & -1 & 3 \\ 3 & 1 & -1 \\ 2 & 2 & 1 \end{pmatrix}.$$

TEST-TOP 14.

Problem 27. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & 3 \\ 1 & -2 & 2 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

Problem 28. Find in $\mathbb{R}_{\min,\times}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}.$$

TEST-TOP 15.

Problem 29. Find in $\mathbb{R}_{\max,\times}$ all solutions of the inequality $\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1/2 & 1/3 \\ 2 & 1/2 & 1/2 \\ 2 & 1 & 1/2 \end{pmatrix}.$$

Problem 30. Find in $\mathbb{R}_{\min,+}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^{-}\boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 3 \\ 1 & 1 & 2 \\ -2 & -1 & 1 \end{pmatrix}.$$

TEST-TOP 16.

Problem 31. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} -2 & 0 & -2 \\ -1 & -1 & 1 \\ -1 & -2 & -1 \end{pmatrix}.$$

Problem 32. Find in $\mathbb{R}_{\min,+}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^{-}\boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & -1 \\ 1 & 1 & 3 \end{pmatrix}.$$

TEST-TOP 17.

Problem 33. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 3 & 3 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 34. Find in $\mathbb{R}_{\min,\times}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^{-}\boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 2 & 1 \\ 1/2 & 3 & 1 \\ 3 & 4 & 3 \end{pmatrix}.$$

TEST-TOP 18.

Problem 35. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/2 & 1/2 \\ 3 & 1/2 & 1/3 \end{pmatrix}.$$

Problem 36. Find in $\mathbb{R}_{\min,+}$ all solutions of the problem $\min_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 2 & 2 \\ 0 & 1 & 0 \end{pmatrix}.$$

TEST-TOP 19.

Problem 37. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 0 & 4 & 1 \\ 2 & 2 & 3 \\ 2 & 1 & 2 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 38. Find in $\mathbb{R}_{\min,+}$ all solutions of the problem $\min_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & -1 \\ 3 & 2 & 1 \end{pmatrix}.$$

TEST-TOP 20.

Problem 39. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 2 & -1 & 2 \end{pmatrix}.$$

Problem 40. Find in $\mathbb{R}_{\max,\times}$ all solutions of the problem $\min_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1/2 & 1/2 \\ 2 & 1/2 & 1/3 \\ 1 & 1/3 & 1 \end{pmatrix}.$$

TEST-TOP 21.

Problem 41. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 3 \\ 3 & 0 & 2 \\ -2 & 1 & 0 \end{pmatrix}.$$

Problem 42. Find in $\mathbb{R}_{\max,+}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & -1 \\ -2 & -2 & -1 \end{pmatrix}.$$

TEST-TOP 22.

Problem 43. Find in $\mathbb{R}_{\min,+}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 0 \\ 4 & 2 & 2 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Problem 44. Find in $\mathbb{R}_{\max,\times}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1/2 & 1 & 1/3 \\ 1/2 & 1/2 & 1 \\ 1 & 1/2 & 1/3 \end{pmatrix}.$$

TEST-TOP 23.

Problem 45. Find in $\mathbb{R}_{\min,\times}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 3 & 2 \\ 1 & 3 & 2 \\ 1 & 1/2 & 3 \end{pmatrix}.$$

Problem 46. Find in $\mathbb{R}_{\max,+}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} -1 & 2 & 3 \\ -2 & 0 & 1 \\ -4 & -2 & -2 \end{pmatrix}.$$

TEST-TOP 24.

Problem 47. Find in $\mathbb{R}_{\max,\times}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1/2 & 1 \\ 2 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/2 \end{pmatrix}.$$

Problem 48. Find in $\mathbb{R}_{\min,+}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 3 & 3 & 1 \\ 2 & 3 & 2 \\ 3 & 0 & 3 \end{pmatrix}.$$

TEST-TOP 25.

Problem 49. Find in $\mathbb{R}_{\min,+}$ all solutions of the inequality $\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{pmatrix}.$$

Problem 50. Find in $\mathbb{R}_{\max,+}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} -2 & 0 & -1 \\ 0 & -1 & 1 \\ -2 & -3 & -1 \end{pmatrix}.$$

TEST-TOP 26.

Problem 51. Find in $\mathbb{R}_{\max,+}$ all solutions of the equation $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & 1 \\ 2 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 52. Find in $\mathbb{R}_{\min,\times}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 1 & 2 & 3 \end{pmatrix}.$$

TEST-TOP 27.

Problem 53. Find in $\mathbb{R}_{\max,+}$ all solutions of the inequality $\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 0 & -2 & -2 \\ 2 & -1 & 1 \\ 0 & -3 & -1 \end{pmatrix}.$$

Problem 54. Find in $\mathbb{R}_{\min,+}$ all solutions of the problem $\min\limits_{\boldsymbol{x}} \boldsymbol{x}^- \boldsymbol{A}\boldsymbol{x}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 3 & 2 \\ 0 & 2 & 2 \end{pmatrix}.$$