Curved integrals

BACKGROUND INFORMATION

1. Curvilinear integrals of the first kind. Let the rectified curve Γ be given by the equation

$$\mathbf{r} = \mathbf{r}(s), \quad 0 \leqslant s \leqslant S \tag{1}$$

where s is the variable arc length of this curve. Then, if the function F is defined on the curve Γ, then the number

$$\int_0^S F(\mathbf{r}(s))ds$$

is called a curvilinear integral of the first kind from the function F along the curve Γ and denotes

$$\int_\Gamma F(x;y;z)ds \text{ or } \int_\Gamma F ds.$$

只是一个符号与F(x,y;z)中的x无关

Thus, by definition

$$\int_\Gamma F(x;y;z)dx = \int_0^S F(x(s);y(s);z(s))ds \tag{2}$$

Integral (2) exists if the function F is continuous on the curve Γ.
Properties of a curvilinear integral of the first kind.

1. The curvilinear integral of the first kind does not depend on the curve orientation.

2. If the curve Γ is the union of a finite number of curves $\Gamma_1, \ldots\ldots, \Gamma_k$, and the function F is continuous on Γ, then

$$\int_\Gamma F(x;y;z)dx = \sum_{i=1}^{k} \int_{\Gamma_i} F(x;y;z)ds \tag{3}$$

3. If the smooth curve Γ is given by the equation

$$\mathbf{r} = \mathbf{r}(t), \quad \alpha \leqslant t \leqslant \beta, \tag{4}$$

and the function F is continuous on the curve Γ, then

$$\int_\Gamma F(x;y;z)ds = \int_\alpha^\beta F(x(t);y(t);z(t))\sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}dt \tag{5}$$

If Γ is a smooth plane curve given by the equation

$$y = f(x), \quad a \leqslant x \leqslant b \tag{6}$$

than

$$\int_\Gamma F(x;y)dx = \int_a^b F(x;f(x))\sqrt{1+(f'(x))^2}dx \tag{7}$$

Similarly, if a smooth plane curve Γ is given by the equation, then

$$x = \varphi(y), \quad c \leqslant y \leqslant d$$

$$\int_\Gamma F(x;y)dx = \int_c^d F(\varphi(y);y)\sqrt{1+(\varphi'(y))^2}dy \tag{8}$$

2. Curved integrals of the second kind. Let the smooth curve Γ be given by equation (1). Then

$$\frac{d\mathbf{r}}{ds} = \tau = (\cos\alpha; \cos\beta; \cos\gamma) \tag{9}$$

- is a unit vector tangent to this curve. Here α, β, γ are the angles formed by the tangent with the coordinate axes Ox, Oy and Oz, respectively.

Let a vector function $\mathbf{F} = (P;Q;R)$ be defined on the curve Γ such that for a scalar function

$$F_\tau = (\mathbf{F}, \boldsymbol{\tau}) = P\cos\alpha + Q\cos\beta + P\cos\gamma$$

exists $\int_\Gamma F_\tau ds$. Then value

$$\int_\Gamma F_\tau ds. = \int_\Gamma (\mathbf{F}, \boldsymbol{\tau})ds. \tag{10}$$

it is called a curvilinear integral of the second kind from the function $\mathbf{F}$ along the curve Γ and denotes

$$\int_\Gamma^{\mathrm{T}} Pdx + Qdy + Rdz$$

Thus, by definition

$$\int_\Gamma Pdx + Qdy + Rdz = \int_0^S (P\cos\alpha + Q\cos\beta + R\cos\gamma)ds \tag{11}$$

where $(\cos\alpha; \cos\beta; \cos\gamma)$ is the unit vector of the tangent to the curve Γ. Formula (11) can be written in vector form:

$$\int_\Gamma (\mathbf{F}, d\mathbf{r}) = \int_0^S (\mathbf{F}(\mathbf{r}(s)), \boldsymbol{\tau}(s))ds \tag{12}$$

where $d\mathbf{r} = (dx; dy; dz)$.
If $Q = R = 0$, then the formula (11) is written as

$$\int_\Gamma P dx = \int_0^S P(x(s); y(s); z(s)) \cos\alpha(s) ds \tag{13}$$

Similarly,

$$\int_\Gamma Q dy = \int_0^S Q \cos\beta ds, \quad \int_\Gamma R dz = \int_0^S R \cos\gamma ds \tag{14}$$

Properties of a curved integral of the second kind.

1. When the orientation of the curve changes to the opposite, the curvilinear integral of the second kind changes sign.

2. If the smooth curve Γ is given by equation (4), and the vector function $\mathbf{F} = (P; Q; R)$ is continuous on Γ, then

$$\int_\Gamma (\mathbf{F}, d\mathbf{r}) = \int_\alpha^\beta (\mathbf{F}, \mathbf{r}'(t))\, dt \tag{15}$$

or

$$\begin{aligned} \int_\Gamma P dx + Q dy + R dz = \int_\alpha^\beta & [P(x(t); y(t); z(t))x'(t) + \\ & + Q(x(t); y(t); z(t))y'(t) + R(x(t); y(t); z(t))z'(t)]\, dt \end{aligned} \tag{16}$$

In the case when Γ is a flat smooth curve given by equation (6), it follows from formula (16) that

$$\int_\Gamma P(x; y) dx = \int_a^b P(x; f(x)) dx \tag{17}$$

$$\int_\Gamma Q(x; y) dy = \int_a^b Q(x; f(x)) f'(x) dx. \tag{18}$$

3. Green's formula. Let the boundary G of a flat bounded domain G consist of a finite set of piecewise smooth curves. Then if the functions $P, Q, \frac{\partial P}{\partial y}, \frac{\partial Q}{\partial x}$ are continuous on $\bar{G}$, then Green's formula is valid

$$\iint_G \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \int_\Gamma P dx + Q dy \tag{19}$$

轮廓 朝向 穿过
where the contour Γ is oriented so that when traversing it, the area G remains on the left.

From formula (19) for $Q = x, P = -y$ we get

$$S = \frac{1}{2} \int_\Gamma x dy - y dx \tag{20}$$

where $S = \iint_G dxdy$ is the area of the G area bounded by the Γ contour (when traversing the Γ contour, the G area remains on the left).

4. Conditions for the independence of a curved integral from the integration path. If the functions $P(x;y)$ and $Q(x;y)$ are continuous in the plane domain G, then the curvilinear integral

$$\int_{\Gamma_{AB}} Pdx + Qdy \tag{21}$$

does not depend on the integration path Γ_{AB} (the curve Γ_{AB} lies in the domain G, A is its beginning, B is its end) if and only if the expression $Pdx+Qdy$ is a complete differential some function $u = u(x;y)$, i.e. in the domain of G the condition is satisfied

$$du = Pdx + Qdy \quad \text{or} \quad \frac{\partial u}{\partial x} = P, \quad \frac{\partial u}{\partial y} = Q. \tag{22}$$

At the same time

$$int_{\Gamma_{AB}} Pdx + Qdy = u(B) - u(A) \tag{23}$$

Here

$$u(x;y) = \int_{\Gamma_{M_0 M}} Pdx + Qdy \tag{24}$$

where $\Gamma_{M_0 M}$ is some curve with the beginning at a fixed point $M_0\,(x_0;y_0)$ and the end at the point $M(x;y)$ lying in the region G.

Let the functions $P, Q, \frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$ be continuous in the flat domain G. Then in order for the curvilinear integral (21) to be independent of the integration path, it is necessary, and in the case when G is a simply connected domain, then it is sufficient that the condition is fulfilled in the domain of G

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \tag{25}$$

5. Some applications of curved integrals. Let a mass with a linear plane $\rho(x;y;z)$ be distributed on a piecewise smooth curve Γ (or $\rho(x;y)$ for a flat curve).

The mass of the curve is calculated by the formula

$$m = \int_{\Gamma} \rho(x;y;z)ds \tag{26}$$

coordinates of the center of mass - according to the formulas

$$x_C = \frac{1}{m}\int_{\Gamma} x\rho ds, \quad y_C = \frac{1}{m}\int_{\Gamma} y\rho ds, \quad z_C = \frac{1}{m}\int_{\Gamma} z\rho ds \tag{27}$$

the moments of inertia relative to the axes Ox, Oy and Oz - according to the formulas

$$I_x = \int_\Gamma \left(y^2+z^2\right)\rho ds, \quad I_y = \int_\Gamma \left(z^2+x^2\right)\rho ds, \quad I_z = \int_\Gamma \left(x^2+y^2\right)\rho ds. \quad (28)$$

Let the vector function $\mathbf{F}(\mathbf{r})$ be set on the domain Ω, where $\mathbf{r}isthe$radius vector of a point from Ω, then it is said that a vector (force) field is set on Ω. Let Γ be a piecewise smooth oriented curve in Q and the vector field $\mathbf{F}$ be continuous on Γ.

The work of the field $\mathbf{F}$ along R is called the integral

$$A = \int_\Gamma \mathbf{F}(\mathbf{r}) d\mathbf{r} \quad (29)$$

EXAMPLES

Example 1. Calculate the curvilinear integral

1. 判断 1/2 类型

$$I = \int_\Gamma (x+y) ds$$

where Γ is the boundary of a triangle (Fig. 10.1) with vertices $O(0;0)$, $A(1;0), B(1;1)$.

Δ Let I_1, I_2, I_3 be curvilinear integrals of the function $x+y$ over the segments AB, BO and OA, respectively. Since the segment AB is given by the equation $x=1, 0 \leqslant y \leqslant 1$, then by formula (8) we get

(1st kind)

$$I_1 = \int_0^1 (y+1) dy = \frac{3}{2}$$

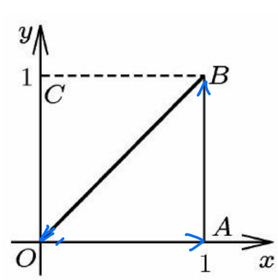

Fig. 10.1

The segments BO and OA are given respectively by the equations $y=x$, $0 \leqslant x \leqslant 1$, and $y=0, 0 \leqslant x \leqslant 1$. By the formula (7) we find

$$I_2 = \int_0^1 2x\sqrt{2}dx = \sqrt{2}, \quad I_3 = \int_0^1 x dx = \frac{1}{2}$$

$\sqrt{1+f'(x)}$

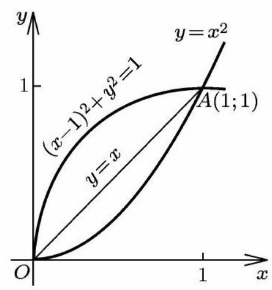

Fig. 10.2

Therefore, $I = I_1 + i_2 + i_3 = 2 + +\sqrt{2}$.

Example 2. Calculate the curvilinear integral

$$I = \int_{\Gamma} ydx + xdy$$

orientation. matters here. (2nd kind).

along the curve Γ with the beginning $O(0;0)$ and the end $A(1;1)$ if (Fig. 10.2):

1. Γ - segment OA;
2. Γ - arc of the parabola $y = x^2$;
3. R is an arc of a circle of radius 1 centered at $(1;0)$.

Δ 1) Since the segment OA is given by the equation $y = x$, $0 \leq x \leq 1$, then applying formulas (17) and (18), we find

$$I = \int_0^1 xdx + \int_0^1 xdx = 1$$

2. If Γ is the arc of a parabola, then

$$\int_{\Gamma} ydx = \int_0^1 x^2 dx, \quad \int_{\Gamma} xdy = \int_0^1 2x^2 dx, \quad I = \int_0^1 3x^2 Dx = 1.$$

3. Since the equation of the arc of a circle can be written as

$$x = 1 + \cos t, \quad y = \sin t$$

where t varies from π to $\pi/2$, then by formula (16) we get

$$I = \int_{\pi}^{\pi/2} \sin t(-\sin t)dt + \int_{\pi}^{\pi/2} (1 + \cos t)\cos t dt =$$

$$= \int_{\pi}^{\pi/2} (\cos t + \cos 2t)dt = 1$$

Example 3. Calculate the curvilinear integral using Green's formula

$$I=\int_{G} x^{2} y d x-x y^{2} d y$$

where Γ is a circle $x^2+y^2=R^2$, run counterclockwise.
And let's use the formula (19), where

$$P=x^{2} y, \quad Q=-x y^{2}, \quad \frac{\partial Q}{\partial x}=-y^{2}, \quad \frac{\partial P}{\partial y}=x^{2}$$

Then

$$I=-\int_{D}\left(x^{2}+y^{2}\right) d x d y$$

where D is a circle of radius R centered at $(0;0)$. Turning to the polar coordinates, we get

$$I=-\int_{0}^{2 \pi} d \varphi \int_{0}^{R} r^{3} d r=-\frac{\pi R^{4}}{2}$$

Example 4. Using formula (20), find the area S bounded by the astroid

$$x=a \cos ^{3} t, \quad y=a \sin ^{3} t, \quad 0 \leqslant t \leqslant 2 \pi .$$

Δ Applying the formulas (20) and (16), we obtain

$$\begin{aligned} S=\tfrac{1}{2} \int_{0}^{2 \pi}\left(x(t) y^{\prime}(t)-y(t) x^{\prime}(t)\right) d t=\tfrac{3 a^{2}}{2} \int_{0}^{2 \pi}\left(\cos ^{4} t \sin ^{2} t+\sin ^{4} t \cos ^{2} t\right) d t= \\ =\tfrac{3 a^{2}}{8} \int_{0}^{2 \pi} \sin ^{2} 2 t d t=\tfrac{3 a^{2}}{16} \int_{0}^{2 \pi}(1-\cos 4 t) d t=\tfrac{3 \pi a^{2}}{8} .\end{aligned}$$

Example 5. Show that the curvilinear integral

$$I=\int_{A B}\left(3 x^{2} y+y\right) d x+\left(x^{3}+x\right) d y$$

where $A(1;-2), b(2;3)$, does not depend on the integration path, and calculate this integral.

Δ Since the functions $P=3x^2y+y, Q=x^3+x, \frac{\partial P}{\partial x}$ and $\frac{\partial Q}{\partial y}$ are continuous in R^2 and the condition (25) is satisfied, then the integral does not depend on the integration path and is expressed by the formula (23).

The function $u(x;y)$ can be found by formula (24). Note, however, that the integrand is a complete differential, since

$$\begin{aligned} & \left(3 x^{2}+y\right) d x+\left(x^{3}+x\right) d y=\left(3 x^{2} y d x+x^{3} d y\right)+(y d x+x d y)= \\ & \quad=d\left(x^{3} y\right)+d(x y)=d\left(x^{3} y+x y\right)=d u\end{aligned}$$

Therefore, $u=x^3y+xy$, and by formula (23) we find

$$I=u(B)-u(A)=30-(-4)=34$$

TASKS

1. Calculate a curvilinear integral of the first kind over a flat curve Γ :

   (a) $\int_\Gamma ds, \Gamma$ - a segment with the ends $(0;0)$ and $(1;2)$;

   (b) $\int_\Gamma (2x+y)ds, \partial$ is a polyline $ABOA$, where $A(1;0), B(0;2), O(0;0)$;

   (c) $\int_G (x+y)ds, \partial-$ border of a triangle with vertices $(0;0)$; $(1;0)$ and $(0;1)$;

   (d) $\int_\Gamma \frac{ds}{y-x}, \Gamma$ is a segment with ends $(0;-2)$ and $(4;0)$;

   (e) $\int_\Gamma \frac{ds}{\sqrt{x^2+y^2+4}}$, $Gamma$ is a segment with the ends $(0;0)$ and $(1;2)$.

2. Calculate the curvilinear integral $\int_\Gamma xyds$ if:

   (a) G - border of a square with vertices $(1;0), (0;1), (-1;0)$, $(0;-1)$; rante;

   (b) Γ is a quarter of the ellipse $x^2/a^2 + y^2/b^2 = 1$ lying in the I quad-

   (c) R — border of a rectangle with vertices $(0;0), (4;0), (4;2)$, $(0;2)$.

Calculate the curvilinear integral over a flat curve $\Gamma(4-11)$.

4. $\int_\Gamma x^2 ds, \Gamma-$ the arc of the circle $x^2+y^2=a^2, y \geqslant 0$.

5. $\int_\Gamma \left(x^2+y^2\right)^n ds, \Gamma$ - circle $x^2+y^2=a^2$.

6. $\int_\Gamma f(x,y)dx, \Gamma$ - circle $x^2+y^2=ax$, if:

   (a) $f(x;y) = x-y$; $(b) f(x;y) = \sqrt{x^2+y^2}$.

7. $\int_\Gamma f(x;y)ds, \Gamma$ - the right lobe of the lemniscate given in polar coordinates by the equation $r^2 = a^2\cos 2\varphi$ if:

   (a) $f(x;y) = x+y$;

   (b) $f(x;y) = x\sqrt{x^2-y^2}$.

8. $\int_\Gamma |y| ds, \Gamma$ - lemniscate $r^2 = a^2 \cos 2\varphi$.

9. $\int_\Gamma \left(x^{4/3}+y^{4/3}\right) ds, \Gamma$ - astroid $x^{2/3}+y^{2/3}=a^{2/3}$.

10. $\int_\Gamma f(x;y)ds, \Gamma-$ the arch of the cycloid $x = a(t-\sin t)$, $y = a(1-\cos t)$, $0 \leq t \leq 2\pi$ if:

    (a) $f(x;y) = y$; (b) $f(x;y) = y^2$.

11. $\int_\Gamma f(x;y)ds, \Gamma-$ circle sweep arc $x = a(\cos t + t\sin t)$, $\quad y = a(\sin t - t\cos t)$, $\quad 0 \leqslant t \leqslant 2\pi$, if

(a) $f(x;y) = x^2 + y^2$;

(b) $f(x;y) = \sqrt{x^2+y^2}$.

Calculate the curvilinear integral over the spatial curve $\Gamma(14-18)$. if:

14. $\int_\Gamma \sqrt{2y^2+z^2}ds, \Gamma$ - circle $x^2+y^2+z^2=a^2, x=y$.

15. $\int_\Gamma xyzds, \Gamma$ is a quarter of the circle $x^2+y^2+z^2=a^2, x=y$, located in the I octant.

16. $\int_\Gamma (x+y)ds, \Gamma$ is a quarter of the circle $x^2+y^2+z^2=a^2, y==x$, located in the I octant.

17. $\int_\Gamma x^2ds, \Gamma-$circle $x^2+y^2+z^2=a^2, x+y+z=0$.

18. $\int_\Gamma zds, \Gamma-$ curve arc $x^2+y^2=z^2, y^2=ax$ from point $(0;0;0)$ to point $(a;a;a\sqrt{2}), a>0$.

Calculate the curvilinear integral of the second kind along the curve Γ, run in the direction of increasing its parameter x (19,20).

19. (a) $\int_\Gamma xydx, \Gamma-$ the arc of the sine wave $y=\sin x, 0 \leqslant x \leqslant \pi$;
    (b) $\int_\Gamma \left(x-\frac{1}{y}\right)dy, \Gamma-$ the arc of the parabola $y=x^2, 1 \leqslant x \leqslant 2$;
    (c) $\int_\Gamma xdy - ydx, \Gamma-$ curve $y=x^3, 0 \leqslant x \leqslant 2$;
    (d) $\int_\Gamma \frac{y}{x}dx + dy, \Gamma-$ curve $y=\ln x, 1 \leqslant x \leqslant e$;
    (e) $\int_\Gamma 2xydx + x^2dy, \Gamma-$ the arc of the parabola $y=\frac{x^2}{4}, 0 \leqslant x \leqslant 2$;
    (f) $\int_\Gamma 2xydx - x^2dy, \Gamma-$ the arc of the parabola $y=\sqrt{\frac{x}{2}}, 0 \leqslant x \leqslant 2$.

20. (a) $\int_\Gamma \cos ydx - \sin ydy, \Gamma-$ straight line segment $y=-x, -2 \leqslant \leqslant x \leqslant 2$
    (b) $\int_\Gamma \left(xy-y^2\right)dx + xdy, \Gamma-$ curve $y=2\sqrt{x}, 0 \leqslant x \leqslant 1$;
    (c) $\int_\Gamma \left(x^2-2xy\right)dx + \left(y^2-2xy\right)dy, \Gamma-$ the arc of the parabola $y=x^2$, $-1 \leqslant x \leqslant 1$
    (d) $\int_\Gamma \left(x^2+y^2\right)dx + \left(x^2-y^2\right)dy, \Gamma-$ curve $y=1-|x-1|, 0 \leqslant \leqslant x \leqslant 2$.

Calculate the curvilinear integral along the curve Γ running from point A to point $B(21-25)$.

21. $\int_\Gamma xdy - ydx, A(0;0), B(1;2)$, if:
    (a) Γ - section AB;
    (b) Γ - the arc of the parabola $y=2x^2$;
    (c) Γ - polyline ACB, where $C(0;1)$.

22. $\int_\Gamma xydx - y^2dy, \Gamma-$ the arc of the parabola $y^2=2x, A(0;0), B(2;2)$.

23. $\int_\Gamma \frac{3x}{y}dx - \frac{2y^3}{x}dy, \Gamma$ - the arc of the parabola $x = y^2, A(4;2), B(1;1)$.

24. $\int_\Gamma \frac{x}{y}dx - \frac{y-x}{x}dy, \Gamma$ - the arc of the parabola $y = x^2, A(2;4), B(1;1)$.

25. $\int_\Gamma xdy, \Gamma$ - semicircle $x^2 + y^2 = a^2, x \geqslant 0, A(0;-a),\ B(0;a)$.

26. Calculate the curvilinear integral along the segment AB oriented in the direction from point A to point B :

    (a) $\int_\Gamma x^3dy - xydx, A(0;-2), B(1;3)$;
    (b) $\int_\Gamma -3x^2dx + y^3dy, A(0;0), B(2;4)$;
    (c) $\int_\Gamma (2x-y)dx + (4x+5y)dy, A(3;-4), B(1;2)$;
    (d) $\int_\Gamma (4x+5y)dx + (2x-y)dy, A(1;-9), B(4;-3)$;
    (e) $\int_\Gamma \left(\frac{x}{x^2+y^2} + y\right)dx + \left(\frac{y}{x^2+y^2} + x\right)dy, A(1;0), B(3;4)$;
    (f) $\int_\Gamma (x+y)dx + (x-y)dy, A(0;1), B(2;3)$.

Calculate the curvilinear integral along the curve Γ, run in the direction of increasing its parameter $t(27, 28)$.

27. (a) $\int_\Gamma xy^2dx, \Gamma-$ the arc of the circle $x = \cos t, y = \sin t, 0 \leqslant t \leqslant \leqslant \pi/2$
    (b) $\int_\Gamma xdy + ydx, \Gamma-$ the arc of the circle $x = R\cos t, y = R\sin t,\ 0 \leqslant t \leqslant \pi/2$
    (c) $\int_\Gamma ydx - xdy, \Gamma-$ ellipse $x = a\cos t, y = b\sin t, 0 \leqslant t \leqslant 2\pi$;
    (d) $\int_\Gamma^\Gamma y^2dx + x^2dy, \Gamma-$ the upper half of the ellipse $x = a\cos t,\ y = b\sin t$.

28. (a) $\int_\Gamma (2a-y)dx + (y-a)dy, \Gamma-$ the arc of the cycloid $x = a(t - -\sin t), y = a(1-\cos t), 0 \leqslant t \leqslant 2\pi$
    (b) $\int_\Gamma \frac{x^2dy - y^2dx}{x^{5/3} + y^{5/3}}, \Gamma-$ the arc of the astroid $x = a\cos^3 t, y = a\sin^3 t,\ 0 \leqslant t \leqslant \pi/2$.

Calculate the curvilinear integral of the second kind by the spatial curve Γ, running in the direction of increasing parameter $t(31-36)$.

31. $\int_\Gamma ydx + zdy + xdz, \Gamma-$ the spiral of the helix $x = a\cos t,\ y = a\sin t, z = bt, 0 \leqslant t \leqslant 2\pi$.

32. $\int_\Gamma \left(y^2 - z^2\right)dx + 2yzdy - x^2dz, \Gamma$ - curve $x = t, y = t^2, z = = t^3, 0 \leqslant t \leqslant 1$.

33. $\int_\Gamma yzdx + z\sqrt{a^2 - y^2}dy + xydz, \Gamma-$ the arc of the helix $x = a\cos t, y = a\sin t, z = at/(2\pi), 0 \leqslant t \leqslant 2\pi$.

34. $\int_\Gamma (y+z)dx + (z+x)dy + (x+y)dz, \Gamma$ - curve $x = a\sin^2 t,\ y = 2a\sin t\cos t, z = a\cos^2 t, 0 \leqslant t \leqslant \pi$.

35. $\int_{\Gamma} x dx+(x+y) dy+(x+y+z) dz, \Gamma-$ curve $x=a \sin t,\ y=a \cos t, z=a(\sin t+\cos t), 0 \leqslant t \leqslant 2 \pi$.

36. $\int_{\Gamma} y dx+z dy+x dz, \Gamma$ - circle $x=a \cos \alpha \cos t, y==a \cos \alpha \sin t, z=a \sin \alpha$ ($\alpha=$ const).

Calculate the curvilinear integral of the second kind from the spatial curve $\Gamma(37-44)$.

37. $\int_{\Gamma} x dx+y dy+(x+y-1) dz, \Gamma$ is the segment AB traversed from the point $A(1;1;1)$ to the point $B(2;3;4)$.

38. $\int_{\Gamma} \frac{x dx+y dy+z dz}{\sqrt{x^{2}+y^{2}+z^{2}-x-y+2 z}}, \Gamma-$ segment AB, run from point $A(1;1;1)$ to the point $B(4;4;4)$.

39. $\int_{\Gamma} x(z-y) dx+y(x-z) dy+z(y-x) dz, \Gamma$ is a polyline $ABCA$, where $A(a;0;0), B(0;a;0), C(0;0;a)$.

40. $\int_{\Gamma} y^{2} dx+z^{2} dy+x^{2} dz, \Gamma-$the intersection line of the sphere $x^{2}++y^{2}+z^{2}=R^{2}$ and the cylinder $x^{2}+y^{2}=R x(R>0, z \geqslant 0)$, run counterclockwise when viewed from a point $(0;0;0)$.

41. $\int_{\Gamma}(y-z) dx+(z-x) dy+(x-y) dz, \Gamma$ is a circle $x^{2}++y^{2}+z^{2}=a^{2}, y=x \operatorname{tg} \alpha(0 \leqslant \alpha \leqslant \pi)$, run counterclockwise when viewed from the positive semi-axis Ox.

42. $\int_{\Gamma}\left(y^{2}-z^{2}\right) dx+\left(z^{2}-x^{2}\right) dy+\left(x^{2}-y^{2}\right) dz, \Gamma-$ the boundary of the part of the sphere $x^{2}+y^{2}+z^{2}=1$ (lying in the I octant), traversed clockwise if viewed from the point $(0;0;0)$.

43. $\int_{\Gamma}(y+z) dx+(z+x) dy+(x+y) dz, \Gamma$ is the circle $x^{2}++y^{2}+z^{2}=a^{2}, x+y+z=0$, run counterclockwise when viewed from the positive semi-axis Oy.

44. $\int_{\Gamma}\left(y^{2}+z^{2}\right) dx+\left(x^{2}+z^{2}\right) dy+\left(x^{2}+y^{2}\right) dz, \Gamma$ is the line of intersection of surfaces

$$x^{2}+y^{2}+z^{2}=2 R x, \quad x^{2}+y^{2}=2 r x, \quad 0<r<R, \quad z \geqslant 0$$

run counterclockwise when viewed from the positive half-axis Oz.

Applying Green's formula, calculate the curvilinear integral over a closed curve Γ, traversed so that its interior remains on the left $(45-55)$.

45. $\int_{\Gamma}(x y+x+y) dx+(x y+x-y) dy$ if:

    (a) Γ - ellipse $x^{2}/a^{2}+y^{2}/b^{2}=1$;
    (b) $Gamma$ - circle $x^{2}+y^{2}=a x$.

46. $\int_{\Gamma}(2 x y-y) dx+x^{2} dy, \Gamma$ is an ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$.

47. $\int_\Gamma \frac{xdy+ydx}{x^2+y^2}, \Gamma$ is a circle $(x-1)^2+(y-1)^2=1$.

48. $\int_\Gamma (x+y)^2 dx - \left(x^2+y^2\right) dy, \Gamma-$ the boundary of a triangle with vertices $(1;1),(3;2),(2;5)$.

49. $\int_\Gamma \left(y-x^2\right) dx + \left(x+y^2\right) dy, \quad \Gamma-$ the boundary of the circular sector $0< r<R, 0<\varphi<\alpha \leqslant \pi/2$, where $(r;\varphi)$ are the polar coordinates.

50. $\int_\Gamma e^x[(1-\cos y)dx+(\sin y-y)dy], \Gamma-$ area boundary $0<<x<\pi, 0< y<\sin x$.

51. $\int_\Gamma e^{y^2-x^2}(\cos 2xydx+\sin 2xydy, \quad \Gamma$ - circle $x^2+y^2==R^2$.

52. $\int_\Gamma \left(e^x \sin y-y\right) dx+\left(e^x \cos y-1\right) dy, \quad \Gamma$ is the boundary of the area $x^2+y^2<ax, y>0$.

53. $\int_\Gamma \frac{dx-dy}{x+y}, \Gamma$ is the border of a square with vertices $(1;0),(0;1),(-1;0),(0;-1)$.

54. $\int_\Gamma \sqrt{x^2+y^2}dx+y\left(xy+\ln\left(x+\sqrt{x^2+y^2}\right)\right)dx, \quad \Gamma$ - circle $x^2+y^2= R^2$.

55. $\int_\Gamma (x+y)^2 dx-(x-y)^2 dy, \Gamma-$ the boundary of the region formed by the segment AB, where $A(1;1), B(2;6)$, and the arc of the parabola $y= ax^2++bx+c$, passing through the points $A, B, O(0;0)$.

After making sure that the integrand is a complete differential, calculate the curvilinear integral along the curve Γ with the beginning at point A and the end at point $B(56-68)$.

56. $\int_\Gamma xdy+ydx, A(-1;3), B(2;2)$.

57. $\int_\Gamma xdx+ydy, A(-1;0), B(-3;4)$.

58. $\int_\Gamma (x+y)dx+(x-y)dy, A(2;-1), B(1;0)$.

59. $\int_\Gamma 2xydx+x^2dy, A(0;0), B(-2;-1)$.

60. $\int_\Gamma \left(x^4+4xy^3\right)dx+\left(6x^2y^2-5y^4\right)dy, A(-2;-1), B(0;3)$.

61. $\int_\Gamma \left(x^2+2xy-y^2\right)dx+\left(x^2-2xy-y^2\right)dy, \quad A(3;0), \quad A(0;-3)$.

62. $\int_\Gamma \left(3x^2-2xy+y^2\right)dx+\left(2xy-x^2-3y^2\right)dy$, A(-1 ; 2), B(1 ;-2) .

63. $\int_\Gamma f(x+y)(dx+dy), f(t)$ - continuous function, $A(0;0)$, $B\left(x_0;y_0\right)$.

64. $\int_\Gamma \varphi(x)dx+\psi(y)dy, \quad \varphi(t), \quad \psi(t)$ - continuous functions, $A\left(x_1;y_1\right), B\left(x_2;y_2\right)$.

65. $\int_\Gamma e^x\cos ydx-e^x\sin ydy, A(0;0), B\left(x_0;y_0\right)$.

66. $\int_\Gamma xdx+y^2dy-z^3dz, A(-1;0;2), B(0;1;-2)$.

67. $\int_\Gamma yzdx+xzdy+xydz, A(2;-1;0), B(1;2;3)$.

68. $\int_\Gamma \frac{xdx+ydy+zdz}{\sqrt{x^2+y^2+z^2}}, A\in S_1, B\in S_2$, where S_1 - sphere $x^2+y^2++z^2=R_1^2, S_2-$ sphere $x^2+y^2+z^2=R_2^2\,(R_1>0, R_2>0)$.