

Lorans series

TASKS

In Problems 1-3, expand the given function in a Laurent series valid for the given annular domain.

1. $f(z) = \frac{\cos z}{z}, 0 < |z|$

Answer: $\frac{1}{z} - \frac{z}{2!} + \frac{z^3}{4!} - \frac{z^5}{6!}$

2. $f(z) = e^{-1/z^2}, 0 < |z|$

Answer: $1 - \frac{1}{1!z^2} + \frac{1}{2!z^4} - \frac{1}{3!z^6}$

3. $f(z) = \frac{e^z}{z-1}, 0 < |z-1|$

Answer: $\frac{e}{z-1} + e + \frac{e(z-1)}{2!} + \frac{e(z-1)^2}{3!}$

In Problems 4 – 6, expand $f(z) = \frac{1}{z(z-3)}$ in a Laurent series valid for the indicated annular domain.

4. $0 < |z| < 3$

Answer: $-\frac{1}{3z} - \frac{1}{3^2} - \frac{z}{3^3} - \frac{z^2}{3^4}$

5. $0 < |z-3| < 3$

Answer: $\frac{1}{3(z-3)} - \frac{1}{3!} + \frac{z-3}{3^2} - \frac{(z-3)^2}{3^4}$

6. $1 < |z-4| < 4$

Answer: $\dots - \frac{1}{3(z-4)^2} + \frac{1}{3(z-4)} - \frac{1}{12} + \frac{z-4}{3 \cdot 4^2} - \frac{(z-4)^2}{3 \cdot 4^3}$

In Problems 7,8, expand $f(z) = \frac{1}{(z-1)(z-2)}$ in a Laurent series valid for the given annular domain.

7. $1 < |z| < 2$

Answer: $\dots - \frac{1}{z^2} - \frac{1}{z} - \frac{1}{2} - \frac{z}{2^2} - \frac{z^2}{2^3}$

8. $0 < |z-1| < 1$

Answer: $\frac{-11}{z-1} - 1 - (z-1) - (z-1)^2$

9. Expand $f(z) = \frac{z}{(z+1)(z-2)}$ in a Laurent series valid for the given annular domain. $0 < |z + 1| < 3$

Answer: $\frac{1}{3(z+1)} - \frac{2}{3^2} - \frac{2(z+1)}{3^3} - \frac{2(z+1)^2}{3^4}$