Concluding Remarks

Governing equations for Mechanics of Solids, Fluids, and Gas.

Short list of engineering solvers.

Mathematical packages

- **Matlab**
- **Scilab**
- **MathCAD**
- **Mathematica**
- **Maple**

are general purpose packages

For solving <u>real-life</u> engineering or research problems, we need advanced packages that can solve more complex equations.

The equations of linear elasticity are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + F_x = \rho \frac{\partial^2 u_x}{\partial t^2}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + F_y = \rho \frac{\partial^2 u_y}{\partial t^2}$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + F_z = \rho \frac{\partial^2 u_z}{\partial t^2}$$

- Strain-displacement equations:

$$\varepsilon_{ij} = \frac{1}{2}(u_{j,i} + u_{i,j})$$

where $\varepsilon_{ij} = \varepsilon_{ji}$ is the strain. These are 6 independent equations relating strains

and displacements with 9 independent

unknowns (strains and displacements).

In engineering notation, they are:

$$\epsilon_x = \frac{\partial u_x}{\partial x} \qquad \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

$$\epsilon_y = \frac{\partial u_y}{\partial y} \qquad \gamma_{yz} = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}$$

$$\epsilon_z = \frac{\partial u_z}{\partial z} \qquad \gamma_{zx} = \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}$$

Constitutive equations. The equation for Hooke's law is:

$$\sigma_{ij} = C_{ijkl}\, \varepsilon_{kl}$$

where C_{ijkl} is the stiffness tensor. These are 6 independent equations

Equations Governing Viscous Fluid with constant density:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \Delta u,$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu \Delta v,$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = Z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \nu \Delta w.$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

u,v,w **– velocity components,** *p* **- pressure**

Equations Governing Viscous heat-conducting gas or plasma

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \\[2mm] \dfrac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u}\vec{u}) = -\nabla p + \nabla \cdot \boldsymbol{\tau} \\[2mm] \dfrac{\partial (\rho E)}{\partial t} + \nabla \cdot (\rho \vec{u} H) = \nabla \cdot [\vec{u} \cdot \boldsymbol{\tau} + \vec{q}] \end{cases}$$

$$\vec{u} = u\,\vec{i} + v\vec{j} + w\,\vec{k}$$

$$p = \rho RT$$

$$E = e + 0.5\left(u^2 + v^2 + w^2\right) = C_v T + 0.5\left(u^2 + v^2 + w^2\right)$$

$$H = E + p/\rho = h + 0.5\left(u^2 + v^2 + w^2\right) = C_p T + 0.5\left(u^2 + v^2 + w^2\right)$$

$$\boldsymbol{\tau} = 2\mu(T)\left(\mathbf{S} - \frac{1}{3}\mathbf{I}\,\nabla \cdot \vec{u}\right)$$

$$\mathbf{S} = \frac{1}{2}\left(\nabla\vec{u} + [\nabla\vec{u}]^t\right) \qquad \vec{q} = -\lambda(T)\nabla T$$

Unknown quantities $u,\ v,\ w,\ \rho,\ T$ depend on $x,\ y,\ z,\ t$,

plus, possibly, equations of chemical reactions, combustion, radiation, . .

Software packages for Engineering Problems

https://wiki2.org/en/List_of_finite_element_software_packages?ysclid=m1zrzn8fkm918773170

Software	Features	Developer	Version	Released	License	Price	Platform
Agros2D	Multiplatform open source application for the solution of physical problems based on the Hermes library	University of West Bohemia	3.2	2014-03-03	GNU GPL	Free	Linux, Windows
CalculiX	It is an Open Source FEA project. The solver uses a partially compatible ABAQUS file format. The pre/post-processor generates input data for many FEA and CFD applications	Guido Dhondt, Klaus Wittig	2.20	2022-08-01	GNU GPL	Free	Linux, Windows
DIANA FEA	General purpose finite element package utilised by civil, structural and geotechnical engineers.	DIANA FEA BV, The Netherlands	10.1	2016-11-14	Proprietary commercial software	Paid	Windows, Linux
deal.II	Comprehensive set of tools for finite element codes, scaling from laptops to clusters with 100,000+ cores. Written in C++, it supports all widely used finite element types, serial and parallel meshes, and h and hp adaptivity.	Wolfgang Bangerth, Timo Heister, Guido Kanschat, Matthias Maier et al.	9.5	2023-07-07	LGPL	Free	Linux, Unix, Mac OS X, Windows

DUNE	Distributed and Unified Numerics Environment, written in C++	DUNE Developer team	2.4.1	2016-02-29	GPL Version 2 with Run-Time Exception	Free	Linux, Unix, Mac OS X
Elmer FEM	Open source multiphysical simulation software developed by Finnish Ministry of Education's CSC, written primarily in Fortran (written in Fortran90, C and C++)	CSC	8.2	2021-03-04	GPL	Free	Linux, Mac OS X, Windows
FEBio	Finite Elements for Biomechanics	University of Utah (MRL), Columbia University (MBL)	3.7	June, 2022	MIT	Free	Linux, Mac OS X, Windows
FEniCS Project	Software package developed by American and European researchers with the goal to enable automated solution of differential equations	FEniCS Team	1.6.0	2015-07-29	LGPL (Core) & GPL/LGPL (Non-Core)[1]	Free	Linux, Unix, Mac OS X, Windows
FEATool Multiphysics	MATLAB FEM and PDE multiphysics simulation toolbox	Precise Simulation	1.10	2019-05-17	Proprietary EULA	Free for personal use[2]	Windows, Mac OS X, Linux, Unix

FreeFEM[3]	FreeFEM is a free and open-source parallel FEA software for multiphysics simulations. The problems are defined in terms of their variational formulation and can be easily implemented using FreeFEM language. Written in C++.	Sorbonne University[4] and Jacques-Louis Lions Laboratory[5]	4.2.1	2019-06-06	LGPL	Free	Linux, MacOS, Windows, Solaris
GOMA	GOMA is an open-source, parallel, and scalable multiphysics software package for modeling and simulation of real-life physical processes, with a basis in computational fluid dynamics for problems with evolving geometry.	Sandia National Laboratories, University of New Mexico	6.1	Aug 28, 2015	GPL Version 2	Free	Linux
GetFEM++	A generic finite element library written in C++ with interfaces for Python, Matlab and Scilab. It focuses on modeling of contact mechanics and discontinuities (e.g. cracks).	Yves Renard, Julien Pommier	5.4.2	2022-07	LGPL	Free	Unix, Mac OS X, Windows

Name	Description	Developer	Version	Date	License	Cost	Platform
Hermes Project	Modular C/C++ library for rapid development of space- and space-time adaptive hp-FEM solvers	hp-FEM group	3.0	2014-03-01	LGPL	Free	Linux, Unix, Mac OS X, Windows
Mathematica[6]	General purpose computation software.	Wolfram Research	14.0.0 (January 9, 2024; 4 months ago) [±]⤢[7]	Regularly	Proprietary		Linux, Mac OS X, Windows, Raspbian, Online service.
MATLAB Partial Differential Equation Toolbox	MATLAB Toolbox for solving structural, thermal, electromagnetics, and other general PDEs	MathWorks	3.3 (R2019b)	2019-09-11	Proprietary commercial software		Linux, Mac OS X, Windows
MFEM	MFEM is a free, lightweight, scalable C++ library for finite element methods that features arbitrary high-order finite element meshes and spaces, support for a wide variety of discretizations, and emphasis on usability, generality, and high-performance computing efficiency.	MFEM team	4.7	2024-05-07	BSD	Free	Linux, Unix, Mac OS X, Windows
MoFEM JosePH	Mesh Oriented hp-FE code, written in C++	University of Glasgow	0.6.8	2017-11-16	LGPL	Free	Unix, Mac OS X

Name	Description	Developer	Version	Released	License	Cost	Platforms
MOOSE	Object Oriented FE framework, written in C++	Idaho National Laboratory		regularly	LGPL	Free	Unix, Mac OS X
OOFEM	Object Oriented Finite EleMent solver, written in C++	Bořek Patzák	2.5	2017-12-30	GPL Version 2	Free	Unix, Windows
OpenSees	Open System for Earthquake Engineering Simulation		3.3.0	2021-05-24	Non Commercial	Free	Unix, Linux, Windows
SESAM (FEM)	Software suite for structural and hydrodynamic analysis of ships and offshore structures	DNV GL		regularly	Proprietary, SaaS		Windows, Web browser
Range Software	Multiphysics Finite Element Analysis Software	Tomáš Šoltys	3.0	2018-04-30	GPL	Free	Linux, Windows
Z88/Z88Aurora	Freeware finite element package; The present version Z88Aurora V5 offers, in addition to static strength analysis modules such as non-linear strength calculations (large displacements), simulations with non-linear materials, natural frequency, static thermal analysis and a contact module.	Frank Rieg	Z88 V15, Z88Aurora V5	2017-07-17, 2019-04-01	GNU GPL, Custom	Free	Linux, Windows, Mac OS X

Закрыть

Name	Description	Developer	Version	Date	License	Notes	OS
Abaqus	Advanced Franco-USA software from SIMULIA, owned by Dassault Systemes	Abaqus Inc.	2023	2022-11	Proprietary commercial software	Free learning edition available, up to 1000 nodes[8]	Linux, Windows
CONSELF	CAE simulation from your browser	CONSELF SRL	2.9	2015-10	SaaS	Freemium	Web browser
FreeCAD	Parametric 3D modeler with a FEM workbench allowing it to use external solvers like CalculiX, Z88, Elmer, and OpenFOAM	FreeCAD Team	0.20.1	10 August 2022	LGPL 2	Free	Linux, Windows, Mac OS X
ADINA	Finite element software for structural, fluid, heat transfer, electromagnetic, and multiphysics problems, including fluid-structure interaction and thermo-mechanical coupling	Adina R&D			Proprietary commercial software		
Advance Design	BIM software for FEM structural analysis, including international design eurocodes	GRAITEC	2014	2013-09	Proprietary commercial software		
Autodesk Simulation	Finite Element software of Autodesk	Autodesk			Proprietary commercial software		Windows
ANSYS	US-based and -developed full CAE software package	Ansys Inc.	2022 R2	2022-07-28	Proprietary commercial software	Free student version available, up to 32,000 nodes/elements[9]	Windows, Linux

Name	Description	Developer	Version	Date	License	Notes	Platform
COMSOL Multiphysics	COMSOL Multiphysics Finite Element Analysis Software (formerly FEMLAB)	COMSOL Inc.	6.1	2022-11-01	Proprietary EULA		Linux, Mac OS X, Windows, Web browser
CosmosWorks	Part of SolidWorks	Dassault Systèmes SolidWorks Corp.			Proprietary commercial software		Windows
Quickfield	EM, Heat Transfer and Stress Analysis [10]	Tera Analysis Ltd	6.4 [11]	2020-04-17	Proprietary EULA	Free Student Edition available[12]	Windows
Pam Crash	Best used for explicit dynamics / crash analysis	ESI	15.5.1	2020-03-05	Proprietary commercial software		Linux, Windows
LS-DYNA	Best known for explicit dynamics / crash analysis	LSTC - Livermore Software Technology Corporation	R10.1	2020	Proprietary commercial software		Linux, Windows
Mecway	Structural, heat transfer, electrostatic, acoustic	Mecway Limited	20.0	2023-07	Proprietary commercial software	Free edition available, up to 1000 nodes[13]	Windows
Nastran	Originally developed for NASA, now available commercially from several software companies	MSC NASTRAN, Siemens PLM NX Nastran[14]	2014	2014	Proprietary EULA		Linux, Mac OS X, Windows

RFEM	3D finite element analysis software	Dlubal Software	6.04.0005	2023-10-30	Proprietary commercial software	Free student license available[15]	Windows
SimScale	German 100% web-based CAE platform	SimScale GmbH	14	2013-07	SaaS	Free community version available[16]	Web browser
VisualFEA	Finite element software for structural, geotechnical, heat transfer and seepage analysis	Intuition Software	5.11	2016-01	Proprietary software	Free educational version available[17]	Mac OS X, Windows
JCMsuite	Finite element software for the analysis of electromagnetic waves, elasticity and heat conduction	JCMwave GmbH	5.4.3	2023-03-09	Proprietary EULA		Linux, Windows
Radioss	Best known for explicit dynamics simulations	Altair Engineering			Proprietary commercial software; OpenRadioss: AGPL-3.0	Open-source version (OpenRadioss) available	Linux, Windows
JMAG	2D and 3D finite element analysis software for electromagnetic field, thermal, structural	JSOL	18.1	2019-06	Proprietary commercial software	Education pack available	Linux, Windows, Web browser

| SDC Verifier | An extension for Ansys Mechanical, Femap and Simcenter with out of the box predefined standards on fatigue, stiffener and plate buckling, beam member checks, joint checks and weld. Such as AISC 360-10, API 2A RP, ISO 19902, Norsok N004, DIN15018, Eurocode 3, FEM 1.001, ABS 2004, ABS 2014, DNV RP-C201 2010, DNV CN30/1995, FKM etc. | SDC Verifier | 2021 R.2 | 2021-11 | Proprietary commercial software | Student version available | Windows |

ANSYS   www.ansys.com

(multinational company, was founded in 1970, now 600+ employees)

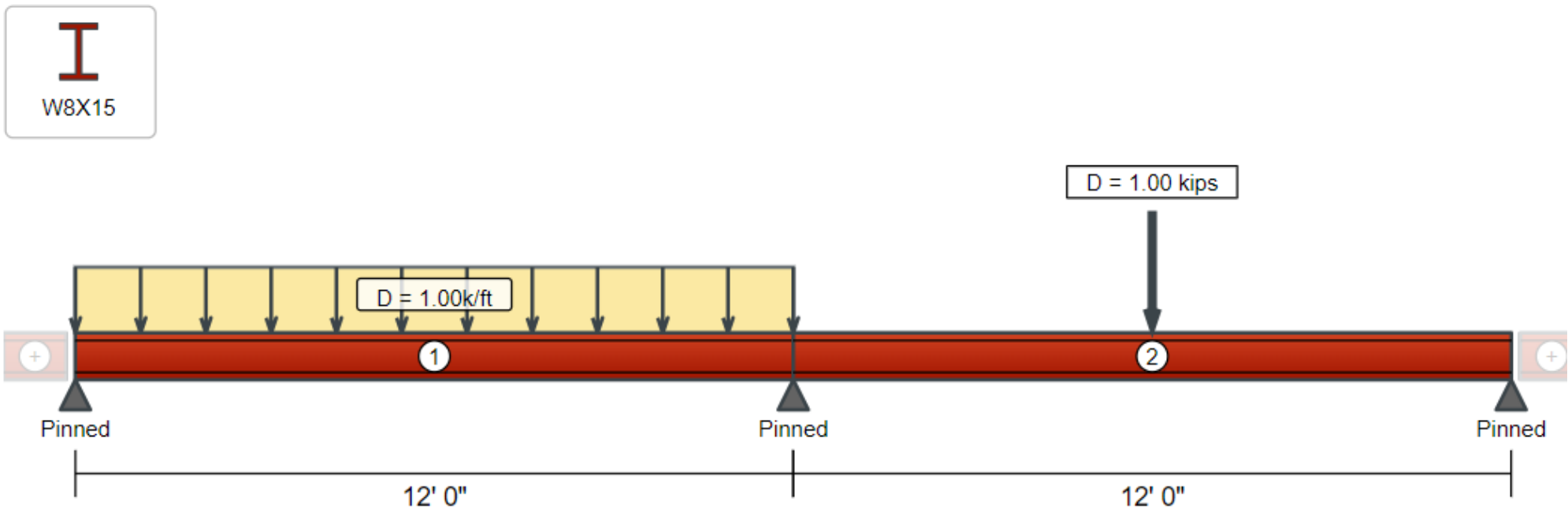for academic and industrial use – it is expensive
student version – free of charge

Available educational software, free of charge:

LISA	www.lisafea.com
EasyCFD	www.easycfd.net
MicroCFD	www.microcfd.com
Flowsquare	www.flowsquare.com

Available educational software (user-friendly, price from CHY 1000):

FEATool Multiphysics https://www.featool.com/

Computation of solid Beams online, for example:

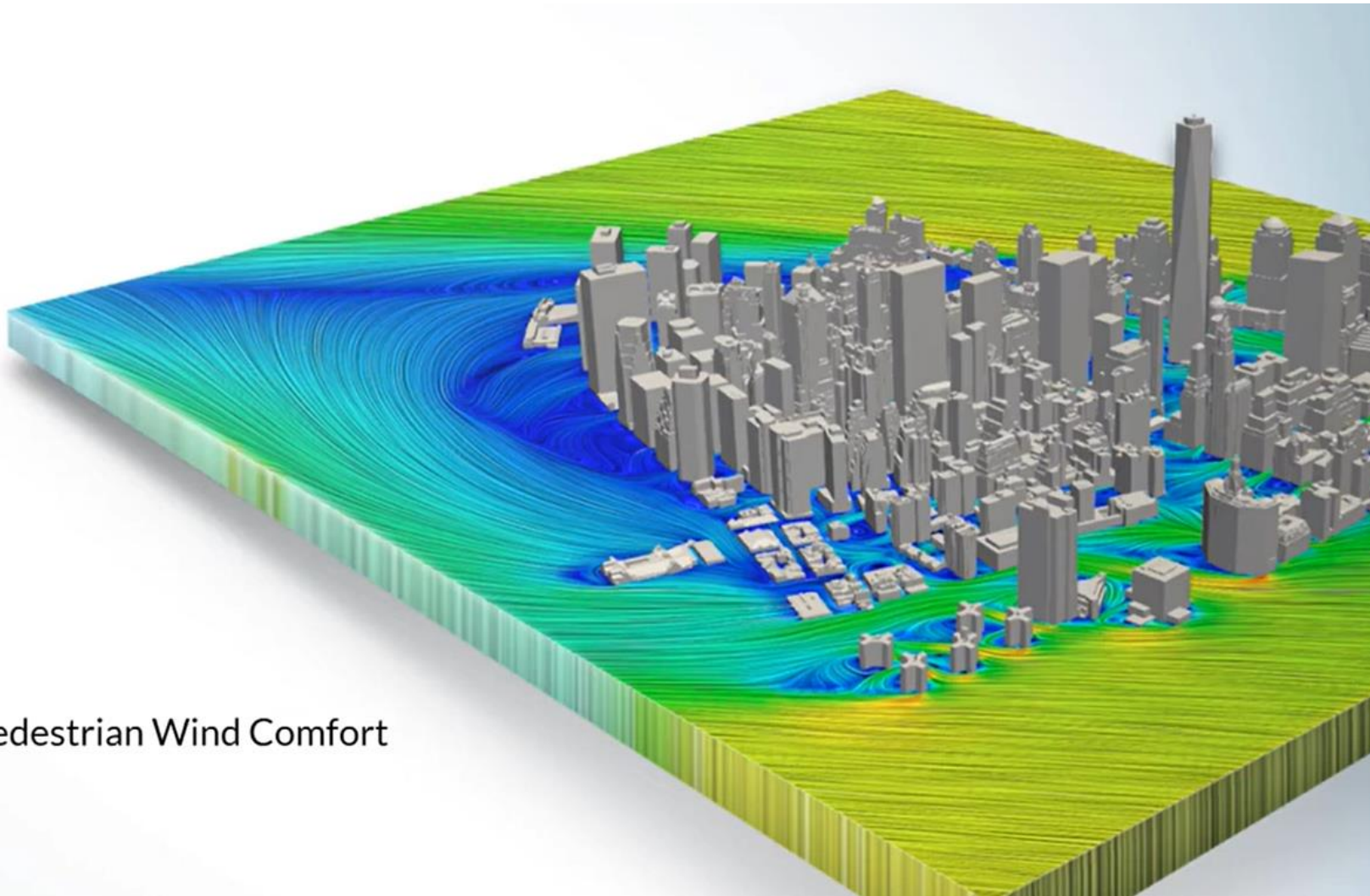https://webstructural.com/beam-designer.html

3 examples of solved up-to-date engineering problems

Wind turbines for power generation

https://en.wikipedia.org/wiki/Wind_power

Pedestrian Wind Comfort

https://www.youtube.com/watch?v=OqRUtRytanI

Mission to Mars