Demonstrating Bell Inequalities through Hardy's Test

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December 2020, PHYS 4052W, Draft Final

Abstract

In this experiment we sought to demonstrate a violation of one of the Bell inequalities, which are inequalities that express the constraints of local realism. The method used was Hardy's test, which is a method of creating a system of two particles in which quantum mechanics predicts a violation of a Bell inequality. This was done through the use of a high-powered (80mW) UV laser and a β-Barium-Borate crystal, which spontaneously down-converts photons, causing entanglement. We expected to see that the results of measurements on the two photons were interrelated in a way that contradicts the principle of local realism, but in the end we were unable to find a configuration that violated local realism.

Introduction

Just after quantum mechanics was introduced to explain the areas classical mechanics could not, many were skeptical of it's validity. In particular, there is a famous question proposed by Einstein, Rosen, and Podolosky, on whether quantum mechanics can be considered to be complete[1]. The issue was based on quantum mechanics violating local realism. Locality is the idea that a measurement in one system cannot be affected by a measurement in another system unless a direct causal relationship is present between the two systems. In the case of the two entangled photons discussed in this paper, we will set up the experiment such that there is no physical connection between the two arms. The reality aspect of local realism is the idea that physically-measureable quantities should have definite values prior to their actual measurement; something that is not true in quantum mechanics.

In this experiment, we used Hardy's Test to look for the violations of local realism that quantum mechanics imply, which would provide further confirmation of the validity and necessity of quantum mechanics. Hardy's test is a special case of the Bell inequalities. The original Bell inequalities demonstrated violations of local realism through a statistical anomaly, before it seemed experimentally possible to do so [2]. This statistical description is, however, somewhat difficult to grasp conceptually. Later developments by Greenberger, Horne, and Zeilinger were able to show violations in a more simple binary test, but required three entangled particles, and were still difficult to conceptualize [3]. Hardy's test is a two-particle binary test of Bell's inequalities, and has the benefit of being much easier to explain to a non-technical audience [4].

The concept of these entangled photons and the violation of local realism is a hot topic in contemporary science as it is being used to theorize on quantum computing techniques, particularly quantum encryption. This is the idea that an individual could be given a specific quantum state as their "key"; if anyone tried to read their encryption, they would inherently change it and the owner would be alerted to the attempt. The attempt at reading another person's data would fail. Thus far, this is not a widely-distributable technique (neither are quantum computers), but this idea may end up being invaluable in the future of computing [5].

Theory

The core of this experiment is the search for a behavior that runs counter to classical physics and the ideas of local realism. We will begin by describing the results we expect based on quantum mechanics, and then proceed to explain the underlying physics that the results come from. This description is adapted from Carlson, Olmstead, and Beck [4]. Our experimental setup includes a nonlinear crystal called a β-barium-borate crystal which should produce pairs of entangled photons [6]. From each pair, one photon goes to “Station A”, and another to “Station B”. At each station there is optical equipment set up to measure whether the polarization of the photon is at a certain angle. Station A can be set up to look for either θ_{A1} or θ_{A2}, and Station B can be set to look for either θ_{B1} or θ_{B2}. These angles are specifically chosen to make the following observations hold; a more complete description of how they are selected can be found in [4].

We will begin by describing the experiment as conceptually-simple as possible, then discuss the modifications required to make it more experimentally viable.

The probability of observing an event at Station A(B) when measuring at the angle $\theta_{A(B)i}$ will be referred to as P($\theta_{A(B)i}$).

With our experiment conducted several times in different configurations, quantum mechanics indicates that we should observe the following [4]:

1. When Station A is at θ_{A1} and Station B is at θ_{B1}, they will observe simultaneous counts only some of the time. Specifically, $P(\theta_{A1}, \theta_{B1}) \cong 0.09$.
2. With Station A at θ_{A2} and Station B at θ_{B1}, we look at the conditional probability that a photon will be observed at Station A given that a photon is observed at Station B. We expect this to occur in essentially all cases, with $P(\theta_{A2}|\theta_{B1}) = 1$.
3. Similarly, with Station A at θ_{A1} and Station B at θ_{B2} we expect to see a photon at Station B whenever a photon is observed at Station A. That is, $P(\theta_{B2}|\theta_{A1}) = 1$.

Given the above is observed, it's important to note that if local realism is assumed (that is, if stations A and B are taken to be independent) simple probability would give us that

$$P(\theta_{A2}, \theta_{B2}) \geq P(\theta_{A1}, \theta_{B1}) \cong 0.09. \tag{1}$$

This is not what quantum mechanics expects. Quantum mechanics would instead have that

4. Stations A and B set to θ_{A2} and θ_{B2} will both observe a photon with a probability of 0. That is, $P(\theta_{A2}, \theta_{B2}) = 0$.

For convenience we will refer to these four outcomes predicted by quantum mechanics as observations 1 through 4.

These results (if observed) are inconsistent with what you'd get if the two photons had set states when they were generated and didn't interact with each other. As such, the outcomes we've described would demonstrate that either the photons did not have set states or did interact with each other, violating local realism. The results described are consistent with a quantum mechanical description of certain entangled states. Specifically, the wave function

$$|\psi\rangle = \sqrt{0.8}\, |H\rangle_A |H\rangle_B + \sqrt{0.2}\, |V\rangle_A |V\rangle_B \,, \tag{2}$$

with H and V referring to horizontal and vertical polarization for a photon, respectively, should produce the above results for measurement angles of magnitudes 55° and 71°[4].

While the above will work in theory, the argument relies on seeing certain outcomes 100% of the time, which is not practical experimentally. The following modification is equivalent and avoids this issue.

Let us start by noting that if under a certain circumstance a photon is observed to be polarized at a certain angle 100% of the time, it must be polarized at that angle, and hence it will never be observed at a polarization 90° offset from that angle under those same circumstances. This can be used to modify observation 2. Suppose we have that if Station B observes a photon polarized at θ_{B1} Station A will always observe a photon polarized at θ_{A2}. This is equivalent to saying that if Station B observes a photon at θ_{B1} Station A will never observe a photon at $\theta_{A2}^{\perp}$, where $\theta_{A2}^{\perp}$ is an angle 90° offset from θ_{A2}.

We can make a similar modification to observation 3, and get

2′. When Station A is set to $\theta_{A2}^{\perp}$ and Station B is set to θ_{B1} Stations A and B will never detect simultaneous photons. We express this $P(\theta_{B1}, \theta_{A2}^{\perp}) = 0$.

3′. Likewise when Station A is set to θ_{A1} and Station B is set to $\theta_{B2}^{\perp}$ Stations A and B will never detect simultaneous photons, so $P(\theta_{A1}, \theta_{B2}^{\perp}) = 0$.

Observations 2′ and 3′ together imply (under local realism) equation 1, that $P(\theta_{A2}, \theta_{B2}) \geq P(\theta_{A1}, \theta_{B1})$, which is violated by observations 1 and 4.

The convenient thing about this formulation is that if observations $2'$ and $3'$ are not observed perfectly we can modify the local realistic expectation of equation 1 to become

$$P(\theta_{A2}, \theta_{B2}) \geq P(\theta_{A1}, \theta_{B1}) - P(\theta_{B1}, \theta_{A2}^{\perp}) - P(\theta_{A1}, \theta_{B2}^{\perp}). \tag{3}$$

Equation 3 is a form of the Bell-Clauser-Horne inequality[4], which is a restriction on any local realistic system.

From equation 3, we will define a parameter H as

$$H = P(\theta_{A1}, \theta_{B1}) - P(\theta_{B1}, \theta_{A2}^{\perp}) - P(\theta_{A1}, \theta_{B2}^{\perp}) - P(\theta_{A2}, \theta_{B2}). \tag{4}$$

This lets us simplify our experimental criterion to saying that if $H \leq 0$ then our results are consistent with local realism, while results of $H > 0$ are inconsistent with local realism, though allowed in some cases by quantum mechanics.

Apparatus and Procedure

Correlated Photons

Before performing Hardy's Test, we must first verify that we are observing correlated photons, and that there are, in fact, single photons being sent to each station. To accomplish the former, we sent a 405nm, 80mW laser beam through the β-Barium-Borate (BBO) crystal, creating two subsequent 810nm down-converted beams (let's refer to them as A and B). An SPCM (single photon counting module) detector was placed in the path of each of the two 810nm beams at stations A and B, respectively, as shown in Figure 1. These detectors send the count rates observed to a desktop computer running a Labview program which records the data. The Labview program records how often detectors A and B register coincident photons. As a note, the coincidence window we chose was 10ns, which is the shortest coincidence window our electronics were able to use.

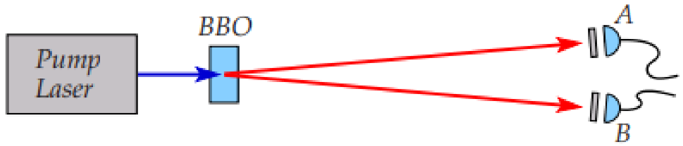

Figure 1: *Diagram with key components of photon correlation setup, pulled from Pearson and Jackson[7].*

What we expect to see is a number of "coincidence counts" greater than what can be explained by uncorrelated events at detectors A and B. Theoretically, the rate we'd expect if the events at A and B are completely uncorrelated is the following, defined here as the rate of "accidental" coincidence counts, or R_{acc}.

$$R_{acc} = 2\tau R_A R_B \tag{5}$$

The pulse width, τ is the duration of the digital pulse produced by the SPCM detectors, and $R_{A(B)}$ represents the count rates detected at A (B). Twice the pulse width is used in this expression because the longest period of time acceptable to consider an event "coincident" between the two detectors occurs when the detected events are within one pulse width (10ns) of each other. As a note this accidental coincidence rate relies on the assumption that the two sources are Poissonian, which should be true of our pump laser.

With knowledge of the statistically-predicted amount of coincident counts, we can now introduce the "anticorrelation parameter", α which gives a measure of how correlated the two down-converted beams are.

$$\alpha = \frac{P_c}{P_A P_B} \tag{6}$$

Here, P_c represents the probability of measuring a coincidence count and $P_{A,B}$ represents the probability of observing a count at detector A or B. This parameter becomes more clear when it is re-written in terms of numbers of counts and ultimately count rates. The probability of an event, P, is determined by the ratio, $\frac{N}{N_P}$, where N_P is the total possible events in a certain time frame.

$$\alpha = \frac{N_c}{N_A N_B} N_P = \frac{R_c}{2\tau R_A R_B} = \frac{R_c}{R_{acc}} \tag{7}$$

The above was simplified using the definitions of total possible events as $N_P = T/2\tau$, and $R_{A,B}$ as the rate of counts observed at detectors A and B. It is expected that we find $\alpha \geq 1$ if our photons are indeed correlated, because the amount of detected coincidences will be greater than the amount of predicted coincidences. This discrepancy can be explained by the splitting of individual (quantized) photons creating pairs of correlated photons; a quantum-mechanical process. We could also get such a result from a source akin to a pulse laser, which sends many photons in bunches, but such a source would not produce single photons, and can be ruled out after the next step of our experiment.

Single Photons

Now that we have established the two 810nm beams as correlated sources, it is possible to verify that single photons are passing through the BBO. This is due to their correlated nature; we know that if there is a photon detected at Station A, there should also be one at Station B. To show this experimentally, it is necessary to introduce a non-polarizing beam-splitter and third detector (B'). These new components will be set up as in Figure 2. Beam B will pass through the beam-splitter, forcing the single photon to be transmitted (toward detector B) or reflected (toward detector B'). The observations from all three detectors are again passed to the Labview program for processing, where the following calculations will be performed.

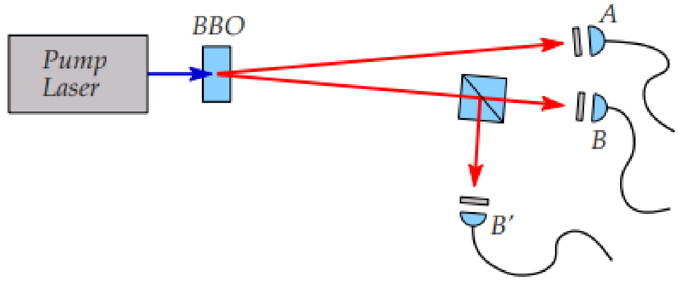

Figure 2: *Diagram with key components of single photon calibration setup, pulled from Pearson and Jackson[7].*

Where we were earlier defining the total possible events as $N_P = T/2\tau$, we can now use $N_P = N_A$ because we will only look for events at Station B when there is an event at Station A. Now, the probability of observing an event at B, conditioned on the observation of an event at A becomes, $P(B|A) = P_B = \frac{N_{AB}}{N_A}$, where N_{AB} represents the amount of AB coincidence counts.

This is where the third detector, B', comes into play; the probability of observing a count at both B and B' (conditioned on A) is given by $P(BB'|A) = P_{BB'} = \frac{N_{ABB'}}{N_A}$ Where $N_{ABB'}$ is the number of triple coincidences between all three detectors. After this, a new anticorrelation parameter, α', for the new setup is defined as

$$\alpha' = \frac{N_{ABB'}}{N_{AB}N_{AB'}}N_A \tag{8}$$

According to the classical notion of light as a wave, we'd expect α' to be ≥ 1. This is due to the ability of waves to quite-literally split at the beam-splitter. Quantum mechanically, single photons are allowed and thus we'd expect α' much less than one because a single photon can only be reflected *or* transmitted, not both. It is also possible for us to tune the laser intensity to increase the probability of single photon emission, if α' is too large initially.

The reason that we use three detectors for this instead of two detectors with a beam-splitter is because the best result we could get from using two detectors is that the detectors were uncorrolated, which is good, but not as strong of a result as we want. By attenuating a two detector setup you could in principle significantly increase the time between coincidences, but background counts can't be attenuated and would become more of an issue, and using a 5 second, or 5 minute, interval in which no coincidence counts were observed would be dishonest, essentially the same as cherry-picking data. A long enough gate time should result in a 2-detector anticorrelation parameter of 1, as with our setup B and B' are not in fact anti-corrolated, and they don't need to be. What we care about is that they're anti-corrolated in the subset of time within 10ns of an event at A, and that's what this test shows.

It is worth noting that now that we have proven the presence of single correlated photons in our apparatus, we can proceed with the tenuous conclusion that they are *entangled*, based upon knowledge of the BBO crystal properties as described thoroughly in [6], though of course to confirm this we would need something similar to the violation of Bell inequalities that is the main thrust of our experiment.

Hardy's Test

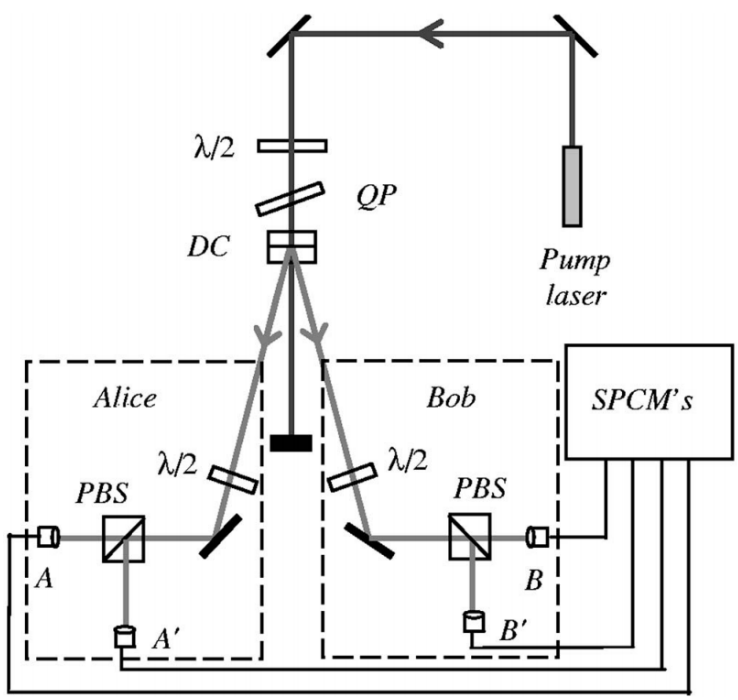

Figure 3: *Diagram of the setup of Hardy's experiment, pulled from Carlson and Beck[4].*

Figure 3 shows a diagram of the setup for Hardy's test. The 405nm beam is sent through a half-wave plate to adjust the polarization, and an angled quarter-wave plate to adjust the relative phase of the horizontal and vertical components of the polarization. Both the half- and quarter-wave plates can be adjusted post-setup for fine tuning. The laser then goes through a pair of BBO crystals that are stacked such that their crystal axes are orthogonal. Due to this orientation, the first crystal converts vertically-polarized photons from the laser into horizontally-polarized entangled pairs and vice versa for the second. Beams A and B (810nm), as described previously will go to stations A and B, respectively, as in the previous steps of the experiment. For Hardy's test the components of station A and station B are the same, so we will just describe the configuration of station A. As beam A reaches Station A, the beam goes through a second half-wave plate to orient a specific incident angle with the polarizing angle of the polarizing beam-splitter that follows the half-wave plate. The orientation of the the half-wave plate is the primary control of the experiment, as it allows the polarizing beam-splitter to select for different angles without rotating the beam-splitter. After the half-wave plate the beam then hits the polarizing beam-splitter, and the photons that pass through or reflect are collected by collimators A and A′, respectively, which are then fed into their corresponding SPCMs. The data from these, as well as from the corresponding SPCMs in Station B, will be fed into Labview for processing.

It is very important that the 405nm beam have a specific polarization configuration as it hits the BBO, so that the entangled photons will have the state $|\psi\rangle$ described in equation 2. This is accomplished by adjusting the half- and quarter- wave plates along with the BBO while using our detectors for calibration. A more complete description of this process can be seen in [4].

The raw data collected via the Labview program will be in the form of count rates, or numbers of events in a given time interval. The types of events that we are looking for are coincidence counts between one SPCM from Station A and one SPCM from Station B, for example the number of events detected in both SPCM A and SPCM B′ in a 10 second interval. This count would be referred to as $N_{AB'}$.

These counts can be converted to probabilities through the formula

$$P(\theta_{Ai}, \theta_{Bj}) = \frac{N_{AB}(\theta_{Ai}, \theta_{Bj})}{N_{AB}(\theta_{Ai}, \theta_{Bj}) + N_{AB'}(\theta_{Ai}, \theta_{Bj}) + N_{A'B}(\theta_{Ai}, \theta_{Bj}) + N_{A'B'}(\theta_{Ai}, \theta_{Bj})} \tag{9}$$

where $P(\theta_{Ai}, \theta_{Bj})$ is the probability of observing an event when Station A is set to measure θ_{Ai} and Station B is set to measure θ_{Bj}. The principle behind this equation is that the denominator is how often a photon is detected in stations A and B simultaneously, and the numerator is how often that event is the one we're looking for.

From here we can find these probabilities for all combinations of θ_{Ai} and θ_{Bj}, and from there find a value for H from equation 4.

Results

Correlated Photons

When initially testing for correlation we simply took single measurements with 20 second gate times for different attenuations and coincidence windows. Were the count rates Poissonian we could extract uncertainties for these measurements after the fact, but we found later that, while somewhat close, the count rates are not quite Poissonian, as the standard deviations for repeated measurements on a given detector are greater than one would expect from a poissonian source. Later on in the experiment we went back and took more measurements, which will be the ones discussed here.

As a note, these measurements were taken in between attempts to take Hardy state data, so the polarizing beamsplitters (PBSs) were left in. To account for this, the 405nm HWP was set to produce mostly horizontal photons from the BBO, and both 810nm HWPs were set to allow horizontal photons through the PBSs. In principle this should be equivalent to having a single BBO in place (instead of the double BBO used for Hardy's test) with no PBS, as all of the light should be polarized so as to go through the polarizing beam splitter.

Additionally, while the attenuations are slightly different from those used in the initial tests for which only one measurement was taken (notably α is very sensitive to attenuation), the values obtained for α for similar attenuations are within about 10% of each other.

All measurements used a 20 second gate time and were taken 11 times. The anticorrelation parameter was calculated for each trial and the set of 11 values were used to calculate a value and an uncertainty.

The first data set was taken with no attenuation of the pump beam, resulting in rates for A and B counts on the order of 100,000. The measured anticorrelation parameter without attenuation was $\alpha = 66.10 \pm 0.08$. This is about 814σ above 1, which indicates strongly that our setup produces corrolated photons. Sigma (σ) refers to the uncertainty in the measurement, in this case 0.08.

The next data set was taken with attenuation of the pump beam to bring the A and B counts down to twice their background level (the rate of counts observed when the laser is blocked). This amounted to A and B counts closer to 1000. The measured anticorrelation parameter with attenuation was $\alpha = (4.0 \pm 0.2) \times 10^3$. This is only 20σ above 1, but the value of α is much higher. The reason that α is so much larger is that when attenuating the pump beam by a factor of 100, the A counts and B counts drop by a factor of 100, and so do the counts that are the result of split photons hitting both detectors, whereas by contrast the expected rate of accidental coincidences R_{acc} is proportional to $R_A \times R_B$, so it drops by a factor of 10,000. This means that if we have split photons the value of α will rise significantly. The reason the uncertainty is so much higher than for the unattenuated trial relative to the value of α is that with such strong attenuation there are relatively few counts in any given run, and so there is a lot more fluctuation. Since this can be compensated for with a larger gate time, this is still preferable to the unattenuated setup for the purposes of data collection, though it remains far easier to deal with count rates in real time without the attenuation.

Single Photons

For single photon statistics we also went back and recollected data.

For single photons we adjusted the polarization of the pump laser so that the BBO would produce mostly horizontal photons, and set the A arm HWP so that detector A would recieve horizontally polarized photons, and adjusted the B arm HWP so that detectors B and C were getting similar count rates.

The first data set taken had no attenuation, and resulted in an alpha value of $\alpha = 0.03 \pm 0.01$. This is 97σ below 1, which indicates we are in fact seeing single photons.

In principle attenuation will improve this number significantly, but with significant attenuation ABC coincidence counts are so rare that getting meaningful results requires extremely long gate times. With 10 trials using a 20 second gate time we got only a single count for ABC coincidence, which means all but one of the trials for alpha produced a value of 0, and the uncertainty is significantly larger than the measured value. The measurement is $\alpha = 0.002 \pm 0.006$, but under the circumstances the value is not very trustworthy.

Since both results indicate the presence of single photons, we can conclude that our setup was able to produce single photons.

Hardy's Test

Unfortunately, we were unable to acquire a Hardy state, and thus could not complete Hardy's test. We tried various methods to try to get it working (see appendix A), but the closest we came to an H value above 0 was an value of $H = -0.18$, which resulted from a state in which $P(\theta_{A1}, \theta_{B1}) = 0.006$, which indicates significant problems with that configuration ($P(\theta_{A1}, \theta_{B1})$ is supposed to be around 0.09, and is the only probability that is expected to be high).

The initial plan was to find a state that worked, then fine tune it and take many more measurements on it. Since it's typically clear that a state will not yield the H values we want without measuring all four probabilities this means that we have very few states that we measured an actual value of H for. What follows is a table containing the values of H we did record. The half-angle section contains the measurement made with the 810nm HWP set to half of the measurement angle, so that the correct polarization will correspond to horizontal when it hits the polarizing beam splitter. The reason there is only one measurement here is that with half angles it was a bit rarer to get even acceptable values for $P(\theta_{A2}, \theta_{B2})$ (the first thing we checked), and we wouldn't typically take a measurement of H unless we were already measuring at least two probabilities. The full-angle section contains the measurements made with the 810nm HWPs set directly to the measuement angles mentioned in the literature, hedging off the possibility that that is what was meant when one of the papers mentioned measuring at a given angle. This is one of several things we tried in the course of our troubleshooting.

	Half Angle	Full Angle			
	State 1	State 1			State 2
$P(\theta_{A2}, \theta_{B2})$	0.0766	0.0358	0.0145	0.7277	0.0129
$P(\theta_{A1}, \theta_{B2}^{\perp})$	0.0837	0.3974	0.5201	0.0416	0.4846
$P(\theta_{A2}^{\perp}, \theta_{B1})$	0.0303	0.4959	0.5698	0.0647	0.4084
$P(\theta_{A1}, \theta_{B1})$	0.0063	0.3177	0.3937	0.1560	0.2804
H	-0.1842	-0.6113	-0.7107	-0.6781	-0.6256

In as far as what went wrong, the aspect of the experiment we are most skeptical of is the choice of angle for the quarter wave plate, though it could also be a result of problems with the alignment of the detectors, or other issues with our apparatus or methodology. Having not isolated the problem, all we have is conjecture.

Near the end, one of the things we attempted was using the 405nm HWP to acquire the correct ratio of horizontal and vertical photons without the quarter wave plate in place, then adding it in and adjusting the angle 180 degrees, stopping every 10 degrees to adjust the tilt to find a minimum for AB coincidences in the θ_{A2}, θ_{B2} configuration and record it. A graph of the outcomes of this can be seen in figure 4. When using the full angles in the

810nm HWPs for this (which I do not think is correct) we found 2 minima from this, and tried looking for H for those states, but $P(\theta_{B1}, \theta_{A2}^{\perp})$ and $P(\theta_{A1}, \theta_{B2}^{\perp})$ were around 50%, and they are meant to be around zero. When using half angles in the 810nm HWPs we were not able to minimize AB coincidences enough to get usable values of $P(\theta_{A2}, \theta_{B2})$.

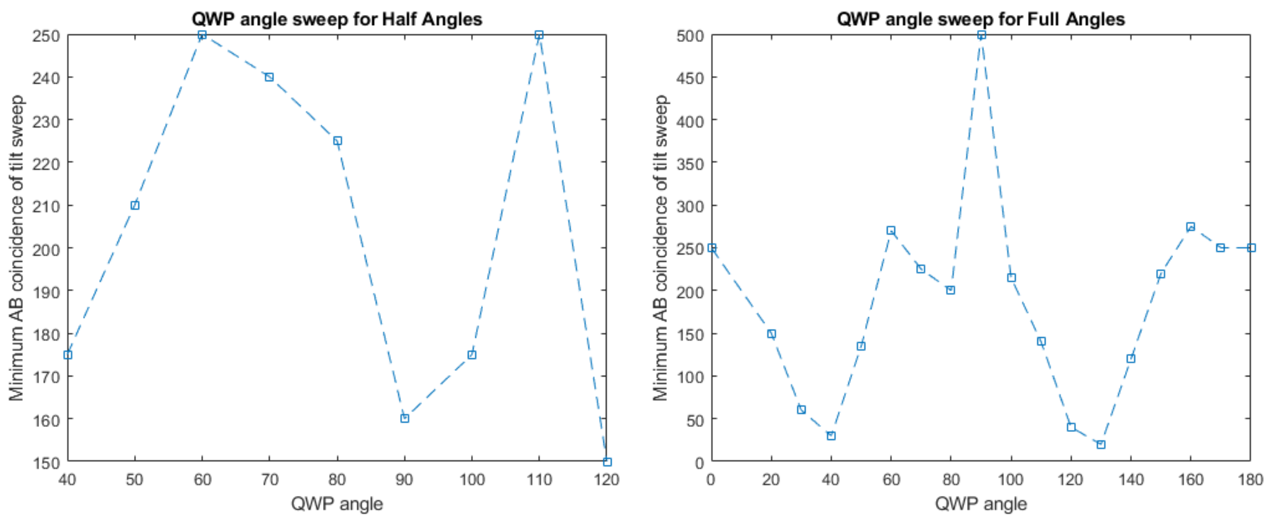

Figure 4: *Graphs of the AB coincidence rates from the QWP angle sweep that was attempted. Note the difference in y axis scale.*

Given more time, one of the next things we might have tried is going through QWP angles in 10 degree increments but setting the 405nm HWP to get the correct ratio of horizontal and vertical photons with the QWP in place and set to the angle being tested, then proceed as normal and try to find an angle that worked.

Ultimately, without having acquired a Hardy state, there's not a lot we can take from this portion of the experiment. That said, in the course of attempting to get the experiment working we adjusted attenuation of different arms of our apparatus a lot, and thinking about this led me to suspect the following:

When adjusting the 405nm HWP to get a 1:4 ratio between horizontal and vertical photons, the aspect that should matter is the configuration of the pump beam when it hits the BBO. As such, if detector inefficiency or attenuation were such that when A and B are fed a signal the AB coincidence is say twice that of CD when C and D are sent the same signal (by adjusting the 810nm HWPs by 45 degrees), then in setting up a 1:4 ratio between AB coincidence and CD coincidences one would in fact be acquiring the wrong state for the pump beam. I'm reasonably confident that such an issue wasn't what prevented us from getting data, as if this were a factor we sufficiently corrected for it, but I feel that verifying this behavior would be an interesting extension to a future project working on Hardy's test.

Conclusion

In the course of this experiment we were able to demonstrate production of single paired photons, as well as lay additional groundwork in preparation for conducting Hardy's test, but were unfortunately unable to acquire a Hardy state, and thus were not able to prove a violation of Bell's inequalities, and thus local realism.

Appendix A: Methods of troubleshooting

The following is a list of some of the methods used to try to get the Hardy state to work. There may have been other methods that I do not recall, as they did not work.

- The measurement angles corresponding to the Hardy's state we were attempting to reach should realistically be acquired by setting the 810nm HWP angles to half of the relevant angle, so as to map polarizations of that angle to the horizontal polarization that would allow it to pass through the polarizing beam splitter to detector A, or B. As a precaution however, all methods attempted were also done using the full angles for the setting of the half wave plates.

- We attempted to acquire the Hardy state with the angle of the quarter plate set such that the fast axis was vertical, and again with the fast axis horizontal.

- The wiki page created by one of the previous groups to do this project lists a procedure for selecting the angle for the quarter wave plate. One of the steps involves picking an angle that causes the magnitude of a certain detector coincidence rate to be a fourth to a third of the way from the minimum that the QWP angle can cause and the maximum. We tried that, as well as angles corresponding to slightly lower and higher magnitudes.

- We tried swapping out the quarter wave plate for a quartz plate, in the hopes that the less specialized quartz plate might have a retardance that was more responsive to tilting.
- We tried putting a linear polarizer in the path of the beam to have a more well-defined initial polarization. We also tried rotating this polarizer to different angles to control attenuation using this.
- We used the quartz plate and the linear polarizer to try to get rid of as much eccentricity from the inital pump beam before it hit the 405nm HWP.
- We adjusted the attenuation of the detectors to try make the coincidence rates between any two detectors under equivalent circumstances as close to identical as we could get it. For example we would want the AB coincidence rate when receiving only horizontal photons to be similar to the AC coincidence rate when the B arm HWP is rotated 45 degrees from that configuration (with the pump beam not changed). The hope here was that this would let the 1:4 ratio we were trying to acquire more trustworthy, as discussed in the Hardy's test portion of the results section.
- We tried various levels of attenuation of the pump beam, mostly using irises. As a note, attenuation used after the BBO should not in principle improve the single and correlated photon properties (i.e. improve the anticorrelation parameter) in the way that attenuation of the pump beam will, though we did not test to confirm this.
- We tried adjusting the detector alignment slightly, but we didn't risk any particularly drastic changes as we might have done had we had more time.

- We tried selecting a HWP angle for the 1:4 ratio without the quarter wave plate in place and then going through 180 degrees on quarter wave plate 10 degrees at a time and seeing how low we could get AB coincidences by adjusting tilt with the 810nm HWPs set to θ_{A2}, θ_{B2}. This was of course repeated with both half and full angles. With full angles we were able to get low values for $P(\theta_{A2}, \theta_{B2})$ and high values for $P(\theta_{A1}, \theta_{B1})$, but the values for $P(\theta_{B1}, \theta_{A2}^{\perp})$ and $P(\theta_{A1}, \theta_{B2}^{\perp})$ were around 50%, so they had to be dismissed. For the half angles the lowest we were able to get the AB coincidences was 140 counts per second, which is not low enough. See 4. Had we more time we would have tried to set a HWP angle for each QWP angle to get 1:4 ratios.

- The BBO tilt that maximized coincidences would sometimes change in different pump-beam configurations, though I'm not sure why. This was noticed in particular when setting the linear polarizer in the pump beam. We would occasionally check to make sure it was set to maximize horizontal and vertical coincidences.

References

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