Practicum 3 Bookwork — Kieran Douglas

4.4

Model: $\log(rent) = \beta_0 + \beta_1 \log(pop) + \beta_2 \log(avinc) + \beta_3 pctstu + u$

(i)

null: $\beta_3 = 0 \longrightarrow H_0 \Longleftrightarrow$ There is no CP effect of student pop. on rents.
alt: $\beta_3 \neq 0 \longrightarrow H_1 \Leftrightarrow$ There is an effect CP of student pop. on rents

(ii) I would expect positive signs for both β_1 and β_2 since generally in the U.S. places with higher populations are more expensive to live in due to increased housing demand. I would expect β_2 to also be positive because increases to average income generally go hand in hand with increased housing cost since land lords would be undercharging if people could or were willing to pay more. Additionally, higher income tends to prefer nicer rentals which may also increase cost. It makes sense that elasticity of rent WRT income and population would be generally increasing.

(iii)

estimated: $\widehat{\ln(rent)} = 0.43 + 0.66 \ln(pop) + 0.507 \ln(avginc) + 0.0056 pctstu$

The statement is wrong because it reflects a misunderstanding of how log-log models should be interpreted. It may work for small percent changes in population, but since elasticities vary at different points on a curve we would expect it to be a bit different at 10% than at a 1% change. The correct interpretation uses the formula $\%\Delta y = ((1+\%\Delta X)^{\beta} - 1) \times 100$ where $\%\Delta y = ((1+0.1)^{0.066} - 1) \times 100 = 0.631\%$ change

(iv) I will test the hypotesis $\{H_0: \beta_1 = 0, H_1: \beta_1 \neq 0\}$ using a two-sided t-test.

$$t = \frac{\hat{\beta}_1 - 0}{SE(\hat{\beta})} \Rightarrow \frac{0.066 - 0}{0.039} = 1.6923$$

; 1% critical value at $n-k-1$
$\hookrightarrow 64-3-1 = 60$
$\hookrightarrow t = 2.66$

$\longrightarrow 1.6923 < 2.66$

I fail to reject the null and determine that there is insufficient evidence to suggest that $\beta_1 \neq 0$.

4.8

Model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$, testing the null that $H_0: \beta_1 - 3\beta_2 = 1$

(i) Since $Var(\alpha X, \beta Y) = \alpha^2 Var(X) + \beta^2 Var(Y) + 2\alpha b \, Cov(X,Y)$

we can write $Var(\hat\beta_1 - 3\hat\beta_2) = 1^2 Var(\hat\beta_1) + 3^2(\hat\beta_2) - 2 \cdot 3 \, Cov(\hat\beta_1, \hat\beta_2)$

$\hookrightarrow \boxed{Var(\hat\beta_1) + 9(\hat\beta_2) - 6\,Cov(\hat\beta_1, \hat\beta_2)}$

To complete the test we need the t stat which requires SE

$$\boxed{SE(\hat\beta_1 - 3\hat\beta_2) = \sqrt{Var(\hat\beta_1) + 9(\hat\beta_2) - 6\,Cov(\hat\beta_1, \hat\beta_2)}}$$

(ii) $\rightarrow \boxed{t = \dfrac{(\hat\beta_1 - 3\hat\beta_2) - 1}{SE(\hat\beta_1 - 3\hat\beta_2)}}$ since $t = \dfrac{\hat\beta_j - \beta_{j,1}}{SE(\hat\beta_j)}$ and we test that $\beta_{j,1}$

we should reparameterize to $\hat\theta = \hat\beta_1 - 3\hat\beta_2$ so $\boxed{t = \dfrac{\hat\theta - 1}{SE(\theta)}}$

(iii) Reparameterizing $\theta = \beta_1 - 3\beta_2$ and $\hat\theta = \hat\beta_1 - 3\hat\beta_2$

We can write the regression equation

$\hookrightarrow y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$ as $y = \beta_0 + (\beta_1 - 3\beta_2)x_1 + \beta_2 x_2 + \beta_3 x_3 + u$

$\Rightarrow \underline{y = \beta_0 + \theta x_1 + \beta_2(3x_1 + x_2) + \beta_3 x_3 + u}$ if we write $\beta_1 = \theta + 3\beta_2$

To obtain $\hat\theta$ and its SE, we need to construct a new variable

$z = 3x_1 + x_2 \longrightarrow \underline{\hat y = \hat\beta_0 + \hat\theta x_1 + \hat\beta_2 z + \hat\beta_3 x_3 + u}$

and the SE for $\hat\theta$ would be $\underline{SE(\hat\theta) = \sqrt{Var(\hat\beta_1) + 9Var(\hat\beta_2) - 6Cov(\hat\beta_1, \hat\beta_2)}}$

(5.4) $y = \beta_0 + \beta_1 x + u_i$ Under GM, OLS is BLUE. Under the GM's 1-4 we show that estimators of the form $\tilde{\beta}_1$ are consistent for the slope β_1.

β_0 estimator $\tilde{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ and $\tilde{\beta}_0 = \beta_0$

↳ This is true because the probability limit is $= \beta_0$:

$E(y_i) = \beta_0 + \beta_1 E(x_i) + E(u_i) \longrightarrow$ Under GM, $E(u_i) = \emptyset$ so...

↳ if $m_y = E(y_i)$ + $m_x = E(x_i)$ then $m_y = \beta_0 + \beta_1 m_x \Rightarrow \beta_0 = m_y - \beta_1 m_x$

↳ LLN states that $\bar{y} \xrightarrow{p} m_y$ and $\bar{x} \xrightarrow{p} m_x$ so...

↳ Since it's consistent for β_1, $\tilde{\beta}_1 \xrightarrow{p} \beta_1$

↳ $\operatorname{Plim}(\tilde{\beta}_0) = \operatorname{plim}(\bar{y} - \tilde{\beta}_1 \bar{x}) = m_y - \beta_1 m_x = \beta_0$

↳ so $\operatorname{plim}(\tilde{\beta}_0) = \beta_0$ thus $\tilde{\beta}_0$ is consistent est. of β_0.

(5.6) Given the equation $y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$ and $E(u|x) = \emptyset$

↳ $E(x) = \emptyset$, $E(x^2) = Var(x) = 1$ + $E(x^3) = 0$

(i) We can write $y = \alpha_0 + \beta_1 x + v$ because...

↳ $y = \beta_0 + \beta_1 x + \beta_2(x^2) + u \longrightarrow y = \underbrace{(\beta_0 + \beta_2 E(x^2))}_{\alpha_0} + \beta_1 x + \underbrace{(\beta_2(x^2 - E(x^2)) + u)}_{v}$

↳ $\alpha_0 = \beta_0 + \beta_2 E(x^2)$ and $v = \beta_2(x^2 - E(x^2)) + u$

↳ Thus $y = \alpha_0 + \beta_1 x + v$

(ii) $E(v|x)$ depends on x unless $\beta_2 = \emptyset$ because...

↳ if $\beta_2 = \emptyset$ then $v = u$ since $v = \beta_2(x^2 - E(x^2)) + u$

↳ otherwise $E(v|x)$ varies with x.

(iii) $Cov(x,v) = \emptyset$ because:

$\hookrightarrow Cov(x,v) = E(xv) - E(x)E(v)$

$\hookrightarrow$ since $E(v) = \emptyset, \rightarrow Cov(x,v) = E(xv)$

$\hookrightarrow E(xv) = E[x\{\beta_2(x^2 - E(x^2))\} + u] \rightarrow E[x(x^2 - E(x^2))]$

$\hookrightarrow$ since $E(x) = \emptyset, E(x^2) = 1, E(x^3) = \emptyset, \Leftrightarrow \emptyset - 1 \cdot 0 = \emptyset$

$\hookrightarrow$ Thus $E(xv) = \emptyset$ and $Cov(x,v) = \emptyset$ as well

(iv) I think that $\hat{\beta}_1$ is consistent for β_1 because the assumptions (minus $E(v|x) = \emptyset$) imply that $\hat{\beta}_1 \xrightarrow{p} \beta_1$ since $Cov(x,v) = \emptyset$ and the $var(x) > \emptyset$, thus $\hat{\beta}_1$ is consistent for β_1. I do think it would be biased though, as unbiasedness necessarily includes $E(v|x) = \emptyset$ which we do not have here. The bias would however shrink asymptotically as n approaches ∞.

(v) Similar to the problem in Chapter 4, even if faced with a nonlinear true relationship we can gauge localized responses in y given some small change to X near its mean. Despite it's biasedness, there can still be some utility in exploring these smaller localized changes in outcome.

(vi) It is generally more valuable to consistently estimate β_1 and β_2 then just β_1 since you can get a sense of the complete slope and quadratic relationship at play. The marginal effect $\beta_1 + 2\beta_2 x$ can then be computed at any point, not just at or around the mean. This can drastically improve both the quality and scope of an analysis.

6.4 Model estimated: $\log(wage) = \beta_0 + \beta_1 educ + \beta_2 educ \cdot pareduc + \beta_3 exper + \beta_4 tenure + u$

(i) In decimal form the return to another year of education in this model is $\Delta \log(wage)/\Delta educ = \beta_1 + \beta_2 \, pareduc$

$\hookrightarrow \dfrac{\partial \log(wage)}{\partial educ} = \beta_1 + \beta_2 \, pareduc \longrightarrow$ another year of educ returns a $(\beta_1 + \beta_2 \, pareduc)100\%$ change in wage where absent the x100 it is a proportion.

I would expect β_2 to be positive, as parents who are more educated probably have children who are more likely to be educated, and both are probably more likely to see higher earnings than those with less education.

(ii) The coefficient on the interaction term states that the returns to education are (0.078% x years of parental education x 4.7% ——— % higher for each additional year that a parent was educated.
In other words, the marginal effect of an additional year of education increases by 0.078% for each additional year of parental education.

(iii) When pareduc is added, the interpretation of the interaction coefficient changes slightly. It now shows the marginal effect of schooling on wages decreases by 0.16% per additional year of parental education. The estimated return to education depends negatively on parental education according to this model, but honestly the true effects are now a bit unclear.

Test:
H_0: return to education does not depend on parental education: $\beta = 0$
H_1: return to education does depend on parental education: $\beta \neq 0$

$t = \dfrac{\beta_j - 0}{SE(\beta_j)} \Rightarrow \dfrac{0.0016}{0.0012} = -1.33 \longrightarrow$ with $n = 722$, crit: ± 1.92

$\longrightarrow -1.33 > -1.92$ so fail to reject the null.

* Hierarchy principal says to include the main effets along with the interaction terms in regression modeling to avoid biasing estimates or misinterpretation.
* Including main effect linearly controls for it's direct impact so the interaction coefficient measures only the relationship we are interested in: closes backdoors