Trading Theory and Market Design

Abstract:
This report details the theory and results of trades executed as an FX (foreign exchange) derivatives market maker. Executions undertaken are for FX forward and European option contracts (call and put) which meet the client's investment objectives whilst endeavouring to reduce or eliminate any risk that may arise in future movements of the exchange rate. This experiment aims to assess the effectiveness of option pricing models and delta hedging to manage foreign currency risk exposures between the contracts and underlying currency. I will consider the 'Greeks' to understand the explained and unexplained PnL (profit and loss) and explore how these values fluctuate over time. After conducting a thorough review of my performance, my research validates that option contracts hold higher risk than forward contracts. Additionally, I found that as contracts move closer to maturity, the spot delta increases and therefore deviates further from the acceptable range of values. It is however important to recognise that the unexplained PnL is high, suggesting an increased risk to investment which highlights the trading strategies and models implemented may not be optimal. Overall, I consider my trading performance to be satisfactory as I was able to minimise the spot delta to an extent where there was negligible risk as a result of effective hedging positions but still acknowledge that there is room for improvement.

Introduction:
This report outlines the analysis conducted on orders made in the FX market using USDMXN currency pairing where USD is the base currency and MXN is the quote currency. The primary objective was to manage a trading portfolio whilst observing the client's requirements, minimising risk, and generating profits. My role as a market maker was in effect to facilitate a market to buy and sell currency pairs – acting as an intermediary (counterparty), providing liquidity (ensuring that there is always a market for a given security) and quoting bid-ask prices, with the aim of profiting from the bid-ask spreads (difference between the bid and ask prices). I focused on forward contracts, which is a contractual agreement made between two counterparties to deliver one currency and receive a predetermined amount of another currency at a specified date in the future (binding obligation). Simultaneously, I worked on European option contracts that provide the buyer with the right, but not the obligation to perform specified transactions, namely, call (long) or put (short), with a seller according to pre-determined terms prior to expiry. I will analyse the delta hedging strategies I have employed in order to minimise spot delta, which measures the sensitivity of a portfolio with respect to changes in the spot price, and evaluate the Black-Scholes model that has been applied to calculate a fair price or theoretical value for options. I provide further explanation of another key component being the Greeks, which are financial metrics/parameters used to estimate the fair value of an option based on changes in a number of different variables, namely, underlying asset price, strike price, volatility of underlying asset, time to expiration and interest rates. These have a direct impact on explained and unexplained PnL, and enable a trader to identify and therefore remedy irregularities within a portfolio. I will also assess Mark-To-Market (MTM), which is a measure of portfolio performance, by valuing the asset based on its current market value. Lastly, I will consider PnL impact which is determined by calculating the difference between the previous day's MTM and the current day.

Main Body:

1. **Investment Theory**

The investment strategy employed by a market maker is triggered by the client's instruction to long/short forward and option orders such that, when the client enters into a contract, the market maker will take the appropriate hedge to mitigate risk exposure that may arise due to currency fluctuations or interest rate movements. Accordingly, where the client enters an FX long/short forward contract, the market maker will take an opposite offsetting position of the quoted currency at expiry. The quoted currency is used to then find the discounted value of the quoted currency at time 0 and applied to the spot rate to calculate the base currency required. Similarly, on an FX put/call option contract, whilst the market maker is not required to take an opposite position (as it may not be exercised), they may get the quoted currency δ such that it offsets the contract value changes. In a situation where the client enters a long FX forward contract, or a long FX call option contract the market maker would short sell the spot and lend the base in the money market. Conversely, for a short FX forward contract or a long FX put option contract, the market maker would act in the opposite direction i.e., long the spot and borrow the base in the money market. This theory has been applied to all the trades as demonstrated in Figure 1.

	Unexplained $20,114	Day1 PnL ($55,640)	Value and Risk							Contract Details					
Contract #	PnL Explained	PnL	MtM	SpotDelta (M MXN)	BSGamma	BSVega	Rho USD	Rho MXN	Carry	Trade Type	Notional (M USD)	Notional (M MXN)	Strike	Maturity	Days to Maturity
	$7,594	($27,932)	$6,373,943	(2.17)	181,103.33	531,977.50	$62	($659)	($12,435)						
1	$26,079	$26,074	$100,193,886	0	0.00	0.00	-9,525	0	$5,216	D	100	102	1.00E+15	17-Feb-24	347
2	($1,706,383)	($1,710,391)	($105,211,961)	(1,891)	0.00	0.00	0	10,002	($27,964)	F	(1,876)	(2,073)	1.00E+15	17-Feb-24	347
3	$1,670,951	$1,674,941	$4,460,832	1,880	0.00	0.00	9,523	-9,947	$22,596	F	102	2,062	20.215	17-Feb-24	347
4	($24,761)	($24,757)	($95,132,311)	0	0.00	0.00	5,943	0	($4,952)	D	(95)	(96)	1.00E+15	21-Oct-23	228
5	$1,484,130	$1,486,454	$97,192,189	1,747	0.00	0.00	0	-6,071	$25,833	F	1,737	1,856	1.00E+15	21-Oct-23	228
6	($1,455,245)	($1,457,567)	($2,050,289)	(1,742)	0.00	0.00	-5,926	6,054	($20,821)	F	(96)	(1,850)	19.275	21-Oct-23	228
7	$2,604	$2,604	$10,005,262	0	0.00	0.00	-143	0	$521	D	10	10.03	1.00E+15	28-Apr-23	52
8	($141,825)	($141,873)	($10,229,862)	(184)	0.00	0.00	0	146	($2,719)	F	(183)	(186)	1.00E+15	28-Apr-23	52
9	$165,615	$169,813	$679,965	267	54,098.65	30,965.66	202	-212	($613)	C	22	408	18.528	28-Apr-23	52
10	($32,890)	($32,885)	($120,046,036)	0	0.00	0.00	1,710	0	($6,578)	D	(120)	(120)	1.00E+15	28-Apr-23	52
11	$1,692,583	$1,694,597	$122,071,601	2,194	0.00	0.00	0	-1,739	$32,445	F	2,189	2,224	1.00E+15	28-Apr-23	52
12	($1,695,469)	($1,697,533)	($2,053,287)	(2,241)	0.00	0.00	-1,747	1,777	($26,424)	F	(123)	(2,272)	18.475	28-Apr-23	52
13	($8,213)	($8,212)	($29,978,145)	0	0.00	0.00	2,965	0	($1,643)	D	(30)	(31)	1.00E+15	2-Mar-24	361
14	$500,659	$501,929	$30,636,622	551	0.00	0.00	0	-3,030	$8,143	F	549	607	1.00E+15	2-Mar-24	361
15	($504,862)	($497,995)	$2,833,401	(502)	64,979.14	256,331.64	-3,039	2,761	($10,310)	P	66	1,302	19.729	2-Mar-24	361
16	$8,213	$8,212	$29,978,145	0	0.00	0.00	-2,965	0	$1,643	D	30	31	1.00E+15	2-Mar-24	361
17	($500,659)	($501,929)	($30,636,622)	(551)	0.00	0.00	0	3,030	($8,143)	F	(549)	(607)	1.00E+15	2-Mar-24	361
18	$527,067	$536,225	$3,716,194	650	62,025.54	244,680.20	3,207	-3,573	$3,460	C	63	1,243	19.729	2-Mar-24	361
19	$0	$9,998,575	$9,998,575	0	0.00	0.00	-142	0	$548	D	10	10	1.00E+15	28-Apr-23	52
20	$0	($10,054,216)	($10,054,216)	(181)	0.00	0.00	0	143	($2,672)	F	(181)	(183)	1.00E+15	28-Apr-23	52

Figure 1 - Portfolio 07/03/23

2. Black-Scholes model

Option pricing models are used to determine an option's fair price/theoretical value. The most widely used option pricing model is the Black-Scholes model which assumes that there are no transaction costs, constant volatility and constant interest rates etc. (Black and Scholes 1973). It is important to note that the Black-Scholes formula is predicated on discrepancies between model assumptions and real-world conditions for example, in real-world markets, volatility constantly fluctuates. The equation for a call and put option is as follows:

$$\mathrm{C}(\mathrm{S},\mathrm{K},\mathrm{r_f},q,T) = Se^{-qT} \cdot N(d_1) - N(d_2) \cdot K \cdot e^{-r_f T},$$

$$P\left(S,K,\sigma,r_f,q,T\right) = N(-d_2) \cdot K \cdot e^{-r_f T} - S_0 e^{-qT} \cdot N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r_f + \frac{\sigma^2}{2}\right) \cdot T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T}$$

where S represents the spot price, K is the strike price, q is the interest rate, r_f is the risk-free rate, T is the maturity, $N(x)$ is the standard normal cumulative distribution function and d_1/d_2 is the probability factors.

3. GREEKS

The Greeks enable investors to make more informed decisions on investments and therefore improve their competency to manage risk and determine what positions to hold. They are defined as partial derivates of an option's price with respect to the corresponding factors. These include (1) **Delta** which measures the sensitivity of an option's price to changes in the underlying asset's spot, defined as $\Delta = \frac{\partial K}{\partial s}$. (2) **Gamma** which measures the sensitivity of the option's delta with respect to the underlying asset's spot price, defined as $\tau = \frac{\partial^2 K}{\partial s^2} = 0$. (3) **Vega** which measures the sensitivity of an option's price to changes in implied volatility of the underlying asset, defined as $Vega = \frac{\partial K}{\partial s}$. (4) **Rho** which measures the sensitivity of an options price to changes in the local currency and foreign interest rate, defined as $\rho_{local} = \frac{\partial K}{\partial r_f}; \rho_{foreign} = \frac{\partial K}{\partial q}$. (5) **Theta** which measures the sensitivity of an option's price to changes in time to-maturity, defined as $\theta = \frac{\partial K}{\partial T}$.

Delta Explain	BSGamma Explain	BSVega Explain	RhoUSD Explain	RhoMXN Explain	Theta	Total Explain
$12,963.60	$42,174.92	$22.29	$0.00	$3,985.17	-$51,552.03	$7,594
$0.00	$0.00	$0.00	$0.00	$0.00	$26,078.54	$26,079
-$1,267,131.07	$0.00	$0.00	$0.00	-$299,431.20	-$139,820.77	($1,706,383)
$1,260,181.72	$0.00	$0.00	$0.00	$297,789.02	$112,980.56	$1,670,951
$0.00	$0.00	$0.00	$0.00	$0.00	-$24,761.11	($24,761)
$1,171,689.57	$0.00	$0.00	$0.00	$183,277.19	$129,162.95	$1,484,130
-$1,168,379.88	$0.00	$0.00	$0.00	-$182,759.48	-$104,105.95	($1,455,245)
$0.00	$0.00	$0.00	$0.00	$0.00	$2,604.18	$2,604
-$123,503.48	$0.00	$0.00	$0.00	-$4,726.12	-$13,594.91	($141,825)
$150,002.88	$12,913.83	$22.29	$0.00	$5,739.54	-$3,063.48	$165,615
$0.00	$0.00	$0.00	$0.00	$0.00	-$32,890.23	($32,890)
$1,473,960.87	$0.00	$0.00	$0.00	$56,395.36	$162,226.29	$1,692,583
-$1,505,737.72	$0.00	$0.00	$0.00	-$57,611.18	-$132,120.02	($1,695,469)
$0.00	$0.00	$0.00	$0.00	$0.00	-$8,213.42	($8,213)
$369,297.92	$0.00	$0.00	$0.00	$90,647.22	$40,714.35	$500,659
-$375,969.40	$14,970.79	$0.00	$0.00	-$92,314.86	-$51,548.22	($504,862)
$0.00	$0.00	$0.00	$0.00	$0.00	$8,213.42	$8,213
-$369,297.92	$0.00	$0.00	$0.00	-$90,647.22	-$40,714.35	($500,659)
$397,850.11	$14,290.30	$0.00	$0.00	$97,626.89	$17,300.15	$527,067

Figure 2 - Greeks 07/03/23

When considering the performance of the portfolio, we observe that the Vega and Gamma for all the forward trades are nil, whereas for the options there are values. For example, on March 7th there are Gamma values for all three option contracts of $12,913.83 etc. (see Figure 2). In addition, the Vega is 22.29 for the 9th contract. This is a consequence of the terms of forward contracts which are fixed and are therefore not subject to as many variations and complications as option contracts. Note that the other two option contracts have a Vega of nil, which is due to the volatility remaining constant from March 2nd (see Figure 2). Furthermore, it is important to recognise that an options risk profile is more complex than that of a forward. The pricing formula is non-linear, therefore producing more unexplained PnL as a result of non-linear changes in price and considerable changes in Greeks. In my portfolio, this inconsistency became more pronounced from March 2nd onwards, as evidenced by a noticeable increase of unexplained PnL from $0 to -$6658. This is evidently a consequence of a change in volatility from 13% to 12.8%. By March 7th, the portfolio had an increase of unexplained PnL from -$6,508 to $20,114, suggesting an inadequacy in the models implemented as a market maker. Upon closer inspection, this drastic change occurred due to the interest rates (MXN TIIE) mid changing from 10% to 9.7% (Figure 4 & 5). To reduce the remaining unexplained PnL it is plausible to take higher order derivatives (2nd order, 3rd order etc.) of the Greeks to explain them further.

Another point to consider is that from March 2nd there is a change of 0.1% in the mid LIBOR rate (Figure 3 & 4). On March 7th, although the LIBOR remains the same, the TIIE rate changes by 0.3%, hence there is a disproportionate difference in unexplained PnL as a result of these changes in the interest rates. At face value, this discrepancy in the unexplained appears to be correlated to a larger difference in the TIIE rate, however, this could be attributed to the non-linear nature of the Greeks.

RATES	Bid / Offer		Mid
USD Libor=	1.90%	2.10%	2.00%
MXN TIIE=	9.90%	10.10%	10.00%

Figure 3 – Risk free rates 28/02/23

RATES	Bid / Offer		Mid
USD Libor=	1.80%	2.00%	1.90%
MXN TIIE=	9.90%	10.10%	10.00%

Figure 4 – Risk free rates 02/03/23

RATES	Bid / Offer		Mid
USD Libor=	1.80%	2.00%	1.90%
MXN TIIE=	9.60%	9.80%	9.70%

Figure 5 – Risk free rates 07/03/23

4. Delta Hedging

The investment strategy required delta hedging on all transactions to mitigate potential risk by taking an offsetting position that is opposite to the delta (rate of change of an option's price) of a forward. For option contracts, we do not need to take the opposite position of quoted currency as it may not be exercised. When the spot rate changes ΔS, our base currency value changes $\delta \cdot \Delta S$; our option value changes $\frac{\partial C}{\partial s}\Delta S\ or \frac{\partial P}{\partial s}\Delta S. Hence, \delta = -\frac{\partial C}{\partial s}\ or\ \delta = -\frac{\partial P}{\partial s}\Delta S\ where\ C\ and\ S$ is the call and put.

In the context of the portfolio (only used forward hedges), I had to implement this strategy, for example, on February 17th the client was long 102M USD. To counteract this, I entered an offsetting position of 100M USD in which 1,876 is the hedge in terms of MXN. Then, 102M is the value of the hedge at maturity in USD and 2073 is the value at maturity in MXN calculated using $Notional_{USD/MXN} \cdot$

$\exp\left(\mathrm{r_f/q} \cdot \frac{\mathrm{T}}{365}\right)$ *where* r_f/q depends on whether you are calculating the amount in USD or MXN. Likewise, I submitted an option contract valued at 63M USD and matched this with a hedge of 30M USD under the same principles. These positions controlled the spot delta considerably keeping it within the desired range. When evaluating the option trades, hedging half the amount of the initial trade reduced the spot delta the most. This is a common strategy employed by traders as it limits their exposure to fluctuations in the market (Natenberg 2015). Subsequently, this further mitigates potential risk whilst maintaining the opportunity for potential gains. Moreover, it is pertinent to acknowledge that you can only partially hedge the trades you submit. This can be attributed to various factors, one being the restrictions imposed by the board lot which stipulate that you can only trade in 5M USD increments in the spot market (see illustration in Figure 1). As you get closer to the maturity of a contract, the spot delta shifts dramatically. This is apparent by March 7th when the spot delta had exceeded the ± 100 range. In order to neutralize this, I submitted another hedging position on the trade with the highest delta which was the European Call option maturing on February 28th, 2023. This returned the spot delta to 2.42 (see Figure 1) thus significantly reducing the risk of the portfolio.

5. Bid-Ask Spread

As a market maker, one of the main objectives is to maximise profits made through an increase in the bid-ask spread. For forward contracts, this can be accomplished by adjusting the strike price formula, $Fwd_{bid/offer} = S_{bid/offer} \cdot \frac{\exp(r_q\delta_q)}{\exp(r_b\delta_b)}$ (δ_q *and* δ_b *are the day count fraction,* r_q *and* r_b is the interest rate of the quoted and base currency). This utilises the best bid (if the client is short) and offer (if the client is long) spot rates. The first order submitted by the client on February 17th (see figure 7), instructed the purchase of 102M USD using MXN at the forward rate of 2,062 expiring in 12 months. This is calculated by multiplying the strike (20.215) by the notional amount of the initial trade. The mid spot price was chosen to compute the strike price, consequently, the PnL was -\$531,636. Whereas on February 21st, when the client was in a short position, I chose the best rate available (best bid) to compute the strike price. Although still not being positive, this dramatically increased the PnL to -\$6704 whilst enabling me to raise profits from the spread from 0.001 to 0.212 (see Figure 6). Accordingly, on February 17th it is reasonable to choose the ask spot rate (18.760), as the client is in a long position. In addition to this, as the market maker, you can further increase or decrease (depending on long or short) the bid-ask spread by slightly adjusting these strike values by a nominal amount. For instance, for the forward contract on February 28th, I decreased the strike to 18.475 as opposed to 18.477 which was the initial optimal rate (see Figure 6). However, it is important to recognise that caution should be taken when adjusting the bid and ask prices as the clients may go elsewhere to receive better rates.

Quotes Blotter

	Quote Date	Trade Type	Bid	Offer	Maturity	Client Level	Traded	Ticket #
1	17-Feb-23	1Y Fwd	20.214	20.215	14-Feb-24	20.215	TRUE	3
2	21-Feb-23	8M Fwd	19.225	19.487	21-Oct-23	19.225	TRUE	6
3	28-Feb-23	2M Call	0.125	0.130	28-Apr-23	0.130	TRUE	9
4	28-Feb-23	2M Fwd	18.475	18.579	28-Apr-23	18.475	TRUE	12
5	2-Mar-23	1Y Put	0.127	0.129	2-Mar-24	0.129	TRUE	15
6	2-Mar-23	1Y Call	0.127	0.129	2-Mar-24	0.129	TRUE	18

Figure 6 - Quote Blotter

Figure 8 shows a European call option contract completed on March 2nd in which the client wanted to long 63M USD at the forward rate of 1,243 MXN expiring 366 days, with the strike price quoted at 19.729 (Note that the strike price is equivalent to the forward rate for option contracts). In terms of options, the bid-ask spread is calculated using the 1Y volatility, therefore it is favourable to choose the best bid and ask volatilities. It is not necessary to adjust the volatility spread as a considerable amount of profit is being made from the option premiums of \$3,716,194 etc. (see Figure 1).

Contract Details					
Trade Type	Notional (M USD)	Notional (M MXN)	Strike	Maturity	Days to Maturity
D	100	102	##########	17-Feb-24	365
F	(1,876)	(2,073)	##########	17-Feb-24	365
F	102	2,062	20.215	17-Feb-24	365

Figure 7 - Trades 17/02/23

Contract Details					
Trade Type	Notional (M USD)	Notional (M MXN)	Strike	Maturity	Days to Maturity
D	30	31	1.00E+15	2-Mar-24	366
F	(549)	(607)	1.00E+15	2-Mar-24	366
C	63	1,243	19.729	2-Mar-24	366

Figure 8 – Trades 02/02/23

6. Discussion

On the one hand, the portfolio can be deemed to be successful as demonstrated by the spot remaining within the desired range for all the trades. This is a result of efficient delta hedging strategies in offsetting most of the risks associated with each trade, through effectively counteracting changes in spot delta as contracts get closer to maturity. However, I could have used option hedges as well as forwards to provide more flexibility in offsetting positions. Likewise, as the MTM value is relatively high and positive by March 7th, it implies that the contracts are performing well. This is evident through inspection of the MTM which has increased from -$531,636 on February 17th, to $6,360,443 on March 7th, where there have been consistent profits from February 28th onwards. However, it is important to note that the MTM is subject to change positively or negatively due to fluctuations in market conditions, therefore is not a reliable indicator of portfolio performance. On the other hand, there are improvements that can be made to increase profitability and reduce the risk of the portfolio. These include choosing the best spot rate depending on whether the client is long or short to improve the bid-ask spread to increase my profits. For instance, if I modified the calculation of the strike price, whereby, I replaced the mid spot with the ask (18.76), it would lead to an increase of the MTM to $2,538. To further enhance the bid-ask spread I could make marginal adjustments to the strike of the forward contracts, specifically, increasing it when the client is long and decreasing it when the client is in a short position. Another drawback of the portfolio is to consider the effectiveness of the Black-Scholes model when valuing European option contracts. Despite being the most revered method, it is limited by the assumptions of the model. To combat this, a trader may use Black-Scholes in conjunction with other pricing models such as the "Heston Model" which makes use of stochastic volatility and has the "ability to generate volatility satisfying market conditions" (Ye 2013). Lastly, the non-linear nature of the GREEKS causes huge sums of unexplained PnL as only the first order derivatives have been taken for all of them. As mentioned previously, taking higher order derivatives will reduce this as it provides more information regarding the price of the option.

Conclusion:

In conclusion, by March 7th, one of the primary objectives of keeping the spot delta within the ± 100 range was achieved convincingly showing that the portfolio was effective. Furthermore, it generated some substantial spread and held a high MTM, and therefore relatively stable profits. I recognize that I missed opportunities to optimise profits by taking advantage of more favourable quote rates, particularly, on February 17th. When analysing a portfolio, being able to explain PnL is essential as it enables inadequacies in the portfolio to be identified and finds ways to rectify them. The limitations of the Greeks render the portfolio susceptible to unexplained PnL therefore it is essential to understand its root cause.

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