# Discussion 10: November 11<sup>th</sup>

## **Operational Semantics**

Formal semantics of a PL: Mathematical description of meaning of programs in a PL.

#### **Terminologies in Operational Semantics**

```
A; e1 => v1 A; e2 => v2 v3 is v1 + v2

A; e1 + e2 => v3
```

- Expression: A program that evaluates to a value
- Value: A result of an expression
- Environment: A mapping from variables to values
- Hypothesis: A set of rules that describe the meaning of expressions
- Judgement: A statement with expressions and values (  $e \Rightarrow v$  ). The expression e evaluates to the value v.

#### **Operational Semantics Exercise**

Solve the following problems using operational semantics:

Evaluate the expression using the given hypotheses: A; let y = 1 in let x = 2 in  $x \Rightarrow 2$ 

A(x) = v  $n \Rightarrow n$   $A; x \Rightarrow v$ 

A; e1 => v1 A, x: v1; e2 => v2 A; e1 => v1 A; e2 => v2 v3 is v1 + v2

A; let x = e1 in  $e2 \Rightarrow v2$  A;  $e1 + e2 \Rightarrow v3$ 

### Lambda Calculus

#### Lambda Calculus Terminologies

```
\x. \y. x + y + a
```

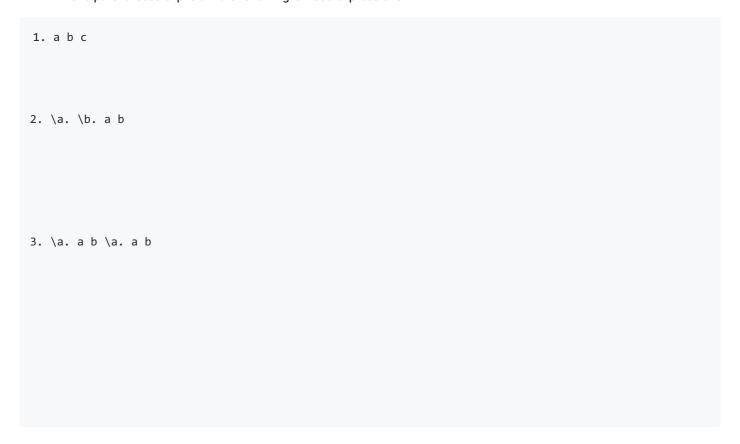
- · Lambda Expression: A lambda expression is a function that takes an argument and returns a value.
- Free Variable: A variable that is not bound by a lambda expression.
- Lambda Abstraction: A lambda abstraction is a function that takes an argument and returns a function.
- Alpha Conversion: A process of renaming a variable in a lambda expression to avoid name conflict. Does not change the meaning of the expression. Do not rename free variables
- Beta Reduction: A process of substituting a lambda expression for a variable in a lambda abstraction.

#### Lambda Calculus Exercise

Things to keep in mind: 1. Alpha conversion: Do not rename free variables 2. Explicit Parentheses: Scope of a variable extends to far right or the first ) seen. 3. Lambda Calculus is left-associative. 4. Beta reduction: Keep applying the functions until you can't anymore.

Solve the following problems using lambda calculus:

1. Make parentheses explicit in the following lambda expressions:



2. Identify the free variables in the following lambda expressions:

