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Question 1

a) $5n^3 + 2n^2 + 3n = O(n^3)$

for $n > n_0$: $5n^3 + 2n^2 + 3n \leq c \cdot n^3$

$5n^3 + 2n^3 + 3n^3 = 10n^3$

for $n > n_0$: $5n^3 + 2n^2 + 3n \leq 10n^3$

$c_1 = 10$

$n_0 = 0$

b) $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

for $n \geq n_0$: $c_2 n \leq \sqrt{7n^2 + 2n - 8} \leq c_1 n$

$\sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n} \leq \sqrt{7n^2 + 2n^2}$

$\sqrt{7n^2 + 2n - 8} \leq \sqrt{9n^2} = 3n$

$\sqrt{7n^2 + 2n - 8} \leq 3n$

$n\sqrt{7} = \sqrt{7n^2} \leq \sqrt{7n^2 + 2n - 8} \leq 3n$

$\sqrt{2n - 8} \geq 0$

$2n - 8 \geq 0$

$\frac{2n}{2} \geq \frac{8}{2}$

$n \geq 4$

$c_1 = 3$

$c_2 = \sqrt{7}$

$n_0 = 4$

for all $n \geq n_0$ we have:

$(\sqrt{7})n \leq \sqrt{7n^2 + 2n - 8} \leq 3n$

c) $d(n) = O(f(n))$ $e(n) = O(g(n))$

for $n > k_0$, $d(n) \leq c_1 \cdot f(n)$

for $n > k_1$, $e(n) \leq c_2 \cdot g(n)$

for $n > k_0 k_1$, $d(n)e(n) \leq (c_1 \cdot f(n))(c_2 \cdot g(n))$

for $n \geq n_0$, $d(n)e(n) \leq c \cdot (f(n)g(n))$

$d(n)e(n) \leq O(f(n)g(n))$

Question 2

1. $O(n^2)$
2. $O(n)$
3. $O(\log_2(n^2))$
4. $O(\log_2(n))$