

# • PROGRAM OF “PHYSICS”

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# **PHYSICS I**

## **(General Mechanics)**

**02 credits (30 periods)**

### **Chapter 1 Bases of Kinematics**

- Motion in One Dimension
- Motion in Two Dimensions

### **Chapter 2 The Laws of Motion**

### **Chapter 3 Work and Mechanical Energy**

### **Chapter 4 Linear Momentum and Collisions**

### **Chapter 5 Rotation of a Rigid Object**

**About a Fixed Axis**

### **Chapter 6 Static Equilibrium**

### **Chapter 7 Universal Gravitation**

# **PHYSICS I**

## **Chapter 2 The Laws of Motion**

Newton's First Law and Inertial Frames

Newton's Second Law

Newton's Third Law

Some Applications of Newton's Laws

The Gravitational Force and Weight

Forces of Friction

Uniform Circular Motion and Nonuniform Circular Motion

Motion in the Presence of Resistive Forces

Motion in Accelerated Frames

# **Classical Mechanics**

Describes the relationship between the motion of objects in our everyday world and the forces acting on them

Conditions when Classical Mechanics does not apply  
very tiny objects (< atomic sizes) → **quantum physics**  
objects moving near the speed of light → **relativity**

# Forces

Usually think of a force as a push or pull

Vector quantity

May be contact or field force

lực trường

## Fundamental Forces

Types

Strong nuclear force

Electromagnetic force \*

Weak nuclear force

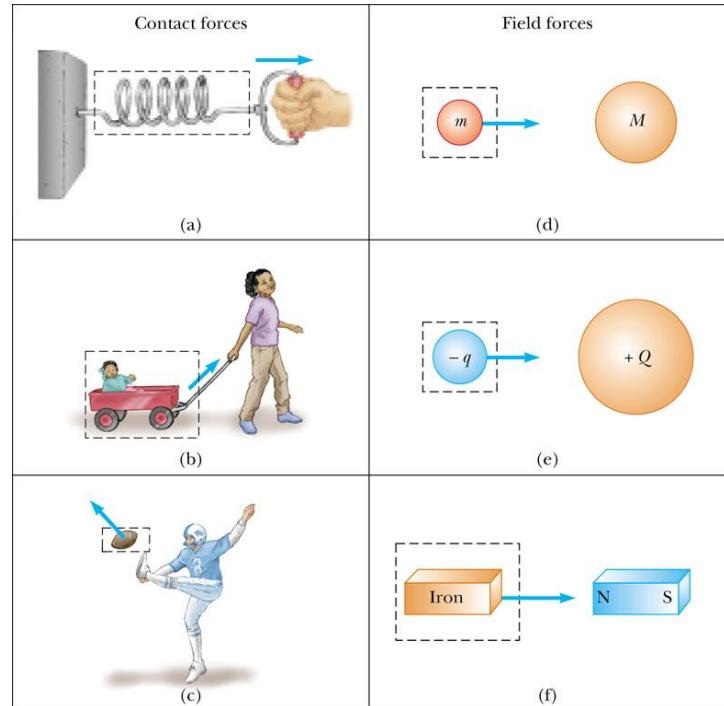
Gravity

Characteristics

All field forces

Listed in order of decreasing strength

Only gravity and electromagnetic in mechanics



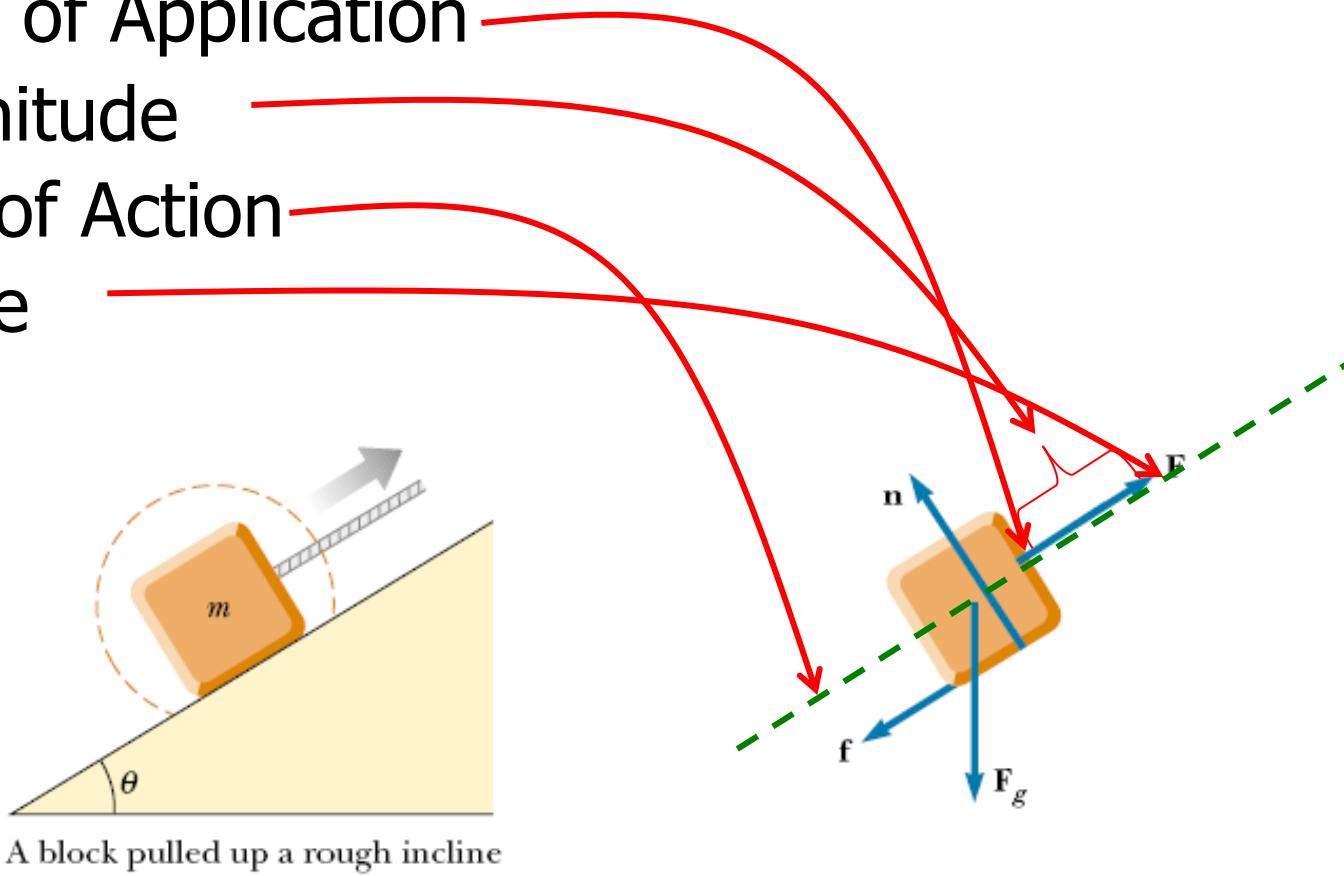
# Force as a vector

## Elements of a Force

Given a single force, one is interested in knowing all of the following:

điểm đặt lực

1. Point of Application
2. Magnitude
3. Line of Action
4. Sense



# **1 Newton's First Law and Inertial Frames**

- "In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line)."
- External force
  - any force that results from the interaction between the object and its environment
- Alternative statement of Newton's 1st Law
  - When there are no external forces acting on an object, the acceleration of the object is zero.

# Inertia and Mass

- Inertia is the tendency of an object to continue in its original motion
- Mass is a measure of the inertia, i.e resistance of an object to changes in its motion due to a force

Recall: mass is a scalar quantity (unit : kilograms-kg)

- **An inertial frame of reference is one that is not accelerating.** by itself: nội tại
- **Mass is an inherent property of an object and is independent of the object's surroundings and of the method used to measure it.**
- **Mass and weight : two different quantities.**  
**( the weight of an object is equal to the magnitude of the gravitational force exerted on the object )**

## 2 Newton's Second Law      net force: hợp lực

- “The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.”

$$\vec{a} = \frac{\sum \vec{F}}{m} \quad (*)$$

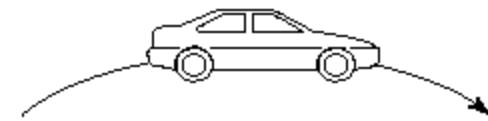
Note:  $\sum \vec{F}$  represents the vector sum of all external forces acting on the object.

(\*) : a vector equation → in terms of components:

$$\sum \vec{F} = m \vec{a} : \begin{cases} F_x = ma_x \\ F_y = ma_y \\ F_z = ma_z \end{cases}$$

# Test 1

A car rounds a curve while maintaining a constant speed. Is there a net force on the car as it rounds the curve?



1. No—its speed is constant.
2. Yes.
3. It depends on the sharpness of the curve and the speed of the car.
4. It depends on the driving experience of the driver.

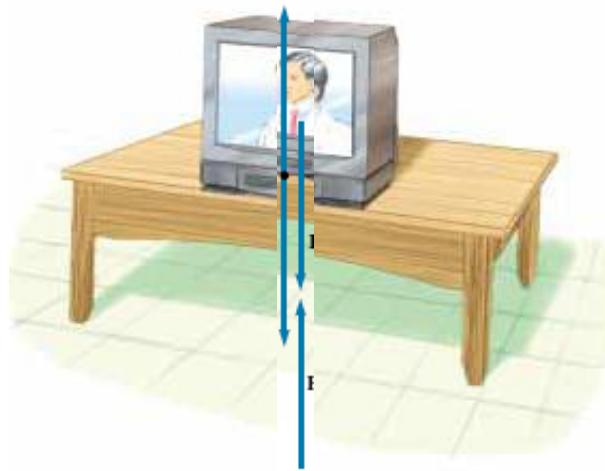
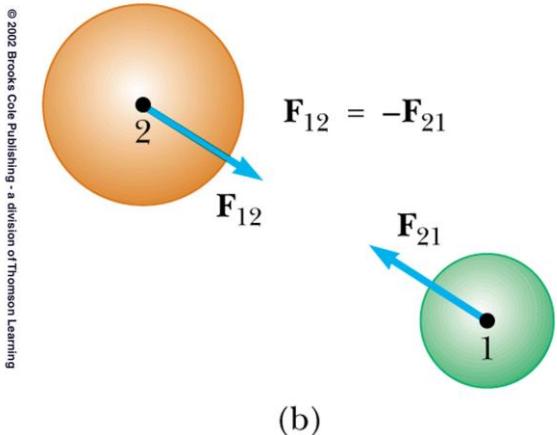
**Note: Acceleration is a change in the speed and/or direction of an object. Thus, because its direction has changed, the car has accelerated and a force must have been exerted on it.**

## 2 Newton's Third Law

- "If two objects interact, the force  $\vec{F}_{12}$  exerted by object 1 on object 2 is equal in magnitude to and opposite in direction to the force  $\vec{F}_{21}$  exerted by object 2 on object 1."

$$\vec{F}_{12} = -\vec{F}_{21}$$

$F_{12}$  may be called the *action* force and  $F_{21}$  the *reaction* force  
The action and reaction forces act on different objects



# Test 2

Consider a person standing in an elevator that is accelerating upward. The upward normal force  $N$  exerted by the elevator floor on the person is

1. larger than
2. identical to
3. smaller than
4. equal to zero, i.e. irrelevant to

the downward weight  $W$  of the person.

Note: In order for the person to be accelerated upward, the normal force exerted by the elevator floor on her must exceed her weight.

# Test 3

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart.

Who moves away with the higher speed?

1. the large man
2. the little boy

Note: Newton's third law : the force exerted by the man on the boy and the force exerted by the boy on the man are an action–reaction pair, and so they must be equal in magnitude → the boy, having the lesser mass, experiences the greater acceleration. Both individuals accelerate for the same amount of time, but the greater acceleration of the boy over this time interval results in his moving away from the interaction with the higher speed.

# **4 Some Applications of Newton's Laws**

## **Assumptions**

Objects behave as particles

→ can ignore rotational motion (for now)

Masses of strings or ropes are negligible

Interested only in the forces acting on the object

→ can neglect reaction forces

## **Free Body Diagram**

Must identify all the forces acting on the object of interest

Choose an appropriate coordinate system

If the free body diagram is incorrect, the solution will likely  
be incorrect

# **STRATEGY**

- 1 Make a sketch of the situation described in the problem, introduce a coordinate frame
- 2 Draw a free body diagram for the isolated object under consideration and label all the forces acting on it
- 3 Resolve the forces into x- and y-components, using a convenient coordinate system
- 4 Apply equations, keeping track of signs
- 5 Solve the resulting equations

## 4.1 The Gravitational Force and Weight

Gravitational Force : Mutual force of attraction between any two objects

Expressed by Newton's Law of Universal Gravitation :

$$F_g = G \frac{m_1 m_2}{r^2}; \quad G = 6.67 \times 10^{-11} \text{ N.m}^2.\text{kg}^{-2}$$

The magnitude of the gravitational force acting on an object of mass  $m$  near the Earth's surface is called the weight  $w$  of the object

$w = mg$  is a special case of Newton's Second Law

$$g = G \frac{M}{r^2}$$

$$g = 6.67 \times 10^{-11} \text{ N.m}^2.\text{kg}^{-2} \times \frac{6.0 \times 10^{24} \text{ kg}}{\{6.4 \times 10^6 \text{ m}\}^2} =$$

## 4.2 Equilibrium

static equilibrium: cân bằng tĩnh

An object either at rest or moving with a constant velocity is said to be in *equilibrium*

The net force acting on the object is zero

$$\sum \vec{F} = 0 \quad \begin{array}{l} \nearrow \\ \searrow \end{array} \quad \begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \end{array}$$

Easier to work with the equation in terms of its components

# Solving Equilibrium Problems

**Make a sketch of the situation described in the problem**

**Draw a free body diagram for the isolated object under consideration and label all the forces acting on it**

**Resolve the forces into x- and y-components, using a convenient coordinate system**

**Apply equations, keeping track of signs**

**Solve the resulting equations**

*Strategy*

# EXAMPLE 1

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of  $37.0^\circ$  and  $53.0^\circ$  with the horizontal.

(a) Find the tension in the three cables.

$$(a) T_3 = F_g = 125 \text{ N}$$

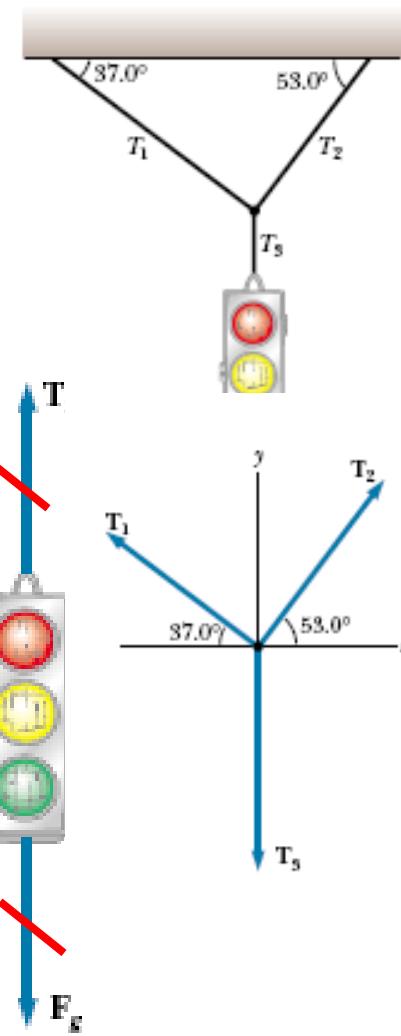
$$(1) \sum F_x = -T_1 \cos 37.0^\circ + T_2 \cos 53.0^\circ = 0$$

$$(2) \sum F_y = T_1 \sin 37.0^\circ + T_2 \sin 53.0^\circ + (-125\text{N}) = 0$$

$$(1) \rightarrow T_2 = 1.33T_1$$

$$(2) \rightarrow T_1 = 75.1 \text{ N}$$

$$(1) \rightarrow T_2 = 99.9 \text{ N}$$



## EXAMPLE 1

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of  $37.0^\circ$  and  $53.0^\circ$  with the horizontal.

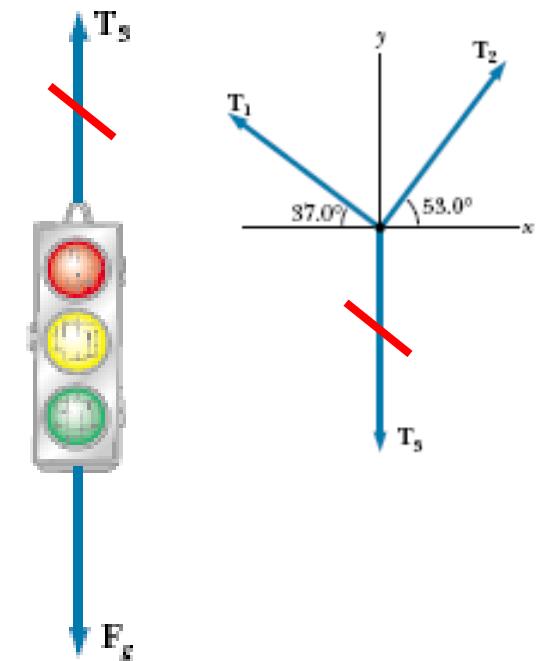
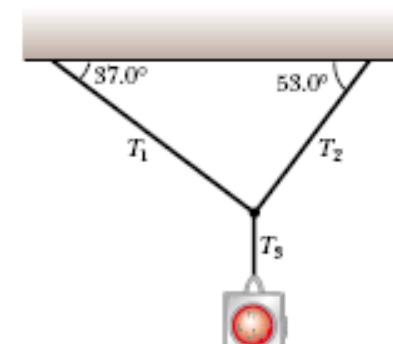
(b) In what situation does  $T_1 = T_2$  ?

$$(b) \quad T_3 = F_g = 125 \text{ N}$$

$$(1) \quad \sum F_x = -T_1 \cos \alpha + T_2 \cos \beta = 0$$

$$(2) \quad \sum F_y = T_1 \sin \alpha + T_2 \sin \beta + (-125\text{N}) = 0$$

$$\alpha = \beta$$



# Newton's Second Law Problems

- ▶ Similar to equilibrium except  $\sum \vec{F} = \vec{ma}$
- ▶ Use components  $\sum F_x = ma_x$ ;  $\sum F_y = ma_y$
- ▶  $a_x$  or  $a_y$  may be zero

## Solving Newton's Second Law Problems

- Make a sketch of the situation described in the problem
- Draw a free body diagram for the isolated object under consideration and label all the forces acting on it
  - If more than one object is present, draw free body diagram for each object
- Resolve the forces into x- and y-components, using a convenient coordinate system
- Apply equations, keeping track of signs
- Solve the resulting equations

## **EXAMPLE 2**

**A child holds a sled at rest on frictionless, snow-covered hill, as shown in figure. If the sled weights 77.0 N, find the force T exerted by the rope on the sled and the force n exerted by the hill on the sled.**



- ▶ Choose the coordinate system with x along the incline and y perpendicular to the incline
- ▶ Replace the force of gravity with its components

Given:

angle:  $\alpha = 30^\circ$

weight:  $w = 77.0 \text{ N}$

Find:

Tension  $T = ?$

Normal  $n = ?$

1. Introduce coordinate frame:

Oy: y is directed perp. to incline

$$F_g = mg$$

Ox: x is directed right, along incline

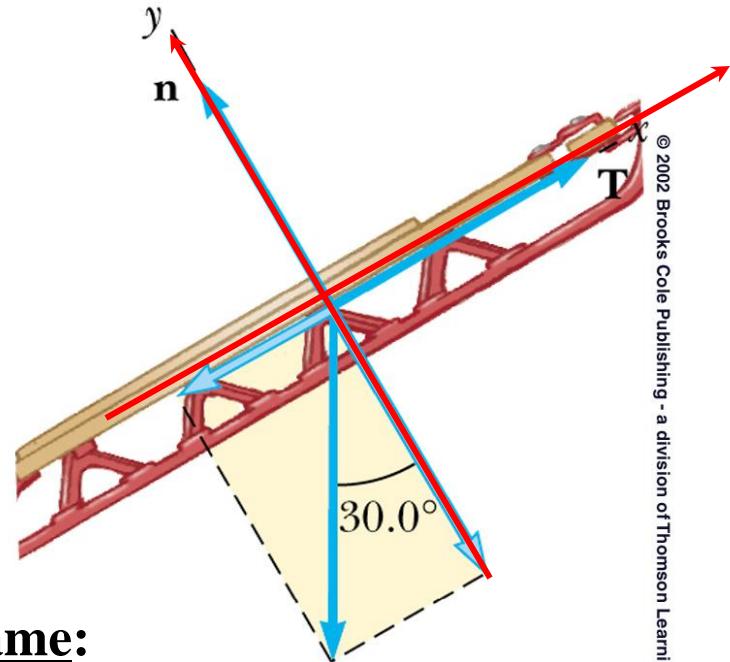
Note:  $\sum \vec{F} = 0$

$$Ox : \sum F_x = T - mg \sin \alpha = 0 ;$$

$$T = mg(\sin 30^\circ) = 77.0 \text{ N} (\sin 30^\circ) = \underline{38.5 \text{ N}}$$

$$Oy : \sum F_y = n - mg \cos \alpha = 0 ;$$

$$n = mg(\cos 30^\circ) = 77.0 \text{ N} (\cos 30^\circ) = \underline{66.7 \text{ N}}$$



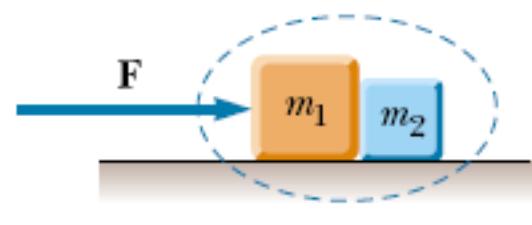
## PROBLEM 1

Two blocks of masses  $m_1$  and  $m_2$  are placed in contact with each other on a frictionless horizontal surface. A constant horizontal force  $F$  is applied to the block of mass  $m_1$ .

(a) Determine the magnitude of the acceleration of the two-block system.

**SOLUTION** (a)

$$\sum F_x = F = (m_1 + m_2)a_x ; \quad a_x = \frac{F}{m_1 + m_2}$$



## PROBLEM 1

Two blocks of masses  $m_1$  and  $m_2$  are placed in contact with each other on a frictionless horizontal surface. A constant horizontal force  $F$  is applied to the block of mass  $m_1$ .

(b) Determine the magnitude of the contact force between the two blocks.

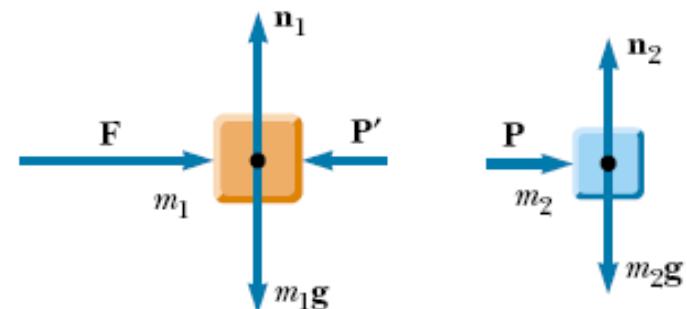
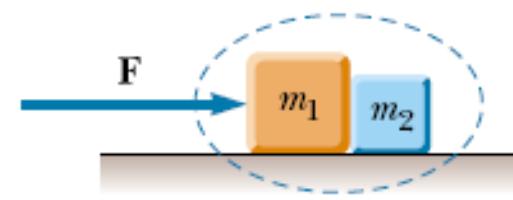
**SOLUTION** (b)

Treat each block separately with its own free-body diagram

$$\text{For } m_2 : \sum F_x = P = m_2 a_x$$

$$\text{For } m_1 : \sum F_x = F - P' = m_1 a_x$$

$$P' = F - m_1 a_x = F - \frac{m_1 F}{m_1 + m_2} = \frac{m_2 F}{m_1 + m_2} \longrightarrow P' = P$$



## PROBLEM 2

Two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass (figure), the arrangement is called an *Atwood machine*. Determine the magnitude of the acceleration of the two objects and the tension in the lightweight cord.

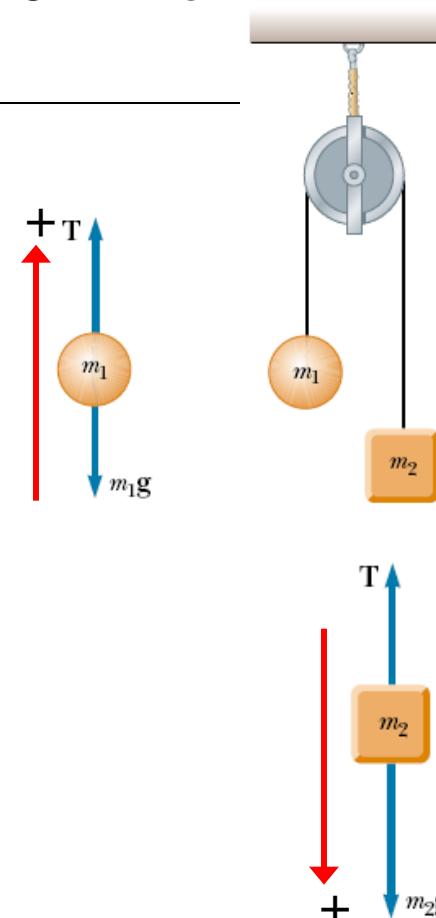
### SOLUTION

$$\text{For } m_1 : \sum F_y = T - m_1 g = m_1 a_y \quad (1)$$

$$\text{For } m_2 : \sum F_y = m_2 g - T = m_2 a_y \quad (2)$$

$$(1) + (2) \longrightarrow a_y = \frac{m_2 - m_1}{m_1 + m_2} g$$

$$(1) \longrightarrow T = \frac{2m_1 m_2}{m_1 + m_2} g$$



## 4.3 Forces of Friction

- ▶ When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion
  - This is due to the interactions between the object and its environment
- ▶ This resistance is called the *force of friction*
- ▶ Friction is proportional to the normal force
- ▶ The force of static friction is generally greater than the force of kinetic friction
- ▶ The coefficient of friction ( $\mu$ ) depends on the surfaces in contact
- ▶ The direction of the frictional force is opposite the direction of motion
- ▶ The coefficients of friction are nearly independent of the area of contact

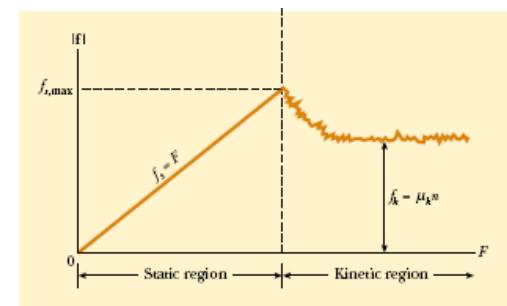
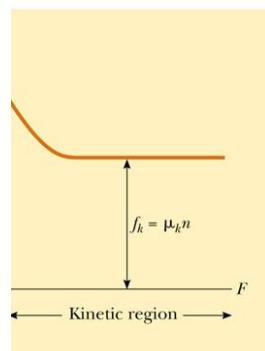
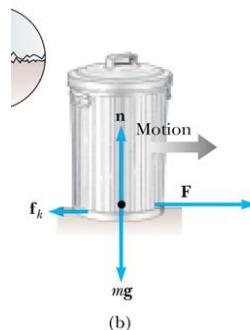
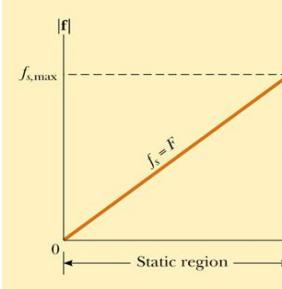
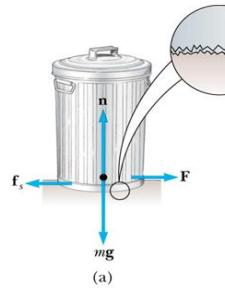
# Static Friction, $f_s$

- ▶ Static friction acts to keep the object from moving
- ▶ If  $F$  increases, so does  $f_s$
- ▶ If  $F$  decreases, so does  $f_s$
- ▶  $f_s \leq \mu n$

# Kinetic Friction, $f_k$

- ▶ The force of kinetic friction acts when the object is in motion

$$f_k = \mu n$$



**TABLE 5.2** Coefficients of Friction<sup>a</sup>

	$\mu_s$	$\mu_k$
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

<sup>a</sup> All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

# Test 4

You are pushing a wooden crate across the floor at constant speed. You decide to turn the crate on end, reducing by half the surface area in contact with the floor. In the new orientation, to push the same crate across the same floor with the same speed, the force that you apply must be about

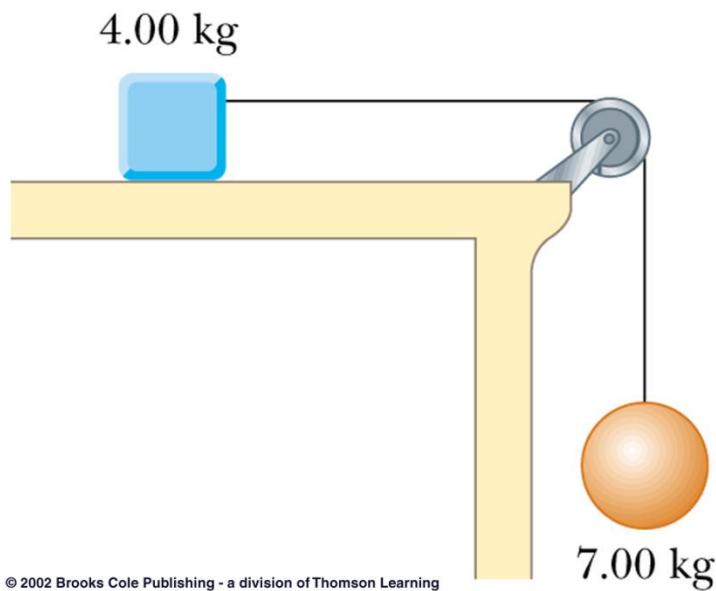
1. four times as great
2. twice as great
3. equally great
4. half as great
5. one-fourth as great

as the force required before you changed the crate's orientation.

Note: The force is proportional to the coefficient of kinetic friction and the weight of the crate. Neither depends on the size of the surface in contact with the floor.

## PROBLEM 3

Two objects  $m_1 = 4.00 \text{ kg}$  and  $m_2 = 7.00 \text{ kg}$  are connected by a light string that passes over a frictionless pulley. The coefficient of sliding friction between the  $4.00 \text{ kg}$  object and the surface is 0.300. Find the acceleration of the two objects and the tension of the string.



Given:

mass1:  $m_1 = 4.00 \text{ kg}$   
mass2:  $m_2 = 7.00 \text{ kg}$   
friction:  $\mu = 0.300$

Find:

Tensions  $T = ?$

Acceleration  $a = ?$

Introduce two coordinate frames:

Oy: y's are directed up

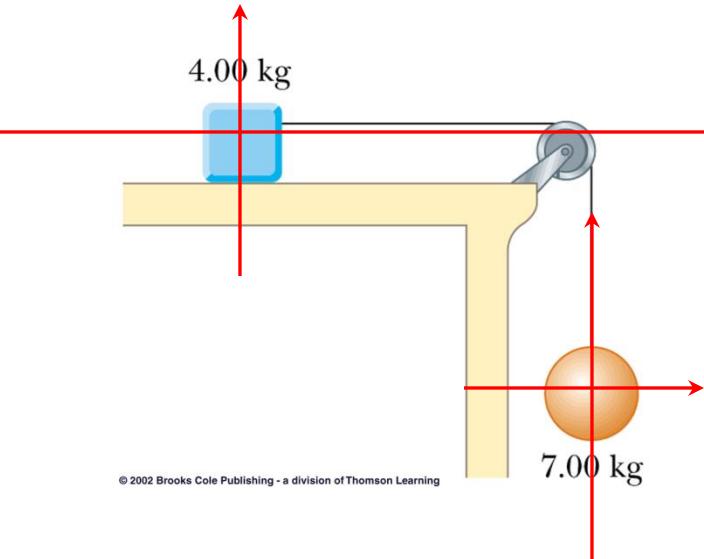
Ox: x's are directed right

Note:  $\sum \vec{F} = m\vec{a}$ , and  $f_k = \mu n$

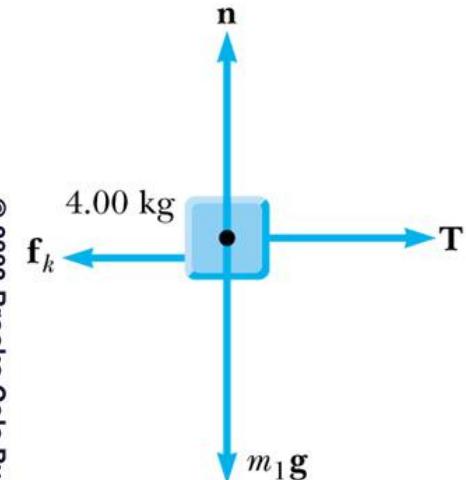
$$\text{Mass1: } O_{x_1}: \sum F_x = T - f_k = m_1 a,$$

$$O_{y_1}: \sum F_y = n - m_1 g = 0.$$

$$\text{Mass2: } O_{y_2}: \sum F_y = m_2 g - T = m_2 a.$$



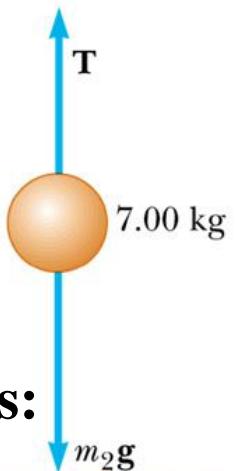
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Solving those equations:

$$a = 5.16 \text{ m/s}^2$$

$$T = 32.4 \text{ N}$$



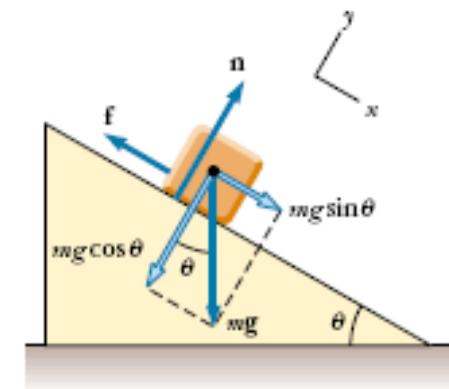
## PROBLEM 4

### Experimental Determination of $\mu_s$ and $\mu_k$

Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in the figure. The incline angle is increased until the block starts to move. Let us show that by measuring the critical angle  $\theta_c$  at which this slipping just occurs, we can obtain  $\mu_s$ .

## SOLUTION

The only forces acting on the block are the force of gravity  $mg$ , the normal force  $n$ , and the force of static friction  $f_s$ .



Static case: (1)  $\sum F_x = mg \sin \theta - f_s = ma_x = 0$

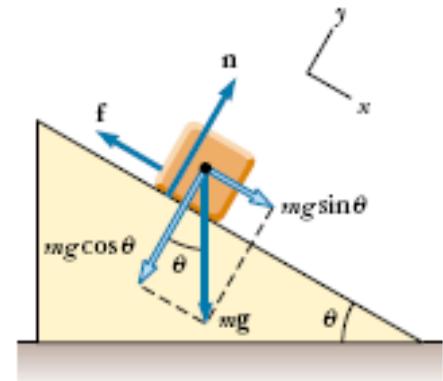
(2)  $\sum F_y = n - mg \cos \theta = ma_y = 0$

$$f_s = mg \sin \theta = \left( \frac{n}{\cos \theta} \right) \sin \theta = n \tan \theta$$

Static case: (1)  $\sum F_x = mg \sin \theta - f_s = ma_x = 0$

(2)  $\sum F_y = n - mg \cos \theta = ma_y = 0$

$$f_s = mg \sin \theta = \left( \frac{n}{\cos \theta} \right) \sin \theta = n \tan \theta$$



At critical angle :  $f_s = f_{SMAX} = \mu_S n$  ;

$$\mu_S n = n \tan \theta ;$$

$$\mu_S = \tan \theta$$

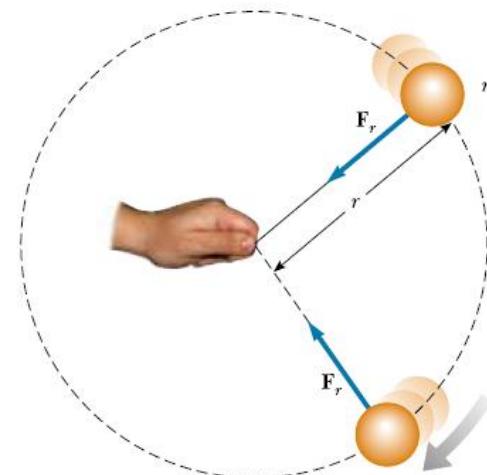
## 4.4 Newton's Second Law Applied to Uniform Circular Motion

A particle moving with uniform speed  $v$  in a circular path of radius  $r$  experiences a *centripetal acceleration*  $a_r$ , that has a magnitude

$$a_r = \frac{v^2}{R} = R\omega^2 = \text{const}$$

Apply Newton's second law along the radial direction, the value of the net force causing the centripetal acceleration :

$$\sum F_r = ma_r = m \frac{v^2}{R}$$



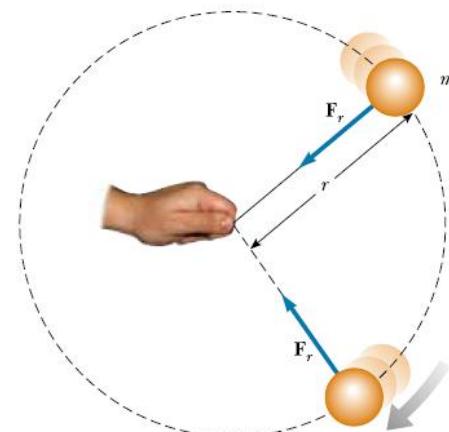
## **EXAMPLE 3**

A ball of mass 0.500 kg is attached to the end of a cord 1.50 m long. The ball is whirled in a horizontal circle as was shown in the figure. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?

$$\sum F_R = T = m \frac{v^2}{r}$$

$$v = \sqrt{\frac{Tr}{m}} ; v_{MAX} = \sqrt{\frac{T_{MAX}r}{m}}$$

$$= \sqrt{\frac{(50.0 \text{ N})(1.50 \text{ m})}{0.500 \text{ kg}}} = 12.2 \text{ m/s}$$

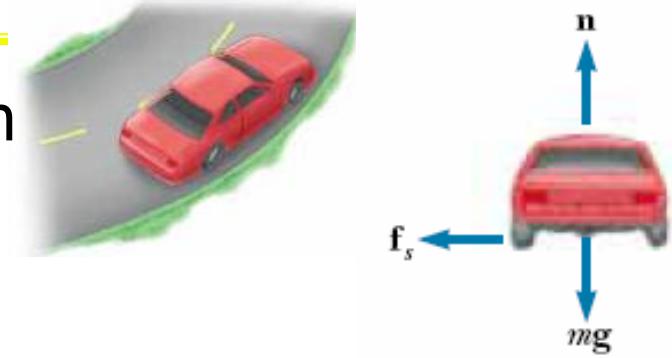


## **EXAMPLE 4**

A 1500-kg car moving on a flat, horizontal road negotiates a curve. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.500, find the maximum speed the car can have and still make the turn successfully.

The force that enables the car to remain in its circular path (centripetal force) is the force of static friction :

$$F_s = m \frac{v^2}{r}$$



The speed the car is maximum : the friction force has its maximum value :

$$\begin{aligned} v_{\max} &= \sqrt{\frac{f_{s,\max} r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r} \\ &= \sqrt{(0.500)(9.80 \text{ m/s}^2)(35.0 \text{ m})} = 13.1 \text{ m/s} \end{aligned}$$

## PROBLEM 5

### The Conical Pendulum

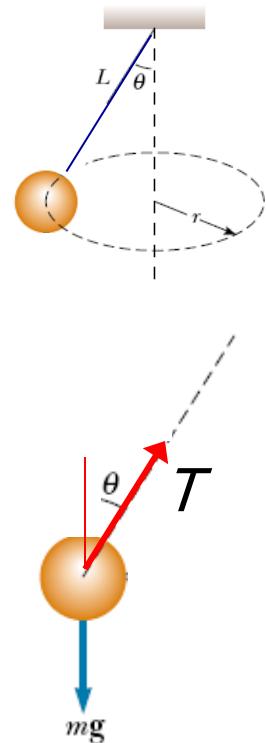
A small object of mass  $m$  is suspended from a string of length  $L$ . The object revolves with constant speed  $v$  in a horizontal circle. The angle made by the string and the vertical is  $\theta$ . Find an expression for  $v$ .

### SOLUTION

$$\begin{aligned}\sum F_Y &= ma_Y = 0 ; T \cos \theta = mg \\ \sum F_R &= T \sin \theta = ma_R ; T \sin \theta = m \frac{v^2}{r} \end{aligned} \quad \left. \tan \theta = \frac{v^2}{rg} \right\}$$

$$v = \sqrt{rg \tan \theta}$$

$$v = \sqrt{Lg \sin \theta \tan \theta}$$



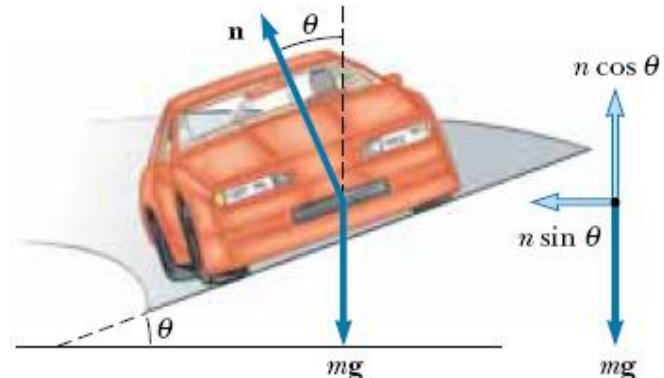
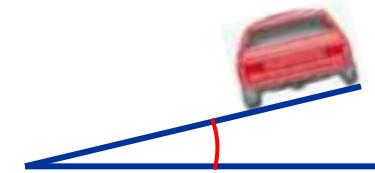
## PROBLEM 6

A car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually *banked*; this means the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s and the radius of the curve is 50.0 m. At what angle should the curve be banked?

## SOLUTION

$$\begin{aligned}\sum F_Y &= ma_Y = 0 ; \quad n \cos \theta = mg \\ n \sin \theta &= m \frac{v^2}{r} \end{aligned}\left. \begin{array}{l} \\ \end{array} \right\} \tan \theta = \frac{v^2}{rg}$$

$$\theta = 20.1^\circ$$



## 4.5 Newton's Second Law Applied to Nonuniform Circular Motion

Acceleration vector :

a radial component vector  $\vec{a}_R$

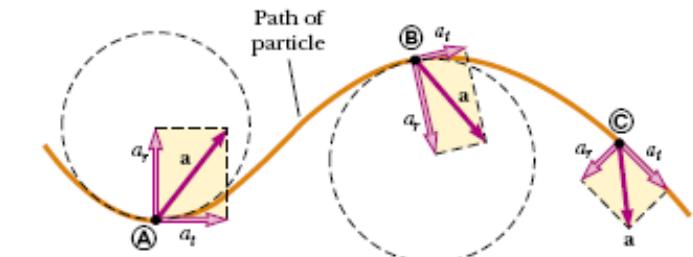
and a tangential component vector  $\vec{a}_T$

$$\vec{a} = \vec{a}_R + \vec{a}_T$$

$$\vec{a}_T = \frac{d|\vec{v}|}{dt}$$

$$\vec{a}_R = \frac{v^2}{R}$$

$$a = \sqrt{a_T^2 + a_R^2}$$



$$\vec{F} = m\vec{a} = m\vec{a}_R + m\vec{a}_T$$

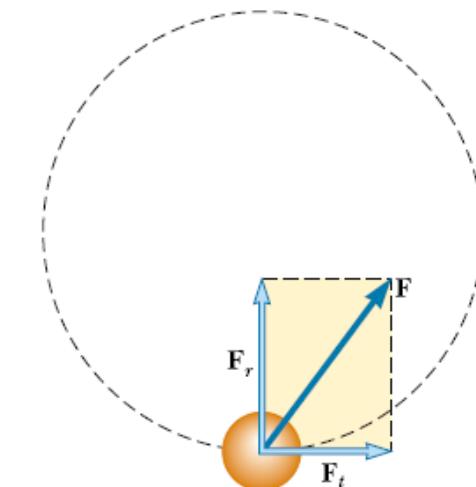
$$\vec{F} = \vec{F}_R + \vec{F}_T$$

$$\vec{F}_R = m\vec{a}_R$$

$$\vec{F}_T = m\vec{a}_T$$

$$F_T = m \frac{d|\vec{v}|}{dt}$$

$$F_R = m \frac{v^2}{R}$$



## **EXAMPLE 5**

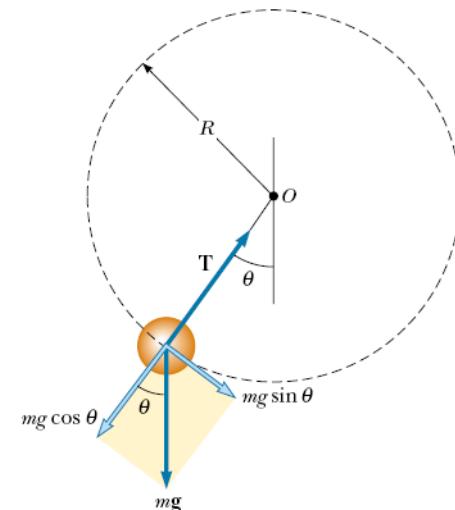
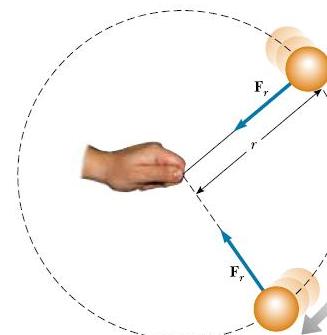
A small sphere of mass  $m$  is attached to the end of a cord of length  $R$  and whirls in a *vertical* circle about a fixed point  $O$ . Determine the tension in the cord at any instant when the speed of the sphere is  $v$  and the cord makes an angle  $\theta$  with the vertical.

$$\sum \vec{F} = m\vec{g} + \vec{T}$$

$$\sum F_t = mg \sin \theta = ma_t ; \quad a_t = g \sin \theta$$

$$\sum F_R = T - mg \cos \theta = ma_r = m \frac{v^2}{R}$$

$$T = m \left( \frac{v^2}{R} + g \cos \theta \right)$$



## **EXAMPLE 5**

A small sphere of mass  $m$  is attached to the end of a cord of length  $R$  and whirls in a *vertical* circle about a fixed point  $O$ . Determine the tension in the cord at any instant when the speed of the sphere is  $v$  and the cord makes an angle  $\theta$  with the vertical.

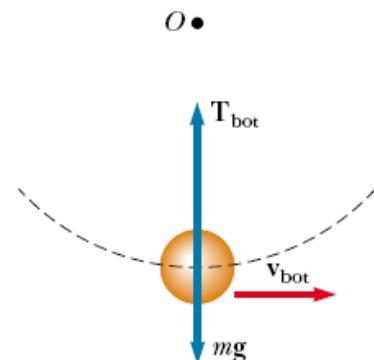
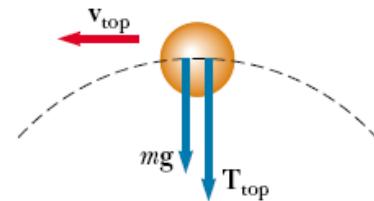
$$T = m \left( \frac{v^2}{R} + g \cos \theta \right)$$

At the top of the path:  $\theta = 180^\circ$

$$T_{top} = m \left( \frac{v_{top}^2}{R} - g \right)$$

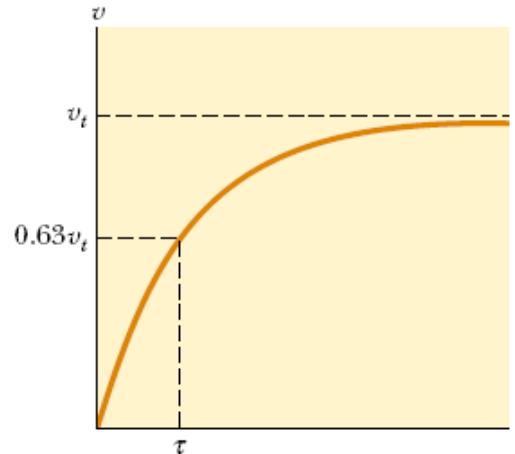
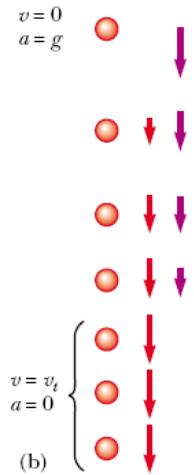
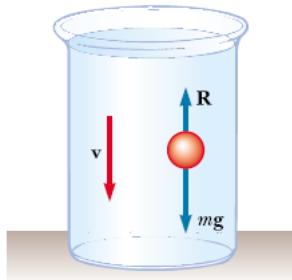
At the bottom:  $\theta = 0^\circ$

$$T_{bot} = m \left( \frac{v_{bot}^2}{R} + g \right)$$



## 4.6 Motion in the Presence of Resistive Forces

- Consider the effect of the interaction between the object and the medium (a liquid or a gas) through which it moves. The medium exerts a resistive force  $R$  on the object moving through it.
- **Examples :** The air resistance associated with moving vehicles (sometimes called *air drag*) and the viscous forces that act on objects moving through a liquid.
- The magnitude of  $R$  depends on such factors as the speed of the object, and the direction of  $R$  is always opposite the direction of motion of the object relative to the medium. The magnitude of  $R$  nearly always increases with increasing speed.



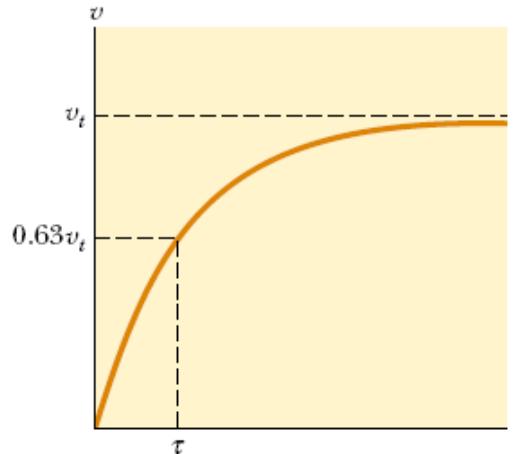
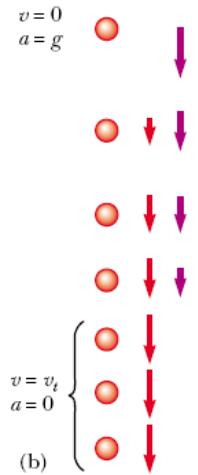
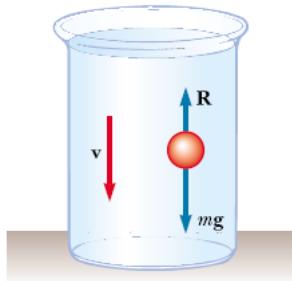
- Assume that the resistive force acting on an object moving through a liquid or gas is proportional to the object's speed :

$$R = bv$$

( $b$  is a constant whose value depends on the properties of the medium and on the shape and dimensions of the object)

- Newton's second law to the vertical motion :

$$\sum F_y = mg - bv = ma$$

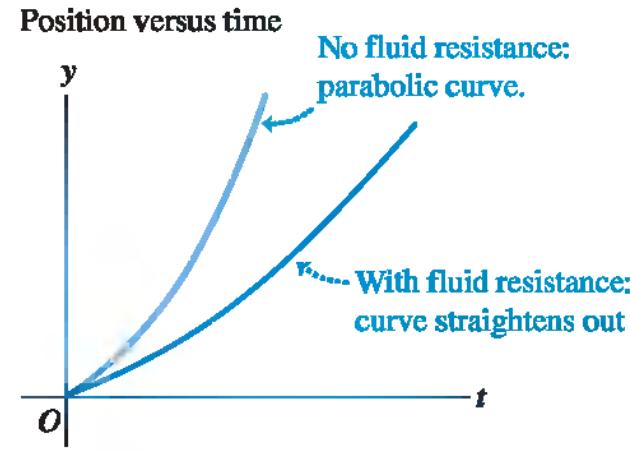
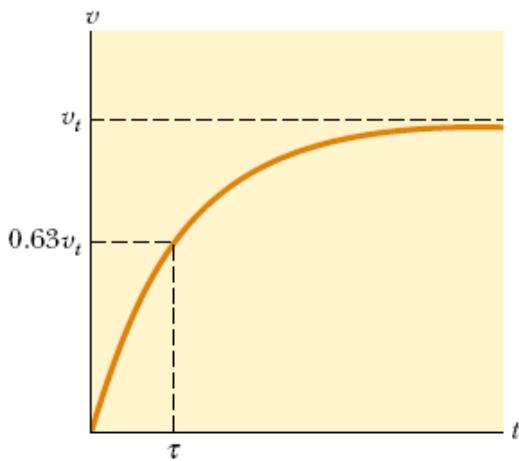
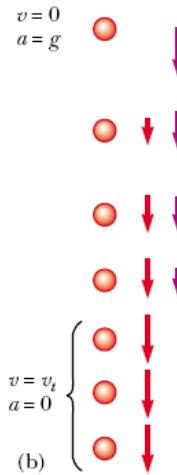


$$\sum F_y = mg - bv = ma = m \frac{dv}{dt}; \quad m \frac{dv}{dt} = -bv + mg \quad (1)$$

*(differential equation)*

When the magnitude of the resistive force equals the object's weight : the sphere reaches its terminal speed  $v_t$  :

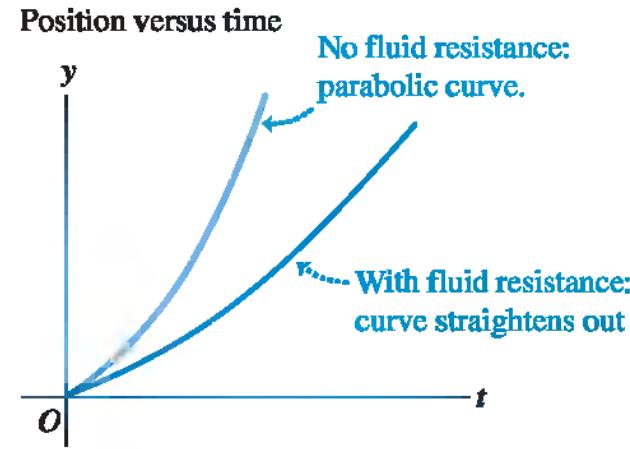
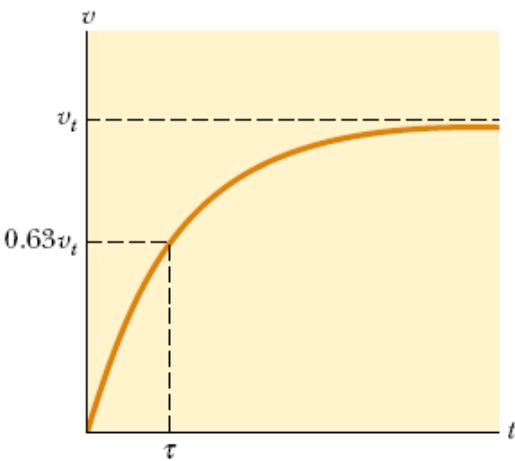
$$(1) \longrightarrow m \frac{dv}{dt} = -bv + bv_t; \quad m \frac{dv}{dt} = -b(v - v_t); \quad \frac{dv}{v - v_t} = -\frac{b}{m} dt;$$



$$\frac{dv}{v - v_t} = -\frac{b}{m} dt ; \quad \int_0^v \frac{dv}{v - v_t} = -\frac{b}{m} \int_0^t dt ; \quad \ln \frac{v - v_t}{v_t} = -\frac{b}{m} t ;$$

$$\frac{v - v_t}{-v_t} = e^{-\frac{b}{m} t} ; \quad v = v_t (1 - e^{-\frac{b}{m} t}) \quad a = \frac{dv}{dt} = g e^{-\frac{b}{m} t}$$

$$y = v_t \left[ t - \frac{m}{b} (1 - e^{-\frac{b}{m} t}) \right]$$



By putting the time constant :  $\tau = \frac{m}{b}$

$$v = v_t (1 - e^{-t/\tau}) \quad a = g e^{-t/\tau}$$

$$y = v_t \left[ t - \frac{m}{b} (1 - e^{-t/\tau}) \right]$$

## **EXAMPLE 6**

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The sphere reaches a terminal speed of 5.00 cm/s. Determine the time constant and the time it takes the sphere to reach 90% of its terminal speed.

$$b = \frac{mg}{v_t} = \frac{(2.00 \text{ g})(980 \text{ cm/s}^2)}{5.00 \text{ cm/s}} = 392 \text{ g/s}$$

$$\tau = \frac{m}{b} = \frac{2.00 \text{ g}}{392 \text{ g/s}} = 5.10 \times 10^{-3} \text{ s}$$

## **EXAMPLE 6**

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The sphere reaches a terminal speed of 5.00 cm/s. Determine the time constant and the time it takes the sphere to reach 90% of its terminal speed.

$$0.900v_t = v_t(1 - e^{-t/\tau})$$

$$1 - e^{-t/\tau} = 0.900$$

$$e^{-t/\tau} = 0.100$$

$$-\frac{t}{\tau} = \ln(0.100) = -2.30$$

$$t = 2.30\tau = 2.30(5.10 \times 10^{-3} \text{ s}) = 11.7 \times 10^{-3} \text{ s}$$

$$= 11.7 \text{ ms}$$

## 5. Motion in Accelerated Frames

- Newton's laws of motion are valid only when observations are made in an **inertial** frame of reference (at rest or moving with constant velocity).
- How an observer in a noninertial frame of reference (one that is accelerating) applies Newton's second law ?
- In a noninertial frame of reference with accelerating  $A$  , we add a **Fictitious Forces (inertial force)**:

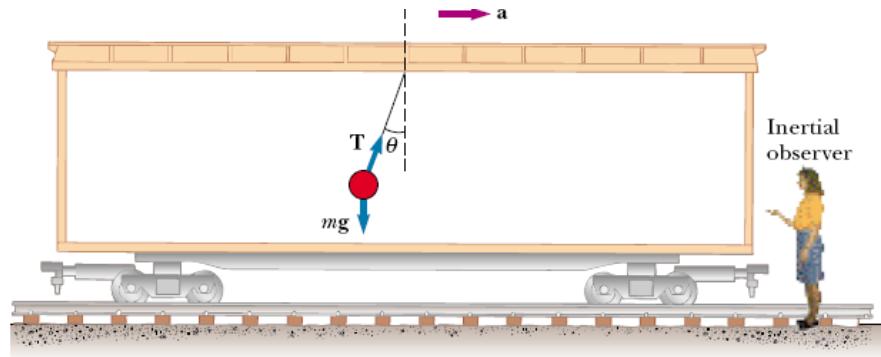
$$- m\vec{A}$$

→ in a noninertial frame of reference, Newton's second law :

$$\vec{ma} = \sum \vec{F} - m\vec{A}$$

## **EXAMPLE 7**

A small sphere of mass  $m$  is hung by a cord from the ceiling of a boxcar that is accelerating to the right with acceleration  $a$ . Find the angle  $\theta$ .



- For the **inertial** observer at rest :

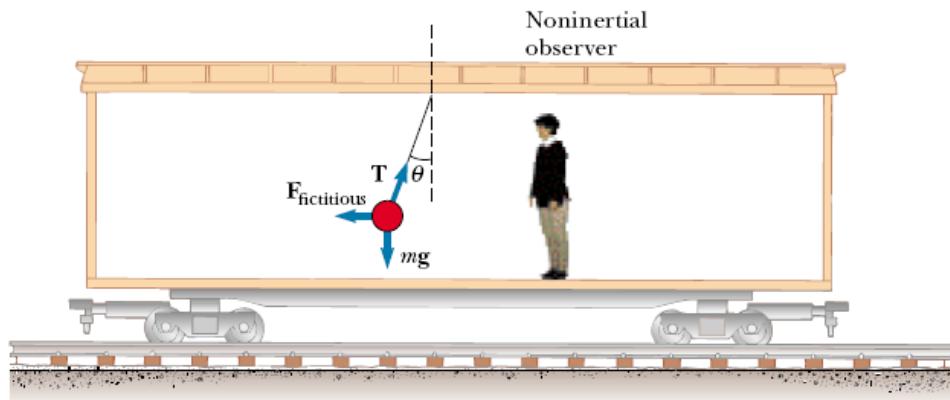
$$\sum F_x = T \sin \theta = ma$$

$$\sum F_y = T \cos \theta - mg = 0$$

$$\tan \theta = \frac{a}{g}$$

## **EXAMPLE 7**

A small sphere of mass  $m$  is hung by a cord from the ceiling of a boxcar that is accelerating to the right with acceleration  $a$ . Find the angle  $\theta$ .



- For the **noninertial** observer riding in the car:

$$\sum F_x = T \sin \theta + F_{\text{FICTIOUS}} = T \sin \theta - (mA) = 0 \quad (1)$$

$$\sum F_y = T \cos \theta - mg = 0$$

$$(1) \longrightarrow T \sin \theta - (ma) = 0 ; T \sin \theta = ma ; \tan \theta = \frac{a}{g}$$

## PROBLEM 7

A person stands on a scale in an elevator. As the elevator starts, the scale has a constant reading of 591 N. As the elevator later stops, the scale reading is 391 N. Assume the magnitude of the acceleration is the same during starting and stopping, and determine the person's mass and the acceleration of the elevator.

## SOLUTION

For the **noninertial** observer in the elevator :

- When the elevator is starting:

$$m(g - a) = n = 591 \text{ N}$$

- When the elevator is stopping :

$$\sum F_y' = mg - n' + (-ma') = mg - n' + (+ma) = 0 ;$$

$$\rightarrow a = 2.00 \text{ m/s}^2 ; \quad m = 50.1 \text{ kg}$$