THE INTERNATIONAL UNIVERSITY – VIETNAM NATIONAL UNIVERSITY – HCMC

CALCULUS 2 – FINAL EXAMINATION

Semester 2, 2023-2024 • Date: June, 2024 • Duration: 120 minutes

SUBJECT: Calculus 2	
Department of Mathematics	Lecturers
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INSTRUCTIONS:

Each student is allowed a scientific calculator and a maximum of TWO double-sided sheets of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.

Each question carries 10 points.

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Question 1. The flow of heat along a thin conducting rod is governed by the one-dimensional *heat equation*

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2},$$

where $u(x,t)$ is the temperature at a location x on the rod at time t. The positive constant k is related to the conductivity of the material. Find the constant k such that the function

$$u(x,t) = 2e^{-4t}\cos(2x)$$

satisfies the heat equation.

Question 2. Let $f(x,y,z) = x^3\sqrt{y^2+z^2}$. Find the linear approximation of the function $f(x,y,z)$ at the point $(2,3,4)$ and use it to approximate $f(2.01, 3.08, 3.95)$.

Question 3. Find the critical points of

$$f(x,y) = x^4 + 4x^2(y-2) + 8(y-1)^2.$$

Determine for each critical point whether it is a local maximum, local minimum or a saddle point.

Question 4. Use Lagrange multipliers to find absolute minimum and absolute maximum values of the function

$$f(x,y) = x - y, \quad \text{subject to} \quad x^2 + y^2 - 3xy = 20.$$

Question 5. Let D be the planar domain bounded by the parabolas $y = (x-1)^2$ and $x = 1 - y^2$. Find the area of D.

Question 6. Evaluate the line integral $\int_C xy^2\,ds$ where C is the right half of the unit circle centered at the origin.

Question 7. Find the volume of the region S that lies between the paraboloid $z = 24 - x^2 - y^2$ and the cone $z = 2\sqrt{x^2 + y^2}$.

Question 8. Evaluate the triple integral $\iiint_E (x - y)dV$, where E is the solid bounded by the three coordinates planes and the plane $x + y + z = 2$.

Question 9. Find a function $f(x, y)$ so that $\nabla f(x, y) = \langle y\cos(xy), x\cos(xy) + 2y\rangle$.

Question 10. Use Green's Theorem to evaluate

$$\oint_C \sqrt{1 + x^3}\, dx + 2xy\, dy,$$

where C is the triangle from $(0, 0)$ to $(1, 0)$ to $(1, 3)$ to $(0, 0)$.

—END OF THE QUESTION PAPER—