

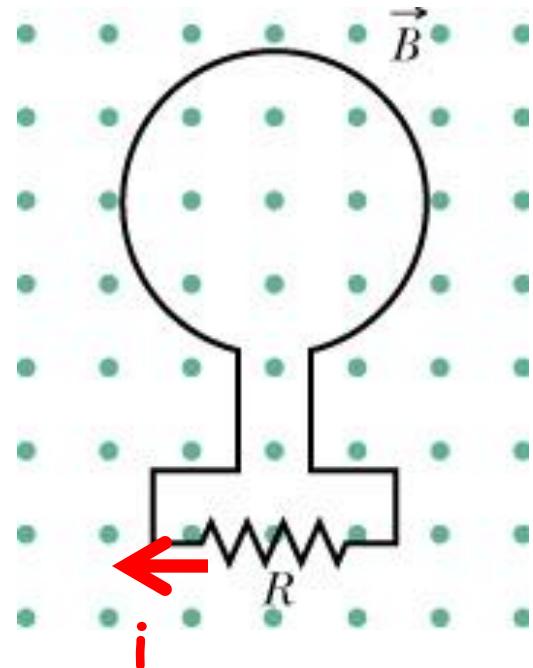
**Homework (lecture 14): 2, 4, 7, 9, 10, 14, 16, 23, 26, 29,
35, 39, 45, 49, 54, 59, 64, 69**

2. In the figure below, the magnetic flux through the loop increases according to the relation $\Phi_B = 6.0t^2 + 7.0t$, where Φ_B is in milliwebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop when $t = 2.0$ s? (b) Is the direction of the current through R to the right or left?

(a) Using Faraday's law: $\varepsilon = -\frac{d\Phi_B}{dt}$

$$|\varepsilon| = (12t + 7) = 31(mV)$$

(b) B increases, using Lenz's law, B_{induced} should be point into the page, so the current through R is to the left



4. A uniform magnetic field B is perpendicular to the plane of a circular loop of diameter 10 cm formed from wire of diameter 2.5 mm and resistivity $1.69 \times 10^{-8} \Omega \cdot \text{m}$. At what rate must the magnitude of B change to induce a 10 A current in the loop?

$$R = \rho \frac{l}{A} = 1.69 \times 10^{-8} \times \frac{2\pi \times 0.05}{\pi \frac{0.0025^2}{4}} = 1.1 \times 10^{-3} (\Omega)$$

$$i = \frac{\mathcal{E}}{R} = -\frac{1}{R} \frac{d\Phi_B}{dt} = -\frac{\pi r^2}{R} \frac{dB}{dt}$$

$$\left| \frac{dB}{dt} \right| = \frac{iR}{\pi r^2} = \frac{10 \times 1.1 \times 10^{-3}}{\pi \times (0.05)^2} = 1.4 (T/s)$$

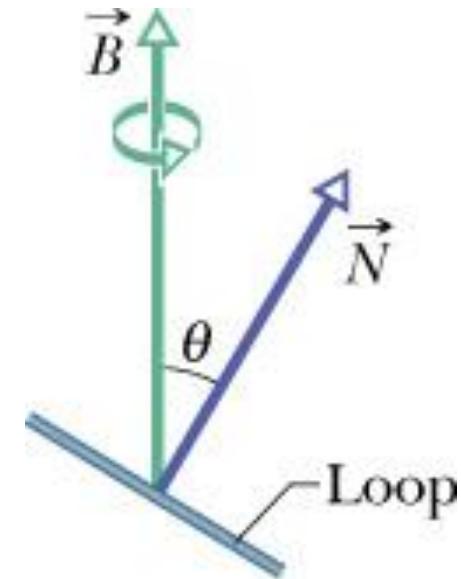
7. In the figure below, a circular loop of wire 10 cm in diameter (seen edge-on) is placed with its normal \vec{N} at an angle $\theta = 30^\circ$ with the direction of a uniform magnetic field \vec{B} of magnitude 0.50 T. The loop is then rotated such that \vec{N} rotates in a cone about the field direction at the rate 100 rev/min; angle θ remains unchanged during the process. What is the emf induced in the loop?

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = B \cdot A \cdot \cos \theta = \text{constant}$$

Therefore:

$$\mathcal{E} = 0$$



9. A square wire loop with 2.00 m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in the figure below. The loop contains an ideal battery with emf $\varepsilon = 20.0$ V. If the magnitude of the field varies with time according to $B = 0.0420 - 0.870t$, with B in tesla and t in seconds, what are (a) the net emf in the circuit and (b) the direction of the (net) current around the loop?

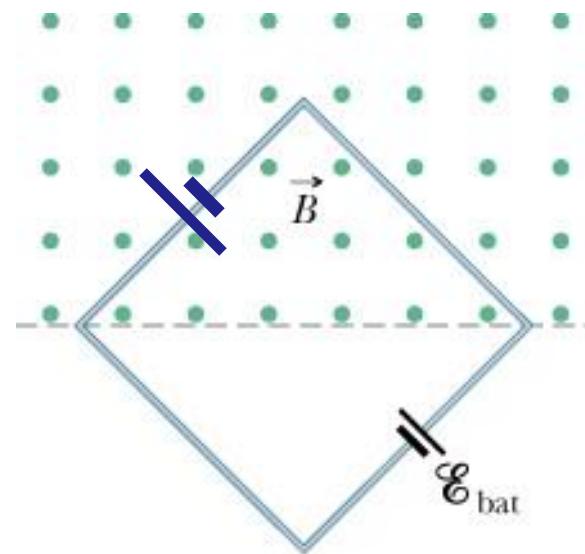
(a) As time goes on, B decreases

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}(-0.87) = 1.74(V)$$

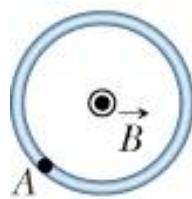
The induced B points out of the page, so the induced current as well as the induced emf is counterclockwise.

(b) The net current is counterclockwise:

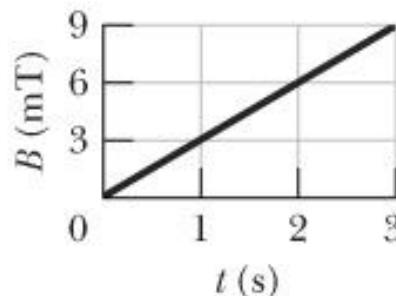
$$\varepsilon_{net} = \varepsilon + \varepsilon_i$$



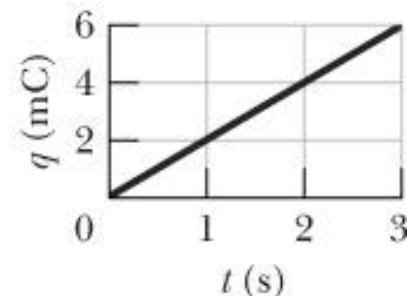
10. In Fig. 30-41a, a uniform magnetic field B increases in magnitude with time t as given by Fig. 30-41b. A circular conducting loop of area $8.0 \times 10^{-4} \text{ m}^2$ lies in the field, in the plane of the page. The amount of charge q passing point A on the loop is given in Fig. 30-41c as a function of t . What is the loop's resistance?



(a)



(b)



(c)

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -A \frac{dB}{dt} = -0.003A \quad (\text{V})$$

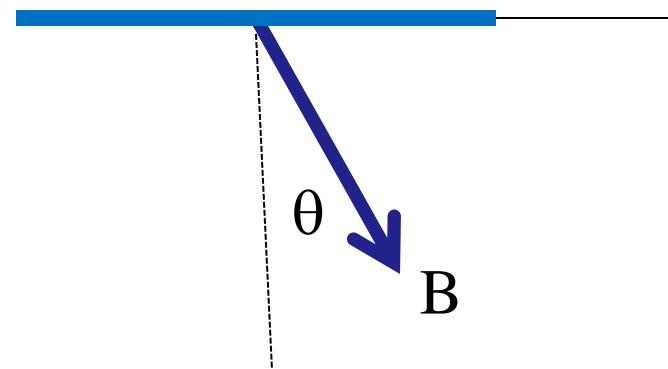
$$i = \frac{dq}{dt} = 0.002 \quad (\text{A})$$

$$\Rightarrow R = \frac{|\mathcal{E}|}{i} = \frac{0.003 \times 8 \times 10^{-4}}{0.002} = 0.0012(\Omega)$$

14. At a certain place, Earth's magnetic field has magnitude $B = 0.590$ gauss and is inclined downward at an angle of 70.0° to the horizontal. A flat horizontal circular coil of wire with a radius of 10.0 cm has 1000 turns and a total resistance of 85.0Ω . It is connected in series to a meter with 140Ω resistance. The coil is flipped through a half-revolution about a diameter, so that it is again horizontal. How much charge flows through the meter during the flip?

$$\mathcal{E} = -\frac{d\Phi_B}{dt}; i = \frac{dq}{dt}$$

$$i = \frac{|\mathcal{E}|}{R} \Rightarrow \frac{dq}{dt} = \left| \frac{\frac{d\Phi_B}{dt}}{R} \right|$$



$$|dq| = \frac{|d\Phi_B|}{R} = \frac{N}{R} (BA \cos \theta - (-BA \cos \theta))$$

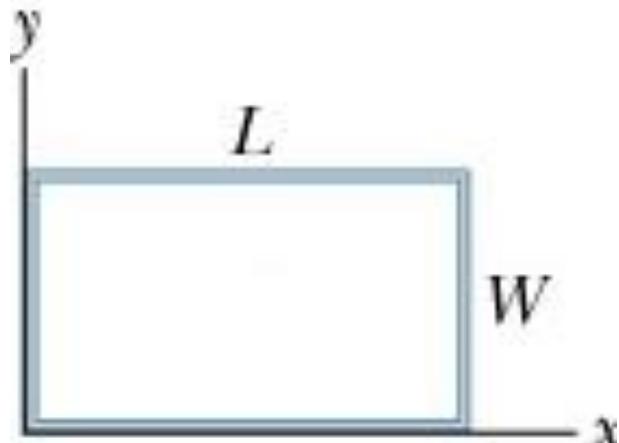
$$|dq| = \frac{2NBA \cos \theta}{R} \quad \text{with } \theta = 20^\circ; \quad 1\text{gauss} = 10^{-4}\text{T}$$

16. In Fig. 30-45, a wire loop of lengths $L = 40.0$ cm and $W = 25.0$ cm lies in a magnetic field B . What are the (a) magnitude ε and (b) direction (clockwise or counterclockwise-or "none" if $\varepsilon = 0$) of the emf induced in the loop if $\vec{B} = (4.00 \times 10^{-2} T/m)\hat{y}\vec{k}$? What are (c) ε and (d) the direction if $\vec{B} = (6.00 \times 10^{-2} T/s)t\hat{k}$? What are (e) ε and (f) the direction if $\vec{B} = (8.00 \times 10^{-2} T/m.s)\hat{y}\vec{t}\hat{k}$? What are (g) ε and (h) the direction if $\vec{B} = (3.00 \times 10^{-2} T/m.s)x\hat{t}\hat{j}$? What are (i) ε and (j) the direction if $\vec{B} = (5.00 \times 10^{-2} T/m.s)y\hat{t}\hat{i}$?

(a) zero; (b) none

$$(c) \varepsilon = -\frac{d\Phi_B}{dt} = -0.06 \times 0.4 \times 0.25 = -0.006(V)$$

(d) dB/dt increases, so the induced emf direction is clockwise



23. As seen in Fig. 30-49, a square loop of wire has sides of length 2.0 cm. A magnetic field is directed out of the page; its magnitude is given by $B = 4.0t^2y$, where B is in teslas, t is in seconds, and y is in meters. At $t = 2.5$ s, what are the (a) magnitude and (b) direction of the emf induced in the loop?

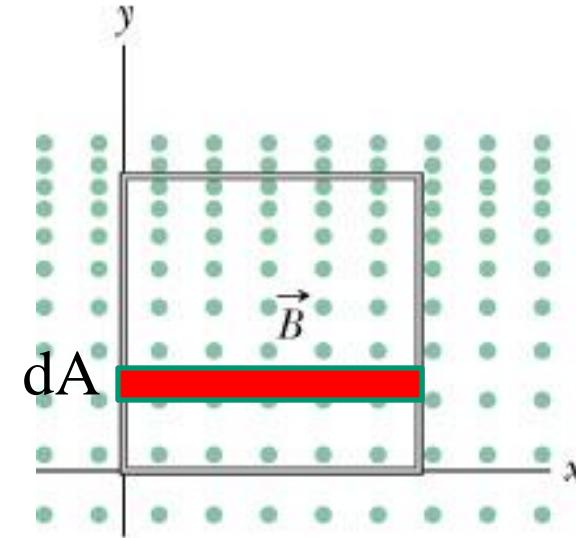
$$(a) \quad \varepsilon = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \int \vec{B} d\vec{A} = \int 4t^2 y (ldy) = 4lt^2 \int_0^l y dy = 2l^3 t^2$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -4l^3 t$$

At $t = 2.5$ s:

$$|\varepsilon| = 4l^3 t = 8 \times 10^{-5} (V)$$



(b) B increases, the current direction is clockwise, so the induced emf direction is clockwise

26. A loop antenna of area 2.00 cm^2 and resistance $5.21 \mu\Omega$ is perpendicular to a uniform magnetic field of magnitude $17.0 \mu T$. The field magnitude drops to zero in 2.96 ms . How much thermal energy is produced in the loop by the change in field?

$$|\varepsilon| = \left| \frac{d\Phi_B}{dt} \right| = \frac{\Delta B \times A}{\Delta t} = \frac{17 \times 10^{-6} \times 2 \times 10^{-4}}{2.96 \times 10^{-3}} = 1.2 \times 10^{-6} (\text{V})$$

$$E_{\text{thermal}} = i^2 R \Delta t = \frac{\varepsilon^2}{R} \Delta t = \frac{1.2^2 \times 10^{-12}}{5.21 \times 10^{-6}} 2.96 \times 10^{-3} = 8 \times 10^{-10} (\text{J})$$

29. In Fig. 30-52, a metal rod is forced to move with constant velocity v along two parallel metal rails, connected with a strip of metal at one end. A magnetic field of magnitude $B = 0.350 \text{ T}$ points out of the page. (a) If the rails are separated by $L = 25.0 \text{ cm}$ and the speed of the rod is 55.0 cm/s , what emf is generated? (b) If the rod has a resistance of 18.0Ω and the rails and connector have negligible resistance, what is the current in the rod? (c) At what rate is energy being transferred to thermal energy?

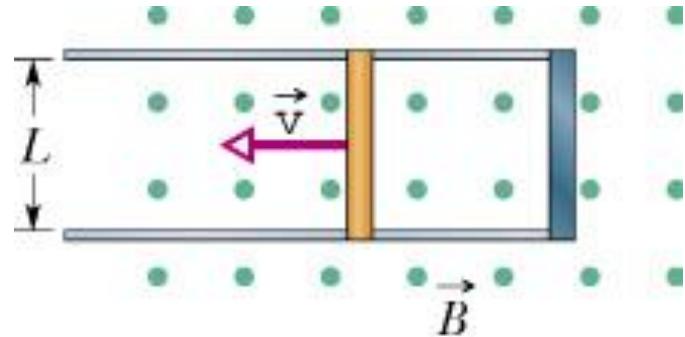
$$(a) \mathcal{E} = \left| -\frac{d\Phi_B}{dt} \right| = \frac{d(BLvt)}{dt} = BvL$$

$$\mathcal{E} = 0.35 \times 0.55 \times 0.25 = 0.048(\text{V})$$

$$(b) i = \frac{\mathcal{E}}{R} = \frac{0.048}{18} = 2.7 \times 10^{-3} (\text{A})$$

using Lenz's law: the current direction is clockwise

$$(c) P = i^2 R = 0.13(\text{mW})$$



35. A long solenoid has a diameter of 12.0 cm. When a current i exists in its windings, a uniform magnetic field of magnitude $B = 30.0 \text{ mT}$ is produced in its interior. By decreasing i , the field is caused to decrease at the rate of 6.50 mT/s . Calculate the magnitude of the induced electric field (a) 2.20 cm and (b) 8.20 cm from the axis of the solenoid (see also Sample Problem 30-4, page 804)

$$(a) \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}; \oint \vec{E} \cdot d\vec{s} = E 2\pi r; \Phi_B = BA = B\pi r^2$$

$$\Rightarrow E = \frac{r}{2} \frac{dB}{dt} = \frac{2.2 \times 10^{-2}}{2} \frac{6.5 \times 10^{-3}}{2} = 7.15 \times 10^{-5} (\text{V/m})$$

$$(b) \oint \vec{E} \cdot d\vec{s} = E 2\pi r; \Phi_B = BA = B\pi R^2$$

$$\Rightarrow E = \frac{R^2}{2r} \frac{dB}{dt} = \frac{6.0^2 \times 10^{-4}}{2 \times 8.2 \times 10^{-2}} \frac{6.5 \times 10^{-3}}{2} = 1.43 \times 10^{-4} (\text{V/m})$$

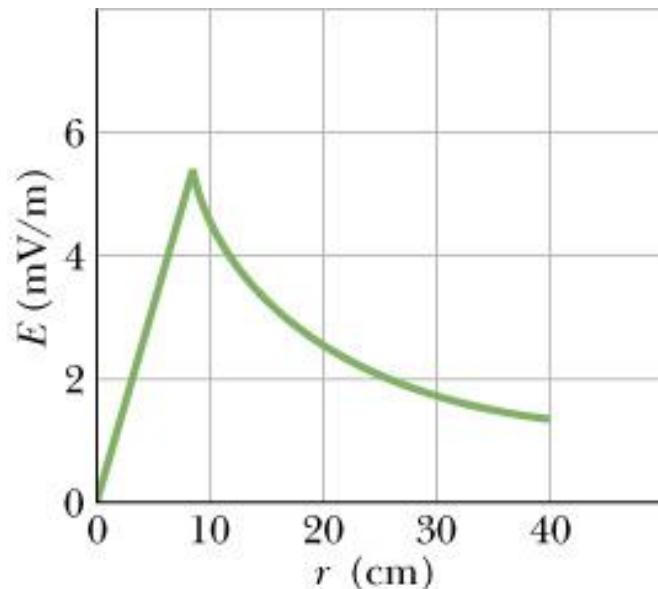


Figure 30-15 of Sample Problem 30-4

39. The inductance of a closely packed coil of 400 turns is 8.0 mH. Calculate the magnetic flux through the coil when the current is 5.0 mA.

- Φ_B : magnetic flux
- $N\Phi_B$: magnetic flux linkage

$$\Phi_B = \frac{Li}{N} = \frac{8.0 \times 10^{-3} \times 5.0 \times 10^{-3}}{400} = 1.0 \times 10^{-7} \text{ (Wb)}$$

45. ***Inductors in series.*** Two inductors L_1 and L_2 are connected in series and are separated by a large distance so that the magnetic field of one cannot affect the other. (a) Show that the equivalent inductance is given by $L_{eq} = L_1 + L_2$. (*Hint:* Review the derivations for resistors in series and capacitors in series. Which is similar here?) (b) What is the generalization of (a) for N inductors in series?

$$\mathcal{E}_L = -L \frac{di}{dt}$$

Consider two inductors in series:

$$\mathcal{E}_{L_1} = -L_1 \frac{di}{dt}; \mathcal{E}_{L_2} = -L_2 \frac{di}{dt}$$

$$\mathcal{E}_{L_{eq}} = -L_{eq} \frac{di}{dt}; \mathcal{E}_{L_{eq}} = \mathcal{E}_{L_1} + \mathcal{E}_{L_2}$$

$$\Rightarrow L_{eq} = L_1 + L_2$$

49. A battery is connected to a series RL circuit at time $t = 0$. At what multiple of τ_L will the current be 0.100% less than its equilibrium value?

$$i = \frac{\mathcal{E}}{R} \left(1 - e^{-t/\tau_L} \right)$$

The equilibrium value:

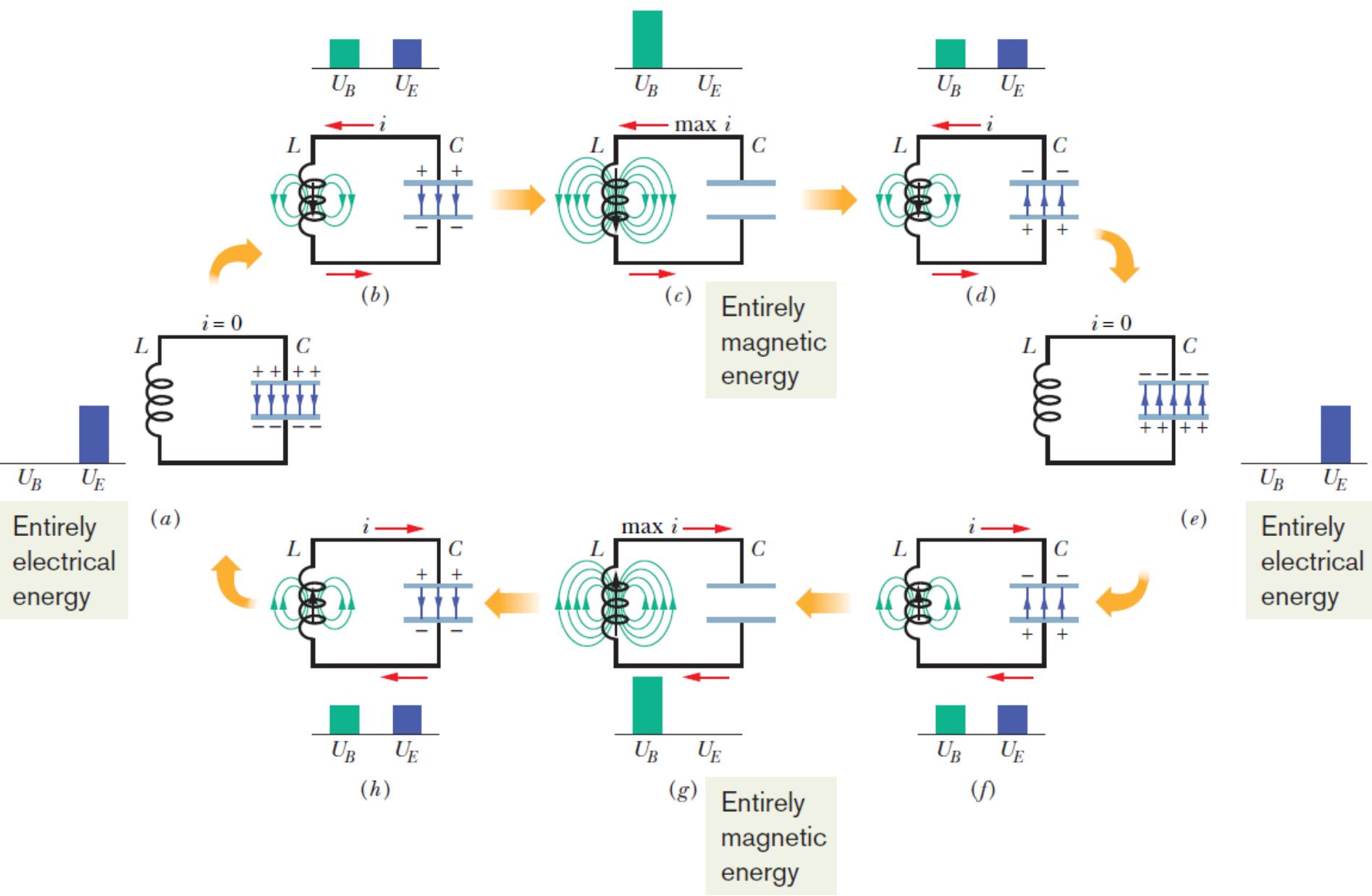
$$i_{equil} = \frac{\mathcal{E}}{R}$$

Time at which:

$$i = 0.999 i_{equil} \Rightarrow 0.999 \frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R} \left(1 - e^{-t/\tau_L} \right)$$

$$t = 6.91\tau_L$$

Homework (lecture 15): 1, 5, 7, 9, 10, 13, 17, 25, 29, 32,
36, 39, 44, 53, 57, 60



1. An oscillating LC circuit consists of a 75.0 mH inductor and a 3.60 μF capacitor. If the maximum charge on the capacitor is 2.90 μC , what are (a) the total energy in the circuit and (b) the maximum current?

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{Li^2}{2}$$

(a) When q is maximum:

$$U = U_{E,\max} = \frac{q_{\max}^2}{2C}$$

(b) i is maximum when $q = 0$:

$$U_{B,\max} = \frac{Li_{\max}^2}{2} = U_{E,\max}$$

5. The frequency of oscillation of a certain LC circuit is 200 kHz. At time $t = 0$, plate A of the capacitor has maximum positive charge. At what earliest time $t > 0$ will (a) plate A again have maximum positive charge, (b) the other plate of the capacitor have maximum positive charge, and (c) the inductor have maximum magnetic field?

$$q = Q \cos(\omega t + \phi)$$

Determine ϕ from the conditions given in the problem, at $t = 0$:

$$q = Q \cos \phi \text{ is maximum, so } \phi = 0$$

(a) So, q is max again as $T = (2\pi/\omega) \times n$

$$T = 2\pi\sqrt{LC} = 1/f = 5 \times 10^{-6} \text{ (s)} = 5(\mu\text{s})$$

(b) plate B has maximum positive charge at:

$$t = \frac{1}{2}T + (n-1)T \Rightarrow t = 2.5(\mu\text{s})$$

(c)

$$U_B = \frac{Q^2}{2C} \sin^2(\omega t + \phi) \Rightarrow U_B \text{ max as } t = \frac{T}{4} + \frac{(n-1)}{2}T$$

$$\Rightarrow t = 1.25(\mu\text{s})$$

7. The energy in an oscillating LC circuit containing a 1.25 H inductor is 5.70 μJ . The maximum charge on the capacitor is 175 μC . For a mechanical system with the same period, find the (a) mass, (b) spring constant, (c) maximum displacement, and (d) maximum speed.

q corresponds to x , $1/C$ corresponds to k ,
 i corresponds to v , and L corresponds to m .

(a) mass $m = 1.25 \text{ kg}$

(b) spring constant $k = 1/C$

$$U = \frac{Q^2}{2C} \Rightarrow C = \frac{Q^2}{2U} = \frac{(175 \times 10^{-6})^2}{2 \times 5.7 \times 10^{-6}} = 2.69 \times 10^{-3} (\text{F})$$

$$k = \frac{1}{2.69 \times 10^{-3}} = 372 (\text{N/m})$$

(c) $x_{\max} = 175 \mu\text{m} = 1.75 \times 10^{-4} (\text{m})$

(d) $\frac{Li^2}{2} = \frac{Q^2}{2C} \Rightarrow i = \frac{Q}{\sqrt{LC}} = 3.02 \times 10^{-3} (\text{A})$

$$v_{\max} = 3.02 \times 10^{-3} (\text{m/s})$$

9. In an oscillating LC circuit with $L = 50 \text{ mH}$ and $C = 4.0 \mu\text{F}$, the current is initially a maximum. How long will it take before the capacitor is fully charged for the first time?

$$i = -I \sin(\omega t + \phi)$$

At $t = 0$, i is max:

$$\phi = \pm\pi/2$$

$$i = -I \sin(\omega t \pm \pi/2)$$

when the capacitor is fully charged, $i = 0$:

$$\omega t = \pi/2 \Rightarrow 2\pi \frac{t}{T} = \frac{\pi}{2} \Rightarrow t = \frac{T}{4}$$

$$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{2\pi\sqrt{50 \times 10^{-3} \times 4 \times 10^{-6}}}{4} = 7 \times 10^{-4} \text{ (s)}$$

10. LC oscillators have been used in circuits connected to loudspeakers to create some of the sounds of electronic music. What inductance must be used with a $6.7 \mu\text{F}$ capacitor to produce a frequency of 10 kHz, which is near the middle of the audible range of frequencies?

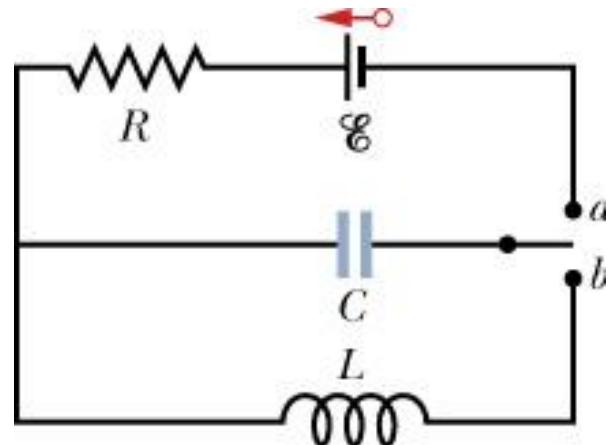
$$T = 2\pi\sqrt{LC} = 1/f \Rightarrow L = \frac{1}{4\pi^2 f^2 C}$$

13. In Fig. 31-25, $R = 14.0 \Omega$, $C = 6.20 \mu\text{F}$, and $L = 54.0 \text{ mH}$, and the ideal battery has emf $\varepsilon = 34.0 \text{ V}$. The switch is kept at a for a long time and then thrown to position b. What are the (a) frequency and (b) current amplitude of the resulting oscillations?

$$(a) T = 2\pi\sqrt{LC} = 1/f \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$$

(b) When the capacitor is fully charged
(the switch is on a):

$$Q = C\varepsilon$$



The maximum current when the switch is on b:

$$I = \omega Q = 2\pi f Q$$

17. In an oscillating LC circuit, $L = 25.0 \text{ mH}$ and $C = 7.80 \mu\text{F}$. At time $t = 0$ the current is 9.20 mA , the charge on the capacitor is $3.80 \mu\text{C}$, and the capacitor is charging. What are (a) the total energy in the circuit, (b) the maximum charge on the capacitor, and (c) the maximum current? (d) If the charge on the capacitor is given by $q = Q\cos(\omega t + \phi)$, what is the phase angle ϕ ? (e) Suppose the data are the same, except that the capacitor is discharging at $t = 0$. What then is ϕ ?

(a) $t = 0$:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{LI^2}{2}$$

(b) the maximum charge:

$$U = U_{E,\max} = \frac{Q^2}{2C} \Rightarrow Q = \sqrt{2CU}$$

(c) the maximum current:

$$U = U_{B,\max} = \frac{LI^2}{2} \Rightarrow I = \sqrt{\frac{2U}{L}}$$

(d) the charge is given:

$$q = Q\cos(\omega t + \phi)$$

At $t = 0$: $q = 3.8 \mu\text{C}$

$$\cos \phi = \frac{q}{Q} \Rightarrow \phi = \pm 47^0$$

The capacitor is charging at $t = 0$:

$$\frac{dq}{dt} = -\omega Q \sin(\omega t + \phi) = -\omega Q \sin \phi > 0$$

$$\phi = -47^0$$

(d) if the capacitor is discharging:

$$\frac{dq}{dt} = -\omega Q \sin(\omega t + \phi) = -\omega Q \sin \phi < 0$$

$$\phi = +47^0$$

25. What resistance R should be connected in series with an inductance $L = 220 \text{ mH}$ and capacitance $C = 12.0 \mu\text{F}$ for the maximum charge on the capacitor to decay to 99.0% of its initial value in 50.0 cycles? (Assume $\omega' \approx \omega$).

$$q = Q e^{-Rt/2L} \cos(\omega't + \phi)$$

$$\omega' = \sqrt{\omega^2 - (R/2L)^2} \quad \omega = 1/\sqrt{LC}$$

We have:

$$\frac{q_{\max}}{Q} = e^{-Rt/2L} = 0.99$$

$$t = 50T = 50 \frac{2\pi}{\omega} = 100\pi\sqrt{LC}$$

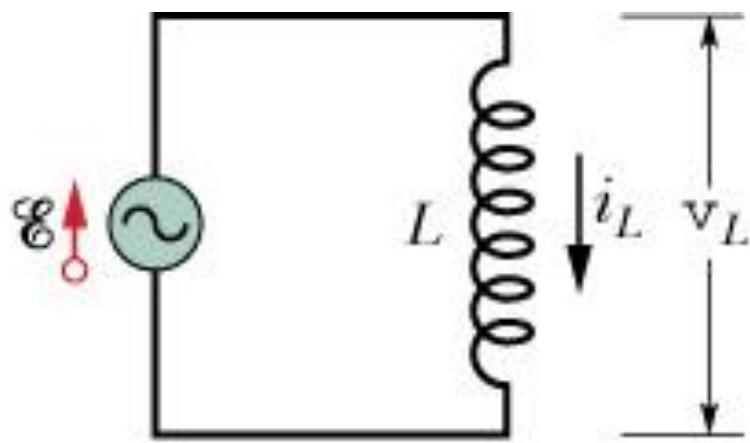
$$-Rt/2L = \ln\left(\frac{q_{\max}}{Q}\right) \Rightarrow R = -\frac{2L}{t} \ln\left(\frac{q_{\max}}{Q}\right)$$

29. A 50.0 mH inductor is connected as in Fig. 31-10a to an ac generator with $\varepsilon_m = 30.0$ V. What is the amplitude of the resulting alternating current if the frequency of the emf is (a) 1.00 kHz and (b) 8.00 kHz?

$$i_L = I_L \sin(\omega_d t - \phi)$$

$$V_L = I_L X_L = \varepsilon_m$$

$$X_L = \omega_d L = 2\pi f L$$



(a)

$$I_L = \frac{\varepsilon_m}{X_L} = \frac{\varepsilon_m}{2\pi f_d L}$$

32. An ac generator has emf $\varepsilon = \varepsilon_m \sin \omega_d t$, with $\varepsilon_m = 25.0 \text{ V}$ and $\omega_d = 377 \text{ rad/s}$. It is connected to a 12.7 H inductor. (a) What is the maximum value of the current? (b) When the current is a maximum, what is the emf of the generator? (c) When the emf of the generator is -12.5 V and increasing in magnitude, what is the current?

$$(a) I_L = \frac{\varepsilon_m}{X_L} = \frac{\varepsilon_m}{\omega_d L} = \frac{25}{377 \times 12.7} = 5.22 \times 10^{-3} \text{ (A)}$$

$$i_L = I_L \sin(\omega_d t - \frac{\pi}{2})$$

$$(b) \text{ when } i_L \text{ is maximum, } \sin(\omega_d t - \frac{\pi}{2}) = 1 \Rightarrow \omega_d t = (2n+1)\pi$$

$$\varepsilon = \varepsilon_m \sin(\omega_d t) = 0$$

$$(c) \sin(\omega_d t) = -\frac{1}{2} \Rightarrow \omega_d t = 2n\pi + \frac{7}{6}\pi \text{ or } \omega_d t = 2n\pi + \frac{11}{6}\pi$$

$$\frac{d\varepsilon}{dt} = \varepsilon_m \omega_d \cos(\omega_d t) < 0 \Rightarrow \omega_d t = 2n\pi + \frac{7}{6}\pi$$

$$i_L = I_L \sin(2n\pi + \frac{2}{3}\pi) = 5.22 \times 10^{-3} \times 0.866 = 4.52 \times 10^{-3} \text{ (A)}$$

36. The current amplitude I versus driving angular frequency ω_d for a driven RLC circuit is given in Fig. 31-26. The inductance is $200 \mu\text{H}$, and the emf amplitude is 8.0 V . What are (a) C and (b) R ?

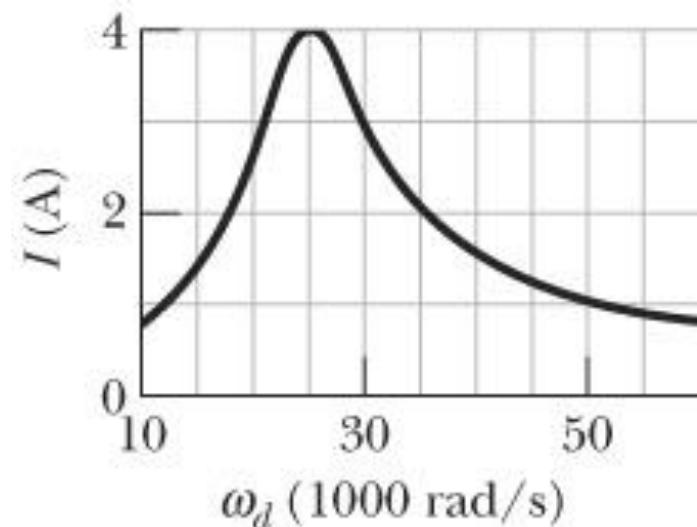
(a) The current I is maximum when:

$$\omega_d = 25 \times 10^3 \text{ (rad/s)}$$

$$\omega_d = \omega = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_d^2 L}$$

(b) At resonance:

$$Z = R$$



$$R = Z = \frac{\mathcal{E}_m}{I} = \frac{8}{4} = 2(\Omega)$$

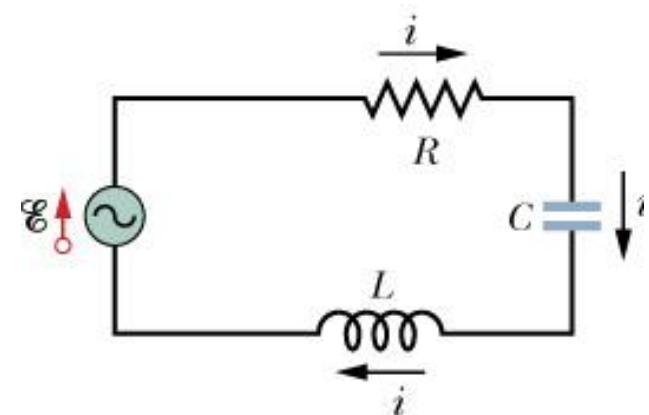
39. In Fig. 31-7, set $R = 200 \Omega$, $C = 70.0 \mu\text{F}$, $L = 230 \text{ mH}$, $f_d = 60.0 \text{ Hz}$, and $\varepsilon_m = 36.0 \text{ V}$. What are (a) Z , (b) ϕ , and (c) I ? (d) Draw a phasor diagram.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan \phi = \frac{X_L - X_C}{R}$$

If $\phi > 0$: ε_m leads the current

If $\phi < 0$: the current leads ε_m



$$I = \frac{\varepsilon_m}{Z}$$

