

**Due date: 3:00 PM, November 05, 2024.**

Submit your solution papers (hard copy) for Q1-Q10 into my mailbox outside the office O2.610 (near the office door of O2.610) and also submit a single-solution-file on Blackboard.

**Instructions:**

- Submit solutions (hard copy) for Questions 1-10. The rest Q11-Q23 are for your practicing and reviewing. These Q11-Q23 are not graded. We will grade questions 1-10 only. The solutions **must be** in the order as listed, that is, Q1, Q2, Q3, ..., Q10.
- The submissions for HW Assignment 2 (in papers) are into my mailbox (outside the office O2.610).

**Exercises for Chapter 2**

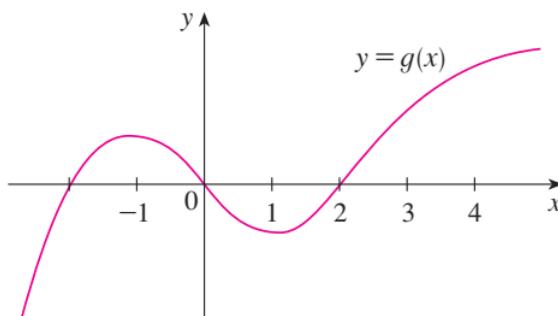
1. Differentiate the following functions

$$(a) f(x) = x^3 \sin(3x^4 + 1) + e^{-4x}.$$

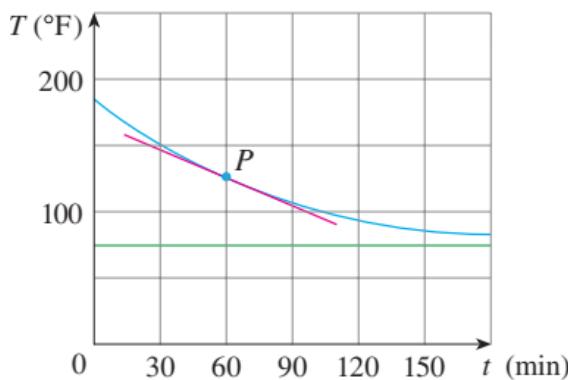
$$(b) f(u) = \frac{1+u^3}{e^{u^4} + \sqrt{u}}.$$

$$(c) y = \sec \theta \tan \theta.$$

2. For the function  $t$  whose graph is given, arrange the following numbers in increasing order and explain your reasoning: 0,  $g'(-2)$ ,  $g'(0)$ ,  $g'(2)$ ,  $g'(4)$ .



3. A roast turkey is taken from an oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F. The graph shows how the temperature of the turkey decreases and eventually approaches room temperature. By measuring the slope of the tangent, estimate the rate of change of the temperature after an hour



4. A ball is thrown vertically upward with an initial velocity 80 ft/s (feet/second), then its height after  $t$  seconds is

$$s = 80t - 16t^2.$$

- (a) At what time does the ball reach the maximum height? What is the maximum height reached by the ball?
- (b) After 2 seconds, what is the height of the ball? Does the ball reach this height at another time?
- (c) What is the velocity of the ball when it is 90 ft above the ground on its way up? on its way down?

5. Find the following limits

(a)  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$ , (b)  $\lim_{x \rightarrow 1} \frac{8^x - 8}{x - 1}$ , (c)  $\lim_{h \rightarrow 0} \frac{\sqrt[4]{16+h} - 2}{h}$ .

Hint: Using the definition of derivative:

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$$

Consequently, one can evaluate a limit above via calculating the derivative  $f'(a)$  with an appropriate choice of  $f(x)$ .

6. Let

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

- (a) Show that  $f(x)$  is continuous everywhere.
- (b) Determine whether  $f'(0)$  exist.

7. Differentiate the following functions

(a)  $f(x) = \sqrt{2x - 1}$ , (b)  $f(x) = \cos\left(\frac{1-e^{2x}}{1+e^{2x}}\right)$ , (c)  $f(x) = (x^3 + x - 1)^{3/2}$ .

8. Use logarithmic differentiation to find the derivative of the function  $f(x) = x^{\cos(\sqrt{2x})}$ ,  $x > 0$ .

9. Given

$$x^2 + 2xy - y^2 + x = 2, \quad (y \neq 0)$$

- (a) Find  $\frac{dy}{dx}$  and  $y'(1)$ .
- (b) Find an equation of the tangent line to the curve at the point  $(1, 2)$ .

10. Find the derivative of the functions [(a) and (b)].

(a)  $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$ , (b)  $y = \sqrt{1 - x^2} \arcsin x$ .

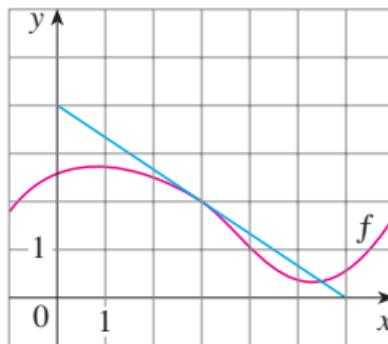
- (c) If  $F(x) = f(g(x))$ , where  $f(-2) = 8$ ,  $f'(-2) = 4$ ,  $f'(5) = 3$ ,  $g(5) = -2$ , and  $g'(5) = 6$ , find  $F'(5)$ .

11. Find  $dy/dx$  by implicit differentiation [(a), (b)]

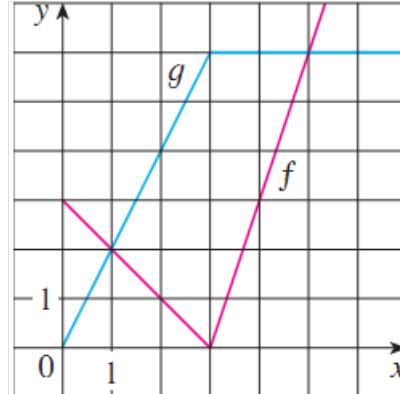
(a)  $\sqrt{xy} = 1 + x^2y$ , (b)  $x \cos y + y \cos x = 1$ ,

- (c) Given  $x \cos(f(x)) + f(x) \cos x = 1$ . Find  $f(0)$  and  $f'(0)$ . Write the equation of the tangent line to the graph of  $y = f(x)$  at the point  $(0, f(0))$ .

12. The cost (in dollars) of producing units of a certain commodity is  $C(x) = 5000 + 10x + 0.05x^2$ .
- Find the average rate of change of  $C(x)$  with respect to  $x$  when the production level is changed from  $x = 100$  to  $x = 105$ .
  - Find the instantaneous rate of change of  $C(x)$  with respect to  $x$  when  $x = 100$ . (This is called the marginal cost.)
13. If  $g(x) = \sqrt{f(x)}$  where the graph of  $f(x)$  is shown, evaluate  $f'(3)$  and  $g'(3)$ .



14. If  $f$  and  $g$  are the functions whose graphs are shown, let  $P(x) = f(x)g(x)$ ,  $Q(x) = \frac{f(x)}{g(x)}$ , and  $C(x) = f(g(x))$ . Find (a)  $P'(2)$ , (b)  $Q'(2)$ , (c)  $C'(2)$ , (d)  $g''(2)$ , and (e)  $(g^{-1})'(2)$ .



15. Under certain circumstances a rumor spreads according to the equation

$$p(t) = \frac{1}{1 + ae^{-kt}},$$

where  $p(t)$  is the proportion of the population that knows the rumor at time  $t$ , and  $a$  and  $k$  are positive constants.

- Find  $\lim_{t \rightarrow \infty} p(t)$ . What does your answer mean?
  - Find the rate of spread of the rumor.
16. Suppose  $f$  and  $f^{-1}$  are both differentiable for all  $x$  with  $f(1) = 2$  and  $f'(1) = -3$ . Find the line tangent to the graph of  $f^{-1}(x)$  at  $(2, 1)$ ?  
Hint: Use derivative of the inverse function.

17. Find the linearization of the function  $f(x) = \tan(x)$  at  $x = \frac{\pi}{4}$  and then use it to approximate  $\tan(46^\circ)$ .

Hint: Note that  $1^\circ = \pi/180$  radian.

18. A particle moves along the x-axis, its position at time  $t$  given by  $x(t) = \frac{t}{1+t^2}, t \geq 0$ , where  $t$  is measured in seconds and  $x$  in meters.

(a) Find the velocity at time  $t$ .

(b) When is the particle moving to the right? When is it moving to the left?

(c) Find the total distance traveled during the first 4 s.

Hint: (c)  $|s(1) - s(0)| + |s(4) - s(1)| = 1/2 + (1/2 - 4/17) = 13/17$ .

19. Consider the following function

$$f(x) = \frac{\sqrt{x+1}(x^2+1)^3}{\sqrt[3]{x^2+2x+8}}, \quad (x > -1).$$

(a) Use logarithmic differentiation to find the derivative.

(b) Find the tangent line to the graph of  $y = f(x)$  at  $(0, 1/2)$ .

20. The radius of a sphere was measured and found to be 21 cm with a possible error in measurement of at most 0.025 cm. What is the maximum error in using this value of the radius to compute the volume of the sphere?

Hint: Use differential.

21. The population of a particular species of insect grows according to the equation

$$P(t) = \frac{5,000}{1 + 4e^{-0.1t}},$$

where  $t$  is measured in weeks.

(a) Find  $\lim_{t \rightarrow \infty} P(t)$ .

(b) Find the rate of growth of the population.

(c) Find the inverse function of  $P(t)$ . Estimate how long it takes for the population to reach 2,000?

22. The cost, in dollars, of producing  $x$  yards of a certain fabric is

$$C(x) = 1200 + 12x - 0.1x^2 + 0.0005x^3$$

a. Find the marginal cost function.

b. Find  $C'(200)$  and explain its meaning. What does it predict?

c. Compare  $C'(200)$  with the cost of manufacturing the 201st yard of fabric.

23. A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420.

a. Find an expression for the number of bacteria after hours.

b. Find the number of bacteria after 3 hours.

c. Find the rate of growth after 3 hours.

Hint:  $P'(t) = kP(t)$ , hence,  $P(t) = P(0)e^{kt}$ , where  $P(0) = 100$ . Use  $P(1) = 100$  to calculate  $k$ .