

Chapter 3: Current and Resistance. Direct Current Circuits

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3.2. Resistance and Resistivity

3.3. Ohm's Law and a Microscopic View of Ohm's Law

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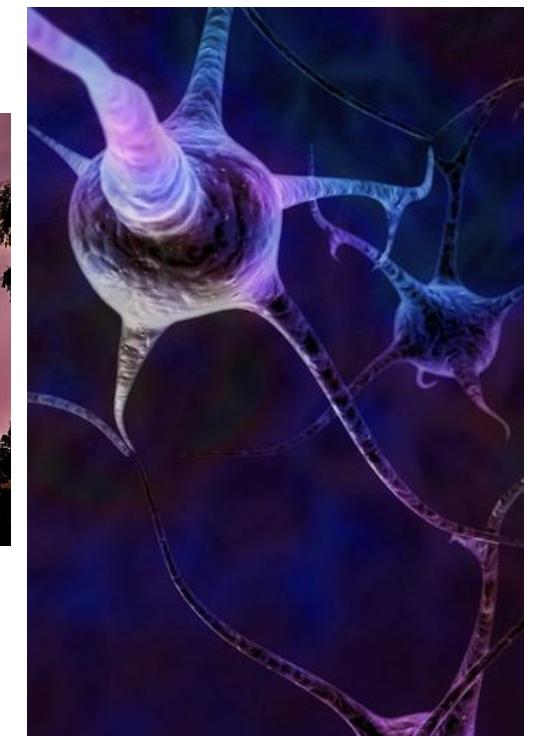
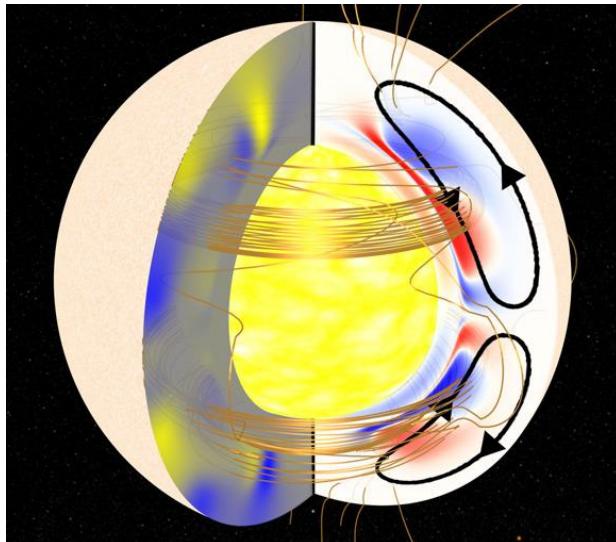
3.8. RC Circuits

Overview

In chapters 1 and 2, we study the behaviour of electric charges at rest - electrostatics. In this chapter, we focus on electric currents - charges in motion.

Examples of electric currents are from large currents in the Sun, in lightning strokes to tiny nerve currents in our muscular activity.

In this chapter, we study only steady currents of conduction electrons moving through metallic conductors.



3.1. Electric Current:

3.1.1. Definition:

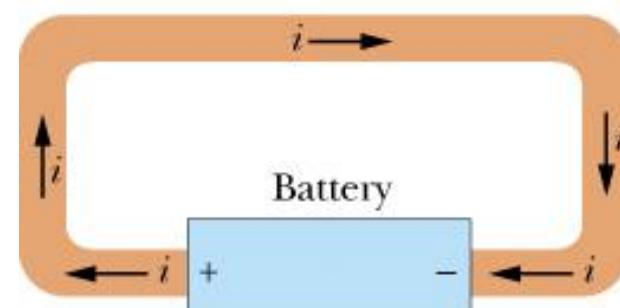
- Figure a shows an isolated conducting loop, all points of the loop have the same potential
- If we insert a battery in the loop (figure b) the loop is no longer at a single potential. Electric fields exert forces on the electrons, causing them to move and thus establishing a current
- In a very short time, the electron flow reaches a constant value and we say "the current is in its steady state", it means it does not vary with time
- The current i is defined:

$$i = \frac{dq}{dt}$$

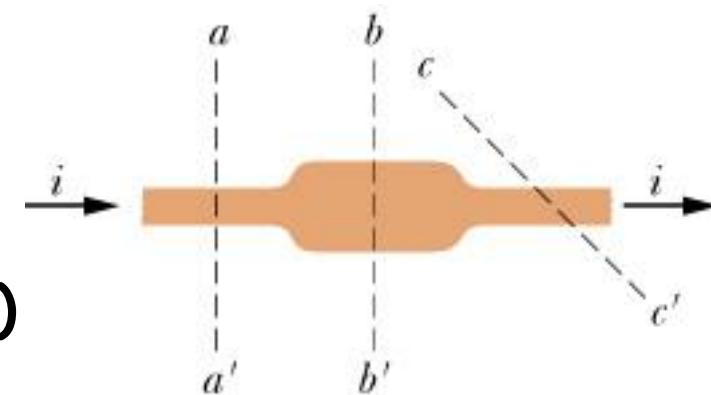
dq : charge passes through a plane (e.g., aa') in time dt



(a)



(b)

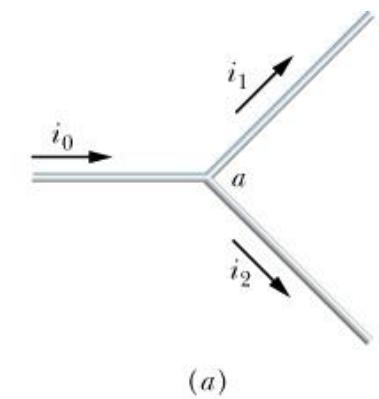


- So, the charge passes through the plane in a time interval t :

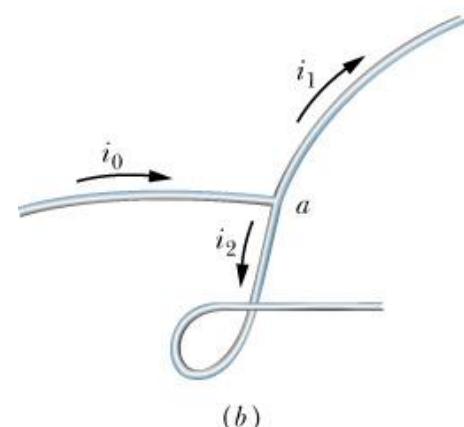
$$q = \int dq = \int_0^t idt$$

- For a steady-state, the current is the same for planes aa', bb', and cc' regardless of their location and orientation.
 - **SI unit:** 1 ampere = 1 A = 1 coulomb per second = 1 C/s
 - If a current i_0 splitting at a junction into two branches, because charge is conserved, therefore:

$$i_0 = i_1 + i_2$$



Note: bending or reorienting the wires in space does not change the validity the equation above

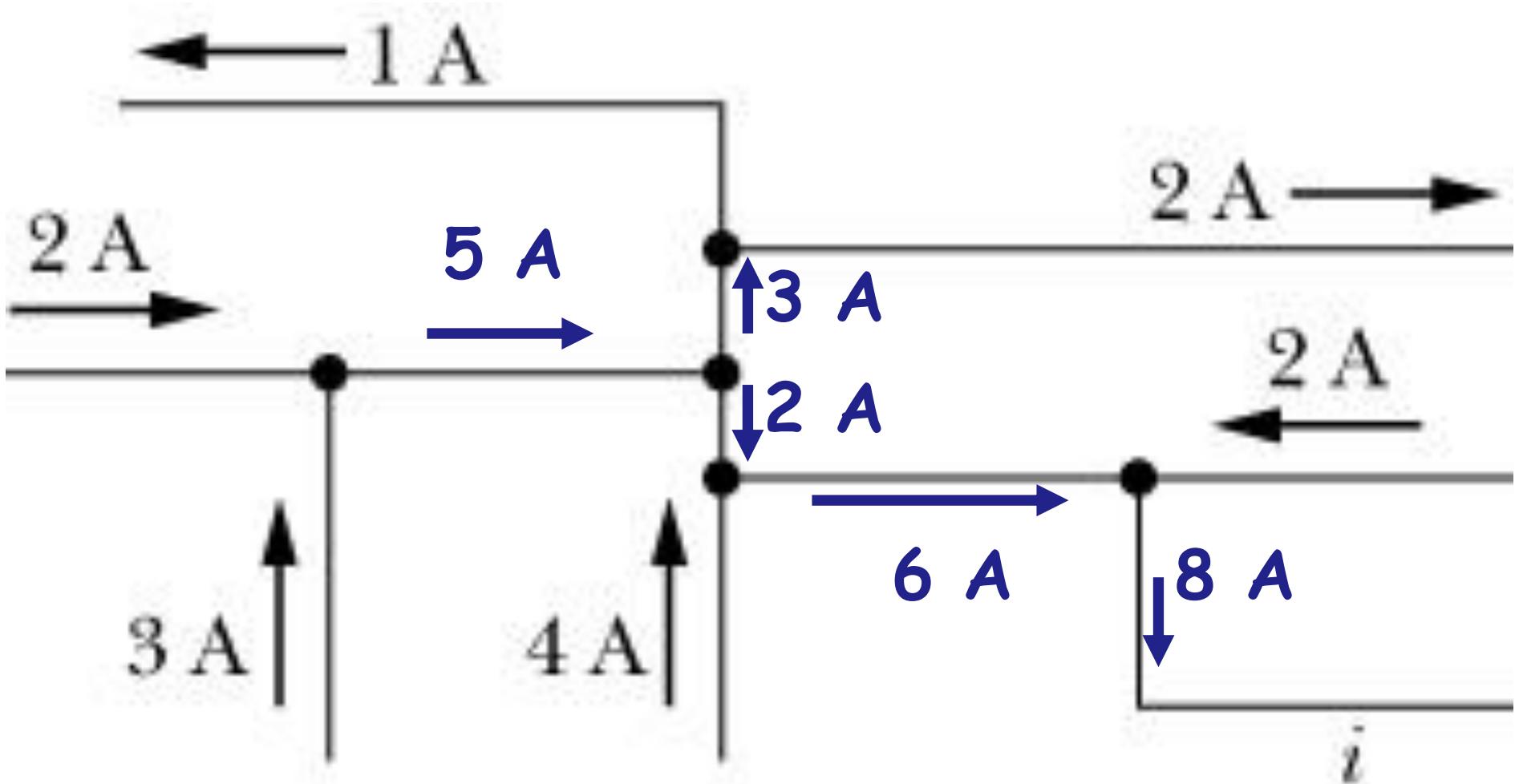


3.1.2. The Directions of Currents:

- We represent a current with an arrow to indicate that charge is moving
- The current arrows show the direction in which positively charged particles, which are called **charge carriers**, move from the positive terminal to the negative terminal of a battery. However, the charge carriers in the wires are electrons (negative charge) and they move in the opposite direction to the current arrows
- But we follow the convention:

A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction

Checkpoint: The figure shows a portion of a circuit. What are the magnitude and direction of the current i in the lower right-hand wire?



$$i_0 = i_1 + i_2$$

3.1.3. Current Density:

$$i = \int \vec{J} d\vec{A}$$

\vec{J} is the current density, has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative

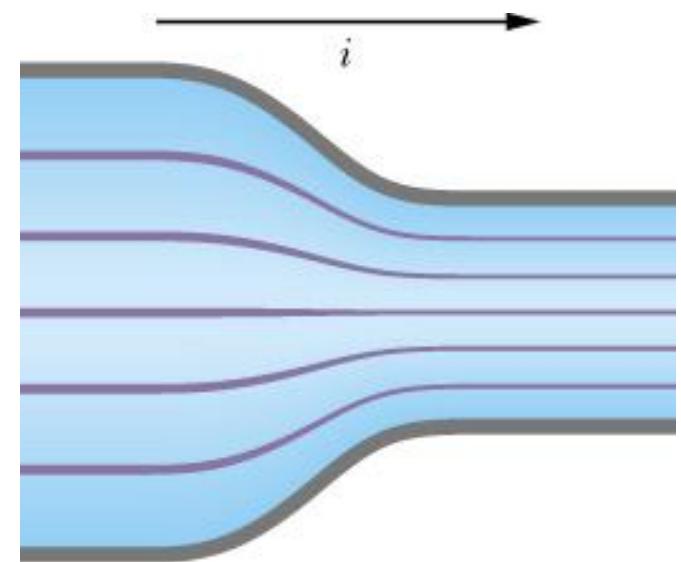
$d\vec{A}$ is the area vector of the element dA

If \vec{J} is uniform and parallel to $d\vec{A}$

$$i = \int J dA = J \int dA = JA$$

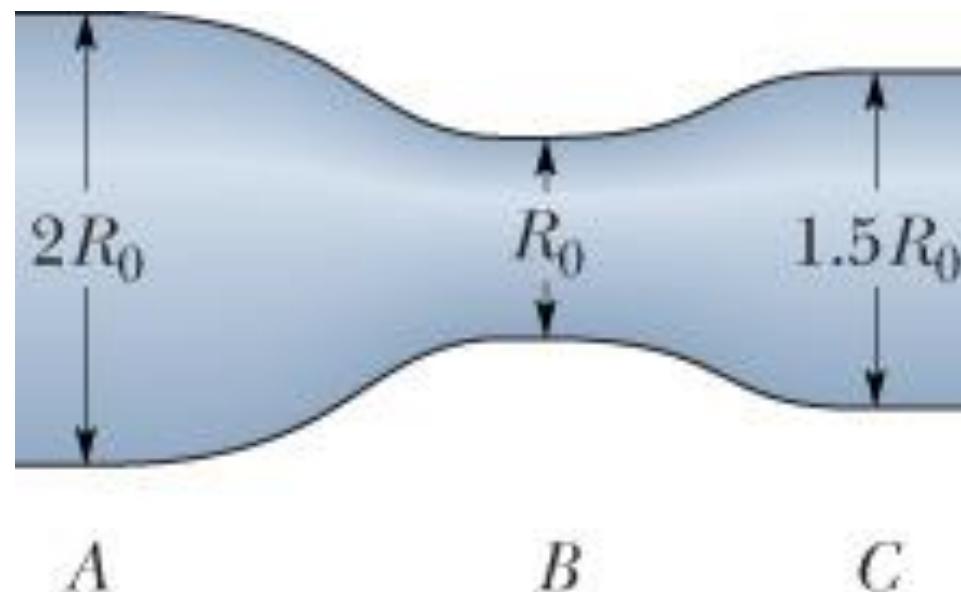
$$J = \frac{i}{A}$$

Note: the current from a wider conductor to a narrower one does not change but the current density does change. It is greater in the narrower conductor



Streamlines represent current density in the flow of charge

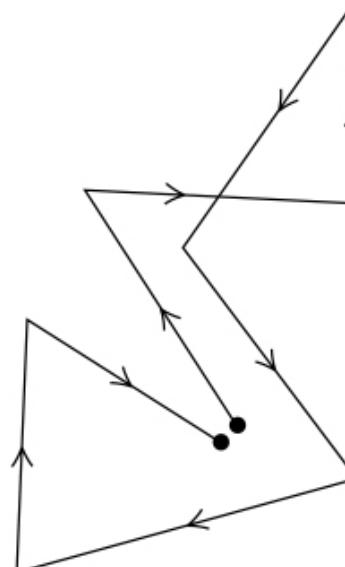
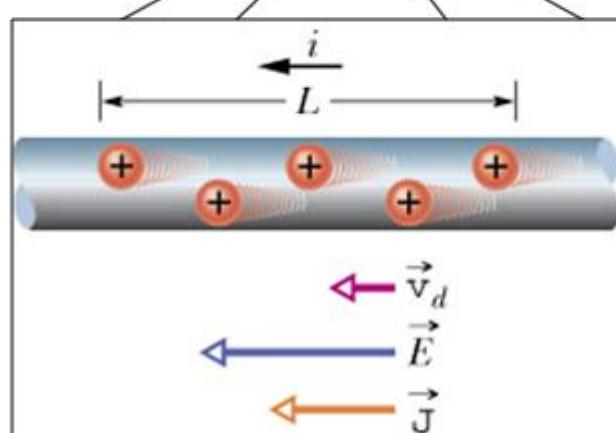
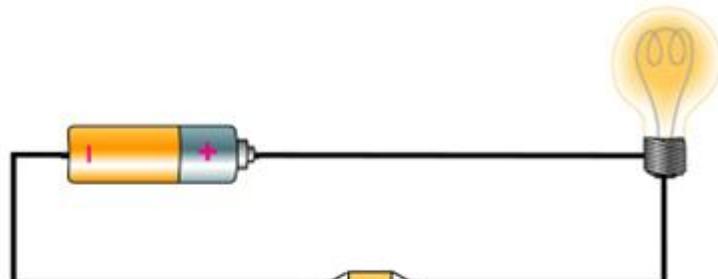
Checkpoint: A wire that carries a current consists of three sections with different radii. Rank the sections according to the following quantities, greatest first: (a) current, (b) magnitude of current density.



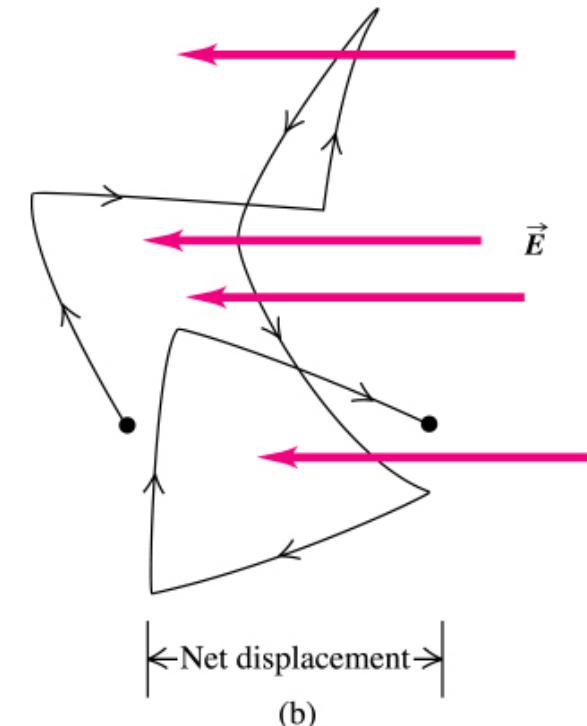
- (a) all tie; (b) B, C, A

3.1.4. Drift Speed:

- If a conductor does not have a current through it, the conduction electrons move randomly (10^6 m/s)
- If an electric field is applied the electrons still move randomly but now they tend to drift with a **drift speed v_d** about 10^{-5} or 10^{-4} m/s



(a)
 $\vec{E} = \mathbf{0}$



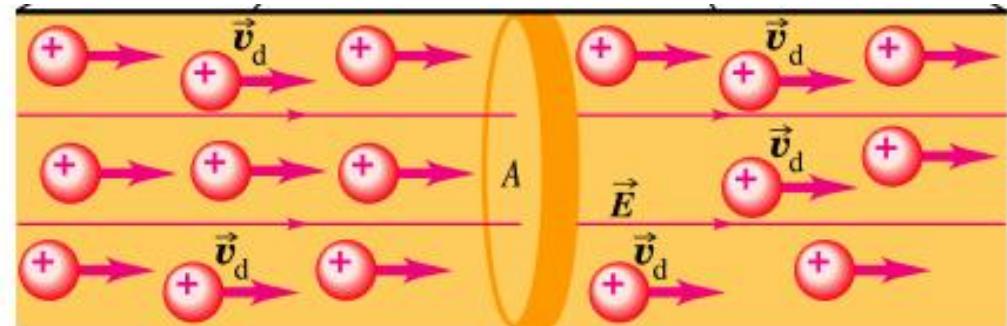
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Animation

Note: the drift speed is very low compared with the speeds of random motions

- Now, we find the relationship between v_d and J :

We consider the **equivalent drift** of positive charge carriers (actually electrons move)



The total charge q of the carriers in a length L of the wire:

$$q = n \times (AL) \times e$$

n : number of carriers per unit volume

the current i :

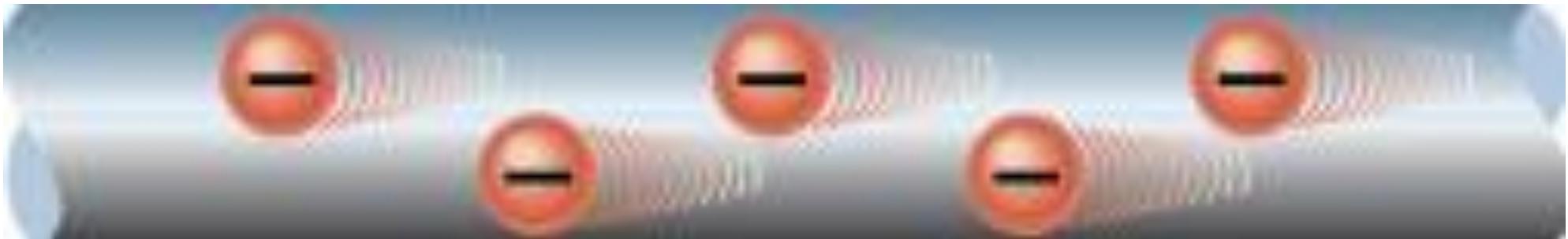
$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$$

$$v_d = \frac{i}{nAe} = \frac{J}{ne}$$

$$\boxed{\vec{J} = (ne)\vec{v}_d}$$

ne : charge density (C/m^3)

Checkpoint: The figure shows conduction electrons moving leftward in a wire. Are the following leftward or rightward:
(a) the current i , (b) the current density J , (c) the electric field E in the wire?



- (a) the direction of current represents the direction in which positive charges move, so the current i is rightward
- (b) J is rightward
- (c) E is rightward

3.2. Resistance and Resistivity:

3.2.1. Definition:

- Resistance R is defined by the following equation:

$$R = \frac{V}{i}$$

SI unit: 1 ohm = 1 Ω = 1 volt per ampere = 1 V/A

- A conductor that provides a specified resistance is called a **resistor** (symbol: 

Usually, we deal not with particular objects (e.g., resistance R of an object) but with materials represented by its resistivity defined by the following equation

- Resistivity ρ of the material:

$$\rho = \frac{E}{J}$$

Unit: $(V/m)/(A/m^2) = (V/A)m = \Omega m$



$$\vec{E} = \rho \vec{J}$$

- The conductivity σ of a material:

$$\sigma = \frac{1}{\rho}$$

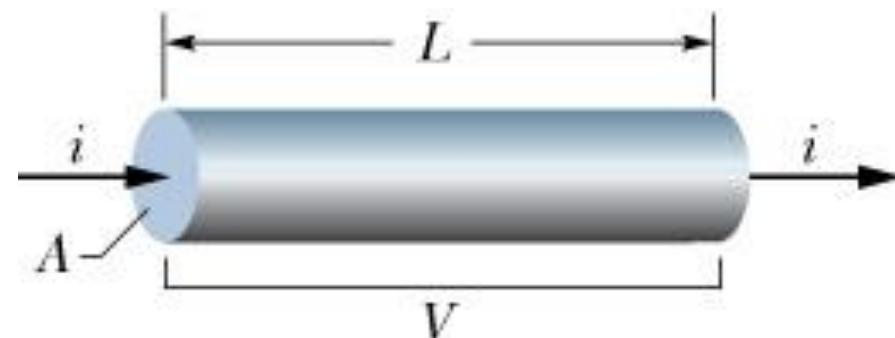
Unit: $(\Omega m)^{-1}$

$$\vec{J} = \sigma \vec{E}$$

3.2.2. Calculating Resistance from Resistivity:

First, we need to discriminate resistance from resistivity:
"Resistance is a property of an object and Resistivity is a property of a material"

- We apply a potential difference V between the ends of a wire of length L and cross section A , resulting in a current i



$$E = \frac{V}{L}; J = \frac{i}{A}$$

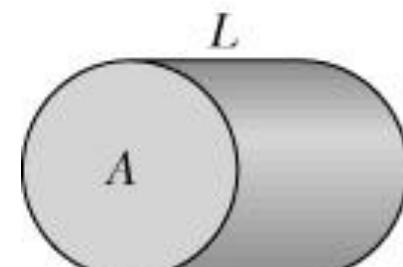
we also have:

$$\rho = \frac{E}{J} = \frac{V/L}{i/A} = R \frac{A}{L}$$

$$R = \rho \frac{L}{A}$$

Checkpoint: The figure shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference V is placed across their lengths

(a) and (c) tie, (b)



(a)



(b)



(c)

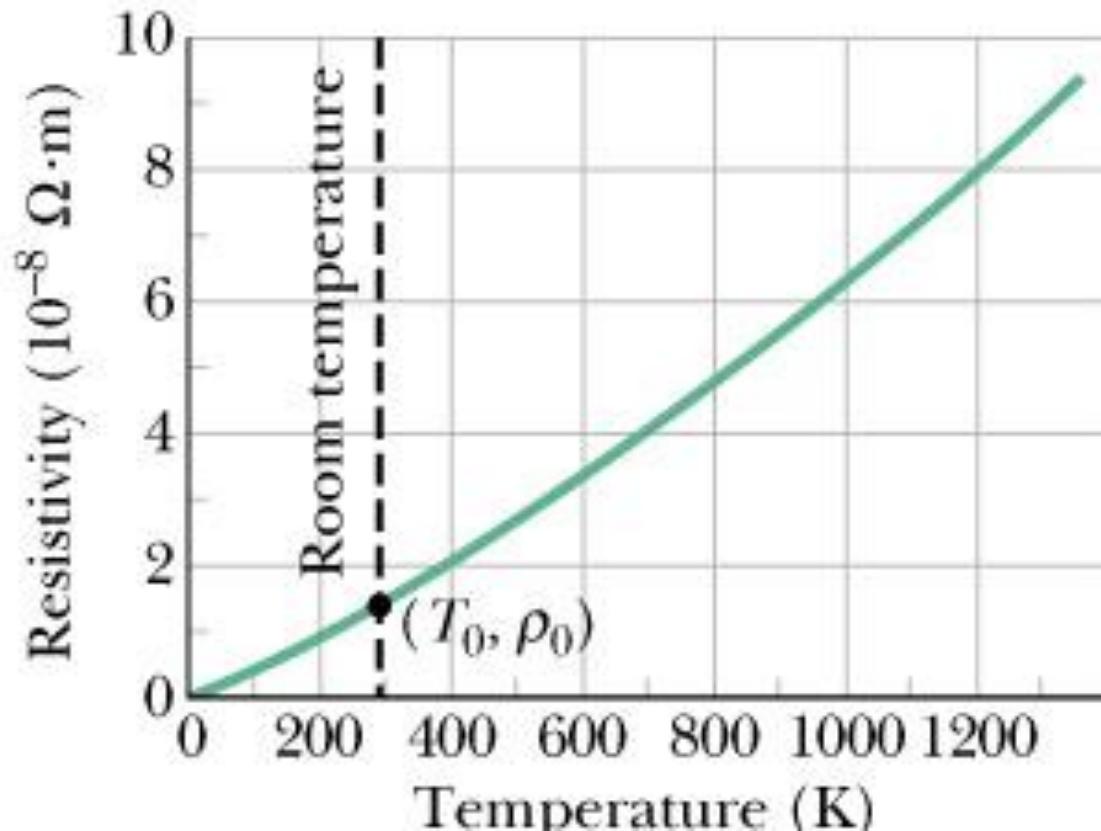
3.2.3. Variation of the Resistivity with Temperature:

- The resistivity of a material depends on temperature. This relation for metals is **fairly linear** over a broad temperature range

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

ρ_0 is the resistivity at temperature T_0

α is the temperature coefficient of resistivity

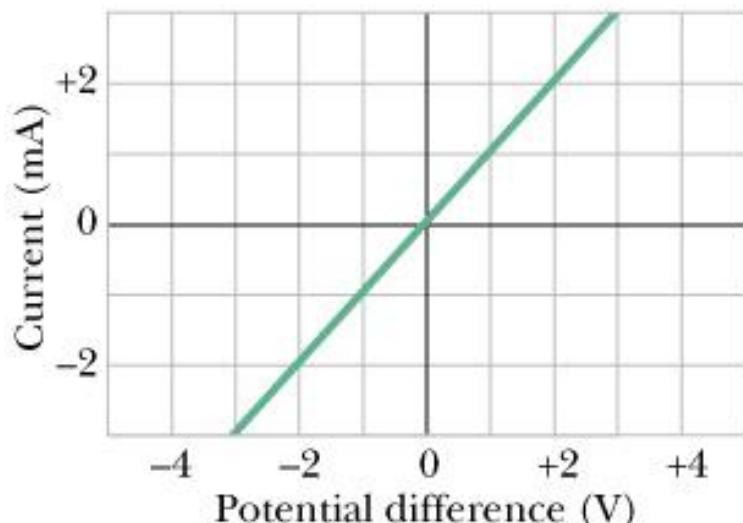


3.3. Ohm's Law and a Microscopic View of Ohm's Law:

3.3.1. Ohm's Law:

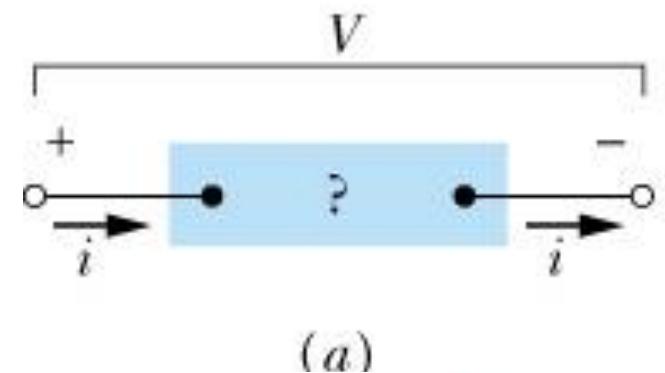
The current through a device is **always** directly proportional to the potential difference applied to the device

$$i = \frac{V}{R}$$

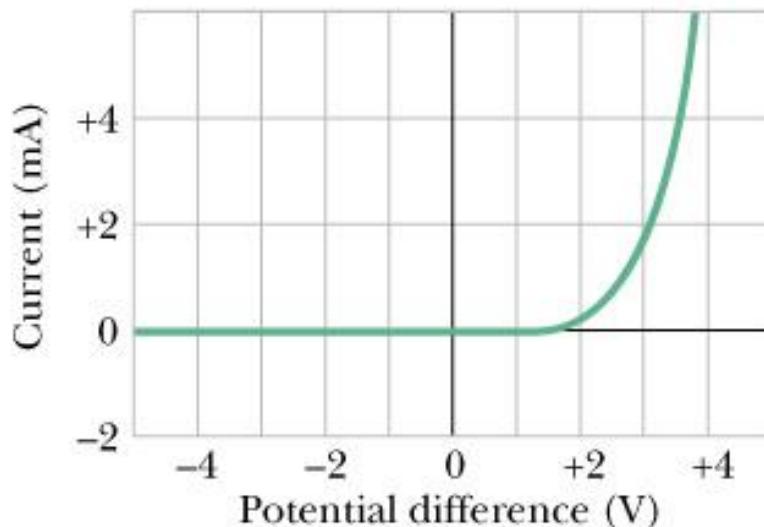


(b)

A linear relation between i and V



(a)



(c)

A non-linear relation

- The essence of Ohm's law is that the i versus V plot is linear, it means that R is independent of V
- Therefore:

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity (direction) of the applied potential difference

- We also have:

$$\vec{E} = \rho \vec{J} \quad (\text{analog of } V = iR)$$

So:

A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and polarity (direction) of the applied electric field

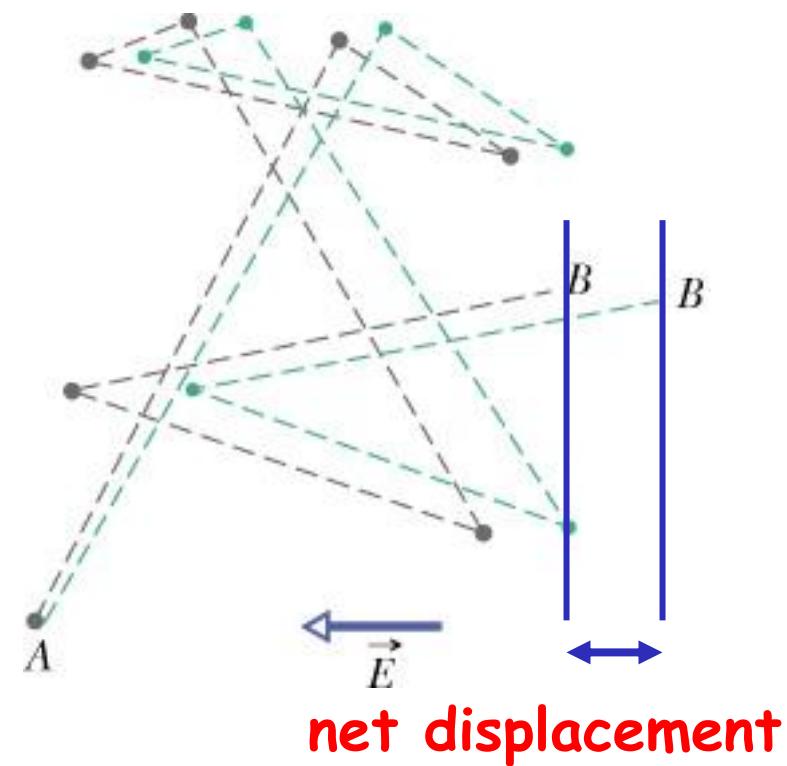
3.3.2. A Microscopic View of Ohm's Law:

Key question: Why do particular materials obey Ohm's law?

- First, we assume that the conduction electrons in the metal are free to move throughout the volume of a sample and they collide not with one another but only with atoms of the metal (**the free-electron model**), this is like the molecules of a gas
- In a conductor, the electrons move randomly with an effective speed $v_{\text{eff}} \approx 10^6 \text{ m/s}$. If we apply an electric field, the electrons modify their random motions slightly and drift very slowly in the opposite direction to the field with a velocity $v_d \approx 10^{-7} \text{ m/s}$, so $v_d \approx 10^{-13} v_{\text{eff}}$

If we only consider the electron motion due to the electric field, an acceleration:

$$a = \frac{F}{m} = \frac{eE}{m}$$



After a typical collision, each electron will completely lose its previous drift velocity, so between the two successive collisions:

$$v_d = a\tau = \frac{eE\tau}{m}$$

τ : the average time (mean free time) between the collisions

We have:

$$\vec{J} = (ne)\vec{v}_d$$

So:

$$v_d = \frac{J}{ne} = \frac{eE\tau}{m}$$

$$E = \left(\frac{m}{e^2 n \tau} \right) J$$

We introduce:

$$\rho = \frac{m}{e^2 n \tau}$$

So, we can rewrite:

$$\vec{E} = \rho \vec{J} \text{ (see the definition of resistivity)}$$

n (the number of carriers per unit volume), e and m are constant for metals

τ is hardly affected by the electric field as v_d caused by the field is too much smaller than v_{eff}

therefore for metals, their resistivity ρ is a constant, independent of the electric field.

3.3.3. Power in Electric Circuits:

- We consider a circuit as shown in the figure
- In time interval dt , an amount of charge dq moves between two terminals with a potential difference V , its potential energy decreases by an amount:

$$dU = dqV = idtV$$

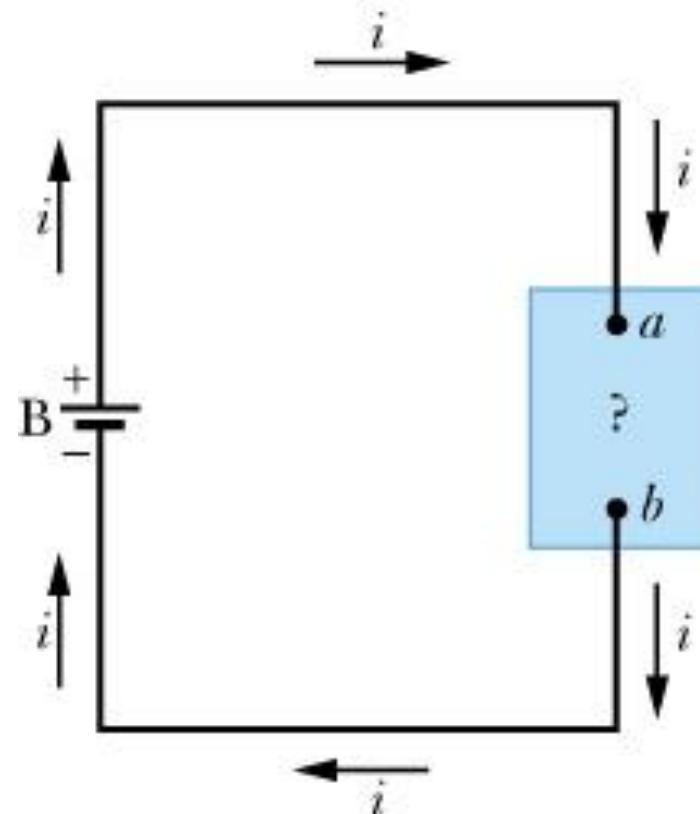
- Power P is the rate of electric energy transfer:

$$P = \frac{dU}{dt} = iV$$

Unit: $1 \text{ V.A} = (1 \text{ J/C}) (1 \text{ C/s}) = 1 \text{ J/s} = 1 \text{ W}$

For a resistor, we have $V = iR$, so:

$$P = i^2 R \text{ or } P = \frac{V^2}{R}$$



3.4. Semiconductors and Superconductors:

3.4.1. Semiconductors: A semiconductor is a material with electrical conductivity σ intermediate in magnitude between that of a conductor and an insulator, e.g., silicon

Property	Copper	Silicon
Type of material	Metal	Semiconductor
Charge carrier density, m^{-3}	8.49×10^{28}	1×10^{16}
Resistivity, Ωm	1.69×10^{-8}	2.5×10^3
Temperature coefficient of resistivity, K^{-1}	$+4.3 \times 10^{-3}$	-70×10^{-3}

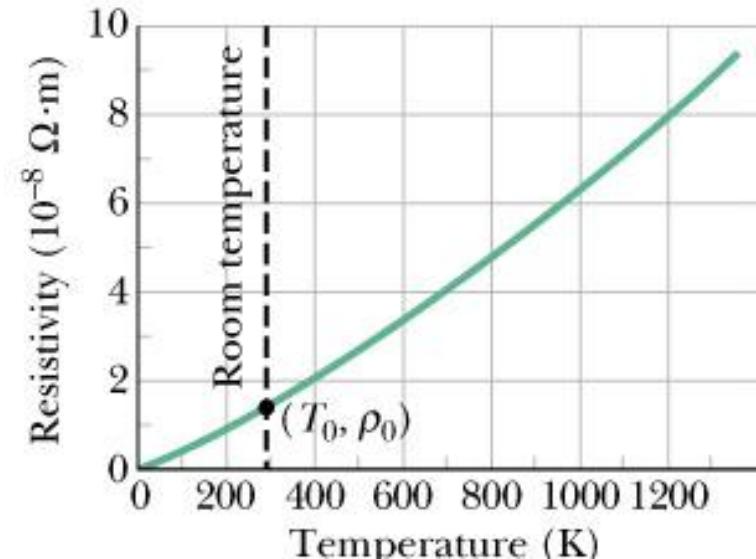
- Silicon has many fewer charge carriers comparing to that of copper and has a much higher resistivity
- Silicon has a negative temperature coefficient of resistivity ($\alpha < 0$), so the resistivity of silicon rapidly decreases with increasing temperature

We have for both conductors and semiconductors:

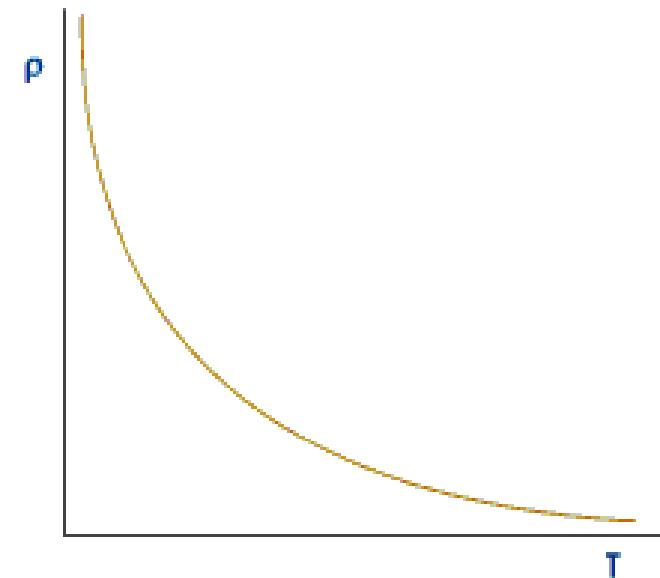
Metals

$$\rho = \frac{m}{e^2 n \tau}$$

- For conductors, n is nearly constant, so the increase of temperature will increase the collision rate, hence will reduce the mean time between collisions, τ decreases leading to an increase in ρ
- For semiconductors, n increases very rapidly with temperature resulting in a rapid decrease in ρ



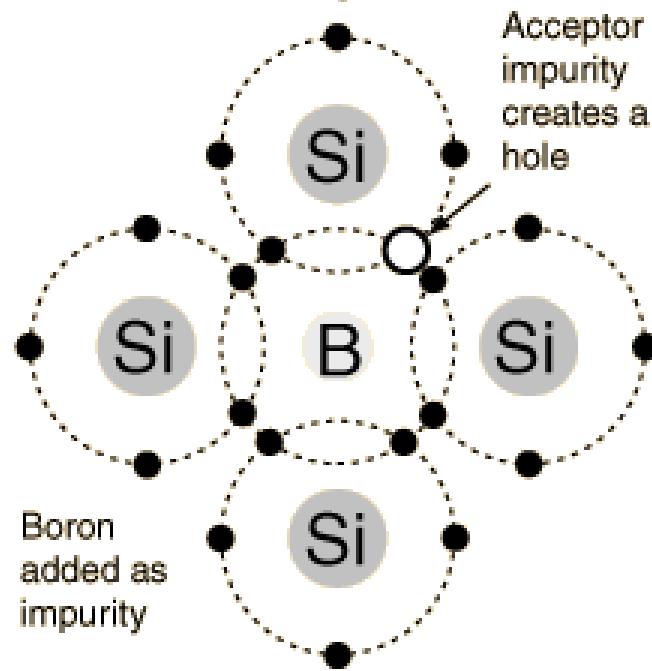
Semiconductors



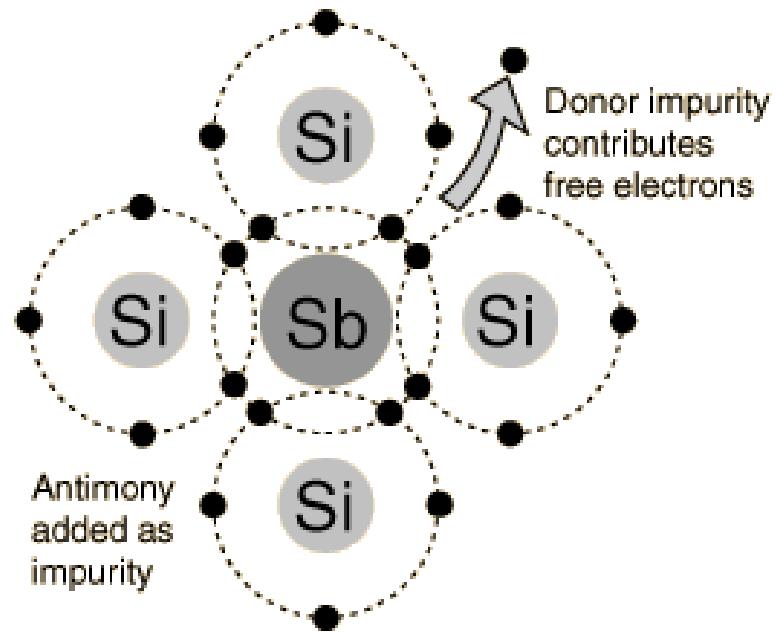
Doping of Semiconductors:

The resistivity of semiconductors can be greatly reduced by adding small amounts of specific "impurity" atoms in a process called **doping**.

P-Type



N-Type



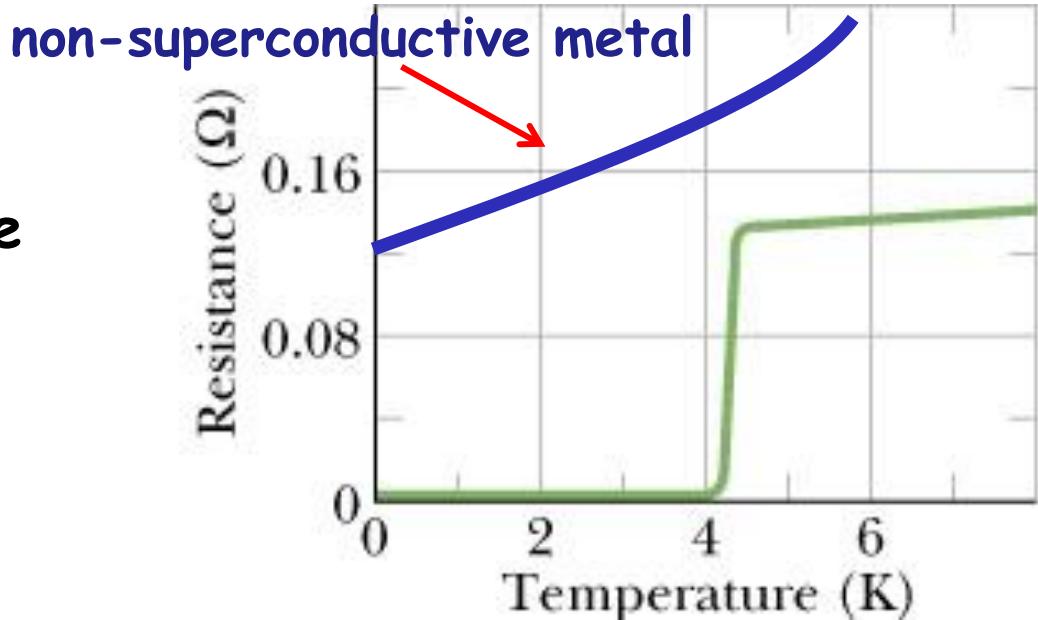
Doping Si with B creates deficiencies of valence electrons (called holes), producing p-type semiconductor

Doping Si with Sb contributes excess electrons, producing n-type semiconductor

3.4.2. Superconductors:

Superconductors are materials that lose all electrical resistance at low temperatures

For example, the resistance of mercury drops to zero at ~4 K



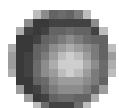
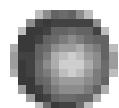
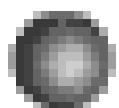
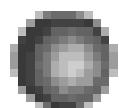
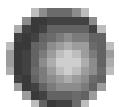
This phenomenon of superconductivity plays a very important role in technology

Explanation:

According to the Bardeen-Cooper-Schrieffer theory, the conduction electrons move in coordinated pairs

One of the electrons in a pair may electrically distort the molecular structure of the material as it moves through, creating a short-lived concentration of positive charge. The other e- in the pair may then be attracted toward this positive charge

This coordination would prevent collisions between electrons and the molecules of the material, completely eliminating resistance



Homework: 1, 5, 10, 12, 19, 24, 26, 31, 37, 43, 47
(pages 700-702)