MIDTERM SAMPLE QUESTIONS, S2 2022-2023

This paper shows sample exam questions. In the actual exam, Part A will contain 20 True/False questions (total 60 points). If you want, you can give the explanation for part A in the exam papers. Part B will contain about 4 regular written questions (40 points). Please fill in your name and student ID in this question sheet.

Name: .... **Student ID:**........................

Department of Mathematics	Lecturers	Proctor(s)

Instructions: *You can use two A4 sheets of notes and a calculator. All other documents and electronic devices are forbidden.*

Part A: True/False Questions. *Each question carries 3 points - Fill your answer in the answer sheet - Only the answer sheet will be graded - Explain your answer if you wish*

ANSWER SHEET OF PART A

1. Ⓣ Ⓕ	6. Ⓣ Ⓕ	11. Ⓣ Ⓕ	16. Ⓣ Ⓕ
2. Ⓣ Ⓕ	7. Ⓣ Ⓕ	12. Ⓣ Ⓕ	17. Ⓣ Ⓕ
3. Ⓣ Ⓕ	8. Ⓣ Ⓕ	13. Ⓣ Ⓕ	18. Ⓣ Ⓕ
4. Ⓣ Ⓕ	9. Ⓣ Ⓕ	14. Ⓣ Ⓕ	19. Ⓣ Ⓕ
5. Ⓣ Ⓕ	10. Ⓣ Ⓕ	15. Ⓣ Ⓕ	20. Ⓣ Ⓕ

1. For any vectors $\mathbf{u}$ and $\mathbf{v}$ and in V_3, $(2\mathbf{u}+\mathbf{v})\times\mathbf{v} = 2\mathbf{u}\times\mathbf{v}$.
2. $(\mathbf{i}\cdot\mathbf{i})^2 + (\mathbf{i}\cdot\mathbf{j})^2 + (\mathbf{k}\cdot\mathbf{k})^2 = 3$, where $\mathbf{i} = \langle 1,0,0\rangle$, $\mathbf{j} = \langle 0,1,0\rangle$, and $\mathbf{k} = \langle 0,0,1\rangle$.
3. If $\mathbf{u} = \langle u_1, u_2, u_3\rangle$ is a three dimensional vector, then the vector projection of $\mathbf{u}$ onto $\mathbf{j} = \langle 0,1,0\rangle$ is u_2.
4. If for some three dimensional vectors $\mathbf{u, v, w}$ we have $\mathbf{u}\cdot\mathbf{v} = \mathbf{u}\cdot\mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.
5. The series $\sum_{n=1}^{\infty} e^{-n}$ is convergent.
6. The series $\sum_{n=1}^{\infty} \dfrac{3}{n^2+3}$ is divergent.
7. The series $\sum_{n=2}^{\infty} (-1)^n \dfrac{1}{\sqrt{n}+1}$ is convergent.
8. The explicit formula of the sequence $\{\dfrac{2}{25}, \dfrac{4}{36}, \dfrac{6}{49}, \dfrac{8}{64}, \dfrac{10}{81}, \dots\}$ is $a_n = \dfrac{2n}{(n+4)^2}$.

9. If $\lim_{n\to\infty} a_n = 0$ then the series $\sum_{n=1}^{\infty} a_n$ is convergent.

10. The Ratio Test can be used to determine whether $\sum_{n=1}^{\infty} \dfrac{1}{n!}$ converges.

11. The series $\sum_{n=1}^{\infty} (-1)^n \dfrac{(2n)!}{(n!)^2}$ diverges.

12. The parametric curve given by $\mathbf{r}(t) = \langle t^2, 3t+1, t-2 \rangle$, pass through $P(1,4,-1)$ and $Q(16,11,2)$.

13. The symmetric equation for the line of intersection between two planes $x+y+z=2$ and $x+2y-4z=3$ is given by $-\dfrac{x-1}{6} = \dfrac{y-1}{5} = z$.

14. $\dfrac{d}{dt}\left[\mathbf{u}(t) \cdot \mathbf{u}(t)\right] = 2\mathbf{u}'(t) \cdot \mathbf{u}(t)$.

15. The domain of $f(x,y) = \sqrt{1-x^2-y^2}$ is $-1 \le x \le 1$ and $-1 \le y \le 1$.

16. The series $\sum_{n=1}^{\infty} \dfrac{1}{n+\sqrt{n}+1}$ is divergent.

17. It can be shown that $0.99999\cdots = 1$.

18. If $\lim_{n\to\infty} a_n = 1$ then $\lim_{n\to\infty} a_{2n} = 1$

19. The linear equation $2x - y + 3z = 1$ represents a line in space.

20. If $a_n \ge b_n \ge 0$ and the series $\sum_{n=1}^{\infty} a_n$ is divergent then $\sum_{n=1}^{\infty} b_n$ is divergent.

Part B: Show your work in details and indicate answers clearly. Each question carries 10 points.

21. Find the limit, if it exists or show that the limit does not exist $\lim_{(x,y)\to(0,0)} \dfrac{2x^2y^2}{2x^2+3y^2}$.

22. The position of a moving robot at time t (in seconds) is determined by the vector function

$$\mathbf{r} = 2t\sqrt{t}\,\mathbf{i} + \cos(\pi t)\,\mathbf{j} + \sin(\pi t)\,\mathbf{k}, \quad t \ge 0.$$

Find the velocity $\mathbf{r}'(t)$ and the unit velocity vector $\dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$ when $t = 1$ (s).

23. Find the radius of convergence and interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n\,(x-1)^n}{n^2\,2^n}$$

24. Find the Maclaurin series for $f(x) = \ln(3+2x)$ and find its radius of convergence.

Remark: You will write the solutions for part B and any explanation for part A (if you wish) in the regular exam papers. You will submit this question sheet which contains the answers for part A as well.