Homework (lecture 1):

- Prove the shell theorem for the electrostatic force
- 3, 6, 9, 10, 16, 23, 33, 35 (page 575-577)

3. What must be the distance between point charge q_1 = 26.0 μC and point charge q_2 = -47 μC for the electrostatic force between them to have a magnitude of 5.70 N?

$$F = k \frac{q_1 q_2}{r^2}$$

$$r = \sqrt{k \frac{|q_1||q_2|}{F}}$$

$$1 \mu C = 10^{-6} C$$

$$r = \sqrt{\frac{8.99 \times 10^9 (26 \times 10^{-6})(47 \times 10^{-6})}{5.7}} = 1.39 \ (m)$$

6. Two equally charged particles are held 3.2×10^{-3} m apart and then released from rest. The initial acceleration of the first particle is observed to be 6.0 m/s^2 and that of the second to be 9.0 m/s^2. If the mass of the first particle is 6.3×10^{-7} kg, what are (a) the mass of the second particle and (b) the magnitude of the charge of each particle.

$$F_{21} = m_1 a_1 \qquad\qquad F_{12} = m_2 a_2$$

$$F_{12} = F_{21}: \qquad m_2 = \frac{a_1}{a_2} m_1$$

$$F_{12} = F_{21} = k \frac{q_1 q_2}{r^2} = m_1 a_1$$

$$q = \sqrt{\frac{m_1 a_1 r^2}{k}} = \sqrt{\frac{6.3 \times 10^{-7} \times 6 \times \left(3.2 \times 10^{-3}\right)^2}{8.99 \times 10^9}} = 6.6 \times 10^{-11} (C)$$

$$q = 66 \, (\text{pC})$$

9. Two identical conducting spheres, fixed in place, attract each other with an electrostatic force of 0.108 N when their center-to-center separation is 50.0 cm. The spheres are then connected by a thin conducting wire. When the wire is removed, the spheres repel each other with an electrostatic force of 0.036 N. Of the initial charges on the spheres, with a positive net charge, what was (a) the negative charge on one of them and (b) the positive charge on the other?

Using the shell theorem:

Before: $\quad F_1 = -k\dfrac{q_1 q_2}{r^2} \Rightarrow q_1 q_2 = -3.0\times 10^{-12} \quad (C)$

After: the net charge is positive

$$F_2 = k\dfrac{q^2}{r^2} = k\dfrac{\left(\dfrac{q_1 + q_2}{2}\right)^2}{r^2} \Rightarrow q_1 + q_2 = 2.0\times 10^{-6} \quad (C)$$

$$q_1^2 - 2.0 \times 10^{-6} q_1 - 3.0 \times 10^{-12} = 0$$

The solutions:

1st: $\quad q_1 = 3.0 \times 10^{-6}(C) \Rightarrow q_2 = -1.0 \times 10^{-6}(C)$

2nd: $\quad q_1 = -1.0 \times 10^{-6}(C) \Rightarrow q_2 = 3.0 \times 10^{-6}(C)$

Note: if the net charge is negative

$$q_1 + q_2 = -2.0 \times 10^{-6} \quad (C)$$

the solutions should be:

$$q_1(\text{or } q_2) = 1.0 \times 10^{-6}(C) \Rightarrow q_2(\text{or } q_1) = -3.0 \times 10^{-6}(C)$$

10. In the figure as shown, four particles form a square. The charges are $q_1 = q_4 = Q$ and $q_2 = q_3 = q$. (a) What is Q/q if the net electrostatic force on particles 1 and 4 is zero? (b) Is there any value of q that makes the net electrostatic force on each of the four particles zero? explain.
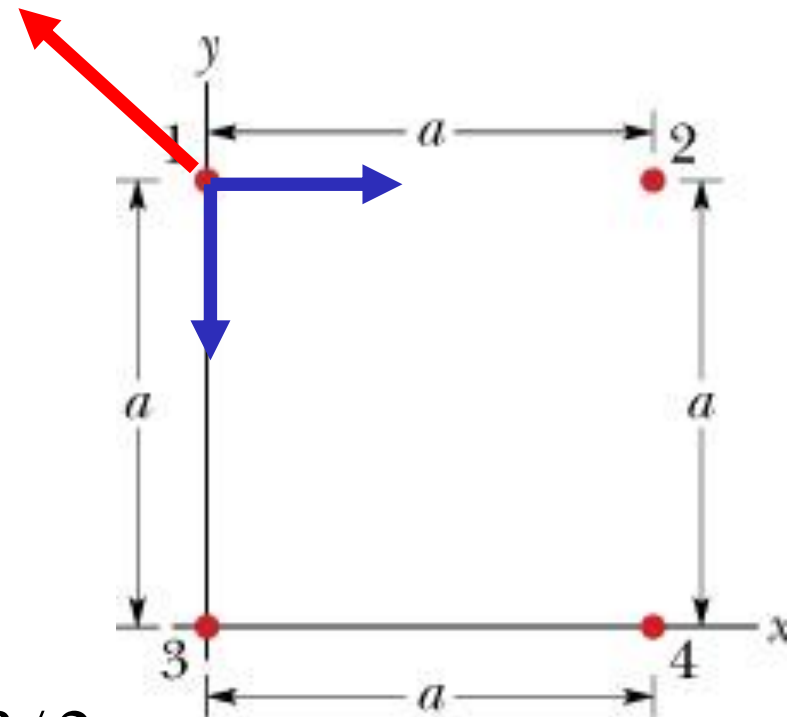
(a) q_1 & q_4 have the same sign, all three forces act on q_1 as shown

$$\vec{F}_{41} = \vec{F}_{21} + \vec{F}_{31}$$

$$F_{41} = \sqrt{2}F_{21}$$

$$k\frac{Q^2}{\left(a\sqrt{2}\right)^2} = \sqrt{2}k\frac{|qQ|}{(a)^2} \Rightarrow \left|\frac{Q}{q}\right| = 2^{3/2} = 2.83$$

(b) if the net force acting on particle 3 is also zero:
➔ this is inconsistent with (a), so the answer is NO
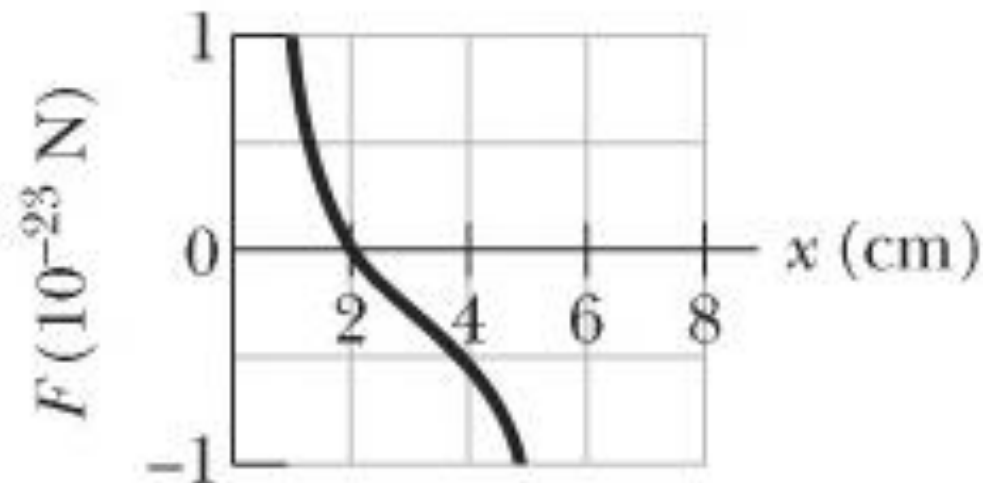
$$\left|\frac{q}{Q}\right| = 2.83$$

16. See the figure as shown, particle 1 (of charge q_1) and particle 2 (of charge q_2) are fixed in place on an x axis, 8.0 cm apart. Particle 3 (of charge $q_3 = +6.0 \times 10^{-19}$ C) is to be placed on the line between particles 1 and 2 so that they produce a net electrostatic force $F_{3,net}$ on it. The diagram gives the x component of that force versus the coordinate x at which particle 3 is placed. What are (a) the sign of charge $q1$ and (b) the ratio q_2/q_1?

- at 2 cm: $F_{net} = 0$ so 1 and 2 must have the same sign.
- when q_3 approaches q1, F_{13} increases in magnitude. F_{net} increases in the positive x direction, so F_{13} is a repulsive force, $q_1 > 0$
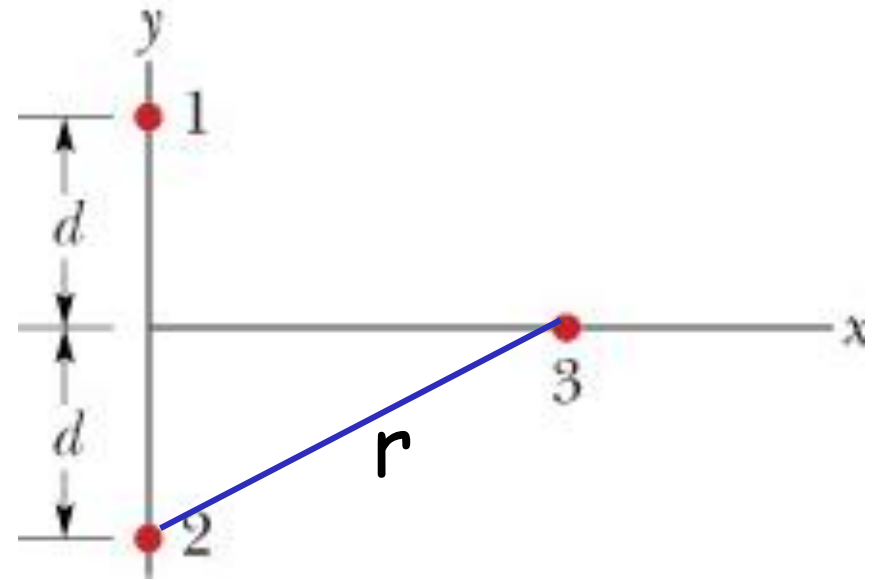
$$k\frac{q_1 q_3}{r^2{}_{13}} = k\frac{q_2 q_3}{r^2{}_{23}}$$

$$\frac{q_2}{q_1} = \left(\frac{r_{23}}{r_{13}}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$$

23. See the figure, particles 1 and 2 of charge $q_1 = q_2 = +3.2\times10^{-19}$ C are on a y axis at distance d = 17.0 cm from the origin. Particle 3 of charge $q_3 = +6.4\times10^{-19}$ C is moved gradually along the x axis from x = 0 to x = +5.0 m. At what values of x will the magnitude of the electrostatic force on the third particle from the other two particles be (a) minimum and (b) maximum? what are the (c) minimum and (d) maximum magnitudes?
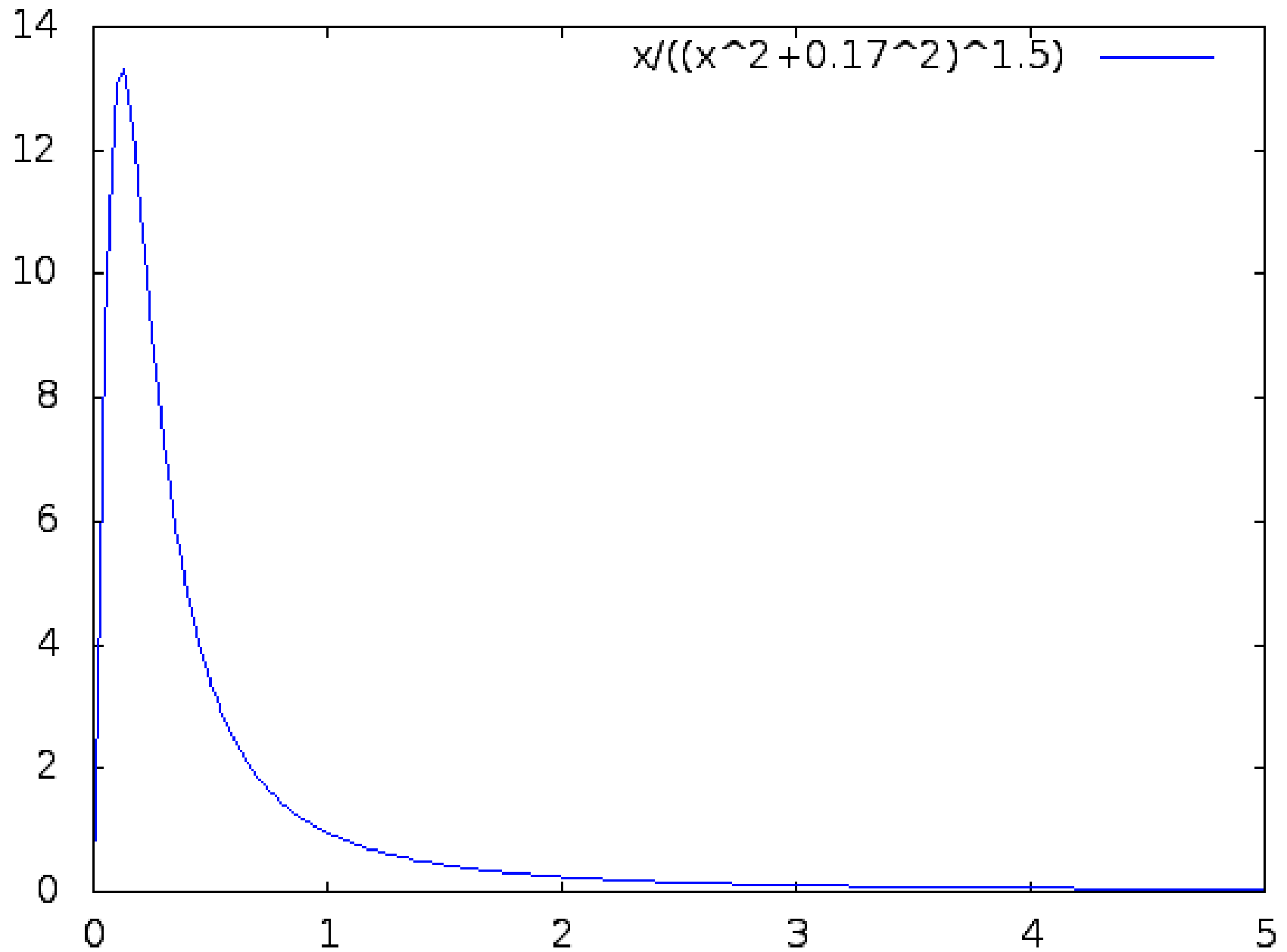
The net force acting on particle 3:

$$F_{net} = 2\times k \frac{qq_3}{r^2}\times \cos\theta$$

$$r = \sqrt{x^2 + d^2}\,; \cos\theta = \frac{x}{r}$$

$$F_{net} = 2kqq_3 \frac{x}{\left(x^2 + d^2\right)^{3/2}} \Rightarrow x = 0 : \text{minimum}$$

$$F'_{net} = 0 \Rightarrow d^2 - 2x^2 = 0 \Rightarrow x = d/\sqrt{2} \;\; :\text{maximum}$$

http://www.function-grapher.com/index.php

33. Calculate the number of coulombs of positive charge in 250 cm³ of (neutral) water. (Hint: A hydrogen atom contains one proton; an oxygen atom contains eight protons)

The mass of the sample:

$$m = \rho V = 1 \times 250 = 250 (g)$$

The number of moles:

$$n = \frac{m}{M_{molar}} = \frac{250}{18} = 13.9$$

The positive charge:

$$Q = nN_A q$$

$$= 13.9 \times 6.023 \times 10^{23} \times 10 \times 1.6 \times 10^{-19} = 1.34 \times 10^7 (C)$$

35. In crystals of the salt cesium chloride, cesium ions Cs^+ form the eight corners of a cube and a chlorine ion Cl^- is at the cube's center. The edge length of the cube is 0.4 nm. The Cs^+ ions are each deficient by one electron (and thus each has a charge of +e), and the Cl^- ion has one excess electron (and thus has a charge of -e). (a) What is the magnitude of the net electrostatic force exerted on the Cl^- ion by the eight Cs^+ ions? (b) If one of the Cs^+ ions is missing, the crystal is said to have a defect, what is the magnitude of the net force exerted on the Cl^- ion by the seven remaining Cs^+ ions?
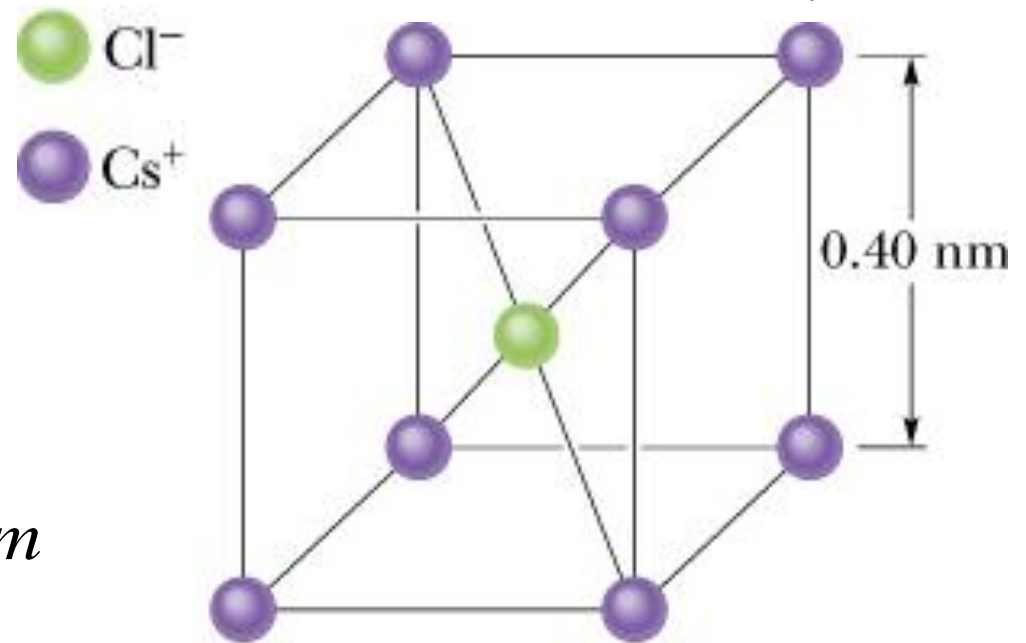
(a) The net force is zero due to the symmetric distribution of the eight Cs^+ around Cl^-

(b)
$$F_{7Cs+} = F_{1Cs+} = k\frac{e^2}{r^2}$$

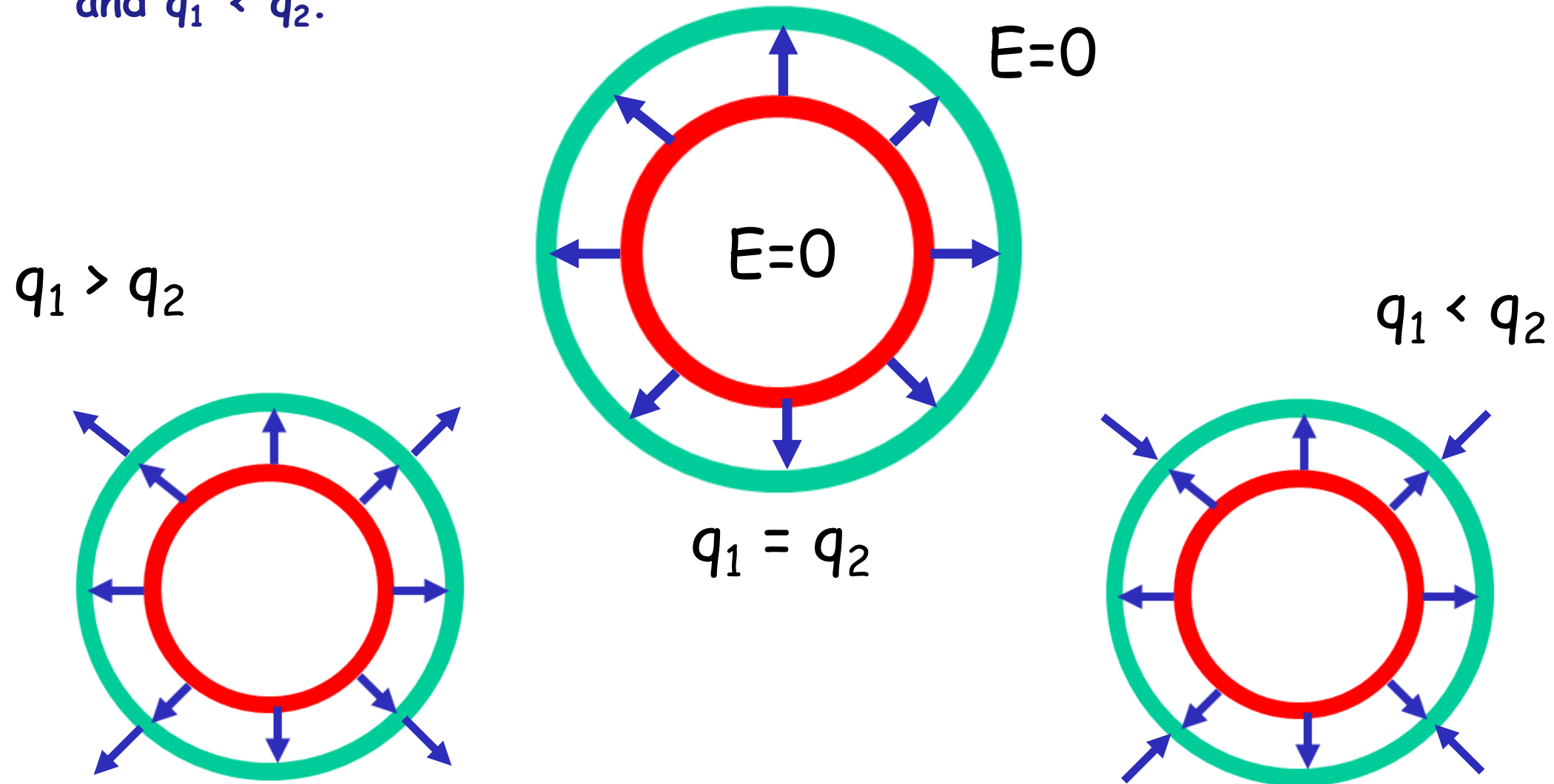$$r = \frac{\sqrt{a^2 + (a\sqrt{2})^2}}{2}; \quad a = 0.4nm$$

$$F_{7Cs+} = k\frac{e^2}{(3/4)a^2}$$

Cl⁻

Cs⁺

0.40 nm

Homework (lecture 2): 1, 5, 14, 15, 19, 23, 27, 31, 35, 44, 54, 56, 59 (page 598-603)

1. Sketch qualitatively the electric field lines both between and outside two concentric conducting spherical shells when a uniform positive charge q_1 is on the inner shell and a uniform negative charge $-q_2$ is on the outer. Consider the cases $q_1 > q_2$, $q_1 = q_2$, and $q_1 < q_2$.
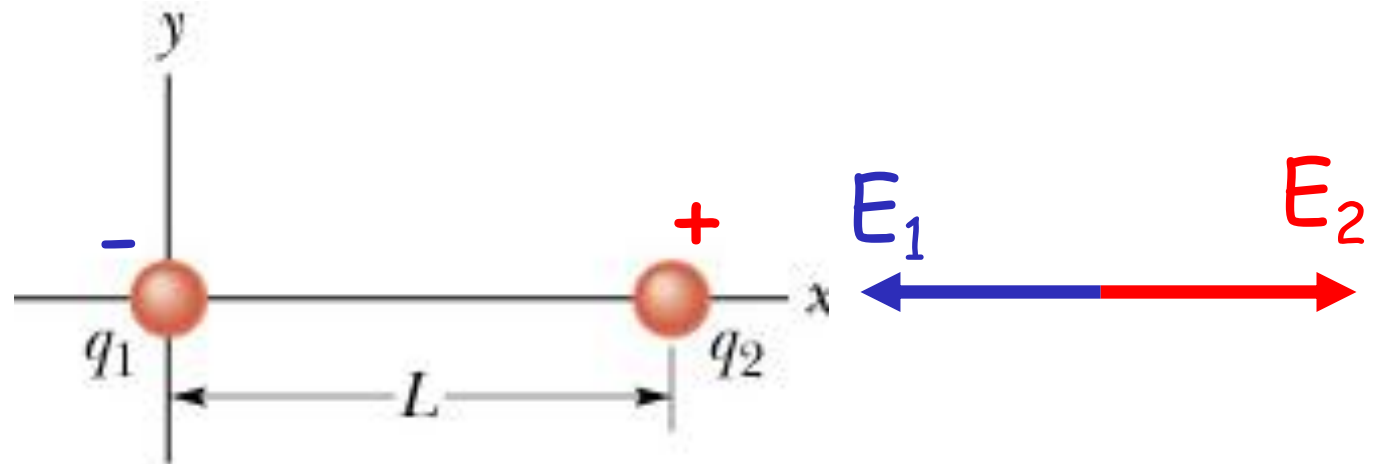
5. What is the magnitude of a point charge whose electric field 50 cm away has the magnitude 2.0 N/C?

$$E = \frac{k|q|}{r^2}$$

$$|q| = \frac{Er^2}{k} = \frac{2 \times 0.5^2}{8.99 \times 10^9} = 5.6 \times 10^{-11} (C)$$

14. See the figure, particle 1 of charge q_1 = -4.0q and particle 2 of charge q_2 = +2.0q are fixed to an x axis. (a) As a multiple of distance L, at what coordinate on the axis is the net electric field of the particles zero? (b) Sketch the net electric field lines.

$$E_1 = \frac{k|q_1|}{x^2} = E_2 = \frac{k|q_2|}{(x-L)^2}$$

$$\frac{(x-L)^2}{x^2} = \frac{2}{4} \Rightarrow x = 3.41L$$

15. The three particles are fixed in place and have charges $q_1 = q_2 = +e$ and $q_3 = +2e$. Distance $a = 6.0$ μm. What are the (a) magnitude and (b) direction of the net electric field at point P due to the particles?
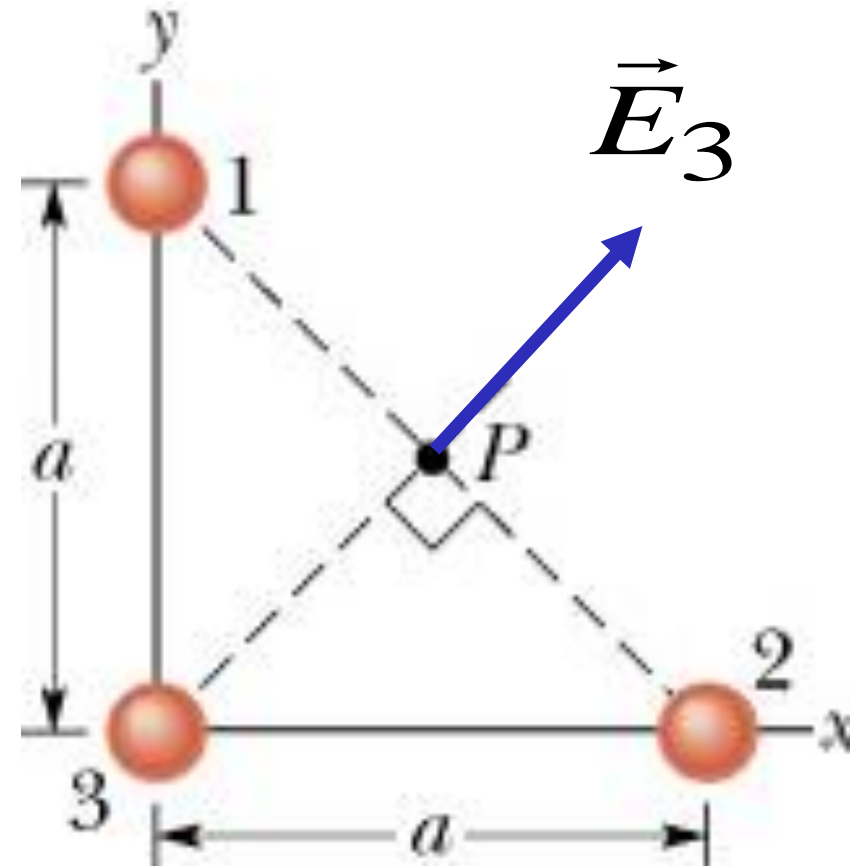
$$\vec{E}_1 + \vec{E}_2 = 0$$

$$\vec{E}_{net} = \vec{E}_3$$

$$E_3 = k\frac{2e}{OP^2}; OP = \frac{a\sqrt{2}}{2}$$

$$E_3 = 8.99 \times 10^9 \frac{4 \times 1.6 \times 10^{-19}}{6.0^2 \times 10^{-12}}$$

$$E_3 = 160(N/C); (\vec{E}_3, Ox) = 45^0$$

19. The figure shows an electric dipole. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the dipole's electric field at point P, located at distance r >> d?

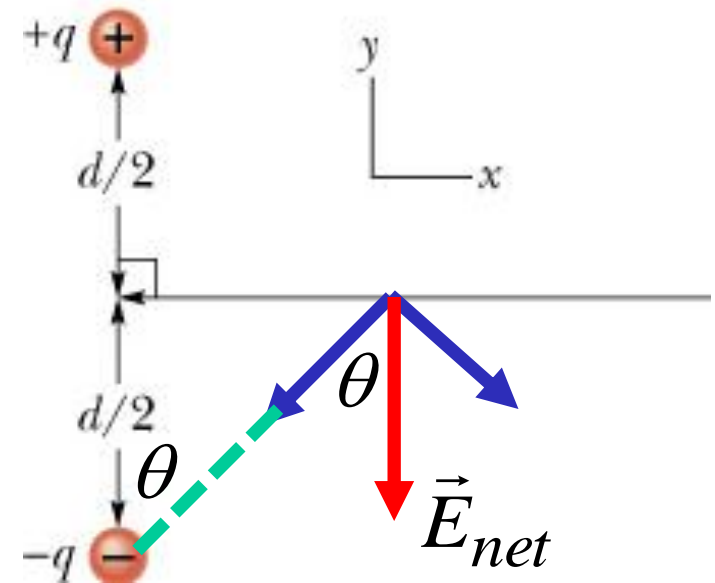$$E_{net} = 2k \frac{q}{r^2} \cos \theta = \frac{kqd}{r^3}$$

$p = qd$: electric dipole moment

$$E_{net} = \frac{kp}{\left[\left(\frac{d}{2} \right)^2 + x^2 \right]^{3/2}}$$

$$E_{net} = \frac{kp}{x^3}$$

- If x>>d:
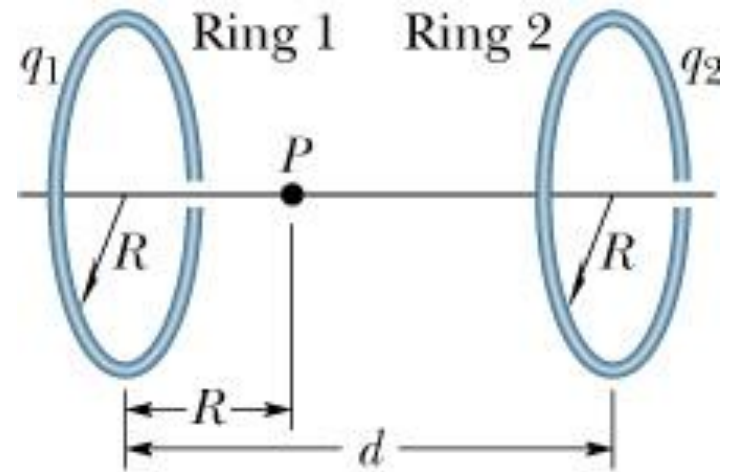- $\vec{E}_{net}$ points in the negative direction of the y axis

23. The figure shows two parallel nonconducting rings with their central axes along a common line. Ring 1 has uniform charge q_1 and radius R; ring 2 has uniform charge q_2 and the same radius R. The rings are separated by distance d = 3.0R. The net electric field at point P on the common line, at distance R from ring 1, is zero. What is the ratio q_1/q_2?

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}$$

$$\frac{kq_1 R}{(R^2 + R^2)^{3/2}} = \frac{kq_2 2R}{(4R^2 + R^2)^{3/2}}$$

$$\frac{q_1}{q_2} = \frac{4\sqrt{2}}{5\sqrt{5}} = 0.51$$

<u>Note:</u> q_1 and q_2 must have the same sign to produce a net electric field equal to zero

27. Two curved plastic rods, one of charge +q and the other of charge –q, form a circle of radius R = 8.5 cm in an xy plane. The x axis passes through both of the connecting points, and the charge is distributed uniformly on both rods. If q = 15.0 pC, what are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field produced at P, the center of the circle?
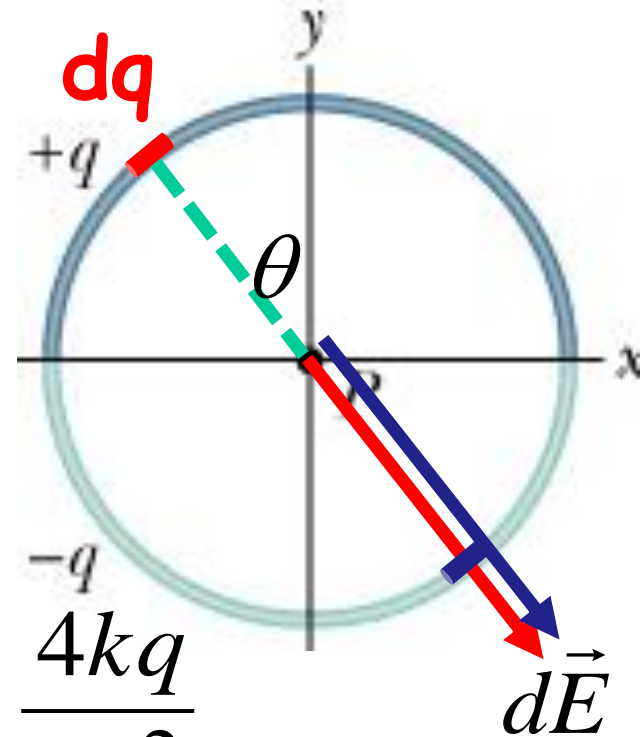
Consider a differential charge dq:

$$dq = \lambda ds = \lambda(R d\theta)$$

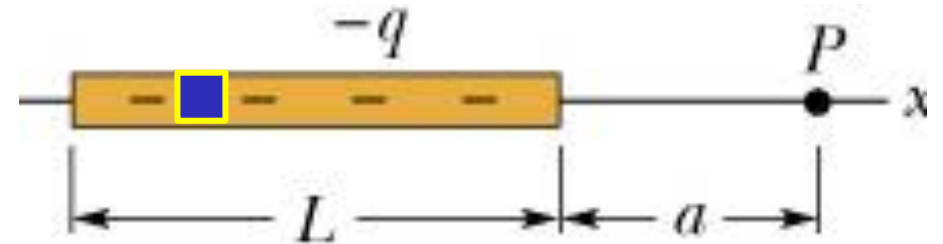$$dE_{1rod} = k\frac{(\lambda ds)}{R^2}\cos\theta = \frac{k\lambda}{R}\cos\theta \, d\theta$$

For two rods:

$$dE_{2rods} = 2\times\frac{k\lambda}{R}\cos\theta \, d\theta$$

$$E_{2rods} = 2\times\frac{k\lambda}{R}\int_{-90}^{90}\cos\theta \, d\theta = \frac{4k\lambda}{R} = \frac{4kq}{\pi R^2}$$

$$E_{2rods} = \frac{4\times 8.99\times 10^9 \times 15\times 10^{-12}}{3.14\times 8.5^2 \times 10^{-4}} = 23.8(N/C)$$

31. A nonconducting rod of length L = 8.15 cm has charge -q = -4.23 fC uniformly distributed along its length. (a) What is the linear charge density of the rod? What are the (b) magnitude and (c) direction (relative to the positive direction of the x axis) of the electric field produced at point P, at a = 12.0 cm from the rod? What is the electric field magnitude at a = 50 m by (d) the rod and (e) a particle of charge -q = -4.23 fC that replaces the rod?

(a)

$$\lambda = \frac{-q}{L} = \frac{-4.23 \times 10^{-15}}{8.15 \times 10^{-2}} = -0{,}519 \times 10^{-13} \, (C/m)$$

(b)

$$dE_x = k \frac{\lambda dx}{(L+a-x)^2}$$

$$E_x = -k \frac{q}{a(L+a)} = -1.6 \times 10^{-3} \, (N/C)$$

(c) the negative direction of the x axis

$L \langle \langle a :$

$$E_x = -k \frac{q}{a^2}$$

$$E_x = -1.5 \times 10^{-8} (N/C)$$

(d) for a distant point, the rod acts like a point charge, so the electric field of the point charge is the same as that of the rod:

$$E_x = -1.5 \times 10^{-8} (N/C)$$

35. At what distance along the central perpendicular axis of a uniformly charged plastic disk of radius 0.6 m is the magnitude of the electric field equal to one-half the magnitude of the field at the center of the surface of the disk?

$$E_z = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

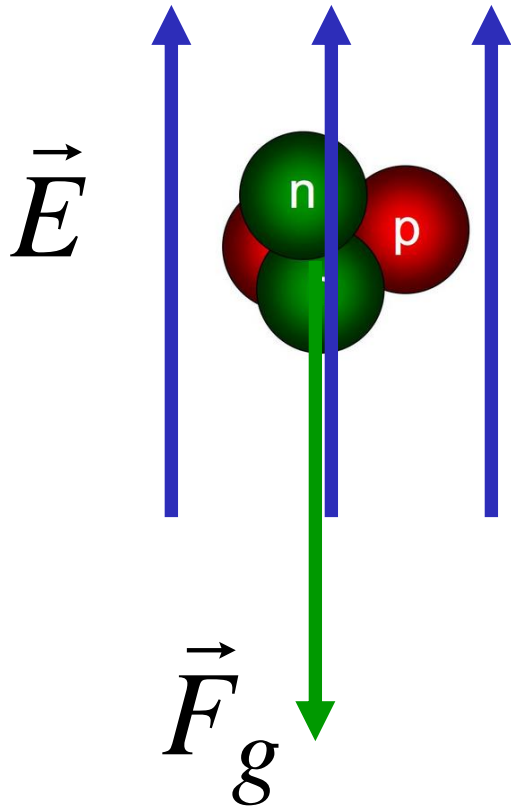At the center (very close to the center):

$$E_c = \frac{\sigma}{2\varepsilon_0}$$

$$E_z = \frac{1}{2}E_c :$$

$$1 - \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2}$$

$$z = R/\sqrt{3} = 0.35 (m)$$

44. An alpha particle (the nucleus of a helium atom) has a mass of 6.64×10^{-27} kg and a charge of $+2e$. What are the (a) magnitude and (b) direction of the electric field that will balance the gravitational force on the particle?

$$qE = mg$$

$$E = \frac{mg}{2e}$$

54. An electron is shot at an initial speed of $v_0 = 4.0 \times 10^6$ m/s, at angle $\theta_0 = 40^0$ from an x axis. It moves through a uniform electric field $\vec{E} = (5.0 \, N/C)\hat{j}$. A screen for detecting electrons is positioned parallel to the y axis, at distance x = 3.0 m. In unit vector notation, what is the velocity of the electron when it hits the screen?

$$F_g \approx 9 \times 10^{-30} (N)$$

$$F = 8 \times 10^{-19} (N)$$

So, we ignore gravity

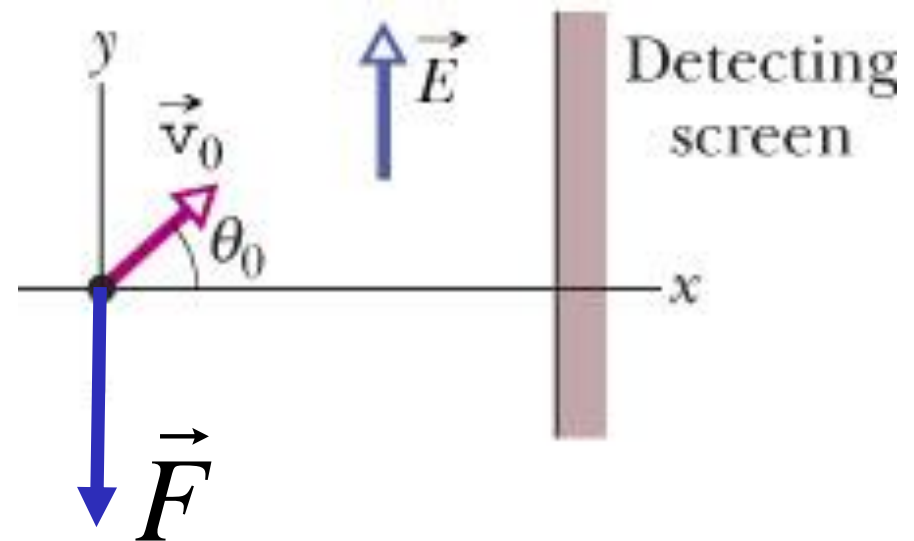$$a_y = \frac{eE}{m} = \frac{1.6 \times 10^{-19} \times 5.0}{9.11 \times 10^{-31}}$$

$$= 8.78 \times 10^{11} (m/s^2)$$

Time to hit the screen:

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{3}{4 \times 10^6 \cos 40^0} = 0.98 \times 10^{-6} (s)$$
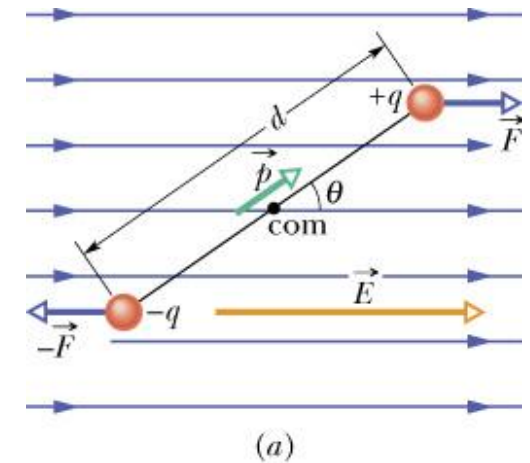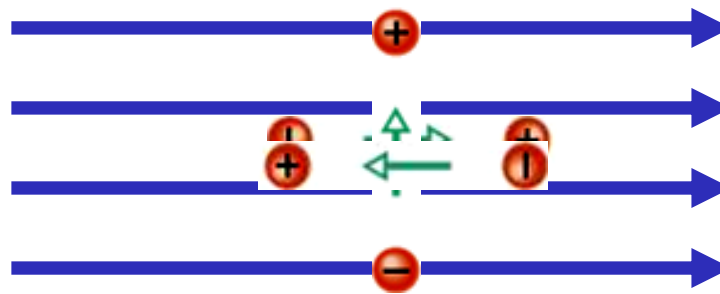
$$v_y = v_{0y} - a_y t = v_0 \sin\theta - at = 1.71 \times 10^6 (m/s)$$

$$\vec{v} = 3.06 \times 10^6 (m/s)\hat{i} + 1.71 \times 10^6 (m/s)\hat{j}$$
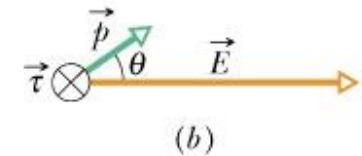
56. An electric dipole consists of charges +2e and -2e separated by 0.85 nm. It is in an electric field of strength 3.4 × 10⁶ N/C. Calculate the magnitude of the torque on the dipole when the dipole moment is (a) parallel to, (b) perpendicular to, and (c) antiparallel to the electric field.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = pE\sin\theta$$

(a) $\tau_{net} = 0 \quad (\theta = 0)$

(b) $\tau_{net} = 2edE = 2 \times 1.6 \times 10^{-19} \times 0.85 \times 10^{-9} \times 3.4 \times 10^{6}$

$$\tau_{net} = 9.3 \times 10^{-22} (N.m)$$

(c) $\tau_{net} = 0 \quad (\theta = 180^{0})$

59. How much work is required to turn an electric dipole 180^0 in a uniform electric field of magnitude E = 46.0 N/C if p = 3.02 × 10⁻²⁵ C.m and the initial angle is 64^0?

$$U = -pE\cos\theta$$

$$W_{applied} = -W_{\vec{E}} = \Delta U$$

$$W_{applied} = U_{\pi+\theta_0} - U_{\theta_0} = -pE\cos(180^0 + 64^0) + pE\cos 64^0$$

$$W_{applied} = 2pE\cos 64^0$$

$$W_{applied} = 2\times 3.02\times 10^{-25}\times 46.0\times 0.438$$

$$W_{applied} = 1.22\times 10^{-23}(J)$$