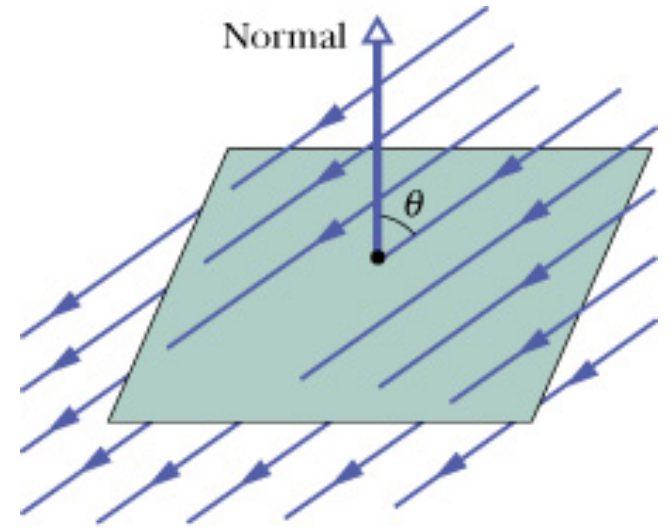


Homework (lecture 4):

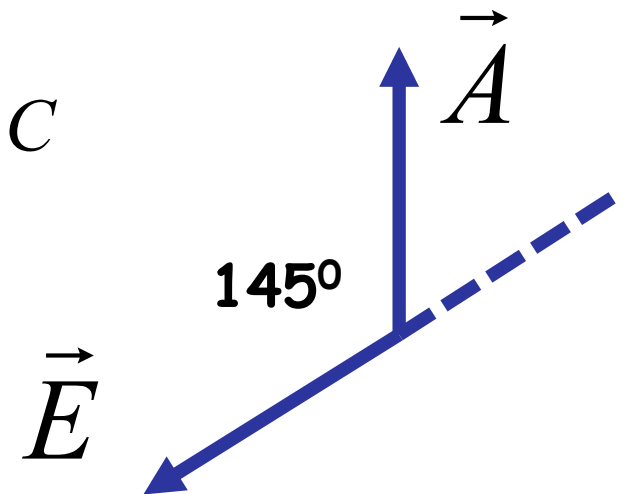
1, 7, 12, 13, 14, 17, 21, 22, 24, 35, 36, 39, 43, 44, 51,
52

1. The square surface as shown measures 3.2 mm on each side. It is immersed in a uniform electric field with $E = 1800 \text{ N/C}$ and with field lines at an angle of $\theta = 35^\circ$ with a normal to the surface. Calculate the electric flux through the surface.



$$\Phi = \vec{E} \cdot \vec{A} = EA \cos \theta = (1800 \text{ N/C}) \times (3.2 \times 10^{-3} \text{ m}) \cos(180^\circ - 35^\circ)$$

$$\Phi = -1.51 \times 10^{-2} \text{ Nm}^2 / \text{C}$$



7. A point charge of $1.8 \mu\text{C}$ is at the center of a cubical Gaussian surface 55 cm on edge. What is the net electric flux through the surface?

Using Gauss's law:

$$\epsilon_0 \Phi = q_{\text{enclosed}}$$

$$\Phi = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{1.8 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2} = 2.0 \times 10^5 \text{ Nm}^2 / \text{C}$$

14. A charged particle is suspended at the center of two concentric spherical shells that are very thin and made of nonconducting material. Figure a shows a cross section. Figure b gives the net flux Φ through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere. (a) What is the charge of the central particle? What are the net charges of (b) shell A and (c) shell B?

$$\epsilon_0 \Phi = q_{\text{enclosed}}$$

(a) For $r < r_A$ (region 1):

$$q_{\text{enclosed}1} = q_{\text{particle}} = \epsilon_0 \Phi_1$$

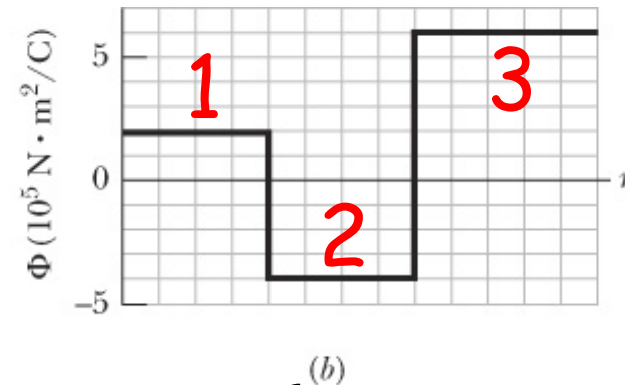
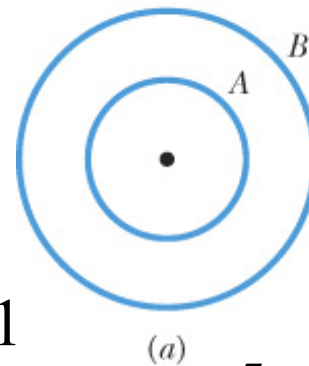
$$q_{\text{particle}} = 8.85 \times 10^{-12} \times 2 \times 10^5 = 1.77 \times 10^{-6} \text{ (C)}$$

$$\approx 1.8 \text{ (}\mu\text{C)}$$

(b) For $r_A < r < r_B$ (region 2): $q_{\text{enclosed}2} = q_{\text{particle}} + q_A = \epsilon_0 \Phi_2$

$$\Phi_2 = -4 \times 10^5 \text{ (Nm}^2\text{/C)} \Rightarrow q_A = -5.3 \times 10^{-6} \text{ (C) or } -5.3 \mu\text{C}$$

(c) For $r_B < r$ (region 3): $\Phi_3 = 6 \times 10^5 \text{ (Nm}^2\text{/C)} \Rightarrow q_B$



17. A uniformly charged conducting sphere of 1.2 m diameter has a surface charge density of $8.1 \mu\text{C}/\text{m}^2$. (a) Find the net charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

(a) charge = area \times surface density

$$q = 4\pi r^2 \sigma = 4 \times 3.14 \times 0.6^2 \times 8.1 \times 10^{-6} = 3.7 \times 10^{-5} (C)$$

(b) We choose a Gaussian surface covers whole the sphere, using Gauss' law:

$$\Phi = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{3.7 \times 10^{-5}}{8.85 \times 10^{-12}} = 4.2 \times 10^6 \text{ Nm}^2 / C$$

21. An isolated conductor of arbitrary shape has a net charge of $+10 \times 10^{-6} \text{ C}$. Inside the conductor is a cavity within which is a point charge $q = +3.0 \times 10^{-6} \text{ C}$. What is the charge (a) on the cavity wall and (b) on the outer surface of the conductor?

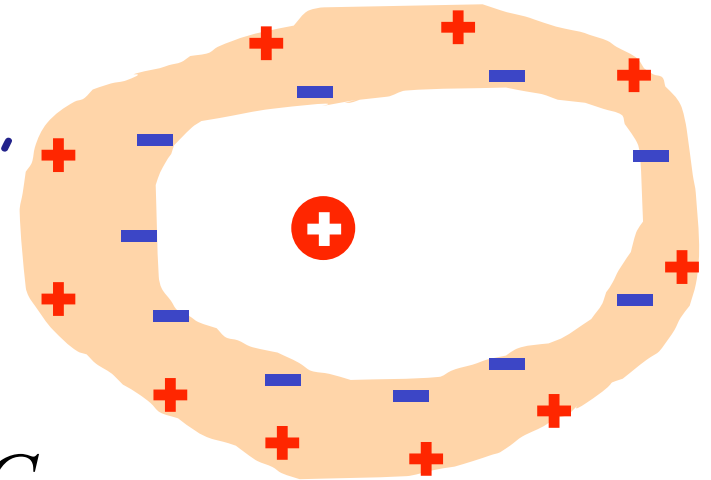
(a) Consider a Gaussian surface within the conductor that covers the cavity wall, in the conductor, $E = 0$:

$$q_{\text{wall}} + q_{\text{point}} = 0$$

$$q_{\text{wall}} = -q_{\text{point}} = -3 \times 10^{-6} \text{ C or } -3 \mu\text{C}$$

(b) the total charge of the conductor:

$$q_{\text{wall}} + q_{\text{outer}} = 10 \times 10^{-6} \Rightarrow q_{\text{outer}} = 13 \times 10^{-6} \text{ C or } 13 \mu\text{C}$$



22. An electron is released from rest at a perpendicular distance of 9 cm from a line of charge on a very long nonconducting rod. That charge is uniformly distributed, with $4.5 \mu\text{C}$ per meter. What is the magnitude of the electron's initial acceleration?

Electric field at point P:

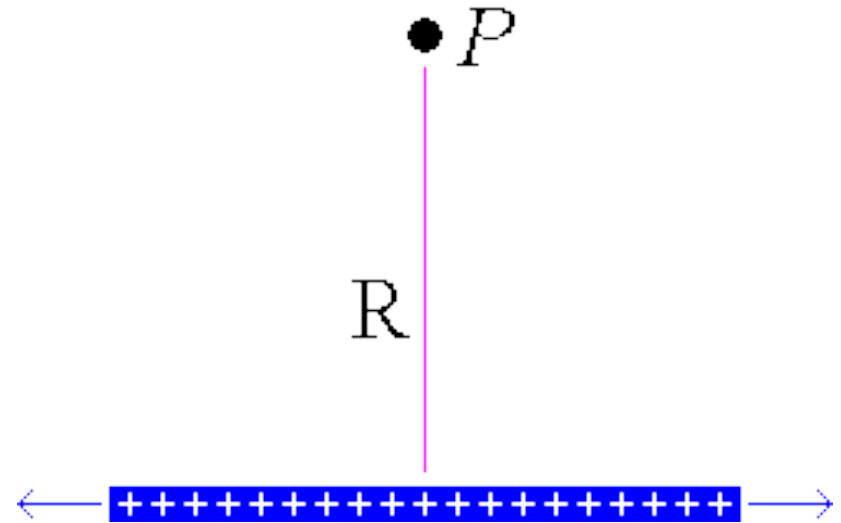
$$E = \frac{\lambda}{2\pi\epsilon_0 R}$$

Force acting on the electron:

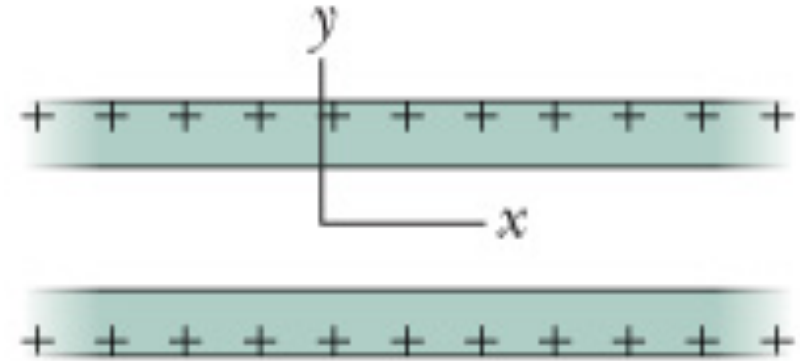
$$F = eE = \frac{e\lambda}{2\pi\epsilon_0 R} = ma \Rightarrow a = \frac{e\lambda}{2\pi\epsilon_0 mR}$$

$$R = 9 \text{ cm} = 0.09 \text{ m}$$

$$\lambda = 4.5 \mu\text{C/m} = 4.5 \times 10^{-6} \text{ C/m}$$



36. The figure shows cross sections through two large, parallel, nonconducting sheets with identical distributions of positive charge with surface charge density $\sigma = 2.31 \times 10^{-22} \text{ C/m}^2$. In unit-vector notation, what is \vec{E} at points (a) above the sheets, (b) between them, and (c) below them?



For **one** non-conducting sheet:

$$E = \frac{\sigma}{2\epsilon_0}$$

Using the superposition to calculate E due to **two** sheets:

(a)

$$E = 2 \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{2.31 \times 10^{-22}}{8.85 \times 10^{-12}} = 2.61 \times 10^{-11} \text{ (N/C)}$$

The net electric field direction is upward $\vec{E} = 2.61 \times 10^{-11} \text{ (N/C)} \hat{j}$

(b) $E = 0$

(c) $\vec{E} = -2.61 \times 10^{-11} \text{ (N/C)} \hat{j}$

The direction is downward

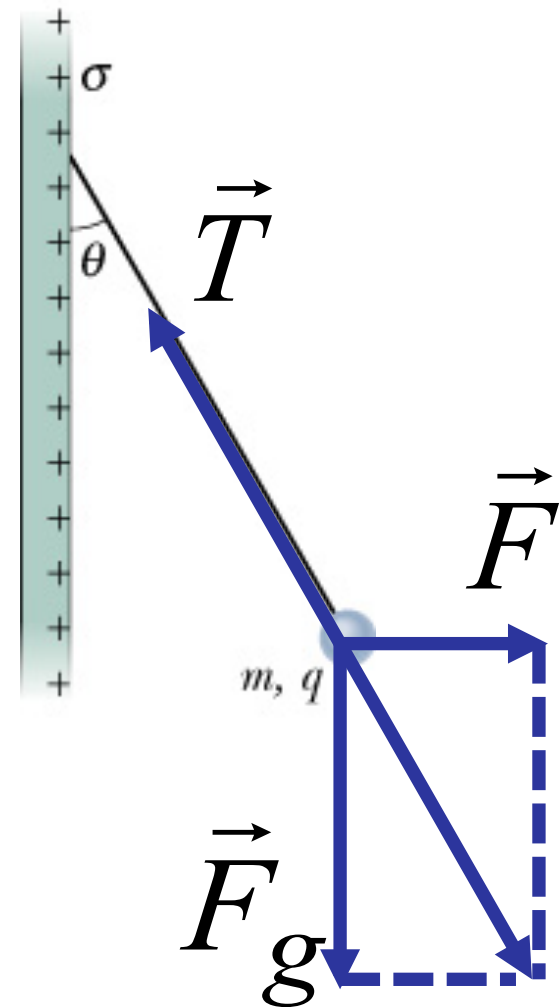
39. A small, nonconducting ball of mass $m = 1 \text{ mg}$ and charge $q = 2 \times 10^{-8} \text{ C}$ hangs from an insulating thread that makes an angle $\theta = 30^\circ$ with a vertical, uniformly charged nonconducting sheet. Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, calculate the surface charge density σ of the sheet.

If the ball is in equilibrium:

$$\vec{F} + \vec{F}_g + \vec{T} = 0$$

$$\tan \theta = \frac{F}{F_g} = \frac{qE}{mg} = \frac{q}{mg} \times \frac{\sigma}{2\epsilon_0}$$

$$\sigma = \frac{2\epsilon_0 mg \tan \theta}{q} = 5 \times 10^{-9} \text{ (C / m}^2\text{)}$$



44. The figure gives the magnitude of the electric field inside and outside a sphere with a positive charge distributed uniformly throughout its volume. What is the charge on the sphere?

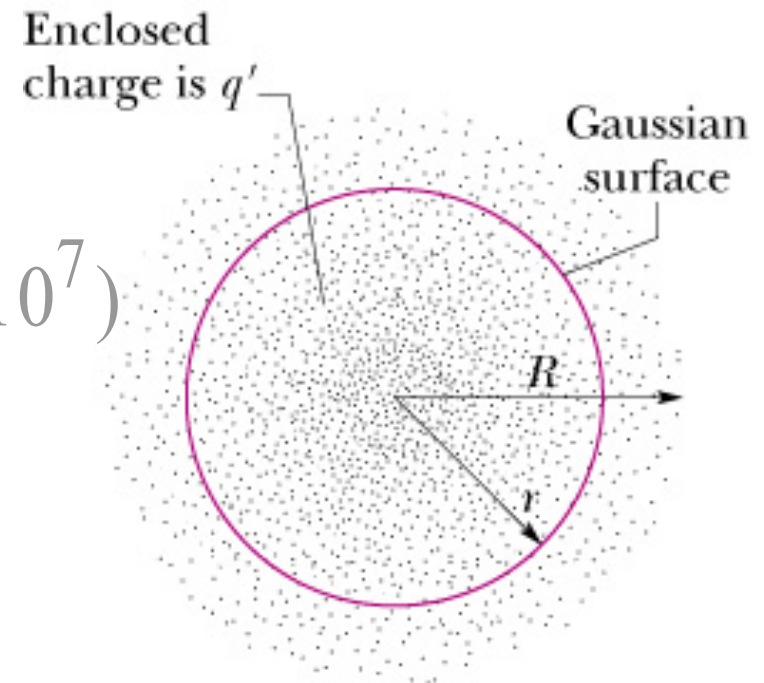
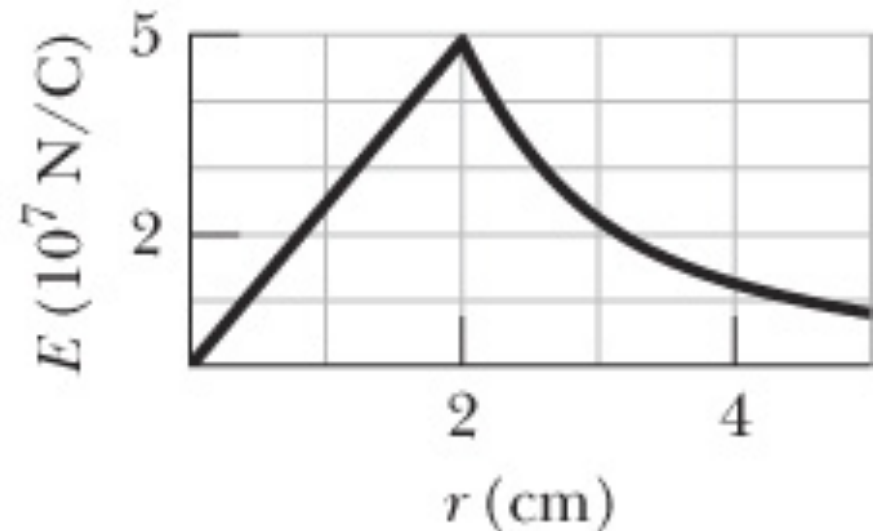
$$E = \left(\frac{q}{4\pi\epsilon_0 R^3} \right) r \quad (r \leq R)$$

- At $r = 2$ cm, E is maximum,
so $R = 2$ cm

$$E = \frac{q}{4\pi\epsilon_0 R^2}$$

$$q = 4 \times 3.14 \times 8.85 \times 10^{-12} \times (0.02)^2 \times (5 \times 10^7)$$

$$= 2.2 \times 10^{-6} \text{ (C)}$$



51. A nonconducting spherical shell of inner radius $a = 2$ cm and outer radius $b = 2.4$ cm has a positive volume charge density $\rho = A/r$, where A is a constant and r is the distance from the center of the shell. In addition, a small ball of charge $q = 45$ fC is located at that center. What value should A have if the electric field in the shell ($a \leq r \leq b$) is to be uniform?

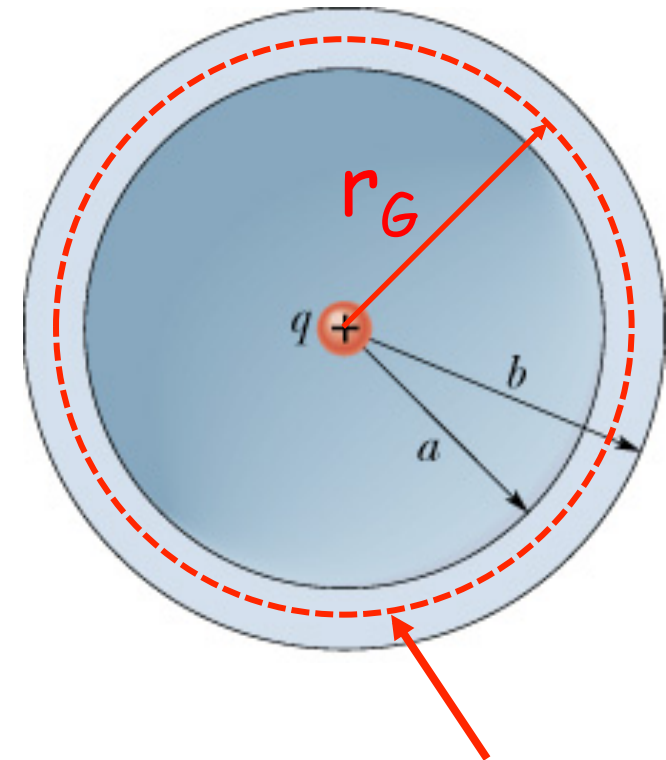
Key idea: First, we need to calculate E inside the shell, if the field is uniform, so E is independent of distance from the center

$$E_r = \frac{1}{4\pi\epsilon_0} \frac{q_{\text{total}}}{r^2}$$

$$q_{\text{total}} = q + q_{\text{shell}}$$

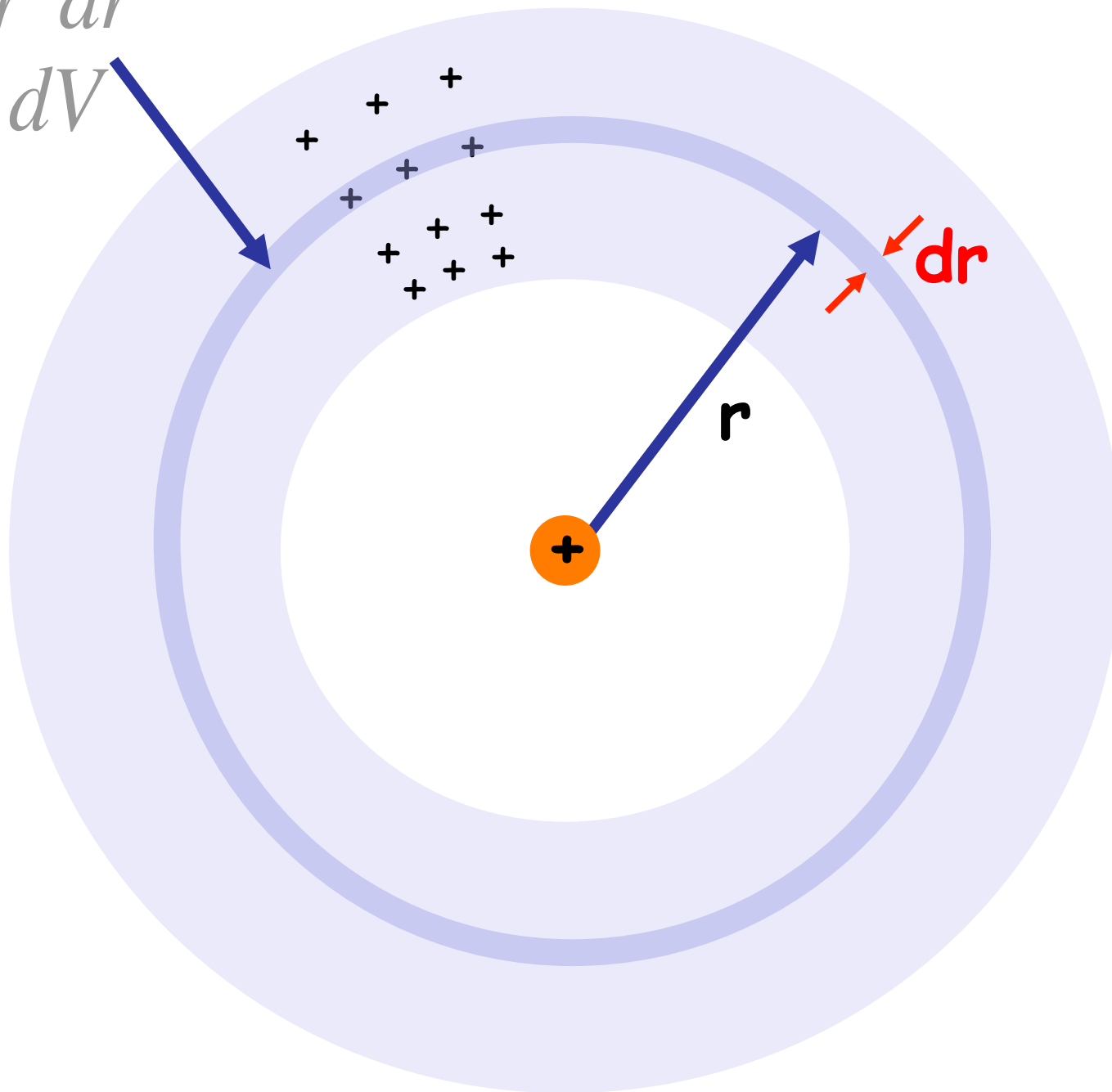
q_{shell} is the enclosed charge in the shell of thickness $r_G - a$: $dq_{\text{shell}} = \rho \times dV = \rho \times 4\pi r^2 dr$

$$q_{\text{shell}} = 4\pi \int_a^{r_G} \frac{A}{r} r^2 dr = 2\pi A(r_G^2 - a^2)$$



Gaussian surface

$$dV = 4\pi r^2 dr$$
$$dq = \rho \times dV$$



Using Gauss' law: $\epsilon_0 \Phi = q_{\text{total}}$

$$\epsilon_0 E 4\pi r_G^2 = q_{\text{total}}$$

$$E = \frac{q_{\text{total}}}{\epsilon_0 4\pi r_G^2} = \frac{q + 2\pi A(r_G^2 - a^2)}{4\pi\epsilon_0 r_G^2}$$

We rewrite:

$$E = \frac{A}{2\epsilon_0} + \frac{1}{2\epsilon_0} \left(\frac{q}{2\pi} - Aa^2 \right) \times \frac{1}{r_G^2}$$

If E is uniform in the shell:

$$\frac{q}{2\pi} - Aa^2 = 0 \Rightarrow A = \frac{q}{2\pi a^2}$$

$$A = \frac{45 \times 10^{-15} \text{ C}}{2 \times 3.14 \times (0.02 \text{ m})^2} = 1.79 \times 10^{-11} (\text{C} / \text{m}^2)$$

52. The figure below shows a spherical shell with uniform volume charge density $\rho = 1.56 \text{ nC/m}^3$, inner radius $a = 10 \text{ cm}$, and outer radius $b = 2a$. What is the magnitude of the electric field at radial distances (a) $r = 0$, (b) $r = a/2$, (c) $r = a$, (d) $r = 1.5 a$, (e) $r = b$, and (f) $r = 3b$?

For (a), (b), (c) using Gauss's law, we find

$$E = 0$$

For (d), (e) $a \leq r \leq b$:

The enclosed charge:

$$q_{enc} = \rho \times V = \rho \left(\frac{4}{3} \pi r^3 - \frac{4}{3} \pi a^3 \right)$$

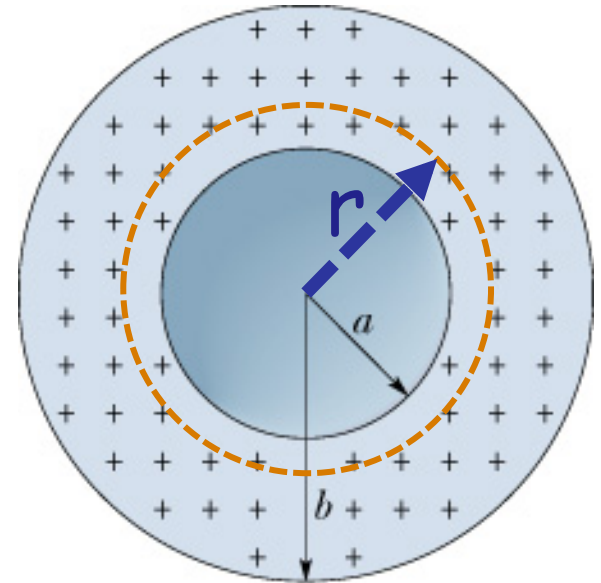
The electric field:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q_{enc}}{r^2}$$

For (f):

$$E = \frac{1}{4\pi\epsilon_0} \frac{q_{total}}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\rho \times \frac{4}{3} \pi (b^3 - a^3)}{r^2}$$

$$E = \frac{\rho}{3\epsilon_0} \frac{b^3 - a^3}{r^2}$$



Homework (lecture 5):

1, 6, 8, 11, 14, 18, 24, 28, 29, 35, 43, 59, 60, 64

1. A particular 12 V car battery can send a total charge of 84 A.h through a circuit, from one terminal to the other. (a) How many coulombs of charge does this represent? (b) If this entire charge undergoes a change in electric potential of 12 V, how much energy is involved?

(a) In the previous lecture, we mentioned that the coulomb unit is derived from ampere for electric current i:

$$i = \frac{dq}{dt} \Rightarrow dq = i dt$$

$$Q = 84(\text{C/s}) \times 3600(\text{s}) = 3 \times 10^5 (\text{C})$$

(b) Energy is computed by:

$$\Delta U = \Delta V \times Q = 12 \times 3 \times 10^5 = 3.6 \times 10^6 (\text{J})$$

6. When an electron moves from A to B along an electric field, see the figure. The electric field does 4.78×10^{-19} J of work on it. What are the electric potential differences (a) $V_B - V_A$, (b) $V_C - V_A$, and (c) $V_C - V_B$?

(a) We have work done by the electric field:

$$W = -q\Delta V$$

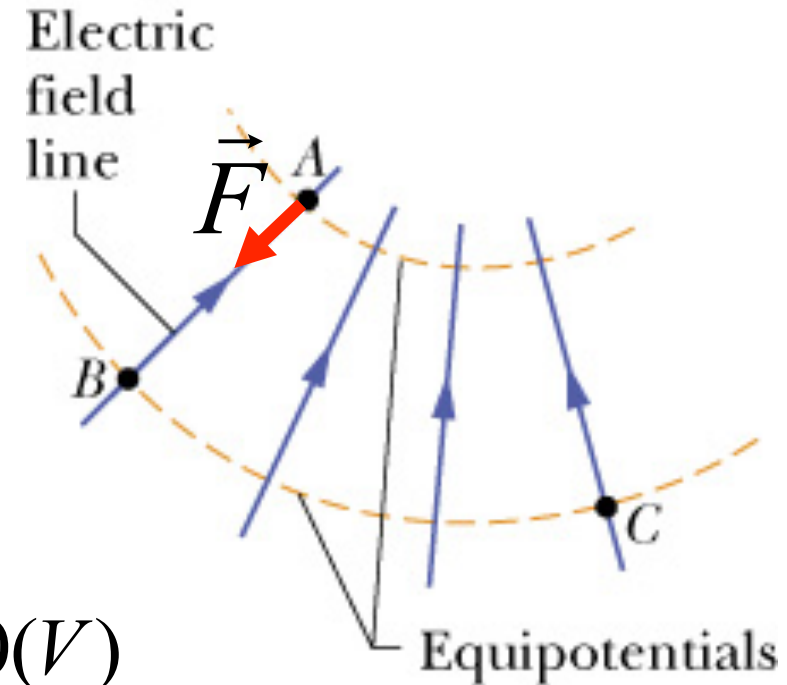
$$W = -(-e)(V_B - V_A)$$

$$V_B - V_A = \frac{W}{e} = \frac{4.78 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.0(V)$$

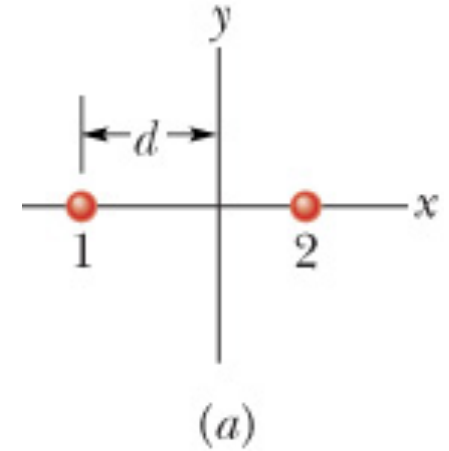
(b)

$$V_C - V_A = V_B - V_A = 3.0(V)$$

(c) $V_C - V_B = 0$: on the same equipotential



18. Two charged particles are shown in Figure a. Particle 1, with charge q_1 , is fixed in place at distance d . Particle 2, with charge q_2 , can be moved along the x axis. Figure b gives the net electric potential V at the origin due to the two particles as a function of the x coordinate of particle 2. The plot has an asymptote of $V = 5.92 \times 10^{-7} \text{ V}$ as $x \rightarrow \infty$. What is q_2 in terms of e ?



Potential due to a point charge: $V = k \frac{q}{r}$

Potential at the origin (O) due to q_1 and q_2 :

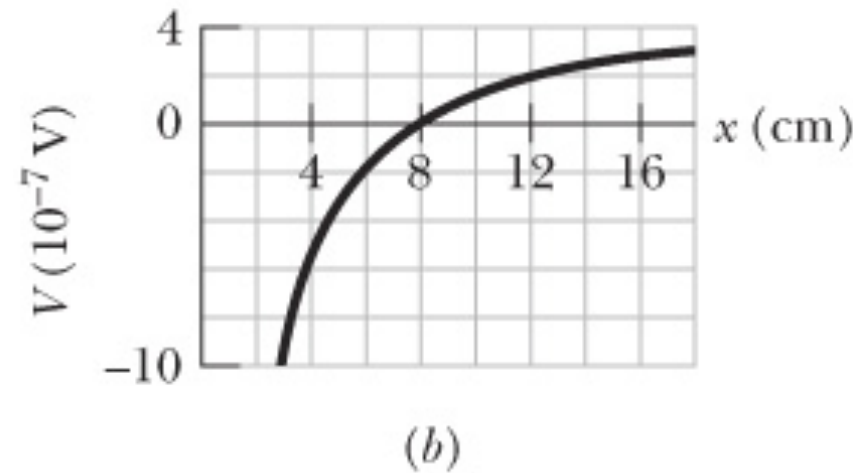
$$V_O = k \frac{q_1}{d} + k \frac{q_2}{x}$$

$$V_{O,x=\infty} = k \frac{q_1}{d} = 5.92 \times 10^{-7} \text{ (V)}$$

At $x = 8 \text{ cm}$, $V_O = 0$:

$$V_{O,x=8} = V_{O,x=\infty} + k \frac{q_2}{x}$$

$$q_2 = -\frac{V_{O,x=\infty} x}{k} = -\frac{5.92 \times 10^{-7} \times 0.08}{8.99 \times 10^9} = -5.27 \times 10^{-18} \text{ (C) or } -33e$$



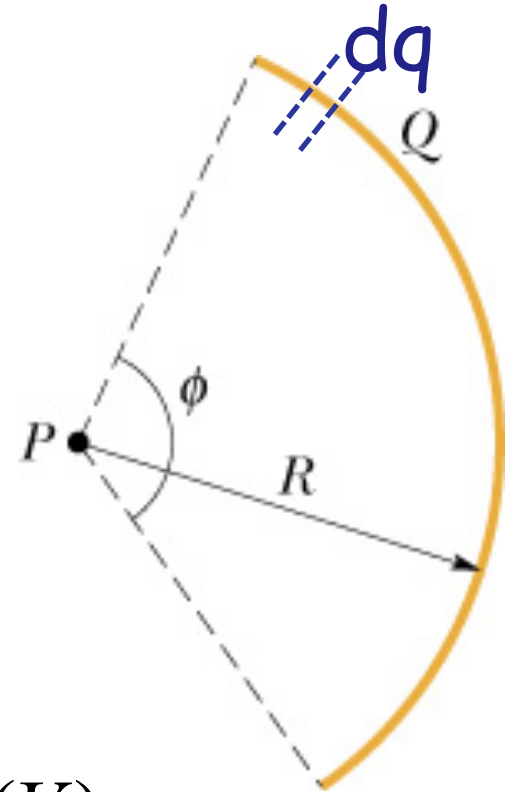
24. The figure shows a plastic rod having a uniformly distributed charge $Q = -28.9 \text{ pC}$ has been bent into a circular arc of radius $R = 3.71 \text{ cm}$ and central angle $\Phi = 120^\circ$. With $V=0$ at infinity, what is the electric potential at P , the center of curvature of the rod?

Consider potential at P due to an element dq :

$$dV = k \frac{dq}{R}$$

$$V = \int k \frac{dq}{R} = k \frac{Q}{R}$$

$$V = \frac{8.99 \times 10^9 \times (-28.9 \times 10^{-12})}{3.71 \times 10^{-2}} = -7.0(V)$$



35. The electric potential at points in an xy plane is given by $V = (2 \text{ V/m}^2)x^2 - (3 \text{ V/m}^2)y^2$. In unit vector notation, what is the electric field at the point (3.0 m, 2.0 m)?

We have:

$$\vec{E} = -\nabla V$$

$$E_x = -\frac{\partial V}{\partial x}; E_y = -\frac{\partial V}{\partial y}$$

$$E_x = -4x = -12(V/m); E_y = 6y = 12(V/m)$$

$$\vec{E} = -12(V/m)\hat{i} + 12(V/m)\hat{j}$$

43. How much work is required to set up the arrangement of the figure below if $q = 2.3 \text{ pC}$, $a = 64 \text{ cm}$, and the particles are initially infinitely far apart and at rest?

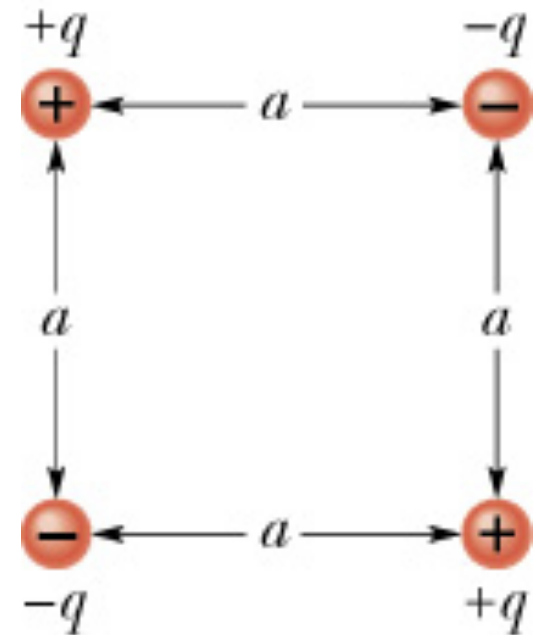
We have 4 charges, so we have $N = 6$ pairs:

$$N = \frac{n(n-1)}{2}$$

$$W_{\text{applied}} = U_{\text{system}}$$

$$U_{\text{system}} = kq^2 \left(-\frac{1}{a} - \frac{1}{a} + \frac{1}{a\sqrt{2}} - \frac{1}{a} + \frac{1}{a\sqrt{2}} - \frac{1}{a} \right)$$

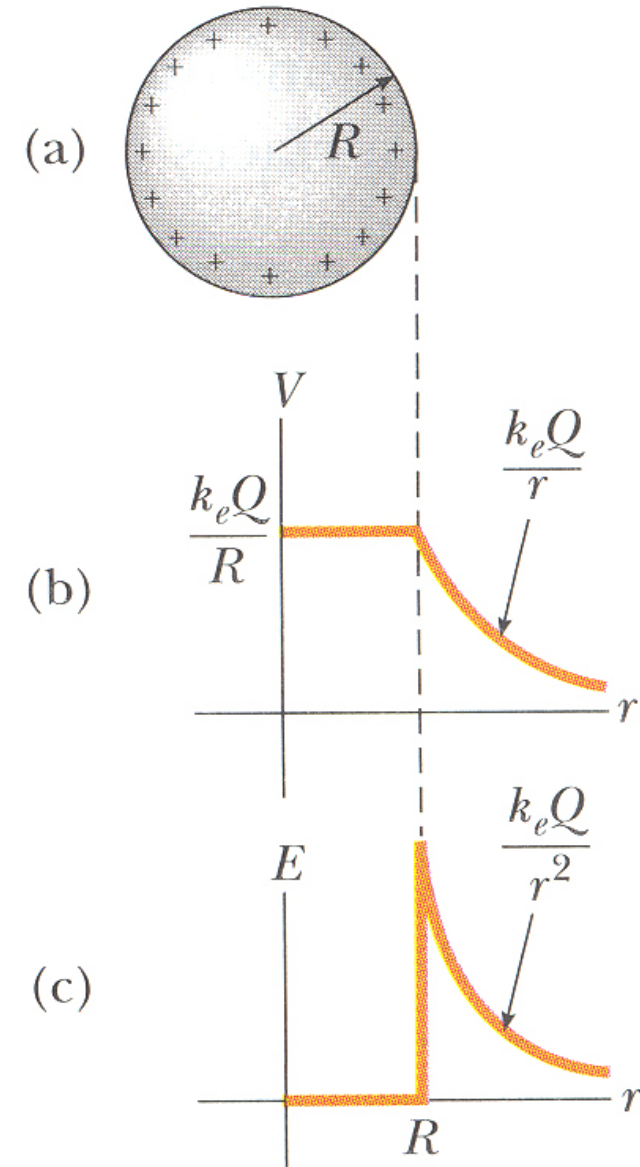
$$U_{\text{system}} = \frac{2kq^2}{a} \left(\frac{1}{\sqrt{2}} - 2 \right)$$



Note: $q = 2.3 \text{ pC} = 2.3 \times 10^{-12} \text{ C}$; $a = 64 \text{ cm} = 0.64 \text{ m}$

64. A hollow metal sphere has a potential of $+300\text{ V}$ with respect to ground (defined to be at $V = 0$) and a charge of $5.0 \times 10^{-9}\text{ C}$. Find the electric potential at the center of the sphere.

$V = \text{constant} = +300\text{ V}$ throughout the entire conductor, this is valid for solid and hollow metal spheres.



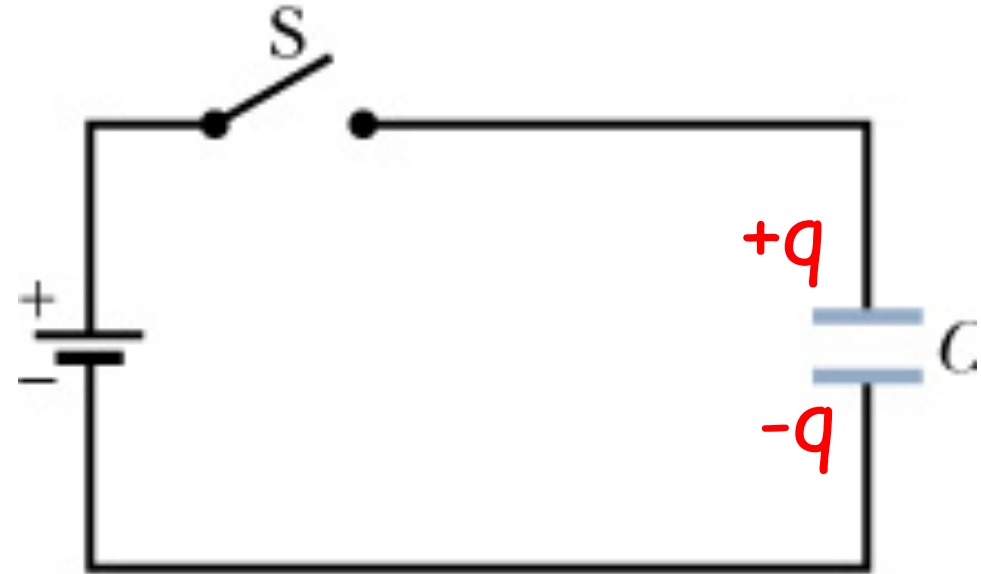
Homework (lecture 6):

2, 4, 6, 11, 14, 16, 26, 31, 33, 42, 48, 51

2. The capacitor in the figure below has a capacitance of $30\ \mu\text{F}$ and is initially uncharged. The battery provides a potential difference of $120\ \text{V}$. After switch S is closed, how much charge will pass through it?

S is closed, the charge on the capacitor plates is:

$$q = CV$$



$$q = 30 \times 10^{-6} \times 120 = 3.6 \times 10^{-3} (C)$$

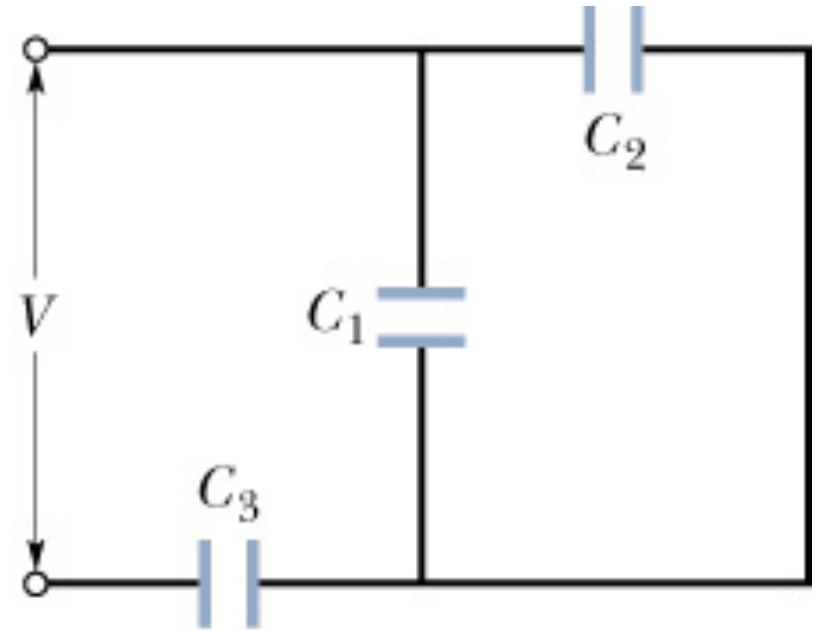
11. In the figure below, find the equivalent capacitance of the combination. Assume that $C_1 = 10.0 \mu\text{F}$, $C_2 = 5.0 \mu\text{F}$, and $C_3 = 4.0 \mu\text{F}$

C_1 and C_2 are in parallel, the equivalent capacitance :

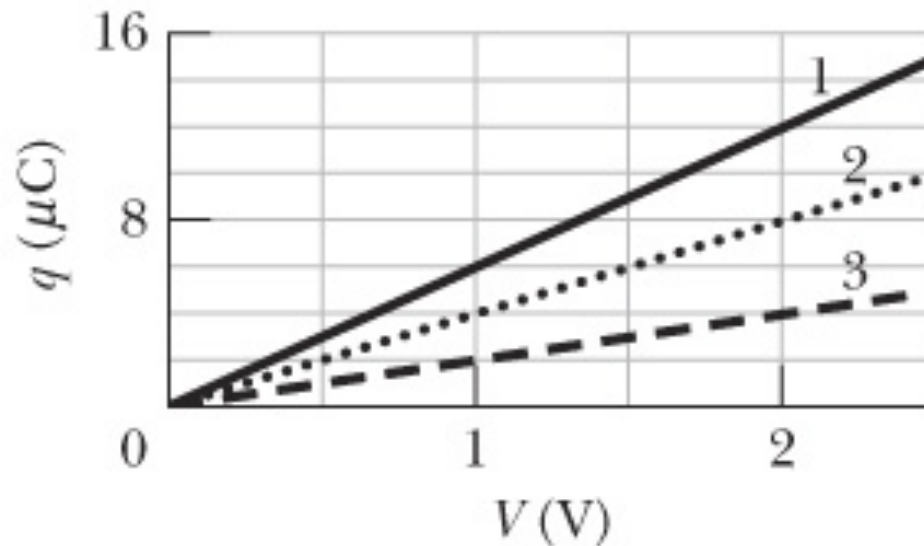
$$C_{12} = C_1 + C_2 = 15(\mu\text{F})$$

C_{12} and C_3 in series:

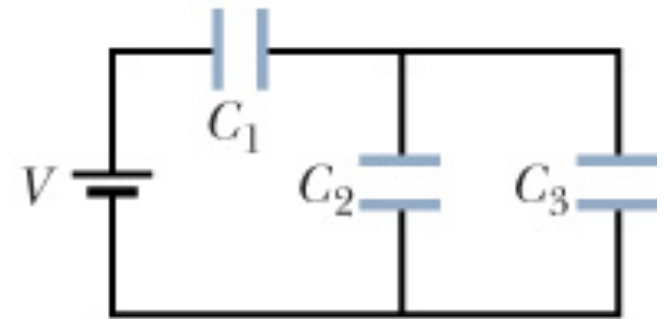
$$C_{123} = \frac{C_{12}C_3}{C_{12} + C_3} = \frac{15 \times 4}{15 + 4} = 3.16(\mu\text{F})$$



16. Plot 1 in Figure a gives the charge q that can be stored on capacitor 1 versus the electric potential V set up across it. Plots 2 and 3 are similar plots for capacitors 2 and 3, respectively. Figure b shows a circuit with those three capacitors and a 10.0 V battery. What is the charge stored on capacitor 2 in that circuit?



(a)



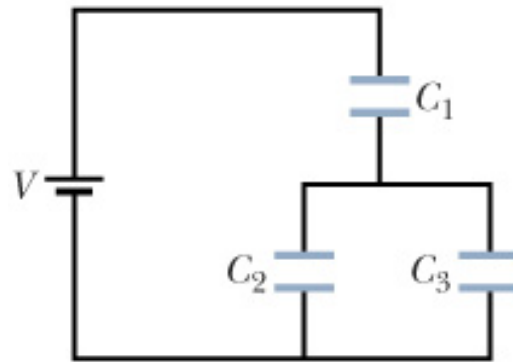
(b)

$$C_1 = \frac{q_1}{V_1} = \frac{12(\mu\text{C})}{2(\text{V})} = 6\mu\text{F}; C_2 = \frac{q_2}{V_2} = \frac{8(\mu\text{C})}{2(\text{V})} = 4\mu\text{F}; C_3 = \frac{q_3}{V_3} = \frac{4(\mu\text{C})}{2(\text{V})} = 2\mu\text{F}$$

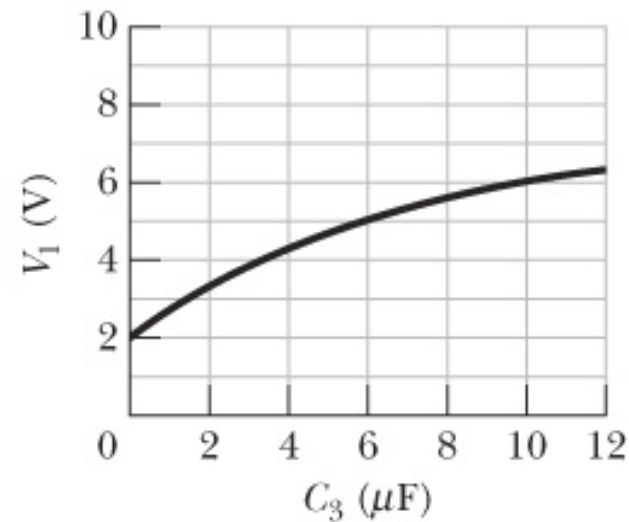
$$C_{123} = 3(\mu\text{F})$$

$$V_1 = \frac{q}{C_1} = \frac{C_{123}V}{C_1} = \frac{1}{2}10 = 5(\text{V}) \Rightarrow q_2 = C_2V_2 = 4\mu\text{F} \times 5\text{V} = 20\mu\text{C}$$

26. Capacitor 3 in Figure a is a variable capacitor (its capacitance C_3 can be varied). Figure b gives the electric potential V_1 across capacitor 1 versus C_3 . Electric potential V_1 approaches an asymptote of 8 V as $C_3 \rightarrow \infty$. What are (a) the electric potential V across the battery, (b) C_1 , and (c) C_2 ?



(a)



(b)

(a) When $C_3 \rightarrow \infty$, $C_{123} = C_1$; so, $V = V_1 = 8$ V

(b)

$$C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{C_1 (C_2 + C_3)}{C_1 + C_2 + C_3};$$

$$V_1 = \frac{q}{C_1} = \frac{C_{123}V}{C_1} = \frac{C_2 + C_3}{C_1 + C_2 + C_3} V$$

• At $C_3 = 0$, $V_1 = 2 \text{ V}$:

$$C_1 = 3C_2$$

• At $C_3 = 6 \mu\text{F}$, $V_1 = 5 \text{ V}$:

$$V_1 = \frac{C_2 + 6}{3C_2 + C_2 + 6} 8 = 5$$

$$C_2 = 1.5 \mu\text{F}; C_1 = 4.5 \mu\text{F}$$

33. A charged isolated metal sphere of diameter 10 cm has a potential of 8000 V relative to $V = 0$ at infinity. Calculate the energy density in the electric field near the surface of the sphere.

In a general case, the energy density is computed by:

$$u = \frac{1}{2} \epsilon_0 E^2$$

For a charged isolated metal sphere:

$$u = \frac{1}{2} \epsilon_0 \left(\frac{V}{R} \right)^2 = \frac{1}{2} 8.85 \times 10^{-12} \left(\frac{8000}{0.05} \right)^2 = 0.113 (\text{J/m}^3)$$

42. A parallel-plate air-filled capacitor has a capacitance of 50 pF: (a) If each of its plates has an area of 0.30 m², what is the separation? (b) If the region between the plates is now filled with material having $\kappa = 5.6$, what is the capacitance?

(a) For parallel-plate capacitors: $C = \frac{\epsilon_0 A}{d}$

$$d = \frac{\epsilon_0 A}{C} = \frac{8.85 \times 10^{-12} \times 0.30}{50 \times 10^{-12}} = 5.3 \times 10^{-2} (m) = 5.3 (cm)$$

(b) With a dielectric: $C' = \kappa C = 5.6 \times 50 = 280 (pF)$

48. The figure below shows a parallel-plate capacitor with a plate area $A = 5.56 \text{ cm}^2$ and separation $d = 5.56 \text{ mm}$. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 7.00$; the right half is filled with material of dielectric constant $\kappa_2 = 12.0$. What is the capacitance?

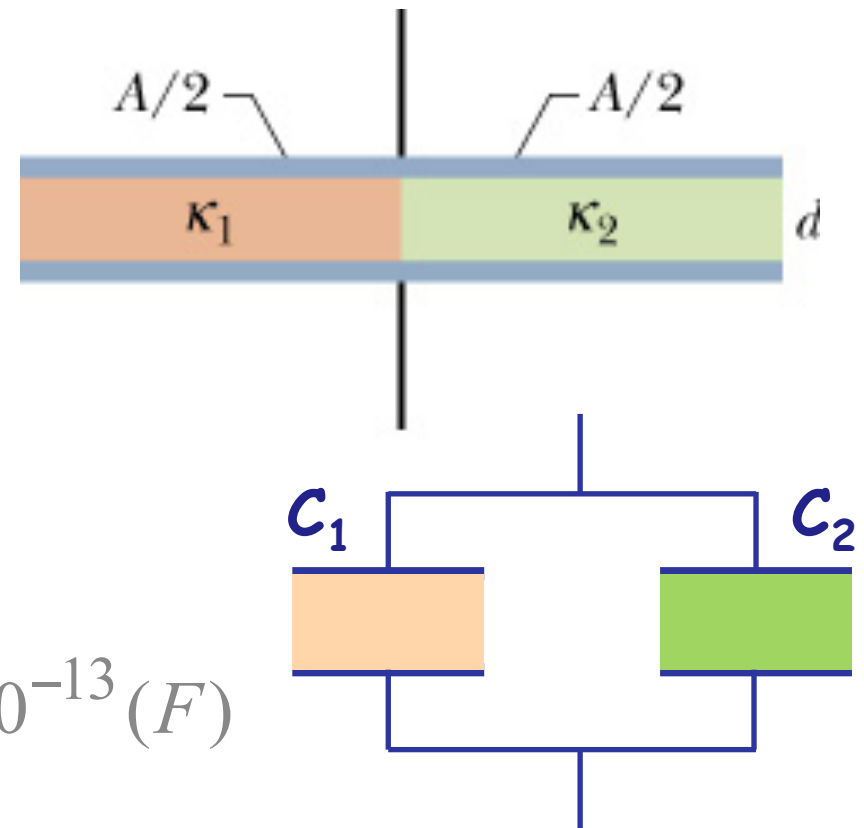
Their configuration is equivalent to a combination of **two capacitors in parallel** with dielectrics κ_1 and κ_2 , respectively

$$C_0 = \frac{\epsilon_0(A/2)}{d} =$$

$$= \frac{8.85 \times 10^{-12} \times 5.56 \times 10^{-4}}{2 \times 5.56 \times 10^{-3}} = 4.43 \times 10^{-13} (F)$$

$$C_0 = 0.443 \text{ pF}$$

$$C_{\text{equivalent}} = C_1 + C_2 = (\kappa_1 + \kappa_2)C_0 = 8.42(\text{pF})$$



equivalent capacitor