

Part B: Electromagnetism

Chapter 4: Magnetism

4.1. The Magnetic Field

4.2. The Hall Effect

4.3. Motion of a Charged Particle in a Magnetic Field

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Overview

In this part, we study the science of magnetic fields, including what produces a magnetic field, how a magnetic field can produce a magnetic force on a moving charged particle. The applications of magnetic fields are countless, e.g., audiotape, videotape, CD, DVD players, TVs, computers, telephones, medical devices,...

4.1. The Magnetic Field

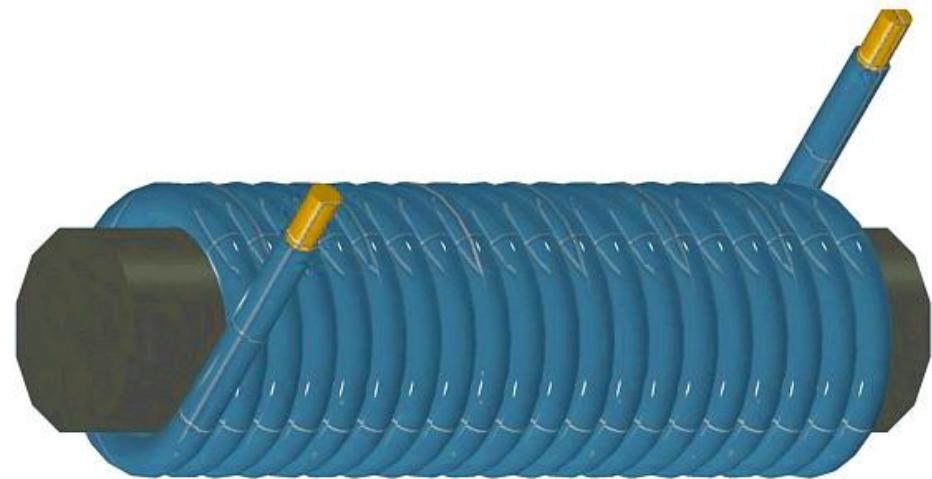
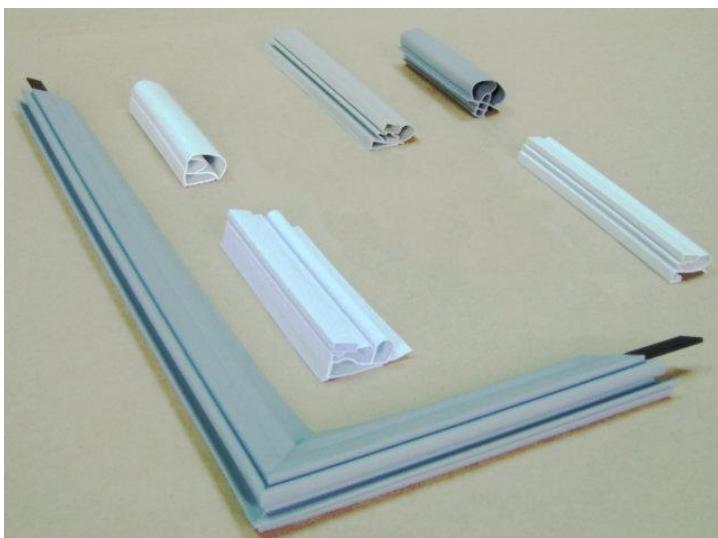
4.1.1. The Magnetic Field:

a. What Produces a Magnetic Field?

- We have discussed how a charged plastic rod produces a vector field - the electric field at all points in the space around the rod
- Here, we have a magnet that produces a vector field - the magnetic field \vec{B} at all points in the space around the magnet

Examples:

- Permanent magnets: magnets at the door of refrigerators
- Electromagnets: a wire core coil is wound around an iron core and a current is sent through the coil. The magnetic field is produced by the flow of electric current, and the magnetic field disappears when the current is turned off



In the previous chapters, we have studied that an electric charge sets up an electric field that affects other electric charges. Here, we might expect that a **magnetic charge** sets up a magnetic field that can then affect other magnetic charges. Such a magnetic charge is called a magnetic monopole (a new elementary particle?) predicted by certain theories but its existence has NOT been confirmed.

So, the key question is “**How then are magnetic fields set up?**”

There are two ways:

- (1) Moving electrically charged particles, e.g., a current in a wire, creating magnetic fields
- (2) Elementary particles such as protons, electrons, have an **intrinsic** magnetic field around them. This field is a basic characteristic of the particles such as their mass, their electric charge.

In some materials, the magnetic fields of electrons add together to give a net magnetic field around the material (magnets)

b. The Definition of \vec{B} :

As the existence of the magnetic charge has not been confirmed, so we can not use the way of definition of electric field $\vec{E} = \frac{\vec{F}}{q}$ here.

We must use another way to define \vec{B} :

- firing a charged particle through the point at which B is to be defined with various directions and speeds
- determining the force \vec{F}_B acting on the particle at that point
- After many such trials, we can find when the particle's velocity is along a particular axis, \vec{F}_B is zero. For all other directions, $F_B \sim v \sin \phi$, where ϕ is the angle between the zero-force axis and \vec{v} . And the direction of \vec{F}_B is always perpendicular to \vec{v}

$$B = \frac{F_B}{|q|v}$$

We can summarize all the results above by:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$F_B = |q| v B \sin \phi$$

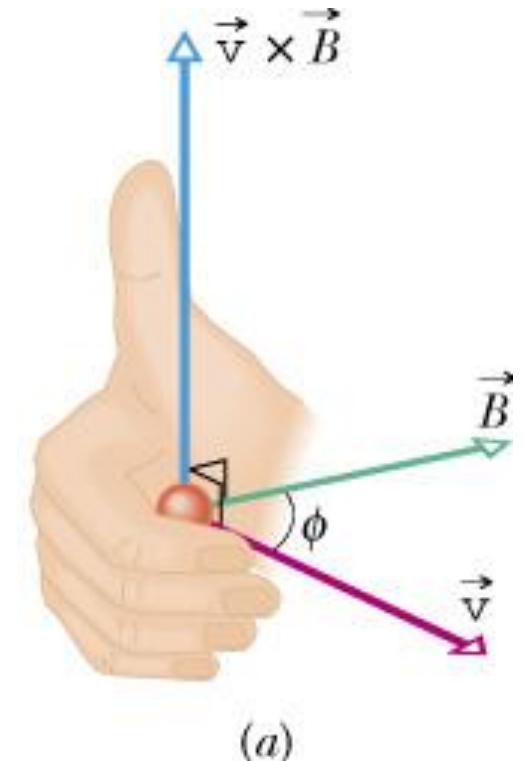
ϕ is the angle between the directions of \vec{v} and \vec{B}

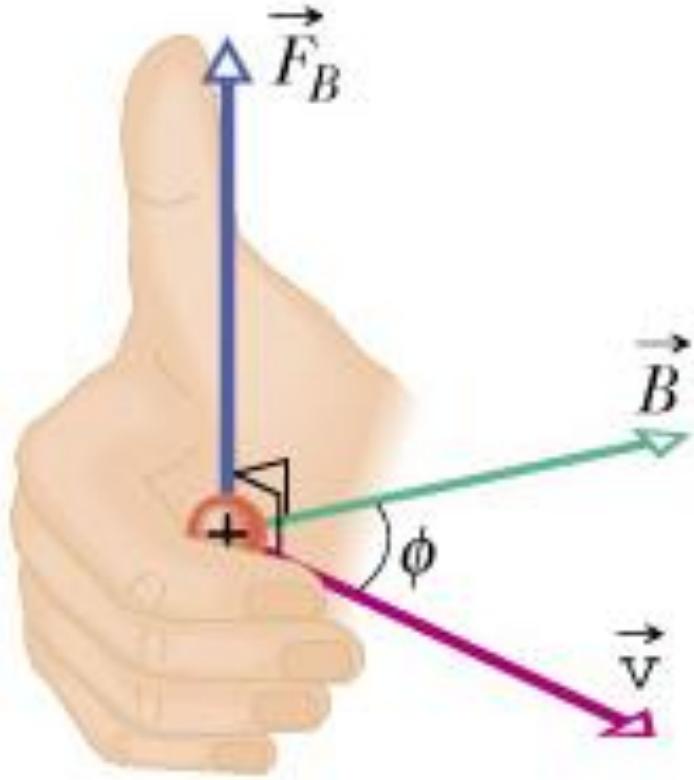
c. The Magnetic Force Acting on a Particle:

We use the right-hand rule to determine the \vec{F}_B direction:
the fingers (of the right-hand) sweep \vec{v} into \vec{B}
through the **smaller** angle ϕ , the thumb points
in the direction of $\vec{v} \times \vec{B}$, then we consider the
sign of charge q

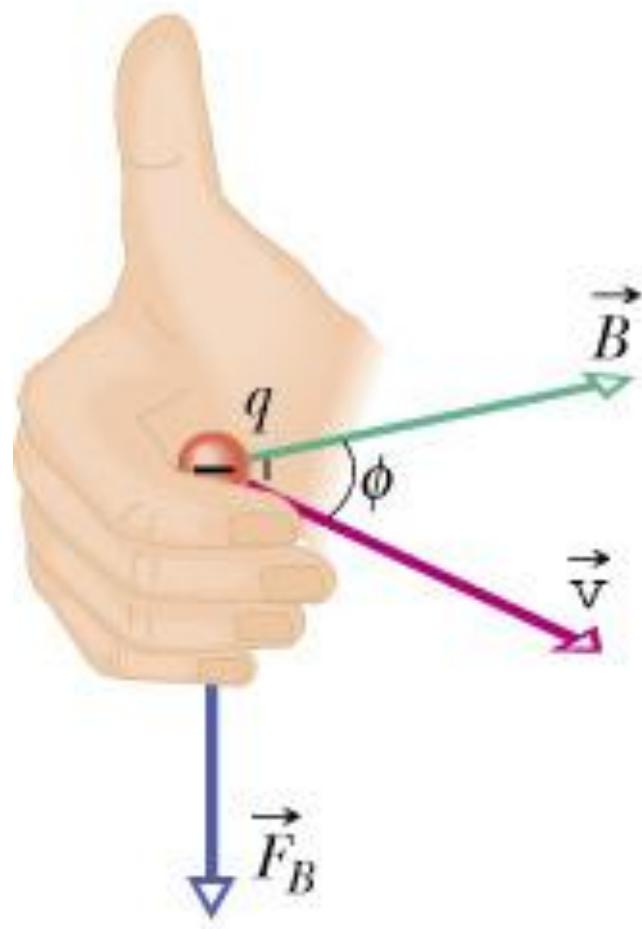
Two special cases:

- $F_B = 0$ if $\phi = 0^\circ$ or $\phi = 180^\circ$
- F_B is maximum if $\phi = 90^\circ$





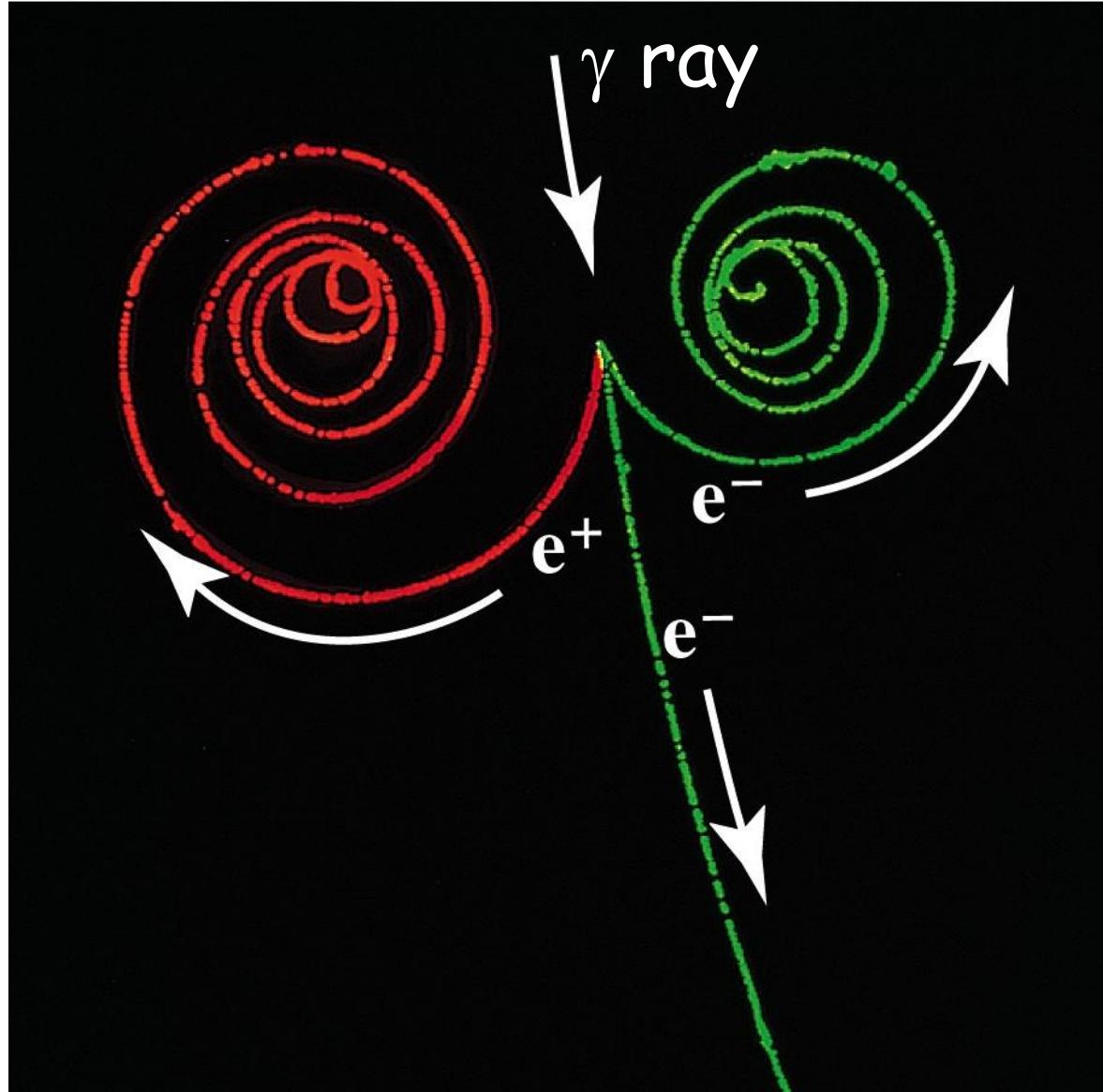
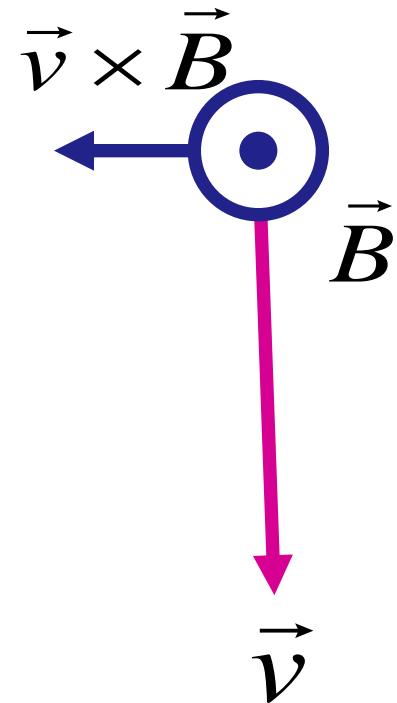
(b)



(c)

- If $q > 0$ (figure b): the force is directed along the thumb
- If $q < 0$ (figure c): the force is directed opposite the thumb

The tracks of two electrons (e^-) and a positron (e^+) in a bubble chamber in a uniform magnetic field, which is directed out of the plane of the page



- The SI unit for B is called the **tesla (T)**:

$$1 \text{ tesla} = 1 \text{ T} = 1 \frac{\text{newton}}{(\text{coulomb})(\text{meter/second})}$$

$$1 \text{ T} = 1 \frac{\text{newton}}{(\text{coulomb/second})(\text{meter})} = 1 \frac{\text{N}}{\text{A.m}}$$

Table 28-1

Some Approximate Magnetic Fields

At surface of neutron star	10^8 T
Near big electromagnet	1.5 T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in magnetically shielded room	10^{-14} T

- An earlier non-SI unit for B is the **gauss (G)**:

$$1 \text{ tesla} = 10^4 \text{ gauss}$$

d. Magnetic Field Lines:

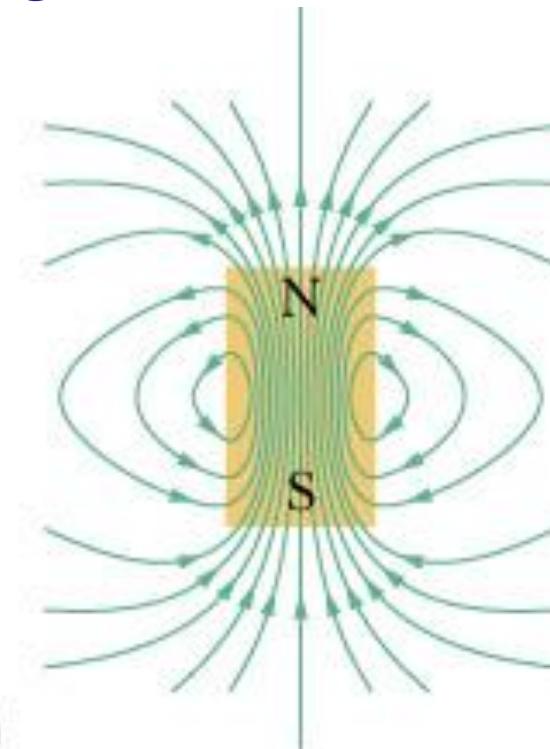
Magnetic fields can be represented by field lines with the following rule:

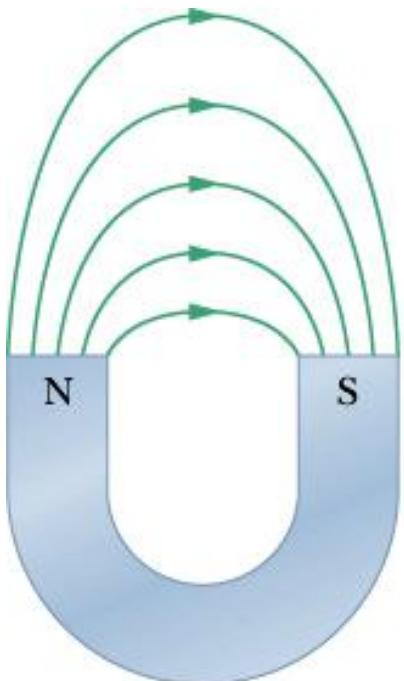
(a) the direction of the tangent to a magnetic field line at any point gives the direction of B at that point

(b) the spacing of the lines represents the magnitude of B , it means the magnetic field is stronger where the lines are closer together, and conversely (see **Figure a** for a bar magnet).

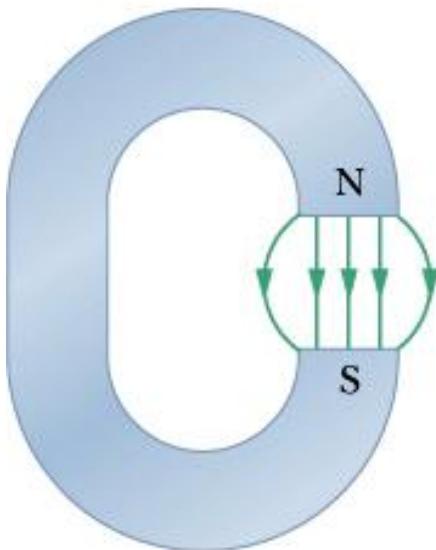
Note that:

- All the lines pass through the magnet and they all form closed loops (even those not shown closed in the figure)
- A magnet has two poles (**magnetic dipole**): north pole and south pole. The field lines emerge from the north pole and enter the south pole
- Opposite magnetic poles attract each other, and like magnetic poles repel each other

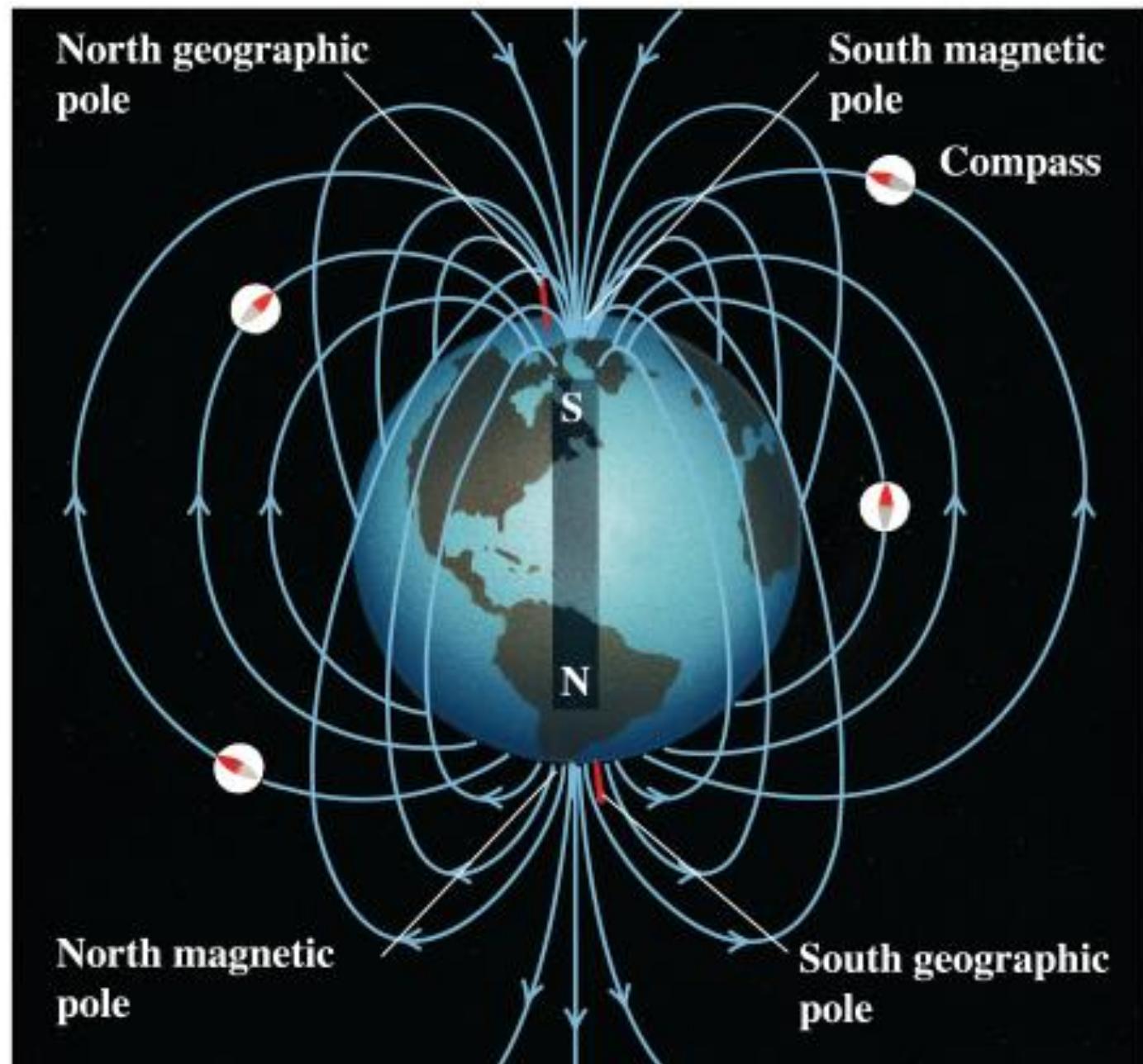




(a)
A horseshoe magnet



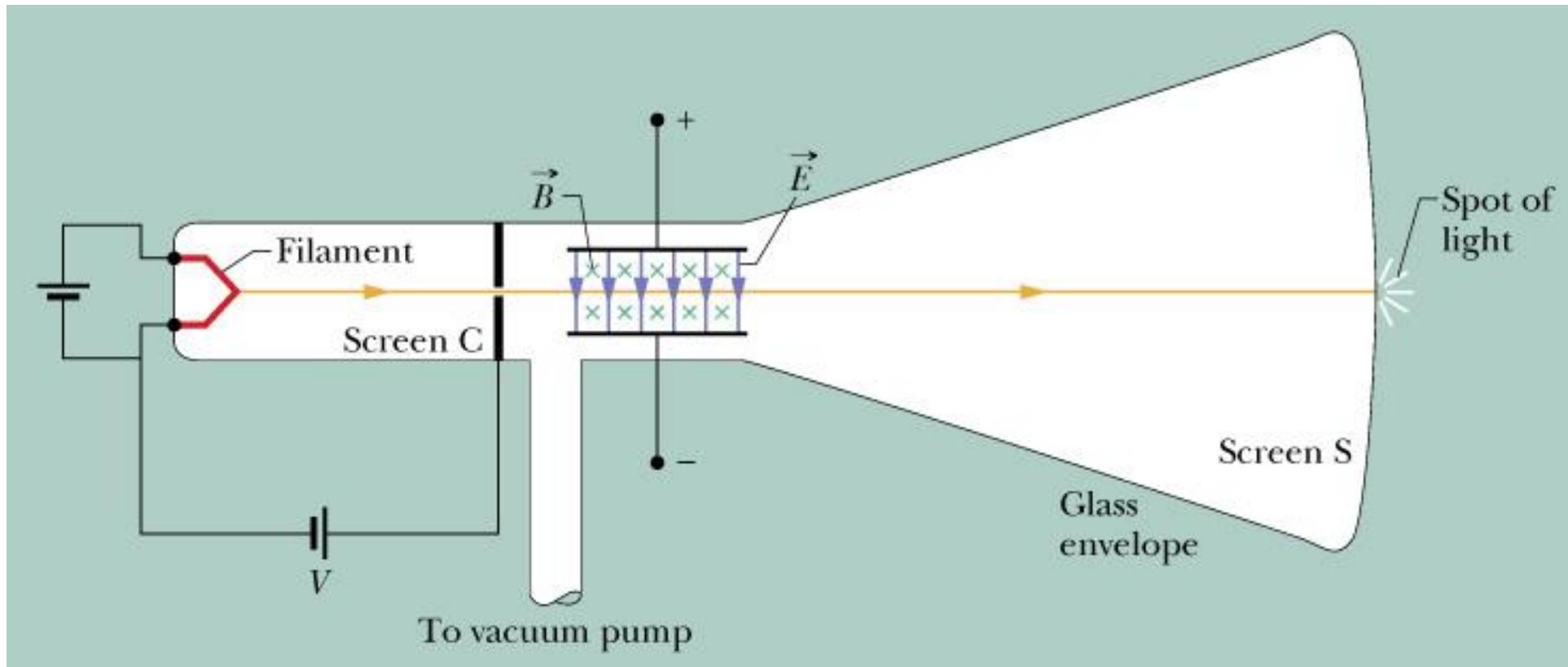
(b)
A C-shaped magnet



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e. Thomson's Experimental Apparatus: Discovery of the Electron

Crossed fields: An electric field E and a magnetic field B can produce a force on a charged particle, when they are perpendicular to each other, they are crossed fields.



A modern version of **Thomson's apparatus** for measuring the ratio of mass to charge for the electron (considered to be the "discovery of the electron")

In this arrangement, electrons from the hot filament are forced up by electric field E and down by magnetic field B , so the forces are in opposition

Thomson carried out the following steps:

1. Set $E = 0$ and $B = 0$, no deflection of the electron beam
2. Turn on E and measure the beam deflection (see Sample problem, p.593)

$$y = \frac{|q| E L^2}{2 m v^2} \quad (1)$$

L: length of the plates

3. Maintaining E , turn on B and adjust its value until the beam returns to the undeflected position, $F_E = F_B$

$$|q| E = |q| v B \sin(90^\circ) = |q| v B$$

So:

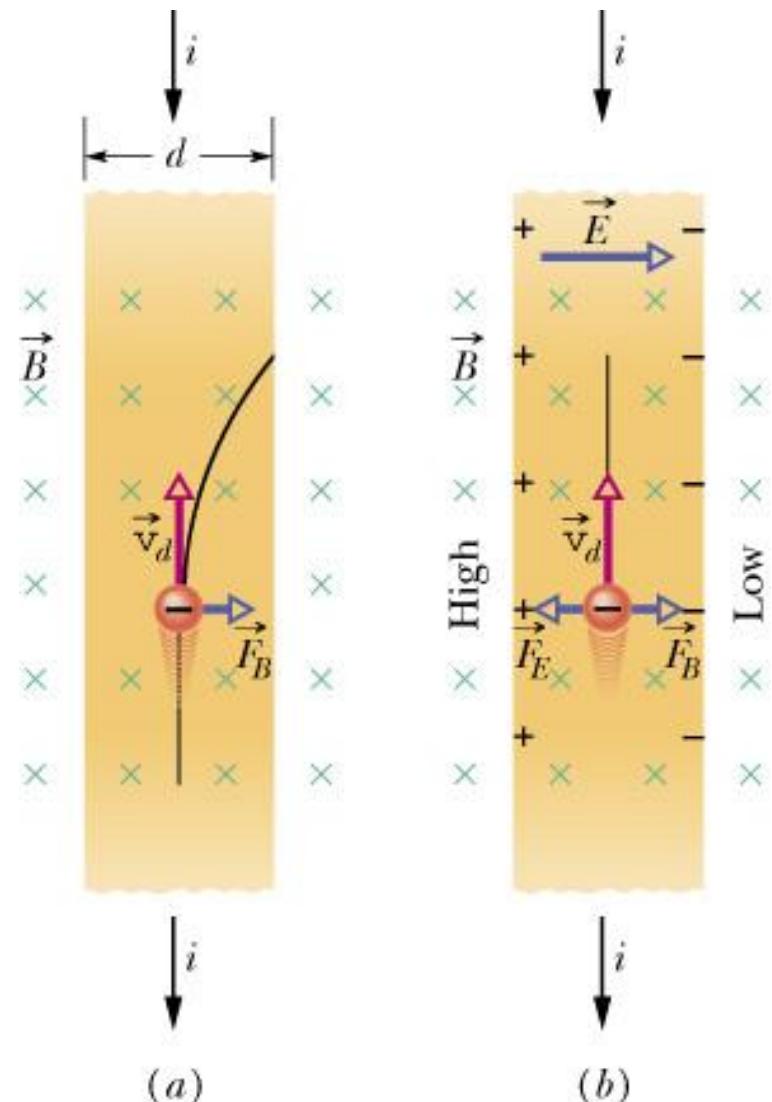
$$v = \frac{E}{B} \quad (2)$$

(1) & (2):

$$\frac{m}{|q|} = \frac{B^2 L^2}{2 y E}$$

4.2. The Hall Effect:

- The effect was discovered by Edwin H. Hall in 1879 when he was a graduate student at the Johns Hopkins University
- The Hall effect mentions that conduction electrons in a wire are deflected by a magnetic field



(a) Electrons in a copper strip are deflected by B , moving to **the right** edge of the strip

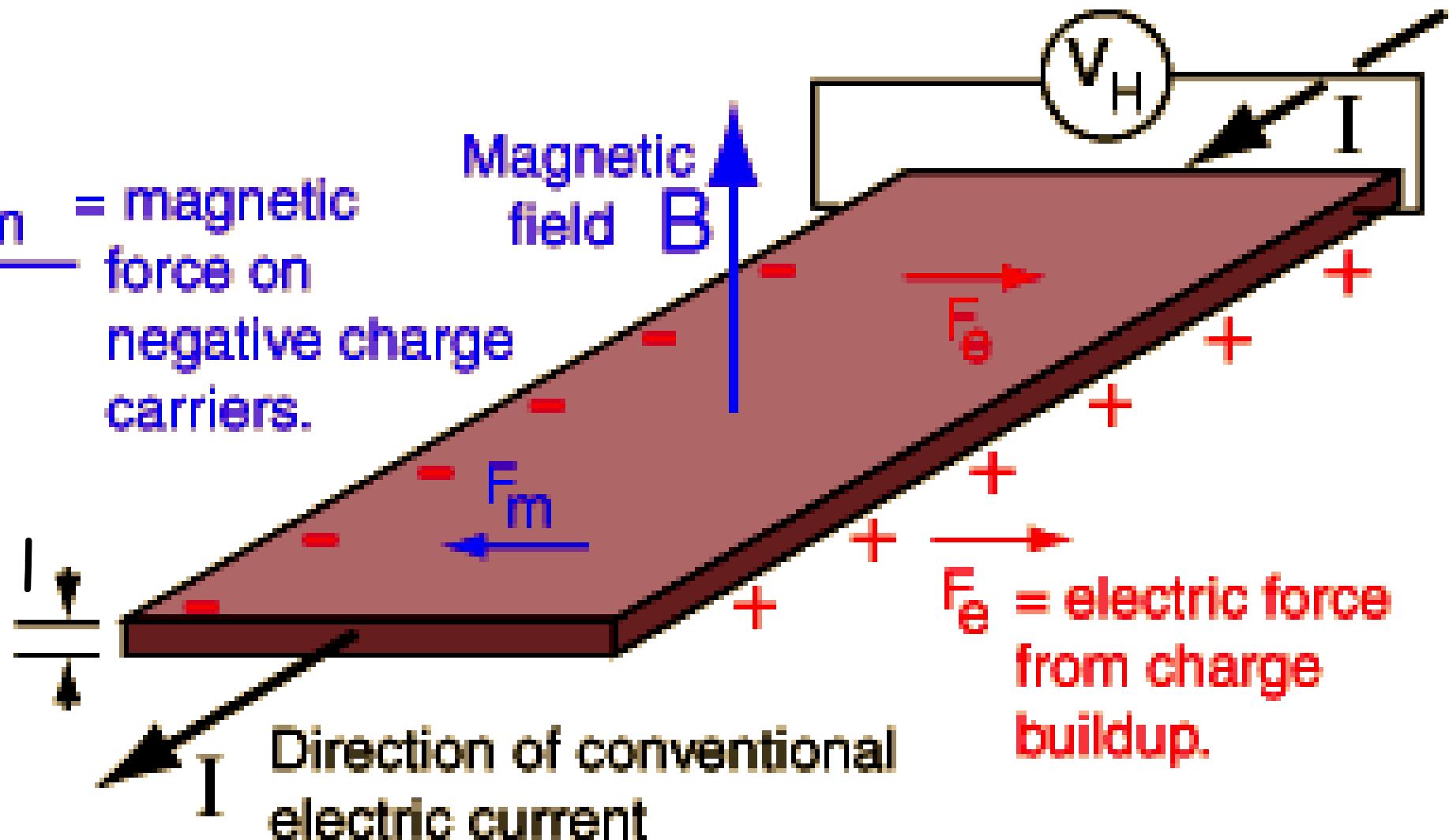
(b) The separation of positive and negative charges produces E as shown, resulting in F_E to **the left**

(c) An equilibrium is established as $F_B = F_E$:

$$V = Ed$$

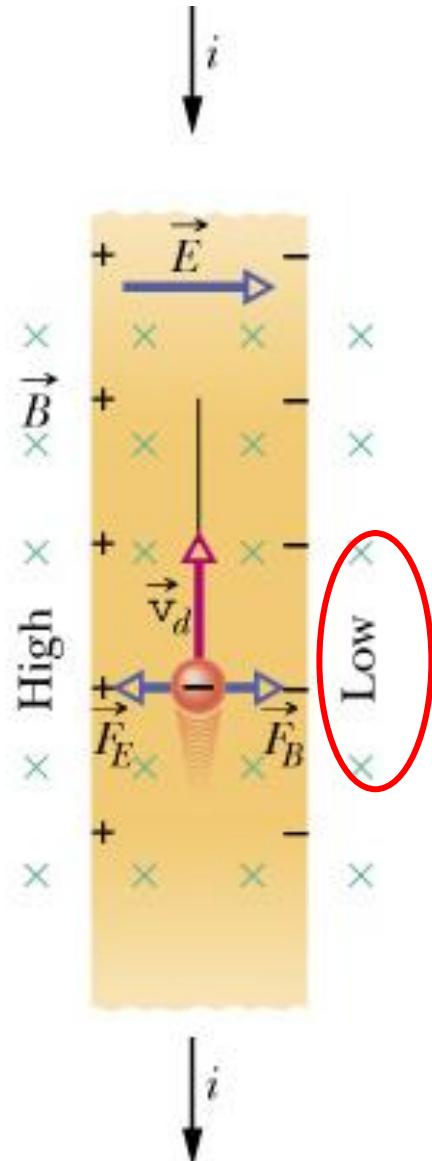
Using a voltmeter, we can measure V and determine which edge is at higher potential to check the electron deflection.

F_m = magnetic force on negative charge carriers.

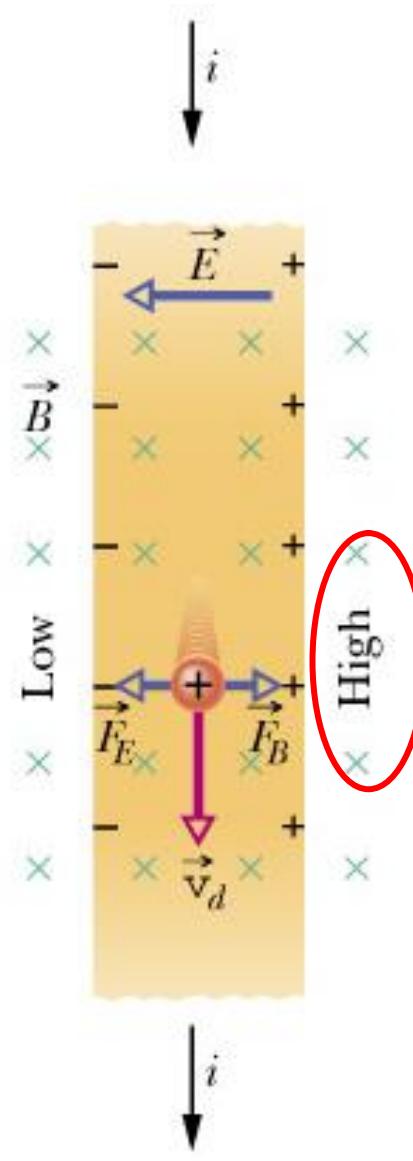


- The Hall effect also allows us to find out whether the charge carriers in a conductor are positive or negative:

negative charge



positive charge



For an equilibrium: $eE = ev_d B$

Drift speed: $v_d = \frac{J}{ne} = \frac{i}{neA}$

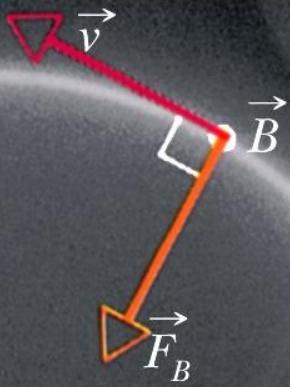
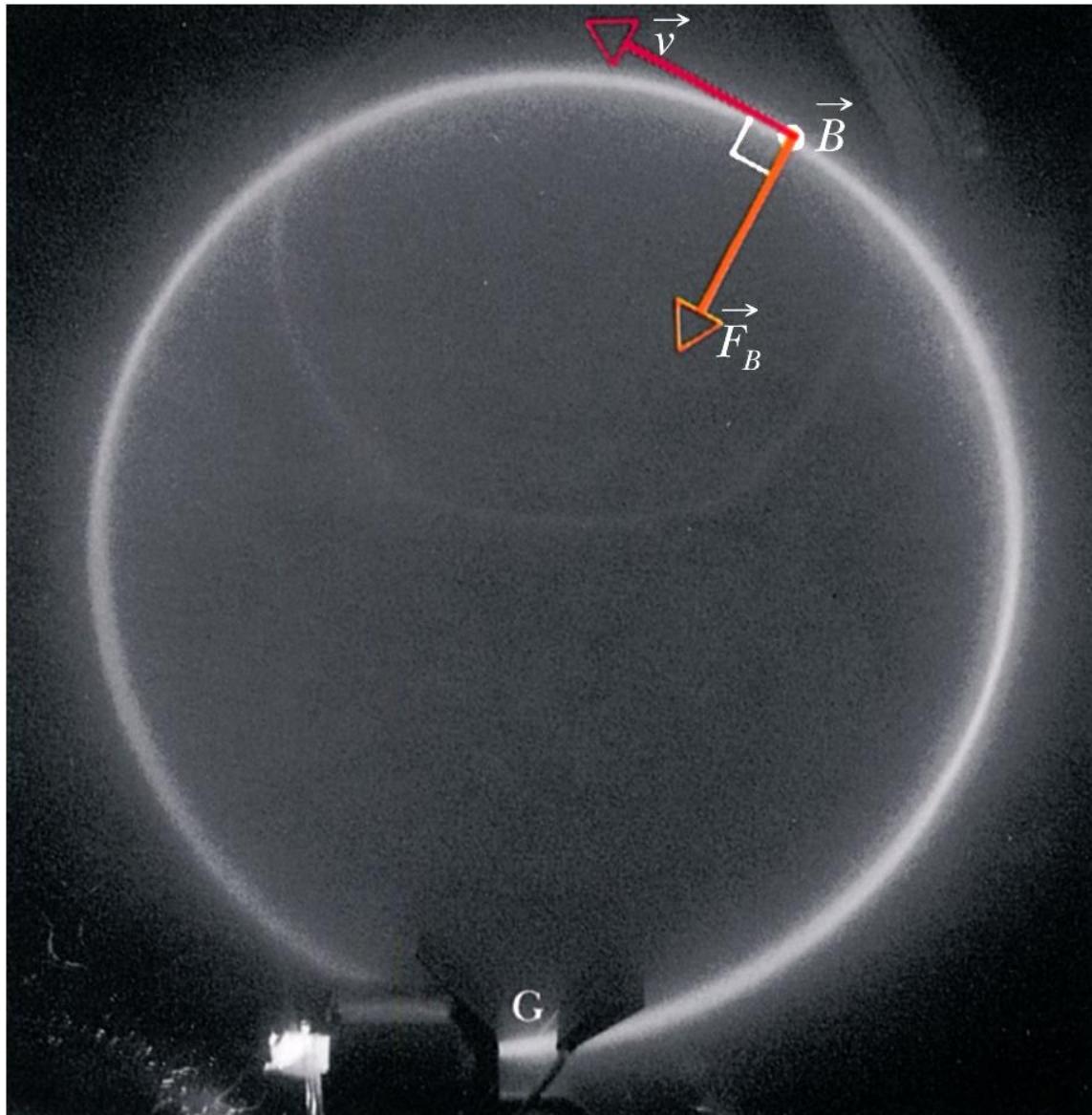
We can derive the number density:

$$n = \frac{Bi}{Vle}$$

where $l = A/d$ is the thickness of the strip

4.3. Motion of a Charged Particle in a Magnetic Field:

- We consider a beam of electrons moving in a region of uniform magnetic field B as shown. The magnetic force acts on an electron, causing the electrons moving along **a circular path**:



$$F_B = qvB = m \frac{v^2}{r}$$

- the radius of the circular path:

$$r = \frac{mv}{qB}$$

- the period:

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

- the angular frequency:

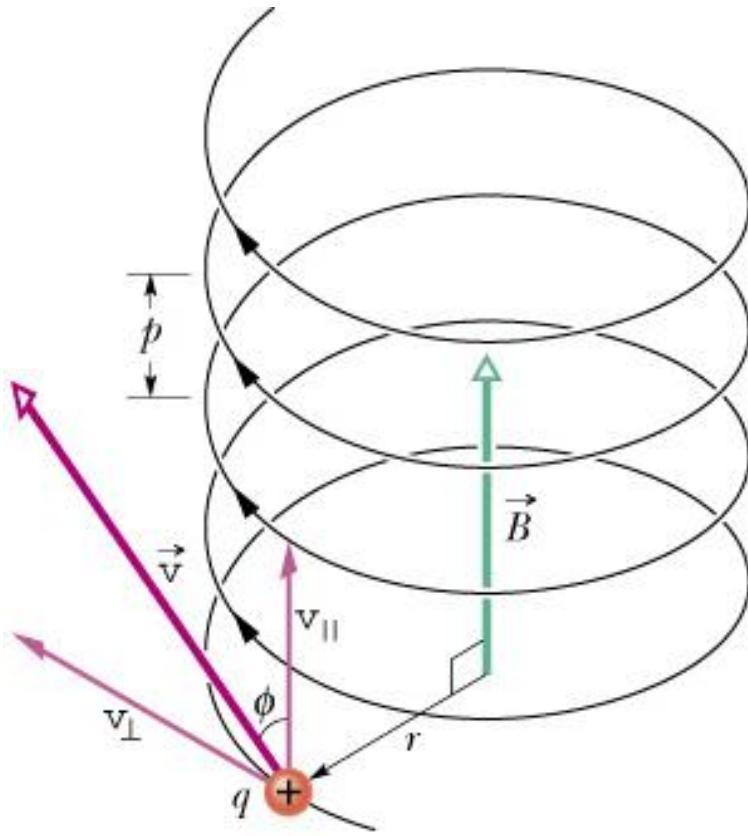
$$\omega = 2\pi f = \frac{qB}{m}$$

Helical Paths:

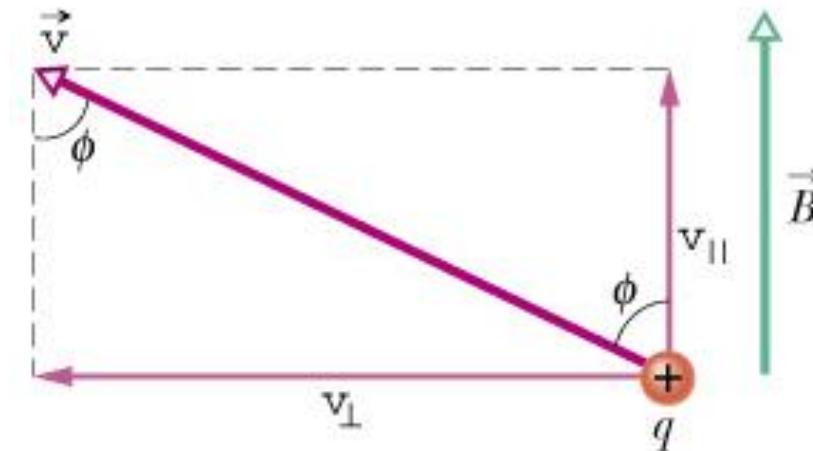
In a general case, if the velocity of the charge has a component parallel to the magnetic field, the charge will move in a **helical path** about the \vec{B} field direction:

$$v_{||} = v \cos \phi \text{ and } v_{\perp} = v \sin \phi$$

The $v_{||}$ component determines the pitch p of the helix

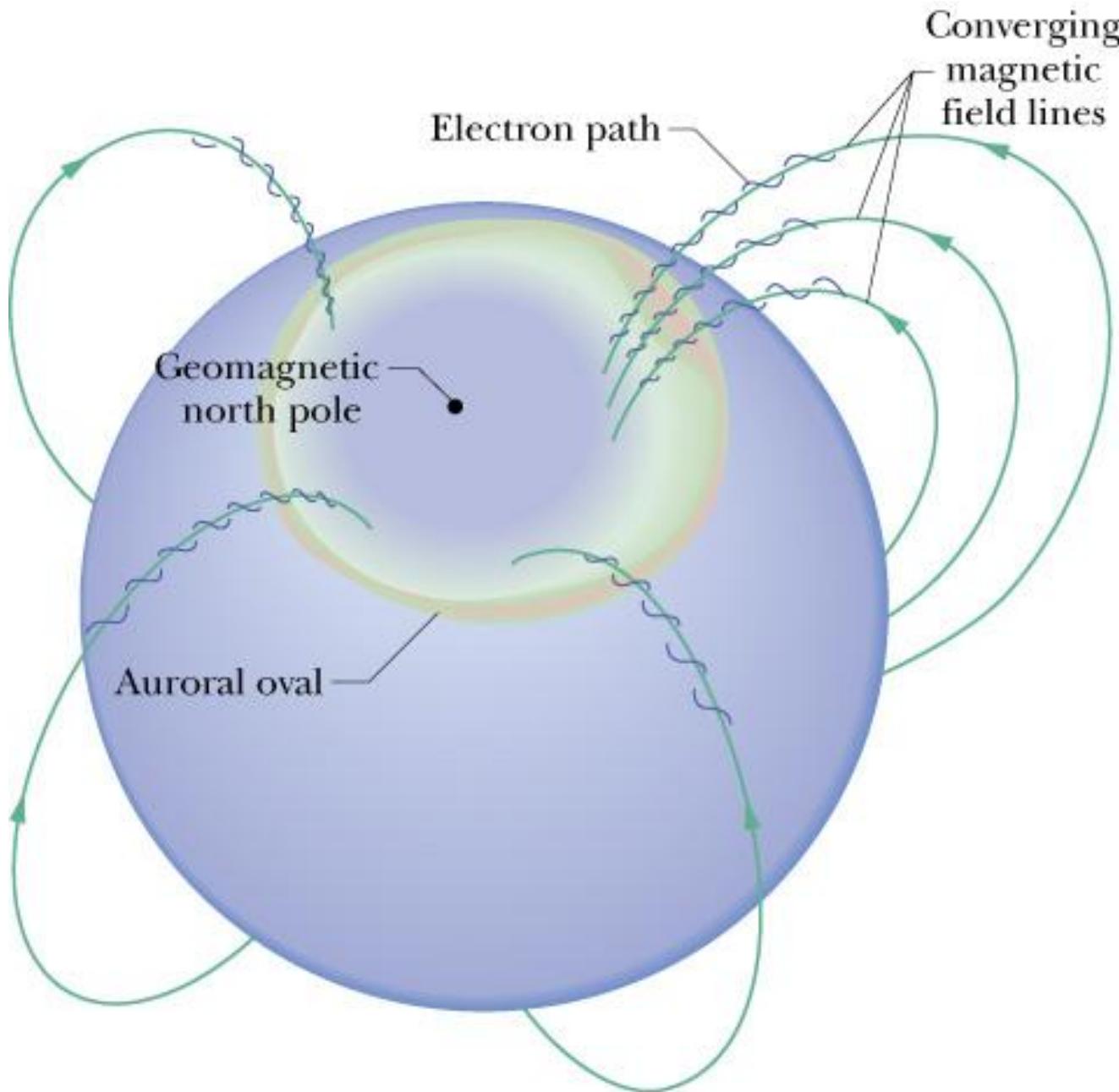


uniform B

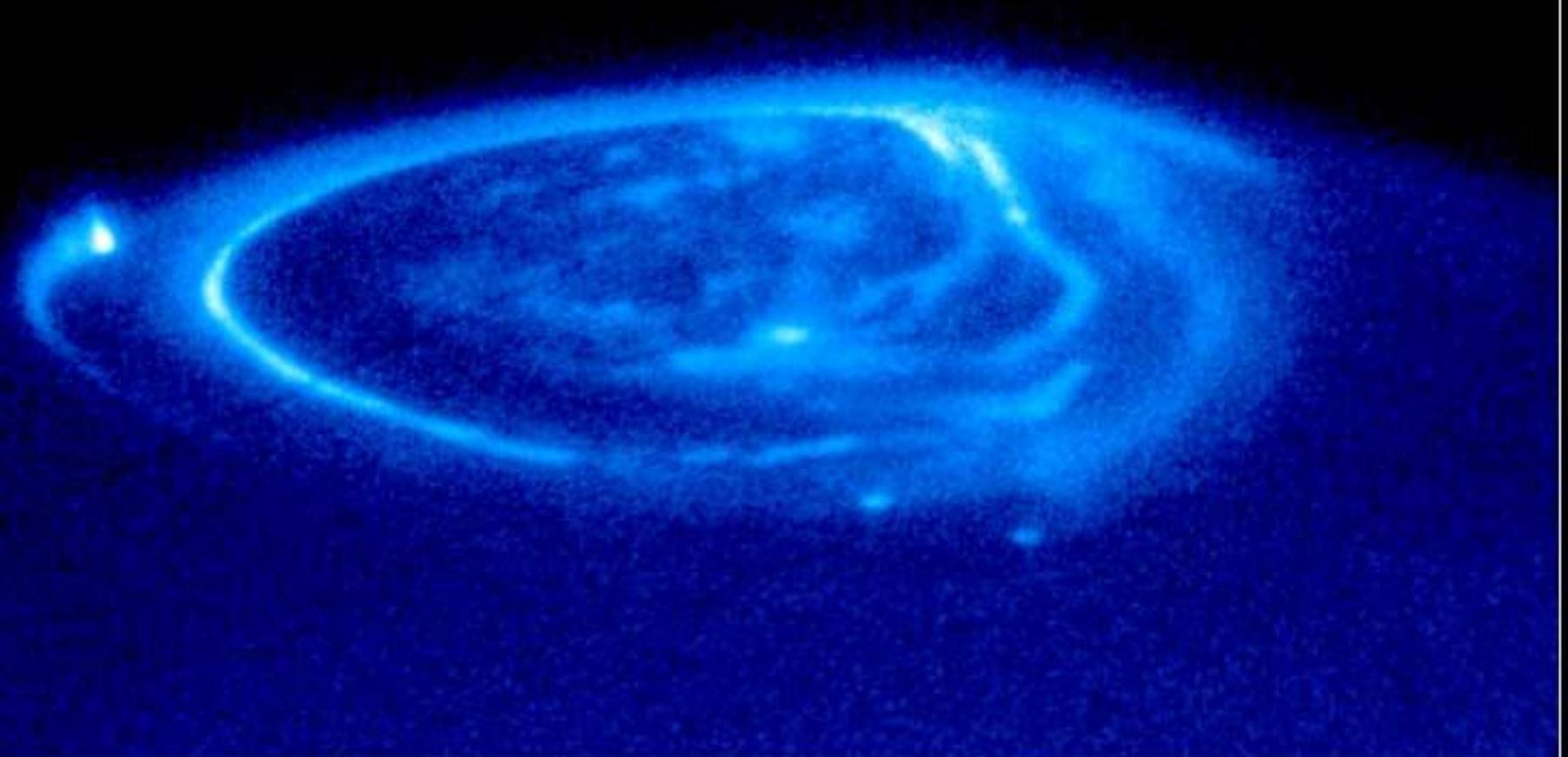


nonuniform B - magnetic bottle

The auroral oval surrounding Earth's geomagnetic north pole



[movie](#)



Jupiter Aurora

NASA and J. Clarke (University of Michigan) • STScI-PRC00-38

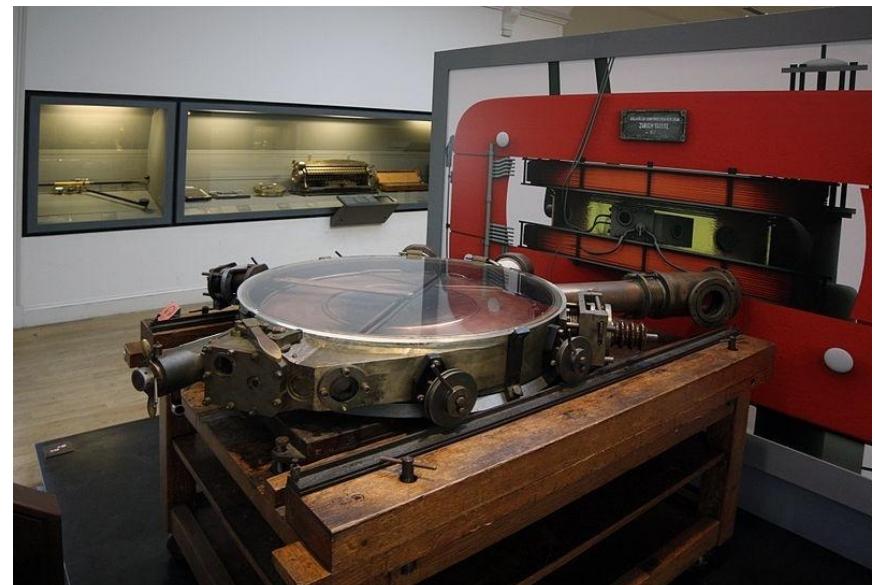
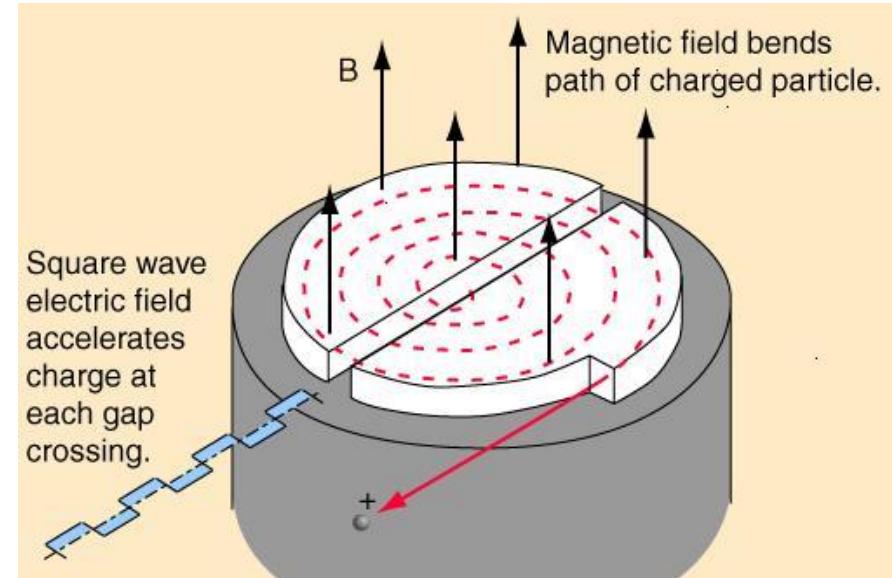
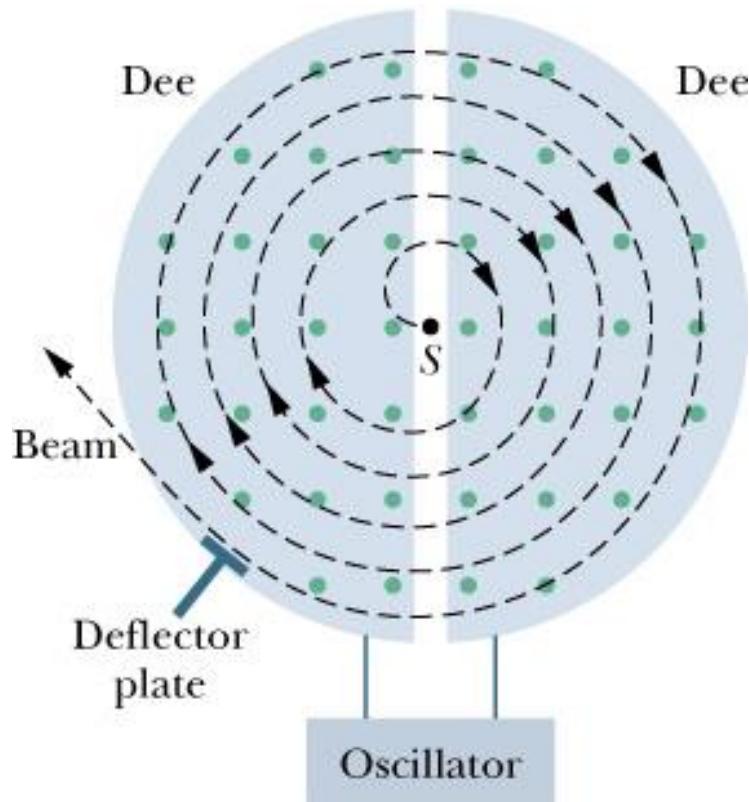
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[movie](#)

Cyclotrons and Synchrotrons:

They are particle accelerators that can give the particles enough kinetic energy to slam into a solid target. Then we analyze the debris of the collisions to study the subatomic particles of matter

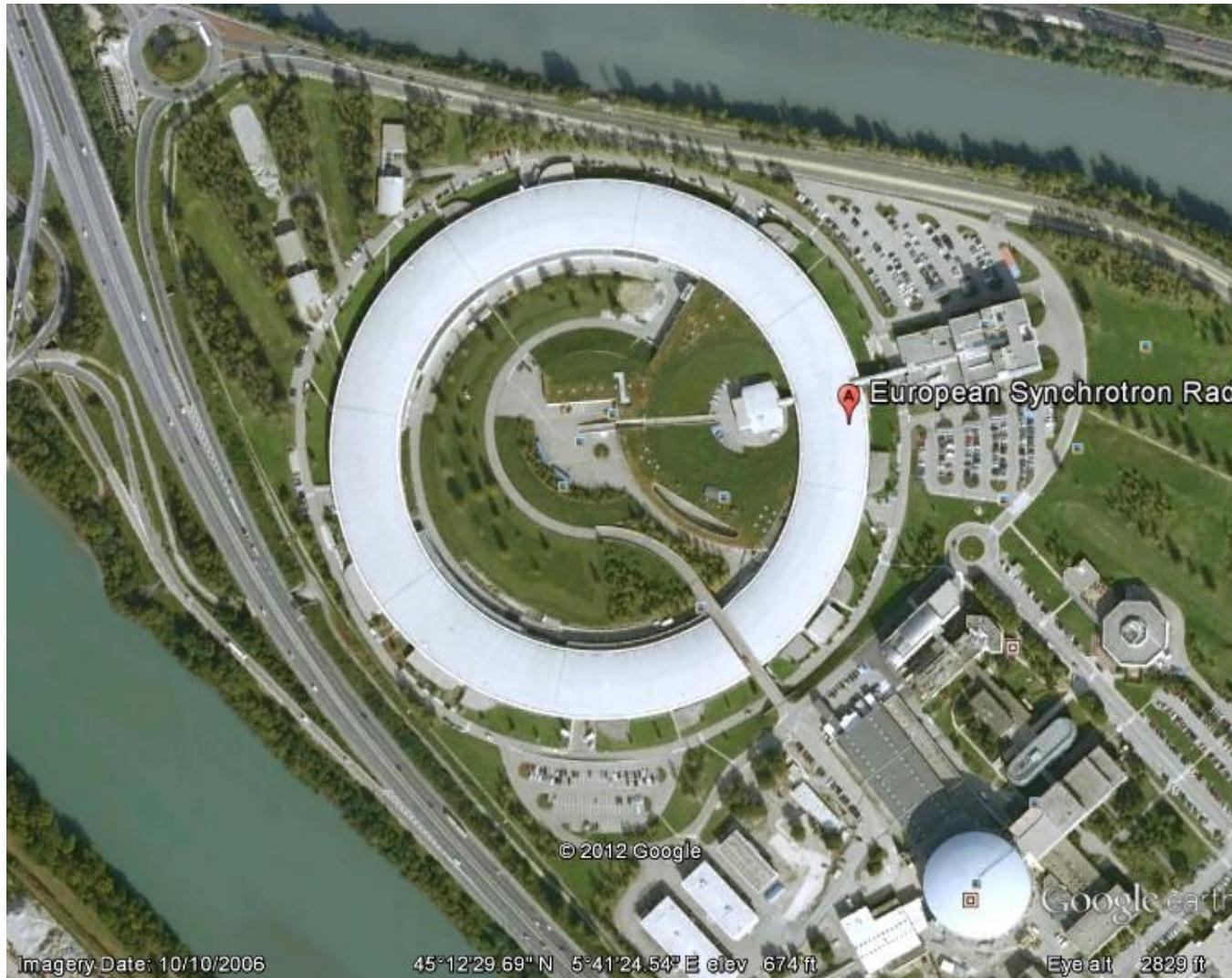
- **The Cyclotron:** A compact type of particle accelerators, in the cyclotron the particles circulate and they are accelerated to **~10 MeV** for a proton



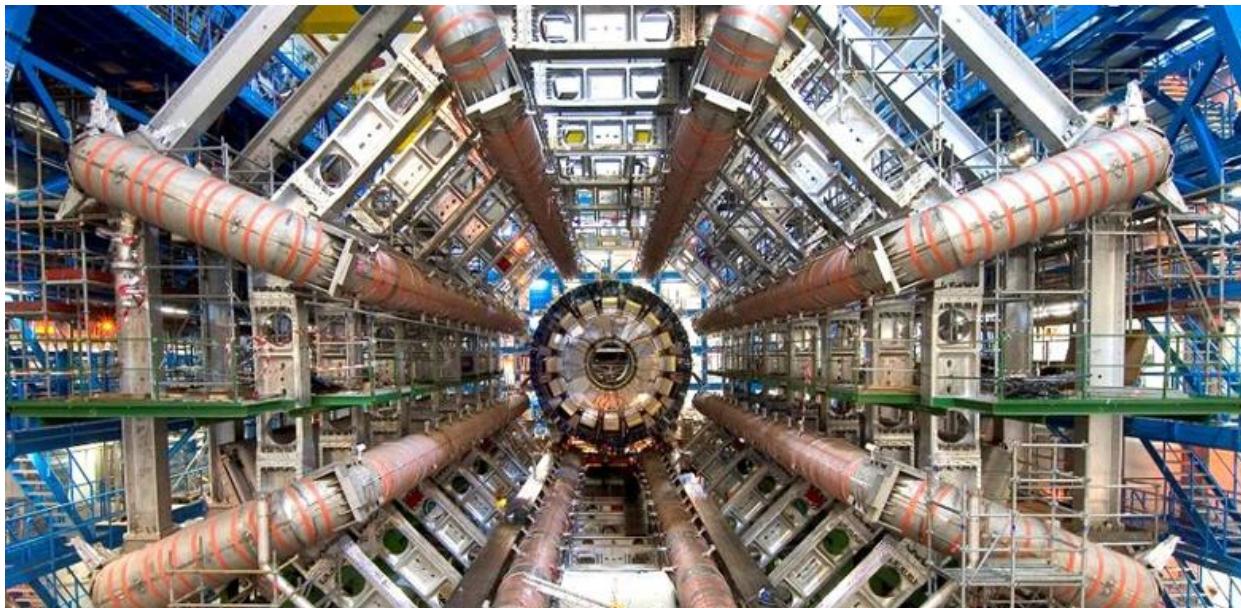
The Proton Synchrotron:

It is a particular type of cyclic particle accelerator, originally from the cyclotron. The protons can be accelerated to ~ 1 TeV (10^{12} eV)

The Grenoble synchrotron



The Large Hadron Collider (LHC) is the world's largest and highest-energy particle accelerator



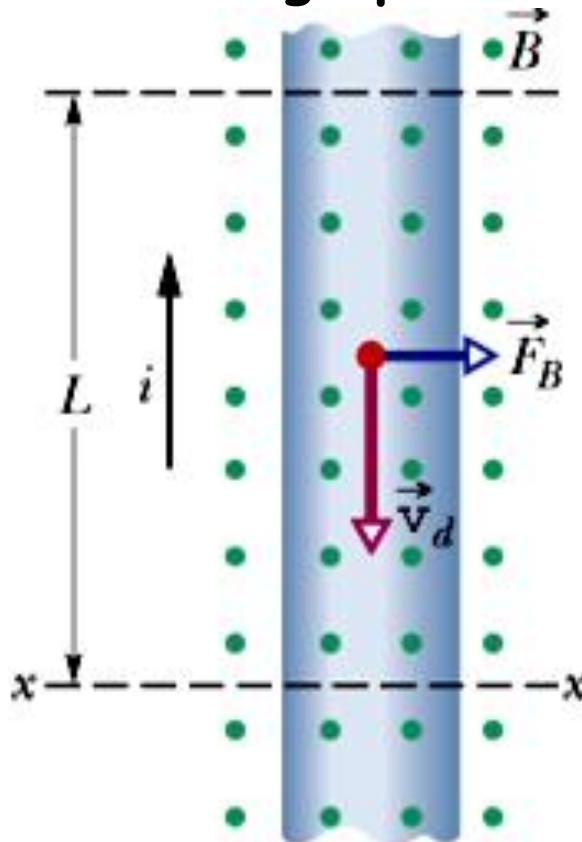
4.4. Magnetic Force on a Current-Carrying Wire:

Since magnetic fields exert a force on electrons moving in a wire, so the force is then transmitted to the wire itself.

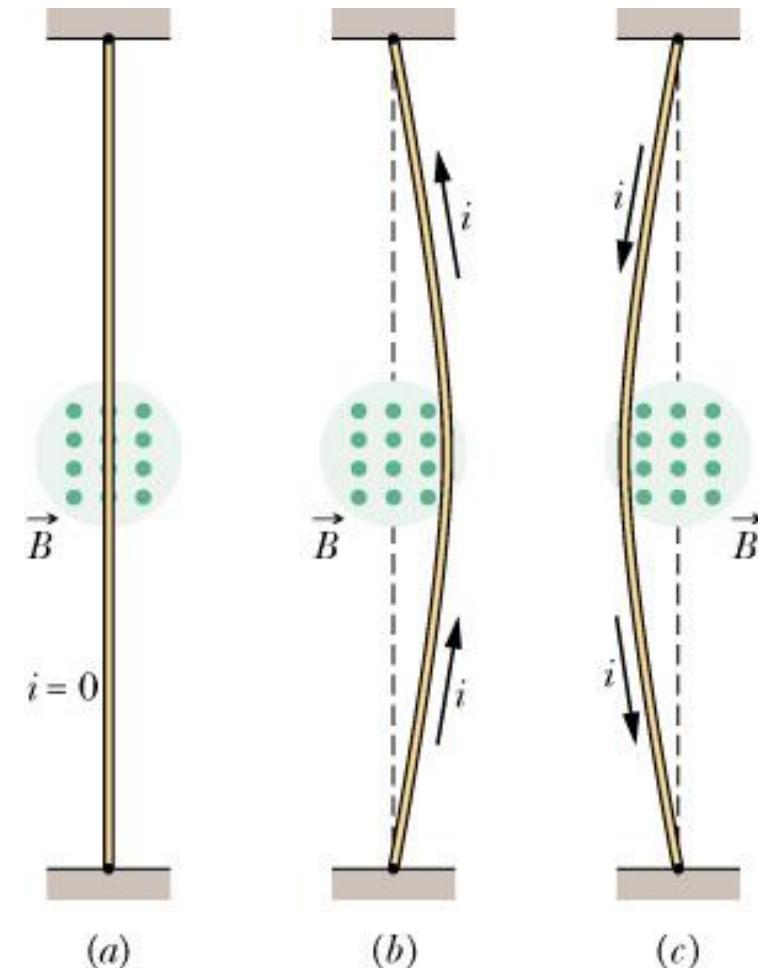
Consider a length L of the wire, all the electrons will drift past plane xx' in a time:

$$t = L / v_d$$

The charge pass through the plane:



$$q = it = i \frac{L}{v_d}$$



Force acting on the wire of length L :

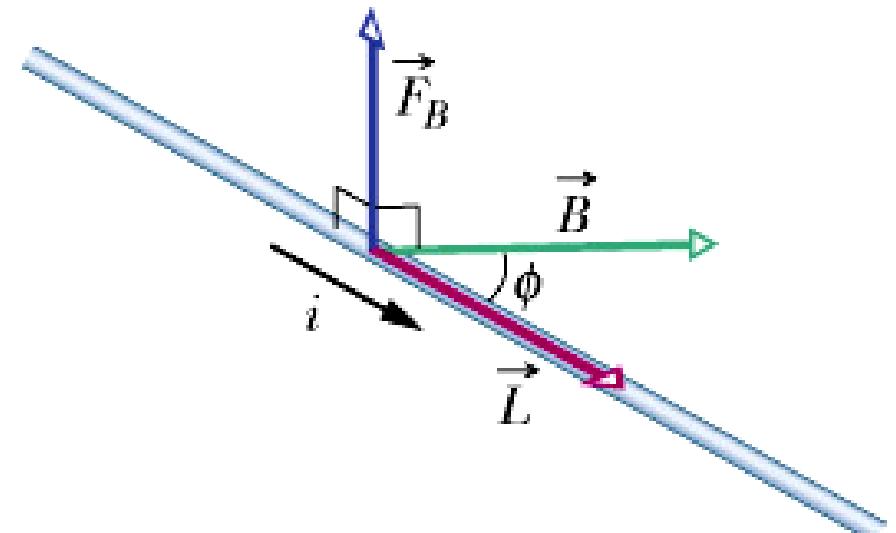
$$F_B = qv_d B \sin \phi = \frac{iL}{v_d} v_d B = iLB$$

In general, force acting on a length L of straight line:

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

\vec{L} is a length vector with magnitude L and direction along the wire segment in the direction of the current

$$F_B = iLB\sin\phi$$



Note: If the field is not uniform or the wire is not straight, we can use the following equation for small segments:

$$d\vec{F}_B = id\vec{L} \times \vec{B}$$

4.5. Torque on a Current-Carrying Coil:

In this section, we study the magnetic forces exerting on a current-carrying loop, a basic element of electric motors.

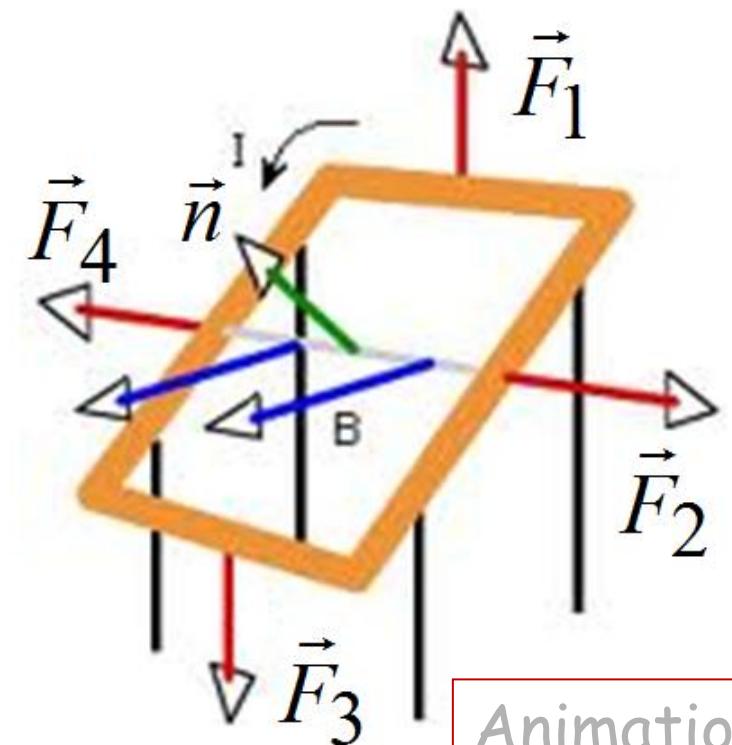
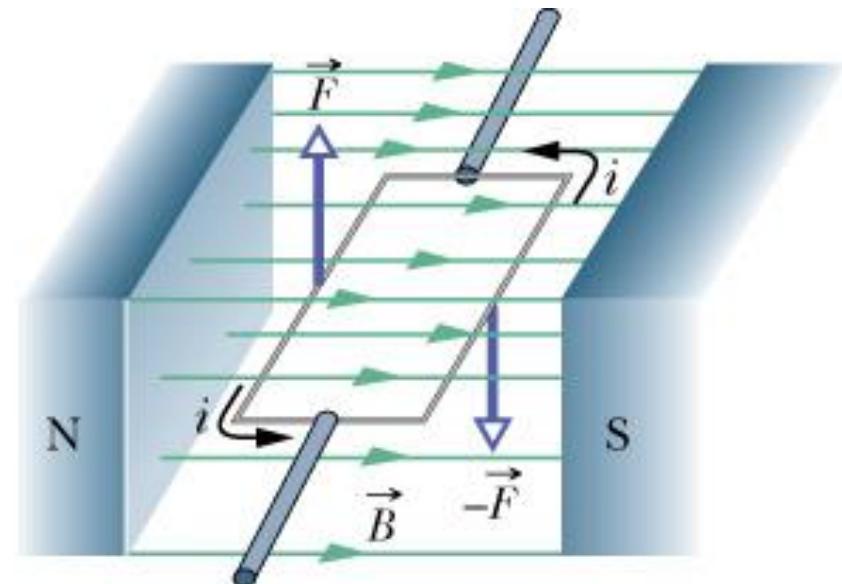
[Animation](#)

Consider a single current-carrying loop immersed in a B field as shown:

Two magnetic forces F and $-F$ produce a torque on the loop

To define the orientation of the loop in the B field, we use a normal vector \vec{n} (curl the fingers of your right hand in the current direction, your thumb points in the \vec{n} direction).

F_2 and F_4 cancel out each other: their net force is zero and their net torque is also zero



[Animation](#)

The torque due to forces

F_1 and F_3 :

$$\tau' = iaB\left(\frac{b}{2}\sin\theta\right) + iaB\left(\frac{b}{2}\sin\theta\right)$$

$$\tau' = iabB\sin\theta$$

$$\boxed{\tau' = iAB\sin\theta}$$

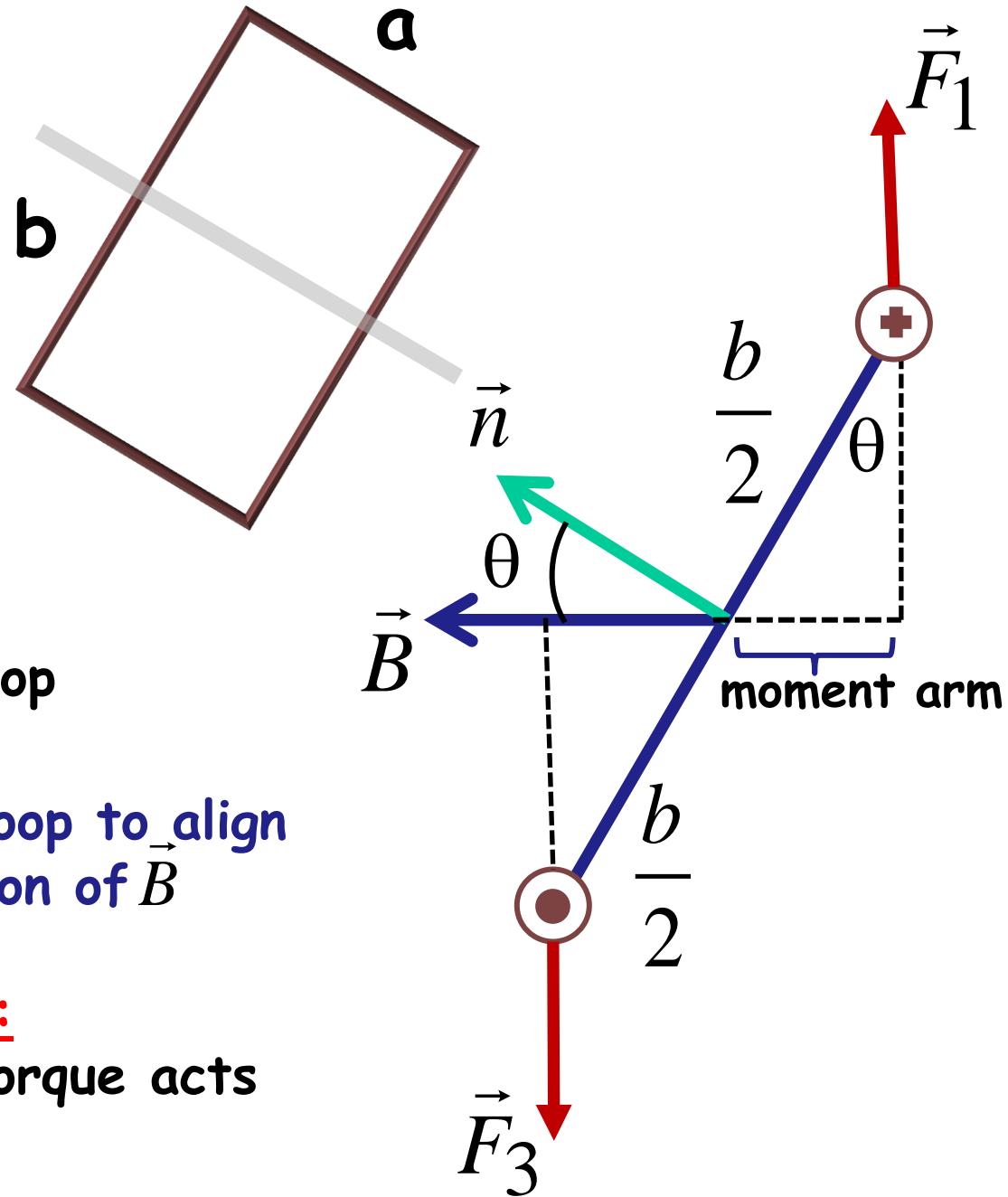
where $A = ab$: the area of the loop

→ Torque τ' tends to rotate the loop to align its normal vector with the direction of \vec{B}

Torque on a current-carrying coil:

For a coil of N loops, the total torque acts on the coil:

$$\boxed{\tau = N\tau' = NiAB\sin\theta}$$



4.6. The Magnetic Dipole Moment:

A current-carrying coil in a magnetic field acts like a bar magnet, so the coil is considered to be a **magnetic dipole**. Thus, we can define a magnetic dipole moment $\vec{\mu}$ of the coil:

$$\mu = NiA$$

$\vec{\mu}$: its direction is the direction of \vec{n}
the fingers of your right hand curl around
the coil in the current direction, your
extended thumb points the $\vec{\mu}$ direction

N: the number of turns in the coil

i: the current

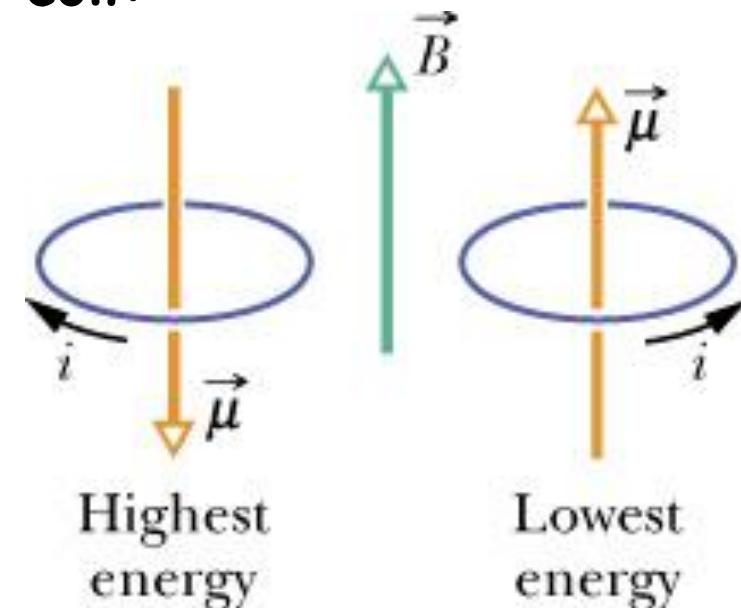
A: the area enclosed by each turn

Unit: A.m²

$$\tau = \mu B \sin \theta$$

θ : the angle between $\vec{\mu}$ and \vec{B}

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



Orientation Energy of a Magnetic Dipole:

A magnetic dipole in an external magnetic field has a magnetic potential energy (**unit: joule**):

$$U(\theta) = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \theta$$

- $\theta = 0^\circ$, lowest energy:

$$U(\theta) = -\mu B$$

- $\theta = 180^\circ$, highest energy:

$$U(\theta) = +\mu B$$

Work done on the dipole by the magnetic field:

$$W = -\Delta U = -(U_f - U_i)$$

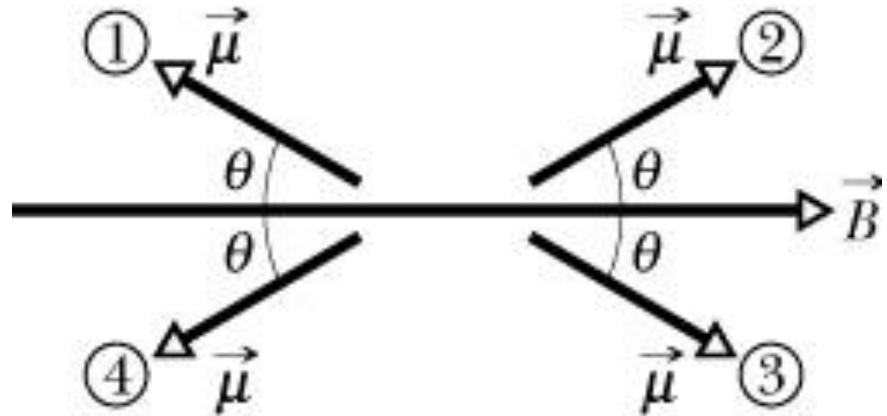
Work done on the dipole by the applied torque:

$$W_a = -W = U_f - U_i$$

Checkpoint: Rank the orientations of a magnetic dipole moment in a magnetic field according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first

$$\tau = \mu B \sin(\vec{\mu}, \vec{B})$$

$$U(\theta) = -\mu B \cos(\vec{\mu}, \vec{B})$$



- (a) all tie
- (b) 1-4, 2-3

Homework: 3, 5, 9, 13, 21, 25, 29, 31, 40, 45, 49, 51,
57, 62 (pages 757-761)