CALCULUS 2 (MA003IU) – FINAL EXAMINATION

Semester 2, 2022-23 • Duration: 120 minutes • Date: August 03, 2023

SUBJECT: CALCULUS 2	
Department of Mathematics	Lecturers
Dr. Nguyen Minh Quan	Assoc. Prof. Tran Vu Khanh, Dr. Nguyen Minh Quan

INSTRUCTIONS:

- Use of calculator is allowed. Each student is allowed two double-sided sheets of notes (size A4 or similar). All other documents and electronic devices are forbidden.
- Write the steps you use to arrive at the answers to each question. No marks will be given for the answer alone.
- There are a total of 10 (ten) questions. Each one carries 10 points.

1. Show that the function $u(x,y) = \ln\left(x^2+y^2\right)$ satisfies the Laplace equation $u_{xx}(x,y) + u_{yy}(x,y) = 0$ for $(x,y) \neq (0,0)$.
2. Let $f(x,y) = \dfrac{e^{-2x}}{1+y^2}$. Find the gradient vector $\nabla f(x,y)$ and the directional derivative $D_{\mathbf{u}} f(0,0)$, where $\mathbf{u}$ is a unit vector of $\mathbf{u} = \dfrac{3}{5}\mathbf{i} - \dfrac{4}{5}\mathbf{j}$.
3. Find an equation of the tangent plane to the surface given by $\sin(xyz) = x+2y+3z$ at the point $(2,-1,0)$.
4. Find the critical points of the function $f(x,y) = 3xy - x^2y - xy^2$, and determine whether each critical point corresponds to a local maximum, local minimum or a saddle point.
5. Find the absolute maximum and minimum values of function $f(x,y) = 5x+2y$ within the domain $D = \{(x,y) : x^2+y^2 \leq 25\}$.
6. Evaluate $\displaystyle\iint_D \frac{dA}{4+x^2+y^2}$, where D is the disk $x^2+y^2 \leqslant 4$.
7. Evaluate $\iiint_E xy\,dV$ where E lies under the plane $z = 1+x+y$ and above the region in the xy-plane bounded by curves $y = \sqrt{x}$, $y = 0$, and $y = 2$.
8. Evaluate the line integral $\int_C yz\cos x\,ds$ where C is parameterized by $x = t$, $y = 3\cos t$, $z = 3\sin t$ with $0 \leq t \leq \pi$.
9. Use Green's theorem to evaluate the line integral $\displaystyle\oint_C \left(xy + e^{-2x}\right)dx + \left(x^2+x+ye^y\right)dy$, where C is the boundary of the triangle D with vertices $(-1,0)$, $(1,0)$, and $(0,1)$, oriented counterclockwise.
10. Let $\mathbf{F} = (2xy+5)\,\mathbf{i} + \left(x^2-4z\right)\mathbf{j} - 4y\mathbf{k}$. Find a function V (if any) such that $\nabla V = \mathbf{F}$.

—END OF THE QUESTION PAPERS. GOOD LUCK!—