International University, VNU-HCMC

School of Computer Science and Engineering

Lecture 7: Keys and Functional Dependencies

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Acknowledgement

- The following slides have been created based on Database system concepts book, 7^{th} Edition.
- And other slides are references from Dr. Sudeepa Roy, Duke University.

Purpose of the Lecture

- Introduce the concepts of functional dependencies (FDs) and their role in relational database design.
- Explain keys, super keys, and candidate keys, and how they are derived from FDs.
- Teach techniques such as attribute closure and minimal cover to reason about FDs.
- Provide a systematic approach to detecting and reducing redundancy in database schemas, serving as a foundation for normalization.

Warm-up Question

- Given a relation that stores user and group information (e.g., UserGroup(uid, uname, gid)),
  What problems might arise from this design, and how can functional dependencies help identify redundancy and anomalies?

Outline

- Functional Dependencies
- Keys/ Super keys
- Attribute closure
- Minimal cover

Motivation

- Why is *UserGroup* (*uid*, *uname*, *gid*) a bad design?
  - It has redundancy user name is recorded multiple times, once for each group that a user belongs to
    - ✓ Leads to update, insertion, deletion anomalies

uid	*uname*	*gid*
142	Bart	dps
123	Milhouse	gov
857	Lisa	abc
857	Lisa	gov
456	Ralph	abc
456	Ralph	gov
…	…	…

- Wouldn't it be nice to have a systematic approach to detecting and removing redundancy in designs?
  - Dependencies, decompositions, and normal forms

Definition of Functional dependency

- A functional dependency (FD) on a relation *R* is a statement of the form $X \rightarrow Y$, where *X* and *Y* are sets of attributes in a relation *R.*
- "If two tuples of R agree on X, then they must also agree Y".
- We write this FD formally as
  $X \rightarrow Y$ and say that X ***functionally determine*** Y

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Definition of Functional dependency

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- DF tells us about any two tuples t and u in the relation R. If two tuples u and t have the same value in the left side then they also have the same value in their right side.
- It's common for the right side of an FD to be a single attribute.
- $A_1, A_2, \ldots, A_n \rightarrow B_1, B_2, \ldots, B_m$ is equivalent to the set of FD's
- $A_1, A_2, \ldots, A_n \rightarrow B_1$
- $A_1, A_2, \ldots, A_n \rightarrow B_2$
- …
- $A_1, A_2, \ldots, A_n \rightarrow B_m$

Functional Dependency (FD)

In a relation *r*, a set of attributes Y is ***functionally dependent*** upon another set of attributes X ($X \rightarrow Y$) iff… for all pairs of tuples t_1 and t_2 in *r*…

if $t_1[X]=t_2[X]$…

it MUST be the case that $t_1[Y]=t_2[Y]$

An example

A	B	C	D
a1	b1	c1	d1
a1	b1	c1	d2
a2	b2	c2	d1
a2	b2	c2	d2

What FDs hold in the current state of this relation?

$A \rightarrow D$; *A,B*$\rightarrow$C; B,C $\rightarrow$ D

FD Example (1)

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	StudentID	Year	Class	Instructor
t_1	1	Sophomore	COMP355	Wu
t_2	2	Sophomore	COMP285	Wu
t_3	3	Junior	COMP355	Wu
t_4	3	Junior	COMP285	Wu
t_5	2	Sophomore	COMP355	Russo
t_6	4	Sophomore	COMP355	Russo

What FDs hold in the current state of this relation?

$StudentID \rightarrow Year$

$\{StudentID, Class\} \rightarrow \{Instructor\}$

FD Example (2)

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	StudentID	Year	Class	Instructor
t_1	1	Sophomore	COMP355	Wu
t_2	2	Sophomore	COMP285	Wu
t_3	3	Junior	COMP355	Wu
t_4	3	Junior	COMP285	Wu
t_5	2	Sophomore	COMP355	Russo
t_6	4	Sophomore	COMP355	Russo

$\{StudentID\} \rightarrow \{Year\}$

$\{StudentID, Class\} \rightarrow \{Instructor\}$

Key(s): $\{StudentID, Class\}$

- *Every student is classified as either a Freshman, Sophomore, Junior, or Senior.*
- *Students can take only a single section of a class, taught by a single instructor.*

FD Example (3)

	StudentID	Year	Class	Instructor
t_1	1	Sophomore	COMP355	Wu
t_2	2	Sophomore	COMP285	Wu
t_3	3	Junior	COMP355	Wu
t_4	3	Junior	COMP285	Wu
t_5	2	Sophomore	COMP355	Russo
t_6	4	Sophomore	COMP355	Russo

{StudentID} ↛ {Instructor}
{StudentID} ↛ {Class}
{Year} ↛ {StudentID}
{Year} ↛ {Instructor}
{Year} ↛ {Class}

{Class} ↛ {Year}
{Class} ↛ {StudentID}
{Class} ↛ {Instructor}
{Instructor} ↛ {Class}
{Instructor} ↛ {Year}
{Instructor} ↛ {StudentID}

FD Example (4)

- Example an instance of the relation Movies1

Title	Year	Length	Genre	studioName	starName
Star war	1977	124	SciFi	Fox	Carrie Fisher
Star war	1977	124	SciFi	Fox	Mark Hamill
Star war	1977	124	SciFi	Fox	Harrison Ford
Gone with the wind	1939	231	Drama	MGM	Vivien Leigh
Wayne's World	1992	95	Comedy	Paramount	Dana Carvey
Wayne's World	1992	95	Comedy	Paramount	Mike Meyers

The relation

- Movies1 (title, year, length, genre, studioName, starName)

FD Example (4)

Title	Year	Length	Genre	studioName	starName
Star war	1977	124	SciFi	Fox	Carrie Fisher
Star war	1977	124	SciFi	Fox	Mark Hamill
Star war	1977	124	SciFi	Fox	Harrison Ford
Gone with the wind	1939	231	Drama	MGM	Vivien Leigh
Wayne's World	1992	95	Comedy	Paramount	Dana Carvey
Wayne's World	1992	95	Comedy	Paramount	Mike Meyers

- The Movies1 is not good design because it holds information of three different relations: Movies, Studio, and StarsIn.

FD Example (4)

Title	Year	Length	Genre	studioName	starName
Star war	1977	124	SciFi	Fox	Carrie Fisher
Star war	1977	124	SciFi	Fox	Mark Hamill
Star war	1977	124	SciFi	Fox	Harrison Ford
Gone with the wind	1939	231	Drama	MGM	Vivien Leigh
Wayne's World	1992	95	Comedy	Paramount	Dana Carvey
Wayne's World	1992	95	Comedy	Paramount	Mike Meyers

- We claim that the following FD holds in this schema

  Title, year → length, genre, studioName (right)
- On the other hand, we observe that the stament:

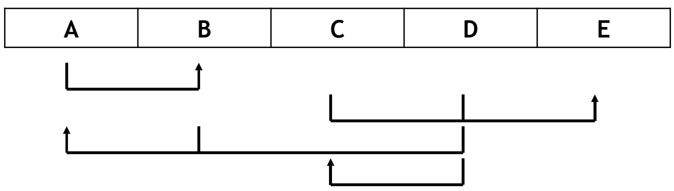
  Title, year → StarName (wrong, not FD)

Ex: Title, year, length → genre, StudioName, StarName

FD. Exercise

Consider the following visual depiction of the functional dependencies of a relational schema.

1. List all FDs in algebraic notation
2. Identify all key(s) of this relation



Answer

Functional Dependencies

$A \rightarrow B$

$CD \rightarrow E$

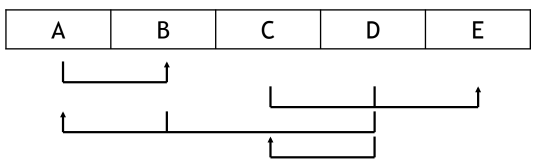
$BD \rightarrow A$

$D \rightarrow C$

Keys

DA

DB



Keys of Relations

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We say a set of one more attributes K is a key for a relation R if:

- Those attributes functionally determine all other attributes of the relation. That is, it's impossible for two distinct tuples of R to agree on all K.
- No proper subset of K functionally determine all other attributes of R, i.e., a key must be minimal.

When a key consists of a single attribute A, we often say that A (rather than {A}) is a key.

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Keys of Relations

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A set of attributes K is a key for a relation R if

- $K \rightarrow$ all (other) attributes of R
  - That is, K is a "super key"
- No proper subset of K satisfies the above condition
  - That is, K is minimal

Keys of Relations

Title	Year	Length	Genre	studioName	starName
Star war	1977	124	SciFi	Fox	Carrie Fisher
Star war	1977	124	SciFi	Fox	Mark Hamill
Star war	1977	124	SciFi	Fox	Harrison Ford
Gone with the wind	1939	231	Drama	MGM	Vivien Leigh
Wayne's World	1992	95	Comedy	Paramount	Dana Carvey
Wayne's World	1992	95	Comedy	Paramount	Mike Meyers

Ex: Attributes {title, year, starName} form a key for the relation Movies1.

- Suppose two tuples agree on title these three attributes: title, year, and starName. Because they agree on title and year, they must agree on other attributes length, genre, and studioName.

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Keys of Relations

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Title	Year	Length	Genre	studioName	starName
Star war	1977	124	SciFi	Fox	Carrie Fisher
Star war	1977	124	SciFi	Fox	Mark Hamill
Star war	1977	124	SciFi	Fox	Harrison Ford
Gone with the wind	1939	231	Drama	MGM	Vivien Leigh
Wayne's World	1992	95	Comedy	Paramount	Dana Carvey
Wayne's World	1992	95	Comedy	Paramount	Mike Meyers

- Argue that no proper subset of {title, year, starName} functionally determines all other attributes. Why title and year do not determine starName, because many movies have more than one star. Thus {title, year} is not a key, similar to {year, starName}, and {title, starName}

Keys of Relations

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- Sometimes a relation has more than one key. If so, it's common to designate one of the keys as the primary key (PK).
- In commercial database systems, the choice of PK can influence some implementation issues such as how the relation is stored on disk. However, the theory of FD's give no special role to PK.

Determining all keys

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- ***Source attributes*** (SA): Those that are appearing only in the left side of the functional dependency (FD), or the ones that are not part of any FDs.
- ***Intermediate attributes*** (IA): Those that are the ones appearing on both sides of the FDs.
- ***Target attributes*** (TA)**:** Those that are only appearing on the right side of the FDs.

Algorithm: Determining all keys

Step 1: Determine source attributes (SA), Intermediate attributes (IA).

Step 2: If IA = ∅ then

K = SA is the only key.

Return K;

Step 3: Determine all subsets of IA.

Step 4: Determine the super keys S_i from $\forall X_i \subset IA$.

IF $(SA \cup X_i)^+ = R^+$ THEN

$S_i = SA \cup X_i$

Step 5: Return all minimal S_i.

Examples: Determining all keys

1. Consider a relation with schema R(A,B,C) and FD's F ={AB →C, C →A}

   What are all the keys of R?

2. Consider a relation with schema R(A,B,C,D,E,G) and FD's F ={E →C, A →D, AB →E, DG →B}

   What are all the keys of R?

Super keys

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- A set of attributes that contains a key is called a super key, short form "superset of a key". Thus, **every key is a super key**.
- Every super key satisfies the first condition of a key: it functionally determines all other attributes of the relation. It does not need to satisfy minimality.

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Super keys

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Title	Year	Length	Genre	studioName	starName
Star war	1977	124	SciFi	Fox	Carrie Fisher
Star war	1977	124	SciFi	Fox	Mark Hamill
Star war	1977	124	SciFi	Fox	Harrison Ford
Gone with the wind	1939	231	Drama	MGM	Vivien Leigh
Wayne's World	1992	95	Comedy	Paramount	Dana Carvey
Wayne's World	1992	95	Comedy	Paramount	Mike Meyers

- Ex: In the relation above, there are many super keys. Not only is the key
- {title, year, starName}: a key
- {title, year, starName, length}
- {title, year, starName, studioName}

are super keys

Exercise

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- Suppose R is a relation with attributes A_1, A_2, ..A_n. As a function of n, tell how many super keys R has, if:

1. The only key is A_1
2. The only keys are A_1 and A_2
3. The only keys are $\{A_1, A_2\}$ and $\{A_3, A_4\}$
4. The only keys are $\{A_1, A_2\}$ and $\{A_1, A_3\}$

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Answer

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1. The only key is A_1: 2^{n-1}
2. The only keys are A_1 and A_2: $3*2^{n-2}$
3. The only keys are $\{A_1, A_2\}$ and $\{A_3, A_4\}$: $7*2^{n-4}$
4. The only keys are $\{A_1, A_2\}$ and $\{A_1, A_3\}$: $3*2^{n-3}$

Answer

The only keys are A_1 and A_2: $3*2^{n-2}$

- The number of super keys that have A_1 is 2^{n-1}. The number of super keys that have A_2 is 2^{n-1}. If we add them, we get $2*2^{n-1}$. But we would have counted some of these super keys twice. The precise number we double-counted are the ones that have both A_1 and A_2. So, we have to subtract these super keys, of which there are 2^{n-2} (since they have both A_1 and A_2 in them). So, the final answer is: $2*2^{n-1} - 2^{n-2} = 3*2^{n-2}$.

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Answer

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1. The only keys are $\{A_1, A_2\}$ and $\{A_3, A_4\}$:
   $7*2^{n-4}$

- The number of super keys that have $\{A_1, A_2\}$ is 2^{n-2}. The number of super keys that have $\{A_3, A_4\}$ is 2^{n-2}.
  If we add them, we get $2*2^{n-2}$. But we would have counted some of these super keys twice. The precise number we double-counted are the ones that have both $\{A_1, A_2\}$ and $\{A_3, A_4\}$. So, we have to subtract these super keys, of which there are 2^{n-4}. So, the final answer is: $2*2^{n-2} - 2^{n-4} = 7*2^{n-4}$.

Answer

1. The only keys are $\{A_1, A_2\}$ and $\{A_1, A_3\}$:
   $3*2^{n-3}$

- The number of super keys that have $\{A_1, A_2\}$ is 2^{n-2}. The number of super keys that have $\{A_1, A_3\}$ is 2^{n-2}.
  If we add them, we get $2*2^{n-2}$. But we would have counted some of these super keys twice. The precise number we double-counted are the ones that have both $\{A_1, A_2\}$ and $\{A_1, A_3\}$. So, we have to subtract these super keys, of which there are 2^{n-3} (since they have A_1,A_2, and A_3 in them). So, the final answer is: $2*2^{n-2} - 2^{n-3} = 3*2^{n-3}$.

Reasoning with FD's

Given a relation R and a set of FD's $\mathcal{F}$

- Does another FD follow from $\mathcal{F}$?
  - Are some of the FD's in $\mathcal{F}$ redundant (i.e., they follow from the others)?
- Is K a key of R?
  - What are all the keys of R?

Rules of $\mathcal{F}$D's

- Armstrong's axioms
  - Reflexivity: If $Y \subseteq X$, then $X \rightarrow Y$
  - Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
  - Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- Rules derived from axioms
  - Splitting: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$
  - Combining: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

$\mathcal{F}$ Using these rules, you can prove or disprove an FD given a set of $\mathcal{F}$Ds

Example

- The set of FD's:

  Title, year $\rightarrow$ length

  Title, year $\rightarrow$ genre

  Title, year $\rightarrow$ studioName

is equivalent to the singe FD

Title, year $\rightarrow$ length, genre, studioName

Example

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- Consider one of the FD's such as:

  Title, year $\rightarrow$ length

  If we try to split the left side into

  Title, year $\rightarrow$ length

  Year $\rightarrow$ length

  Then we get false FD's. That is, title does not functionally determine length, since there can be several movies with the same title.

Trivial Functional Dependencies

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- They are the FD's $X \rightarrow Y$ such that $Y \subseteq X$.

  That is, a trivial FD has a right side that is a subset of its left side.
- For example:

  Title, year $\rightarrow$ title or title $\rightarrow$ title, are trivial FD's

Equivalence rules

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- Given FD $X \rightarrow Y$ is equivalent to $X \rightarrow Z$ where the Z is a subset of Y, that are not belong to X, such that Z $\subset$ Y.

For example:

Title, year $\rightarrow$ title, genre; equivalent to

Title, year $\rightarrow$ genre

Attribute closure

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- Given R, a set of FD's $\mathcal{F}$ that hold in R, and a set of attributes X in R:
- The closure of X (denote X$^+$) with respect to $\mathcal{F}$ is the set of all attributes $\{A_1, A_2, \ldots\}$ functionally determined by X (that is, X $\rightarrow A_1A_2 \ldots$)

Algorithm: Attribute closure

- Input: A set of attributes X and set of FD's of $\mathcal{F}$
- Output: The closure X^+

1. Start with closure = X
2. If $Z \rightarrow Y$ is in $\mathcal{F}$ and Z is already in the closure, then also add Y to the closure
3. Repeat until no new attributes can be added.

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Algorithm: Attribute closure

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Input: R, $\mathcal{F}$, $X \subseteq R^+$
Output: X^+
Step 1: Set $X^+ = X$
Step 2: temp = X^+
$\forall f \quad Z \rightarrow Y \in \mathcal{F}$
if($Z \subseteq X^+$)
$X^+ = X^+ \cup Y$
$\mathcal{F} = \mathcal{F} - f$
Step 3: if (X^+=Temp)
"X^+ is a result" stop
else
Return step 2

Examples

- Let us consider a relation R(A,B,C,D,E,G) and FD's
  $\mathcal{F}$ ={AB $\rightarrow$ C, BC $\rightarrow$ AD, D $\rightarrow$ E, CG $\rightarrow$ B} .
  What is $\{A,B\}^+$?
- X ={A,B}
- Next, X = {A,B,C} based on AB $\rightarrow$ C
- X = {A,B,C,D} based on BC $\rightarrow$ D
- X = {A,B,C,D,E} based on D $\rightarrow$ E and no more changes to X are possible.
- Thus, $\{A,B\}^+$ = {A,B,C,D,E}

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Example

- Let us consider a relation R(A,B,C,D,E,G) and FD's
  $\mathcal{F}$ = {AB $\rightarrow$ C, BC $\rightarrow$ AD, D $\rightarrow$ E, CG $\rightarrow$ B} .

Suppose we wish to test whether AB $\rightarrow$ D follows from these FD's.

- We computer $\{A,B\}^+$ = {A,B,C,D,E}. Since D is a member of the closure, we conclude that AB $\rightarrow$ D does follow.
- However, D $\rightarrow$ A does not follow.

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Exercises

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1. Let us consider a relation R(A,B,C,D,E,G) and FD's
$\mathcal{F}$ ={AB → D, AC → BD, D → G, CG →A} .
What is $\{A,C\}^+$, $\{B,D\}^+$?

2. Let us consider a relation R(A,B,C,D,E,G) and FD's
$\mathcal{F}$ ={AB → C, BC → D, D → EG, BE →C}
Suppose we wish to test whether AB → EG follows from these FD's.

Minimal cover

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- If we are given a set of FD's F, then any set of FD's equivalent to *F* is said to be a basic for *F*. A minimal basic for a relation is a minimal cover *B* that satisfies three conditions.

1. All the FD's in *B* have singleton right sides.
2. If for any FD in *B* we remove one or more attributes from the left side of FD, the result is no longer a minimal cover.
3. If any FD is removed from B, the result is no longer a minimal cover

Example

- Ex: Consider a relation R(A,B,C) and FD's
  F= {$A \to B, B \to A, B \to C, C \to A, C \to B, AB \to C, AC \to B, BC \to A$}.
- Relation R and its FD's have several minimal cover such as:
  {$A \to B, B \to A, B \to C, C \to B$} and
  {$A \to B, B \to C, C \to A$}

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Minimal cover

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1. Functional dependencies with a right-hand side attribute:

- Every set of functional dependencies F is equivalent to a set of functional dependencies G, where the right-hand side of each functional dependency in G consists of only one attribute.
- Example: For F = {$A \to BC, B \to C, AB \to D$}

It follows that F ≡ {$A \to B, A \to C, B \to C, AB \to D$} = G

Minimal cover

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2. The left-hand side of PTH is redundant:

- Let F be a set of functional dependencies on the relation schema R, X be a set of attributes, and X→Y ∈ F. It is said that the functional dependency X → Y has a redundant left-hand side (incomplete dependency) if there exists an A ∈ X such that:

  $F \equiv (F - \{X \rightarrow Y\}) \cup \{(X - A) \rightarrow Y\}$

- Conversely, X → Y is a functional dependency with a non-redundant left-hand side, or Y depends fully on X, or it is a complete functional dependency.

Example

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- Example: Given R(A,B,C), and F = {AB → C; B → C} Determine if F contains a functional dependency with a redundant left-hand side:
- Solution: $F \equiv (F - \{AB \rightarrow C\}) \cup \{(AB - A) \rightarrow C\} = \{B \rightarrow C\}$

  AB → C is an incomplete functional dependency

  B → C is a complete functional dependency

Example

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Given R(A,B,C,D) and the set of functional dependencies F = {A → BC, B→C, AB → D}.

- Determine if F contains a functional dependency with a redundant left-hand side:

Solution: The functional dependency AB → D has a redundant left-hand side B because:

- F ≡ (F – {AB → D}) ∪ {A → D} ≡ {A → BC, B → C, A → D} Thus, the functional dependency AB → D with A^+ = ABCD ⇒ A → D ∈ F^+.
- In F, replace AB → D with A → D ⇒ F ≡ {A → BC, B → C, A → D}

Minimal cover

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3. Non-redundant functional dependencies:

- It is said that F is a set of non-redundant functional dependencies if there does not exist F’ ⊆ F such that F’ ≡ F.
- Conversely, F is a set of redundant functional dependencies.

Example: For R(ABCD), & F = {A → BC, B → D, AB → D}.

- Determine if F contains redundant functional dependencies.
- F is redundant because F ≡ F’= {A → BC, B → D}

Minimal cover

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F is called a minimal cover of functional dependencies if F simultaneously satisfies the following three conditions:

1. F is a set of functional dependencies with a right-hand side attribute.
2. F is a set of functional dependencies with a non-redundant left side.
3. F is a set of non-redundant functional dependencies.

Example

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Given R(A,B,C,D) and F = {A → BC, B → C, AB → D}.
Find minimal cover of F (set of functional dependencies).

HW

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Given R(A,B,C,D) and F = {AB → CD, B → C, C → D}.
Find minimal cover of F (set of functional dependencies).

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Projecting a set of FD's

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- Given R(A,B,C,D) has FD's F = {A→B, B →C, C →D}. Suppose also that we wish to project out the attribute B, leaving a relation R_1(A,C,D).
- Find F_1?

Algorithm: Projecting a set of FD’s

- Input: Two relations R, R_1, computed by the projection $R_1 = \pi_L(R)$. Also, a set of FD’s F that hold in R.
- Output: The set of FD’s that hold in R_1.

1. Let F_1 be the eventual output set of FD’s. Initially, F_1 is empty
2. For each set of attributes X that is a subset of the attributes of R_1, compute X^+. This computation is performed with respect to the set of FD’s F, and may involve attributes that are in the schema of R but not R_1. Add to F_1 all nontrivial FD’s $X \rightarrow A$ such that A both in X^+ and an attribute of R_1.

Algorithm: Projecting a set of FD’s

3. Now, F_1 is a basic for the FD’s that hold in R_1, but may not be a minimal cover. We may construct a minimal basic by modifying F_1 as follows:
   a. If there is an FD f_1 in F_1 that follows from the other FD’s in F_1, remove f_1 from F_1.
   b. Let $Y \rightarrow B$ be an FD in F_1, with at least two attributes in Y, and let Z be Y with one of its attributes removed. If $Z \rightarrow B$ follows from the FD’s in F_1 (including $Y \rightarrow B$), then replace $Y \rightarrow B$ with $Z \rightarrow B$.
   c. Repeat the above steps in all possible ways until no more changes to F_1 can be made.

Example

- Given R(A,B,C,D) has FD's F={A→B, B →C, C →D}. Suppose also that we wish to project out the attribute B, leaving a relation R_1(A,C,D). Find F_1?

Answer:

- In principle, to find the FD's for R_1, we need to take the closure of all eight subsets of {A,C,D}, using the full set of FD's including those involving B.

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Example

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- Subset of {A,C,D}: ∅, A,C,D,AC,AD,CD, ACD.
- Closures of all subsets: $\{\emptyset\}^+ = \emptyset$, $\{A\}^+ = \{A,B,C,D\}$, $\{C\}^+ = \{C,D\}$, $\{D\}^+ = \{D\}$, $\{AC\}^+ = \{A,B,C,D\}$, $\{CD\}^+ = \{C,D\}$, $\{ACD\}^+ = \{A,B,C,D\}$.
- F={A→B, B →C, C →D}
- $F_1^+ = \Pi_{R1}(F) = \{A\to C, A\to D, C\to D\}$. We can observe that A→D follows from the other two by transitivity. Therefore, a minimal cover for the FD's of R_1 is $F_1 = \{A\to C, C\to D\}$.

Exercises

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1. Consider a relation with schema R(A,B,C,D) and FD's
   F ={AB →C, C →D, D →A}
   a. What are all the nontrivial FD's that follow from the given FD's? You should restrict yourself to FD's with single attribute on the right side.
   b. What are all the keys of R?
   c. What are all the super keys for R that are not keys?

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Exercises

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2. Consider a relation with schema R(A,B,C,D) and FD's
   F ={AB →C, BC →D, CD →A, AD →B}
   a. What are all the nontrivial FD's that follow from the given FD's? You should restrict yourself to FD's with single attribute on the right side (Restrict to one attribute on right side).
   b. What are all the keys of R?
   c. What are all the super keys for R that are not keys?

Exercises

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3. Suppose we have relation R(A,B,C,D,E), with some set of FD's, F ={AB →DE, C →E, D →C, E →A} and we wish to project those FD's onto relation S(A,B,C). Find F_1?

Exercises

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Suppose we have relation R(A,B,C,D,E), with some set of FD's,
F ={AB →D, AC→E, BC →D, D →A, E → B}

1. Find key(s) of R
2. Find a minimal cover of F
3. And we wish to project those FD's onto relation S(A,B,D). Find F_1?

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Thank you for your attention!