DETERMINANT AND INVERSE OF 3 x 3 MATRICES ANSWER SHEET

Exercise 1

a) $\begin{bmatrix} 6 & 2 & 3 \\ 4 & 1 & 7 \\ 0 & 8 & 4 \end{bmatrix} = A$

$|A| = 6(4-56) - 2(16-0) + 3(32-0)$

$= -248$

b) $\begin{bmatrix} 1 & -5 & 3 \\ 7 & 4 & -9 \\ 4 & 8 & 2 \end{bmatrix} = B$

$|B| = 1(8+72) + 5(14+36) + 3(56-16)$

$= 450$

Exercise 2

a) matrix of minors $= \begin{bmatrix} -52 & 16 & 32 \\ -16 & 24 & 48 \\ 11 & 30 & -2 \end{bmatrix}$

cofactor matrix $= \begin{bmatrix} -52 & 16 & 11 \\ -16 & 24 & -30 \\ 32 & -48 & -2 \end{bmatrix}$

$|A| = -248$ (from above)

$A^{-1} = -\frac{1}{248} \begin{bmatrix} -52 & 16 & 11 \\ -16 & 24 & -30 \\ 32 & -48 & -2 \end{bmatrix}$

b) matrix of minors = $\begin{bmatrix} 80 & 50 & 40 \\ -34 & -10 & 28 \\ 28 & -30 & 39 \end{bmatrix}$

cofactor matrix = $\begin{bmatrix} 80 & 34 & 28 \\ -50 & -10 & 30 \\ 40 & -28 & 39 \end{bmatrix}$

$|B| = 450$ (from above)

$B^{-1} = \frac{1}{450} \begin{bmatrix} 80 & 34 & 28 \\ -50 & -10 & 30 \\ 40 & -28 & 39 \end{bmatrix}$

Exercise 3

a) $A = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 7 \\ 2 & -1 & 6 \end{bmatrix}$

$|A| = 1(24+7) - 0 + 2(3-8)$

$= 21$

Matrix of minors = $\begin{bmatrix} 31 & -32 & -5 \\ 2 & 2 & -1 \\ -8 & 13 & 4 \end{bmatrix}$

cofactor matrix = $\begin{bmatrix} 31 & -2 & -8 \\ 32 & 2 & -13 \\ -5 & 1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{21} \begin{bmatrix} 31 & -2 & -8 \\ 32 & 2 & -13 \\ -5 & 1 & 4 \end{bmatrix}$

b)

$$B = \begin{bmatrix} 0 & 0 & 3 \\ 4 & 5 & 3 \\ 2 & 7 & 1 \end{bmatrix}$$

$$|B| = 0(5-21) - 0(4-6) + 3(28-10)$$
$$= 54$$

Matrix of minors $= \begin{bmatrix} -16 & -2 & 18 \\ -21 & -6 & 0 \\ -15 & -12 & 0 \end{bmatrix}$

Cofactor matrix $= \begin{bmatrix} -16 & 21 & -15 \\ 2 & -6 & 12 \\ 18 & 0 & 0 \end{bmatrix}$

$$B^{-1} = \begin{bmatrix} \frac{8}{27} & \frac{7}{18} & -\frac{5}{18} \\ \frac{1}{27} & -\frac{1}{9} & \frac{2}{9} \\ \frac{1}{3} & 0 & 0 \end{bmatrix}$$