Matrix Applications: The Markov Model

Ecology is the study of the relations and interactions between organisms and their environment, including other organisms. Understanding how populations change over time is a key aspect of ecological studies. The Markov model uses matrices to predict how populations fluctuate and in turn, how conservationists can anticipate the effect this will have on their environment. For example; in the case of Loggerhead turtles, if there are too many older turtles there will be fewer eggs, however, too many younglings results in not enough egg laying.

The Markov model uses Markov chains in order to model and calculate population changes over time. In order to use this model, we need to have prior knowledge and assume that every individual contributes in exactly the same way to the population. Here is an example of the Markov Model:

Classic Observations on a cohort:
↳ Need to know age-specific survivorship + fecundity data

Fecundity: ability to produce an abundance of offspring

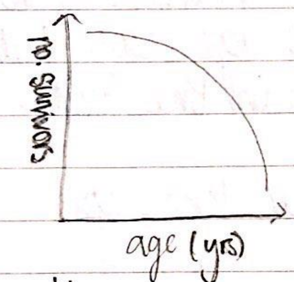
Look at female fecundity only - we assume that there are enough males to fertilise the females
Find the survivorship schedule - % percentages are relative to the previous age class

P_i is the proportion of individuals surviving stage i e.g. grads/entrants

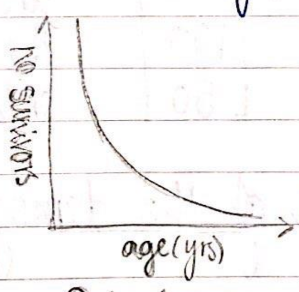
E.g.

Age	No. survived	P_i
0	500	-
1	400	400/500 = 0.8
2	200	200/400 = 0.5
3	50	50/200 = 0.25

Survivorship curves - humans have good juvenile survival ship then deterioration into old age.



Humans



Oak trees

Calculating F_i from cohort data

Fi is the fecundity in the stage weighted by the probability of realizing it.

E.g.

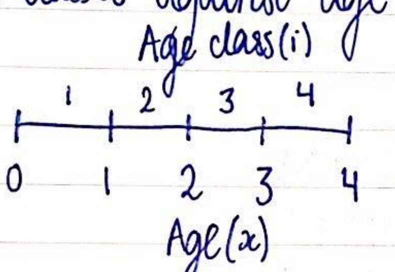
Life stage	Pi	Fecundity	Fi
1	0.8	2	$0.8 \times 2 = 1.6$
2	0.5	3	$0.5 \times 3 = 1.5$
3	0.25	1	$0.25 \times 1 = 0.25$

Model wraps up all of this

Markov chains: a classical maths device for modelling change over time

Make age classes against age

e.g.



Describing a population's age structure at time = t

$$n(t) = \begin{bmatrix} n_1(t) \\ n_2(t) \\ \cdots \\ n_k(t) \end{bmatrix} \qquad n(5) = \begin{bmatrix} 600 \\ 270 \\ 100 \\ 50 \end{bmatrix}$$

In this example there are 600 individuals in the first age class but only 50 in the fourth

Then we need to summarise all of the demographic knowlege

LESLIE MATRIX

$$A = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 \\ P_1 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 \\ 0 & 0 & P_3 & 0 \end{bmatrix}$$

F_1 is the fecundity of people in age class 1 etc.

P_1 is the probability of surviving through age classes

Using the Leslie Matrix to update the age structure

$$n(t+1) = An(t)$$

$$n(t+1) = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 \\ P_1 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 \\ 0 & 0 & P_3 & 0 \end{bmatrix} \begin{bmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \\ n_4(t) \end{bmatrix}$$

This produces next years pop matrix

1. Start by calculating the no. of newborns - individuals entering age class 1

$\therefore n_1(t+1) = F_1 n_1(t) + F_2 n_2(t) + F_3 n_3(t) + F_4 n_4(t)$ → Next years newborns

If there were 10 individuals in each age class the matrix would be...

Year One

$$\begin{bmatrix} 1.6 & 1.5 & 0.25 & 0 \\ 0.8 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 10 \\ 10 \end{bmatrix} = \begin{array}{ll} \begin{bmatrix} 33.5 \\ 8 \\ 5 \\ 2.5 \end{bmatrix} & \begin{array}{l} \text{stage 1 newborns} \\ \text{age class 2} \\ \text{age class 3} \\ \text{age class 4} \end{array} \end{array}$$

Total = 40 Total = 49 → Pop increasing

Year Two

$$\begin{bmatrix} 1.6 & 1.5 & 0.25 & 0 \\ 0.8 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \end{bmatrix} \begin{bmatrix} 33.5 \\ 8 \\ 5 \\ 2.5 \end{bmatrix} = \begin{bmatrix} 66.85 \\ 26.8 \\ 4 \\ 1.25 \end{bmatrix}$$

Total 49 Total = 98.9 → Pop is ↑ exponential

Can use a log scale for the exponential growth
Can use excel spreadsheets to do this easily - adjust graph by changing survivorship

Leslie Matrix doesn't have to be age based → can be size

THEREFORE...

can have 2 P_i values per line as they can stay the same size e.g. loggerhead turtles

$$A = \begin{bmatrix} 0 & 0 & 0 & 4.7 & 61.9 \\ 0.68 & 0.7 & 0 & 0 & 0 \\ 0 & 0.05 & 0.66 & 0 & 0 \\ 0 & 0 & 0.02 & 0.68 & 0 \\ 0 & 0 & 0 & 0.61 & 0.81 \end{bmatrix}$$

Can use this to work out which life stage to direct conservation efforts. Simply assessing the Leslie Matrix can reveal vital information about the population