INVARIANT POINTS AND LINES ANSWER SHEET

Exercise 1:

$$\begin{pmatrix} 4 & 5 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 4x+5y \\ x+2y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad \begin{aligned} 4x+5y &= x \\ x+2y &= y \end{aligned} \quad \text{simplify to} \quad \begin{aligned} x &= -\tfrac{5}{3}y \\ x &= -y \end{aligned}$$

the 2 equations can only be true if $x=0$ and $y=0$ so the invariant point is $(0,0)$

Exercise 2:

$$\begin{pmatrix} 2 & 4 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 2x+4y \\ 7x+3y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad \begin{aligned} 2x+4y &= x \\ 7x+3y &= y \end{aligned} \quad \text{simplify to} \quad \begin{aligned} x &= -4y \\ x &= -\tfrac{2}{7}y \end{aligned}$$

the 2 equations can only be true if $x=0$ and $y=0$ so the invariant point is $(0,0)$

Exercise 3:

$$\begin{pmatrix} 5 & 1 \\ 8 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 5x+y \\ 8x+3y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad \begin{aligned} 5x+y &= x \\ 8x+3y &= y \end{aligned} \quad \text{simplify to} \quad \begin{aligned} y &= -4x \\ y &= -4x \end{aligned}$$

the invariant points would lie on the line $y=-4x$ and be of the form $(\lambda, -4\lambda)$

Exercise 4:

a) i) all 3 dashed lines are invariant lines

ii) the 3 parallel dashed lines are invariant lines

b) i) any line through $(0,0)$ is an invariant line, so those in the form $y=kx$

ii) any line perpendicular to the mirror line ($y=x$ in this case) so those in the form $y=-x+c$ or $x+y=c$.