EIGENVECTORS + EIGENVALUES
FOR 2×2 MATRICES ANSWER SHEET

Exercise 1:

a) $|A - \lambda I| = \left| \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$\left| \begin{bmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{bmatrix} \right| = 0$

$\lambda^2 + 3\lambda + 2 = 0$

$\lambda_1 = -1 \quad \lambda_2 = -2$

b) $|B - \lambda I| = \left| \begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$\left| \begin{bmatrix} 2-\lambda & 2 \\ 5 & -1-\lambda \end{bmatrix} \right| = 0$

$\lambda^2 - \lambda - 12 = 0$

$\lambda_1 = -3 \quad \lambda_2 = 4$

Exercise 2:

a) Eigenvector v_1 associated with eigenvalue $\lambda_1 = -1$

$(A - \lambda_1) \cdot v_1 = 0$

$\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \cdot v_1 = 0 \quad v_1 = \begin{bmatrix} v_{1,1} \\ v_{1,2} \end{bmatrix}$

$v_{1,1} + v_{1,2} = 0$

$v_{1,1} = -v_{1,2}$

$\therefore \quad v_1 = k_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$