Problem set 3

This problem set involves plotting functions. Please refer to `01_PythonIntro/part_3/python_intro_pt3.ipynb` and `01_PythonIntro/part_3/python_intro_pt3.py` for help on how to create plots in Python.

Function approximation

1. Write down the Taylor series expansion for $f(x+h)$, expressed in terms of powers of h for each of the following functions

   - $f(x) = e^x$
   - $f(x) = \sin(x)$
   - $f(x) = 1/(1+x), \quad \text{for } |x| < 1$
   - $f(x) = x^6$

   Truncate each to within $O(h)$, $O(h^2)$ and $O(h^4)$

2. Identify the order of the truncation error in each of the following approximate expressions obtained from expanding the function about the base point $a = 0$.

   - $xe^x \approx x + x^2$
   - $\sin(2\epsilon) \approx 2\epsilon$
   - $(1+\alpha)^3 \approx 1 + 3\alpha$

3. Write down the Taylor series expansion $f(x) = \cos(x)$ about the base point $a = 0$. Let the truncated approximation be denoted by $\hat{f}_N(x)$ where N indicates the number of non-zero terms in the Taylor series.

   - Write a Python function which evaluates the Taylor series expansion $\hat{f}_N(x)$ given an `ndarray` defining different values for x, and for a specified number of series terms N.
   - Plot the function $f(x)$ and the approximations for $f(x)$ using $N = 4$, $N = 8$ and $N = 16$ terms on a single graph. Plot all functions over the range $x \in [-2\pi, 2\pi]$. Save the plot to a PDF file.

- Using your Python function, determine (programmatically) how many terms K you need to ensure that that

$$|f(0.33) - \hat{f}_K(0.33)| < 10^{-5}$$

is satisfied.