Problem set 3

[Revised Jan 28, 2:15 PM]

> This problem set involves plotting functions. Please refer to `01_PythonIntro/part_3/python_intro_pt3.ipynb` and `01_PythonIntro/part_3/python_intro_pt3.py` for help on how to create plots in Python.

> This homework contains a mixture of math questions and questions requiring you to write Python code. If you need to use Python, the question will indicate this. Unless otherwise stated, please do not use Python to answer the question.

Function approximation

1. Write down the Taylor series expansion for $f(x+h)$, expressed in terms of powers of h for each of the following functions

   - $f(x) = e^x$
   - $f(x) = \sin(x)$
   - $f(x) = 1/(1+x), \quad \text{for } |x| < 1$
   - $f(x) = x^6$

   Truncate each to within $O(h)$, $O(h^2)$ and $O(h^4)$

2. Identify the order of the truncation error in each of the following approximate expressions obtained from expanding the function about the base point $a = 0$.

   - $xe^x \approx x + x^2$
   - $\sin(2\epsilon) \approx 2\epsilon$
   - $(1+\alpha)^3 \approx 1 + 3\alpha$

3. Write down the Taylor series expansion $f(x) = \cos(x)$ about the base point $a = 0$. Let the

truncated approximation be denoted by $\hat{f}_N(x)$ where N indicates the number of non-zero terms in the Taylor series.

- Write a Python function which evaluates the Taylor series expansion $\hat{f}_N(x)$ given an `ndarray` defining different values for x, and for a specified number of series terms N.
- Using Python, plot the function $f(x)$ and the approximations for $f(x)$ using $N = 4$, $N = 8$ and $N = 16$ terms on a single graph. Plot all functions over the range $x \in [-2\pi, 2\pi]$. Save the plot to a PDF file.
- Using your Python function, determine (programmatically) how many terms K you need to ensure that that

  $$|f(0.33) - \hat{f}_K(0.33)| < 10^{-5}$$

  is satisfied.