Problem set 6

[Revised Jan 28, 9:22 PM]

Polynomial least squares regression

For questions 1, 2 and 3 you will be required to solve a 2×2 matrix problem. You can use the provided code in `tinysolve.py` to solve this matrix problem. To perform the solve $\mathbf{Ax} = \mathbf{b}$ do the following

```
import tinysolve

x = tinysolve.solve2x2(A, b)
```

where `A` is your 2×2 matrix, `b` is the right hand side vector and `x` is the solution.

---

1. Use the least squares method to fit a straight line of the form

   $$f(x) = a_0 + a_1 x$$

   to the data provided below

   ```
   x = np.array( [0.0, 0.1, 0.2, 0.3, 0.4] )

   y = np.array( [0.58, 0.90, 1.31, 1.92, 2.51] )
   ```

   - Print the computed values of a_0 and a_1 to the screen.
   - Plot the data points `y` and the function $f(x) = a_0 + a_1 x$ on the same graph.

2. Fit a curve of the form

   $$f(x) = a_0 + a_1 x^2$$

   to the data provided below

```
x = np.array( [-2.0, -1.0, 0.0, 1.0, 2.0] )

y = np.array( [6.17, 1.92, 1.51, 2.12, 5.08] )
```

- Write down (on paper) the matrix problem $\mathbf{Qa} = \mathbf{m}$ that's required to be solved to obtain $\mathbf{a} = (a_0, a_1)$. Use

$$S = \sum_{i=1}^{5} [y_i - f(x_i)]^2.$$

- Print the computed values of a_0 and a_1 to the screen.
- Plot the data points `y` and the function $f(x) = a_0 + a_1 x^2$ on the same graph.

3. Fit a curve of the form

$$f(x) = a_0 + a_1 \sin(x)$$

to the data provided below

```
x = np.array( [-numpy.pi, -0.5*numpy.pi, 0.0, 0.5*numpy.pi, numpy.pi] )

y = np.array( [2.10, 5.14, 3.28, 1.16, 2.51] )
```

- Write down (on paper) the matrix problem $\mathbf{Qa} = \mathbf{m}$ that's required to be solved to obtain $\mathbf{a} = (a_0, a_1)$. Use

$$S = \sum_{i=1}^{5} [y_i - f(x_i)]^2.$$

- Print the computed values of a_0 and a_1 to the screen.
- Plot the data points `y` and the function $f(x) = a_0 + a_1 \sin(x)$ on the same graph.

4. Consider using the least squares method to fit a curve of the form

$$f(x) = a_0 + a_1 \sin(x) + a_2 \cos(x)$$

to data consisting of x_i with $i = 1, 2, \ldots, N$ which defines N points in space, and y_i with $i = 1, 2 \ldots, N$ which defines the function values at each x_i.

- Write down (on paper) the matrix problem $\mathbf{Qa} = \mathbf{m}$ that's required to be solved to obtain $\mathbf{a} = (a_0, a_1, a_2)$. Use

$$S = \sum_{i=1}^{N} [y_i - f(x_i)]^2.$$