# Distance Between Point and Line, Ray, or Line Segment

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## 1 Discussion

The following construction applies in any dimension, not just in 3D. Let the test point be $\mathbf{P}$. A line is parameterized as $\mathbf{L}(t) = \mathbf{B} + t\mathbf{M}$ where $\mathbf{B}$ is a point on the line, $\mathbf{M}$ is the line direction, and $t \in \mathbb{R}$. A *ray* is of the same form but with restriction $t \geq 0$. A *line segment* is restricted even further with $t \in [0, 1]$. The end points of the line segment are $\mathbf{B}$ and $\mathbf{B} + \mathbf{M}$.

The closest point on the line to $\mathbf{P}$ is the projection of $\mathbf{P}$ onto the line, $\mathbf{Q} = \mathbf{B} + t_0\mathbf{M}$, where

$$t_0 = \frac{\mathbf{M} \cdot (\mathbf{P} - \mathbf{B})}{\mathbf{M} \cdot \mathbf{M}}.$$

The distance from $\mathbf{P}$ to the line is

$$D = |\mathbf{P} - (\mathbf{B} + t_0\mathbf{M})|.$$

If $t_0 \leq 0$, then the closest point on the ray to $\mathbf{P}$ is $\mathbf{B}$. For $t_0 > 0$, the projection $\mathbf{B} + t_0\mathbf{M}$ is the closest point. The distance from $\mathbf{P}$ to the ray is

$$D = \left\{ \begin{array}{ll} |\mathbf{P} - \mathbf{B}|, & t_0 \leq 0 \\ |\mathbf{P} - (\mathbf{B} + t_0\mathbf{M})|, & t_0 > 0 \end{array} \right\}.$$

Finally, if $t_0 > 1$, then the closest point on the line segment to $\mathbf{P}$ is $\mathbf{B} + \mathbf{M}$. The distance from $\mathbf{P}$ to the line segment is

$$D = \left\{ \begin{array}{ll} |\mathbf{P} - \mathbf{B}|, & t_0 \leq 0 \\ |\mathbf{P} - (\mathbf{B} + t_0\mathbf{M})|, & 0 < t_0 < 1 \\ |\mathbf{P} - (\mathbf{B} + \mathbf{M})|, & t_0 \geq 1 \end{array} \right\}.$$

The division by $\mathbf{M} \cdot \mathbf{M}$ is the most expensive algebraic operation. The implementation should defer the division as late as possible.