Task 1

依题意，两条曲线分别表示为：

$$\theta_1(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3 \quad (0 \le t \le t_{f1}) \tag{1-1}$$

$$\theta_2(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3 \quad (0 \le t \le t_{f2}) \tag{1-2}$$

分别取微分有：

$$\dot{\theta}_1(t) = a_{11} + 2a_{12}t + 3a_{13}t^2 \quad (0 \le t \le t_{f1}) \tag{1-3}$$

$$\dot{\theta}_2(t) = a_{21} + 2a_{22}t + 3a_{23}t^2 \quad (0 \le t \le t_{f2}) \tag{1-4}$$

$$\ddot{\theta}_1(t) = 2a_{12} + 6a_{13}t \qquad \le t \le t_{f1}) \tag{1-5}$$

$$\ddot{\theta}_2(t) = 2a_{22} + 6a_{23}t \quad (0 \le t \le t_{f2}) \tag{1-6}$$

将已知角度分别代入式(1-1)、(1-2)有：

$$\theta_1(0) = a_{10} = \theta_0$$

$$\theta_1(t_{f1}) = a_{10} + a_{11}t_{f1} + a_{12}t_{f1}^2 + a_{13}t_{f1}^3 = \theta_1$$

$$\theta_2(0) = a_{20} = \theta_1$$

$$\theta_2(t_{f2}) = a_{20} + a_{21} \cdot t_{f2} + a_{22} \cdot {t_{f2}}^2 + a_{23} \cdot {t_{f2}}^3 = \theta_2$$

由连接点处的速度和加速度连续，即：

$$\dot{\theta}_1(t_{f1}) = \dot{\theta}_2(0), \quad \ddot{\theta}_1(t_{f1}) = \ddot{\theta}_2(0)$$

分别代入式(1-3)、(1-4)、(1-5)、(1-6)有：

$$a_{11} + 2a_{12}t_{f1} + 3a_{13}{t_{f1}}^2 = a_{21}$$

$$2a_{12} + 6a_{13}t_{f1} = 2a_{22}$$

补充自然边界条件有：

$$\dot{\theta}_1(0) = 2a_{11} = 0$$

$$\dot{\theta}_2(t_{f2}) = a_{21} + 2a_{22} \cdot t_{f2} + 3a_{23}{t_{f2}}^2 = 0$$

当$t = t_{f1} = t_{f2}$时，上式可解得：

$$a_{10} = \theta_0,\quad a_{11} = 0,\quad a_{12} = \frac{3(-\theta_0 + 4\theta_1 - \theta_2)}{4t^2},\quad a_{13} = \frac{5\theta_0 - 8\theta_1 + 3\theta_2}{4t^3}$$

$$a_{20} = \theta_1,\quad a_{21} = \frac{3(\theta_2 - \theta_0)}{4t},\quad a_{22} = \frac{3(\theta_0 - 2\theta_1 + \theta_2)}{2t^2},\quad a_{23} = \frac{-3\theta_0 + 8\theta_1 - 5\theta_2}{4t^3}$$

Task 2

a) 程序：Rhw5_1_main.m

b) 程序：Rhw5_2_main.m