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COSMIC MICROWAVE BACKGROUND RADIATION ANISOTROPIES FOUND IN FOUR-YEAR COBE-DMR DATA

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ABSTRACT

Presented is a maximum likelihood analysis of four-year COBE-DMR observations in the 53GHz and 90GHz frequencies. The analysis will attempt to statistically determine the cosmological parameters Q and n, corresponding to the amplitude and tilt of the CMB spectrum. Maximum likelihood analysis estimates values of $n_{53} = 0.93 \pm 0.32$, $n_{90} = 1.24 \pm 0.51$, $Q_{53} = 18.31 \pm 4.41$ and $Q_{90} = 19.58 \pm 6.61$. This suggests alignment with Harrison-Zeldovich spectrum and primordial density perturbations.

Subject headings: cosmic microwave background — cosmology: observations — methods: statistical

1. INTRODUCTION

Current cosmological theories state that about 380000 years after the Big Bang (at redshift $z = 1100$) the universe was in an inflationary epoch where it consisted of a plasma of protons, free electrons and photons. Due to frequent scattering, the photons are effectively trapped in this plasma. When the temperature decreased to 4000K, recombination occurred causing electrons and atomic nuclei to bind and form neutral atoms. The universe becomes transparent and the photons released at this last scattering become the Cosmic Microwave Background (CMB). Since the universe has expanded by a factor of 1100, the CMB temperature today is about 2.7K. The variations in temperature corresponds to variations in matter density shortly after the Big Bang.

Perturbations in the CMB could be an explanation for large scale structures in the Universe. Standard inflationary models require certain values for Q and n to be correct. By analyzing observational measurements of these parameters one could potentially confirm inflationary theories. The angular power spectrum ties together theory and observations. By comparing those predictions to that measured from the real CMB, we can determine which models best fit with the real universe.

In order to obtain the these cosmological parameters we will in this paper perform a maximum likelihood analysis of COBE-DMR data using Fourier transformations of spherical harmonics decomposition. Spherical harmonics follow a Gaussian distribution which allows us to find a likelihood function for the parameters. We will use the maximum-likelihood framework to determine the best-fit values of the parameters Q and n from the COBE-DMR observations.

2. METHOD

To analyze the CMB temperature as a function of sky position and understand the statistical properties of these fluctuations, we will use spherical harmonic decomposition. The spherical harmonic decomposition corresponds to a spherical equivalent to the Fourier transform

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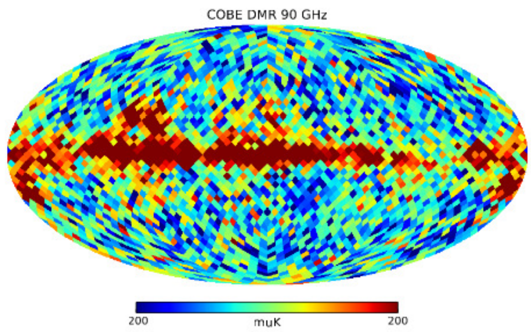

Fig. 1.— Map of the COBE-DMR 90GHz channel.

$$\Delta T(\hat{n}) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\hat{n}), \tag{1}$$

where a_{lm} is a wave mode amplitude describing the strength of a wavelength. Each mode is described by two characteristic numbers, l and m, which respectively represents the effective wavelength of the mode and the phase of the mode. In a statistically isotropic and homogeneous field m does not carry any important information. We are instead more interested in the amplitude of the signal as a function of wavelength, or equivalently its variance, which is the angular power spectrum defined as

$$\langle a_{lm} a^{*}_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l. \tag{2}$$

In order to estimate the variance of the CMB fluctuations, we need to know something about the variance of the instrumental noise. We assume that the noise per pixel is Gaussian distributed with zero mean and standard deviation equal to σ_p, where $\sigma_p = \sigma_0 / \sqrt{N_{obs,p}}$, where σ_0 is the noise per observation and $N_{obs,p}$ is how many times the instrument has looked at pixel p. We assume that the noise is independent between any two pixels such that the noise covariance matrix can be written as

$$N_{pp'} = \sigma_p^2 \delta_{pp'}. \tag{3}$$

Figure 1 shows the map of the COBE-DMR 90GHz channel. There are some regions especially around the galactic plane that are strongly dominated by foreground noise. To reduce contamination of this noise, we use a

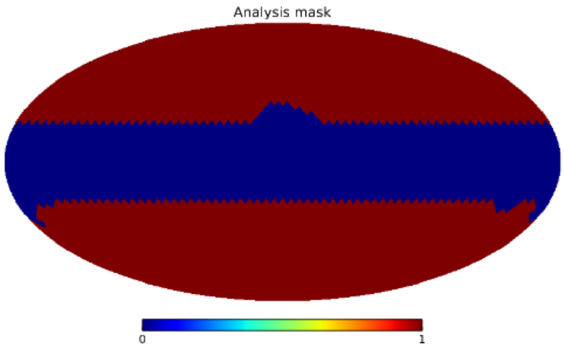

FIG. 2.— Mask used to exclude the most contaminated pixels of the COBE-DMR 90GHz channel.

mask to exclude the most contaminated pixels. The mask is shown in figure 2.

We have to account for the Instrumental beam. This is a point spread function that tells us how large area of the sky is seen by the instrument at any given time. The COBE-DMR beam extends 7 degrees on the sky, effectively suppressing small angular scales. We describe this operation mathematically as a convolution in pixel space, which turn into a multiplication in harmonic space due to the Fourier convolution theorem thus replacing equation 1 with

$$\hat{T}(\hat{n}) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} b_l a_{lm} Y_{lm}(\hat{n}). \qquad (4)$$

Since the CMB observations are well approximated as Gaussian distribution, we find their covariance matrix using the instrumental noise as a component along with a CMB component,

$$d(\hat{n}) = s(\hat{n}) + n(\hat{n}) + f(\hat{n}). \qquad (5)$$

The observed signal in direction $\hat{n}$ is d, s is the true CMB signal, n is the noise, and f is possible non-cosmological foreground signals. We assume none of these are correlated with each other, making all the cross-products have zero mean. The covariance matrix of d is then given by

$$\mathbf{C} \equiv \mathbf{S} + \mathbf{N} + \mathbf{F}, \qquad (6)$$

where $\mathbf{F}$ is the foreground contamination and $\mathbf{N}$ is the noise covariance matrix given by $N_{ij} = \sigma_i^2 \delta_{ij}$ (Górski et al. 1994).

We assume that the CMB field isotropic and Gaussian, but correlated between pixels. We can then find that the pixel-pixel covariance matrix is

$$S_{ij} = \frac{1}{4\pi} \sum_{l=0} (2l+1)(b_l p_l)^2 C_l P_l(\cos\theta_{ij}), \qquad (7)$$

where b_l is the instrumental beam and p_l is the pixel window. The power spectrum C_l is a function that depends critically on cosmological parameters. We use a class of models which are parametrized by an amplitude Q and a spectral index n $(P(k)) \propto k^n$(Bond, Efstathion 1987) on the form

$$C_l = \frac{4\pi}{5} Q^2 \frac{\Gamma(l + \frac{n-1}{2})\Gamma(\frac{9-n}{2})}{\Gamma(l + \frac{5-n}{2})\Gamma(\frac{3+n}{2})}. \qquad (8)$$

Since the expression for $l = 2$ may be simplified to $C_2 = 4\pi/5Q^2$, we will calculate C_{l+1} recursively.

To determine the best-fit values for the parameters Q and n we use the likelihood function

$$\mathcal{L}(Q, n) = p(\mathbf{d}|Q, n). \qquad (9)$$

The log likelihood then becomes

$$-2\log\mathcal{L}(Q, n) = \mathbf{d}^T\mathbf{C}^{-1}\mathbf{d} + \log|\mathbf{C}| + \text{constant}. \qquad (10)$$

We use brute-force grid evaluation as the datasets are small enough not to require Markov Chain Monte Carlo (MCMC).

3. DATA

We analyze two frequencies (53GHz and 90GHz) of COBE-DMR 4-year datasets. They contain DMR map and RMS values for their respective frequency. Both datasets have been downgraded to HEALPy resolution $N_{\text{side}} = 16$ from $N_{\text{side}} = 32$. We use data for the DMR beam function b_l, given for each multipole l. The beam function is shown in figure 3.

Each map has $N_{\text{side}} = 16$, which means that they contain $12 * N_{\text{side}}^2 = 3072$ pixels. 1131 of the total 3072 pixels have been removed by the mask shown in figure 2. The linear size of each pixel is about $220'$ ($3.7°$).

4. RESULTS

Figure 4 and figure 5 show the 68%, 95% and 99.7% levels for the likelihood function for 53GHz and 90GHz respectively.

The maximum likelihood resulted in best-fit values for the parameters Q and n. These values are shown in Table 1.

Figure 6 shows the probability distribution for the parameter Q for both frequencies analyzed. Figure 7 shows the probability distribution for the parameter n for both frequencies analyzed.

5. CONCLUSIONS

The results show a likelihood of $n = 1$, which is consistent with the Harrison-Zeldovich spectrum. Most inflationary models prefer $n = 1$. Our statistical analysis of the CMB-DMR 4-year data could confirm inflationary theories. The primordial perturbation amplitude is $Q \approx 20\mu$K. These perturbations could be the origin of large scale structures in the universe.

Further investigation over a wider range of frequencies are needed to estimate the values of the parameters with a greater statistical significance. The upcoming Wilkinson Microwave Anisotropy Probe (WMAP) which is due to launch in 2001 could provide this data.

TABLE 1

Frequency [GHz]	n	Q
53	0.93 ± 0.32	18.31 ± 4.41
90	1.24 ± 0.51	19.58 ± 6.61

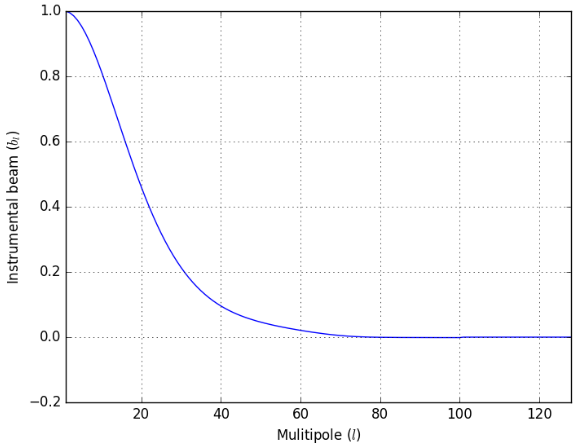

FIG. 3.— DMR beam function b_l as a function of multipole l.

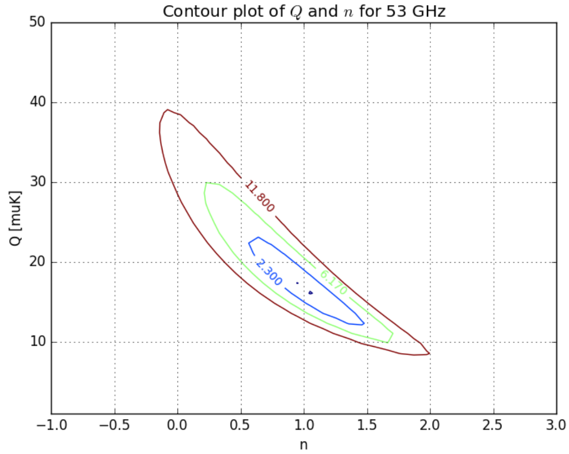

FIG. 4.— Contour plot of the likelihood function for 53GHz. The contours represent 68%, 95% and 99.7% levels for the likelihood function.

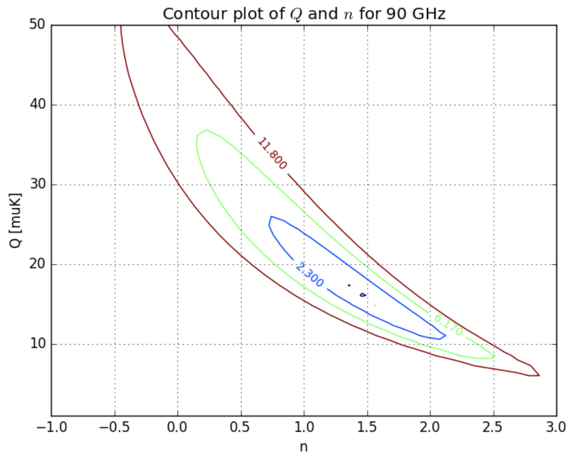

FIG. 5.— Contour plot of the likelihood function for 90GHz. The contours represent 68%, 95% and 99.7% levels for the likelihood function.

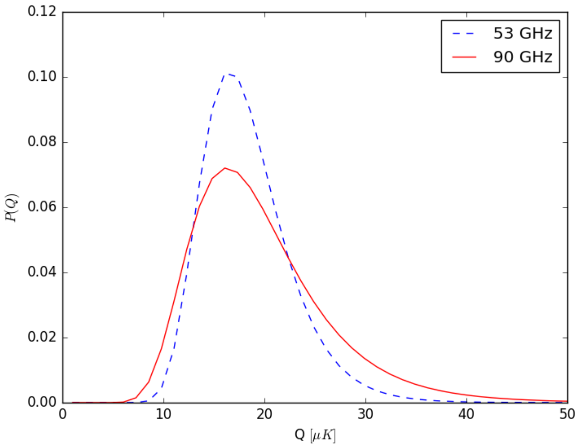

FIG. 6.— Likelihood distribution for Q for 53GHz and 90GHz.

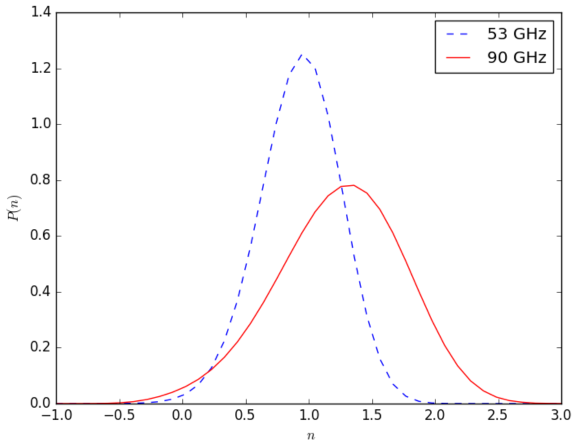

FIG. 7.— Likelihood distribution for n for 53GHz and 90GHz.

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