Exploration of potential parametric forms of conditional extreme value models

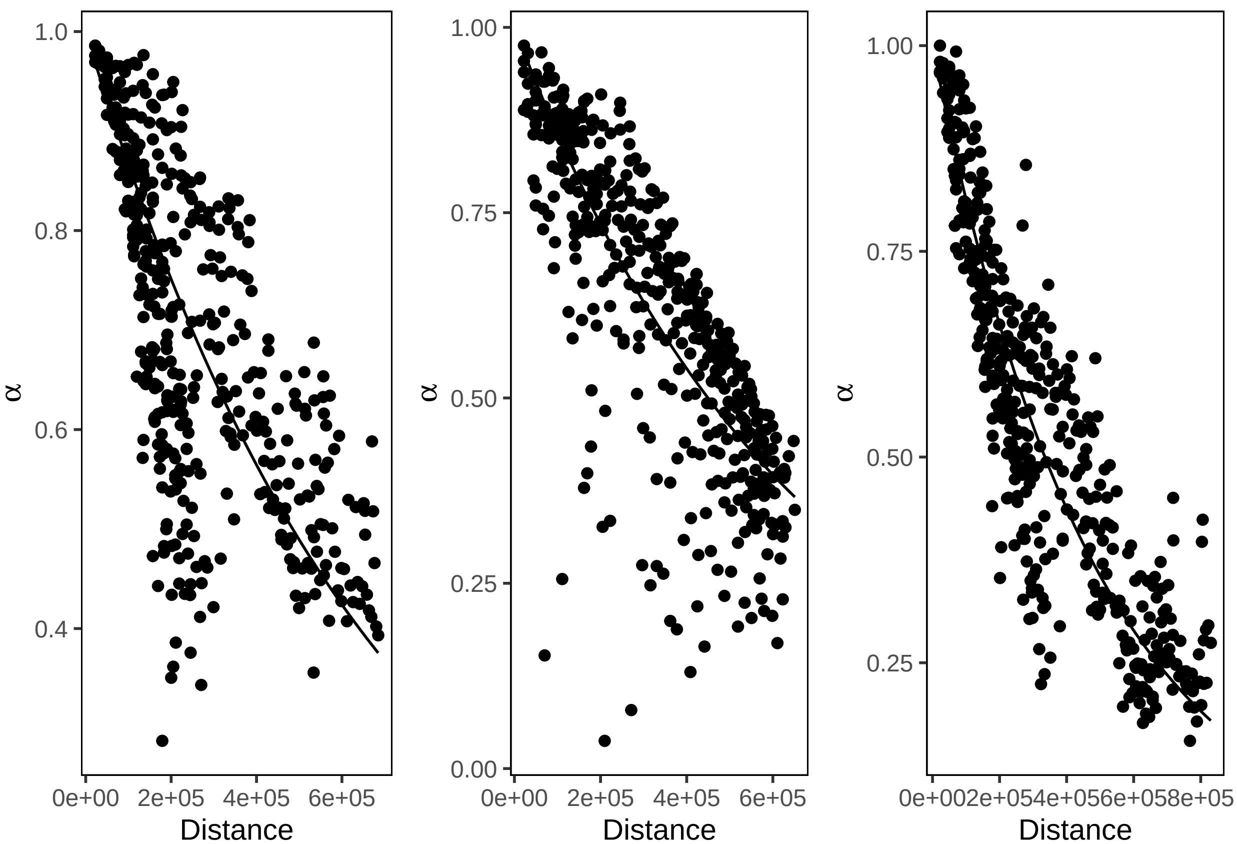
2024-10-25

Set-up

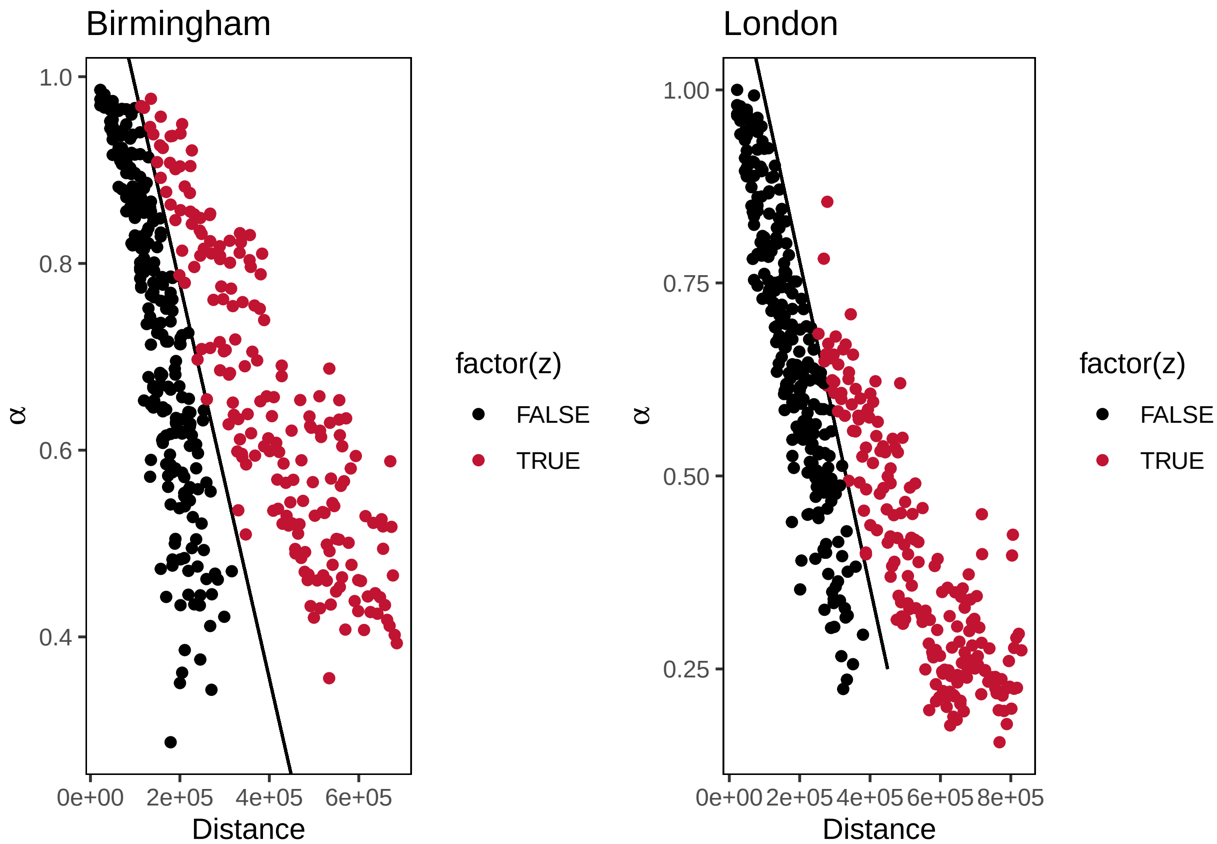
Currently, we work with 19 years of daily maximum temperature data over summer months (June, July, August). (The issue with year 20 remains unresolved.)

Follow-up from meeting with Simon: exploring α against distance

Recall the plot of marginal estimates of α parameter againt distance[m] for the three conditioning sites. This is done using the sequential method (see below).



We look at Birmingham (left) and London (right).

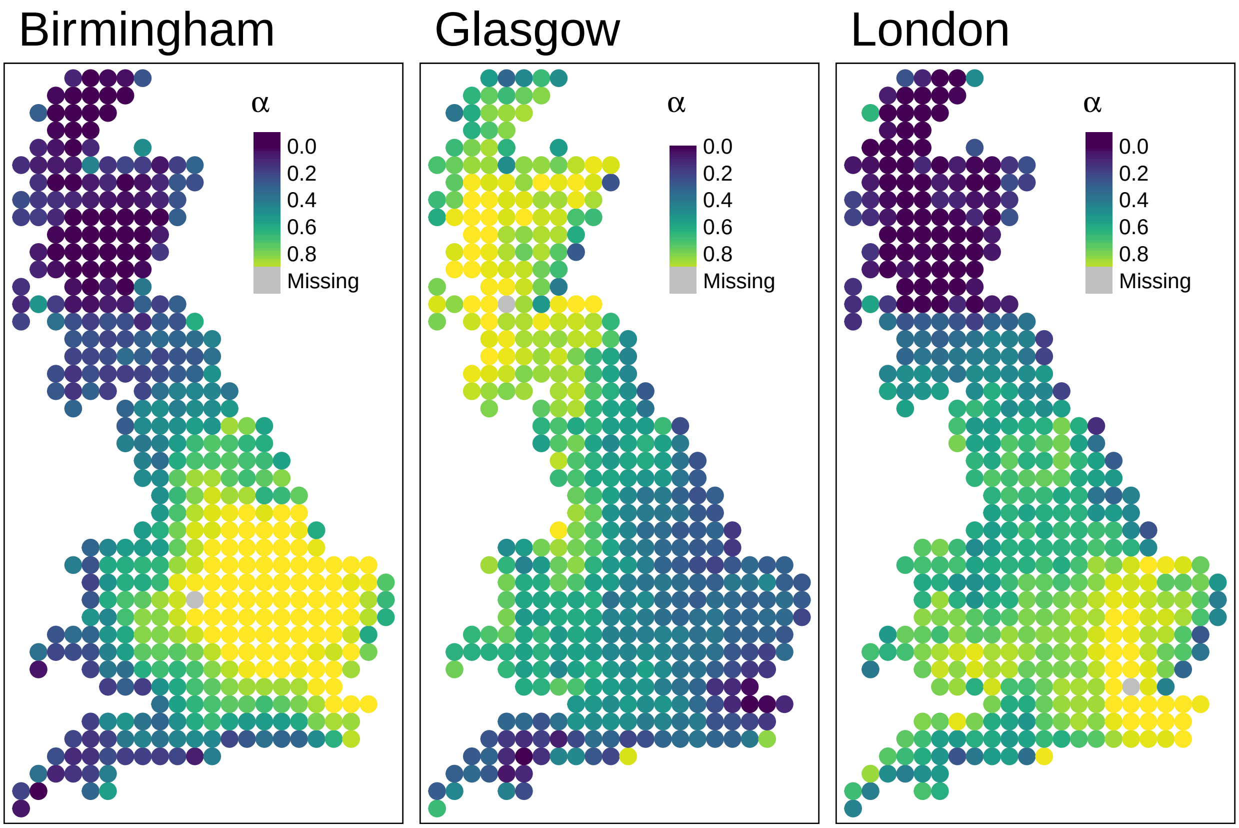

Birmingham
London
α
Distance
factor(z)
FALSE
TRUE
1.0
0.8
0.6
0.4
1.00
0.75
0.50
0.25
0e+00
2e+05
4e+05
6e+05
8e+05

Link both back to spatial locations to explore any potential patterns.
Both conditioning sites show a similar pattern, which suggest higher dependence decay with distance (black) in the south of the mainland UK and in the vicinity of the conditioning sites. The dividing line could be moved to explore this result further.

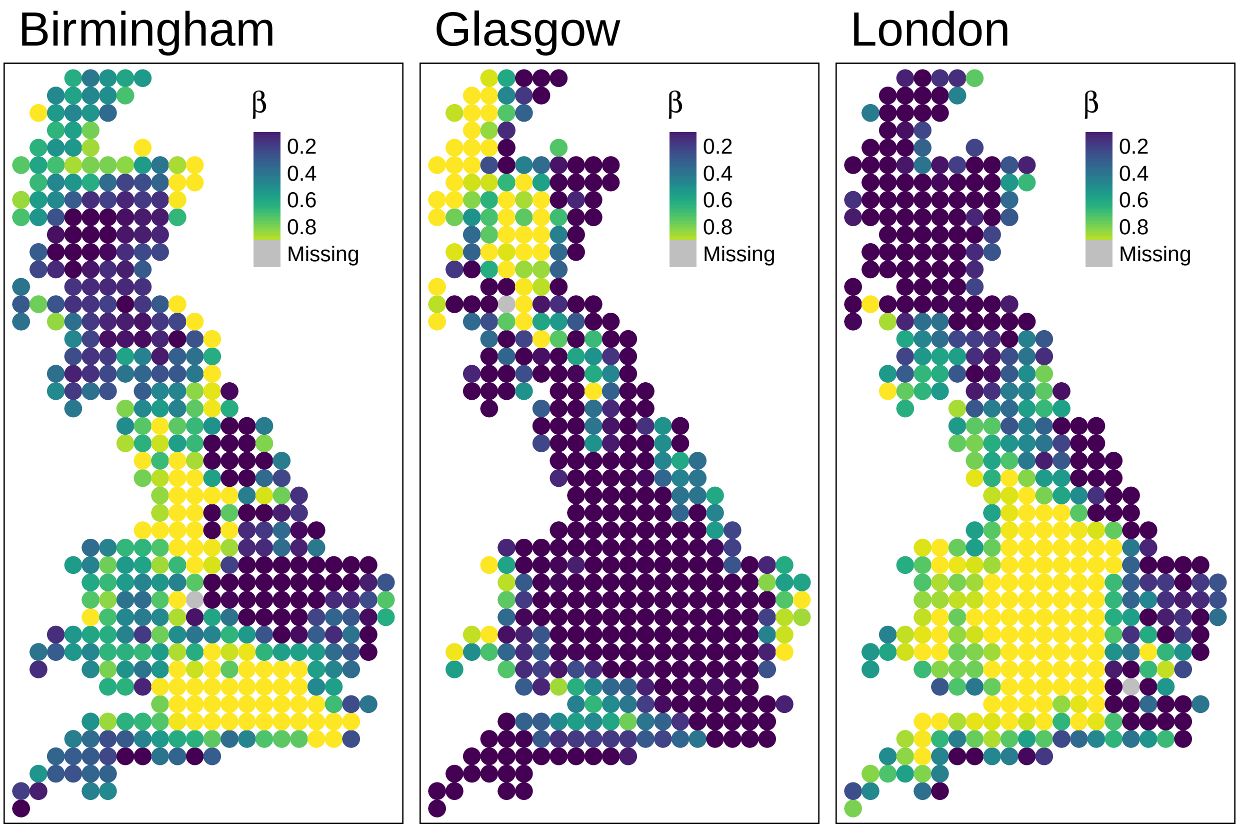
Sequential parameter estimation

1. Fix $\beta = 0$, estimate $\hat{\alpha}$.
2. Fix $\alpha = \hat{\alpha}$, estimate $\hat{\beta}$.
3. Fix $\alpha = \hat{\alpha}$ and $\beta = \hat{\beta}$, estimate $\hat{\mu}$ and $\hat{\sigma}$.
4. Calculate observed residuals, estimate $\hat{\mu}_{AGG}$, $\hat{\sigma}_{AGG}$, $\hat{\delta}_l$ and $\hat{\delta}_u$.

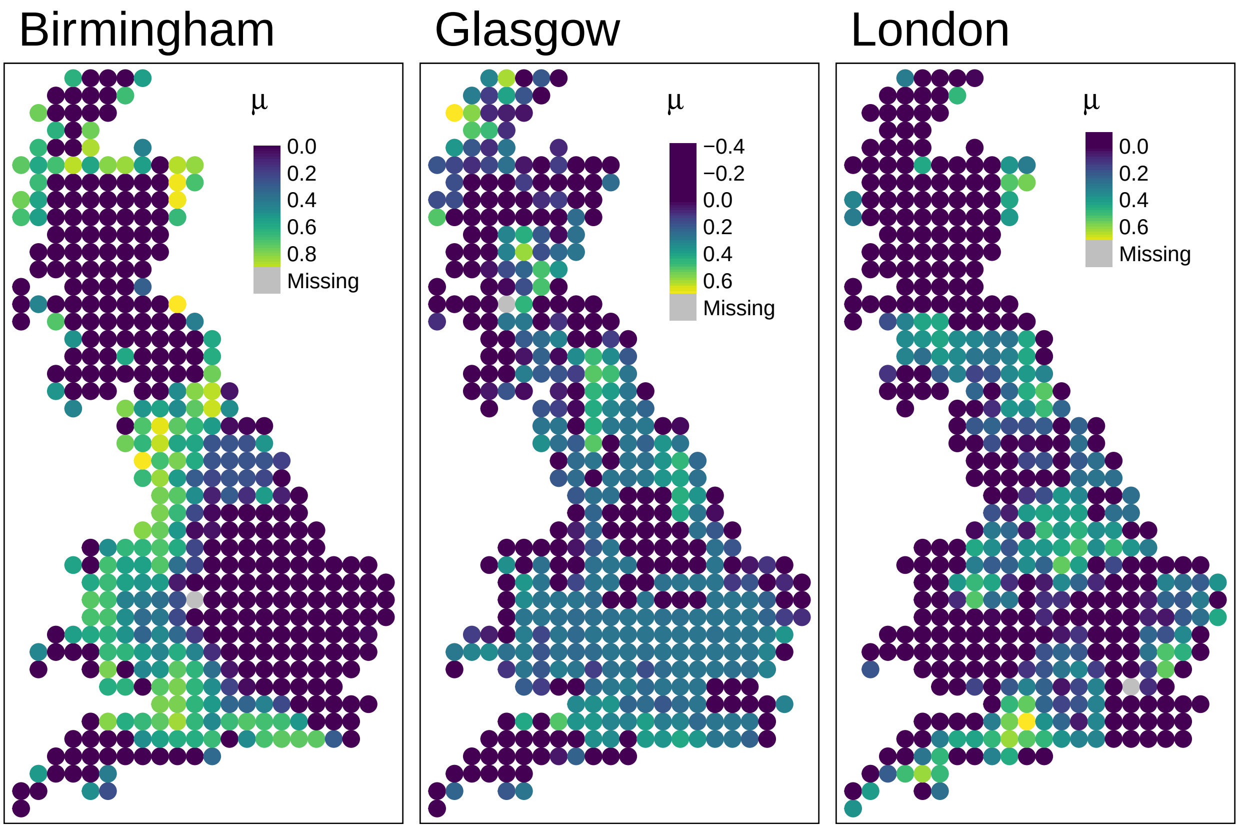
```
## [[1]]
```




```
##
## [[2]]
```

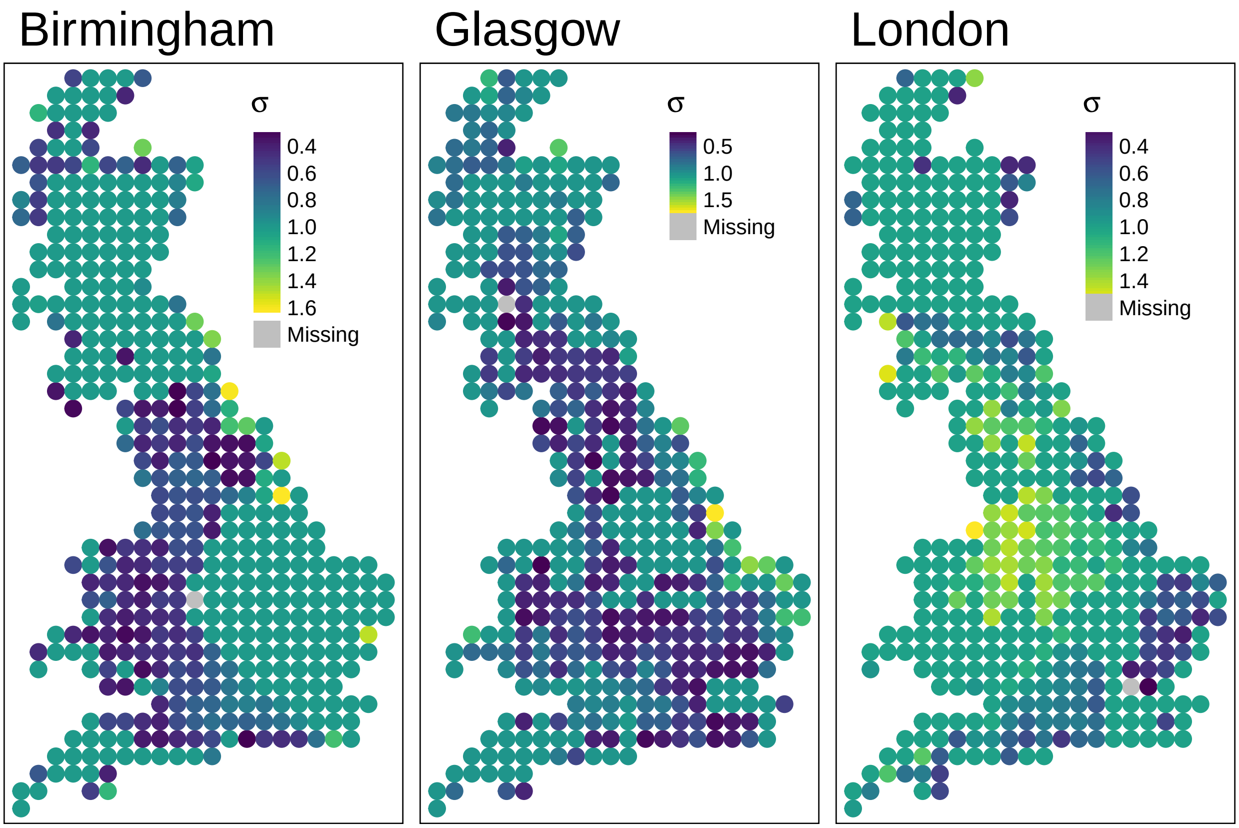



```
##
## [[3]]
```

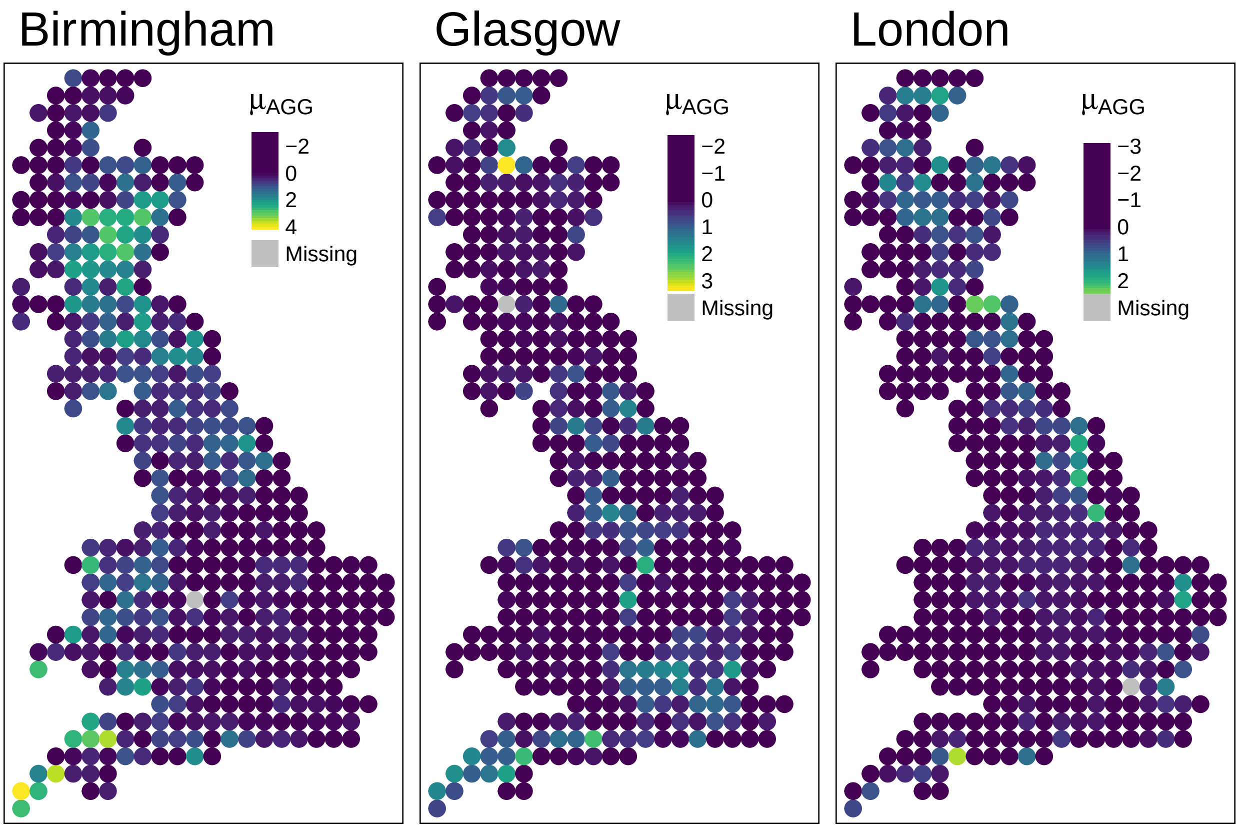


Birmingham
Glasgow
London
μ
0.0
0.2
0.4
0.6
0.8
Missing
μ
−0.4
−0.2
0.0
0.2
0.4
0.6
Missing
μ
0.0
0.2
0.4
0.6
Missing


```
##
## [[4]]
```

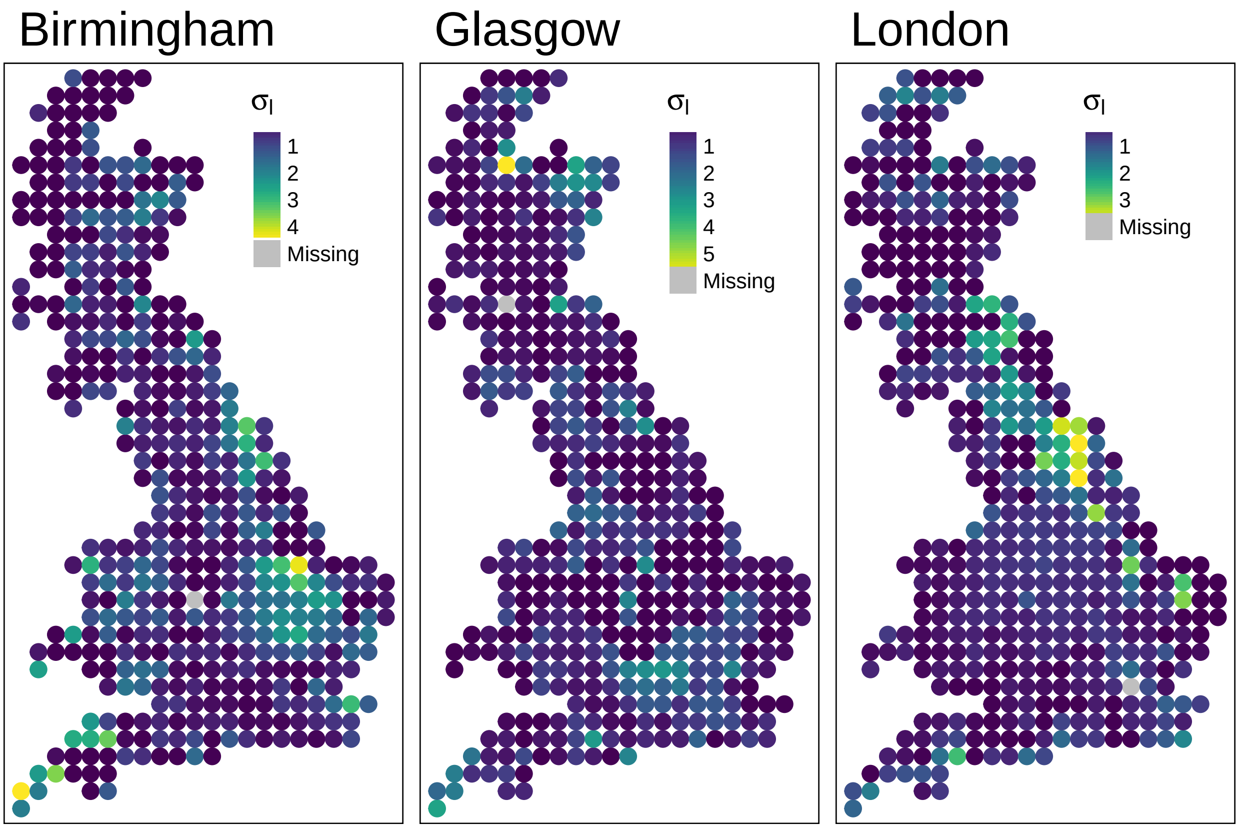

Birmingham
Glasgow
London
σ
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Missing
σ
0.5
1.0
1.5
Missing
σ
0.4
0.6
0.8
1.0
1.2
1.4
Missing

```
##
## [[5]]
```

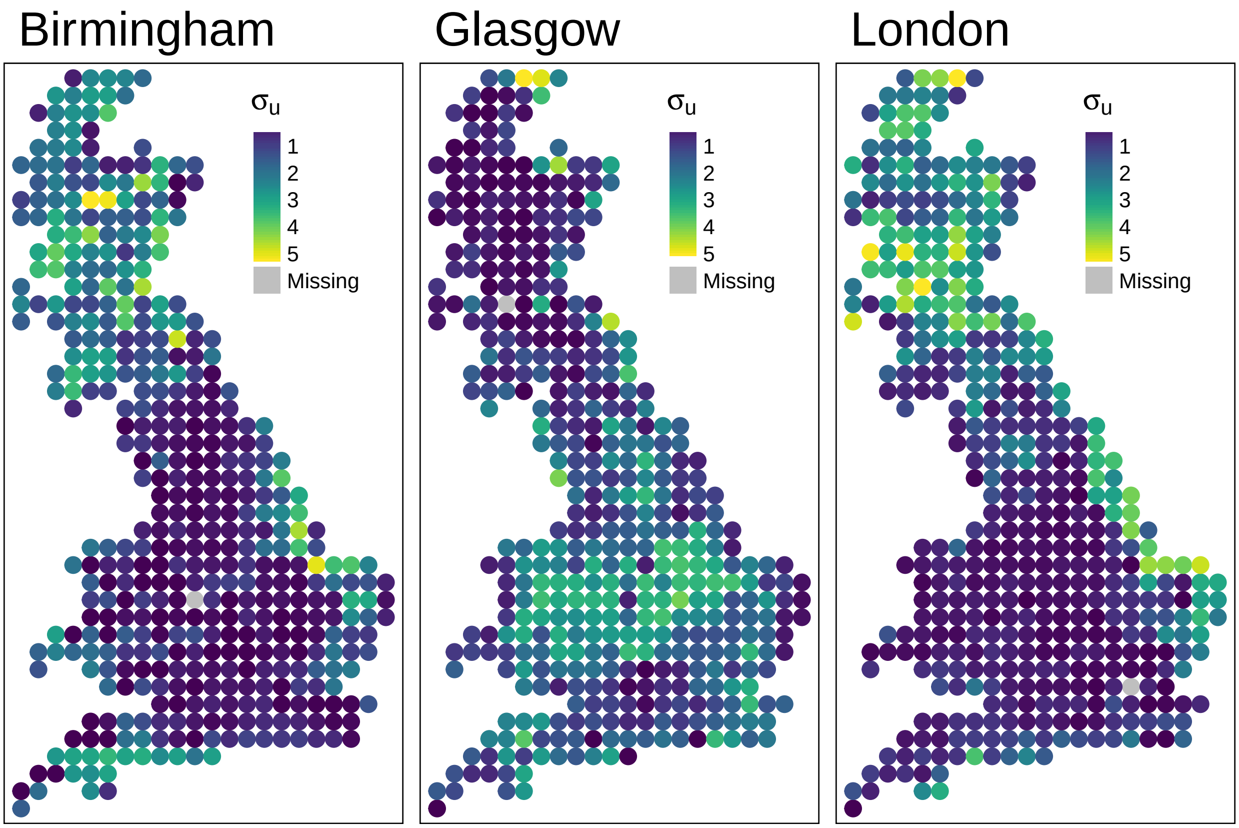


Birmingham
Glasgow
London
μ_{AGG}
−2
0
2
4
Missing
μ_{AGG}
−2
−1
0
1
2
3
Missing
μ_{AGG}
−3
−2
−1
0
1
2
Missing

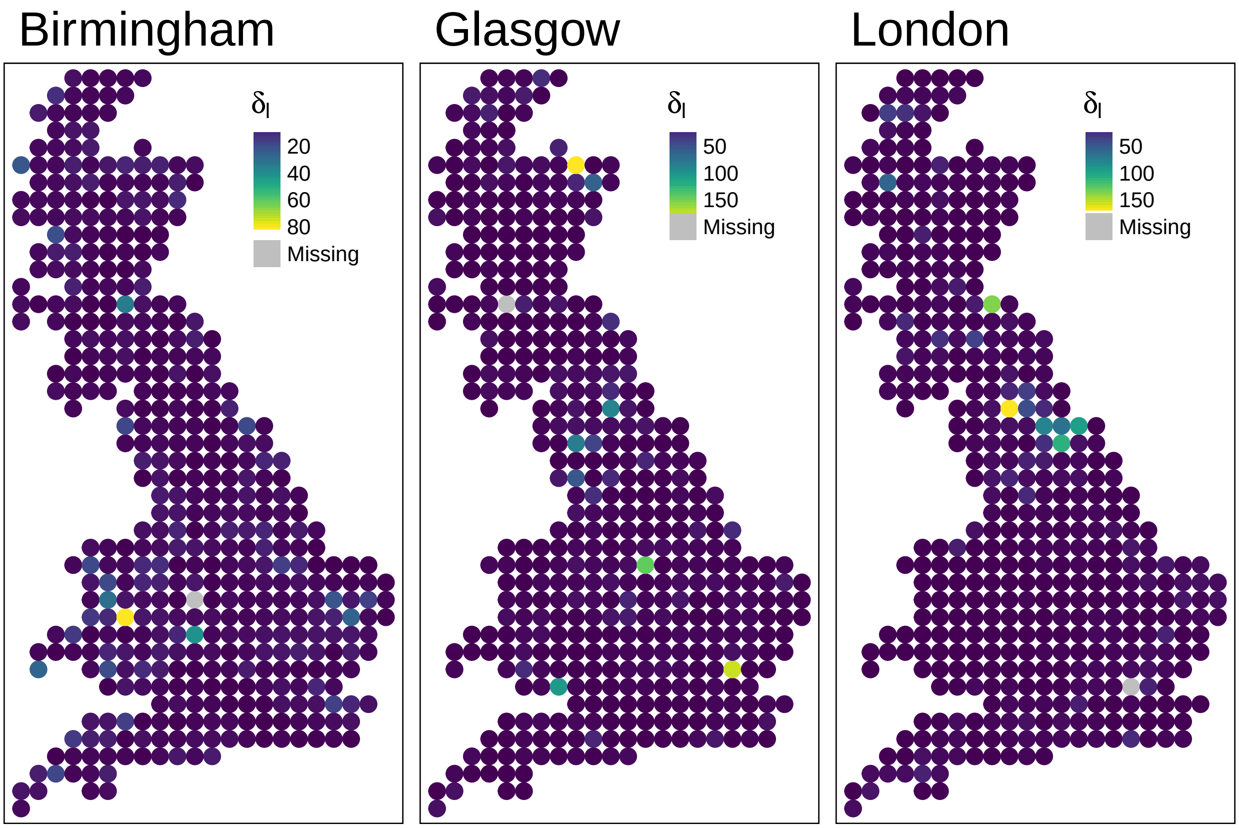

```
##
## [[6]]
```


Birmingham
σl
1
2
3
4
Missing
Glasgow
σl
1
2
3
4
5
Missing
London
σl
1
2
3
Missing

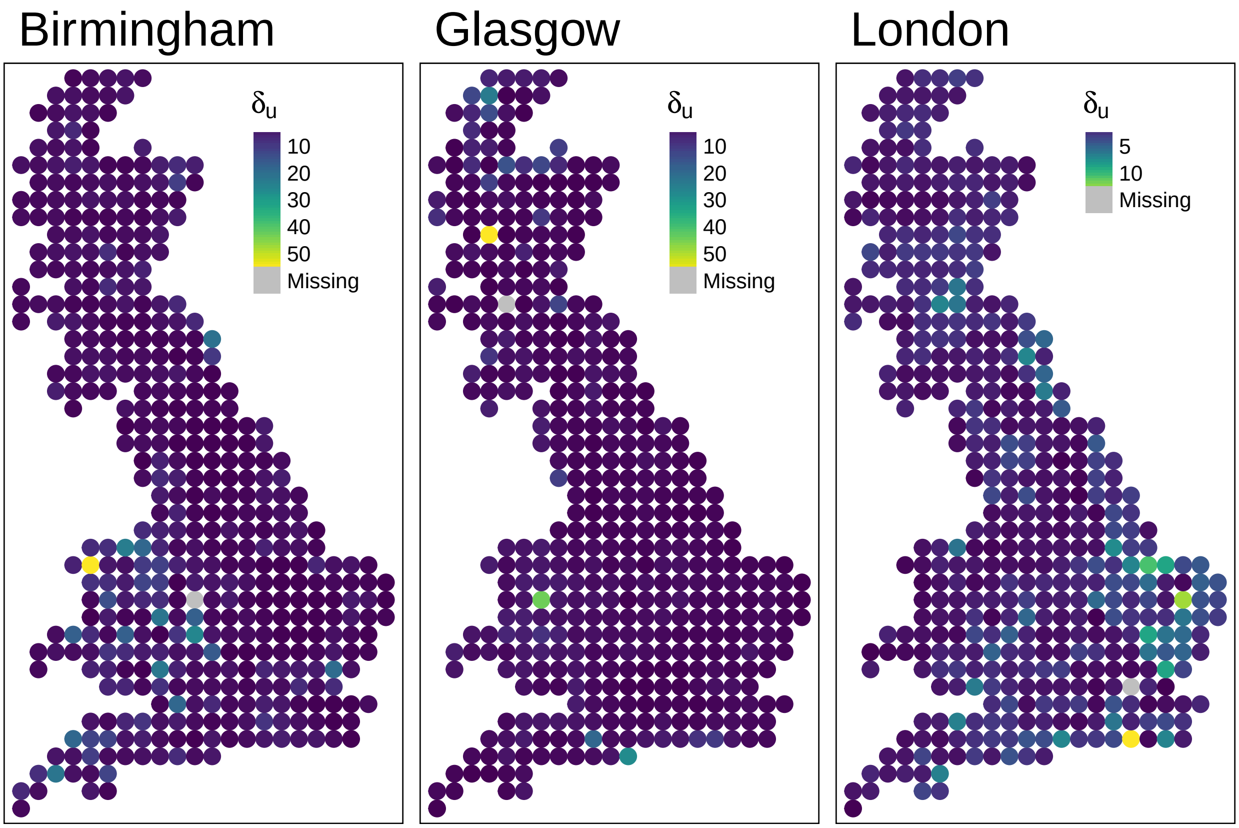
```
##
## [[7]]
```




```
##
## [[8]]
```


Birmingham
δl
20
40
60
80
Missing
Glasgow
δl
50
100
150
Missing
London
δl
50
100
150
Missing

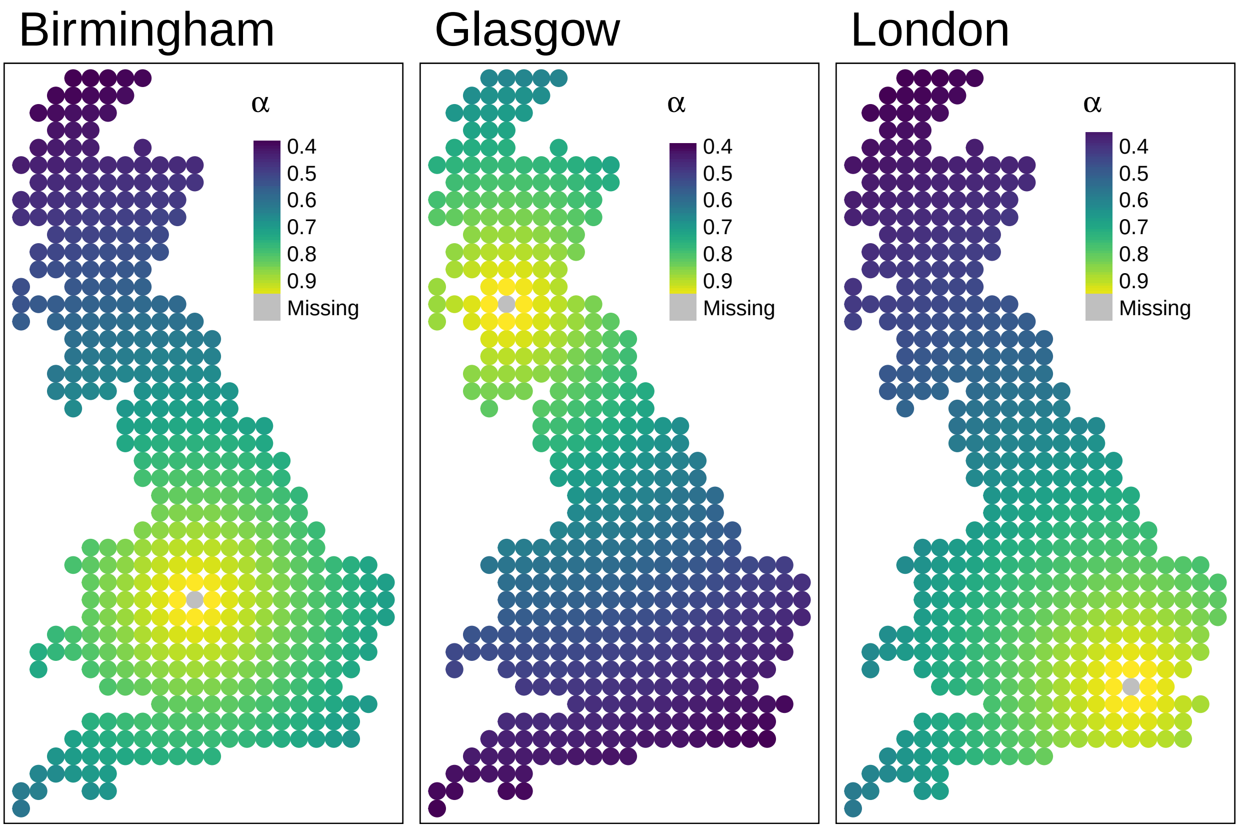
```
##
## [[9]]
```



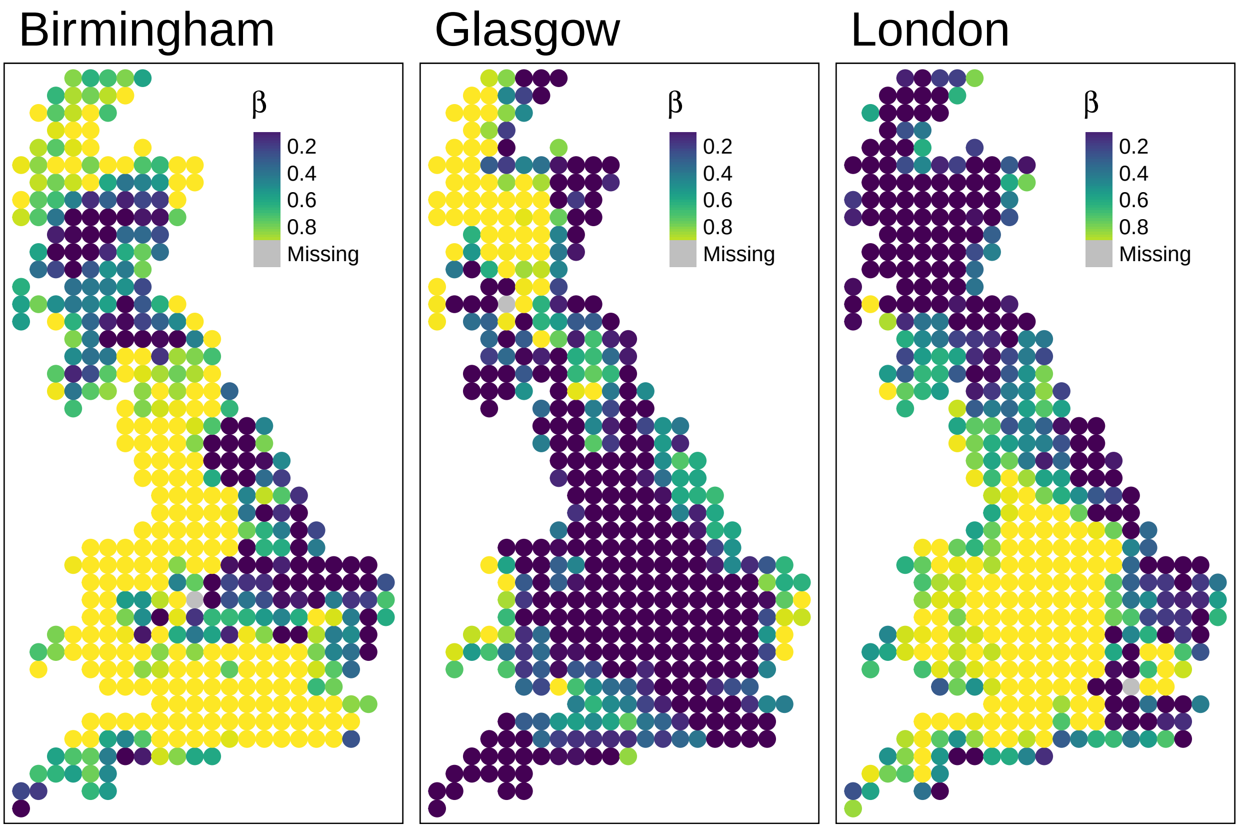
Now, the same approach with alpha values on the fitted exponential curve with distance, so we use estimates of $\hat{\alpha}$ to fit an exponential curve to as a function of distance of site i from the conditioning site j.

1. Fix $\alpha = \alpha(d_{ij}) = \exp\{-\phi d_{ij}\}$, estimate $\hat{\beta}$.
2. Fix $\alpha = \alpha(d_{ij})$ and $\beta = \hat{\beta}$, estimate $\hat{\mu}$ and $\hat{\sigma}$.
3. Calculate observed residuals, estimate $\hat{\mu}_{AGG}$, $\hat{\sigma}_{AGG}$, $\hat{\delta}_l$ and $\hat{\delta}_u$.

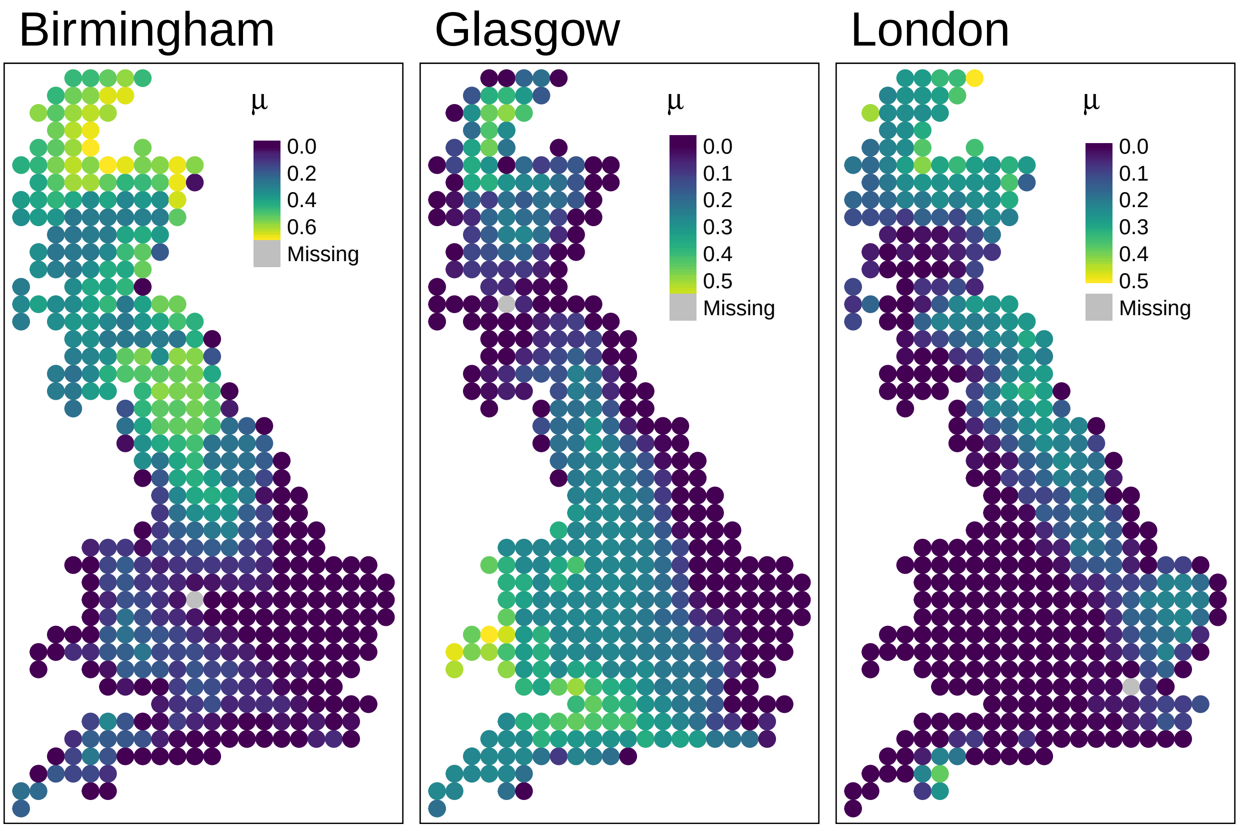
```
## [[1]]
```



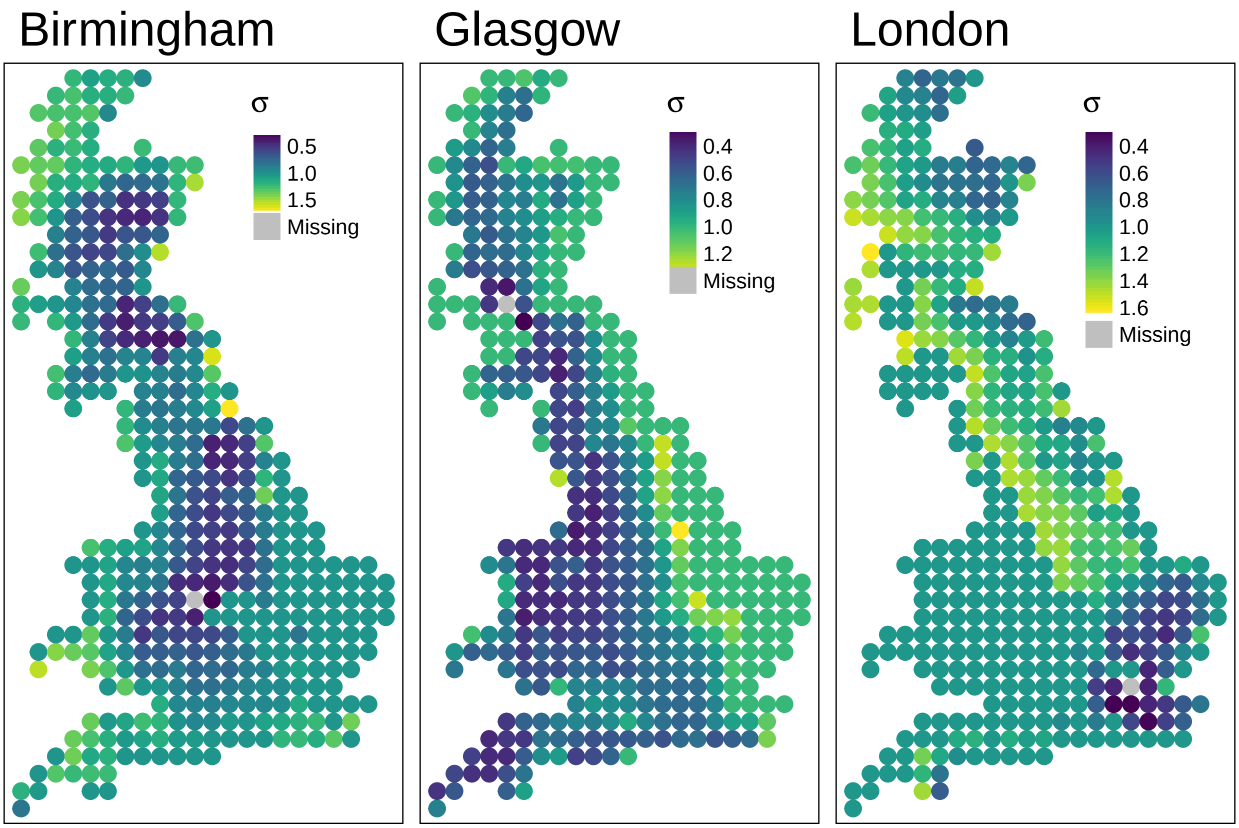

```
##
## [[2]]
```




```
##
## [[3]]
```

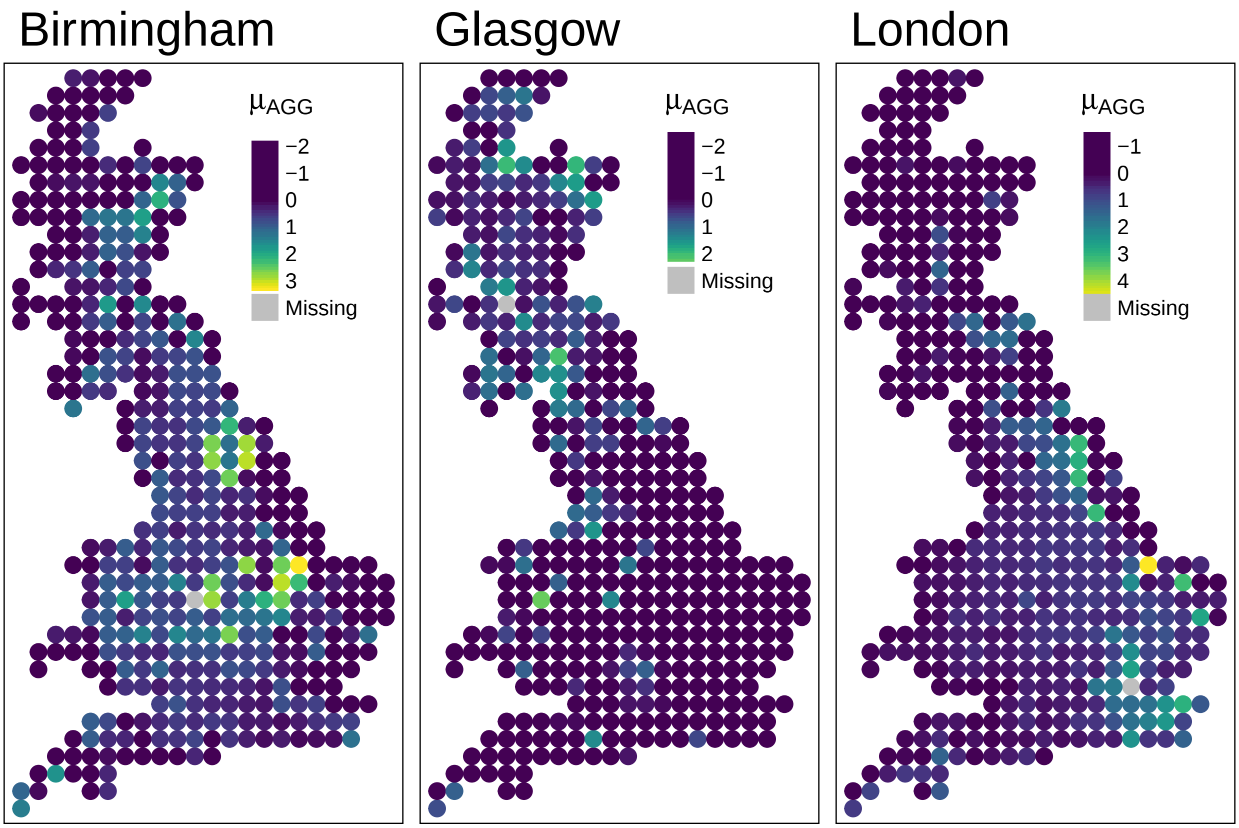

Birmingham
Glasgow
London
μ
0.0
0.2
0.4
0.6
Missing
μ
0.0
0.1
0.2
0.3
0.4
0.5
Missing
μ
0.0
0.1
0.2
0.3
0.4
0.5
Missing

```
##
## [[4]]
```

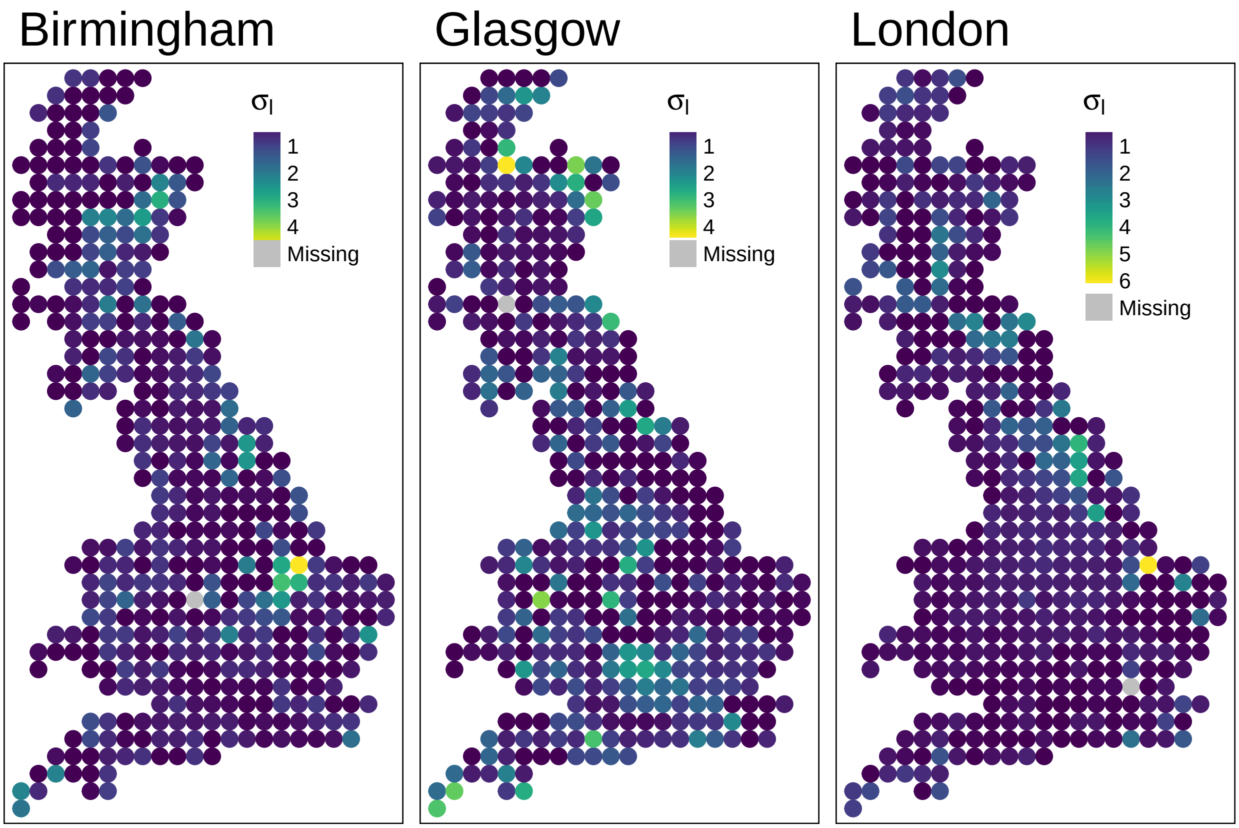


Birmingham
σ
0.5
1.0
1.5
Missing
Glasgow
σ
0.4
0.6
0.8
1.0
1.2
Missing
London
σ
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Missing

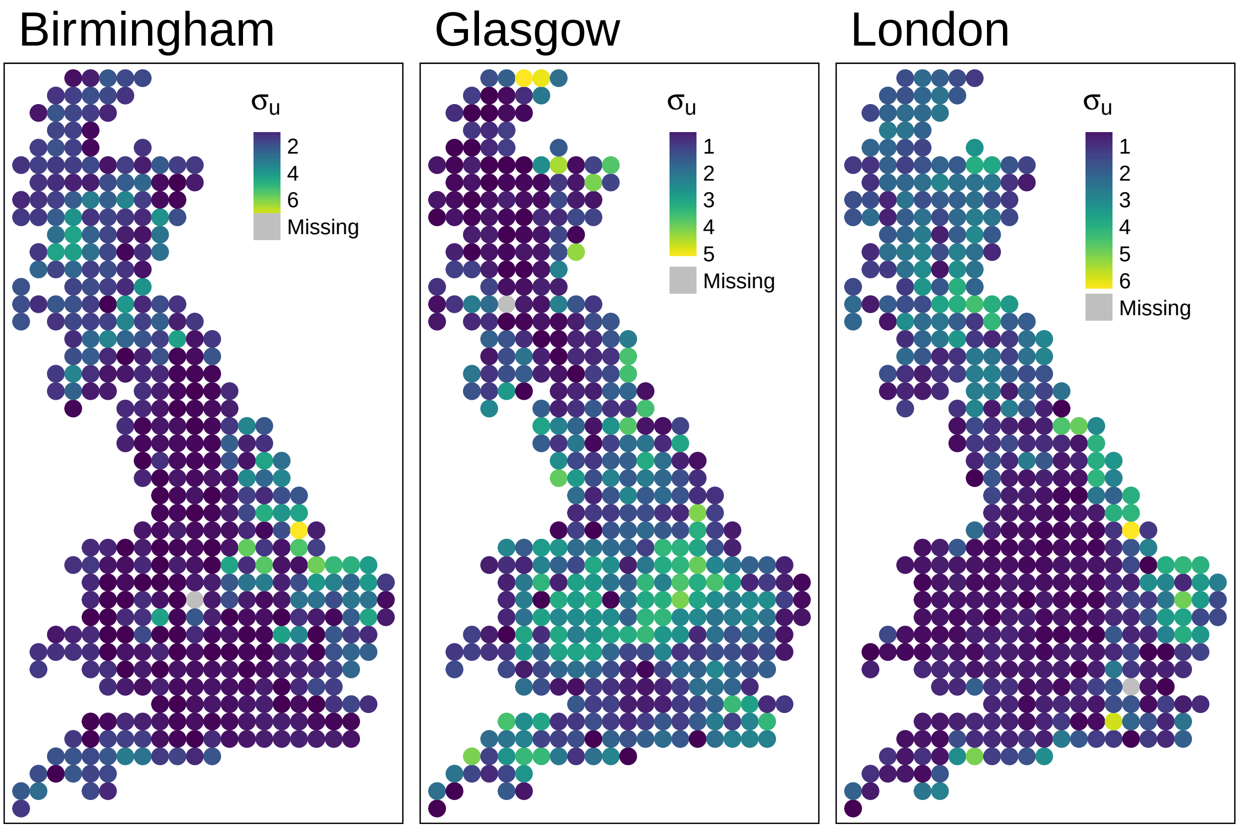

```
##
## [[5]]
```


Birmingham
Glasgow
London
μAGG
−2
−1
0
1
2
3
Missing
μAGG
−2
−1
0
1
2
Missing
μAGG
−1
0
1
2
3
4
Missing

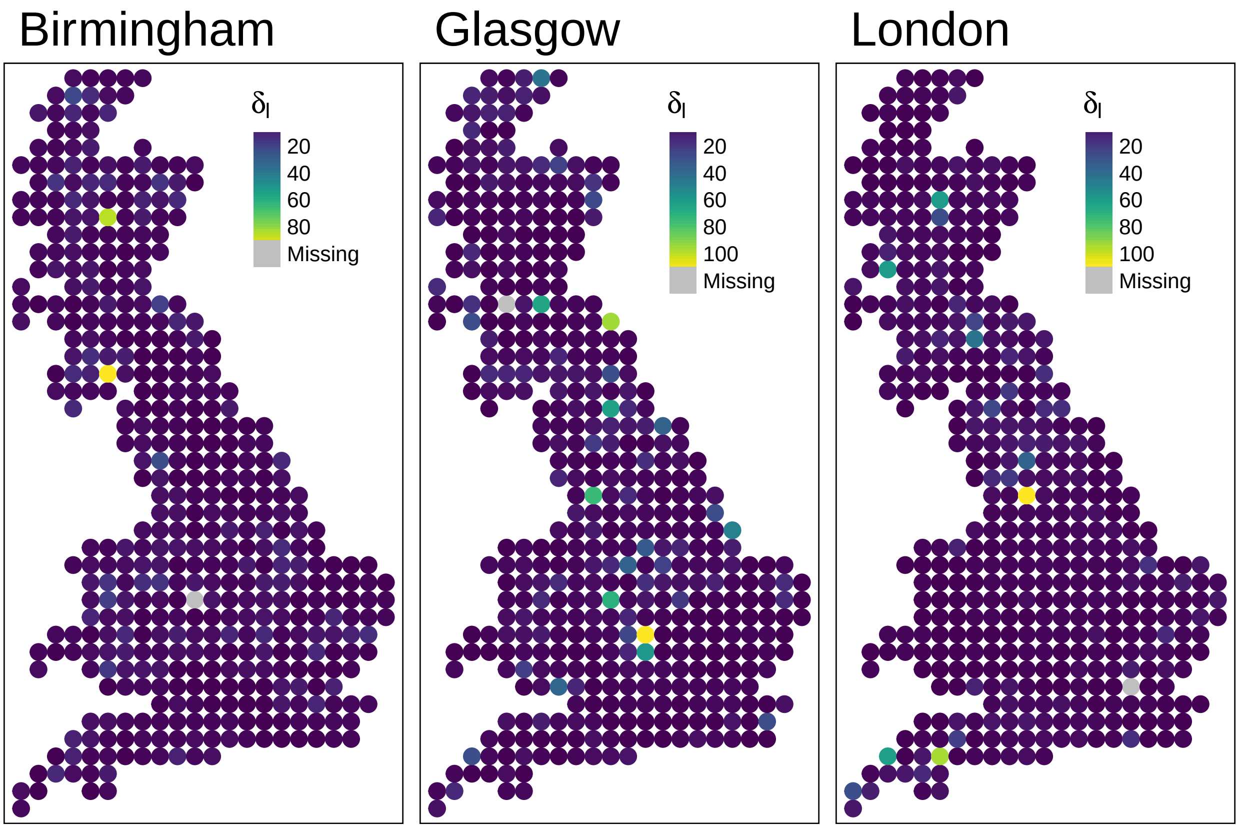
```
##
## [[6]]
```


Birmingham
Glasgow
London
σl
1
2
3
4
Missing
σl
1
2
3
4
Missing
σl
1
2
3
4
5
6
Missing

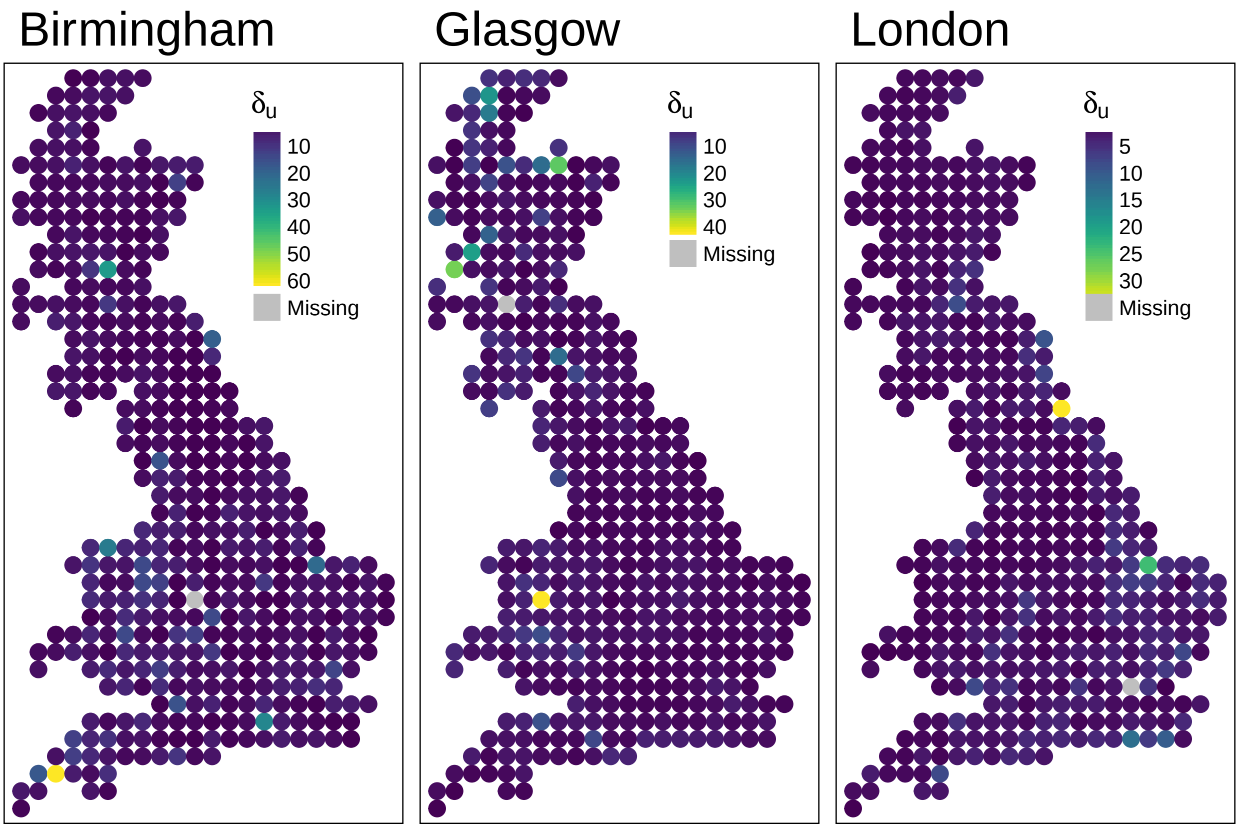
```
##
## [[7]]
```


Birmingham
σu
2
4
6
Missing
Glasgow
σu
1
2
3
4
5
Missing
London
σu
1
2
3
4
5
6
Missing

```
##
## [[8]]
```


Birmingham
Glasgow
London
δl
20
40
60
80
Missing
δl
20
40
60
80
100
Missing
δl
20
40
60
80
100
Missing

```
##
## [[9]]
```



We can observe that imposing a parametric form on $\hat{\alpha}$ that only changes with distance leads to some unexpected values of $\hat{\beta}$ close to 1. For the AGG distribution estimates, $\hat{\mu}_{AGG}$ is the mode of the distribution whereas $\hat{\mu}$ is the median as well as the mode for the symmetric Gaussian distribution. Therefore, lower values for $\hat{\mu}_{AGG}$ suggest heavier tails for the upper tail of the distribution. This is also shown by a positive difference of $\hat{\sigma}_u - \hat{\sigma}_l$.

Conditioning on any random site

Currently, the analysis is conditioning on Birmingham, Glasgow and London, ordered as first 3 columns of the temperature data for an easy link.