Using Subjective Expectations Data to Estimate Educational Choice Models

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Education Choices and Forward-Looking Agents

Education choices are dynamic in nature :

- Higher-ed : lost earnings today vs labor market returns tomorrow
- School choice : private school fees (or location-premium) today vs achievement tomorrow
- College admissions : effort cost today vs higher admission chances tomorrow

Let us characterize the future state of nature as γ^*.

The problem of identification

A student wants to make the education choice that maximizes her utility given a state of nature γ^* :

$$\max_{c \in C} u(c, \gamma^*; \theta)$$

- γ^* are the future returns to each choice c

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- γ^* are the future returns to each choice c
- **Problem** : the student does not observe γ^*

The problem of identification

Assume the student holds a subjective probability function over the states of nature, $Q(\gamma;\beta)$, and maximizes subjective expected utility :

$$\max_{c \in C} \int u(c, \gamma; \theta) dQ(\gamma; \beta)$$

- With data on choices but not on $Q(\gamma;\beta)$, θ and β are not separately identified without further assumptions.

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- With data on choices but not on $Q(\gamma;\beta)$, θ and β are not separately identified without further assumptions.
- Typical assumption : $Q(\gamma;\beta)$ is determined by rational expectations
  - Specify a model for the Data Generating Process
  - Assume $Q(\gamma;\beta)$ coincides with the objective probability distribution of γ^*

The problem of identification

Is the assumption of Rational Expectations reasonable ?

“I would particularly stress that decision makers and empirical economists alike must contend with the logical unobservability of counterfactual outcomes. Much as economists attempt to infer the returns to schooling on schooling choices and outcomes, youth may attempt to learn through observation of the outcomes experienced by family, friends, and others who have made their own past schooling decisions. However, youth cannot observe the outcomes that these people would have experienced had they made other decisions. The possibilities for inference, and the implications for decision making, depend fundamentally on the assumptions that youth maintain about these counterfactual outcomes."

Measuring Expectations, Manski, *Econometrica* 2004

Subjective Belief Data

One solution to the problem of identification is to use subjective belief data on :

- $Q(\gamma; \beta)$
- Stated choices, i.e., probability of making various choices in future

See Giustinelli (2022) for a survey.

Subjective Belief Data

These data can serve various purposes when estimating educational choice models :

1. Inform the model (Arcidiacono, Hotz, Maurel, and Romano (2020))
2. Validate the model (Delavande and Zafar (2019); Van der Klaauw (2012); Wolpin and Gonul (1985))
3. Model learning using panel of beliefs (Stinebrickner and Stinebrickner (2014); Wiswall and Zafar (2015))
4. Separately identify θ and β, and compare outcomes/welfare to rational expectations benchmark (Kapor, Neilson, and Zimmerman (2020); Tincani, Miglino, and Kosse (2025))

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I will give an overview of 1-3, and focus on 4.

Purpose 1 : Inform the Model

Belief Data Collection

Arcidiacono et al. (2020) elicit, for each student i,

- $\gamma_i(c)$: expected earnings in occupation $c \in \{0, 1\}$
- $\pi_i(c)$: stated probability of choosing c in the future

Defining Returns

$$\Delta\gamma_i = \gamma_i(1) - \gamma_i(0)$$

Average Treatment Effects

$$\text{ATE} = \frac{1}{N}\sum_{i=1}^{N}\Delta\gamma_i, \quad \text{ATT} = \frac{1}{N}\sum_{i=1}^{N}\pi_i(1)\,\Delta\gamma_i.$$

Key Insights

- ATT $>$ ATE
- $\Rightarrow$ Students *positively sort* into occupations offering higher *perceived* returns.
- But 73% of students did *not* place all mass on the highest-return occupation.

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Roy-Model of occupational choice based on expected earnings and non-pecuniary factors.

Purpose 1 : Test of Sorting on Gains

Inversion : Under Type I Extreme-Value shocks,

$$\Delta u_i \;=\; \underbrace{\ln \pi_i(1) - \ln \pi_i(0)}_{=\; U_{i1} - U_{i0}} \implies \Delta u_i = U_{i1} - U_{i0}.$$

Relate utilities to expected earnings and other factors :

$$\Delta u_i = \alpha\big(\ln Y_{i1} - \ln Y_{i0}\big) + \beta' X_i + \varepsilon_i \,, \tag{1}$$

where X_i are controls for demographics and major. Assume error orthogonal to earnings term conditional on X_i $\rightarrow$ violated if beliefs and preferences not captured by X_i correlate.

Key estimates : $\widehat{\alpha} > 0$, implying sorting into occupations based on ex-ante gains, but $\beta \neq 0$, implying sorting also on other factors. Mean elasticity across occupations : $\rightarrow$ 10% in expected earnings $\rightarrow \uparrow$ 7.9% subj prob of choosing that occupation.

Conclusion : $\alpha > 0$ confirms positive sorting on *perceived* returns, but $\beta \neq 0$ and large share who "leave money on the table" show that non-pecuniary considerations (credentials, tastes, switching-costs) play a quantitatively important role.

Purpose 2 : Model Validation

Let the (random) utility that student i attaches to choice $c \in C = \{0, 1\}$ be

$$U_i(c, \gamma^*; \theta) \; = \; v(c, \gamma^*; \theta) \; + \; \varepsilon_i(c),$$

where $v(c, \gamma^*; \theta)$ is the deterministic payoff for choice c under state γ^*, and ε_i an i.i.d. shock.
The student does not know the true γ^*, but holds subjective beliefs

$$\gamma^* \; \sim \; Q(\, \gamma \,; \beta).$$

Then the probability she chooses $c = 1$ is

$$\begin{aligned} P_i(c = 1 \mid \theta, \beta) &= \Pr\Big(U_i(1, \gamma; \theta) \; > \; U_i(0, \gamma; \theta)\Big) \\ &= \int F_{\epsilon(0)-\epsilon(1)} \left(v(1, \gamma; \theta) - v(0, \gamma; \theta) \right) \; dQ(\gamma; \beta). \end{aligned} \tag{1}$$

Purpose 2 : Model Validation

If $\varepsilon \sim \text{Type I EV}$, this becomes

$$P_i(c = 1 \mid \theta, \beta) = \int \frac{\exp\big(v(1, \gamma; \theta)\big)}{\sum_{k \in C} \exp\big(v(k, \gamma; \theta)\big)} \, dQ(\gamma; \beta). \qquad (2)$$

$\Rightarrow$ *The expected choice probabilities $P_i(c|\theta, \beta)$ depend on both the structural utility parameters θ and on what we assumed about the belief distribution ($Q(\gamma; \beta)$).*

Purpose 2 : Model Validation

If we have data $\pi(c)$ on the subjective choice probabilities, a.k.a. *stated (probabilistic) choices*, we can validate our model (Wolpin and Gonul (1985)) :

$$\int \frac{\exp\big(v(c,\gamma;\theta)\big)}{\sum_{k\in C}\exp\big(v(k,\gamma;\theta)\big)}\, dQ(\gamma;\beta) \stackrel{?}{=} \pi(c)$$

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If model and expectations are not consistent with each other :

- Check if expectations are better than model at predicting actual future choices
- If so, expectation data contain information about choice determinants not captured in the model (e.g. biased beliefs if model assumed RE)

Purpose 2 : Model Validation

Delavande and Zafar (2019) is a recent example using stated choices to validate a model of university choice in Pakistan. In the model :

- Monetary returns, non-pecuniary factors, and financial constraints determine choice
- Elicit stated choices at baseline and in a hypothetical choice scenario in which students do not have financial constraints
- Estimate life-cycle model using stated choices under baseline financial situation
- Use model to predict choice probabilities in counterfactual without credit constraints
- Compare predicted choice probabilities to elicited stated choices to validate the model

Purpose 3 : Modelling Learning

- **Beliefs evolve :** Students start with priors on their own ability and on returns to different educational choices, then revise these as they gain information.
- **Empirical examples :**
  - *Process of choosing a major :* Stinebrickner and Stinebrickner (2014) use 12 surveys per year to show how beliefs about graduating in STEM vs. non-STEM evolve over time.
  - *Information shocks on earnings :* Wiswall and Zafar (2015) randomly provide undergraduates with true population earnings distributions and document how their beliefs and stated major-choice probabilities update pre- vs. post-treatment
- *Key idea :* With a panel of beliefs $\{Q_{i,t}\}_{t=1}^{T}$, we can explicitly model and estimate belief updating within our structural choice framework $\rightarrow$ shed light on how beliefs evolve.

Purpose 3 : Modelling Learning

Recall for binary choices $C = \{0, 1\}$ at time $t = 1$:

$$\pi_i(c = 1 \mid Q_{i,1}, \theta) = \int F_{\varepsilon(0)-\varepsilon(1)}\big(v(1, \gamma; \theta) - v(0, \gamma; \theta)\big)\ dQ_{i,1}(\gamma; \beta_1).$$

Observing repeated beliefs $\{Q_{i,t}\}$ allows us to write

$$Q_{i,t+1} \ = \ \mathcal{U}\big(Q_{i,t}, \mathrm{signal}_{i,t+1}\big), \quad \beta_{t+1} \neq \beta_t.$$

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- **Stinebrickner and Stinebrickner (2014) :** Q_t evolves as students take courses and observe grades. Model
  - Only grades in STEM courses affect beliefs about STEM performance, hence, students revise science graduation probabilities more slowly than in other courses.
- **Wiswall and Zafar (2015) :** Experimental information treatment provides exogenous shocks to beliefs ; within-individual pre/post comparisons identify the causal $\Delta Q \to \Delta\pi$ mapping. Model
  - Student-specific taste parameters are the main driver of major choices.

Purpose 4 : Comparisons to the RE Benchmark

- With data on $Q(\gamma; \beta)$, we can separately identify beliefs and preferences.
- If we reject Rational Expectations (RE), we can do counterfactual simulations that replace $Q(\gamma; \beta)$ with its RE counterpart
- By comparing counterfactual to baseline model simulations, we can quantify :
  - effect of belief errors on choices and outcomes
  - interaction of beliefs with effects of policies
  - welfare cost from belief biases
  - interaction of beliefs with welfare assessment of policies

Purpose 4 : Comparisons to the RE Benchmark

- **Kapor, Neilson, and Zimmerman (2020)** use a school-choice model and data on subjective admission likelihoods to show how the presence of belief errors can reverse the welfare assessment of two alternative policies
- **Tincani, Miglino, and Kosse (2025)** use a model of pre-college effort and college admissions and data on perceived returns to pre-college effort to measure how belief errors shape the incentive response to preferential college admissions.

Purpose 4 : School-Choice with Subjective Admission Beliefs

Students submit a rank-order list a to a Boston-style mechanism (= get admission advantage at schools you rank higher) in New Haven. Let j index a school. Define :

p^s_{ija} = student i's subjective belief of admission at j under application a,

p^r_{ija} = true (RE) admission probability at j under application a.

Student i's decision solves :

$$\max_a \left(\sum_{j=1}^{J} p^s_{ija}(\nu_{ij}) \right)$$

$\rightarrow$ Trade-off between admission prob and preference : a strategic

applicant may not want to rank truly most preferred choice first if chances of admission there are low.

Purpose 4 : School-Choice with Subjective Admission Beliefs

Define optimism :

$$p_{ijs}^{s} - p_{ijs}^{r}$$

Evidence of errors in beliefs :

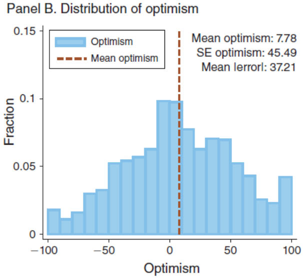

Figure – Distribution of optimism in the New haven Boston-style mechanism (Kapor, Neilson, and Zimmerman (2020))

Beliefs are off by 30-40 p.p. on average.

Purpose 4 : School-Choice with Subjective Admission Beliefs

Evidence of strategic play based on biased beliefs :

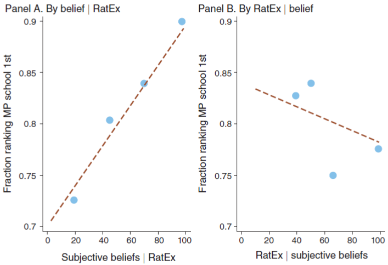

Figure – Fraction listing truly most-preferred first by subjective and RE beliefs (Kapor, Neilson, and Zimmerman (2020))

Purpose 4 : Welfare Under Subjective Beliefs vs. RE

Define aggregate welfare under mechanism M using realized allocation probabilities π_{ij}^{M} and utilities ν_{ij}. Consider $M = \{Boston, DA\}$. Under DA, truthfulness is a dominant strategy.
Then

$$W^{M} = \sum_{i} \sum_{j} \pi_{ij}^{M} \, \nu_{ij}.$$

Findings :

- Under observed *subjective* beliefs, $W^{\mathrm{DA}} > W^{\mathrm{Boston}}$ (DA gains $\approx 27\%$).
- Under RE, $W^{\mathrm{Boston}} > W^{\mathrm{DA}}$ (Boston gains $\approx 3\%$)
- Ignoring belief errors reverses the policy ranking.

Purpose 4 : Subjective Beliefs and Affirmative Action

- Preferential admissions to college for disadvantaged students are adopted to reduce SES-gaps in college enrollment
- They change the incentives to exert pre-college effort by changing the admission rules (e.g. waive entrance exam requirement)
- Tincani, Miglino, and Kosse (2025) ask : How do students' subjective beliefs about their admissions chances and their future college success shape these incentive effects and, ultimately, these policies' college enrollment impacts ?

Empirical Context : PACE (Chile)

Program Overview

- **PACE** (Programa de Acompañamiento y Acceso Efectivo a la Educación Superior) targets disadvantaged high schools.
- Graduates in the *top 15%* of their cohort receive guaranteed college admission.
- Students may take the national exam and apply via the regular selection process.

Study Population Characteristics

- *Academic preparedness :* Grade-10 test scores on average 1.5 SD below those of regular admittees.
- *Socioeconomic status :* Household income $\approx$ 33% of the mean for regular admitees.

Policy Rollout & Identification of Incentive Effects

- Random introduction of PACE across eligible schools in 2016.
- Creates exogenous variation in guaranteed admission incentives.

Impacts of PACE on Effort and Enrollment

1. **College Enrollment Effects**
   - Δ First-year enrollment : $+3.0$ pp ($+35\%$ of control)
   - Δ Fifth-year enrollment : $+1.5$ pp ($+30\%$ of control)
   - $\Rightarrow$ college entrants from PACE schools drop out at higher rate
2. **Pre-College Effort & Achievement**
   - Δ Grade 12 test scores : -0.10 SD
   - Δ Self-reported study effort : -0.9 SD (for national exam prep and school work)

Source : Administrative and survey data.

Impact Mechanisms

1. Waning College Enrollment Effects

- Δ college match (major, geographic location, selectivity) : ✗
- Worse measured ability of college entrants : ✗
- Worse college preparedness due to lower pre-college effort : ✓
- Worse unmeasured ability of college entrants : ✓

2. Negative Pre-College Effort & Achievement Effects

- Teacher and school factors (grading, teaching effort, focus of instruction, support classes) : ✗
- Lower perceived returns to college : ✗
- Response to perceived incentives : ✓
- Perception that pre-college effort does not matter for college success : ✓

Dynamic Structural Model of Education Choices

Objectives

- Replicate experimental findings via a tractable model
- Quantify mechanism contributions
- Evaluate counterfactual policy designs

Key Model Mechanisms

Pre-College Effort
Affects true admission & persistence probabilities

Selection into college
Based on observed and unobserved characteristics

Subjective Beliefs
Perceived link between effort and admission/persistence chances

Allocation Rule
Objective admission likelihoods determine seat assignment

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Next : Formalize these mechanisms in a dynamic programming framework.

Model Sketch : Beliefs about Regular Channel Admissions

At time 0, students form beliefs about :

- The entrance exam score PSU_i^b :

$$\begin{aligned} PSU_i^b &= \overline{PSU}_i^b + \epsilon_i^{Pb} \\ &= \beta_0^{Pb} + \beta_1^{Pb} e_i + \beta_2^{Pb} X_i + \epsilon_i^{Pb} \quad \epsilon_i^{Pb} \sim N(0, \sigma^2_{PSU^b}) \end{aligned}$$

- The chances of regular admission :

$$\begin{aligned} Pr^b(A_i^R = 1 | \overline{PSU}_i^b) &= Pr\left(\overline{PSU}_i^b + \epsilon_i^{Pb} \geq \bar{c}^{Rb} + \epsilon_i^{cRb}\right) \\ &= \Phi\left(\frac{\overline{PSU}_i^b - \bar{c}^{Rb}}{\sqrt{\sigma^2_{PSU^b} + \sigma^2_{c^{Rb}}}}\right) \\ &= \Phi\left(\gamma_0^b + \gamma_1^b \overline{PSU}_i^b\right) \end{aligned}$$

where $c_i^{Rb} \sim N(\bar{c}^{Rb}, \sigma^2_{c^{Rb}})$.

Model Sketch : Beliefs about Preferential Admissions

At time 0, students form beliefs about :

- The GPA GPA_i^b :

$$\begin{aligned} GPA_i^b &= \overline{GPA}_i^b + \epsilon_i^{Pb} \\ &= \beta_0^{Gb} + \beta_1^{Gb} e_i + \beta_2^{Gb} X_i + \epsilon_i^{Gb} \quad \epsilon_i^{Gb} \sim N(0, \sigma^2_{GPA^b}) \end{aligned}$$

- The chances of preferential admission :

$$\begin{aligned} Pr^b(A_i^P = 1 | \overline{GPS}_i^b, \bar{c}_i^{15b}) &= Pr\left(\overline{GPA}_i^b + \epsilon_i^{Gb} \geq \bar{c}_i^{15b} + \epsilon_i^{c15b}\right) \\ &= \Phi\left(\frac{\overline{GPA}_i^b - \bar{c}^{15b}}{\sqrt{\sigma^2_{GPA^b} + \sigma^2_{c^{15b}}}}\right) \\ &= \Phi\left(\pi_1^b(\overline{GPA}_i^b - \bar{c}_i^{15b})\right) \end{aligned}$$

where $c_i^{15b} \sim N(\bar{c}^{15b}, \sigma^2_{c^{15b}})$.
Assume students best-respond to elicited perceived cutoff without imposing equilibrium beliefs (Stahl and Wilson (1995)).

Estimate Perceived Production Functions

We exploit linked survey-administrative data to estimate the perceived production functions of PSU and GPA outside of the model.

$$Y_i^b = \beta_0^b + \beta_1^b e_i + \beta_2^b X_i + \epsilon_i^b$$

From admin data, we observe X_i.

From survey data, we measure :

- β_1^b Details
- e_i Details
- $\bar{Y}_i^b$ Details

Estimate Perceived Production Functions

$$Y_i^b = \bar{Y}_i^b + \epsilon_i^b = \beta_0^b + \beta_1^b e_i + \beta_2^b X_i + \epsilon_i^b \quad \epsilon_i^b \sim N(0, \sigma_b^2)$$

As a function of observed effort $e_i^o = e_i + \epsilon_i^{mee}$, $\bar{Y}_i^b$ is equal to :

$$\begin{aligned} \bar{Y}_i^b &= \beta_o^b + \beta_1^b(e_i^o - \epsilon_i^{mee}) + \beta_2^b X_i \\ &= \beta_o^b + \beta_1^b e_i^o + \beta_2^b X_i + \nu_i^b, \quad \nu_i^b = -\beta_1^b \epsilon_i^{mee} \end{aligned}$$

Define the expected score net of the measured contribution of effort : $\bar{Y}_i^{b,net} = \bar{Y}_i^b - \beta_1^b e_i^o$, which is data, then :

$$\bar{Y}_i^{b,net} = \beta_0^b + \beta_2^b X_i + \nu_i^b \tag{1}$$

Under orthogonality of ϵ_i^{mee}, OLS estimation of (1) gives us consistent estimates of β_0^b, β_2^b.

Estimate Perceived Production Functions

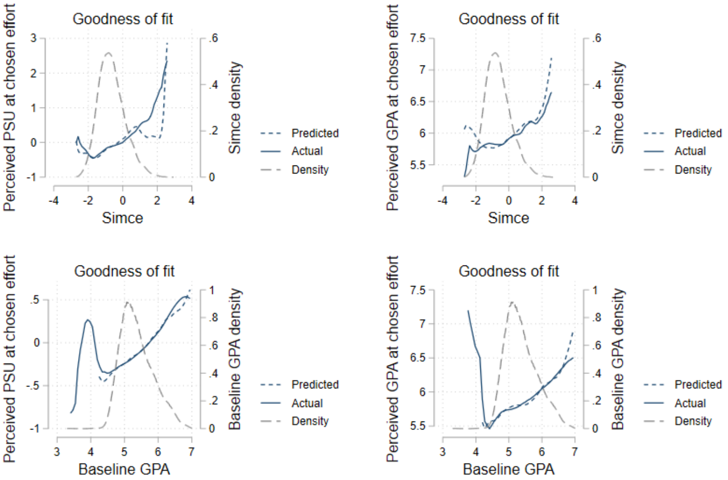

Figure – Goodness of fit of perceived PSU and GPA production functions at exerted effort levels.

Model Periods

Students characterized by baseline characteristics X_i and unobserved type $k_i \in \{1, 2, ..., K\}$.

- Time 1 : choose effort, $u_{i1}(d_{i1}, \Omega_{i1})$
- Time 2 : choose whether to take the entrance exam, $u_{i2}(d_{i2}, \Omega_{i2})$
- Time 3 : admissions occur, based on *actual* admission chances
- Time 4 : choose whether to enroll, $u_{i4}(d_{i4}, \Omega_{i4})$

$$u_{i4}(d_{i4}, \Omega_{i4}) = \begin{cases} \lambda_{0k_i} + pgrad_i^b(\lambda_0^G + q_i^R) + \nu_i^R & \text{if } d_{i4} = \text{“Enroll, regular"} \\ \lambda_{0k_i} + \delta + pgrad_i^b(\lambda_0^G + q_i^P) + \nu_i^P & \text{if } d_{i4} = \text{“Enroll, preferential"} \\ 0 & \text{if } d_{i4} = \text{“Do not enroll"} \end{cases} \tag{2}$$

- Time 5 : actual persistence realizes according to $Pr(Persist_i = 1 | k_i, d_{i1}, X_i) = \Phi(\rho_{0k_i} + \rho_1 d_{i1} + \rho_2 X_i)$

Subjective value function with *objective* laws of motion :

$$V_t^b(\Omega_{it}) = \max_{d_{it} \in D_{it}} \left\{ u(d_{it}, \Omega_{it}) + E^b \left[V_{t+1}\left(\Omega_{it+1} | \Omega_{it}, d_{it}\right) \right] \right\}. \tag{3}$$

Estimation

We estimate the model by Indirect Inference (34 structural parameters, 52 auxiliary model parameters), and achieve good model fit.

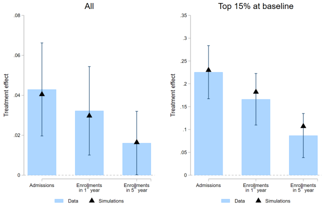

Figure – Model fit—*targeted* moments.

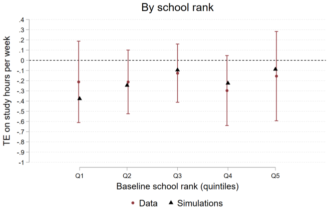

Figure – Model fit—*untargeted* moments.

Model Results I : Role of Effort

Table – Returns to pre-college effort in persistence and omitted variable bias

	Simulated college persistence probability	
	(1)	(2)
Simulated study hours	0.013	0.010
Unobserved type 1		0.035
Outcome mean	0.370	0.370

Note.– The coefficients are OLS estimates of regressions of the simulated persistence probability on baseline Simce test scores and on simulated weekly study hours in high school. The second column includes a dummy for the unobserved student type as control variable. Simulations are performed using the structural-model estimation sample and the estimated model parameters. Outcome mean for this sample reported.

77% of the correlational returns to effort represent a causal effect.

Model Results II : Role of Beliefs

Counterfactual with Rational Expectations

If beliefs were accurate (RE) in both the treatment and the control group, PACE would *not* have reduced pre-college effort because :

- PACE $\uparrow$ returns to effort for admission
- RE $\downarrow$ utility of enrolling in both groups since
  - True persistence rate (mean) : 37%
  - Perceived persistence rate (mean) : 77%

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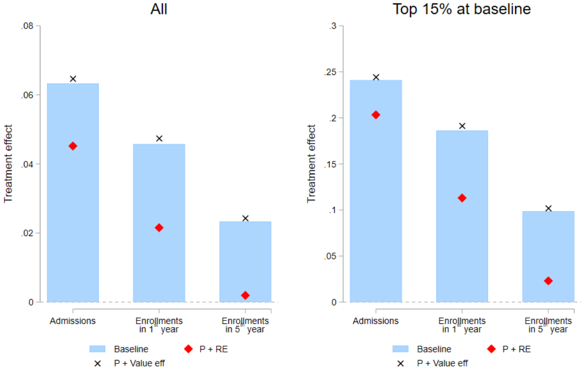
Belief errors are the key driver of PACE's effort disincentives.

Model Results III : Effects of Information Add-ons

All beliefs : Correcting beliefs in treatment group ⇒ further *reductions* in pre-college effort and *compressed* PACE enrollment take-ups.

Beliefs about effort returns in persistence : Correcting beliefs about *persistence returns* in treatment group ⇒ avoids effort reductions *without* dampening the enrollment gains of PACE.



Model Results IV : Alternative PACE cutoffs

We simulate the effects of PACE under alternative cutoffs.

Problem : we elicited beliefs about the within-school GPA distribution under the current policy (top 15% cutoff). Under a different policy cutoff, students would exert different efforts and, presumably, they would perceive a different within-school GPA distribution.

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How can we build counterfactual beliefs ?

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We need to make some assumptions about how beliefs would be formed in the counterfactual.

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We assume that belief errors are constant in the counterfactuals.

Errors with respect to which benchmark ?

- $\bar{c}_i^{15b}$ vs. the actual cutoff that is realized at baseline ?
- $\bar{c}_i^{15b}$ vs. the RE cutoff ?

Model Results IV : Detour on Counterfactual Beliefs

We need to make some assumptions about how beliefs would be formed in the counterfactual.

We assume that belief errors are constant in the counterfactuals.

Errors with respect to which benchmark ?

- $\bar{c}_i^{15b}$ vs. the actual cutoff that is realized at baseline ?
- $\bar{c}_i^{15b}$ vs. the RE cutoff ?

The RE cutoff, because we must be able to simulate the benchmark also in the counterfactual policy !

Model Results IV : Detour on Counterfactual Beliefs

Algorithm to construct counterfactual beliefs and solve the model under counterfactual cutoffs :

1. Assume students would exhibit same belief bias relative to the RE cutoff as they do under the top 15% rule.
2. Compute this bias as the difference between the perceived and the RE cutoff in the baseline.
3. For each alternative cutoff rule, first solve for the RE equilibrium to obtain the corresponding RE cutoff.
4. Construct the counterfactual beliefs by adding the estimated belief bias (from point 2) to this RE cutoff.
5. Solve the model under these counterfactual beliefs.

Model Results IV : Alternative PACE cutoffs

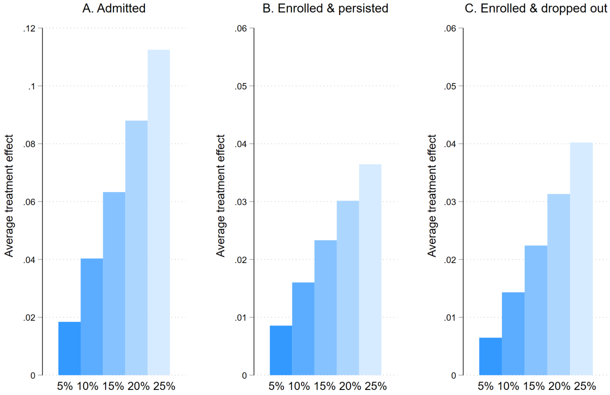

Figure – Simulated effects of PACE with alternative cutoffs.

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Belief Updating in Stinebrickner and Stinebrickner (2014)

$$E\left(\mathrm{AGPA}_{i,j}^{t^*}\right) = \alpha_j + \beta_j E\left(\mathrm{AGPA}_{i,j}^{1}\right) + \gamma_j \mathrm{Sig}_{i,j} + u_{i,j}$$

- $E\left(\mathrm{AGPA}_{i,j}^{t^*}\right)$: posterior mean belief of student i's average GPA in major j once outcomes are fully "resolved"
- $E\left(\mathrm{AGPA}_{i,j}^{1}\right)$: prior mean belief at initial survey (upon entry)
- $\mathrm{Sig}_{i,j}$: imperfect signal (e.g. realized GPA in early major j courses)
- α_j : major-specific intercept capturing systematic bias in priors
- β_j : weight on the prior belief (persistence of initial expectations)
- γ_j : weight on the new signal (speed of learning)
- $u_{i,j}$: mean-zero idiosyncratic error term

$\rightarrow$ Parameters $\alpha_j, \beta_j, \gamma_j$ identified from observations of prior and posterior beliefs and signals.

Return

Belief Updating in Wiswall and Zafar (2015)

Assume two majors $j \in \{0, 1\}$. Student i's utility from major j is

$$u_{ij} = x'_{ij}\theta_x \; + \; \theta_e \ln(\gamma_{ij}) \; + \; \theta_{ij} \; + \; \epsilon_{ij},$$

where

- x_{ij} : observed covariates (e.g. demographics, non-major tastes)
- γ_{ij} : student i's *perceived* future earnings in major j
- θ_e : preference weight on (perceived) earnings
- θ_{ij} : time-invariant taste shock for major j
- $\epsilon_{ij} \sim$ Type I EV i.i.d. across i, j

Taking log-odds of choosing major 1 vs. 0 :

$$\ln \pi_{i1} \; - \; \ln \pi_{i0} = \theta_e \big[\ln \gamma_{i1} - \ln \gamma_{i0}\big] \; + \; \big[\theta_{i1} - \theta_{i0}\big] + \epsilon_{i1}. \qquad (1)$$

Where ϵ_{i1} is an additional orthogonal error. We observe π_{ij} and γ_{ij}, but not θ_{ij}. OLS estimation of (1) would give biased estimates of θ_e if $\Delta\theta_i$ is correlated with $\ln \gamma_{i1} - \ln \gamma_{i0}$.

Belief Updating in Wiswall and Zafar (2015)

Experimental fix : Randomize an information treatment at time t to shift γ_{ij} exogenously, and measure γ_{ij} pre- and post.
Define pre- and post-treatment log-odds L_i^{pre} and L_i^{post}. Then

$$\Delta L_i = L_i^{\mathrm{post}} - L_i^{\mathrm{pre}} = \theta_e \big[\Delta \ln(\gamma_{i1}) - \Delta \ln(\gamma_{i0})\big] \ + \ \Delta\epsilon_i, \qquad (2)$$

where the terms containing θ_{ij} cancel out, so θ_e is identified from the slope on $\Delta \ln \gamma$, which is uncorrelated with the error term.
Where :

$$\Delta \ln \gamma_i \ = \ \big[\ln(\gamma_{i1}^{\text{post}}) - \ln(\gamma_{i1}^{\text{pre}})\big] \ - \ \big[\ln(\gamma_{i0}^{\text{post}}) - \ln(\gamma_{i0}^{\text{pre}})\big].$$

Key estimate : $\uparrow$ 10% in beliefs about self-earnings in a major $\uparrow 3.6 - 6.2\%$ subjective probability of choosing that major, older students have lower elasticities.

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Pre-college Effort Predicts College Persistence

Table – Pre-college Outcomes and Persistence in Selective Colleges

	College persistence or graduation five years after high school graduation			
	(1)	(2)	(3)	(4)
GPA in 12^{th} grade (std)	0.142***			
	(0.019)			
GPA in 12^{th} grade tested subjects (std)		0.108***		
		(0.022)		
GPA in 12^{th} grade untested subjects (std)		0.057**		
		(0.026)		
PSU score (std)	0.047	0.072		
	(0.041)	(0.047)		
Study effort in last high school year (std)			0.075***	
			(0.023)	
Hours of study per week in last high school year				0.017***
				(0.006)
Baseline test score in 10^{th} grade (std)	0.001	-0.032	0.053**	0.052**
	(0.028)	(0.029)	(0.025)	(0.024)
Observations	1013	740	735	748
R^2	0.079	0.085	0.054	0.051

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Over-optimism about absolute and relative ability

Table – Description of Subjective Beliefs

	Mean (1)	Std. Deviation (2)	N (3)
Believed entrance exam score (σ)	-.033	.92	2413
Believed minus actual entrance exam score | took exam (σ)	.591	.916	1853
Believed minus actual 12^{th} grade GPA (GPA points)	-.075	.552	2558
Expected top 15% cutoff	5.82	.846	3326
Actual minus believed top 15% cutoff in school (GPA points)	.401	.854	3326
Believes is in top 15% of school	.431	.495	2469

$\longrightarrow$ Effort reductions concentrated among those who believed they were well above the PACE admission cutoff.

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Perceived persistence likelihood across treatment groups

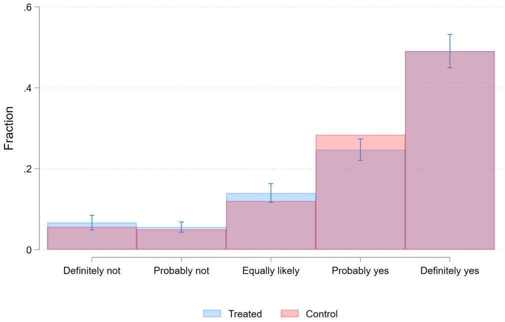

Figure – Distribution of survey responses regarding beliefs about the likelihood of graduating from a selective college conditional on enrolling, by treatment status (i.e., being in a PACE or control school). The figure includes 95% confidence intervals for the difference between the proportion of treated and of control students giving each answer. The confidence intervals were obtained from standard errors clustered at high school level.

→ No effects on perceived graduation likelihood even among those who experienced considerable reductions in effort. Back

Measuring perceived returns to effort in achievement

We use hypothetical effort scenarios to measure perceived returns :

$$\beta_1^b = \frac{Y(X) - Y(X')}{e(X) - e(X')}$$

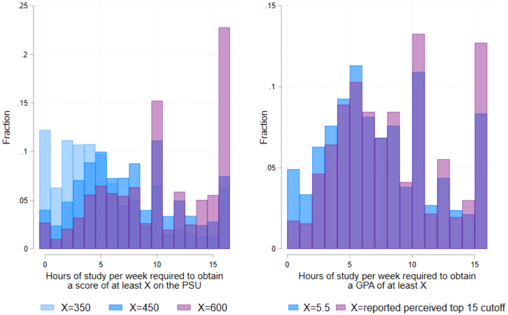

Figure – Distribution of answers to survey questions on perceived returns to effort. The reported perceived top 15 cutoff is on average 5.85 in the sample used to construct these histograms. The sample sizes for the left-side graph are : 5,344 for X=350, 5,442 for X=450, 5,469 for X=600. The sample sizes for the right-side graph are : 5,451 for X=5.5, 5,443 for X=reported perceived top 15 cutoff.

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Measuring effort

We use the following survey question :

“On average, how many hours a week did you study or do homework outside of class time during the first semester of this school year ?"

We assume we measure $e_i^o = e_i + \epsilon_i^{mee} \quad \epsilon_i^{mee} \sim N(0, \sigma^2_{mee})$ i.i.d. and independent of all model initial conditions and shocks.

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Measuring perceived achievement

We use the following survey questions :

$\overline{PSU}_i^b$ "Suppose that you will take the PSU entrance exam this year. What do you think your PSU score will be ?"

$\overline{GPA}_i^b$ "Thinking of yourself, what do you think your grade point average (GPA) will be at the end of high school ?"

We assume we measure $\bar{Y}_i^b$.

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