# Subjective expectations in life-cycle models

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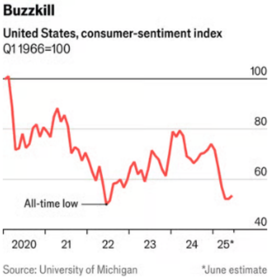
# History of subjective expectations (Bruine de Bruin et al., 2023)




Source: The Economist, 13-06-2025

1940s: University of Michigan's Surveys of Consumers and Survey of Consumer Finances by George Katona and others

- Idea: past behavior not always good guide for future
- Subjective expectations incorporated in aggregate measures relevant for research and policy

# Why measure subjective expectations?

Household expectations valuable to (Katona and Klein, 1952; Manski, 2004)

1. Predict choices
2. Understand how those choices are made (preferences, expectations, constraints)
3. Understand formation of expectations

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Economic theory models decision making based on **probabilistic expectations** (Manski, 2004)

- Inter-personal comparability
- Internal (logical) consistency
- Different moments (location, dispersion)

# Popularity grew since early 1990s

Empirical evidence in favor of subjective expectations

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Additional drivers

- Subjective measures (e.g., Michigan Survey of Consumers) predict changes in consumption and inflation (Carroll et al., 1994; Ludvigson, 2004; Ang et al., 2007)
- Doubts about rationality of expectations (Tversky and Kahneman, 1974)

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Currently many (most?) major household surveys include expectations questions

- 6 in U.S., multiple in Europe (U.K.) and Canada

# Roadmap

# Life-cycle models

Decisions with long-term consequences

- **Saving and retirement**
- Health investments
- Education, marriage, ...

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Basic ingredients

- Choices
- Preferences
- **Expectations** (Manski, 2004; Koşar and O'Dea, 2023)
  - States of nature (health, survival, wage)
  - Future choices
- Budget constraint (institutions)

# A sketch of a life-cycle model (based on Koşar and O'Dea, 2023, and Van der Klaauw and Wolpin, 2008)

Structure: discrete time (age) $t$ in years, certain death at age $T$

**Choices**

- Consumption $c_t$ (continuous)
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**Utility function** (consumption $c_t$ and leisure $l_t$)

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**Budget constraint**

$$\begin{cases} a_{t+1} & = (a_t + y_t \mathbb{1}\{h_t = 1\} - c_t)(1+r) \text{ if } t < T^R \\ a_{t+1} & = (a_t + \kappa \times \text{pension}(\mathbf{h}_{T^R}, \mathbf{y}_{T^R}) - c_t)(1+r) \text{ if } t \geq T^R \\ l_t & = 1 - \eta \mathbb{1}\{h_t = 1\} \end{cases}$$

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Recursive specification

$$V_t(\chi_t) = \max_{c_t, h_t} U(c_t, l_t) + \beta s_{i,t+1} \mathbb{E}_{i,t}\left[V_{t+1}(\chi_{t+1})\right]$$

subject to the budget constraint

state $\chi_t = \{a_t, \mathbf{y}_t, \mathbf{h}_t\}$; $\mathbf{y}_t = (y_1, ..., y_t)$

# Solving the model

Collect unknown model features in $\boldsymbol{\theta}$

- Preference parameters $\{\beta, \gamma, \nu\}$, expectations $\kappa$

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Model solution implies trajectories of choices and of **expectations of future choices**

$$\mathbb{P}\left[\widetilde{h}_{t+\tau} = 1 \middle| t; \chi_t, \boldsymbol{\theta}\right] \qquad \mathbb{P}\left[\widetilde{a}_{t+\tau} > a \middle| t; \chi_t, \boldsymbol{\theta}\right]$$

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**Identification:** different $\boldsymbol{\theta}$ lead to different distributions of observables under the model

- Binary property of model + joint distribution of observables
- More data allow less restrictive model

# Estimation

**Step 1:** Estimate processes outside the model if possible (exogenous)

- All objects that do not depend on decision rules $\widetilde{c}_t$ and $\widetilde{h}_t$
- E.g., interest rate $r$

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- E.g., interest rate $r$

**Step 2:** Estimate remaining unknown model features in $\boldsymbol{\theta}$ by matching model implications to observed data

- Preference parameters $\{\beta, \gamma, \nu\}$ and endogenous aspects of expectations (formation, learning)
- Maximum Likelihood or Method of Moments

# Rational expectations

**Rational expectations**: expectations equal to our estimates

$$s_{i,t+1}\mathbb{E}_{i,t}\left[V_{t+1}|\chi_t\right] = \widehat{s}_{t+1}\widehat{\mathbb{E}}_t\left[V_{t+1}|\chi_t\right]\ \forall i$$

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Good approximation if

- Decision makers have the **same information** as econometrician
- Decision makers **process that information** as econometrician (inference)

# Subjective expectations

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Good approximation if

- We know how respondent $i$ **understands the survey questions**
  - Tries to report $s_{i,t+1}$ (meta-data: time stamps, self-evaluations)
- We know how respondent $i$ **uses the response format**
  - Transformation $s_{i,t+1} \rightarrow \widetilde{s}_{i,t+1}$ (peculiarities in responses)
- Face/construct/predictive validity

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Best practice in survey design (Bruine de Bruin et al., 2023, see also Dillman, 2011; Bergman et al., 2020):

- Cognitive interviews
- Simple language

# Subjective vs rational expectations

More data allow **less restrictive model**

- Identification problem: observed choices compatible with multiple combinations of preferences and expectations (Manski et al., 1991; Manski, 2004)
- Observing expectations directly allows greater flexibility:
  *"...it is enough to assume that elicited expectations faithfully describe persons' perceptions of their environments"*
  *– Manski (2004)*
  - Expectations may not follow observed realization of states
  - Processes may be non-stationary
  - Level and heterogeneity
  - Important question of measurement
- More data, not degrees of freedom

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Expectation formation in status quo and in counterfactual

# Considerations for survey design

**Probabilistic** survey items that elicit **direct analogues of model objects**

- Inter-personal comparability (Manski, 2004; Bruine de Bruin et al., 2023)
- Choice probabilities are more informative than discrete choice (Manski, 1999)
  - **Resolvable uncertainty** in stated preferences for incomplete scenarios
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Example: three questions on future pension claiming

1. *"Agreement with the following statement: I will collect pension benefits by age 65.*
   *1. completely agree, ..., 5. completely disagree"*
2. *"At what age do you expect to start collecting pension benefits?*
   *... years old"*
3. *"What is the chance that you will collect pension benefits by age 65?*
   *0, ..., 100%"*

# Roadmap

# Two types of subjective expectations (Manski, 2004)

Over **states of nature**

- Economic/policy environment
  - Replacement rate income at retirement
  - Income (wage) next year
- Personal characteristics
  - Survival: *"What is the percent chance that you will live to be age 75 or more?"*

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Over **choices**

- Labor supply: *"What are the chances that you will be working full-time after you reach age 62?"*
- Bequests: *"Including property and other valuables, what are the changes that you (and your [partner]) will leave an inheritance totaling $10,000 or more?"*

# Subjective expectations in life-cycle models: states of the nature

**Expectation formation not modeled**

- Survival
  - Saving and bequests: Gan et al. (2015), Bissonnette et al. (2017), Heimer et al. (2019), de Bresser (2024)
  - Annuity demand: O'Dea and Sturrock (2023), Pang (2025)
  - Annuity supply: Hosseini (2015)
- Income shocks: Pistaferri (2001), Attanasio et al. (2020), Arellano et al. (2024)
- Policy environment: Van der Klaauw and Wolpin (2008)

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**Models with expectation formation**

- Survival: Groneck et al. (2016), Caliendo et al. (2020)
- Pension entitlements: Hentall-MacCuish (2019)

# Subjective expectations in life-cycle models: choice expectations I

Features

- Actual choices not available (e.g., life-care annuities in NL)
- Conditional choices allow fully specified economic environment that maps into model
  - Choice set and characteristics
- Identification through experimental variation in attributes (e.g., price)
- Combination with actual choices: estimation efficiency or validation

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**Unconditional** choice expectations (large HH surveys)

$$\widetilde{\mathbb{P}}_i\left(\widetilde{h}_{i,t+\tau} = 1\right)$$

- Expected behavior given **current information set** of respondent
- Labor supply/retirement: Wolpin and Gonul (1985), Van der Klaauw and Wolpin (2008)
- Bequests: McGee (2021), Erdenesuren (2023)

# Subjective expectations in life-cycle models: choice expectations II

**Conditional** choice expectations (bespoke surveys)

$$\widetilde{\mathbb{P}}_i \left( \widetilde{h}_{i,t+\tau} = 1 | \chi_{t+\tau} = \chi^j_{t+\tau} \right)$$

- Expected behavior in **hypothetical scenario**(-s)
  - E.g., labor supply given health: Giustinelli and Shapiro (2024)
- Behavior in multiple states of the world: **no selection**
- Conditions that may never prevail
- Multiple scenarios create within-individual variation

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Examples (discrete and/or probabilistic)

- Retirement: Van Soest and Vonkova (2014), Ameriks et al. (2020)
- Long-term care insurance: Boyer et al. (2020), Michaud and St. Amour (2023)

# Roadmap

# Complications of working with subjective expectations

Complex mapping expectations $\rightarrow$ survey reports

- Logically inconsistent probabilities (monotonicity)
- Choice probabilities inconsistent with our model (recall error)
- Reported probabilities a-priori implausible (e.g., 100% certain to survive past future age)

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**Rounding**

- Bunching of reported probabilities at multiples of 10%
- Latent construct (40% multiple of 1 and 10)
- Survey-induced or not (scale 0-10 or 0-100%)

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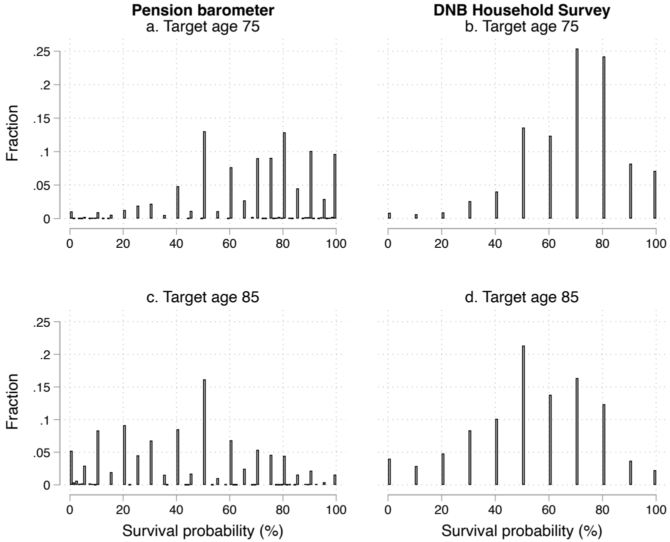
**Focal answers**

- Excess bunching at 0, 50, 100%
- Epistemic uncertainty rather than subjective risk (Fischhoff and Bruine De Bruin, 1999; De Bruin et al., 2000)

# Example: bunching in survival probabilities




Stated probability to live to age 75/85 (de Bresser, 2019)

# How to model subjective expectations?

**Partial identification** (Manski and Molinari, 2010; Bissonnette and de Bresser, 2018)

- For continuous variable: limited number of grid points
- Rounding:
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  - Inferred from probabilities one-by-one (worst case)

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- Linear or higher order splines

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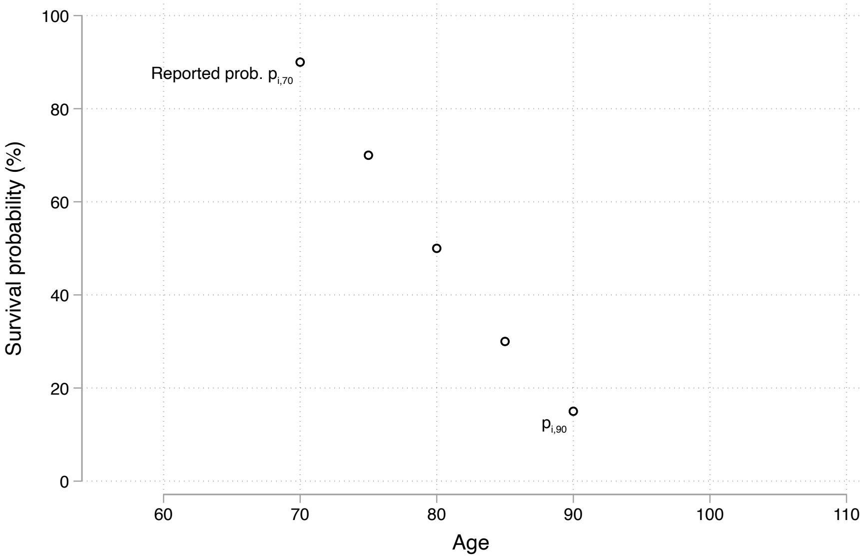
**Parametric mixing model** (Kleinjans and Soest, 2014; de Bresser, 2019, 2024)

- Handles both focal answers and rounding
- Conditional means provide partial pooling for individual-specific parameters

# Classification different modeling approaches

| | Point identification | Partial identification |
|---|---|---|
| Expectations constructed individual-by-individual | Parametric distr. by NLLS<br>Spline interpolation | No rounding, no smoothing<br>Rounding, no smoothing<br>Rounding, smoothing |
| Partial pooling of expectations | Parametric mixing model | |

# Different modeling approaches – data for respondent *i*



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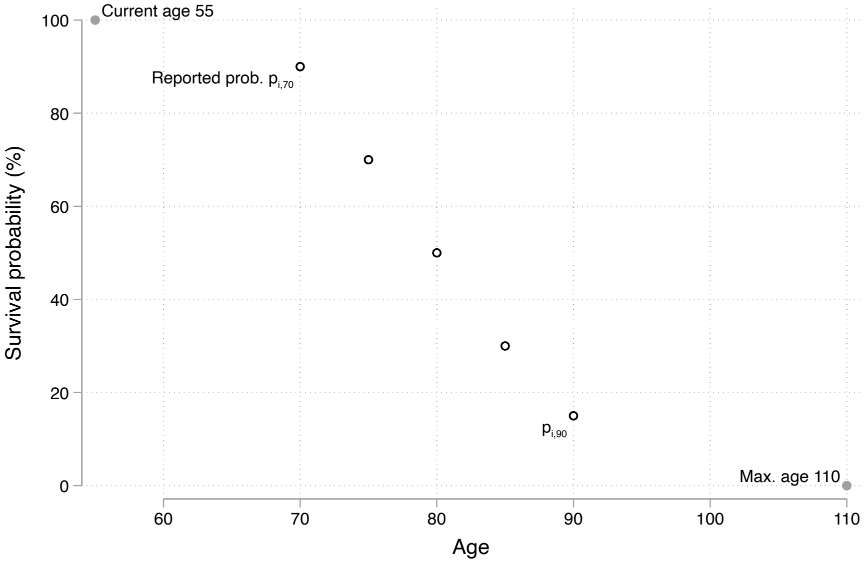

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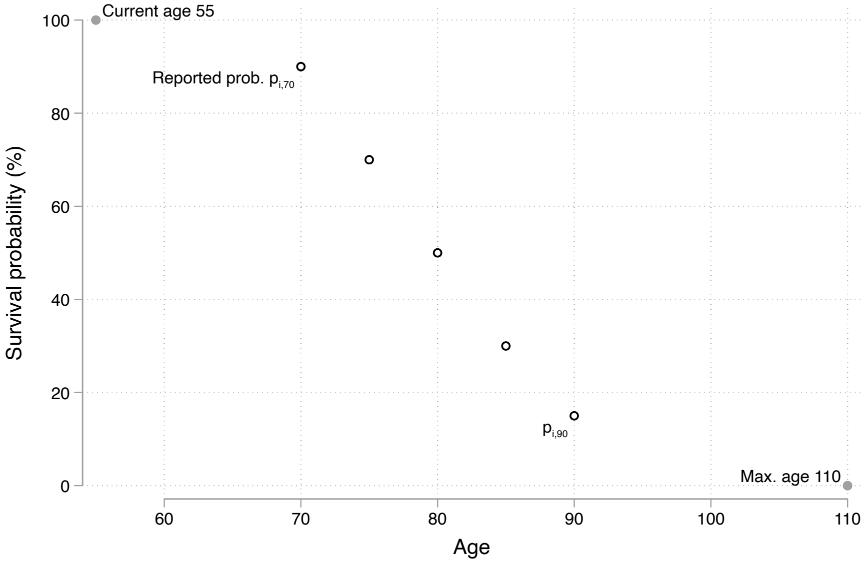

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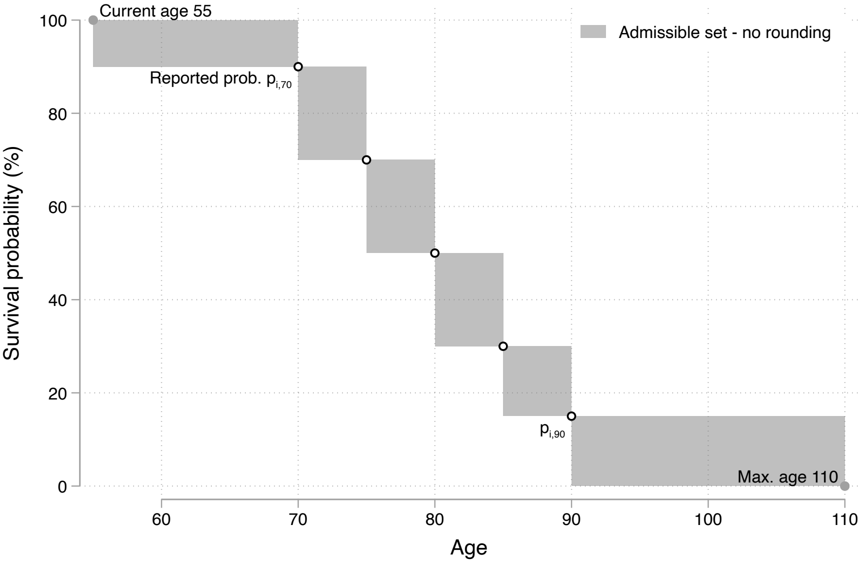

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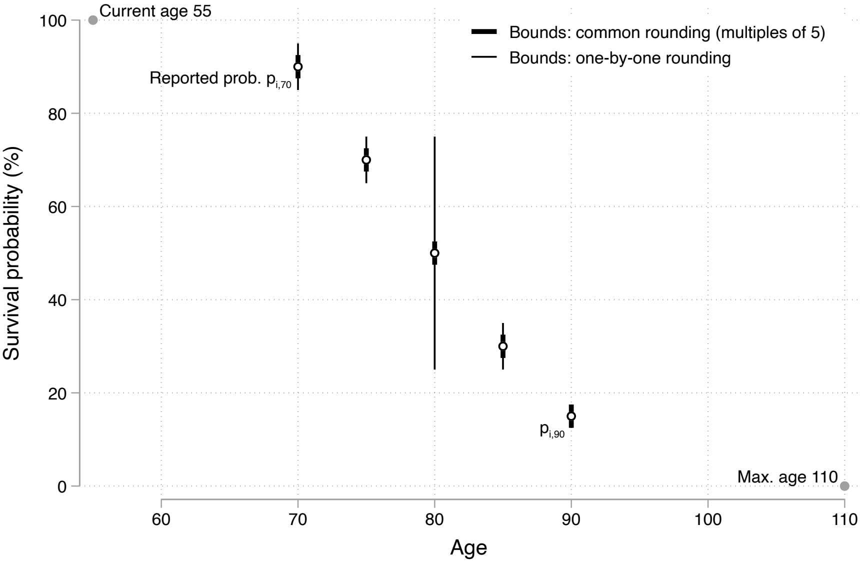

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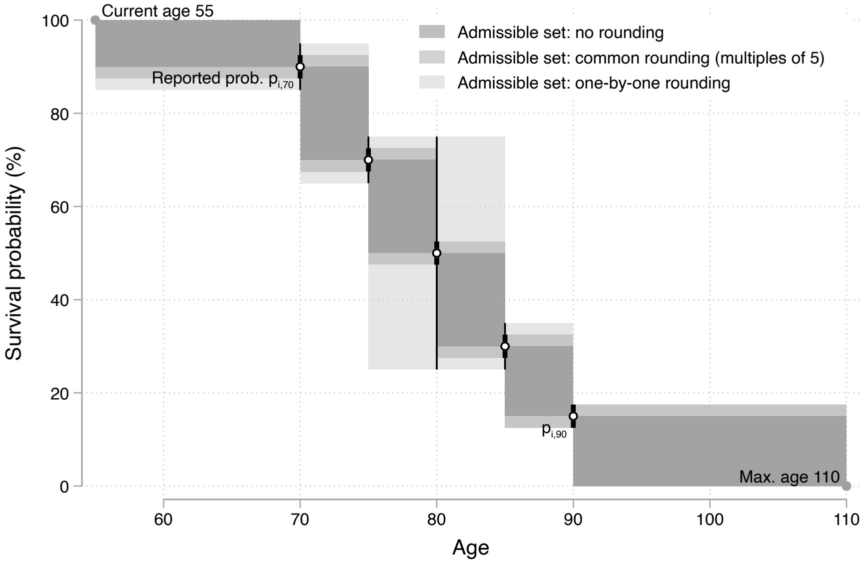

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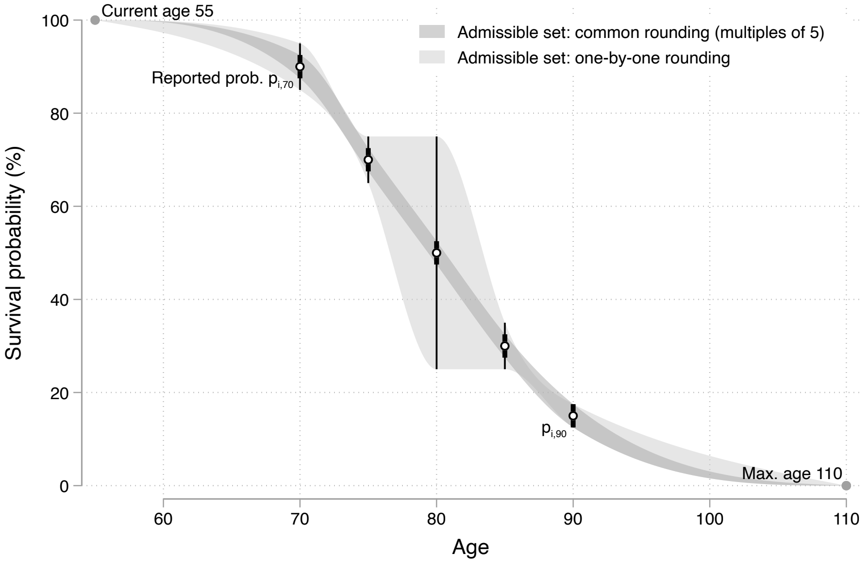

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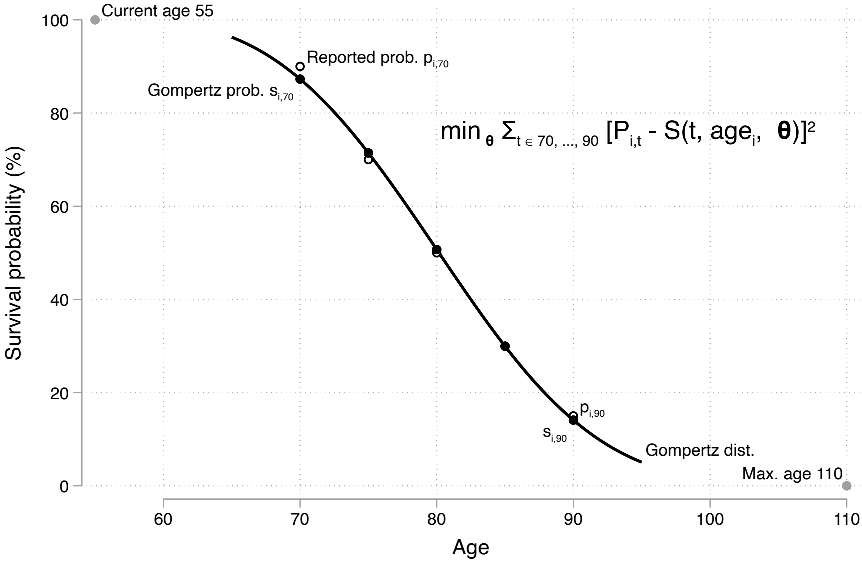

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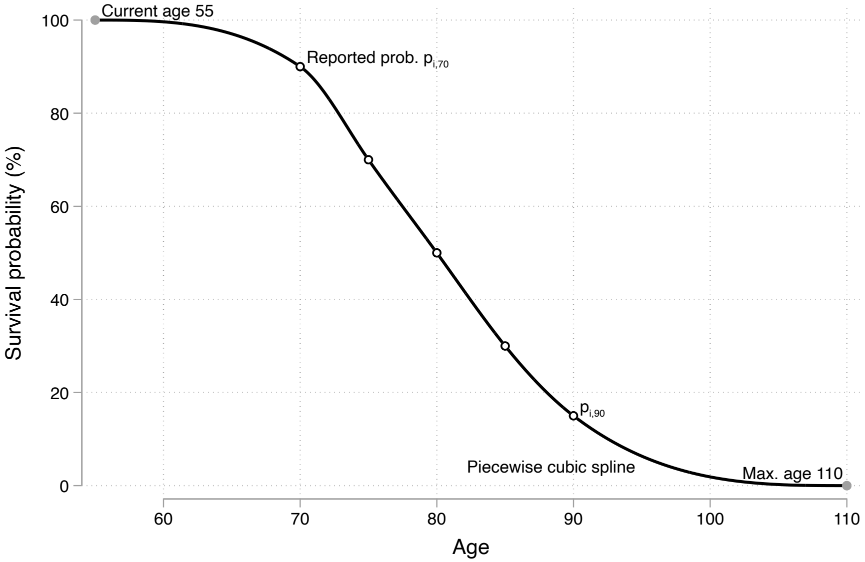

# Partial pooling – parametric mixing model I

Data (DNB Household Survey):

> *"Please indicate your answer on a* **scale of 0 thru 10**, *where 0 means 'no chance at all' and 10 means 'absolutely certain'. How likely is it that you will attain (at least) the age of [65]?"*

- Answers can be interpreted as probabilities (de Bresser, 2019)
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Model:

- Expectations follow **Gompertz distribution**
- Common baseline hazard, multiplied by health and individual effects
- Ordinal rounding model
- **Conditional means** for individual effects capture variation in expectations

# Partial pooling – parametric mixing model II

True expectations follow Gompertz distribution:

$$S_{i,t_i+\tau_k} = \mathbb{P}\left(T \geq t_i + \tau_k \middle| T \geq t_i\right) = g\left(t_i, \tau_k; \gamma_i, \alpha\right)$$
$$\gamma_i = \exp\left(\mathbf{x}_i' \boldsymbol{\beta} + \zeta_i + \eta_{it}\right); \; \zeta_i \sim \mathcal{N}\left(0, \sigma_\zeta^2\right), \; \eta_{it} \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$$

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1. Add recall error $\varepsilon_{i,t_i+\tau_k}$

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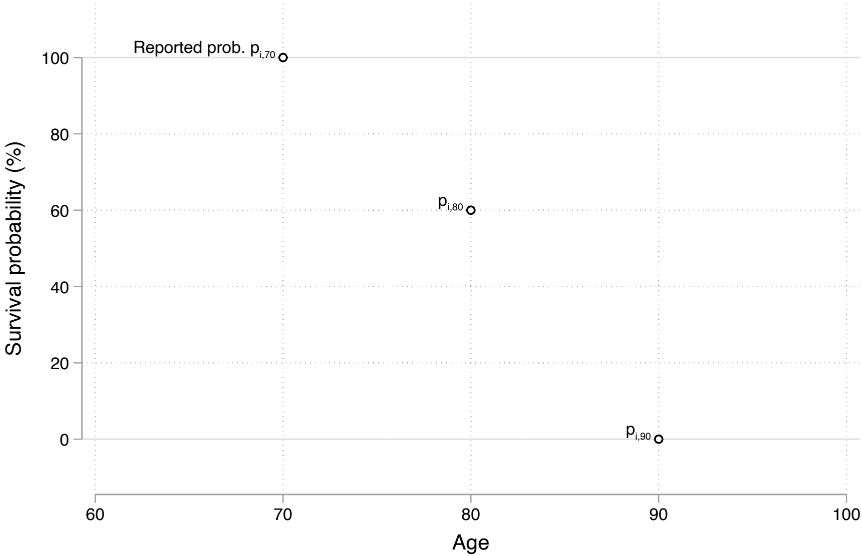
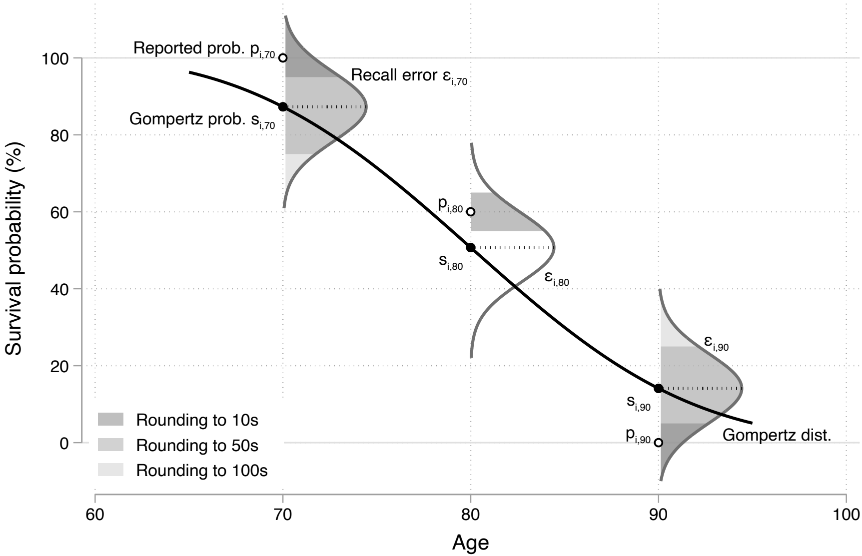
2. $P^*_{i,t_i+\tau_k}$ rounded (10, 50, 100) and censored between 0 and $\min\left[100, P_{i,t_i+\tau_{k-1}}\right]$

# Partial pooling – parametric mixing model III



# Partial pooling – parametric mixing model III



# Parameter estimates for subjective survival

| | a. Pre-reform (1993-2001) | | | | b. Post-reform (2006-2016) | | | |
|---|---|---|---|---|---|---|---|---|
| | Expectations – hazard rates | | Heteroskedasticity recall error | | Expectations – hazard rates | | Heteroskedasticity recall error | |
| Poor health | 1.363*** | (0.0443) | 0.00979 | (0.0509) | 1.244*** | (0.0187) | 0.155*** | (0.0201) |
| Educ. middle[a] | 1.005 | (0.0423) | -0.161*** | (0.0536) | 0.927*** | (0.0249) | -0.133*** | (0.0215) |
| Educ high[a] | 1.058 | (0.0423) | -0.162*** | (0.0456) | 1.104*** | (0.0320) | -0.0993*** | (0.0192) |
| Constant | 0.00875*** | (6.56e-4) | 2.355*** | (0.0391) | 0.0185*** | (4.01e-4) | 2.485*** | (0.0161) |
| Var $[\zeta_i]$ (ind. effects) | 0.406*** | (0.0181) | | | 0.503*** | (0.0687) | | |
| Var $[\eta_{it}]$ (seq. effects) | 0.0626*** | (0.00476) | | | 0.0203*** | (0.00159) | | |
| Var $[\zeta_i]$/Var $[\zeta_i]$ + Var $[\eta_{it}]$ | 0.866*** | (0.0103) | | | 0.961*** | (0.00606) | | |
| Baseline hazard | 7.788*** | (0.100) | | | 6.693*** | (0.0280) | | |
| Number of individuals | 1,371 | | | | 1,557 | | | |
| Number of probabilities | 4,858 | | | | 15,789 | | | |
| Log-likelihood | -8,393.77 | | | | -25,947.56 | | | |

[a] Baseline: low education level (lower secondary education). Educ. middle: intermediate vocational. Educ. high: (applied) university.
Standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$

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| Baseline hazard | 7.788*** | (0.100) | | | 6.693*** | (0.0280) | | |
| | Rounding | | | | Rounding | | | |
| $\mu_1$ | 1.461*** | (0.0895) | | | 2.478*** | (0.110) | | |
| $\mu_2$ | 3.326*** | (0.179) | | | 3.996*** | (0.141) | | |
| Var $[\zeta_i^r]$ (ind. effects) | 0.821*** | (0.184) | | | 2.729*** | (0.320) | | |
| Var $[\eta_{it}^r]$ (seq. effects) | 0.114 | (0.0917) | | | 0.00863 | (0.00962) | | |
| Var $[\zeta_i^r]$/Var $[\zeta_i^r]$ + Var $[\eta_{it}^r]$ | 0.878*** | (0.0883) | | | 0.997*** | (0.00348) | | |
| Corr $[\zeta_i; \zeta_i^r]$ | 0.0909 | (0.0740) | | | 0.149*** | (0.0438) | | |
| Corr $[\eta_{it}; \eta_{it}^r]$ | 0.264 | (0.231) | | | -0.397 | (0.488) | | |
| Number of individuals | 1,371 | | | | 1,557 | | | |
| Number of probabilities | 4,858 | | | | 15,789 | | | |
| Log-likelihood | -8,393.77 | | | | -25,947.56 | | | |

[a] Baseline: low education level (lower secondary education). Educ. middle: intermediate vocational. Educ. high: (applied) university.
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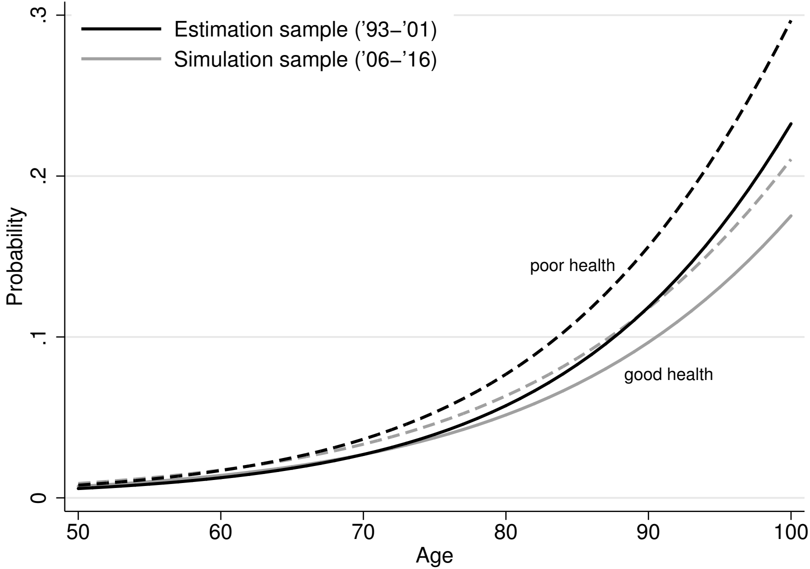
# Subjective mortality estimates – level



# Individual-level coefficients: conditional means

**Mixing distribution:** distribution of subjective survival in population

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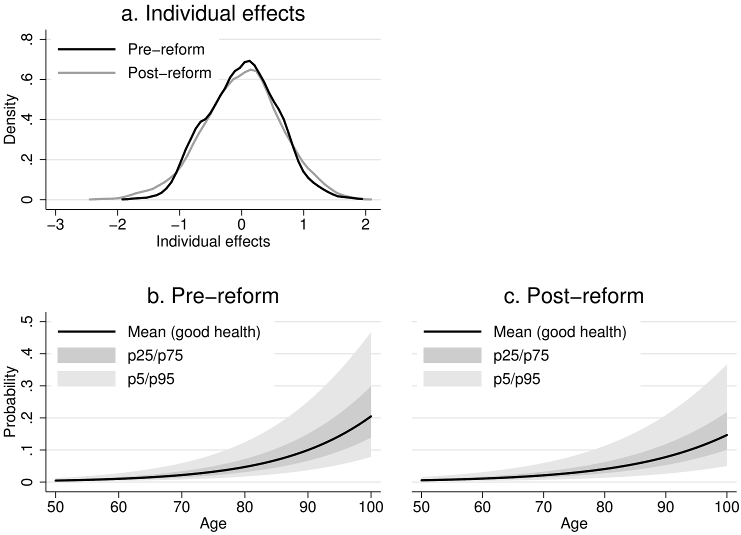
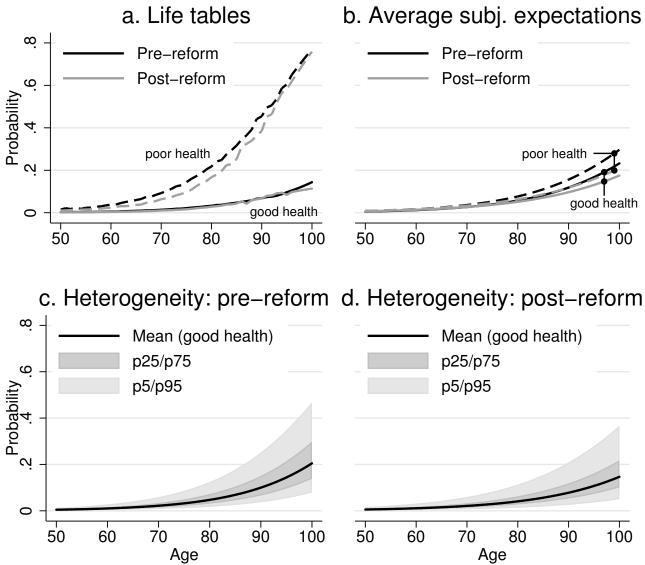
**Conditional means:** approximations of individual-specific subjective survival

- Mixing distribution (prior) + probabilities reported by individual $i$
  $\Longrightarrow$ posterior distribution for individual and sequence effect $i$
- Means of posterior distributions as approximation of $\zeta_i$ and $\eta_{it}$

# Conditional means of unobserved heterogeneity



# Different mortality estimates – levels and variation



# Roadmap

# Can subjective survival expectations improve performance of structural retirement model? (de Bresser, 2024)

Population aging necessitates pension reform

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Long-running panels allow **validation of counterfactual predictions**

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Different combinations of preferences and expectations may...

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2. Average subjective survival
3. Heterogeneous subjective survival

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**Measurement model of subjective survival**

- Heterogeneity captured through single parameter

Validation of counterfactual model predictions is important

# Model outline

Unitary model for men aged 50-100 (yearly)

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**Preferences:** consumption, leisure, bequests

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Pre-reform pension regime (1993-2001) and post-reform regime (2006-2016)

# Preferences ▸ Details ▸ State variables ▸ Expectations

Instantaneous utility from **consumption** and **leisure**

- CRRA utility
- Time costs of
  - Work (fixed cost) (French, 2005)
  - Poor health
  - Claiming DI or UI (stigma/lack of knowledge)

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Utility from leaving **bequests** (French, 2005)

- Weight allowed to depend on household size

# Budget constraint ▸ Details budget ▸ Pensions ▸ Taxes

Sources of income:

- Earnings
- Disability and unemployment benefits (up to age 65)
- Public pension (automatic at age 65)
- **Occupational pension**
  - Replacement rate of final earnings, function of years worked
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Cash-on-hand = wealth + net income – medical expenditures

- Government transfers guarantee $c_{\text{min}} = n_t \times 7,000$

# Policy reform: actuarial adjustments to pensions ▸ Effect

**Pre-reform (1993-2001)**

- Early retirement age 59-64; regular benefits afterwards

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- Actuarial adjustments: benefits lowered by 6% per year for claiming before age 65; raised by 7% for delayed claiming
- No continued accrual of pension entitlements
- Cohorts: born from 1950 onwards

# Estimation

**Method of Simulated Moments** estimation algorithm (French, 2005; De Nardi et al., 2010; French and Jones, 2011):

1. Estimate all processes that do not require model (mortality etc.)
2. Take initial conditions (state variables) from data
3. Simulate life cycles for 5,000 workers
4. Compare target moments and simulations (MM objective function)
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Target moments: age profiles for

- Labor supply (hrs/participation) by health
- DI, UI and occ. pension claiming
- Quartiles of wealth

# Data

Data taken from **DNB Household Survey (DHS)**

- Administered by CentERdata at Tilburg University
- 5,000 individuals in 3,000 households
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- Auxiliary processes
  - Men aged 50+ during relevant waves (1993-2001 and 2006-2016)
- Target moments
  - Sample for auxiliary processes, but never self-employed; 15+ years work experience; relevant cohort (before 1950/1950 and later)

# Parameter estimates

| | (1) Life tables | (2) Average subj. exp. | (3) Heterogeneous exp. |
|---|---|---|---|
| **Utility function** | | | |
| Concavity ($\sigma$) | 4.64 | 4.64 | 5.18 |
| | (0.000023) | (0.000028) | (0.000038) |
| Consumption weight ($\kappa$) | 0.68 | 0.65 | 0.42 |
| | (0.000061) | (0.000063) | (0.000065) |
| Discount factor ($\beta$) | 1.04 | 1.04 | 0.98 |
| | (0.000026) | (0.000035) | (0.000028) |
| Fixed cost of work; hrs/yr ($\gamma$) | 1085 | 1013 | 1101 |
| | (27.5) | (4.5) | (8.0) |
| Time cost of bad health; hrs/yr ($\delta$) | 131 | 249 | 251 |
| | (13.9) | (34.6) | (6.9) |
| Stigma cost DI; hrs/yr ($\phi$) | 1681 | 2029 | 2140 |
| | (441.9) | (345.5) | (145.4) |
| Stigma cost UI; hrs/yr ($\xi$) | 3620 | 3509 | 3568 |
| | (8.9) | (1.9) | (5.9) |
| **Bequests** | | | |
| Intercept bequest weight ($\theta_0$) | -2.45 | -3.91 | -9.27 |
| | (0.00014) | (0.00015) | (0.00010) |
| Slope bequest weight ($\theta_1$) | 4.73 | 5.56 | 2.51 |
| | (0.00012) | (0.00014) | (0.000082) |
| Concavity bequests ($K$) | 675,282 | 913,915 | 441,061 |
| | (29.0) | (34.3) | (6.0) |
| Function value | 473.27 | 473.21 | 483.89 |

Standard errors in parentheses.

[a] $u\left(c_t, l_t\right) = \frac{n_t}{1-\sigma}\left[\left(\left(\frac{c_t}{n_t}\right)^{\kappa} l_t^{1-\kappa}\right)^{1-\sigma} - 1\right]$; $b\left(w_t, n_t\right) = \exp\left[\theta_0 + \theta_1 n_t\right] \times \frac{\left(w_t+K\right)^{(1-\sigma)\kappa}}{1-\sigma}$

$l_t = 4,000 - h_t - \gamma \mathbb{I}\left\{h_t > 0\right\} - \delta \mathbb{I}\left\{\text{health} = \text{bad}\right\} - \phi \mathbb{I}\left\{\text{claim DI}\right\} - \xi \mathbb{I}\left\{\text{claim UI}\right\}$

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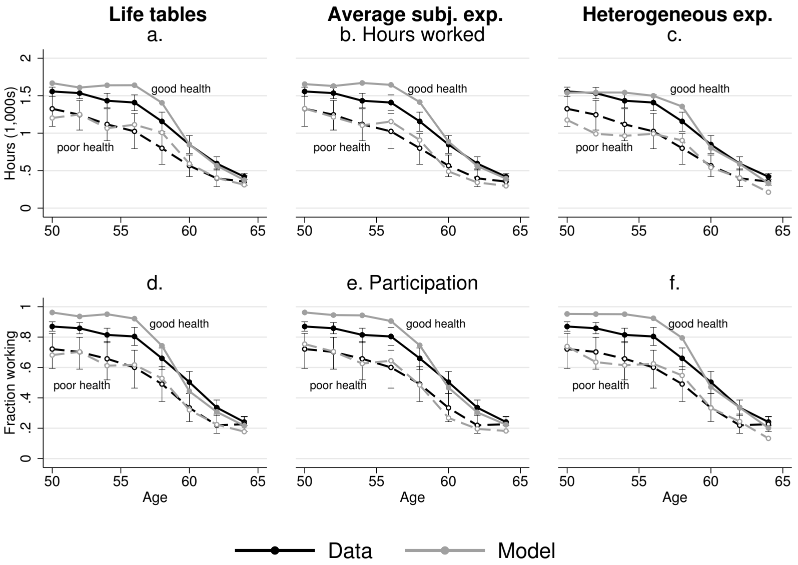
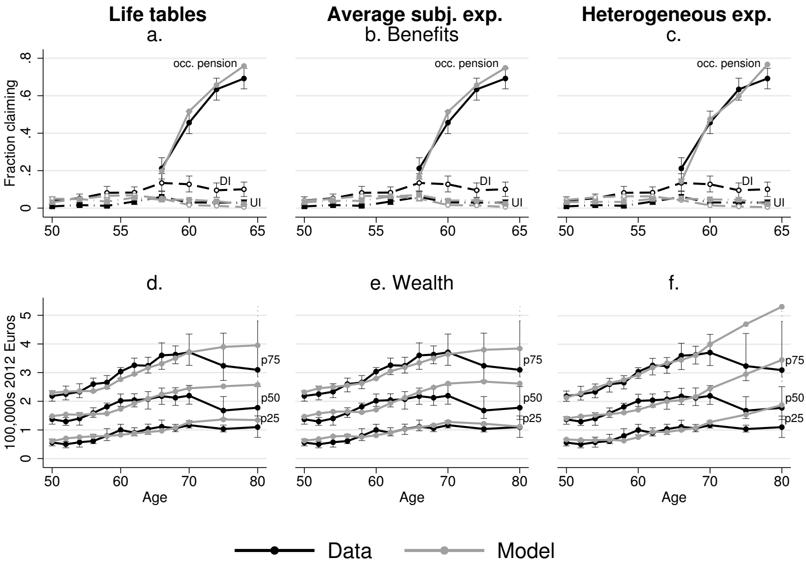
# Model fit I


Life tables
a.
Average subj. exp.
b. Hours worked
Heterogeneous exp.
c.
Hours (1,000s)
0
.5
1
1.5
2
good health
poor health
50
55
60
65
d.
e. Participation
f.
Fraction working
0
.2
.4
.6
.8
1
Age
Data
Model

# Model fit II


Life tables
a.
Average subj. exp.
b. Benefits
Heterogeneous exp.
c.
Fraction claiming
0
.2
.4
.6
.8
occ. pension
DI
UI
50
55
60
65
d.
e. Wealth
f.
100,000s 2012 Euros
0
1
2
3
4
5
p75
p50
p25
50
60
70
80
Age
Data
Model

# Counterfactual predictions – average retirement age

**a. Data**

| | Pre-reform sample ('93-'01) | Post-reform sample ('06-'16) | Difference (yrs) |
|---|---|---|---|
| Average pension age | 61.2<br>(60.97 – 61.41) | 63.8<br>(63.49 – 64.03) | 2.6<br>(2.23 – 2.92) |
| N | 756 | 467 | 1223 |

95% confidence intervals based on robust standard errors in parentheses.

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| **b. Simulations** | | | |
| | Pre-reform sample ('93-'01) | Post-reform sample ('06-'16) | Difference (yrs) |
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| Average subj. exp. | 60.9 | 65.1 | 4.2 |

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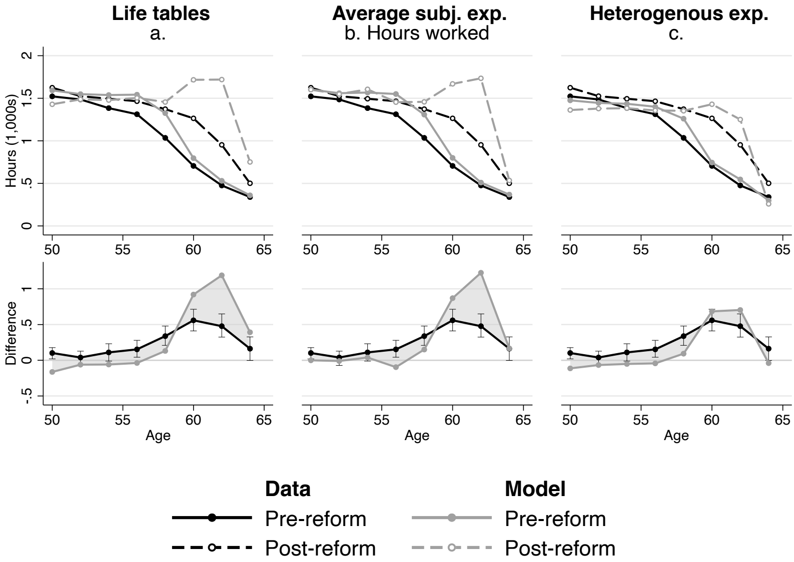
**b. Simulations**

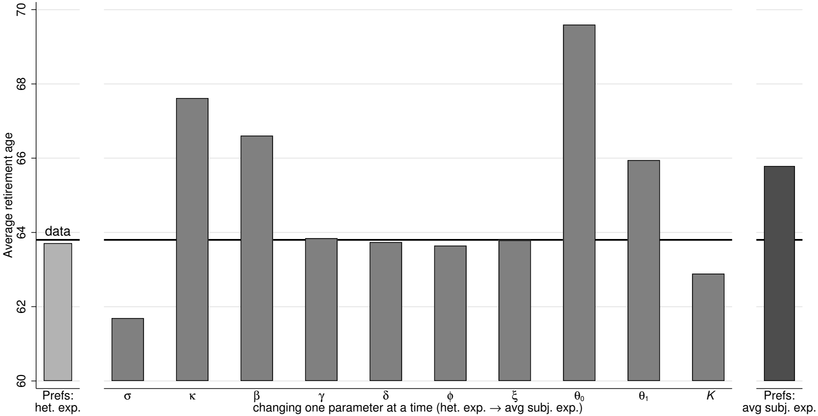
| | Pre-reform sample ('93-'01) | Post-reform sample ('06-'16) | Difference (yrs) |
|---|---|---|---|
| Life tables | 60.7 | 65.9 | 5.2 |
| Average subj. exp. | 60.9 | 65.1 | 4.2 |
| Heterogeneous exp. | 61.0 | 63.7 | 2.7 |

95% confidence intervals based on robust standard errors in parentheses.

# Counterfactual predictions – average hours worked



# Influence of estimates on out-of-sample predictions: average retirement age



# Can subjective survival expectations improve performance of structural retirement model?

**The <u>level</u> of expectations affects preference estimates**

- Actuarial and subjective mortality yield different preference estimates
- Similar model fit
- Poor counterfactual predictions
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No improvement from limited heterogeneity based on health behaviors

# Moving forward

Elicitation of **conditional probabilities**

- Perceptions of causal effects (e.g., Giustinelli and Shapiro, 2024)
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Together with PhDs we will work on:

- Model of life-care annuities with subjective expectations on both states and choices
- Income processes estimated on subj. probabilities, survey measure of income, and admin income

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# Maximum Likelihood estimation ▸ back

Add error $\varepsilon_t$ with CDF $F(\varepsilon)$ to single-period utility

$$U(c_t, l_t, \varepsilon_t) = U(c_t, l_t) + \varepsilon_t$$

Data: $N$ individuals for $T^d$ periods

- Choices (e.g., wealth, labor supply) and/or expectations (e.g., prob. of working at future ages)
- $\boldsymbol{\mathcal{D}} = \left\{a_{i,t}; \mathbb{P}_{i,t}\left[h_{i\tau} = 1\right]; i = 1, ..., N; t = 1, ..., T^d\right\}$

$\widehat{\boldsymbol{\theta}}_{\text{ML}}$ maximizes the likelihood function $\mathcal{L}(\boldsymbol{\mathcal{D}}, \boldsymbol{\theta}) = \prod_{i=1}^{N} L(\mathcal{D}_i, \boldsymbol{\theta})$:

$$\widehat{\boldsymbol{\theta}}_{\text{ML}} = \arg\max_{\boldsymbol{\theta}} \prod_{i=1}^{N} L(\mathcal{D}_i, \boldsymbol{\theta})$$

# Maximum Likelihood estimation

back

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$$\widehat{\boldsymbol{\theta}}_{\text{ML}} = \arg\max_{\boldsymbol{\theta}} \prod_{i=1}^{N} L\left(\mathcal{D}_i, \boldsymbol{\theta}\right)$$

If agent's problem does not have analytical solution and/or there are missing state variables: **Maximum Simulated Likelihood**

- Solve model at $\boldsymbol{\theta}$ to obtain optimal decisions $\check{\tilde{a}}_t\left(\chi_t, \boldsymbol{\theta}\right)$ and $\check{\tilde{\mathbb{P}}}_t\left(\chi_t, \boldsymbol{\theta}\right)$
- Simulate behavior using those decision rules

$$\widehat{\boldsymbol{\theta}}_{\text{MSL}} = \arg\max_{\boldsymbol{\theta}} \prod_{i=1}^{N} L\left(\check{\mathcal{D}}_i, \boldsymbol{\theta}\right)$$

# Method of Simulated Moments back

Moments $\check{\mathbf{m}}(\boldsymbol{\theta})$ summarize behavior simulated from the model

- Should be informative about parameters to be estimated
  - E.g., (expected) labor supply moments to estimate relative weights on consumption and leisure
- Mean / quantiles
- Conditional / unconditional

Estimate $\boldsymbol{\theta}$ bringing simulated behavior as close as possible to the data $\mathbf{m}$:

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left(\mathbf{m} - \check{\mathbf{m}}(\boldsymbol{\theta})\right)' \mathbf{W} \left(\mathbf{m} - \check{\mathbf{m}}(\boldsymbol{\theta})\right)$$

$\mathbf{W}$ is symmetric positive definite weighting matrix

Generalization: **Indirect Inference** (Gourieroux et al., 1993; Smith Jr, 1993)

# Indirect Inference ▸ back

Specify **auxiliary model** of observables

- Easy to compute
- Estimator when applied to data: $\widehat{\boldsymbol{\beta}}_A$
- Constant-only model equivalent to MSM

Estimate parameters by minimizing distance between auxiliary parameters from observed and simulated data

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left(\widehat{\boldsymbol{\beta}}_A - \widehat{\boldsymbol{\beta}}_S\left(\boldsymbol{\theta}\right)\right)' \mathbf{W} \left(\widehat{\boldsymbol{\beta}}_A - \widehat{\boldsymbol{\beta}}_S\left(\boldsymbol{\theta}\right)\right)$$

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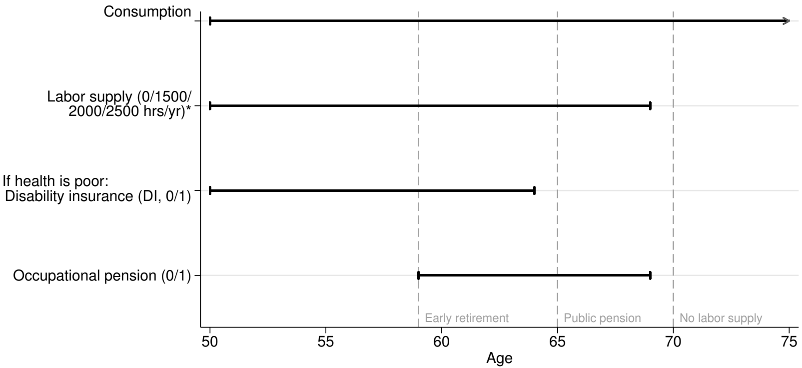
# Decisions at different ages ▸ Back




*If labor supply equals zero, individual does not claim DI or occupational pension and labor market history entitles individual to unemployment insurance (UI), UI benefits are received automatically. UI benefits stop in case the agent chooses to work or claim DI or occupational pensions; when the entitlement is exhausted; or at age 65.

# Life cycle model: state variables Back

| Variable | Values |
|---|---|
| Wealth[a] | 0 – 1,000,000 |
| Wage[a] | 5 – 100 |
| Health | Good/bad |
| Occupational pension | $\{1 =$ no occupational pension scheme,<br>$2 =$ occupational pension scheme – not claimed,<br>$3 =$ occupational pension scheme – claimed$\}$ |
| *Benefit level occ. pension* | |
| Years worked | 15 – 50 years |
| Final earnings[a] | 0 – 250,000 |
| DI eligibility | not eligible/eligible |
| UI eligibility | $\{1 =$ not eligible (0 years),<br>...<br>$4 =$ fully eligible (3 years)$\}$ |

[a] All monetary amounts in 2012 Euros

# Preferences ▸ Model ▸ Estimates

Utility:

$$u\left(c_t, l_t\right) = \frac{n_t}{1-\sigma}\left[\left(\left(\frac{c_t}{n_t}\right)^{\kappa} l_t^{1-\kappa}\right)^{1-\sigma} - 1\right]$$

- $c_t$ consumption; $l_t$ leisure; $n_t$ equivalence scale for the number of individuals in the household
- $\sigma$ concavity of utility function; $\kappa$ relative weight of consumption in utility function

Leisure:

$$l_t = 4,000 - h_t - \gamma \mathbb{I}\left\{h_t > 0\right\} - \delta \mathbb{I}\left\{\text{health} = \text{bad}\right\} - \text{stigma costs}$$

- 4,000 hours time endowment
- $h_t$ hours worked (0/1500/2000/2500 hours per year)
- $\gamma$ fixed leisure cost of work; $\delta$ leisure cost of being in bad health
- Stigma costs for DI and UI expressed in hours of leisure

# Bequest motive (adapted from French, 2005) ▸ Model ▸ Estimates

$$b\left(w_t, n_t\right) = \exp\left[\theta_0 + \theta_1 n_t\right] \times \frac{\left(w_t + K\right)^{(1-\sigma)\kappa}}{1-\sigma}$$

- $w_t$ wealth; $n_t$ equivalence scale (HH size)
- $\theta_1$: weight of bequests in utility depends on survival of spouse
- $K$ determines curvature of bequest utility

# Expectations ▸ Back

**Survival**

1. Actuarial tables, use Bayes' rule to condition on health $m_t$:

$$\Pr(T = t+1 | m_t = 2) = \frac{\Pr(m_t = 2 | T = t+1) \times \Pr(T = t+1)}{\Pr(m_t = 2)}$$

2. Subjective expectations – no heterogeneity
3. Subjective expectations – heterogeneity
4. Life tables conditional on smoking and drinking

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**Health**

- AR(1) logit estimated on observed health $m_t$:

$$\Pr\left(m_{t+1} = 2 | m_t, t\right) = \frac{\exp\left[\mu_0 + \mu_1 t + \mu_2 \mathbb{I}\left\{m_t = 2\right\}\right]}{1 + \exp\left[\mu_0 + \mu_1 t + \mu_2 \mathbb{I}\left\{m_t = 2\right\}\right]}$$

# Expectations ‣ Back

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**Wage**

- AR(1) estimated on observed wages (Gourinchas and Parker, 2002)

# Budget constraint ▸ Back

Consumption cannot exceed cash-on-hand $x_t$:

$$x_t = w_t + netinc_t + inc_t^{\text{sp}} - OOP_t$$

$w_t$ is wealth; $netinc_t$ is net income; $inc_t^{\text{sp}}$ is (exogenous) net income of the spouse; $OOP_t$ are out-of-pocket medical expenditures

- $netinc_t = \tau\left(earn_t, DIinc_t, UIinc_t, pubpens_t, occpens_t, w_t, t\right)$
- 20% wage penalty for working part time (1500 hrs/yr)
- DI and UI replace 70% of earnings (capped at 50,400 Euros)
- Automatic transfer from UI/DI to public pension at age 65
- Government transfers guarantee minimum consumption $c_{\text{min}}$: $x_t = \max\left\{x_t, n_t c_{\text{min}}\right\}$ (Hubbard et al., 1995)

# Pensions ▸ Back

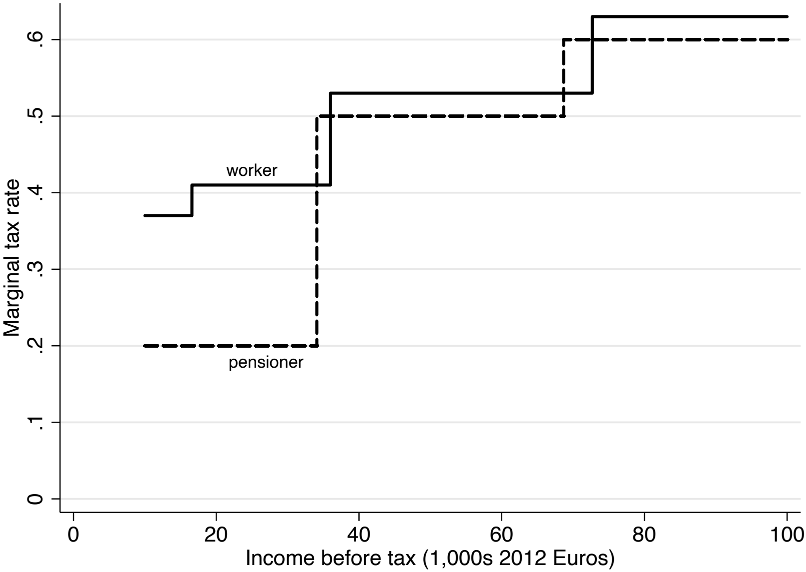
Public pension

- Flat rate subsistence income
- Automatic onset at age 65
- 9,000 Euros/year for individual in couple

Occupational pension

- 90% of employees covered
- Mandatory participation with fixed contribution rate
- Two phases (pre-reform):
  1. Age 59-65: early retirement pension (VUT)
     - ★ 85% replacement rate; no actuarial adjustment; continued accumulation of entitlements
     - ★ Phased out around year 2000 for cohorts 1950 and later
  2. Age 65 and older: regular occupational pension
     - ★ Replacement rate is function of contribution years (1.75pp per year)
     - ★ $occpens_t = 0.0175 \times yrswrk_t \times (prevearn_t - 19,000)$

# Tax function ▸ Back



# Effect of abolishment early retirement (VUT)

Back