Matrices

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Matrices (1ª parte)

```
M = matrix(1:12, nrow=4)
M
```

```
##      [,1] [,2] [,3]
## [1,]    1    5    9
## [2,]    2    6   10
## [3,]    3    7   11
## [4,]    4    8   12
```

```
M = matrix(1:12, nrow=4, byrow = T)
M
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    4    5    6
## [3,]    7    8    9
## [4,]   10   11   12
```

```
M = matrix(1:12, nrow=3)
M
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    4    7   10
## [2,]    2    5    8   11
## [3,]    3    6    9   12
```

```
M = matrix(1:12, nrow=5)
```

```
## Warning in matrix(1:12, nrow = 5): la longitud de los datos [12] no es un
## submúltiplo o múltiplo del número de filas [5] en la matriz
```

```
M
```

```
##      [,1] [,2] [,3]
## [1,]    1    6   11
## [2,]    2    7   12
## [3,]    3    8    1
## [4,]    4    9    2
## [5,]    5   10    3
```

```r
M = matrix(1, nrow = 4, ncol = 6)
M
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    1    1    1    1    1
## [2,]    1    1    1    1    1    1
## [3,]    1    1    1    1    1    1
## [4,]    1    1    1    1    1    1
```

Ejercicio

```r
M = matrix(0, nrow = 3, ncol = 5)
M
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    0    0    0    0
## [2,]    0    0    0    0    0
## [3,]    0    0    0    0    0
```

```r
vec = 1:12
M = matrix(vec, nrow = 3)
M
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    4    7   10
## [2,]    2    5    8   11
## [3,]    3    6    9   12
```

Matrices (2ª parte)

```r
M = rbind(M, c(1,2,3,4), -4:-1)
M
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    4    7   10
## [2,]    2    5    8   11
## [3,]    3    6    9   12
## [4,]    1    2    3    4
## [5,]   -4   -3   -2   -1
```

```r
M = cbind(M, seq(0, 100, length.out = 5))
M
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    4    7   10    0
## [2,]    2    5    8   11   25
## [3,]    3    6    9   12   50
## [4,]    1    2    3    4   75
## [5,]   -4   -3   -2   -1  100
```

```r
M_diag = diag(1:9)
M_diag
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    1    0    0    0    0    0    0    0    0
##  [2,]    0    2    0    0    0    0    0    0    0
##  [3,]    0    0    3    0    0    0    0    0    0
##  [4,]    0    0    0    4    0    0    0    0    0
##  [5,]    0    0    0    0    5    0    0    0    0
##  [6,]    0    0    0    0    0    6    0    0    0
##  [7,]    0    0    0    0    0    0    7    0    0
##  [8,]    0    0    0    0    0    0    0    8    0
##  [9,]    0    0    0    0    0    0    0    0    9
```

Propiedades matrices

```r
m = matrix(1:20, nrow = 4)
m
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    5    9   13   17
## [2,]    2    6   10   14   18
## [3,]    3    7   11   15   19
## [4,]    4    8   12   16   20
```

```r
diag(m)
```

```
## [1]  1  6 11 16
```

```r
nrow(m)
```

```
## [1] 4
```

```r
ncol(m)
```

```
## [1] 5
```

```r
dim(m)
```

```
## [1] 4 5
```

```r
sum(m)
```

```
## [1] 210
```

```r
prod(m)
```

```
## [1] 2.432902e+18
```

```r
mean(m)
```

```
## [1] 10.5
```

```r
colSums(m)
```

```
## [1] 10 26 42 58 74
```

```r
rowSums(m)
```

```
## [1] 45 50 55 60
```

```r
colMeans(m)
```

```
## [1]  2.5  6.5 10.5 14.5 18.5
```

```r
rowMeans(m)
```

```
## [1]  9 10 11 12
```

```r
apply(m, MARGIN = 1, FUN = function(x){sqrt(sum(x^2))})
```

```
## [1] 23.76973 25.69047 27.65863 29.66479
```

```r
apply(m, MARGIN = 2, FUN = function(x){sqrt(sum(x^2))})
```

```
## [1]  5.477226 13.190906 21.118712 29.086079 37.067506
```

```r
apply(m, MARGIN = c(1,2), FUN = function(x){x^2})
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1   25   81  169  289
## [2,]    4   36  100  196  324
## [3,]    9   49  121  225  361
## [4,]   16   64  144  256  400
```

Repaso álgebra lineal (1ª parte)

```
m = matrix(0:99, nrow = 10)
m
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0   10   20   30   40   50   60   70   80    90
##  [2,]    1   11   21   31   41   51   61   71   81    91
##  [3,]    2   12   22   32   42   52   62   72   82    92
##  [4,]    3   13   23   33   43   53   63   73   83    93
##  [5,]    4   14   24   34   44   54   64   74   84    94
##  [6,]    5   15   25   35   45   55   65   75   85    95
##  [7,]    6   16   26   36   46   56   66   76   86    96
##  [8,]    7   17   27   37   47   57   67   77   87    97
##  [9,]    8   18   28   38   48   58   68   78   88    98
## [10,]    9   19   29   39   49   59   69   79   89    99
```

```
tm = t(m)
tm
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0    1    2    3    4    5    6    7    8     9
##  [2,]   10   11   12   13   14   15   16   17   18    19
##  [3,]   20   21   22   23   24   25   26   27   28    29
##  [4,]   30   31   32   33   34   35   36   37   38    39
##  [5,]   40   41   42   43   44   45   46   47   48    49
##  [6,]   50   51   52   53   54   55   56   57   58    59
##  [7,]   60   61   62   63   64   65   66   67   68    69
##  [8,]   70   71   72   73   74   75   76   77   78    79
##  [9,]   80   81   82   83   84   85   86   87   88    89
## [10,]   90   91   92   93   94   95   96   97   98    99
```

```
m + tm
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0   11   22   33   44   55   66   77   88    99
##  [2,]   11   22   33   44   55   66   77   88   99   110
##  [3,]   22   33   44   55   66   77   88   99  110   121
##  [4,]   33   44   55   66   77   88   99  110  121   132
##  [5,]   44   55   66   77   88   99  110  121  132   143
##  [6,]   55   66   77   88   99  110  121  132  143   154
##  [7,]   66   77   88   99  110  121  132  143  154   165
##  [8,]   77   88   99  110  121  132  143  154  165   176
##  [9,]   88   99  110  121  132  143  154  165  176   187
## [10,]   99  110  121  132  143  154  165  176  187   198
```

```
m * tm
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0   10   40   90  160  250  360  490  640   810
##  [2,]   10  121  252  403  574  765  976 1207 1458  1729
```

```
##  [3,]   40  252  484  736 1008 1300 1612 1944 2296  2668
##  [4,]   90  403  736 1089 1462 1855 2268 2701 3154  3627
##  [5,]  160  574 1008 1462 1936 2430 2944 3478 4032  4606
##  [6,]  250  765 1300 1855 2430 3025 3640 4275 4930  5605
##  [7,]  360  976 1612 2268 2944 3640 4356 5092 5848  6624
##  [8,]  490 1207 1944 2701 3478 4275 5092 5929 6786  7663
##  [9,]  640 1458 2296 3154 4032 4930 5848 6786 7744  8722
## [10,]  810 1729 2668 3627 4606 5605 6624 7663 8722  9801
```

```
m %*% tm
```

```
##        [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9] [,10]
##  [1,] 28500 28950 29400 29850 30300 30750 31200 31650 32100 32550
##  [2,] 28950 29410 29870 30330 30790 31250 31710 32170 32630 33090
##  [3,] 29400 29870 30340 30810 31280 31750 32220 32690 33160 33630
##  [4,] 29850 30330 30810 31290 31770 32250 32730 33210 33690 34170
##  [5,] 30300 30790 31280 31770 32260 32750 33240 33730 34220 34710
##  [6,] 30750 31250 31750 32250 32750 33250 33750 34250 34750 35250
##  [7,] 31200 31710 32220 32730 33240 33750 34260 34770 35280 35790
##  [8,] 31650 32170 32690 33210 33730 34250 34770 35290 35810 36330
##  [9,] 32100 32630 33160 33690 34220 34750 35280 35810 36340 36870
## [10,] 32550 33090 33630 34170 34710 35250 35790 36330 36870 37410
```

```
library(Biodem)
mtx.exp(m, 2)
```

```
##       [,1] [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9] [,10]
##  [1,] 2850 7350 11850 16350 20850 25350 29850 34350 38850 43350
##  [2,] 2895 7495 12095 16695 21295 25895 30495 35095 39695 44295
##  [3,] 2940 7640 12340 17040 21740 26440 31140 35840 40540 45240
##  [4,] 2985 7785 12585 17385 22185 26985 31785 36585 41385 46185
##  [5,] 3030 7930 12830 17730 22630 27530 32430 37330 42230 47130
##  [6,] 3075 8075 13075 18075 23075 28075 33075 38075 43075 48075
##  [7,] 3120 8220 13320 18420 23520 28620 33720 38820 43920 49020
##  [8,] 3165 8365 13565 18765 23965 29165 34365 39565 44765 49965
##  [9,] 3210 8510 13810 19110 24410 29710 35010 40310 45610 50910
## [10,] 3255 8655 14055 19455 24855 30255 35655 41055 46455 51855
```

```
library(Matrix)
library(expm)
```

```
##
## Attaching package: 'expm'
```

```
## The following object is masked from 'package:Matrix':
##
##     expm
```

```
tm %^% 2
```

```
##        [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9] [,10]
##  [1,]  2850  2895  2940  2985  3030  3075  3120  3165  3210  3255
##  [2,]  7350  7495  7640  7785  7930  8075  8220  8365  8510  8655
##  [3,] 11850 12095 12340 12585 12830 13075 13320 13565 13810 14055
##  [4,] 16350 16695 17040 17385 17730 18075 18420 18765 19110 19455
##  [5,] 20850 21295 21740 22185 22630 23075 23520 23965 24410 24855
##  [6,] 25350 25895 26440 26985 27530 28075 28620 29165 29710 30255
##  [7,] 29850 30495 31140 31785 32430 33075 33720 34365 35010 35655
##  [8,] 34350 35095 35840 36585 37330 38075 38820 39565 40310 41055
##  [9,] 38850 39695 40540 41385 42230 43075 43920 44765 45610 46455
## [10,] 43350 44295 45240 46185 47130 48075 49020 49965 50910 51855
```

Ejercicio

Si $A = \begin{pmatrix} 2 & 0 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & 3 \end{pmatrix}$ y $B = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$, calcular $A \cdot B$, A^2 y B^3

```
A = matrix(c(2, 0, 2, 1, 2, 3, 0, 1, 3), nrow = 3, byrow = TRUE)
B = matrix(c(3, 2, 1, 1, 0, 0, 1, 1, 1), nrow = 3, byrow = TRUE)
A * B
```

```
##      [,1] [,2] [,3]
## [1,]    6    0    2
## [2,]    1    0    0
## [3,]    0    1    3
```

```
A %*% A
```

```
##      [,1] [,2] [,3]
## [1,]    4    2   10
## [2,]    4    7   17
## [3,]    1    5   12
```

```
B %*% B %*% B
```

```
##      [,1] [,2] [,3]
## [1,]   47   28   16
## [2,]   12    7    4
## [3,]   20   12    7
```

Repaso álgebra lineal (2ª parte)

```
library(magic)
```

```
## Loading required package: abind
```

```r
m = magic(3)
m
```

```
##      [,1] [,2] [,3]
## [1,]    2    7    6
## [2,]    9    5    1
## [3,]    4    3    8
```

```r
det(m)
```

```
## [1] -360
```

```r
qr(m)
```

```
## $qr
##             [,1]      [,2]      [,3]
## [1,] -10.0498756 -7.064764 -5.273697
## [2,]   0.8955335  5.752313  5.865914
## [3,]   0.3980149  0.290118  6.227293
##
## $rank
## [1] 3
##
## $qraux
## [1] 1.199007 1.956991 6.227293
##
## $pivot
## [1] 1 2 3
##
## attr(,"class")
## [1] "qr"
```

```r
qr(m)$rank
```

```
## [1] 3
```

```r
solve(m) # Inversa de la matriz
```

```
##             [,1]        [,2]        [,3]
## [1,] -0.10277778  0.10555556  0.06388889
## [2,]  0.18888889  0.02222222 -0.14444444
## [3,] -0.01944444 -0.06111111  0.14722222
```

```r
solve(m, c(1, 2, 3)) # Resuelve sistema de ecuaciones
```

```
## [1]  0.3 -0.2  0.3
```

```
eigen(m)
```

```
## eigen() decomposition
## $values
## [1] 15.000000 -4.898979  4.898979
##
## $vectors
##            [,1]        [,2]        [,3]
## [1,] -0.5773503 -0.74158162 -0.07491496
## [2,] -0.5773503  0.66666667 -0.66666667
## [3,] -0.5773503  0.07491496  0.74158162
```

```
eigen(m)$values
```

```
## [1] 15.000000 -4.898979  4.898979
```

```
eigen(m)$vectors
```

```
##            [,1]        [,2]        [,3]
## [1,] -0.5773503 -0.74158162 -0.07491496
## [2,] -0.5773503  0.66666667 -0.66666667
## [3,] -0.5773503  0.07491496  0.74158162
```

Ejercicio

Si $M = \begin{pmatrix} 2 & 6 & -8 \\ 0 & 6 & -3 \\ 0 & 2 & 1 \end{pmatrix}$ comprobar que $M = P \cdot D \cdot P^{-1}$

```
M = rbind(c(2,6,-8), c(0,6,-3), c(0,2,1))
M
```

```
##      [,1] [,2] [,3]
## [1,]    2    6   -8
## [2,]    0    6   -3
## [3,]    0    2    1
```

```
P = eigen(M)$vectors
P
```

```
##           [,1]       [,2] [,3]
## [1,] 0.2672612 -0.8164966    1
## [2,] 0.8017837  0.4082483    0
## [3,] 0.5345225  0.4082483    0
```

```
D = diag(eigen(M)$values)
D
```

```
##      [,1] [,2] [,3]
## [1,]    4    0    0
## [2,]    0    3    0
## [3,]    0    0    2
```

```
Pinv = solve(P)
Pinv
```

```
##      [,1]      [,2]      [,3]
## [1,]    0  3.741657 -3.741657
## [2,]    0 -4.898979  7.348469
## [3,]    1 -5.000000  7.000000
```

```
P%*%D%*%Pinv
```

```
##      [,1] [,2] [,3]
## [1,]    2    6   -8
## [2,]    0    6   -3
## [3,]    0    2    1
```

Repaso álgebra lineal (3ª parte)

```
A = matrix(c(3-2i, 5+3i, 1+2i, 2-1i), nrow = 2, byrow = T)
A
```

```
##      [,1] [,2]
## [1,] 3-2i 5+3i
## [2,] 1+2i 2-1i
```

```
A%*%A
```

```
##       [,1]  [,2]
## [1,]  4+1i 34+0i
## [2,] 11+7i  2+9i
```

```
eigen(A)
```

```
## eigen() decomposition
## $values
## [1] 4.902076+1.101916i 0.097924-4.101916i
##
## $vectors
##                       [,1]                  [,2]
## [1,] 0.8483705+0.000000i  0.8519823+0.000000i
## [2,] 0.4695014+0.244614i -0.5216168-0.045189i
```

```
solve(A, c(1-1i, 4))
```

```
## [1] 0.4823529-1.0705882i 0.5294118+0.3176471i
```

```r
prod(eigen(A)$values) # det(A)
```

```
## [1] 5-20i
```