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Algorithm

We are going to use Breadth First Search to implement our algorithm.

- Firstly, add a new node m, and connect m with all hospital nodes. Therefore, we get a new graph $G_{\text{new}} = (V_{\text{new}}, E_{\text{new}})$, where

$$V_{\text{new}} = \{m\} \cup V,$$

$$E_{\text{new}} = E \cup \{(m, h) \mid h \text{ is a hospital node}, h \in V\}.$$

- After updating G, we use BFS on G_{new} starting at m, i.e., $\texttt{BFS}(G_{\text{new}}, m)$. For each house u, we take the minimum distance to some hospital as $d(u) - 1$, where $d(u)$ is the distance found by `BFS`. This is valid because, according to the lecture slides, after performing `BFS` from s, for every $v \in V$, $d(s, v) = \delta(s, v)$, where $\delta(s, v)$ represents the shortest path distance from s to v.

Time Complexity

Adding extra node m and connecting it with all hospital nodes will take $O(|V|)$, since we only need to traverse through each node. Using `BFS` to find the shortest distance for each house will take $O(|V_{\text{new}}| + |E_{\text{new}}|)$. Since $|V_{\text{new}}| = |V| + 1$ and $|E_{\text{new}}| \leq |E| + |V|$, we finally have:

$$O(|V|) + O(|V_{\text{new}}| + |E_{\text{new}}|) \leq O(|V|) + O(|V| + |E| + |V|) = O(|E| + |V|).$$