Introduction to Algorithm Analysis June 13, 2017
University of California Santa Cruz CMPS 102, Spring 2017
Professor Abhradeep Guha Thakurta 12-3pm (Pacific time)

Practice Final Exam

- **Do not open this exam booklet until you are directed to do so.** Read all the instructions on this page.
- When the exam begins, write your UCSC ID on every page of this exam booklet. You are given 2 minutes at the start of the exam to do precisely this.
- This exam contains five problems. You should answer all.
- No calculators, programmable devices or cellphones are permitted.
- This a closed book, and closed notes exam. You are allowed one hand-written (double sided) A4 sheet.
- Write your solutions in the space provided.
- Do not waste time and paper re-deriving facts that we have studied. It is sufficient to cite known results.
- Any time you are asked to give an algorithm you must also provide an *analysis of its running time* and a *proof of correctness.*
- Show your work, as partial credit will be given. You will be graded not only on the **correctness** of your answer, but also on the **clarity** with which you express it. Be neat.
- Good luck!

Name: ______________________________ **ID**: __________

Problem	1	2	3	4	5	Total
Points	25	25	25	25	25	125
Grade						

Problem 1 (5 * 5 = 25 points). There are fives parts to this problem. Answer all parts.

(a) Implement the following function in $O(n)$ time, and $O(1)$ space.

FIB(n)
if $n == 1$ or $n == 0$:
    return n
$x = $ FIB$(n - 1)$
$y = $ FIB$(n - 2)$
$z = x + y$
return z

Solution:

(b) *True or false?* Decide whether the following is always true, never true, or sometimes true for asymptotically nonnegative functions f and g. If it is always true or never true, give a proof. If it is sometimes true, give one example for which it is true, and one for which it is false: $f(n) = \omega(g(n))$ and $f(n) = O(g(n))$
Answer:

Justification:

(c) Suppose you are given a graph G with non-negative edges, and you want to find shortest path from a source node, to every other node. Which algorithm will you use?

(d) *True or false?* Every k-regular bipartite graph has a perfect matching.
Answer:

Justification:

(e) *True or false?* The class P is a subset of the class NP.
Answer:

Justification:

Problem 2 (25 points, Homework 1 Problem 4 (verbatim)). Suppose you are given two sorted arrays, each with n elements. There are $2n$ values in total, and you may assume that no two values are the same. You would like to determine the median of this set of $2n$ values. We define the median as the n^{th} smallest value.

Give a divide and conquer algorithm which finds the median in asymptotic time $\Theta(\log n)$. Argue why your algorithm is correct. Write down and solve the running time recurrence.

Problem 3 (25 points). Solved Exercise 1 from Chapter 6 of the text book.

Problem 4 (25 points, Dynamic programming). Chapter 6 Problem 3 from the book.

Problem 5 (25 points, Network Flow). Solved Exercise 1 from Chapter 7 of the text book.

— **End of the exam** —

— Scratch space. —

— Scratch space. —

— Scratch space. —

— Scratch space. —

— Scratch space. —