Practice Final Exam

- Do not open this exam booklet until you are directed to do so. Read all the instructions on this page.
- When the exam begins, write your UCSC ID on every page of this exam booklet. You are given 2 minutes at the start of the exam to do precisely this.
- This exam contains five problems. You should answer all.
- No calculators, programmable devices or cellphones are permitted.
- This a closed book, and closed notes exam. You are allowed one hand-written (double sided) A4 sheet.
- Write your solutions in the space provided.
- Do not waste time and paper re-deriving facts that we have studied. It is sufficient to cite known results.
- Any time you are asked to give an algorithm you must also provide an analysis of its running time and a proof of correctness.
- Show your work, as partial credit will be given. You will be graded not only on the **correctness** of your answer, but also on the **clarity** with which you express it. Be neat.
- Good luck!

Name:	ID:

Problem	1	2	3	4	5	Total
Points	25	25	25	25	25	125
Grade						

Problem 1 (5 * 5 = 25 points). There are fives parts to this problem. Answer all parts.

(a) Implement the following function in O(n) time, and O(1) space.

```
FIB(n)

1 if n == 1 or n == 0:

2 return n

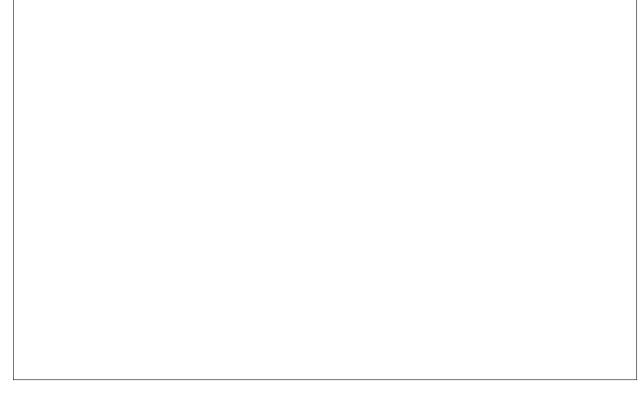
3 x = \text{FIB}(n-1)

4 y = \text{FIB}(n-2)

5 z = x + y

6 return z
```

Solution:



(b) True or false? Decide whether the following is always true, never true, or sometimes true for asymptotically nonnegative functions f and g. If it is always true or never true, give a proof. If it is sometimes true, give one example for which it is true, and one for which it is false: $f(n) = \omega(g(n))$ and f(n) = O(g(n)) Answer:

	Justification:
(c)	Suppose you are given a graph G with non-negative edges, and you want to find shortest path from a source node, to every other node. Which algorithm will you use?
(d)	True or false? Every k -regular bipartite graph has a perfect matching. Answer: Justification:
(e)	True or false? The class P is a subset of the class NP. Answer: Justification:

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Problem 2 (25 points, Homework 1 Problem 4 (verbatim)). Suppose you are given two sorted arrays, each with n elements. There are 2n values in total, and you may assume that no two values are the same. You would like to determine the median of this set of 2n values. We define the median as the nth smallest value.

Give a divide and conquer algorithm which finds the median in asymptotic time $\Theta(\log n)$. Argue why your algorithm is correct. Write down and solve the running time recurrence.

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Problem 3 (25 points). Solved Exercise 1 fr	rom Chapter 6 of the text book.

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Problem 4 (25 points, Dynamic program	aming). Chapter 6 Problem 3 from the book.

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Problem 5 (25 points, Network Flow). book.	Solved Exercise 1 from Chapter 7 of the text

— End of the exam — $\,$