ASSIGNMENT

MATRICES AND DETERMINANTS

SECTION - A

1. If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$ , find the values of $x \ and \ y$ .

2. Using determinants , show that the points $(1,0), (6,0), (0,0)$ are collinear.

3. Evaluate : $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

4. Find the co-factor of $a_{12}$ in the following : $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

5. If $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$ , find the values of $x$ .

6. A matrix $A$ of order 3x3 has determinant 5 . What is the value of $|3A|$ .

7. For what value of $x$ , the following matrix is singular?

$$\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$$

8. If A = $\begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$ write the order of AB and BA .

10. If $A$ is a square matrix of order 3 such that $|adj\ A\,| = 64$ , $find\ |A|$ .

11. If A , B , C are three non-zero square matrices of same order, find the condition on A such that AB = AC ⇒ B = C.

12. Give an example of two non-zero 2x2 matrices A , B such that AB = 0.

13. If $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$ , find $x$ , 0< $x < \frac{\pi}{2}$ when $A + A' = I$ .

14. If B is a skew-symmetric matrix , write whether the matrix ( ABA’) is symmetric or Skew-symmetric.

15. On expanding by first row , the value of a third order determinant is $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ . Write the expression for its value on expanding by 2nd column. Where $A_{ij}$ is the co-factor of the element $a_{ij}$ .

16. For what value of $k$ , the matrix $A = \begin{bmatrix} 4 & 3-k \\ 1 & 2 \end{bmatrix}$ is not invertible ?

17. If the matrix A is both symmetric and skew-symmetric , then what is the matrix A ?

18. If $A$ is any matrix such that $|A| = 12 , find\ A.(Adj\ A)$ .

19. Find $x\ if\ \begin{bmatrix} x & -3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} = 0.$

20. Without expanding , evaluate $\begin{vmatrix} x & 4 & 2y \\ 2x & -3 & 4y \\ 3x & 2 & 6y \end{vmatrix}$ .

21. Find the condition that the matrices A and B will be the inverse of each other.

22. When will a matrix $A = [a_{ij}]_{mxn}$ be a square matrix ?

23. If $A$ is a square matrix of order 3 and $|A| = 5$ , find $|A^{-1}|$ .

24. If A and B are square matrices of same order , such that $A^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$ and

$B^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$ $find\ \ (AB)^{-1}$ .

25. If A is square matrix of order 3 such that $|A| = -6$ , find $|adj\ A|$ .

26. If A is a 3x4 matrix and B is a matrix such that $A^TB$ and $BA^T$ are both defined , then find the order of matrix B .

## SECTION - B

1. Using properties of determinants , prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

2. Using properties of determinants , prove that $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$

3. If a, b and c are all positive and distinct, show that $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ has a negative value.

4. Let A = $\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as sum of two matrices such that one is symmetric and the other is skew-symmetric.

5. if $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ , verify that $A^2 - 4A - 5I = 0$

6. Using properties of determinants , prove that

$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$$

7. Solve for $x$ : $\begin{vmatrix} 3x - 8 & 3 & 3 \\ 3 & 3x - 8 & 3 \\ 3 & 3 & 3x - 8 \end{vmatrix} = 0$

8. Using properties of determinants , prove that

$$\begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix} = (ab + bc + ca)^3.$$

9. If a , b , and c are real numbers and $\begin{vmatrix} b + c & c + a & a + b \\ c + a & a + b & b + c \\ a + b & b + c & c + a \end{vmatrix} = 0$ , show that $a + b + c = 0 \;\; or \;\; a = b = c$ .

10. Using elementary transformations , find the inverse of $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

11. Prove that $\begin{vmatrix} a + b & b + c & c + a \\ b + c & c + a & a + b \\ c + a & a + b & b + c \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ .

12. Show that $\begin{vmatrix} 1 + a & 1 & 1 \\ 1 & 1 + b & 1 \\ 1 & 1 & 1 + c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

13. If $x , y , z$ are different numbers and $\Delta = \begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0$ , then show that $1 + xyz = 0.$

14. If $a$ , $b$ , $c$ are in A.P , show that $\begin{vmatrix} 2x + 4 & 5x + 7 & 8x + a \\ 3x + 5 & 6x + 8 & 9x + b \\ 4x + 6 & 7x + 9 & 10x + c \end{vmatrix} = 0.$

15. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ , show that $A^2 - 5A + 7I = 0$ and hence find $A^{-1}$.

## SECTION - C

1. Using matrices , solve the following system of linear equations

$2x - y + z = 3$ ; $-x + 2y - z = -4$ ; $x - y + 2z = 1$ .

2. Using elementary transformations , find the inverse of the following matrix : $\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$

3. Find the matrix $P$ satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

4. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ , find $A^{-1}$ and hence solve the system of equations

$x + 2y + z = 4$ ; $-x + y + z = 0$ $and$ $x - 3y + z = 2$.

5. Using elementary transformations , find the inverse of the following matrix $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

6. Using matrix method , solve $x + 2z = 1$ ; $2x - y - z = -3$ ; $x + 2y = 3z = 2$

7. If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ $and$ $B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ , $find$ $AB$ and hence solve the system,

$x - y + 2z = 1$ ; $2y - 3z = 1$ ; $3x - 2y + 4z = 2$.

8. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ , $find$ $A^{-1}$ and hence solve the system

$x + 2y = 5$ ; $-2x + 4y + z = 9$ ; $3x - y + 2z = 7$ .

9. Using elementary transformations obtain the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

10. Show that $\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (z+x)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$.