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AI24BTECH11018 - Sreya

14) Consider the following topoligies on the set $\mathbb{R}$ of all real numbers:
T_1=$U \subset \mathbb{R} : 0 \notin U$ or $U = \mathbb{R}$,
T_2=$U \subset \mathbb{R} : 0 \in U$ or $U = \phi$
$T_3 = T_1 \cap T_2$
Then the closure of the set 1 in $(\mathbb{R}, T_3)$
    a) 1
    b) $0, 1$
    c) $\mathbb{R}$
    d) $\mathbb{R}\backslash 0$
15) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a differentiable.Let $D_u f(0,0)$ and $D_v f(0,0)$ be the directinal derivatives at of f at $(0,0)$ in the direction of unit vectors $u = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$ and $v = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$. respectvely. if $D_u f(0,0) = \sqrt{5}$ and $D_v f(0,0) = \sqrt{2}$ then $\frac{\partial f}{\partial x}(0,0)+\frac{\partial f}{\partial y}(0,0) =$
16) let r denote the boundary of the square region R with vertices $(0,0)$,$(2,0)$,$(2,2)$ and $(0,2)$ oriented in the counter-clockwise direction. then

$$\oint_r \left(1 - y^2\right) dx + xdy = \tag{1}$$

17) The number of $5 - sylow$ subgroups in symmetric groups S_5 of degree 5 is
18) let I be the generated by x^2+x+1 in the polynomial ring $R = \mathbb{Z}[x]$,where $\mathbb{Z}_3$ denotes the ring of integers modulo 3. Then the number of units in the quotient ring $\frac{R}{I}$ is
19) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ bea linear transforation such that
$T\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $T^2\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$,$T^2\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$
20) Let $y(x)$ be the solution of the following intial value problem

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0, x>0 \tag{2}$$

$$y(2) = 0, \frac{dy}{dx}(2) = 4. \tag{3}$$

Then $y(4) =$
21) Let

$$f(x) = x^4 + 2x^3 - 11x^2 - 12x + 36 for x \in \mathbb{R} \tag{4}$$

The order of convergence of the newton-raphson method

$$x_n + 1 = x_n - \frac{f(x_n)}{f'(x_n)}, n \geq 0, \tag{5}$$

with $x_0 = 2.1$ for finding the root $\alpha = 2$ the equaton $f(x) = 0$ is

22) if the polynomial

$$p(x) = \alpha + \beta(x+2) + \gamma(x+2)(x+1) + \delta(x+2)(x+1)x \tag{6}$$

interpolates the data

x	-2	-1	0	1	2
f(x)	2	-1	8	5	-34

then $\alpha + \beta + \gamma + \delta$

23) Consider the linear programming problem P:

$$Maximise 2x_1 + 3x_2 \tag{7}$$

subject to

$$2x_1 + x_2 \leq 6, \tag{8}$$

$$-x_1 + x_2 \leq 1, \tag{9}$$

$$x_1 + x_2 \leq 3, \tag{10}$$

$$x_1 \geq 0, and x_2 \geq 0 \tag{11}$$

Then the optional of the dual of P is equal to

24) Consider the linear programming problem P:

$$Maximise 2x_1 + 3x_2 \tag{12}$$

subject to

$$2x_1 + 3x_2 + s_1 = 12, \tag{13}$$

$$-x_1 + x_2 + s_1 = 1, \tag{14}$$

$$x_1 + 2x_2 + s_3 = 3, \tag{15}$$

$$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0, s_2 \geq 0, and s_3 \geq 0. \tag{16}$$

if $\begin{pmatrix} x_1 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$ is a basic feasable solution of p , then $x_1 + s_1 + s_2 + s_2 + s_4$

25) Let H be a complex Hilbert space. Let $u, v \in H$ be a such that $\langle u, v \rangle = 2$. Then

$$\frac{1}{2\pi} \int_0^{2\pi} \left\| u + e^{it} v \right\|^2 e^{it} dt = \tag{17}$$

26) Let $\mathbb{Z}$ denote the ring of integers. consider the ring
$R = a + b\sqrt{-17} : a, b \in \mathbb{R}$ of the feild $\mathbb{C}$ of complex numbers
Consider the following statements:
$P : 2 + \sqrt{-17}$ is an irreducible element.
$Q : 2 + \sqrt{-17}$ is a prime element.
Then

a) both P and Q is TRUE
b) P is TRUE and Q is FALSE
c) P is FALSE and Q is TRUE
d) both P and Q are FALSE