

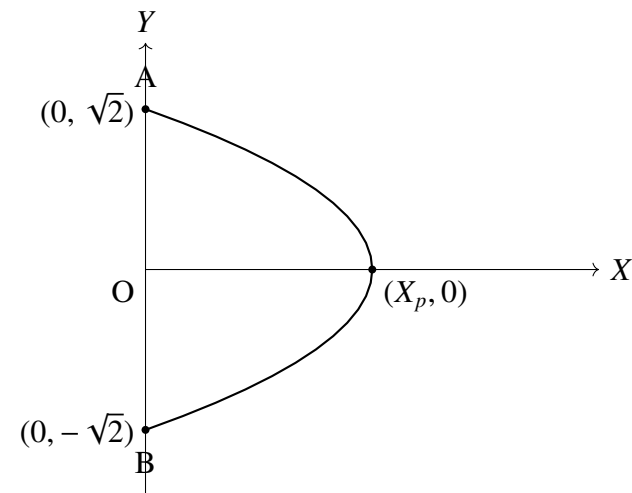
Subjective questions

AI24BTECH11018 - Sreya

21. Sketch the curves and identify the region bounded by $x = \frac{1}{2}$, $x = 2$, $y = \ln x$ and $y = 2^x$. Find the area of region.

(1991 – 4Marks)

22. if f is a continuous function with $\int_0^x f(t)dt \rightarrow \infty$ as $|x| \rightarrow \infty$, then show that every line $y = mx$



intersects the curve $y^2 + \int_0^x f(t)dt = 2!$

(1991 – 4Marks)

23. Evaluate $\int_0^\pi \frac{x \sin 2x \sin(\frac{\pi}{2} \cos x)}{2x - \pi} dx$

(1991 – 4Marks)

24. Sketch the region bounded by the curves $y = x^2$ and $y = \frac{2}{1+x^2}$. Find the area.

(1992 – 4Marks)

25. Determine a positive integer $n \leq 5$, such that $\int_e^x (x-1)^n = 16 - 6e$

(1992 – 4Marks)

26. Evaluate $\int_2^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{(x^2 + 1)(x^4 - 1)} dx$

(1993 – 5Marks)

27. Show that $\int_0^{n\pi+v} |\sin x| dx = 2n + 1 - \cos v$ where n is a positive integer and $0 \leq v < \pi$

(1994 – 4Marks)

28. In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x - x^2$ and $y = x^2 - x$?

(1994 – 5Marks)

29. $I_m = \int_0^\pi \frac{1 - \cos mx}{1 - \cos x} dx$. Use mathematical induction to prove that $I_m = m\pi$, $m = 0, 1, 2, 3, \dots$

(1995 – 5marks)

- 30 Evaluate the definite integral:

$$\int_{\frac{-1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \left(\frac{x^4}{1-x^4} \right) \cos^{-1} \left(\frac{2x}{1+x^2} \right) dx$$

(1995 – 5Marks)

31. Consider a square with vertices at $(1,1)$, $(-1,1)$, $(-1,-1)$ and $(1,-1)$. Let S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area.

(1995 – 5Marks)

32. Let A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0$, $y = 0$ and $x = \frac{\pi}{4}$. Prove that for $n > 2$, $A_n + A_{n-2} = \frac{1}{n-2}$ and deduce

$$\frac{1}{2n+2} < A_n < \frac{1}{2n-2}.$$

(1996 – 3Marks)

33. Determine the value of $\int_{-\pi}^\pi \frac{2x(1+\sin x)}{1+\cos^2 x} dx$.

(1997 – 5Marks)

34. Let $f(x) = \text{Maximum } x^2, (1-x)^2, 2x(1-x)$, Where $0 \leq x \leq 1$. Determine the area of the region bounded by the curves $y = f(x)$, x-axis, $x = 0$ and $x = 1$.

(1997 – 5Marks)

35. Prove that $\int_0^1 \tan^{-1} \left(\frac{1}{1-x+x^2} \right) dx = 2 \int_0^1 \tan^{-1} x dx$. Hence or otherwise, evaluate the integral $\int_0^1 \tan^{-1}(1-x+x^2) dx$.

(1998 – 8Marks)