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AI24BTECH11018 - Sreya

- 14) Consider the following topologies on the set \mathbb{R} of all real numbers:

$$T_1 = U \subset \mathbb{R} : 0 \notin U \text{ or } U = \mathbb{R},$$

$$T_2 = U \subset \mathbb{R} : 0 \in U \text{ or } U = \emptyset$$

$$T_3 = T_1 \cap T_2$$

Then the closure of the set 1 in (\mathbb{R}, T_3)

- a) 1
- b) 0, 1
- c) \mathbb{R}
- d) $\mathbb{R} \setminus 0$

- 15) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a differentiable. Let $D_u f(0,0)$ and $D_v f(0,0)$ be the directional derivatives at f at $(0,0)$ in the direction of unit vectors $u = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$ and $v = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$, respectively. if $D_u f(0,0) = \sqrt{5}$ and $D_v f(0,0) = \sqrt{2}$ then $\frac{\partial f}{\partial x}(0,0) + \frac{\partial f}{\partial y}(0,0) =$

- 16) let r denote the boundary of the square region R with vertices $(0,0), (2,0), (2,2)$ and $(0,2)$ oriented in the counter-clockwise direction. then

$$\oint_r (1 - y^2) dx + x dy = \quad (1)$$

- 17) The number of 5 - sylow subgroups in symmetric groups S_5 of degree 5 is

- 18) let I be the generated by $x^2 + x + 1$ in the polynomial ring $R = \mathbb{Z}[x]$, where \mathbb{Z}_3 denotes the ring of integers modulo 3. Then the number of units in the quotient ring $\frac{R}{I}$ is

- 19) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation such that

$$T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, T^2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, T^2 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

- 20) Let $y(x)$ be the solution of the following initial value problem

$$x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = 0, x > 0 \quad (2)$$

$$y(2) = 0, \frac{dy}{dx}(2) = 4. \quad (3)$$

Then $y(4) =$

- 21) Let

$$f(x) = x^4 + 2x^3 - 11x^2 - 12x + 36 \text{ for } x \in \mathbb{R} \quad (4)$$

The order of convergence of the newton-raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, n \geq 0, \quad (5)$$

with $x_0 = 2.1$ for finding the root $\alpha = 2$ the equation $f(x) = 0$ is

22) if the polynomial

$$p(x) = \alpha + \beta(x+2) + \gamma(x+2)(x+1) + \delta(x+2)(x+1)x \quad (6)$$

interpolates the data

x	-2	-1	0	1	2
f(x)	2	-1	8	5	-34

then $\alpha + \beta + \gamma + \delta$

23) Consider the linear programming problem P :

$$\text{Maximise } 2x_1 + 3x_2 \quad (7)$$

subject to

$$2x_1 + x_2 \leq 6, \quad (8)$$

$$-x_1 + x_2 \leq 1,$$

$$x_1 + x_2 \leq 3, \quad (9)$$

$$x_1 \geq 0, \text{ and } x_2 \geq 0 \quad (10)$$

Then the optional of the dual of P is equal to

24) Consider the linear programming problem P :

$$\text{Maximise } 2x_1 + 3x_2 \quad (11)$$

subject to

$$2x_1 + 3x_2 + s_1 = 12, \quad (12)$$

$$-x_1 + x_2 + s_1 = 1,$$

$$x_1 + 2x_2 + s_3 = 3, \quad (13)$$

$$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0, s_2 \geq 0, \text{ and } s_3 \geq 0. \quad (14)$$

if $\begin{pmatrix} x_1 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$ is a basic feasible solution of p , then $x_1 + s_1 + s_2 + s_4$

25) Let H be a complex Hilbert space. Let $u, v \in H$ be a such that $\langle u, v \rangle = 2$. Then

$$\frac{1}{2\pi} \int_0^{2\pi} \|u + e^{it}v\|^2 e^{it} dt = \quad (15)$$

26) Let \mathbb{Z} denote the ring of integers. consider the ring

$R = a + b\sqrt{-17} : a, b \in \mathbb{R}$ of the field \mathbb{C} of complex numbers

Consider the following statements:

$P : 2 + \sqrt{-17}$ is an irreducible element.

$Q : 2 + \sqrt{-17}$ is a prime element.

Then

- a) both P and Q is TRUE
- b) P is TRUE and Q is FALSE
- c) P is FALSE and Q is TRUE
- d) both P and Q are FALSE