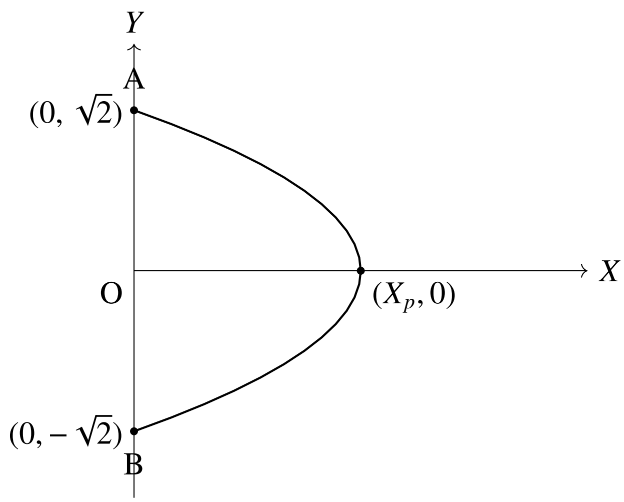
Subjective questions

AI24BTECH11018 - Sreya

21. Sketch the curves and identify the region bounded by $x = \frac{1}{2}$,$x = 2$,$y = \ln x$ and $y = 2^x$. Find the area of region.
(1991 − 4$Marks$)

22. if f is a continous function with $\int_0^x f(t)\,dt \rightarrow \infty$ as $|x| \rightarrow \infty$, then show that every line $y = mx$



intersects the curve $y^2 + \int_0^x f(t)\,dt = 2$!
(1991 − 4$Marks$)

23. Evaluate $\int_0^\pi \frac{x \sin 2xs \sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi}$
(1991 − 4$Marks$)

24. Sketch the region bounded by the curves $y = x^2$ and $y = \frac{2}{1+x^2}$. Find the area.
(1992 − 4$Marks$)

25. Determine a positive integer $n \leq 5$, such that $\int_e^x (x-1)^n = 16 - 6e$
(1991-4 Marks)

26. Evaluate $\int_2^3 \frac{2x^5+x^4-2x^3+2x^4+1}{(x^2+1)(x^4-1)}dx$
(1993 − 5$Marks$)

27. Show that $\int_0^{n\pi+v} |\sin x|\,dx = 2n+1-\cos v$ where n is a positive integer and $0 \leq v < \pi$
(1994 − 4$Marks$)

28. In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x-x^2$ and $y = x^2 - x$?
(1994 − 5$Marks$)

29. $I_m = \int_0^\pi \frac{1-\cos mx}{1-\cos x}dx$. Use mathematical induction to prove that $I_m = m\pi$, $m = 0, 1, 2, 3, ...$
(1995 − 5$marks$)

30. Evaluate the definite integral: $\int_{\frac{-1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \left(\frac{x^4}{1-x^4}\right) cos^{-1}\left(\frac{2x}{1+x^2}\right)$
(1995 − 5$Marks$)

31. Consider a square with vertices at $(1, 1)$,$(-1, 1)$,$(-1, -1)$ and $(1, -1)$. Let s be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area.
(1995 − 5$Marks$)

32. Let A_n be the area bounded by the curve $y = (tanx)^n$ and the lines $l = 0$,$y = 0$ and $x = \frac{\pi}{4}$. Prove that for $n > 2$, $A_n + A_n - 2 = \frac{1}{n-2}$ and deduce $\frac{1}{2n+2} < A_n < \frac{1}{2n-2}$.
(1996 − 3$Marks$)

33. Determine the value of $\int_{-\pi}^\pi \frac{2x(1+\sin x)}{1+\cos^2 x}dx$.
(1997 − 5$Marks$)

34. Let $f(x)$= Maximum $x^2, (1-x)^2, 2x(1-x)$, Where $0 \leq x \leq 1$. Determine the area of the region bounded by the curves $y = f(x)$,x-axis,$x = 0$ and $x = 1$. (1997 − 5$Marks$)

35. Prove that $\int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$=$2\int_0^1 tan^{-1}xdx$. Hence or otherwise, evaluate the integral $\int_0^1 \tan^{-1}\left(1 - x + x^2\right)dx$.
(1998 − 8$Marks$)