Q1) Show that k(x,x') can be expressed as the inner product of an infinite feature space

$$k(x,x') = exp(-||x-x'||^2/2\,\sigma^2)$$

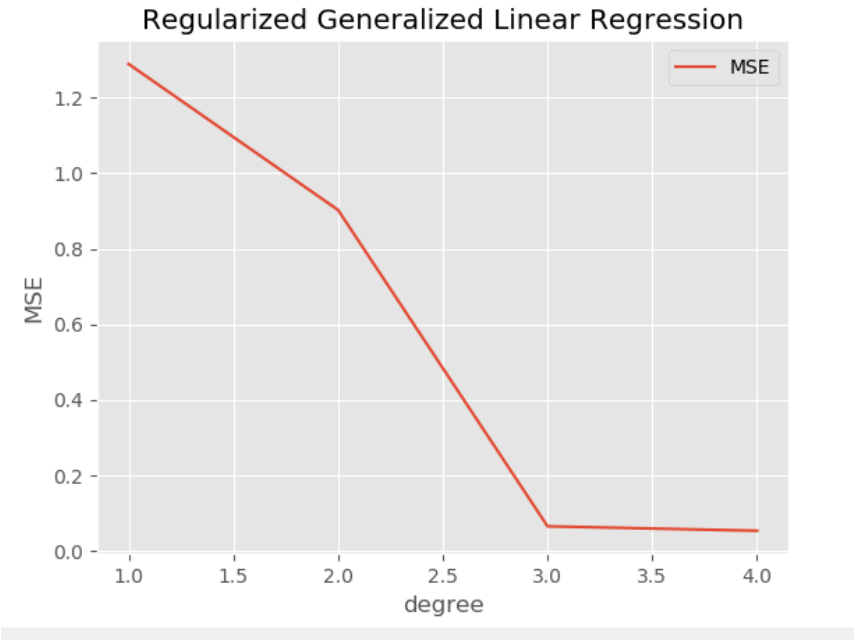
$$= (e^{-xTx/2\sigma\hat{}2})(e^{xTx'/\sigma\hat{}2})(e^{-(x')Tx'/2\sigma\hat{}2})$$

$$= (e^{-xTx/2\sigma\hat{}2})(e^{-(x')Tx'/2\sigma\hat{}2})(\sum_{n=1}^{\infty}\frac{(x^Tx\prime)^n}{\sigma^n n!})$$

In the summation of the final factor, the numerator is equivalent to the polynomial kernel of degree n. The polynomial kernel projects a given vector into a higher dimensional space, as demonstrated in question 2c. Since n extends to infinity in this summation, the Gaussian kernel k(x,x') is the inner product of a feature space with infinite dimensions.

Q2) Non-linear regression techniques

a) **<u>Regularized generalized linear regression</u>**



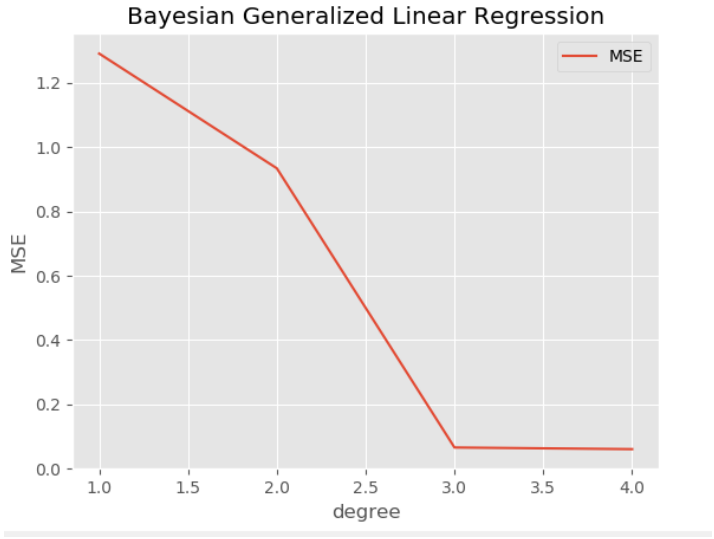
Best degree: 4

Squared error for test set using 4 degrees: 0.0446196

Running time variance for cross validation:

degree 1: 0.0052904 s
degree 2: 0.0086279 s
degree 3: 0.0135583 s
degree 4: 0.0188342 s

b) Bayesian generalized linear regression



Best degree: 4

Squared error for test set using 4 degrees: 0.050440

Running time variance during cross validation:

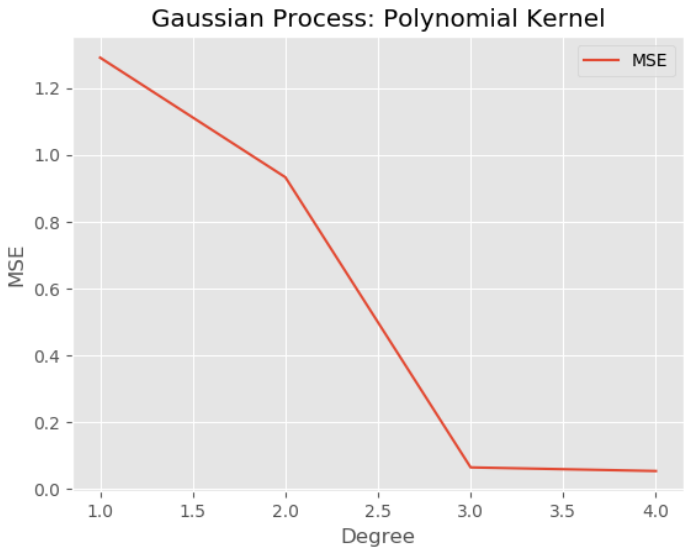
degree 1: 0.00521 s
degree 2: 0.00800 s
degree 3: 0.01286 s
degree 4: 0.01726 s

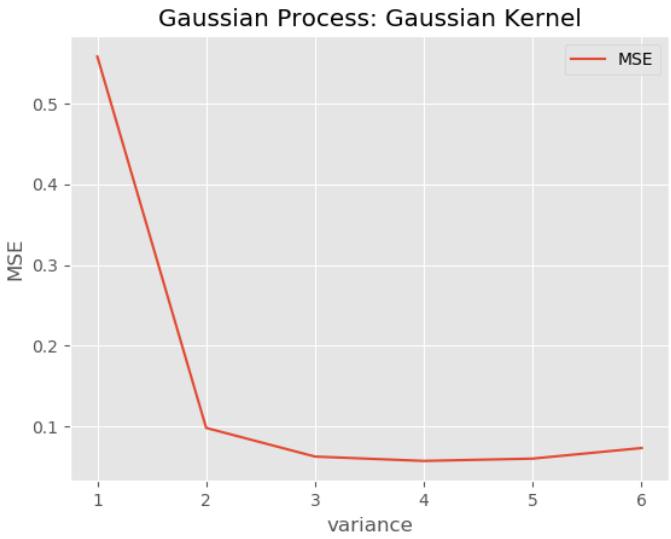
Regularized generalized linear regression vs Bayesian generalized linear regression:

Similarities	Differences
-Both are non-linear regression models -Both use non-linear basis functions to add dimensions to data	In Bayesian learning, instead of making predication according to w*, we compute the weighted average prediction according to Pr(**w**|**X,y**)

c) Gaussian process regression

Identity Kernel: Squared error of test set for identity kernel: 3.59062069





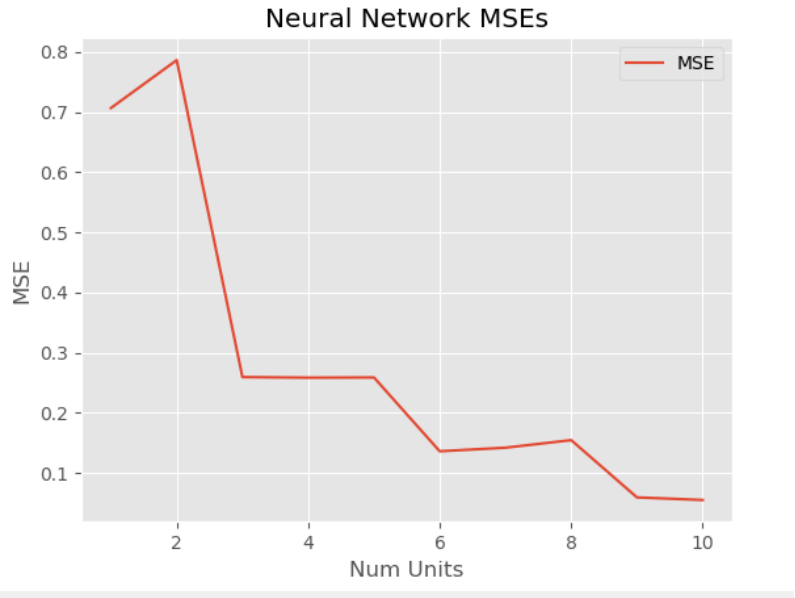
SE of gaussian kernel using best variance (4): 0.04302516

SE of test set for polynomial kernel using best degree (4): 0.04577037

Run time variance on test set:

Identity	Gaussian	Polynomial
0.7518 s	0.9580 s	0.6426 s

d) Neural network



Best number of hidden units: 10

Squared error of test set using best number of units: 0.02947

Run time variance:

1 unit: 142.51 s
2 units: 161.45 s
3 units: 183.18 s
4 units: 210.26 s
5 units: 244.19 s
6 units: 278.83 s
7 units: 313.85 s
8 units: 348.27 s
9 units: 387.42 s
10 units: 432.11 s