Computer Vision lab 1

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1 Lab 1

1.1 1.3

Question 1: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9), (17, 121), (5, 1) and (125, 1) respectively. What do you observe?

We observe that there is a change of imaginary and real parts of the inverse fourier transform due to the point (p,q) (white pixel) being placed in different parts of the 128x128 image, according to slide 28 lecture 3. Further away from origo will result in a higher frequency since the point (0,0) corresponds to frequency equal to zero.

Question 2: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Using the inverse fourier transform on the point p which has a value $\hat{F} = 1$ which can be viewed as a dirac pulse will lead to f being described by two exponential terms which can be rewritten as cos and sine terms with the help of eulers formula.

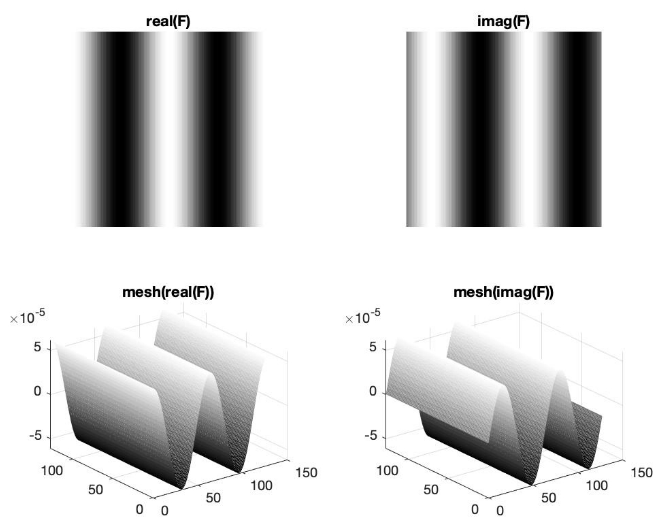

Figure 1: Plots for p(u=1,v=3)

$$f(x, y) = \iint_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} dudv \tag{1}$$

using euler identity

$$e^{j2\pi(ux+vy)} = cos(2\pi(ux + vy)) + jsin((2\pi(ux + vy)) \tag{2}$$

Question 3: How large is the amplitude? Write down the expression derived from Equation (4) in these notes. Complement the code (variable amplitude) accordingly.

$$F(x) = \mathcal{F}_D^{-1}(\hat{F})(x) = \frac{1}{N} \sum_{u \in [0...N-1]^2} \hat{F}(u) e^{\frac{\pm 2\pi i u^T x}{N}} \tag{3}$$

$\hat{F} = d$ where d is a Dirac pulse and using Euler's identity in equation 2, equation 3 can be written as

$$F(x) = \frac{1}{N} \sum_{u} cos(2\pi u^T x) + isin(2\pi u^T x) \tag{4}$$

where then it is easy to see that the amplitude is given by

$$Amplitude = \frac{1}{N} \tag{5}$$

Question 4: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

The length of the sine wave is given by:

$$\lambda = \frac{1}{\sqrt{p^2 + q^2}} \tag{6}$$

The direction of the function is given by:

$$\vec{n} = \tan^{-1}(q/p) \tag{7}$$

Question 5: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Since when applying Fourier transform we assume that the image being observed extends in the same pattern to infinity periodically the point will be projected down according to:

$$uc = u - 1 - sz; \tag{8}$$

example above is for the u axis (p) where sz is the width in the for the current axis, for our example the image has the same width and height therefor M=N=sz=128.

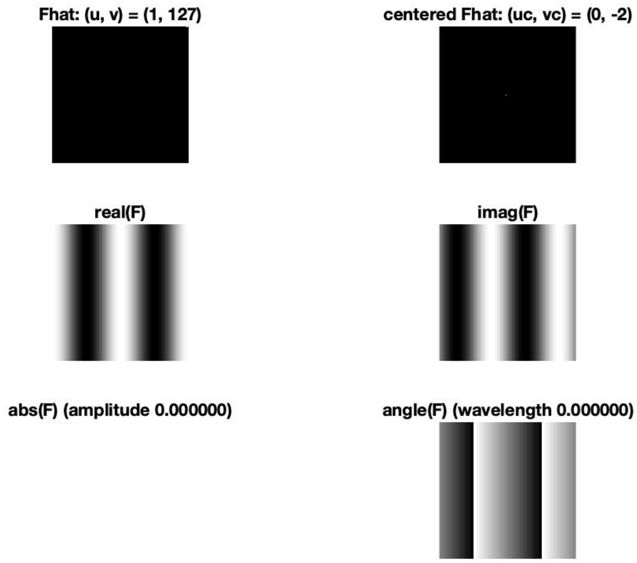

Figure 2: Projection of point p(1,127) when centering is done

Question 6: What is the purpose of the instructions following the question What is done by these instructions? in the code?
The purpose is to project the points with the help of periodicity described in question 5.

1.2 1.4 Linearity

Question 7: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!
Each image Fhat, Ghat has frequency content only along one of the axis, the u and v axis, we get the result shown in Hhat due to linearity. With the axis then shifted we get the results shown in Hhat shift.

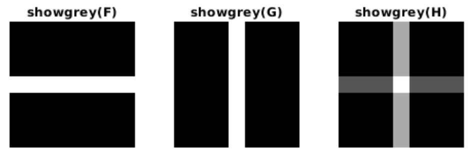

Figure 3: Images in the spatial domain

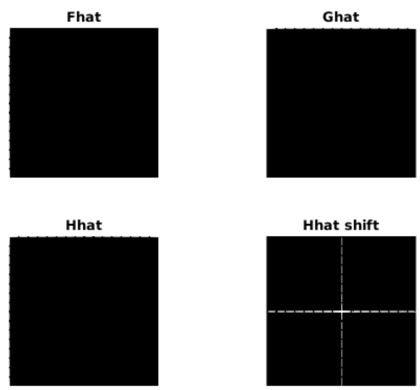

Figure 4: Images in the Fourier

Question 8: Why is the logarithm function applied?
To compress the gray scale to be able to identify more minor frequencies in the Fourier image.
Question 9: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?
We can state that the resulting image Hhat is a result of linearity. What can be seen is a result of the scalar rule and superposition according to:

$$\mathcal{F}(H) = \mathcal{F}(F + 2G) = \mathcal{F}(F) + 2\mathcal{F}(G) \tag{9}$$

1.3 1.5 Multiplication

Question 10: Are there any other ways to compute the last image? **Remember what multiplication in Fourier domain equals to in the spatial domain!** Perform these alternative computations in practice.
Yes, we can take the fourier transform of each image and perform a convolution between them in the fourier domain according to:

$$\mathcal{F}\{f \cdot g\} = \mathcal{F}\{f\} * \mathcal{F}\{g\} \tag{10}$$

Since a multiplication in the spatial domain corresponds to a convolution in the fourier domain.

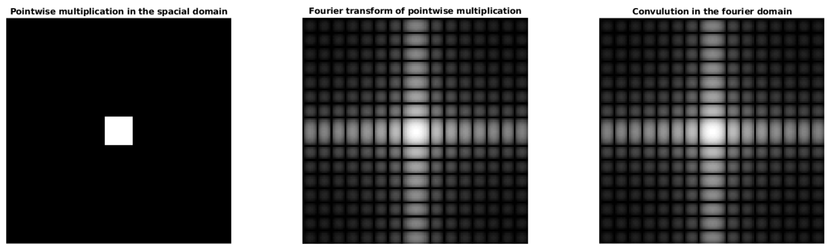

Figure 5: Multiplication for question 10

1.4 1.6 Scaling

Question 11: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.
Expansion in spatial domain is the same as compression in the fourier domain and vice versa. Comparing results shown in Fig. 6 with Fig. 5 it can be seen that we expanded in the v direction and compressed in the u direction in the spatial domain, therefor its compressed in the v direction and expanded in the u direction in fourier domain.

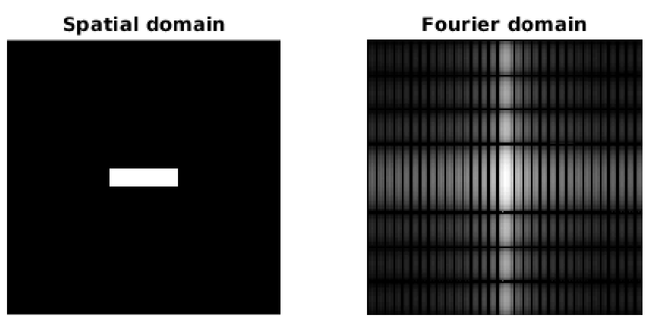

Figure 6: Effects of scaling

1.5 1.7 Rotation

Question 12: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.
The rotation in one domain becomes a rotation in the other domain. The frequencies will have a similiar look with the exception of being rotated. However due to the discretization, i.e rotation of the rectangle won't result in a perfect rectangle, there is noise added when performing the rotations.

1.6 1.8 Information in Fourier phase and magnitude

Question 13: What information is contained in the phase and in the magnitude of the Fourier transform?
Keeping the same phase will show the edges of the picture. Whereas if you keep the same magnitude and change the phase the picture will only tell what grey-levels are on either side of the edge.

1.7 2.3 Filtering procedure

Question 14: Show the impulse response and variance for the above mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and 100.0?

t-value	0.1	0.3	1	10	100
variance	$\begin{pmatrix} 0.013 & 0 \\ 0 & 0.013 \end{pmatrix}$	$\begin{pmatrix} 0.281 & 0 \\ 0 & 0.281 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$	$\begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix}$

Question 15: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.
The variance for the sampled Gaussian kernal is the same as the ideal Gaussian kernal for higher t-values. For t-values 0.1 and 0.3 the sampled kernal becomes non Gaussian, i.e if $t \geq 1$ the covariance for the discretized gaussian kernel follows:

$$C(g(\cdot, \cdot; t)) = t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{11}$$

Question 16: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

We observe the picture becoming more blurry the higher the variance t is because the gaussian becomes smaller in the fourier domain and therefor only includes lower frequencies.

1.8 3.1 Smoothing of noisy data

Question 17: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

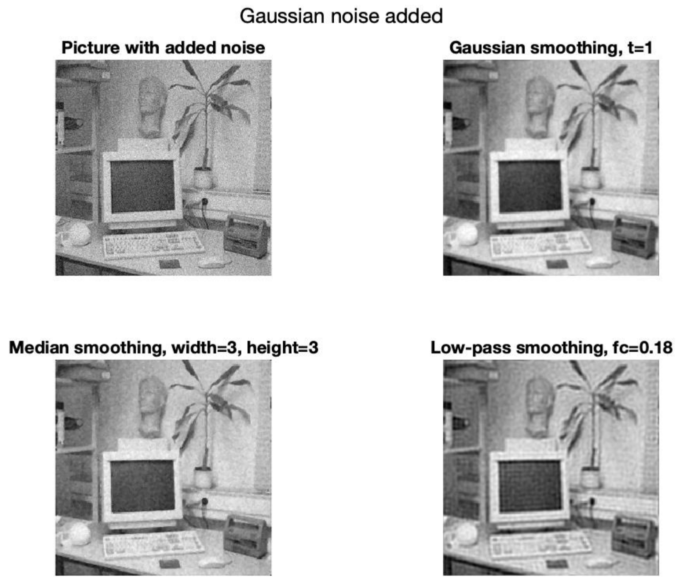

Figure 7

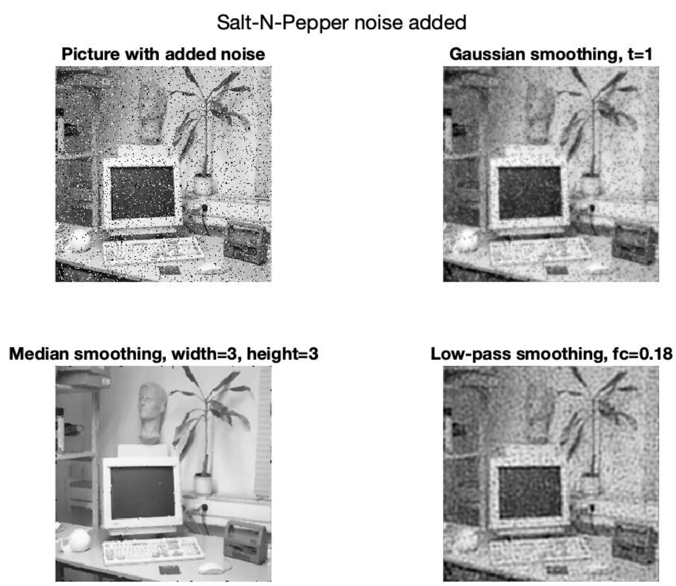

Figure 8

The Gaussian filtered image gets blurrier with higher values of the variance. The median filter looks more like a painting if the window size is bigger and the low pass filter filters out more frequencies if the cut-off is lower.

Gaussian Filter

- Positive
  - Good if the noise is Gaussian
- Negative
  - Blurs the image for larger variance
  - Bad if the noise is salt and pepper

Median Filter

- Positive
  - Good it the noise is Gaussian or salt and pepper
- Negative
  - Large window size will make the image unrecognizable
  - Makes the image look like a painting

Low-pass Filter

- Positive
  - Removes high frequencies
- Negative
  - Bad at Gaussian noise and not very good at salt and pepper noise

Question 18: What conclusions can you draw from comparing the results of the respective methods? The Gaussian and the low pass filter works in basically the same way, they both filter out higher frequencies. The median filter has the best performance, it works well with both types of noise. The low pass filter removes high frequencies like the salt and pepper noise and also information about the edges. The low pass filter is the worst of the three.

1.9 3.2 Smoothing and subsampling

Question 19: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.
In the subsampled image it appears more noise than in the subsampled image that has a gaussian filter applied. As for the low-pass filter after a few iterations a ripple effect can be seen and also some added noise.

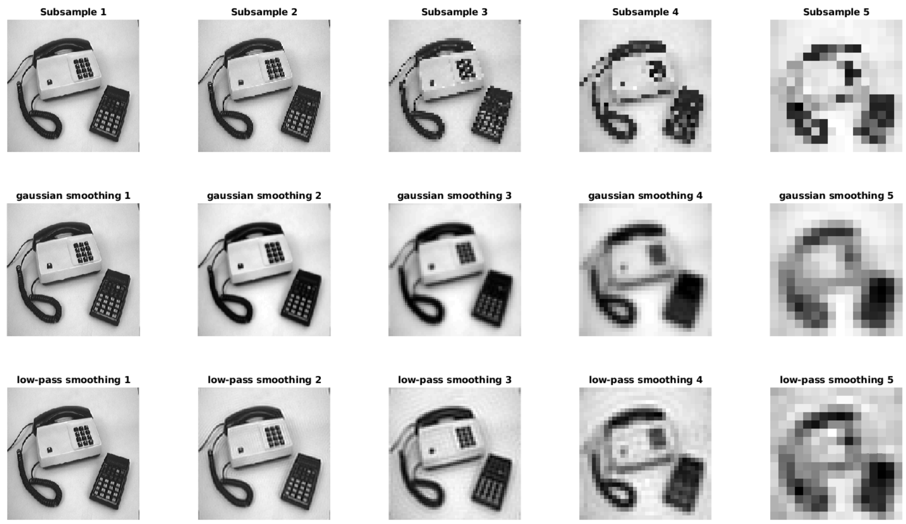

Figure 9

Question 20: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.
The original image has high frequencies that gets resampled and aliases to other frequiencies.
The filtered images removes some higher frequencies so that the frequencies doesn't get aliased when resampled.